CS 480

Introduction to Artificial Intelligence

September 7th, 2021

Announcements / Reminders

- **Contribute to the discussion on Blackboard, please**
- **Please follow the Week 02 To Do List instructions**

Plan for Today

- **Problem Solving: Searching**

Designing the Searching Problem

Analyze and define the Problem / Task	Model and build the State Space	Select searching algorithm	Search

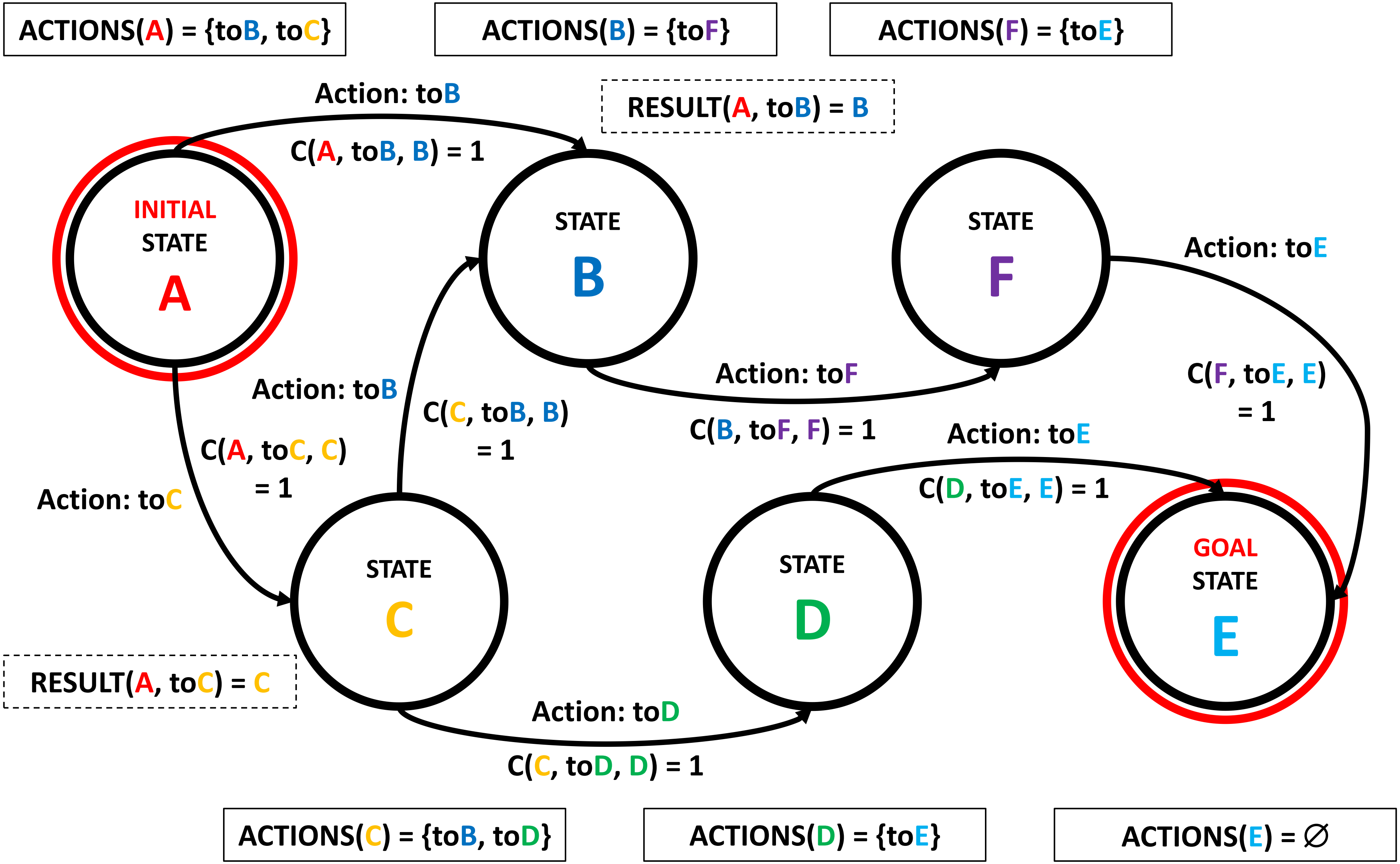
Defining Search Problem

- **Define a set of possible states: State Space**
- **Specify Initial State**
- **Specify Goal State(s) (there can be multiple)**
- **Define a FINITE set of possible Actions for EACH state in the State Space**
- **Come up with a Transition Model which describes what each action does**
- **Specify the Action Cost Function: a function that gives the cost of applying action *a* in state *s***

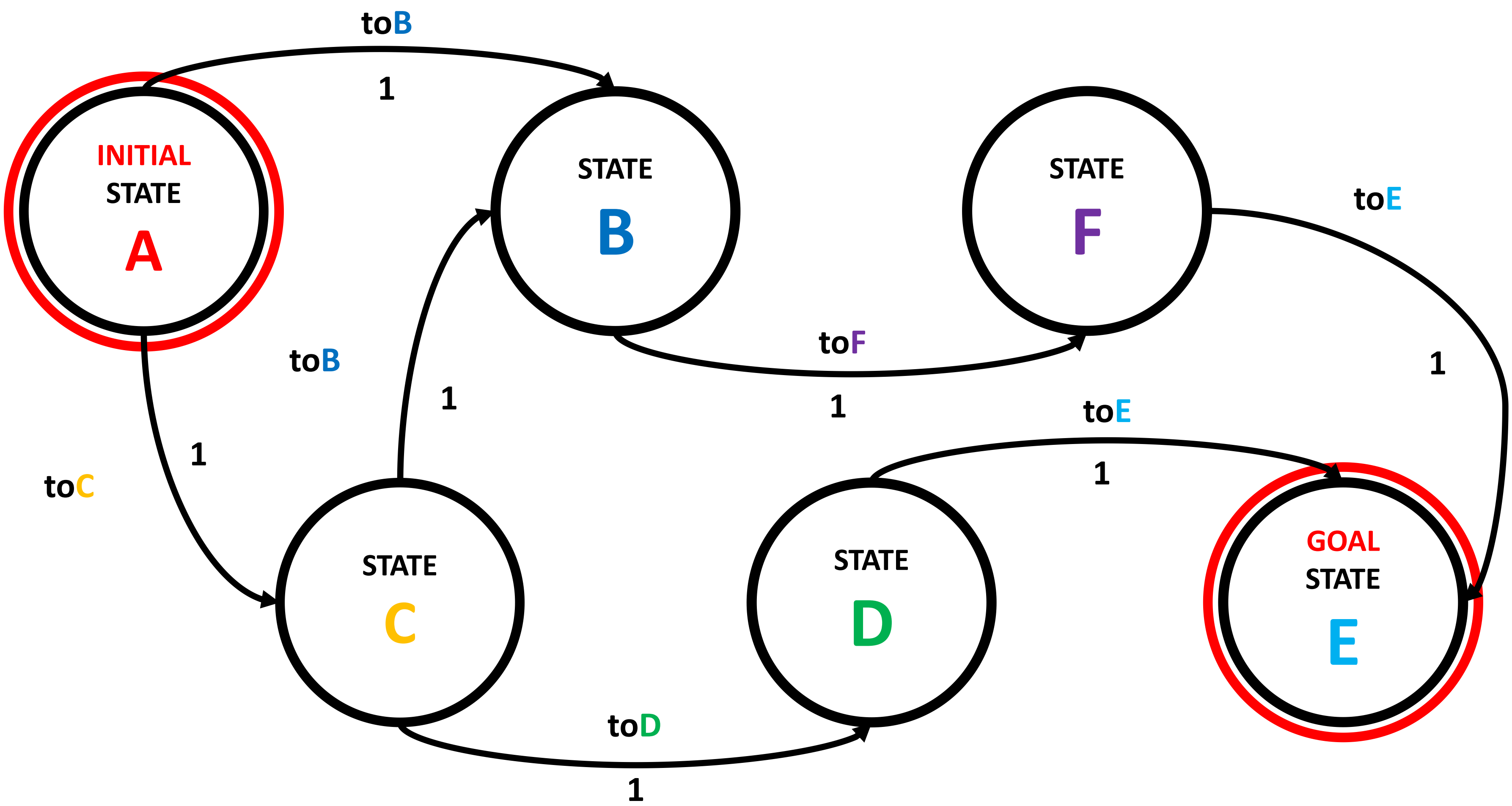
Designing the Searching Problem

Analyze and define the Problem / Task	Model and buid the State Space	Select searching algorithm	Search

