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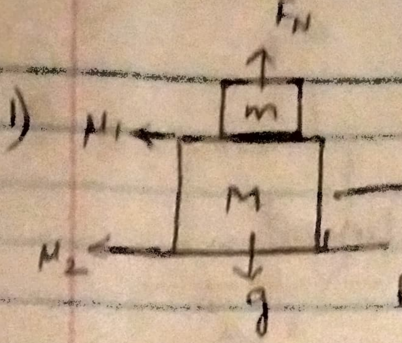
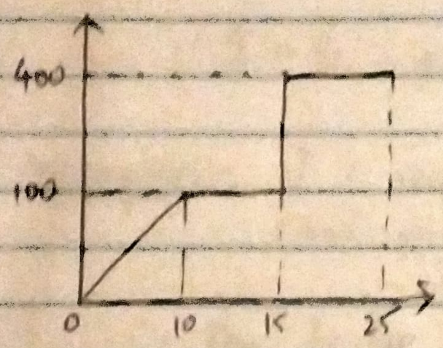
1) 

Free body Diagram

Given $M = 20kg$, $m = 10kg$
$g = 10 m/s^2$ $t_1 = 10s$
$t_2 = 15s$
$t_3 = 25s$
$\mu_1 = 0.6$ $\mu_2 = 0.4$

Force Plot:-



$F_{N_1} = -(Mg + mg) = -(30 \times 10)$
$= -300N$

$F_{N_2} = -mg = -10 \times 10$
$= -100N$

Frictional Forces:- $\mu_2 F_{N_1} = 0.4 \times (-300) = 120 (-)$
$\mu_1 F_{N_2} = 0.6 \times (-100) = 60 (-)$

$F(t_1) = 100N$

$\Sigma F_{net} = F(t) - \mu F$
$= 100 - 120 - 60 = -80N$

$\therefore$ There is no motion in the body at t_1.

$F(t_2) = 400N$

$\Sigma F_{net} = F(t_2) - \mu_2 F_{N_1} - \mu_1 F_{N_2} = 400 - 120 - 60$
$= 220N$

$\therefore$ The block on top falls down because the net force is more than limiting friction.

At $t_3 \rightarrow F(t_3) = 400N$

Here the Block has already fallen off

so $\mu F = 0.4 \times (-Mg)$
$= 0.4 \times (-200)$
$= -80N$.

Values missing at t1 and t2

$\Sigma F_{Net} = F(t_3) - \mu F$
$= 400 - 80$
$= 320N$

Finding x'', x' and x

$$x'' = \frac{\Sigma F_{Net}}{M} = \frac{320}{20} = \underline{16 \, m/s^2}$$

$x' \Rightarrow$ $v = a(t_2 - t_1)$ ← Formula

so $v = 16(25 - 15)$
$= 16 \times 10$
$= \underline{160 \, m/s}$.

$x \Rightarrow s = \frac{v(t_2 - t_1)}{2}$ ← Formula

so $x = \frac{160(25-15)}{2}$
$= \frac{160 \times 10}{2}$
$= \underline{800 \, m}$

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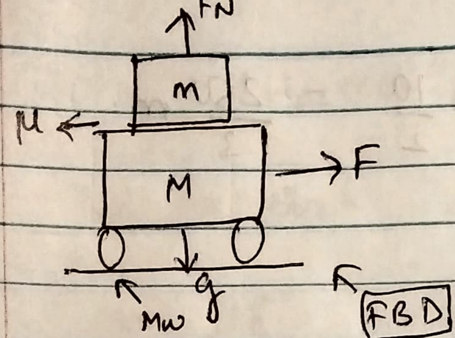
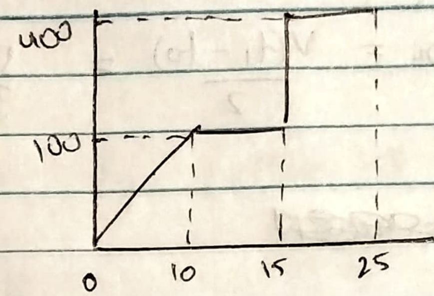
2)





Given $M = 20kg$, $m = 10kg$ $M_w = 1kg$ $n_w = 4$

$I = 0$ $\mu_1 = 0{\cdot}6$ $\mu_2 = 0$ $r = ?$ (not given).

