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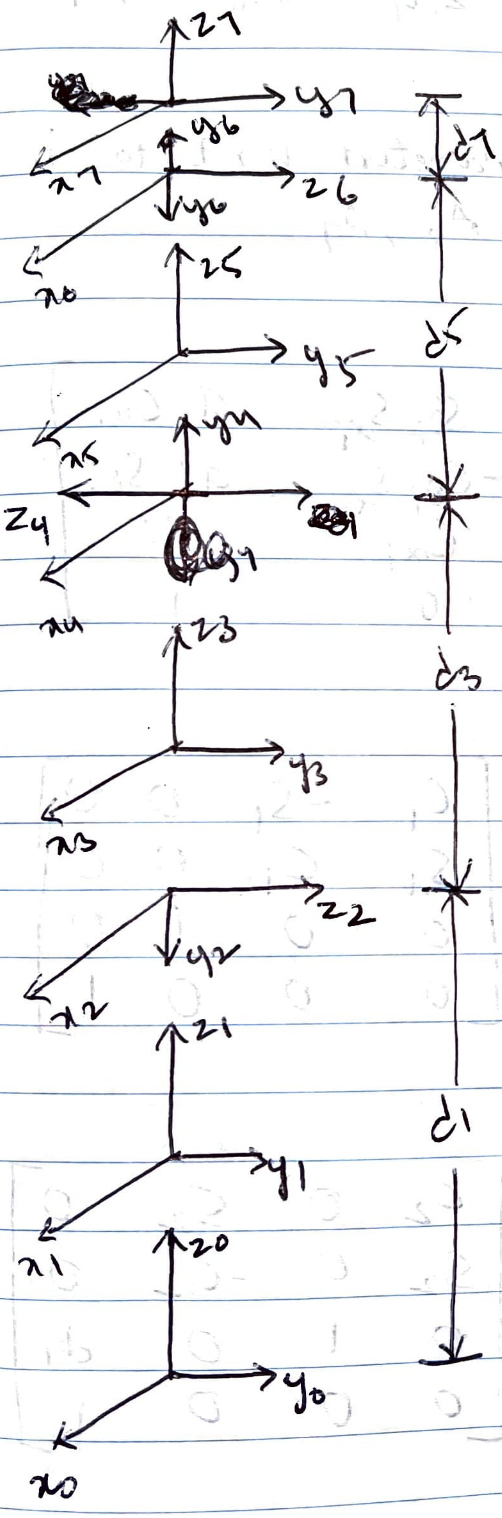
HOMEWORK-3

Problem 1

a) Show all co-ordinates :-



b) DH - Parameters

Link	α	θ_i	d	a_i
1	0	θ_1	0	0
2	$\pi/2$	θ_2	d_1	0
3	$-\pi/2$	θ_3	0	0
4	$-\pi/2$	θ_4	d_3	0
5	$\pi/2$	θ_5	0	0
6	$\pi/2$	θ_6	d_5	0
7	$-\pi/2$	θ_7	d_7	0

Transformation matrices on the other side.

c) Transformation Matrices derived from D-H Parameters

w.r.t $A_i = R_{z,\theta_i} \cdot T_{z,d_i} \cdot T_{x,a_i} \cdot R_{x,\alpha_i}$

so we use the DH parameters w.r.t to i and find $A_1, A_2, A_3, A_4, A_5, A_6, A_7$.

$$A_i = \begin{bmatrix} C_{\theta_i} & -S_{\theta_i} C_{\alpha_i} & S_{\theta_i} S_{\alpha_i} & a_i C_{\theta_i} \\ S_{\theta_i} & C_{\theta_i} C_{\alpha_i} & -C_{\theta_i} S_{\alpha_i} & a_i S_{\theta_i} \\ 0 & S_{\alpha_i} & C_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A_1

$\Rightarrow \theta_i = \theta_1$

$\alpha_i = 0$

$d = 0$

$a_i = 0$

$$A_1 = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A_2

$\Rightarrow \theta_i = \theta_2$

$\alpha = \pi/2$

$d = d_i$

$a_i = 0$

$$A_2 = \begin{bmatrix} C_2 & 0 & S_2 & 0 \\ S_2 & 0 & -C_2 & 0 \\ 0 & 1 & 0 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A_3