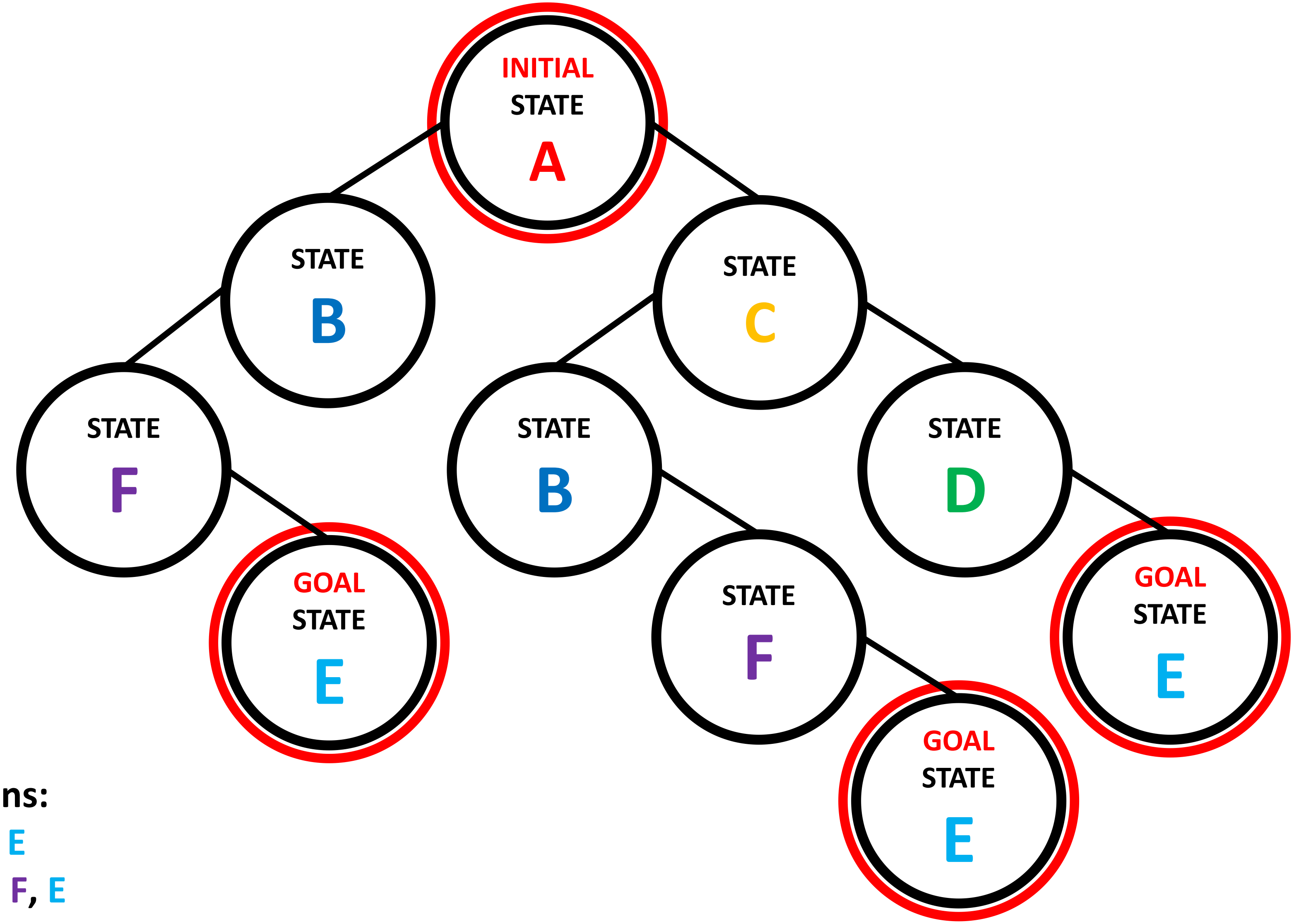
State Space Model: A Graph



State Space Model: A Graph



Searching State Space: Search Tree



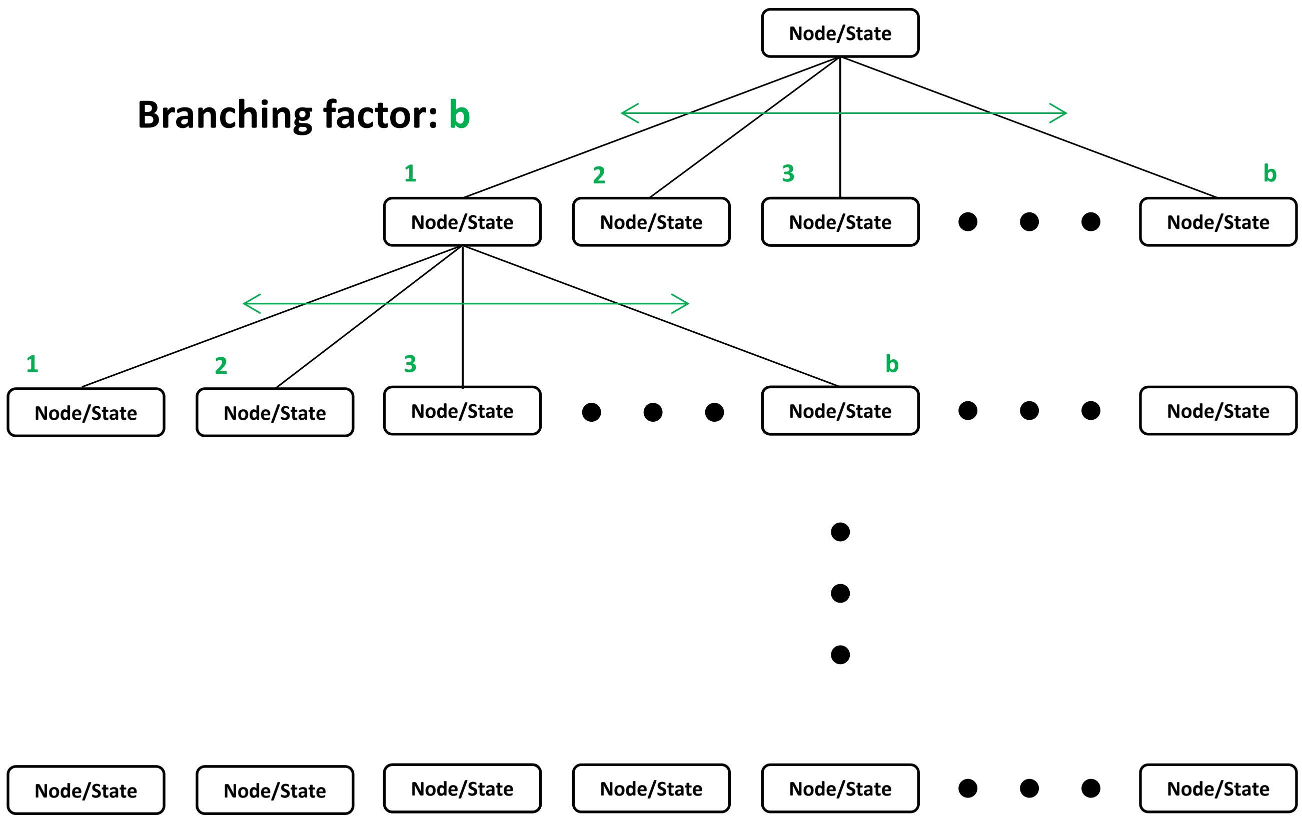
Solutions:

A, B, F, E

A, C, B, F, E

A, C, D, E

Search Tree Challenges: Size



Depth: 0 | $N_0 = 1$

Depth: 1 | $N_1 = b$

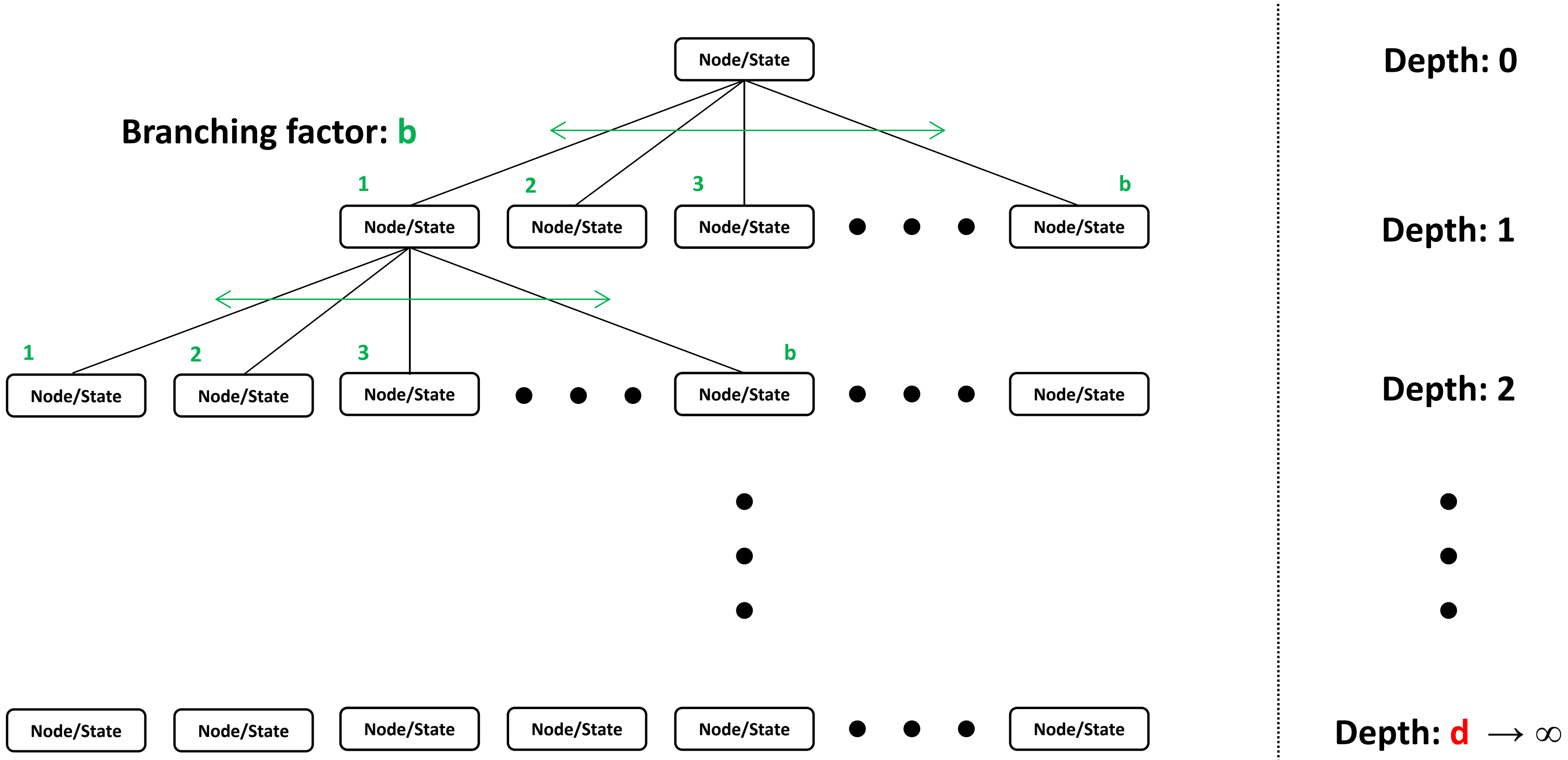
Depth: 2 | $N_2 = b^2$

Depth: d | $N_d = b^d$

Total number of nodes / states: $1 + b + b^2 + b^3 + \ldots + b^d \rightarrow O(b^d)$

Quickly becomes unmanageable and impossible to search with brute force!

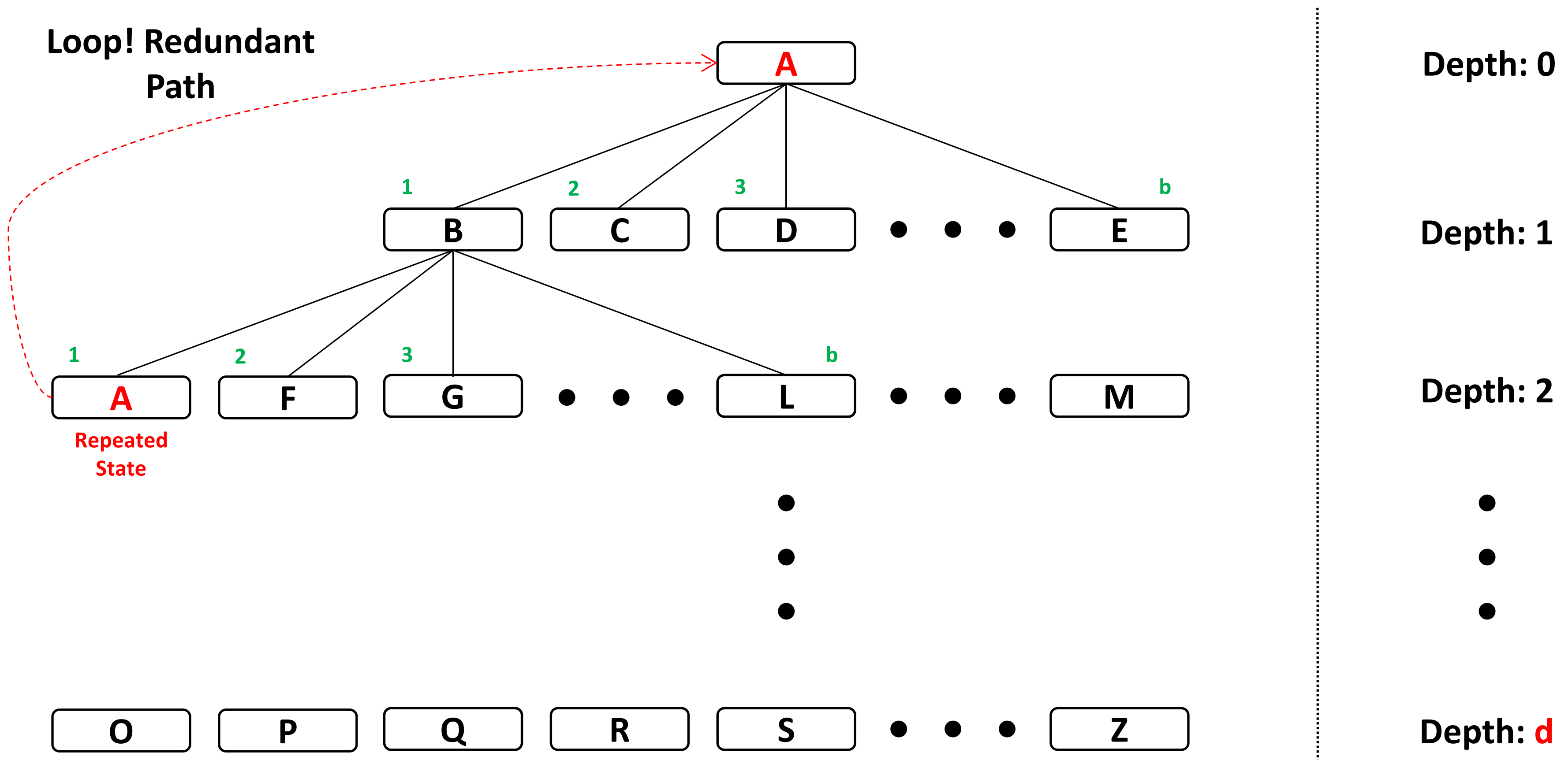
Search Tree Challenges: Infiniteness



Unmanageable and impossible to search with brute force!