Assumption:- I is mass equivalent but is zero in this case and having a unit radius

∴ The body is stationary and the wheels are rigid all throughout (no kinetic energy)

$F(t_1) = 100N$ $F_{N_1} = m \times g$
$= 10 \times 10 = 100N$

$\mu F_N = 0{\cdot}6 \times (10 \times 10)$
$= 60N$

$\Sigma F_{net} = F(t_1) - \mu F_N = 100 - 60 = 40N$

1 so moving from $t_0 \rightarrow t_1$ the man will topple down and the body moves with a velocity and has an accelaration.

$$\ddot{x} = \frac{F_{net}}{(M + M_w)} = \frac{40}{(20+4)} = \frac{40}{24} = \frac{5}{3} m/s^2$$

$$\dot{x} = a(t_1 - t_0) = \frac{5}{3} \times (10-0) = \frac{50}{3} m/s^2$$

$$x = \frac{V(t_1 - t_0)}{2} = \frac{50}{3} \times \frac{10}{2} = \frac{250}{3}\ m.$$

(2)

No friction up and below

But from $t_1 \to t_2$ the body has moved a distance

$$\Sigma F_{net} = 100\,N$$

$$\ddot{x} = a = \frac{100}{24} = \frac{25}{6}\ m/s^2$$

$$\dot{x} \quad v = a(t_2 - t_1) = \frac{25}{6}(15 - 10) = \frac{125}{6}\ m/s^2$$

$$x = \frac{v(t_2 - t_1)}{2} = \frac{125}{6} \times \frac{5}{2} = \frac{625}{12}\ m.$$

(3) $F(t_3) = 400\,N = F(t_2)$ $\quad f = \mu F = 0$

$$\Sigma F_{net} = 400\,N.$$

$$\ddot{x} = a = \frac{400}{24} = \frac{50}{3}\ m/s^2$$

$$\dot{x} = a(t_2 - t_1) = \frac{50}{3} \times 10 = \frac{500}{3}\ m/s$$

$$x = \frac{v(t_2 - t_1)}{2} = \frac{500}{3} \times \frac{10}{2} = \frac{2500}{3}$$

Now the total distance traveled by the body will be

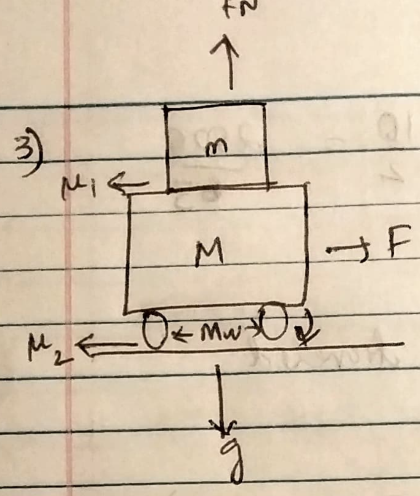
$$\Sigma X = \left(\frac{250}{3} + \frac{625}{12} + \frac{2500}{3} \right) m$$

$$= \frac{1000 + 625 + 10000}{12} m$$

$$= \underline{\underline{968.75 m}}$$

Wrong values in all the cases

3)



Given :-

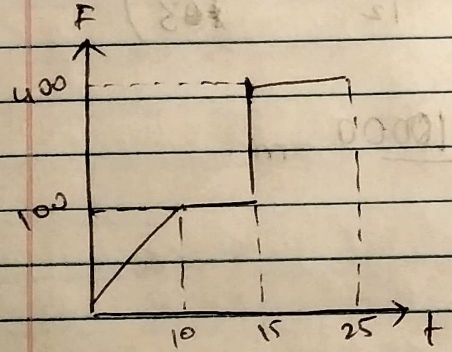
$M = 20\ kg$ $\quad$ $g = 10\ m/s^2$ $\quad$ $t_1 = 10\ s$

