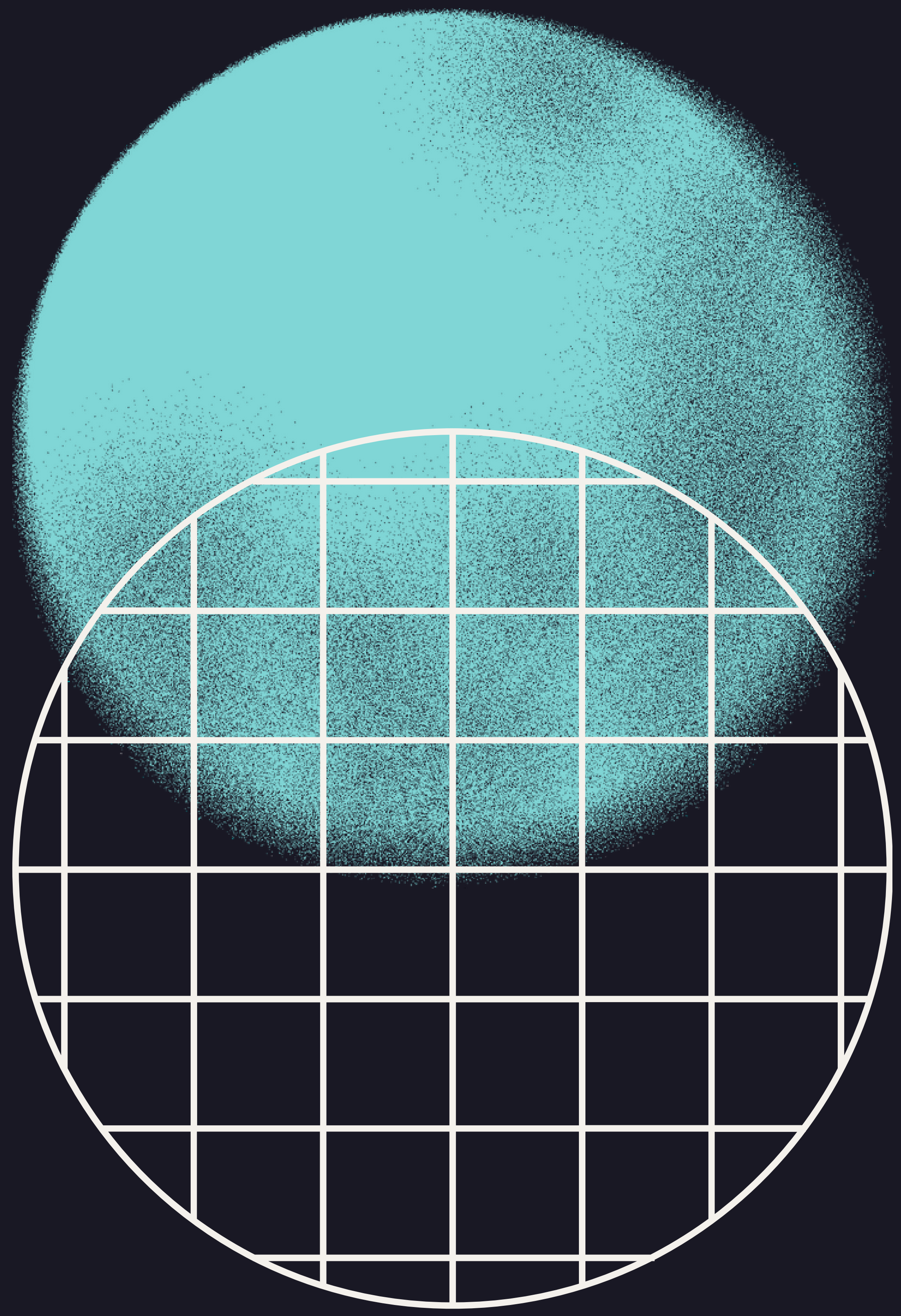


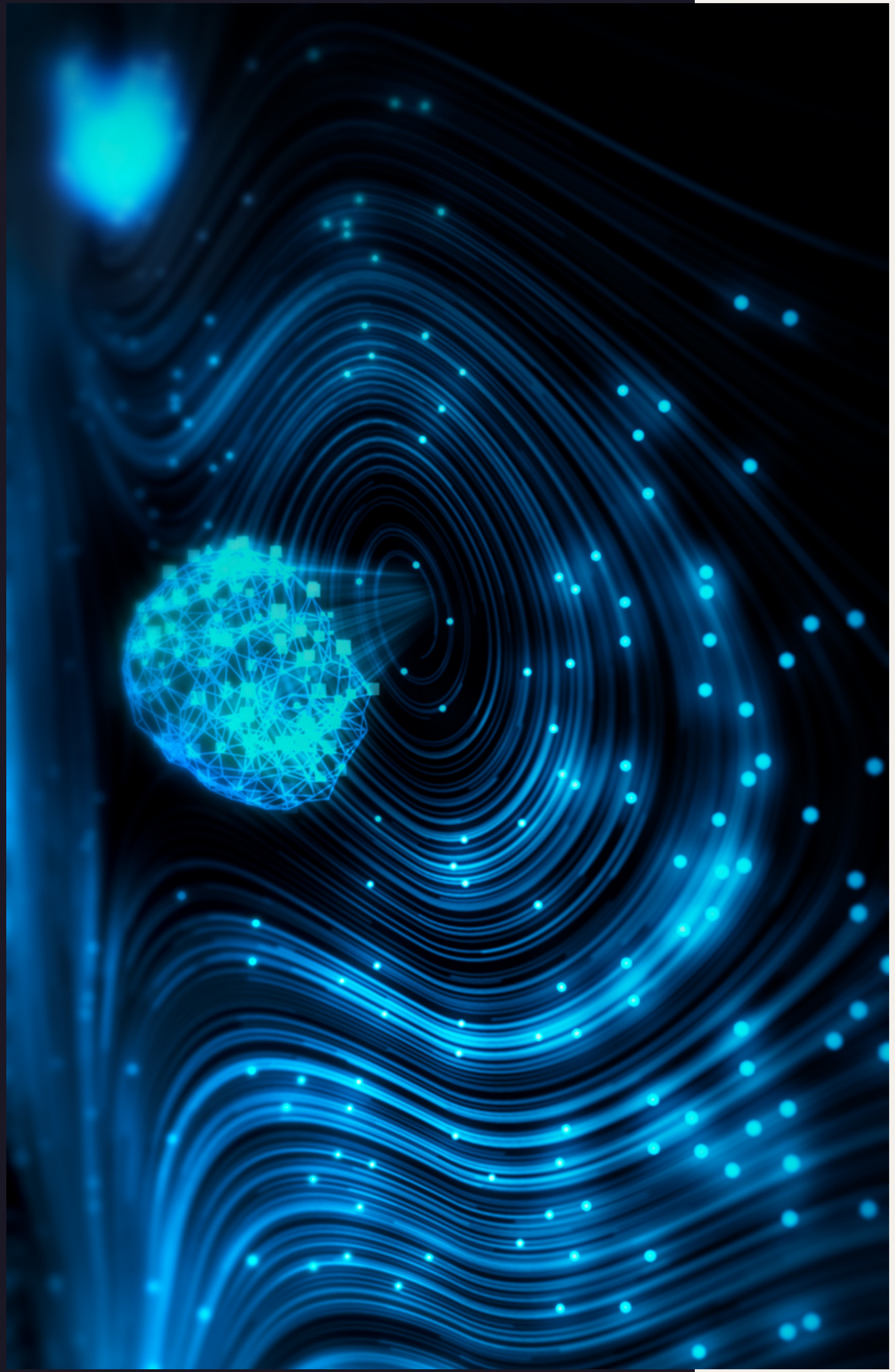
Optimizing Experimentation

An In-Depth Look at Factorial Design

CONTENTS

INTRODUCTION

Welcome to Optimizing Experimentation: *An In-Depth Look at Factorial Design. This presentation will explore the principles and applications of factorial design in experimentation, providing valuable insights for researchers and practitioners.*