Memory and time use grows quickly!

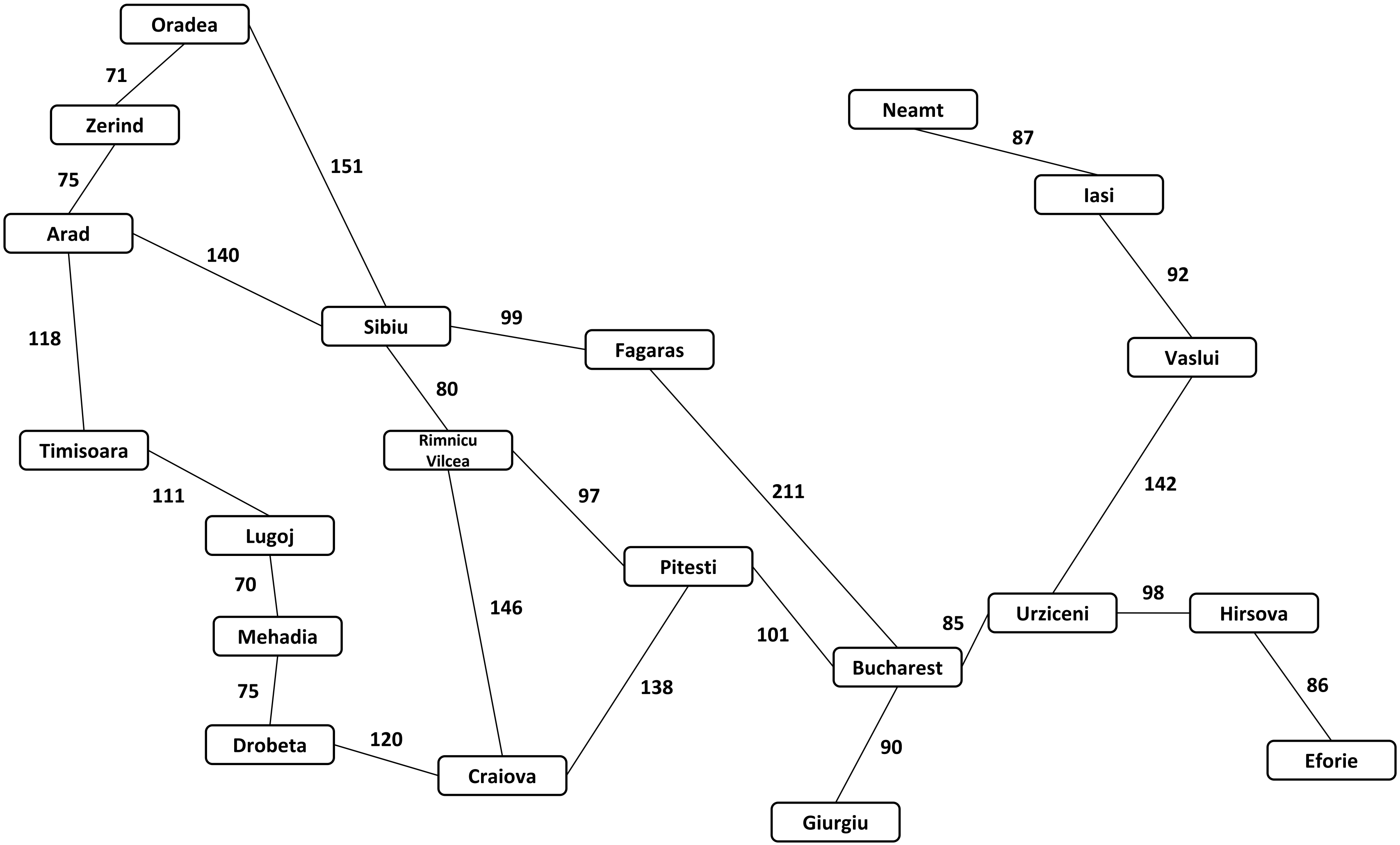
Search Tree Challenges: Loops



This would lead to an infinite state sequence repetition if not handled!

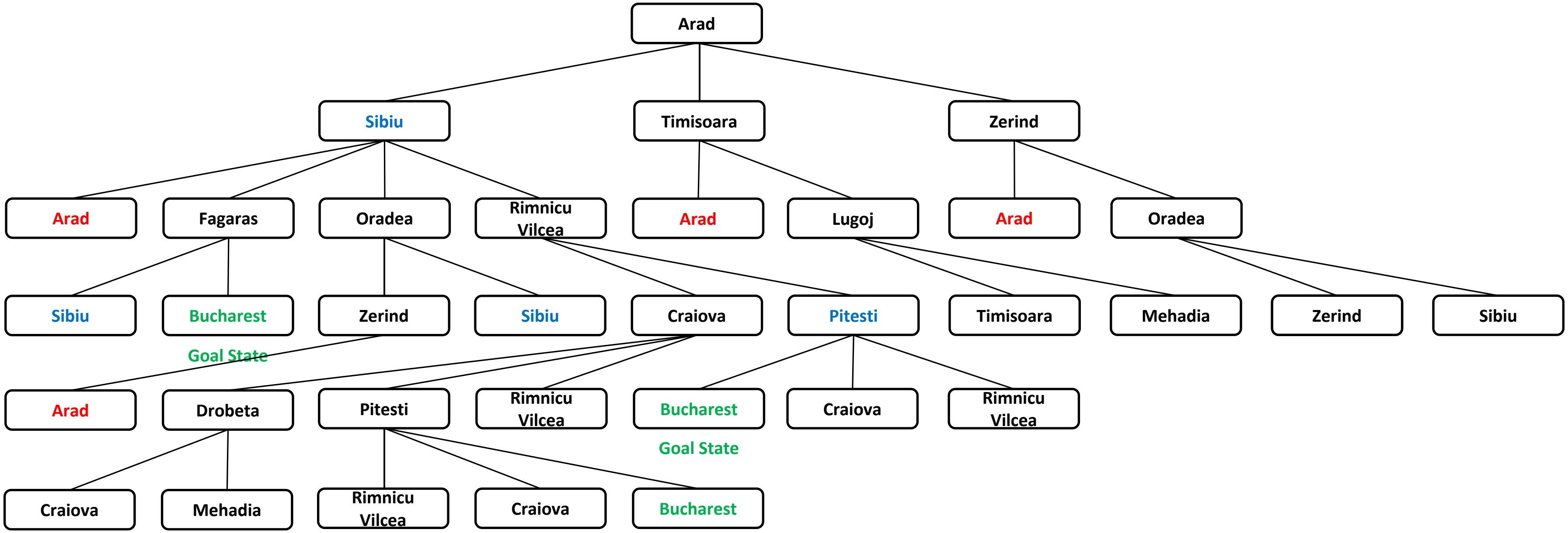
Memory and time use grows quickly!