$m = 10\ kg$ $\quad$ $t_2 = 15\ s$

$M_w = 1\ kg$ $\quad n_w = 4$ $\quad$ $t_3 = 25\ s$

$R_w = 0.02\ m$

$$I = \frac{1}{2} M_w R_w^2 \quad \leftarrow \text{one wheel.}$$



$$\mu_1 F_{N_1} = 0.6 \times (10 \times 10) = 60N$$

$$\mu_2 F_{N_2} = 0.4 \times (34 \times 10) = 136$$

$$F_{N_1} = mg$$
$$F_{N_2} = (M + m + M_w n_w)$$

$$I = \frac{1}{2} \times M_w \times R_w^2$$
$$= \frac{1}{2} \times 1 \times (0.02)^2 = \frac{4 \times 10^{-4}}{2} = 2 \times 10^{-4}$$

4 wheels $\quad \Sigma I = 4(2 \times 10^{-4}) = \underline{8 \times 10^{-4}\ N \cdot m^2}$

For this problem we have to use energy method, to get a non linear differential eqn to solve for distance, from which we will get velocity and accelaration.

Kinetic Energy $\rightarrow \frac{1}{2}mv^2$ (Linear.)

Kinetic Energy $\rightarrow \frac{1}{2}I\omega^2$ (Rotational).

work = $F \cdot x$ ~~Work is integral Fdx and not just Fdx~~

to calculate the work at dt.

$$W = Fdx.$$

w.r.t $\rightarrow$ $v = \frac{dx}{dt}$ $\Rightarrow$ $dx = vdt$

so

$$W = f\,v\,dt$$

In the Energy method.
To calculate Distance we have to equate work done with the kinetic energy.

$$\Sigma K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2.$$

$v = \frac{dx}{dt}$ $\qquad$ $\omega = \frac{V}{R} \Rightarrow \frac{1}{R}\frac{dx}{dt}.$

$$\Sigma k = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}m_2\left(\frac{dx}{dt}\right)^2$$ $\qquad$ $\underline{I = mR^2}$

Now the mass for linear K and rotational K is different

$$\Sigma K = \frac{1}{2} m_1 \left(\frac{dx}{dt}\right)^2 + \frac{1}{2} m_2 \left(\frac{dx}{dt}\right)^2$$

Equating with work.

$$\Sigma K = W$$

$$\boxed{\frac{1}{2} m_1 \left(\frac{dx}{dt}\right)^2 + \frac{1}{2} m_2 \left(\frac{dx}{dt}\right)^2 = F v \, dt}$$

Now to simplify the terms on Right hand side

$$F = m \frac{d^2x}{dt^2}.$$

$$v = \frac{dx}{dt}$$

$$W = m \cdot \ddot{x} \cdot \dot{x} \cdot dt$$

At t_1, Force $F(t_1) = 100\,N$.

substituting in ΣF_{Net}

$$\Sigma F_{Net} = F(t_1) - \mu_1 F_{N1} - \mu_2 F_{N2}$$

$$= 100 - 60 - 136$$

$$= -96N.$$

There is no acceloration, velocity and displacement due to no motion.

At t_2, the force increases to 400 N and the body topples.

$$F(t_2) = 400$$

$$\Sigma F_{net} = F(t_2) - \mu_1 F_{N1} - \mu_2 F_{N2}$$
$$= 400 - 196$$
$$= 204\,N$$

Substituting in the eqn with finite time intervals.

$$\frac{1}{2}(30)\cdot(x')^2 + \frac{1}{2}(4)(x')^2 = 204 \cdot x$$

displacement and integration

$$15(x')^2 + 2(x')^2 = 204(x)$$

$$17(x')^2 = 204x$$

$$(x')^2 = 12x$$

$$\frac{dx}{dt} = \sqrt{12x}$$

$$\int dx = \int_{10}^{15} \sqrt{12x}\, dt$$

$$x = \sqrt{12x}\,(15-10)$$

$$x = 12\cdot(25)$$
$$= \underline{300\,m}$$

At t_3 the same procedure will be followed
but

$$\Sigma F_{net} = 400\,N - 136\,N$$

because the upper block has fallen down

$$\Sigma F_{net} = 264 .$$

$$15(x')^2 + 2(x')^2 = 264\,x$$ Same mistake as case 2.

