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PROBLEM-SET 5

1) Let $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{m\times m}$. Show that $X(t) = e^{At}X(0)e^{Bt}$ is the solution to the following eqn

$$\dot{x}(t) = Ax(t) + x(t)B$$

A) We know that if $X(t) = e^{At}X(0)e^{Bt}$ is a solution of $\dot{x}(t) = Ax(t) + x(t)B$, then it should satisfy the equation.

Differentiating $X(t) = e^{At}X(0)e^{Bt}$

$$\dot{x}(t) = \frac{d}{dt}\left[e^{At}X(0)e^{Bt}\right] = Ae^{At}X(0)e^{Bt} + e^{At}X(0)Be^{Bt} \quad -①$$

w.k.t $\frac{d}{dt}e^{Bt} = Be^{Bt} = e^{Bt}B$, we can write above equation as $\dot{x}(t) = Ae^{At}X(0)e^{Bt} + e^{At}X(0)e^{Bt}B \quad -②$

Now $X(t) = e^{At}X(0)e^{Bt}$

when $t=0$

$X(0) = e^{0}X(0)e^{0} = X(0)$

$\therefore X(t) = x(t) \rightarrow$ substitute in 2

② $\rightarrow$ $\dot{x}(t) = Ax(t) + x(t)B$

$\downarrow$

Hence proved that $X(t) = e^{At}X(0)e^{Bt}$ is a solution of

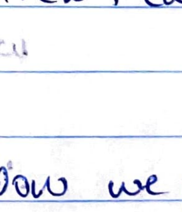
2) A Euclidean Ball, $B(x_c, r)$, in R^n is given by $B(x_c, r) = \{x \in R^n / \|x - x_c\| \leq r\}$ where $r > 0$ and $\|\cdot\|$ is the euclidean norm, $\|x\| = \sqrt{x^T x}$. The vector x_c is the center of the ball and r is the radius. Prove that $B(x_c, r)$ is a convex set.

A)

Euclidean Ball :- In Euclidean 3D space, a ball is taken to be the volume bounded by a 2-D sphere. (In 2D space, a ball is a line segment.

for a sphere (ball) with center $\bar{x}_c$ and radius r. when we take an n dimensional vector $\bar{x}$, which is an arbitrary point inside the sphere.

$$\|\bar{x} - \bar{x}_c\| \leq r$$



If the ball with center x_c is convex, then the shift will also be convex. Cas shifting doesn't effect the convexity.

Proof :- Consider 2pts $\bar{x}_1$ & $\bar{x}_2$, Now we prove that the convex combination lies in the same set.

By the definition of norm, the interior of the Ball is given by $\|x_1\| \leq r$ and $\|x_2\| \leq r$

Considering the convex combination

$$\|\theta \bar{x}_1 + (1-\theta)\bar{x}_2\|$$

Now using the triangular inequality
$$\|\bar{a}\| + \|\bar{b}\| \geq \|\bar{a}+\bar{b}\|$$
we get that

$$\|\theta \bar{x}_1 + (1-\theta)\bar{x}_2\| \leq \|\theta \bar{x}_1\| + \|(1-\theta)\bar{x}_2\|$$
where $\theta \in [0,1]$ and $\theta, (1-\theta) \geq 0$.

so

$$\Rightarrow \theta_1 \|\bar{x}_1\| + (1-\theta)\|\bar{x}_2\|$$

By definition we know that

$$\|\bar{x}_1\| \leq r \text{ and } \|\bar{x}_2\| \leq r.$$

$$\therefore \theta_1 \|\bar{x}_1\| + (1-\theta)\|\bar{x}_2\| \leq \theta_1 r + (1-\theta) r$$
$$\Rightarrow \theta_1 \|\bar{x}_1\| + (1-\theta)\|\bar{x}_2\| \leq r$$

w.k.t $\|\theta_1 \bar{x} + (1-\theta)\bar{x}_2\| \leq \theta_1 \|\bar{x}_1\| + (1-\theta)\|\bar{x}_2\|$

$$\therefore \|\theta_1 \bar{x} + (1-\theta)\bar{x}_2\| \leq r.$$

Hence proved that $\|\theta \bar{x}_1 + (1-\theta)\bar{x}_2\| \in B$

so B is convex Q.E.D

$\downarrow$

$B(x_c, r)$

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3) Let $x(t)$ be a zero mean Gaussian random process with covariance function given by $C_x(t_1, t_2)$ here $C_x(t_1, t_2) = \sigma^2 e^{-|t_1 - t_2|}$.
Find the joint pdf of $X(t)$ and $X(t+s)$

A) When $X_1 = X(t_1)$, $X_2 = X(t_2)$ .... $X_k = X(t_k)$ are samples of a ~~Gaussian~~ random process $X(t_k)$, then that is called a Gaussian random process.