Sample Problem: Dracula's Roadtrip



Problem: Get from Arad to Bucharest efficiently (for example: quickly or cheaply).

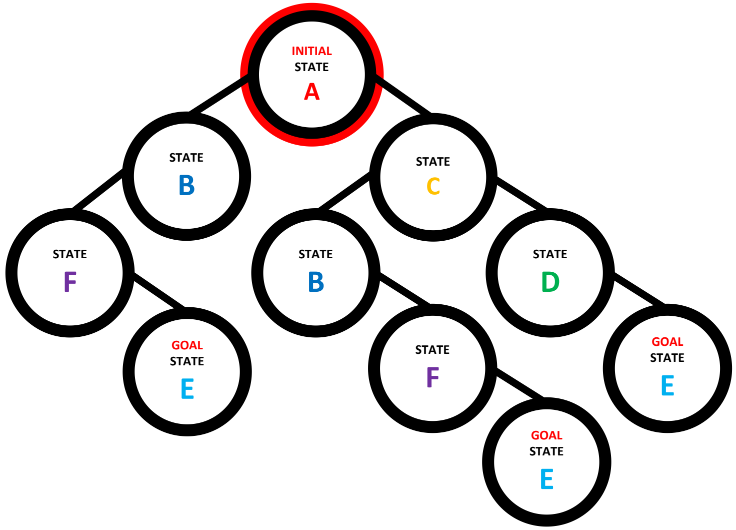
Dracula's Roadtrip as a Tree



INCOMPLETE! I need to redraw it in smarter way

Search Tree: Implementations

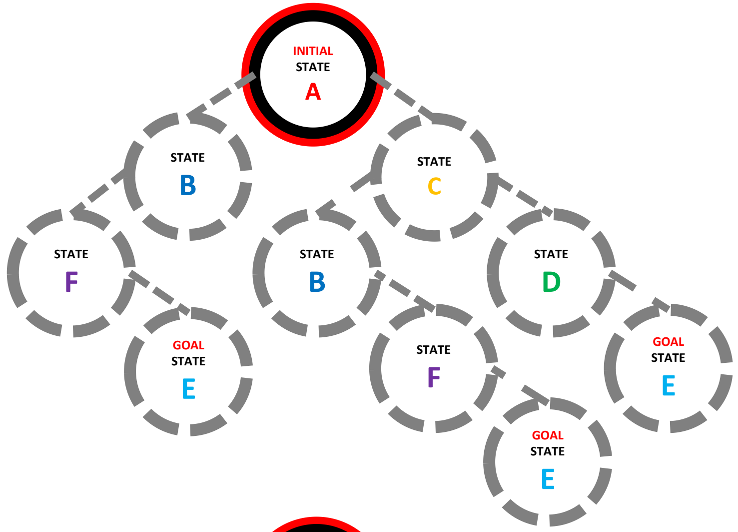
Build entire search tree

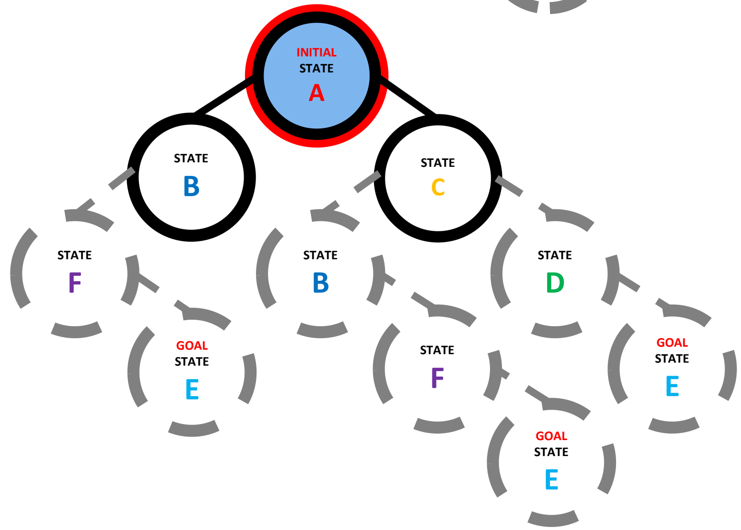


Challenges:

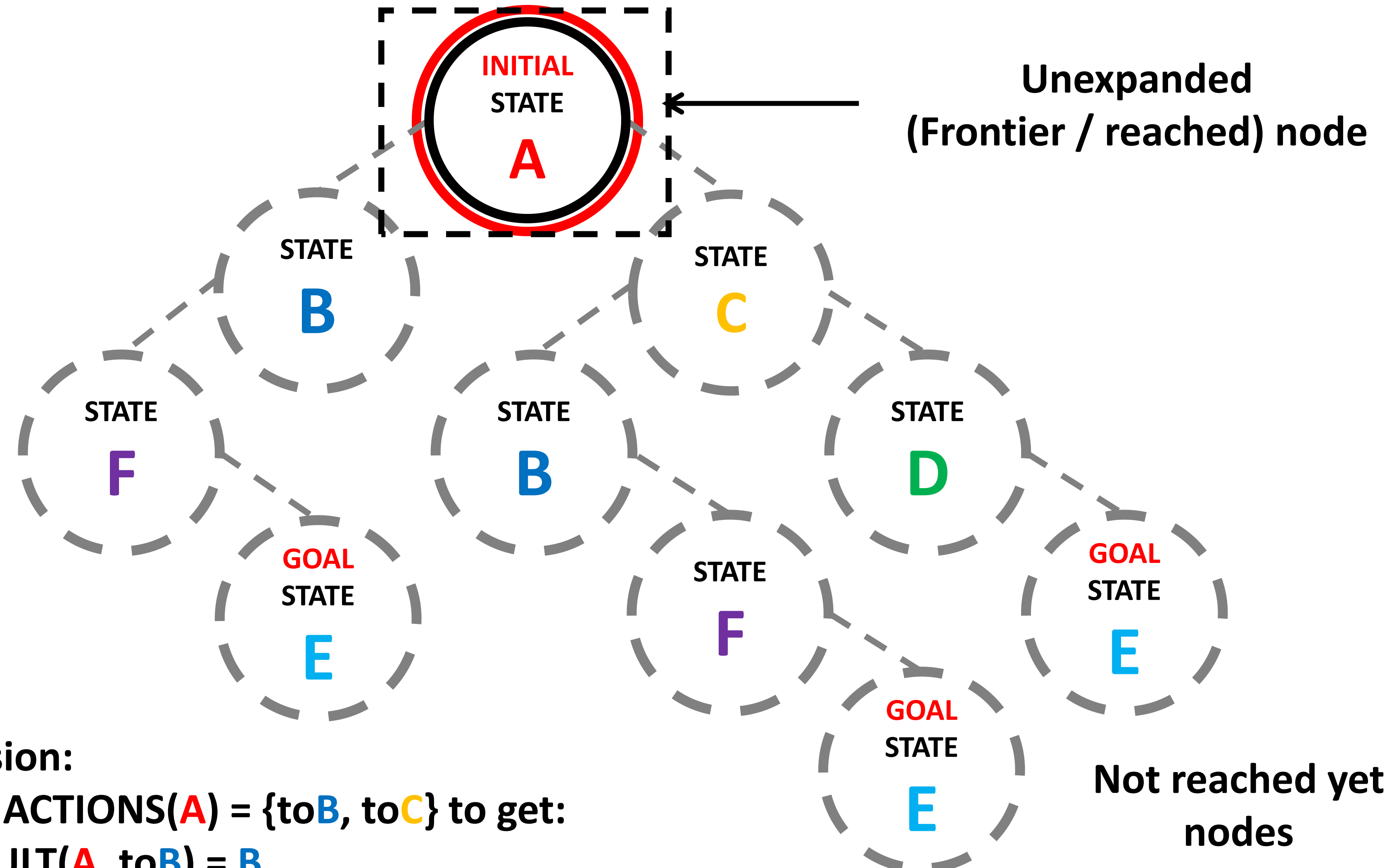
- memory requirements
- impossible for infinite number of states