$$17(x')^2 = 264\,x$$

$$(x')^2 = \frac{264}{17}\,x$$

$$x' = \sqrt{\frac{264}{17}}\,x^{1/2}$$

$$\frac{dx}{dt} = \sqrt{\frac{264}{17}}\,x^{1/2}$$

$$\int dx = \int_{15}^{25} \sqrt{\frac{264}{17}}\,x^{1/2}\,dt .$$

$$x = \sqrt{\frac{264}{17}}\,x^{1/2}\,(10) .$$

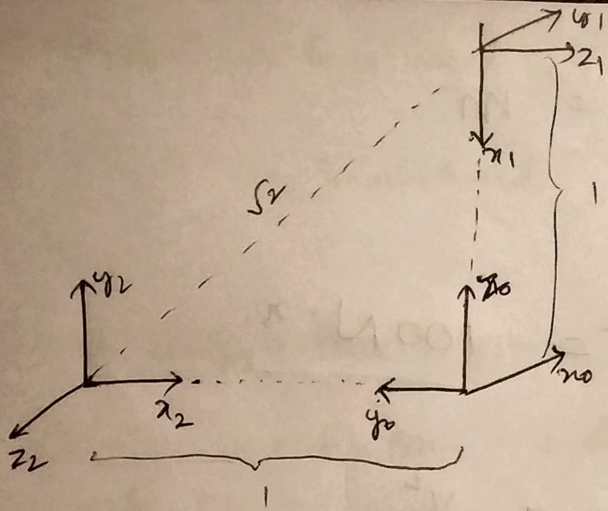
$$x = \frac{264}{17} \times 100$$

$$= 1552.94\,m$$

Total distance $= 1552.94\,m + 300\,m$

$$= 1852.94\,m .$$

4)



Using method of framal reference we will find Homogeneous transformation matrix H_1^0

Homogeneous Transformation Rotational matrix (3X3)

$$R_1^0 = \begin{bmatrix} x_0x_1 & x_0y_1 & x_0z_1 \\ y_0x_1 & y_0y_1 & y_0z_1 \\ z_0x_1 & z_0y_1 & z_0z_1 \end{bmatrix}$$

Upon comparing frames 0 and 1

$$R_1^0 = [X_1^0 \quad Y_1^0 \quad Z_1^0]$$

$$R_1^0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$

from 0 (frame) to 1 (frame) we have a translation of 1 in z direction

Adding Translation to the Homogenous Transformation matrix to make it a 4x4 matrix

$$H_1^0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Homogenous Transformation

Rotational matrix (3X3)

$$R^1_2 = \begin{bmatrix} x_1x_2 & x_1y_2 & x_1z_2 \\ y_1x_2 & y_1y_2 & y_1z_2 \\ z_1x_2 & z_1y_2 & z_1z_2 \end{bmatrix}$$

Upon comparing frames 1 and 2.

$$R^1_2 = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

Here the translation happened diagonally at $\theta = 45^0$, the distance translated was $\sqrt{2}$.

~~thus~~ $\therefore$ $\sqrt{x^2+z^2} = \sqrt{2}$.

translation happened in the x direction is 1

translation happened in the z direction is -1

Adding them to Rotational Homogenous Matrix to make it a 4x4 Homogeneous matrix

$$H^1_2 = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Homogenous Transformation Matrix

(3X3

$$R^0_2 = \begin{bmatrix} x_0 x_2 & x_0 y_2 & x_0 z_2 \\ y_0 x_2 & y_0 y_2 & y_0 z_2 \\ z_0 x_2 & z_0 y_2 & z_0 z_2 \end{bmatrix}$$

$$R^0_2 = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Translation $dy = 1$

$$H^0_2 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now to see if $H^0_1 H^1_2$ will give ~~give~~ H^0_2

Multiplying H^0_1 and H^1_2

The Resultant Matrix is equal to H_2^0

$$\therefore \boxed{H_1^0 H_2^1 = H_2^0}$$

Proof:-

$$H_1^0 \times H_2^1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= H_2^0$$

LHS = RHS

Hence proved