UNDERSTANDING FACTORIAL DESIGN

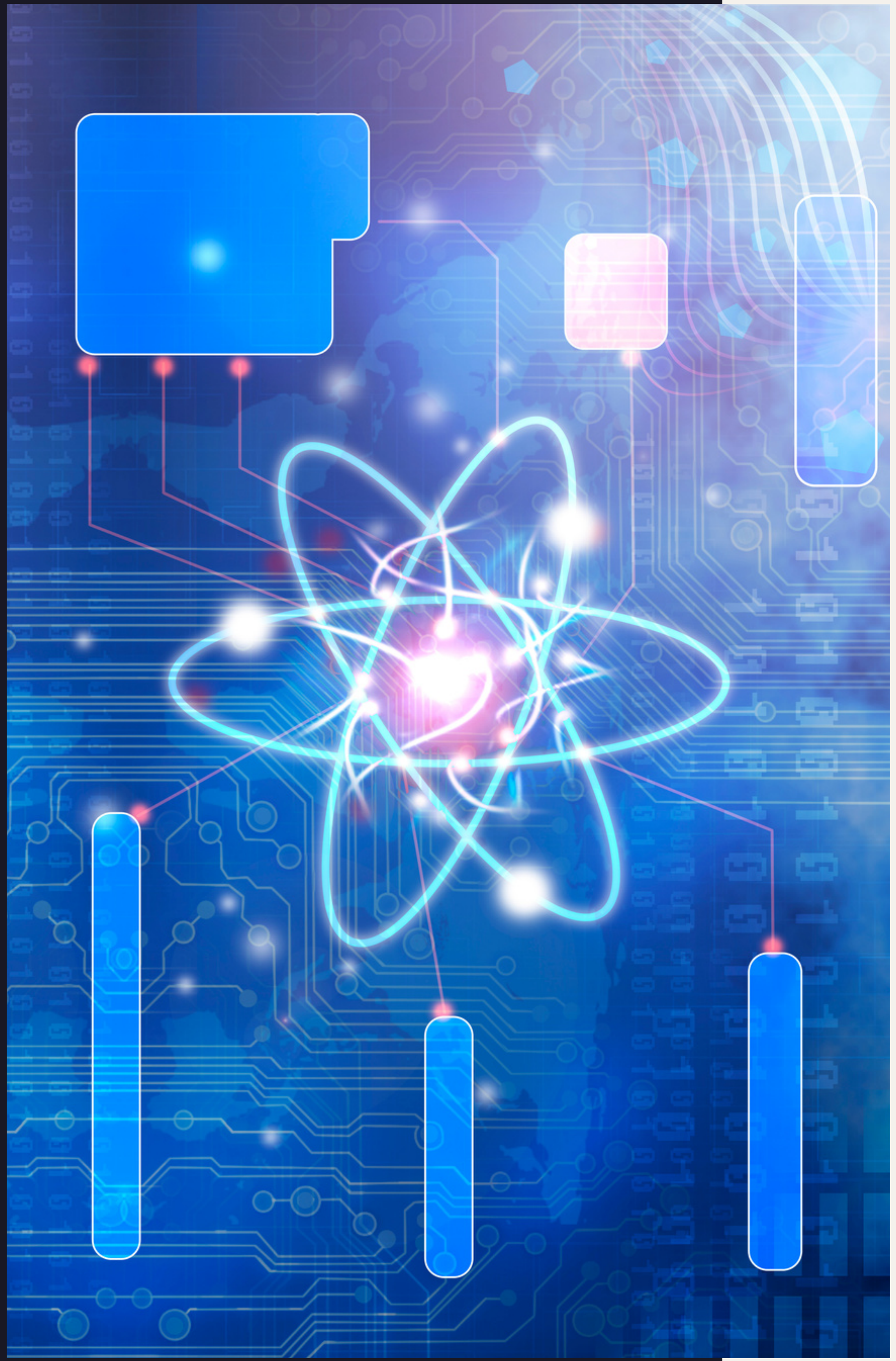
Factorial design is a powerful method for ***optimizing*** *experimentation by systematically varying multiple factors. It allows for the investigation of main* ***effects*** *and* ***interactions*** *between factors, providing a comprehensive understanding of the experimental system.*