Expand/generate nodes as you go





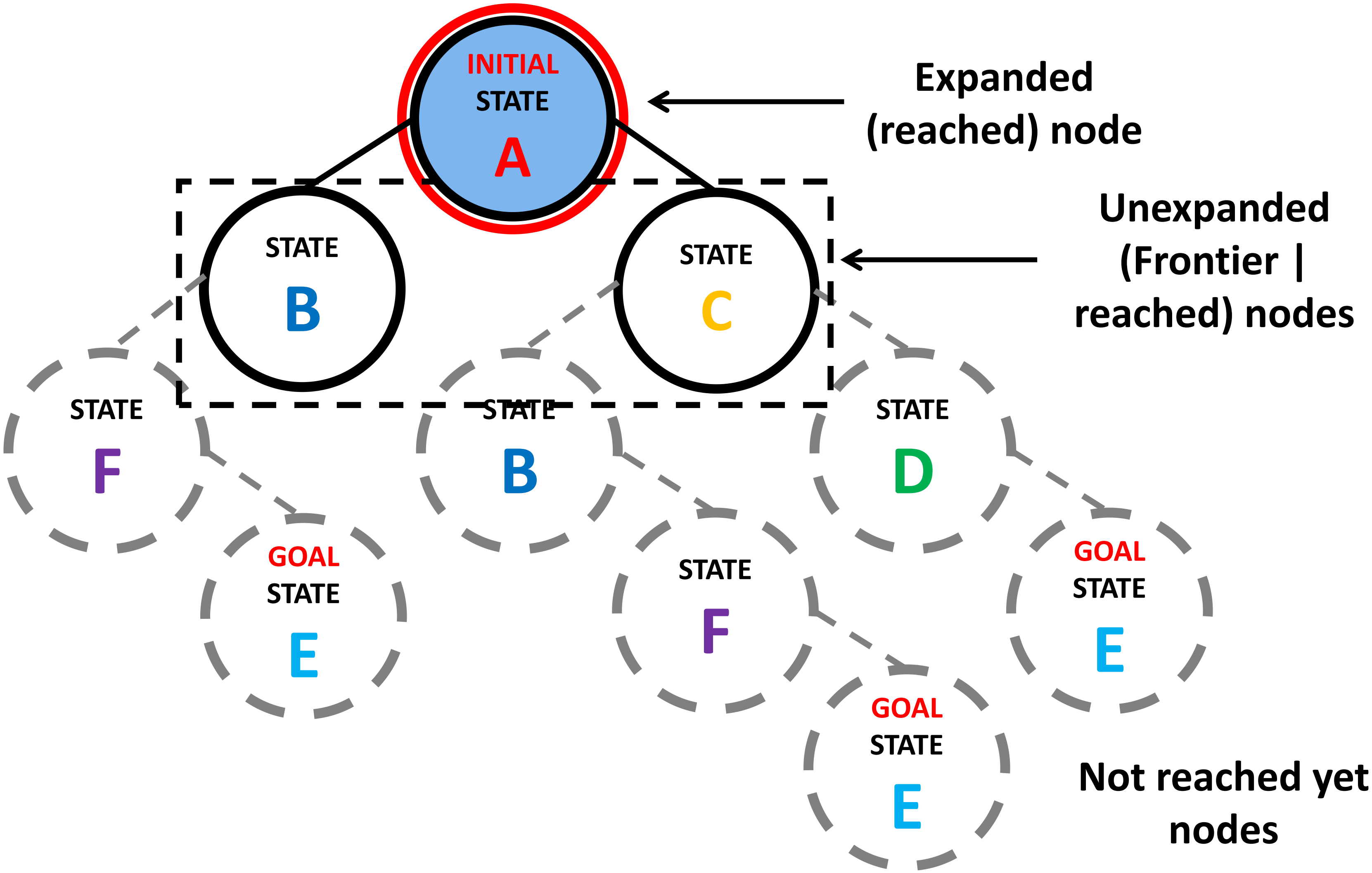
Search Tree: Node Expansion



Expansion:

- Use ACTIONS(A) = {toB, toC} to get:
- RESULT(A, toB) = B
- RESULT(A, toC) = C

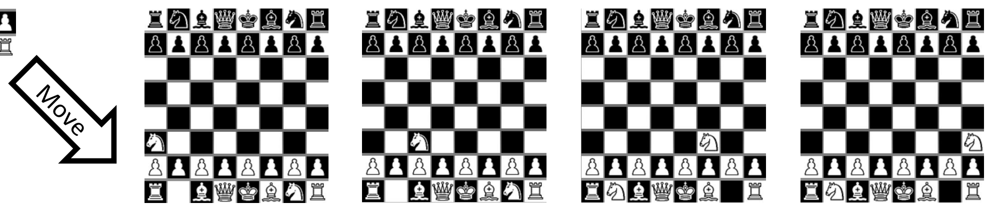
Search Tree: Node Expansion



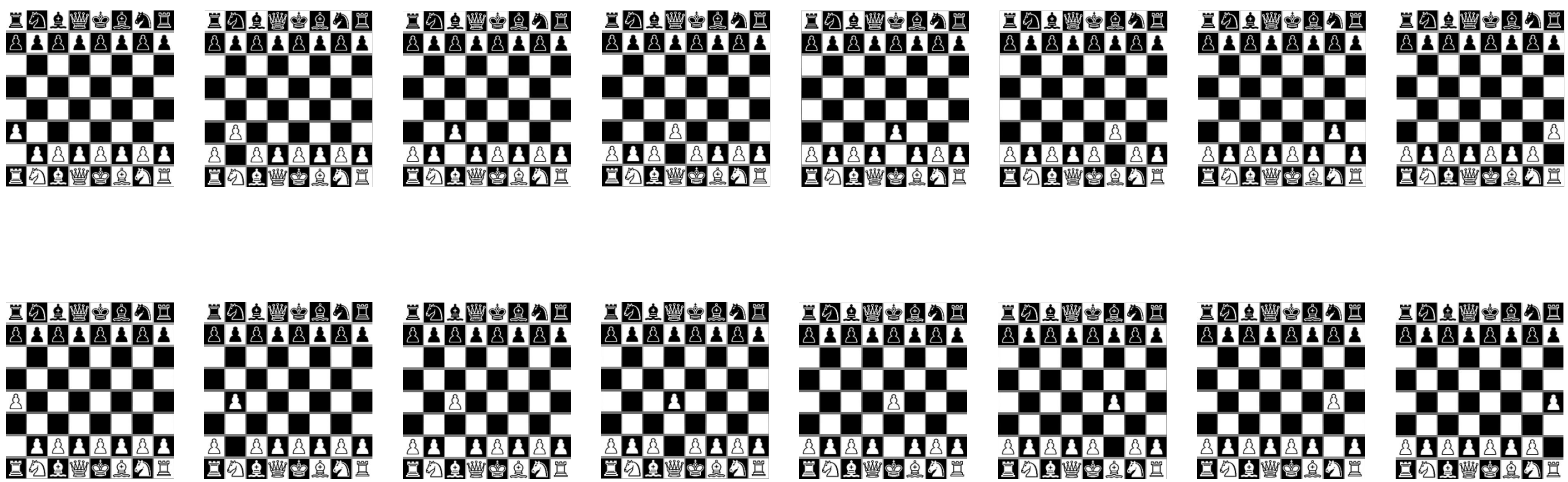
Chess: State Node Expansion

Initial State

Use game rules to generate subsequent possible game tree states / nodes!