$\rightarrow$ $\theta_i = \theta_3$
$\alpha = -\pi/2$
$d = 0$
$a_i = 0$

$$A_3 = \begin{bmatrix} C_3 & 0 & -S_3 & 0 \\ S_3 & 0 & C_3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A_4

$\rightarrow$ $\theta_i = \theta_4$
$\alpha = -\pi/2$
$d = d_3$
$a_i = 0$

$$A_4 = \begin{bmatrix} C_4 & 0 & -S_4 & 0 \\ S_4 & 0 & C_4 & 0 \\ 0 & -1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A_5

$\rightarrow$ $\theta_i = \theta_5$
$\alpha = \pi/2$
$d = 0$
$a_i = 0$

$$A_5 = \begin{bmatrix} C_5 & 0 & S_5 & 0 \\ S_5 & 0 & -C_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A_6

$\rightarrow$ $\theta_i = \theta_6$
$\alpha = \pi/2$
$d = d_5$
$a_i = 0$

$$A_6 = \begin{bmatrix} C_6 & 0 & S_6 & 0 \\ S_6 & 0 & -C_6 & 0 \\ 0 & 1 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A_7

$\rightarrow$ $\theta_i = \theta_7$
$\alpha = -\pi/2$
$d = d_7$
$a_i = 0$

$$A_7 = \begin{bmatrix} C_7 & 0 & -S_7 & 0 \\ S_7 & 0 & C_7 & 0 \\ 0 & -1 & 0 & d_7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now to get the transformation Matrices:-

$${}^{0}_{1}T = A_1$$

$${}^{0}_{2}T = A_1 \cdot A_2$$

$${}^{0}_{3}T = A_1 \cdot A_2 \cdot A_3$$

$${}^{0}_{4}T = A_1 \cdot A_2 \cdot A_3 \cdot A_4$$

$${}^{0}_{5}T = A_1 \cdot A_2 \cdot A_3 \cdot A_4 \cdot A_5$$

$${}^{0}_{6}T = A_1 \cdot A_2 \cdot A_3 \cdot A_4 \cdot A_5 \cdot A_6$$

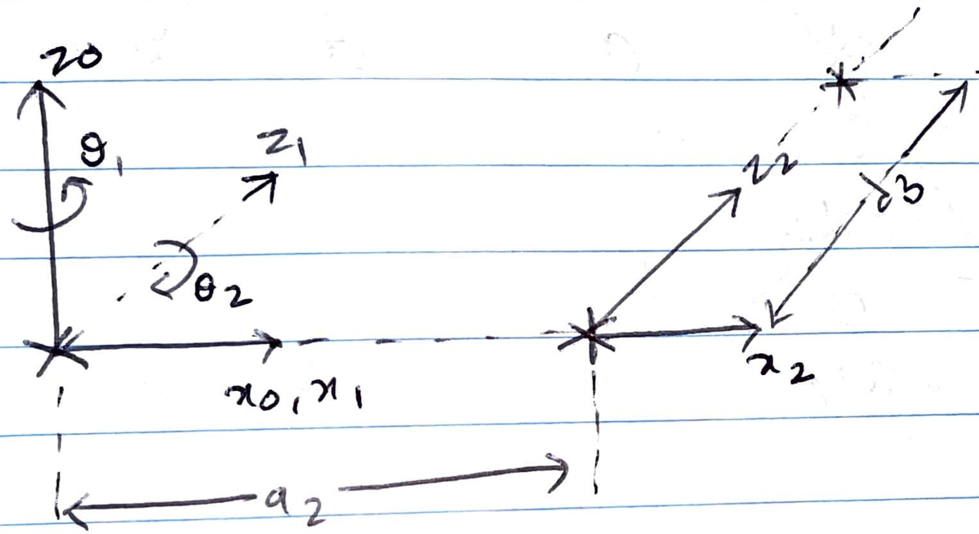
$${}^{0}_{7}T = A_1 \cdot A_2 \cdot A_3 \cdot A_4 \cdot A_5 \cdot A_6 \cdot A_7$$