The joint pdf of jointly Gaussian random variables is determined by the vector of means (m) and the covariance matrix Σ:

$$f_{x_1, x_2, x_3 \ldots}(x_1, x_2, x_3 \ldots) = \frac{1}{(2\pi)^{k/2}} \cdot \frac{1}{\sqrt{|\Sigma|}} \exp\left(\frac{1}{2}(x-m)^T \Sigma^{-1} (x-m)\right)$$

where $X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \end{bmatrix}_{n\times 1}$ $m = \begin{bmatrix} m_x(t_1) \\ m_x(t_2) \\ \vdots \end{bmatrix}_{n\times 1}$ $\Sigma_X = \begin{bmatrix} C_x(t_1,t_1) & C_x(t_1,t_2) \ldots \\ \vdots & \ddots \\ C_x(t_1,t_k) \ldots & C_x(t_k,t_k) \end{bmatrix}_{n\times n}$

$\Rightarrow k = n$ (dimension of X)

$\Rightarrow m_x = E(X)$

$\Rightarrow \Sigma_X = E[(X - m_x)(X - m_x)^T]$

$= E[(X(t_i) - m_x(t_i))(X(t_j) - m_x(t_j))]$

$\Rightarrow C_x(t_i, t_j) = E[X(t_i)X(t_j)] - m_x(t_i)\, m_x(t_j)$

$C_x(t_i, t_j)$ is a covariance function of Σ.

$C_X(t_1, t_2) = \sigma^2 e^{-|t_1 - t_2|}$ is given. $t_1 = t$

$t_2 = t+s$

$$X = \begin{bmatrix} x(t) \\ x(t+s) \end{bmatrix} \qquad m = \begin{bmatrix} m_X(t) \\ m_X(t+s) \end{bmatrix}$$

$$\Sigma_X = \begin{bmatrix} C_X(t,t) & C_X(t, t+s) \\ C_X(t+s, t) & C_X(t+s, t+s) \end{bmatrix}$$

here

$$C_X(t,t) = \sigma^2 e^0 = \sigma^2$$
$$C_X(t, t+s) = \sigma^2 e^{-s}$$
$$C_X(t+s, t) = \sigma^2 e^{-s}$$
$$C_X(t+s, t+s) = \sigma^2 e^0 = \sigma^2$$

$$\Sigma_X = \begin{bmatrix} \sigma^2 & \sigma^2 e^{-s} \\ \sigma^2 e^{-s} & \sigma^2 \end{bmatrix} = \cancel{\begin{bmatrix} 1 & e^{-s} \\ e^{-s} & 1 \end{bmatrix}}$$

$$\Sigma_X^{-1} = \frac{1}{\sigma^2(1-e^{-2s})} \begin{bmatrix} 1 & -e^{-s} \\ -e^{-s} & 1 \end{bmatrix} \qquad |\Sigma| = \underline{\sigma^4 (1 - e^{-2s})}$$

The joint pdf $\Rightarrow$ (for a zero mean process) $\boxed{m_X = 0}$

$$f_{X_1 X_2}(x_1, x_2) = \frac{1}{2\pi\sigma^2\sqrt{1-e^{-2s}}} \exp\left(-\frac{1}{2}\begin{bmatrix} x(t) & x(t+s) \end{bmatrix} \Sigma_X^{-1} \begin{bmatrix} X(t) \\ X(t+s) \end{bmatrix}\right)$$

$$f_{X_1X_2}(x_t, x_{t+s}) = \frac{1}{2\pi\sigma^2\sqrt{1-e^{-2s}}} \exp\left(\frac{1}{2\sigma^2\sqrt{1-e^{-2s}}} \begin{bmatrix} x(t) & x(t+s) \end{bmatrix} \begin{bmatrix} x(t) - x(t+s)e^{-s} \\ -e^{-s}x(t) + x(t+s) \end{bmatrix}\right)$$

$$= \frac{1}{2\pi\sigma^2\sqrt{1-e^{-2s}}} \exp\left(\frac{1}{2\sigma^2\sqrt{1-e^{-2s}}} \left[(x(t))^2 - 2e^{-s}x(t)(x(t+s)) + (x(t+s))^2\right]\right)$$

Joint pdf =

$$\frac{1}{2\pi\sigma^2\sqrt{1-e^{-2s}}} \exp\left(\frac{1}{2\sigma^2\sqrt{1-e^{-2s}}} \left[x(t)^2 - 2e^{-s}x(t)x(t+s) + x(t+s)^2\right]\right)$$

Sorry for cramming everything into 1 line.

