Instance Based Learning

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- k-Nearest Neighbor
- Locally weighted regression
- Radial basis functions
- Case-based reasoning
- Lazy and eager learning

Instance-Based Learning

Key idea: just store all training examples $\langle x_i, f(x_i) \rangle$

Nearest neighbor:

- Given query instance x_q, first locate nearest training example x_n, then estimate $\hat{f}(x_q) \leftarrow f(x_n)$

k-Nearest neighbor:

- Given x_q, take vote among its k nearest nbrs (if discrete-valued target function)
- take mean of f values of k nearest nbrs (if real-valued)

$$\hat{f}(x_q) \leftarrow \frac{\sum_{i=1}^{k} f(x_i)}{k}$$

When To Consider Nearest Neighbor

- Instances map to points in $\Re^n$
- Less than 20 attributes per instance
- Lots of training data

Advantages:

- Training is very fast
- Learn complex target functions
- Don't lose information

Disadvantages:

- Slow at query time
- Easily fooled by irrelevant attributes

Voronoi Diagram

Behavior in the Limit

Consider $p(x)$ defines probability that instance x will be labeled 1 (positive) versus 0 (negative).

Nearest neighbor:

- As number of training examples $\rightarrow \infty$, approaches Gibbs Algorithm

  Gibbs: with probability $p(x)$ predict 1, else 0

k-Nearest neighbor:

- As number of training examples $\rightarrow \infty$ and k gets large, approaches Bayes optimal

  Bayes optimal: if $p(x) > .5$ then predict 1, else 0

Note Gibbs has at most twice the expected error of Bayes optimal

Distance-Weighted kNN

Might want weight nearer neighbors more heavily...

$$\hat{f}(x_q) \leftarrow \frac{\sum_{i=1}^{k} w_i f(x_i)}{\sum_{i=1}^{k} w_i}$$

where

$$w_i \equiv \frac{1}{d(x_q, x_i)^2}$$

and $d(x_q, x_i)$ is distance between x_q and x_i

Note now it makes sense to use *all* training examples instead of just k

$\rightarrow$ Shepard's method

Curse of Dimensionality

Imagine instances described by 20 attributes, but only 2 are relevant to target function

Curse of dimensionality: nearest nbr is easily mislead when high-dimensional X

One approach:

- Stretch jth axis by weight z_j, where $z_1, \ldots, z_n$ chosen to minimize prediction error
- Use cross-validation to automatically choose weights $z_1, \ldots, z_n$
- Note setting z_j to zero eliminates this dimension altogether

see [Moore and Lee, 1994]

Locally Weighted Regression

Note kNN forms local approximation to f for each query point x_q

Why not form an explicit approximation $\hat{f}(x)$ for region surrounding x_q

- Fit linear function to k nearest neighbors
- Fit quadratic, ...
- Produces "piecewise approximation" to f

Several choices of error to minimize:

- Squared error over k nearest neighbors

$$E_1(x_q) \equiv \frac{1}{2} \sum_{x \in \ k \ nearest \ nbrs \ of \ x_q} (f(x) - \hat{f}(x))^2$$

- Distance-weighted squared error over all nbrs

$$E_2(x_q) \equiv \frac{1}{2} \sum_{x \in D} (f(x) - \hat{f}(x))^2 \ K(d(x_q, x))$$

- ...

Radial Basis Function Networks

- Global approximation to target function, in terms of linear combination of local approximations
- Used, e.g., for image classification
- A different kind of neural network
- Closely related to distance-weighted regression, but “eager” instead of “lazy”

Radial Basis Function Networks

where $a_i(x)$ are the attributes describing instance x, and

$$f(x) = w_0 + \sum_{u=1}^{k} w_u K_u(d(x_u, x))$$

One common choice for $K_u(d(x_u, x))$ is

$$K_u(d(x_u, x)) = e^{-\frac{1}{2\sigma_u^2} d^2(x_u, x)}$$

Training Radial Basis Function Networks

Q1: What x_u to use for each kernel function $K_u(d(x_u, x))$

- Scatter uniformly throughout instance space
- Or use training instances (reflects instance distribution)

Q2: How to train weights (assume here Gaussian K_u)

- First choose variance (and perhaps mean) for each K_u
  - e.g., use EM
- Then hold K_u fixed, and train linear output layer
  - efficient methods to fit linear function

Case-Based Reasoning

Can apply instance-based learning even when $X \neq \Re^n$

$\rightarrow$ need different "distance" metric

Case-Based Reasoning is instance-based learning applied to instances with symbolic logic descriptions

Case-Based Reasoning in CADET

CADET: 75 stored examples of mechanical devices

- each training example: $\langle$ qualitative function, mechanical structure$\rangle$
- new query: desired function,
- target value: mechanical structure for this function

Distance metric: match qualitative function descriptions

Case-Based Reasoning in CADET

Case-Based Reasoning in CADET

- Instances represented by rich structural descriptions
- Multiple cases retrieved (and combined) to form solution to new problem
- Tight coupling between case retrieval and problem solving

Bottom line:

- Simple matching of cases useful for tasks such as answering help-desk queries
- Area of ongoing research

Lazy and Eager Learning

Lazy: wait for query before generalizing

Neighbor , Case based reasoning

Eager: generalize before seeing query

- Radial basis function networks, ID3, Backpropagation, NaiveBayes, ...

Does it matter?

- Eager learner must create global approximation
- Lazy learner can create many local approximations
- if they use same H, lazy can represent more complex fns (e.g., consider H = linear functions)