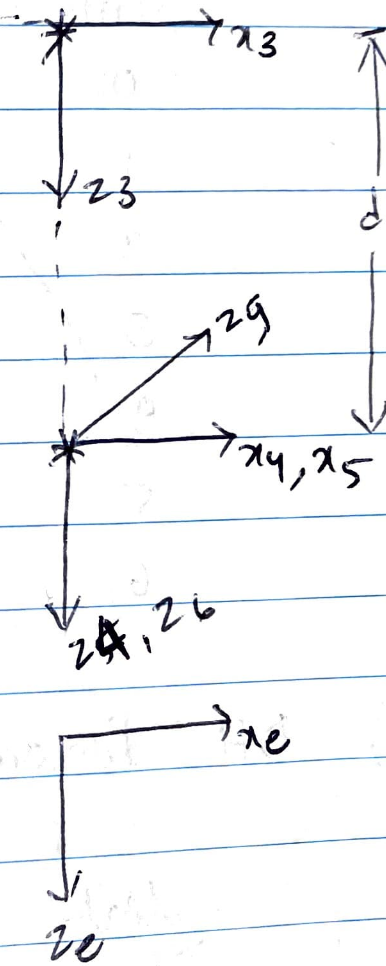
We can use a mathematical program from matlab or python to compute the values of Transformation Matrices.

Like asked in the question on homework. ${}^{0}_{7}T$ will be calculated and then validated using the sympy library of python.

Problem 2

a)





Co-ordinate Frame Assignment

b

b) <u>DH Parameters</u>

d_e – end effector offset (assumption)

Link	α	a	d	θ
1	0	0	0	θ_1
2	$-\pi/2$	0	0	θ_2
3	0	a_2	d_3	θ_3
4	$-\pi/2$	a_3	d_4	θ_4
5	$\pi/2$	0	0	θ_5
6	$-\pi/2$	0	d_e	θ_6

c) To compute Transformation Matrices.

$$A_1 = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{0}_{1}T = A_1$$

$$A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -S_2 & -C_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{0}_{2}T = A_1 \cdot A_2$$

$$A_3 = \begin{bmatrix} C_3 & -S_3 & 0 & a_2 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{0}_{3}T = A_1 \cdot A_2 \cdot A_3$$

$$A_4 = \begin{bmatrix} C_4 & -S_4 & 0 & a_3 \\ 0 & 0 & -1 & -d_4 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{0}_{4}T = A_1 \cdot A_2 \cdot A_3 \cdot A_4$$

$$A_5 = \begin{bmatrix} C_5 & -S_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -S_5 & -C_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{0}_{5}T = A_1 \cdot A_2 \cdot A_3 \cdot A_4 \cdot A_5$$

$$A_6 = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

${}^{0}_{6}T \rightarrow$ The final transformation matrix is going to be

$${}^{0}_{6}T = A_1 \cdot A_2 \cdot A_3 \cdot A_4 \cdot A_5 \cdot A_6$$

$${}^{0}_{6}T = \begin{bmatrix} r_{11} & r_{21} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

After computation we get value for all the r's and p's.

here

$$r_{11} = c_1 [C_{23}(C_4 C_5 C_6 - S_4 S_5) - S_{23} S_5 C_5]$$
$$+ S_1 (S_4 C_5 C_6 + C_4 S_6)$$

$$r_{21} = S_1 [C_{23}(C_4 C_5 C_6 - S_4 S_6) - S_{23} S_5 C_6$$
$$- C_1 (S_4 C_5 C_6 + C_4 S_6)$$

$$r_{31} = -S_{23} (C_4 C_5 C_6 - S_4 S_6) - C_{23} S_5 C_6$$

$$r_{12} = C_1 [C_{23} (-C_4 C_5 S_6 - S_4 C_6) + S_{23} S_5 S_6]$$
$$+ S_1 (C_4 C_6 + S_4 C_5 S_6)$$

$$r_{22} = S_1 [C_{23} (-C_4 C_5 S_6 - S_4 C_6) + S_{23} S_5 S_6]$$
$$- C_1 (C_4 C_6 - S_4 C_5 S_6)$$

$$r_{32} = -S_{23} (-C_4 C_5 S_6 - S_4 C_6) + C_{23} S_5 S_6$$

$$r_{13} = -C_1 (C_{23} C_4 S_5 + S_{23} C_5) - S_1 S_4 S_5$$

$$r_{23} = -S_1 (C_{23} C_4 S_5 + S_{23} C_5) + C_1 S_4 S_5$$

$$r_{33} = S_{23} C_4 S_5 - C_{23} C_5$$

$$p_x = c_1 [a_2 c_2 + a_3 c_{23} - d_4 S_{23}] - d_3 S_1$$

$$p_y = S_1 [a_2 c_2 + a_3 c_{23} - d_4 S_{23}] + d_3 c_1$$

$$p_z = -a_3 S_{23} - a_2 S_2 - d_4 c_{23}$$

d) Now to find the Jacobians

$$z_1 = \begin{bmatrix} -S_1 \\ c_1 \\ 0 \end{bmatrix} \quad z_2 = \begin{bmatrix} -S_1 \\ c_1 \\ 0 \end{bmatrix} \quad z_3 = \begin{bmatrix} -S_{23} c_1 \\ -S_1 S_{23} \\ -c_{23} \end{bmatrix}$$

