Homework 4

Friday, December 2, 2022 5:31 PM

Problem 1 [Exercise 7.8]

weights for given data:

$$w^{(1)} = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.4 \end{bmatrix}, \quad w^{(2)} = \begin{bmatrix} 0.2 \\ 1 \\ -3 \end{bmatrix}, \quad w^{(3)} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

For the data point $x = 2$, $y = 1$, we have

Applying Forward Propagation

$$x^{(0)} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad s^{(1)} = \begin{bmatrix} 0.7 \\ 1 \end{bmatrix}$$

so $$x^{(1)} = \begin{bmatrix} 1 \\ 0.7 \\ 2 \end{bmatrix}, \quad s^{(2)} = [-2.1]$$

$$x^{(2)} = \begin{bmatrix} 1 \\ -2.1 \end{bmatrix}, \quad s^{(3)} = [-3.2]$$

$$x^{(3)} = -3.2$$

Apply Backpropagation and note that $\theta'(s^{(l)}) = 1$ to compute

$$\delta^{(3)} = 2(x^{(3)} - y) = -8.4$$

$$\delta^{(2)} = \theta'(s^{(2)}) \times [W^{(3)} \delta^{(3)}] = -16.8$$

$$\delta^{(1)} = \begin{bmatrix} -16.8 \\ 50.4 \end{bmatrix}$$

$$\frac{\partial e}{\partial W^{(1)}} = x^{(0)} (\delta^{(1)})^T = \begin{bmatrix} -16.8 & 50.4 \\ -33.6 & 100.8 \end{bmatrix}$$

$$\frac{\partial e}{\partial W^{(2)}} = x^{(1)} (\delta^{(2)})^T = \begin{bmatrix} -16.8 \\ -11.76 \\ -33.6 \end{bmatrix}$$

$$\frac{\partial e}{\partial W^{(3)}} = x^{(2)} (\delta^{(3)})^T = \begin{bmatrix} -8.4 \\ -17.64 \end{bmatrix}$$

Problem 3 [Problem 8.2]

A dataset with three data points in $\mathbb{R}^2$

$$X = \begin{bmatrix} 0 & 0 \\ 0 & -1 \\ -2 & 0 \end{bmatrix} \qquad y = \begin{bmatrix} -1 \\ -1 \\ +1 \end{bmatrix}$$

for this data, we let $w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$, we have the objective value.

$$E = \frac{1}{2} w^T w = \frac{1}{2}(w_1^2 + w_2^2)$$

The constraints are

$$y_1(w_1 x_{11} + w_2 x_{12} + b) = -b \geq 1$$
$$y_2(w_1 x_{21} + w_2 x_{22} + b) = w_2 - b \geq 1$$
$$y_3(w_1 x_{31} + w_2 x_{32} + b) = -2w_1 + b \geq 1$$

combine the first and third inequalities, we have $w_1 \leq -1$.
combine the first and the second, we have $w_2 \geq 0$.
so the objective achieves minimal at $w_1 = -1$, $w_2 = 0$
where $E = \frac{1}{2}$. The optimal $b = -1$

The margin is thus $\frac{1}{|w|} = \frac{1}{\sqrt{w_1^2 + w_2^2}} = 1$