ADVANTAGES OF FACTORIAL DESIGN

*Factorial design offers several advantages, including the ability to efficiently explore **multiple variables** and their interactions, leading to a more **comprehensive analysis** of the experimental system and **enhanced insight** into the underlying processes.*

PRACTICAL APPLICATIONS

*Factorial design is widely used in various fields such as **pharmaceutical research**, **manufacturing**, and **social sciences**. Its application enables researchers to gain a deeper understanding of complex systems and optimize experimental outcomes.*



KEY PRINCIPLES

*Understanding the **principles** of factorial design, including **randomization**, **replication**, and **blocking**, is essential for ensuring the validity and reliability of experimental results. These principles are fundamental for robust experimentation.*

DEFINATIONS

1. **Replication**

Replication means repetition of treatments under study. If treatment is repeated r times in the experimental units then it is said to be replicated r times with r replications.

2. **Randomization**

The method of sending treatments to the experimental units at random is called randomization. Treatments can be allotted by lottery or random numbers method. In randomization one can avoid bias.

DEFINATIONS

3. **Blocking**

Blocking is a technique used in design of experiments methodology to deal with the systematic differences to ensure that all the factors of interest and interactions between the factors can be assessed in the design.

RANDOMIZED BLOCK DESIGN (RBD)

Definition

A design in which the experimental material is divided into some homogeneous blocks and the treatments are assigned to the experimental units in each blocks by randomization method is called Randomized Block Design. In this design, we use three basic principles like replication, randomization and local control.

STEPS TO FOLLOW FOR CONSTRUCTION OF RBD

Main Steps

Calculate total G of all the observations in all samples

$$G = \Sigma X_{11} + \Sigma X_{21} + \Sigma X_{31} + \ldots + \Sigma X_{rs}$$

Correction factor (CF) $= \dfrac{(G)^2}{N}$

Calculate total sum of squares (TSS) $= \Sigma\Sigma X_{ij}^2 - \dfrac{(G)^2}{N}$

Sum of squares of treatments (SST) $= \left[\dfrac{X_1^2}{r_1} + \dfrac{X_2^2}{r_2} + \ldots + \dfrac{X_s^2}{r_s}\right] - \dfrac{(G)^2}{N}$

Sum of squares of blocks (SSB) $= \left[\dfrac{X_1^2}{s_1} + \dfrac{X_2^2}{s_2} + \ldots + \dfrac{X_r^2}{s_r}\right] - \dfrac{(G)^2}{N}$

Error sum of squares (SSE) = TSS − SST − SSE

Mean sum of squares of treatments (MST) $= \dfrac{\text{SST}}{s-1}$

Mean sum of squares of blocks (MSB) $= \dfrac{\text{SSB}}{r-1}$

Mean sum of squares of errors (MSE) $= \dfrac{\text{SSE}}{(s-1)(r-1)}$

Calculated F (treatment) value $= \dfrac{\text{MST}}{\text{MSE}}$

Table F (treatment) value $= F_{(s-1),\,(r-1)(s-1)}$

Calculated F (blocks) value $= \dfrac{\text{MSB}}{\text{MSE}}$

Table F (blocks) value $= F_{(r-1),\,(r-1)(s-1)}$

STEPS TO FOLLOW FOR CONSTRUCTION OF RBD

Analysis of Variance Table

Source	*Degrees of freedom*	*Sum of squares*	*Mean sum of squares*	*F-calculated value*	*F-table value* (α%)
Treatment	$s-1$	SST	MST	$F = \dfrac{MST}{MSE}$	$F_{(s-1),\,(r-1)(s-1)}$
Blocks	$r-1$	SSB	MSB		
Error	$(r-1)\,(s-1)$	SSE	MSE	$F = \dfrac{MSB}{MSE}$	$F_{(r-1),\,(r-1)(s-1)}$
Total	$rs-1$	TSS			

At α% level of significance if calculated value $\leq$ table value then we accept H_{01}.
If calculated value $>$ table value then we reject H_{01}.