$$z_4 = \begin{bmatrix} -S_1 c_4 + S_4 c_1 c_{23} \\ S_1 S_4 c_{23} + c_1 c_4 \\ -S_4 c_{23} \end{bmatrix} \quad z_5 = \begin{bmatrix} -(S_1 S_4 + c_1 c_4 c_{23}) S_5 - S_{23} c_1 c_5 \\ (-S_1 c_4 c_{23} + S_4 c_1) S_5 - S_1 S_{23} c_5 \\ S_5 S_{23} c_4 - c_5 c_{23} \end{bmatrix}$$

$$z_6 = \begin{bmatrix} -(S_1 S_4 + c_1 c_4 c_{23}) S_5 - S_{23} c_1 c_5 \\ (-S_1 c_4 c_{23} + S_4 c_1) S_5 - S_1 S_{23} c_5 \\ S_5 S_{23} c_4 - c_5 c_{23} \end{bmatrix} \quad z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad o_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad o_2 = \begin{bmatrix} a_2 c_1 c_2 \\ a_2 S_1 S_2 \\ -a_2 S_2 \end{bmatrix}$$

$$o_3 = \begin{bmatrix} a_2 c_1 c_2 + a_3 c_1 c_{23} - d_3 S_1 \\ a_2 S_1 c_2 + a_3 S_1 c_{23} + d_3 c_1 \\ -a_2 S_2 - a_3 S_{23} \end{bmatrix}$$

$$O_4 = \begin{bmatrix} a_2 c_1 c_2 + a_3 c_1 c_{23} - d_3 s_1 - d_4 s_{23} c_1 \\ a_2 s_1 c_2 + a_3 s_1 c_{23} + d_3 c_1 - d_4 s_1 s_{23} \\ -a_2 s_2 - a_2 s_{23} - d_4 c_{23} \end{bmatrix}$$

$$O_5 = \begin{bmatrix} a_2 c_1 c_2 + a_3 c_1 c_{23} - d_3 s_1 - d_4 s_{23} c_1 \\ a_2 s_1 c_2 + a_3 s_1 c_{23} + d_3 c_1 - d_4 s_1 s_{23} \\ -a_2 s_2 - a_3 s_{23} - d_4 c_{23} \end{bmatrix}$$

$$O_6 = \begin{bmatrix} a_2 c_1 c_2 + a_3 c_1 c_{23} - d_3 s_1 - d_4 s_{23} c_1 - d_e s_1 s_4 s_5 \\ - d_e s_5 c_1 c_4 c_{23} - d_e s_{23} c_1 c_5 \\ a_2 s_1 c_2 + a_3 s_1 c_{23} + d_3 c_1 - d_4 s_1 s_{23} - d_e s_1 s_5 c_4 c_{23} \\ - d_e s_1 s_{23} c_5 + d_e s_4 s_5 c_1 \\ -a_2 s_2 - a_{23} s_{23} - d_4 c_{23} - d_4 c_{23} + d_e s_5 s_{23} c_4 \\ - d_e c_5 c_{23} \end{bmatrix}$$

here in the eqns $d_e \rightarrow$ offset of end effector can be 0 as well. so it is just an assumption.

Jacobians (1st method)

$$J_i = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} \qquad J_i = \begin{bmatrix} z_{i-1} \times (O_n - O_{i-1}) \\ z_{i-1} \end{bmatrix}$$

$$J_1 = \begin{bmatrix} z_0 \times (O_6 - O_0) \\ z_0 \end{bmatrix} \qquad J_2 = \begin{bmatrix} z_1 \times (O_6 - O_1) \\ z_1 \end{bmatrix}$$

$$J_3 = \begin{bmatrix} z_2 \times (O_6 - O_2) \\ z_2 \end{bmatrix} \qquad J_4 = \begin{bmatrix} z_3 \times (O_6 - O_3) \\ z_3 \end{bmatrix}$$

$$J_5 = \begin{bmatrix} z_4 \times (O_6 - O_4) \\ z_4 \end{bmatrix} \qquad J_6 = \begin{bmatrix} z_5 \times (O_6 - O_5) \\ z_5 \end{bmatrix}$$

Now the Jacobian will be

$$\therefore J = [J_1 \ J_2 \ J_3 \ J_4 \ J_5 \ J_6]$$

* We can compute this using a mathematical program or a matrix calculator

* The resultant is very large, hence it is being omitted, the inconvinience is regretted.