4) Consider the following state space eqn:-

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

consider the following cost function

$$J = \int_0^{\infty} (x^T Q + u^2)\, dt$$ where Q is

given by $Q = \begin{bmatrix} 1 & 0 \\ 0 & \gamma \end{bmatrix}$

Design an LQR and then provide a state feedback of $u = Kx$

A) $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $Q = \begin{bmatrix} 1 & 0 \\ 0 & \gamma \end{bmatrix}$

We know that the gain matrix can be represented as $K = -R^{-1} B_K^T P$ which is the optimal feedback solution.

Here P is the solution of the Ricatti equation $A^T P + PA - PB_K R^{-1} B_K^T P = -Q$

The General cost function is

$$J(K, \vec{X}(0)) = \int_0^{\infty} \vec{X}^T(t) Q \vec{X}(t) + \vec{U}_K^T(t) R \vec{U}_K(t)\, dt$$

Comparing the General cost function with the given cost function.

We can see that $R = I$.

Ricatti Eqn: $A^T P + PA - PB_K R^{-1} B^{-1} P = -Q$.

Substituting the known matrices in the Ricatti Eqn:-

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} + \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} I \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} = -Q$$

$$\Rightarrow \begin{bmatrix} 0 & 0 \\ P_{11} & P_{12} \end{bmatrix} + \begin{bmatrix} 0 & P_{11} \\ 0 & P_{12} \end{bmatrix} - \begin{bmatrix} P_{12} \\ P_{22} \end{bmatrix}\begin{bmatrix} P_{12} & P_{22} \end{bmatrix} = -\begin{bmatrix} 1 & 0 \\ 0 & \gamma \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & P_{11} \\ P_{11} & 2P_{12} \end{bmatrix} - \begin{bmatrix} P_{12}^2 & P_{12}P_{22} \\ P_{12}P_{22} & P_{22}^2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -\gamma \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -P_{12}^2 & P_{11} - P_{12}P_{22} \\ P_{11} - P_{12}P_{22} & 2P_{12} - P_{22}^2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -\gamma \end{bmatrix}$$

$P_{11} = P_{12}P_{22}$ $\quad -P_{12}^2 = -1 \Rightarrow P_{12} = \pm 1$

Now when $P_{12} = 1$ **Case 1**

$\Rightarrow P_{11} = P_{22}$

essentially $2P_{12} - P_{22}^2 = -\gamma$

$\Rightarrow P_{22} = \pm\sqrt{\gamma + 2}$

when $P_{12} = -1 \Rightarrow P_{11} = -P_{22}$ **Case 2**

essentially $-2P_{12} - P_{22}^2 = -\gamma \Rightarrow P_{22} = \pm\sqrt{\gamma - 2}$

when we look at case 2, there is a clear possibility for P_{22} to be complex when $\gamma > 0$ which is not desirable.

So we consider only case 1:

In case 1 we will have 2 sub cases

case 1.1 ⇒ when $P_{22} = +\sqrt{\gamma+2}$
case 1.2 ⇒ when $P_{22} = -\sqrt{\gamma+2}$.

→ <u>1.1</u> $P_{22} = +\sqrt{\gamma+2}$

$\therefore P = \begin{bmatrix} \sqrt{\gamma+2} & 1 \\ 1 & \sqrt{\gamma+2} \end{bmatrix}$ since P should always be +ve definite matrix.

The eigen values of P are

$$-1 + \left((\sqrt{\gamma+2}) - \lambda\right)\left((\sqrt{\gamma+2}) - \lambda\right) = 0.$$

$$-1 + \left((\sqrt{\gamma+2}) - \lambda\right)^2 = 0.$$

$$\lambda = \sqrt{\gamma+2} \pm 1$$

→ 1.2 $P_{22} = -\sqrt{\gamma+2}$

$\therefore P = \begin{bmatrix} -\sqrt{\gamma+2} & 1 \\ 1 & -\sqrt{\gamma+2} \end{bmatrix}$

∴ The eigen values are

$$(\lambda + \sqrt{\gamma+2})^2 = 1 \Rightarrow \lambda = \pm 1 - \sqrt{\gamma+2}$$

In case 1.2, there is a choice that upon choosing certain values, the value of P will be negative definite which is not desirable so we move ahead with case 1.1

In case 1.1 $\lambda_1, \lambda_2 > 0$, $\therefore$ P is +ve definite.