At α% level of significance if calculated value $\leq$ table value then we accept H_{02}.
If calculated value $>$ table value then we reject H_{02}.

EXAMPLE

Example: Analyse the following design.

A	C	E	B	D
5	3	10	7	8
D	C	E	A	B
3	4	11	7	8
A	E	D	B	C
5	13	7	9	11
D	C	B	A	E
7	18	7	10	8

Solution:-

Let s denote no. of treatments i.e. $s = 5$

and r be the no. of blocks i.e. $r = 4$

Blocks	Treatments					Total B_j
	A	B	C	D	E	
1	5	7	3	8	10	33
2	7	8	4	3	11	33
3	5	9	11	7	13	45
4	10	7	18	7	8	50
Total T_i	27	31	36	25	42	161

Here, $\boxed{s = 5, \ r = 4}$

Grand Total $G = 161$

1) Correction Factor, $(CF) = \frac{(G)^2}{r \times s} = \frac{(161)^2}{20} = \boxed{1296.05}$

2) Total SUM of SQUARES

$$(TSS) = (5)^2 + (7)^2 + \cdots (7)^2 + (8)^2 - C.F$$

$$= 1537 - 1296.05 = \boxed{240.95}$$

3) SUM of squares of treatments

$$(SSB) = \frac{\left[(27)^2 + (31)^2 + (36)^2 + (25)^2 + (42)^2\right]}{4} - CF$$

$$= 1343.75 - 1296.05 = \boxed{47.7}$$

4) Sum of square of blocks (SSB) =

$$\frac{(33)^2 + (33)^2 + (45)^2 + (50)^2}{5} - CF$$

$$= 1340.6 - 1296.05 = \boxed{44.55}$$

5) Error Sum of squares

$$(SSE) = TSS - SST - SSB$$

$$= 240.95 - 47.7 - 44.55$$

$$= \boxed{148.7}$$

EXAMPLE

6) Mean Sum of square of treatments (MST) $= \frac{SST}{5-1}$

$$= \frac{47.7}{4} = \boxed{11.925}$$

7) Mean sum of squares of Blocks (MSB) $= \frac{SSB}{4-1}$

$$= \frac{44.55}{3} = \boxed{14.85}$$

8) Mean sum of square of Errors (MSE) $= \frac{SSE}{(5-1)(4-1)}$

$$= \frac{148.7}{12} = \boxed{12.391667}$$

9) Calculated F value (Treatments)

$$= \frac{11.925}{12.391667} = \boxed{0.963340}$$

10) Calculated F value (Blocks) $= \frac{14.85}{12.391667} = \boxed{1.198386}$

11) Table F (treatment) value $= F_{(5-1),(5-1)(4-1)}$

$$= F_{4,12} = \boxed{3.36}$$

12) Table F (Blocks) value $= F_{(4-1),(5-1)(4-1)}$

$$= F_{3,12} = \boxed{3.59}$$

Analysis of Variance Table

Source	Degrees of freedom	Sum of Squares	Mean sum of squares	F-Calculated	F-table
Treatments	4	47.7	11.925	0.96234	3.36
Blocks	3	44.55	14.85	1.198386	3.59
Errors	12	148.7	12.391667		
Total	19	240.95			

As, F(calculated value) < F (table value)

$0.96234 < 3.36$
& $1.198386 < 3.59$

[We conclude that all the treatments and the blocks effects are equal.]

CONCLUSION

*In conclusion, **factorial design** provides a powerful framework for **optimizing experimentation** and gaining deeper insights into complex systems. By embracing its principles and applications, researchers can drive **innovation** and enhance the **impact** of their experimental endeavors.*

Thank you!

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