2nd Method.

This method needs us to isolate the final column of transformation matrix and derivate it w.r.t $\theta_1, \dots \theta_6$.

$\uparrow x_p$ $({}^{0}_{6}T)$

we need to calculate

$$\frac{\partial x_p}{\partial \theta_1}, \frac{\partial x_p}{\partial \theta_2}, \frac{\partial x_p}{\partial \theta_3}, \frac{\partial x_p}{\partial \theta_4}, \frac{\partial x_p}{\partial \theta_5}, \frac{\partial x_p}{\partial \theta_6}$$

$\rightarrow$ X_p is nothing but O_6.

$$\frac{\partial x_p}{\partial \theta_1} = \begin{bmatrix} -a_2 S_1 C_2 - a_3 S_1 C_{23} - d_3 C_1 + d_4 S_1 S_{23} + d_e S_1 S_{23} S_5 C_4 \\ - d_e S_4 S_5 C_1 + d_e S_1 S_{23} C_5 \\ a_2 C_1 C_2 + a_3 C_1 C_{23} - d_3 S_1 - d_4 S_{23} C_1 - d_e S_1 S_4 S_5 \\ - d_e S_5 C_1 C_4 C_{23} - d_e S_{23} C_1 C_5 \\ 0 \end{bmatrix}$$

$$\frac{\partial x_p}{\partial \theta_2} = \begin{bmatrix} (-a_2 S_2 - a_3 S_{23} - d_4 C_{23} + d_e S_5 S_{23} C_4 - d_e C_5 C_{23}) C_1 \\ (-a_2 S_2 - a_3 S_{23} - d_4 C_{23} + d_e S_5 S_{23} C_4 - d_e C_5 C_{23}) S_1 \\ - a_2 C_2 - a_3 C_{23} + d_4 S_{23} + d_e S_5 C_4 C_{23} + d_e S_{23} C_5 \end{bmatrix}$$

$$\frac{\partial x_p}{\partial \theta_3} = \begin{bmatrix} (-a_2 S_{23} - d_4 C_{23} + d_e S_5 S_{23} C_4 - d_e C_5 C_{23}) C_1 \\ (-a_2 S_{23} - d_4 C_{23} + d_e S_5 S_{23} C_4 - d_e C_5 C_{23}) S_1 \\ -a_2 C_2 - a_3 C_{23} + d_4 S_{23} + d_e S_5 C_4 C_{23} + d_e S_{23} C_5 \end{bmatrix}$$

$$\frac{\partial x_p}{\partial \theta_4} = \begin{bmatrix} (-S_1 C_4 + S_4 C_1 C_{23}) S_5 \\ (S_1 S_4 C_{23} + C_1 C_4) S_5 \\ -d_e S_4 S_5 S_{23} \end{bmatrix}$$

$$\frac{\partial x_p}{\partial \theta_5} = \begin{bmatrix} (-S_1 S_4 C_5 + S_5 C_{23} C_1 - C_1 C_4 C_5 C_{23}) \\ (S_1 S_5 S_{23} - S_1 C_4 C_5 C_{23} + S_4 C_1 C_5 \\ (S_5 C_{23} + S_{23} C_4 C_5 \end{bmatrix}$$

$$\frac{\partial x_p}{\partial \theta_6} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore J = \begin{bmatrix} \frac{\partial x_p}{\partial \theta_1} & \frac{\partial x_p}{\partial \theta_2} & \frac{\partial x_p}{\partial \theta_3} & \frac{\partial x_p}{\partial \theta_4} & \frac{\partial x_p}{\partial \theta_5} & \frac{\partial x_p}{\partial \theta_6} \\ z_1 & z_2 & z_3 & z_4 & z_5 & z_6 \end{bmatrix}$$

if we plug in the values in J we will get the resultant matrix.