$$P = \begin{bmatrix} \sqrt{\gamma+2} & 1 \\ 1 & \sqrt{\gamma+2} \end{bmatrix}$$

Now we can compute Gain matrix K

$$K = -R^{-1}B^TP$$

$$K = -I^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix}^T \begin{bmatrix} \sqrt{\gamma+2} & 1 \\ 1 & \sqrt{\gamma+2} \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{\gamma+2} & 1 \\ 1 & \sqrt{\gamma+2} \end{bmatrix}$$

$$K = -\begin{bmatrix} 1 & \sqrt{\gamma+2} \end{bmatrix}$$

providing state feed back of $u = Kx$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} -1 & -\sqrt{\gamma+2} \end{bmatrix} x$$

$$= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ -1 & -\sqrt{\gamma+2} \end{bmatrix} x$$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & -\sqrt{\gamma+2} \end{bmatrix} x$$ is the required state eqn with LQR controller provided with a state feedback $u = Kx$

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5) Consider the following state space representation

$$\dot{x}(t) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u(t)$$

The order of A matrix is $n \times n = 3 \times 3$
The order of B matrix is $n \times m = 3 \times 2$

w.k.t
the order of controllability Matrix C is $n \times nm = 3 \times 6$

and we know $C = [B \mid AB \mid A^2B]$

$$B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad AB = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$$A^2B = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 1 & 1 & 2 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 2 & 1 \end{bmatrix}$$

{Used. Symbolab}

We can manually the matrix to echelon form too.

Now the rank(C) after comparing some rows and some manipulation turned out to be 2

$$\text{rank}(C) = 2 \Rightarrow 2 \le n \Rightarrow 2 \le 3$$

It shows that the system is uncontrollable

Now doing Similarity Transformation

$$S_{n-nr} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow S^{-1} = \frac{1}{|S|} Adj(S) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 11 & 1 \end{bmatrix}$$

$$\hat{A} = S^{-1}AS = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\hat{A} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}$$

since $A_{11} = n_r \times n_r = 2\times 2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$$\hat{B} = S^{-1} B_K = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}$$

$B_1 = n_r \times n_r = 2\times 2$

$\Rightarrow B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

The standard form of uncontrollable systems is given by

$$\dot{X} = \hat{A} X + \hat{B} U$$

$$\boxed{\dot{X} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} X + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} U}$$

Here, the controllable part is given by (A_{11}, B_1)

State space form of that

$$\dot{x} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} u$$

6) Consider the system $\dot{x} = \begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix} x$. Investigate the stability of the system using the Lyapunov equation

$A^T P + PA = -Q$ $\qquad Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

A). Lyapunov stability theorem states that, a system is stable iff, for any symmetric +ve definite matrix Q, there exists a symmetric +ve definite matrix P such that

$$A^T P + PA = -Q$$

Assumption:-

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} \Rightarrow (P_{11} - \lambda)(P_{22} - \lambda) = P_{12}^2$$

$$A = \begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix} \qquad A^T = \begin{bmatrix} -3 & -1 \\ 2 & -1 \end{bmatrix}$$

$$A^T P = \begin{bmatrix} -3 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} = \begin{bmatrix} -3P_{11} - P_{12} & -3P_{12} - P_{22} \\ 2P_{11} - P_{12} & 2P_{12} - P_{22} \end{bmatrix}$$

$$\cancel{PA \quad \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} = \begin{bmatrix} -3P_{11} - P_{12} & 2 \end{bmatrix}}$$

$$PA = \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} \begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} -3P_{11} - P_{12} & 2P_{11} - P_{12} \\ -3P_{12} - P_{22} & 2P_{12} - P_{22} \end{bmatrix}$$

$$A^T P + PA = \begin{bmatrix} -6P_{11} - 2P_{12} & 2P_{11} - 4P_{12} - P_{22} \\ 2P_{11} - 4P_{12} - P_{22} & 4P_{12} - 2P_{22} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

We get 3 eqns:

$$-6P_{11} - 2P_{12} = -1 \quad -① $$
$$2P_{11} - 4P_{12} + P_{22} = 0 \quad -② $$
$$4P_{12} - 2P_{22} = -1 \quad -③ $$

* We can solve manually or use cramers rule

Upon solving we get (I did it manually)

$P_{11} = \frac{7}{40}$ $\quad P_{12} = -\frac{1}{40}$ $\quad P_{22} = \frac{18}{40}$

$$\therefore \quad P = \frac{1}{40}\begin{bmatrix} 7 & -1 \\ -1 & 18 \end{bmatrix}$$

finding eigen values of P.

$$(7-\lambda)(18-\lambda) - 1 = 0$$

$$126 - 7\lambda - 18\lambda + \lambda^2 - 1 = 0$$

$$\Rightarrow \quad \lambda^2 - 25\lambda + 125 = 0$$

$$\Rightarrow \quad \lambda = \frac{25}{2} \pm \frac{\sqrt{625-500}}{2}$$

$$\Rightarrow \quad \lambda = \frac{25}{2} \pm \frac{3\sqrt{5}}{2}$$

It is evident that $\lambda_1, \lambda_2 > 0$

$\therefore$ The system is stable.

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