20 Possible legal first moves:
16 pawn moves
4 knight moves



Designing the Searching Problem

Analyze and define the Problem / Task	Model and buid the State Space	Select searching algorithm	Search

Measuring Searching Performanc

Search algorithms can be evaluated in four ways:

- **Completeness: Is the algorithm guaranteed to find a solution when there is one, and to correctly report failure when there is not?**
- **Cost optimality: Does it find a solution with the lowest path cost of all solutions?**
- **Time complexity: How long does it take to find a solution? (in seconds, actions, states, etc.)**
- **Space complexity: How much memory is needed to perform the search?**

Selected Searching Algorithms

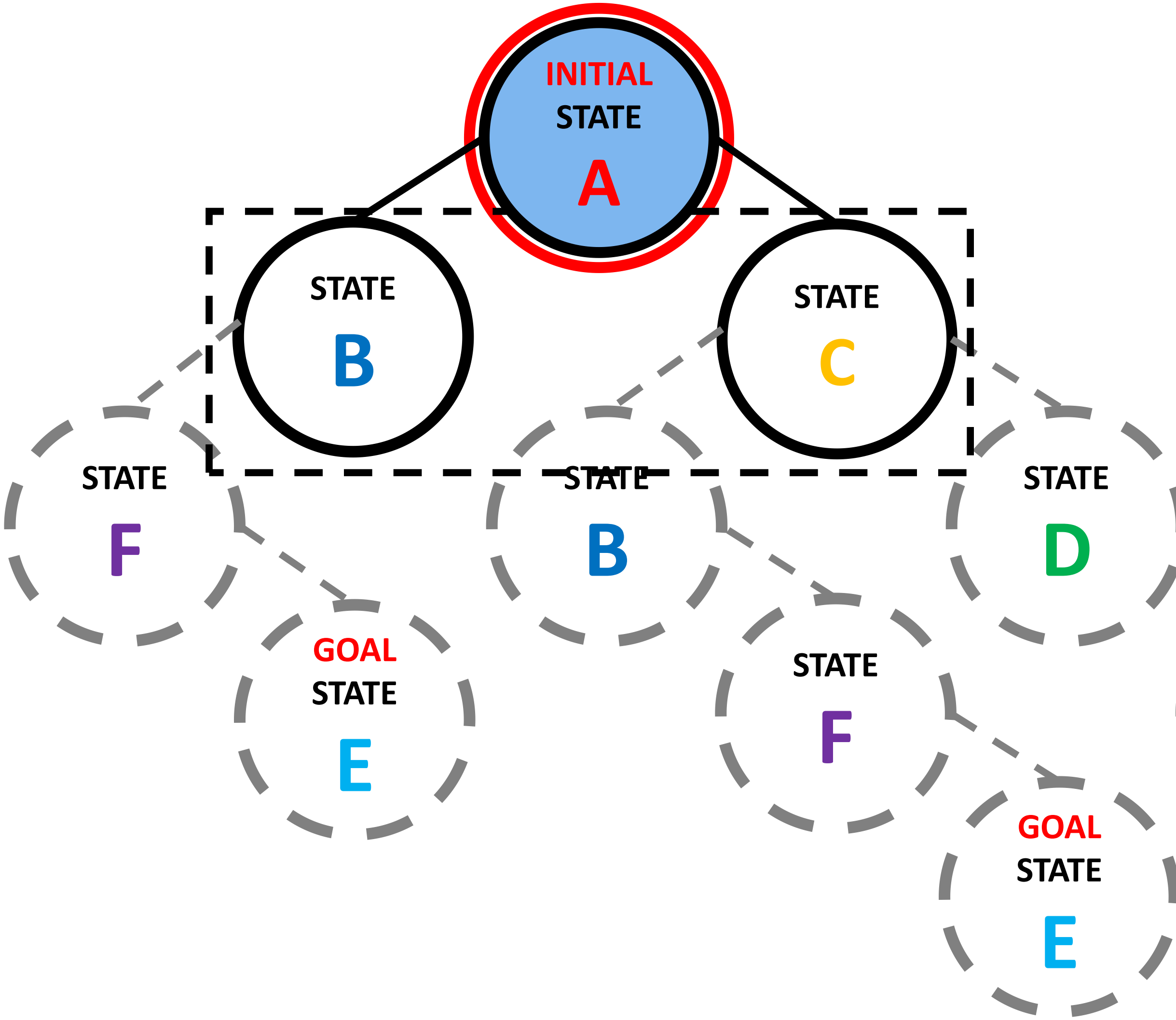
- Searching Algorithms
  - **Uninformed** Search
    - Breadth First Search
    - Depth First Search
    - Brute Force Search
    - Uniform Cost Search
  - **Informed** Search
    - Best First Search
    - Branch and Bound
    - A* Algorithm
    - Hill Climbing Algorithm
    - Constraint Satisfaction Search
    - α - β Pruning
    - Min-Max Search

Uninformed search: an agent <u>has no estimate</u> how far it is from the goal
Informed search: an agent <u>can estimate</u> how far it is from the goal

Uninformed Searching

- Breadth First Search (BFS):
  - Will find a solution with a minimal number of actions
  - Large memory requirement
  - Only relatively small problem instances are tractable
- Depth First Search:
  - May NOT find a solution with a minimal number of actions
  - Requires less memory than BFS (for tree search
  - Backtracking (one child / successor generated at a time)
- Brute Force Search: depends on the approach -> bad
- Uniform Cost Search: minimize solution / path cost

Expansion: Which Node to Expand?



Not reached yet nodes

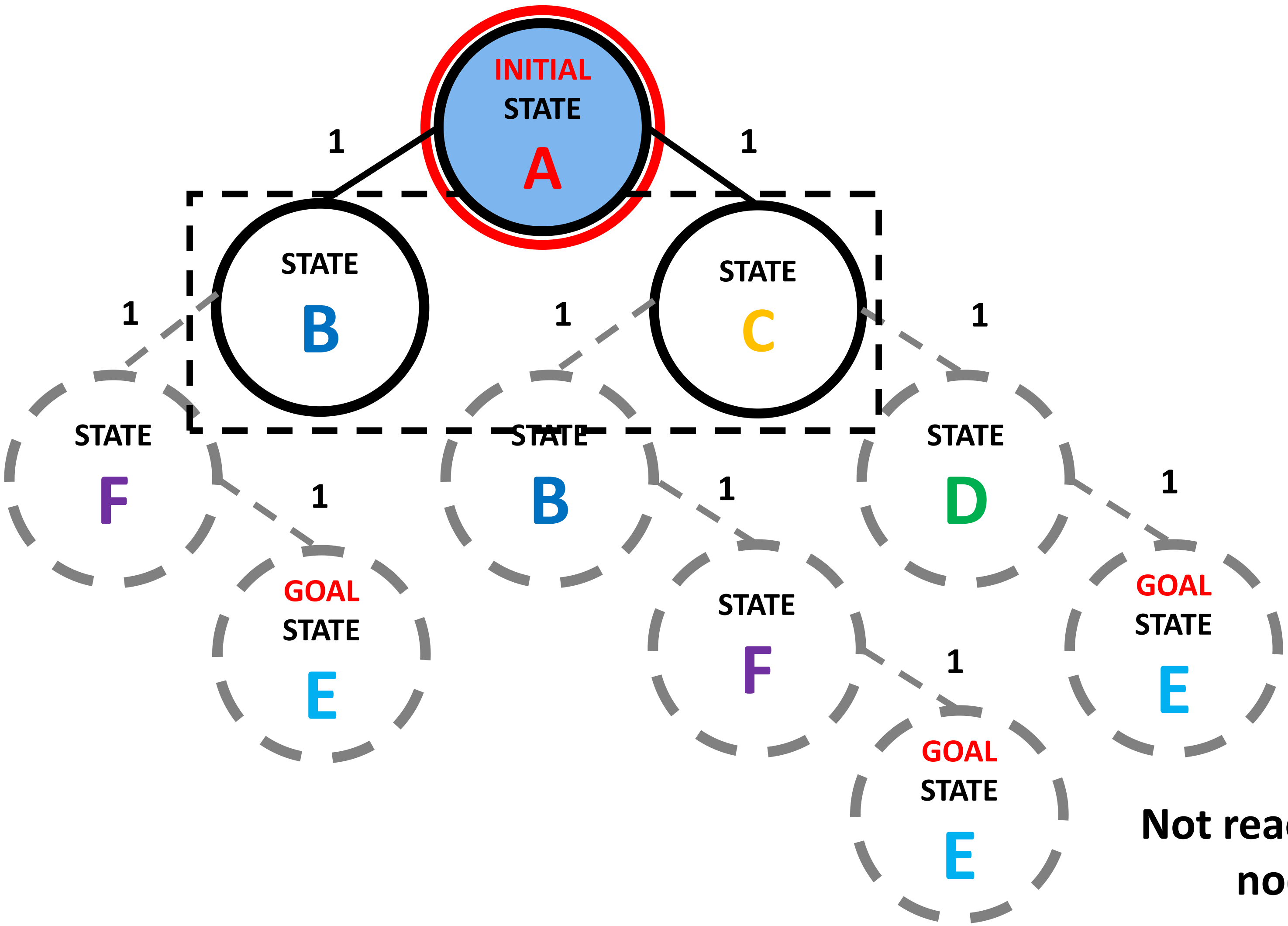
Evaluation function

Calculate / obtain:

f(n) = f(State n)
f(n) = f(relevant information about State n)

A state n with minimum f(n) should be chosen for expansion
What about ties?

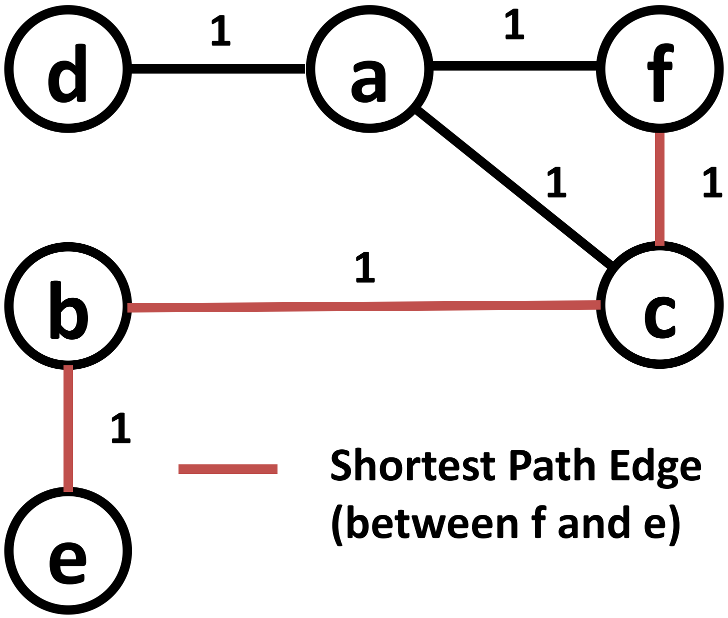
Search Tree: Uniform Action Cost



Not reached yet nodes

Uniform Cost Search | Dijkstra's Algo

Weighted Graph G



Popular algorithms:

- **Dijkstra's algorithm**

Shortest Path Problem

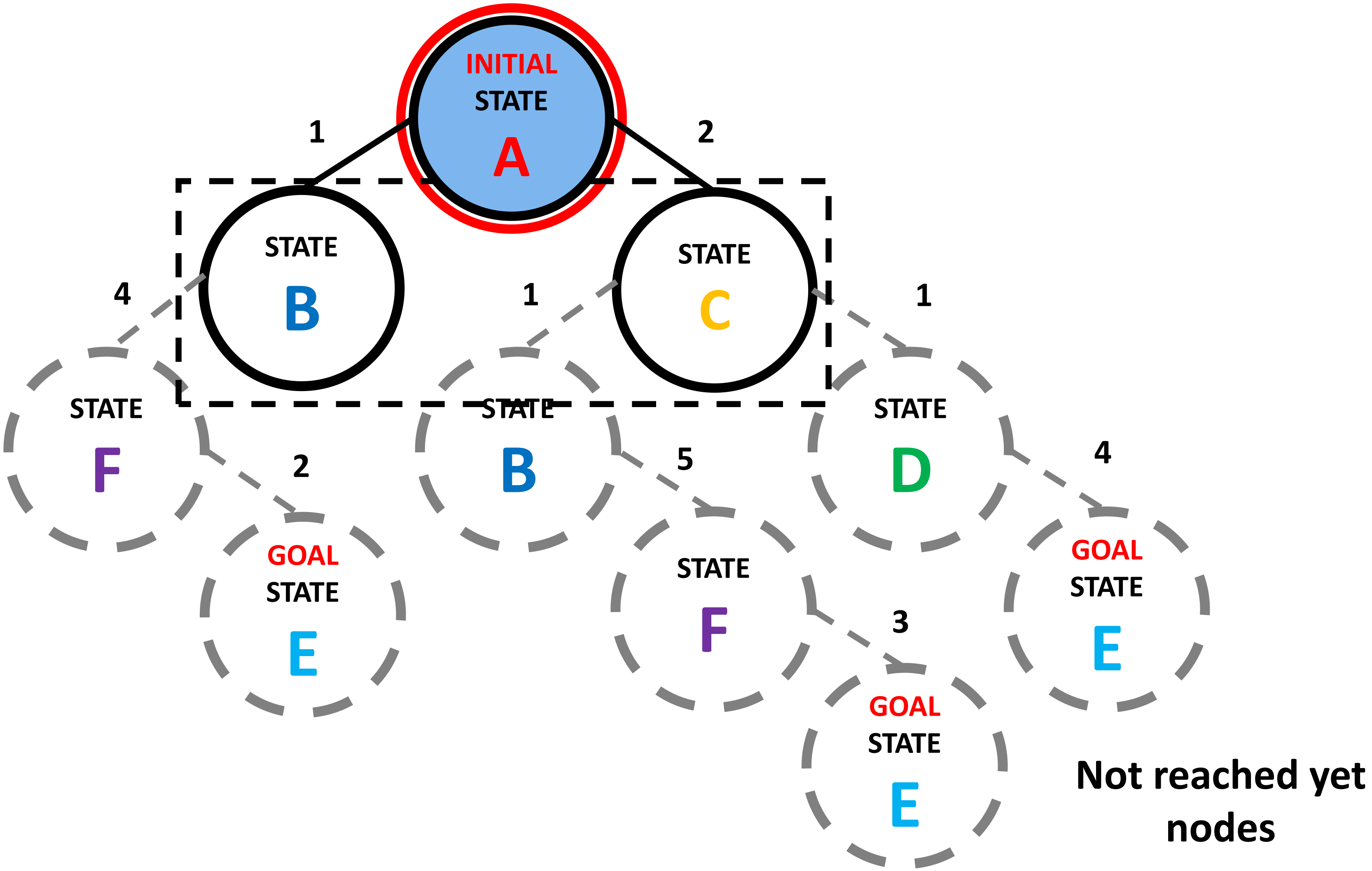
Shortest path problem:
Given a weighted graph G(V, E, w) and two vertices a, b in V, find the shortest path between vertices a and b (**all edge weights are equal**).

BFS and UCS: Pseudocode

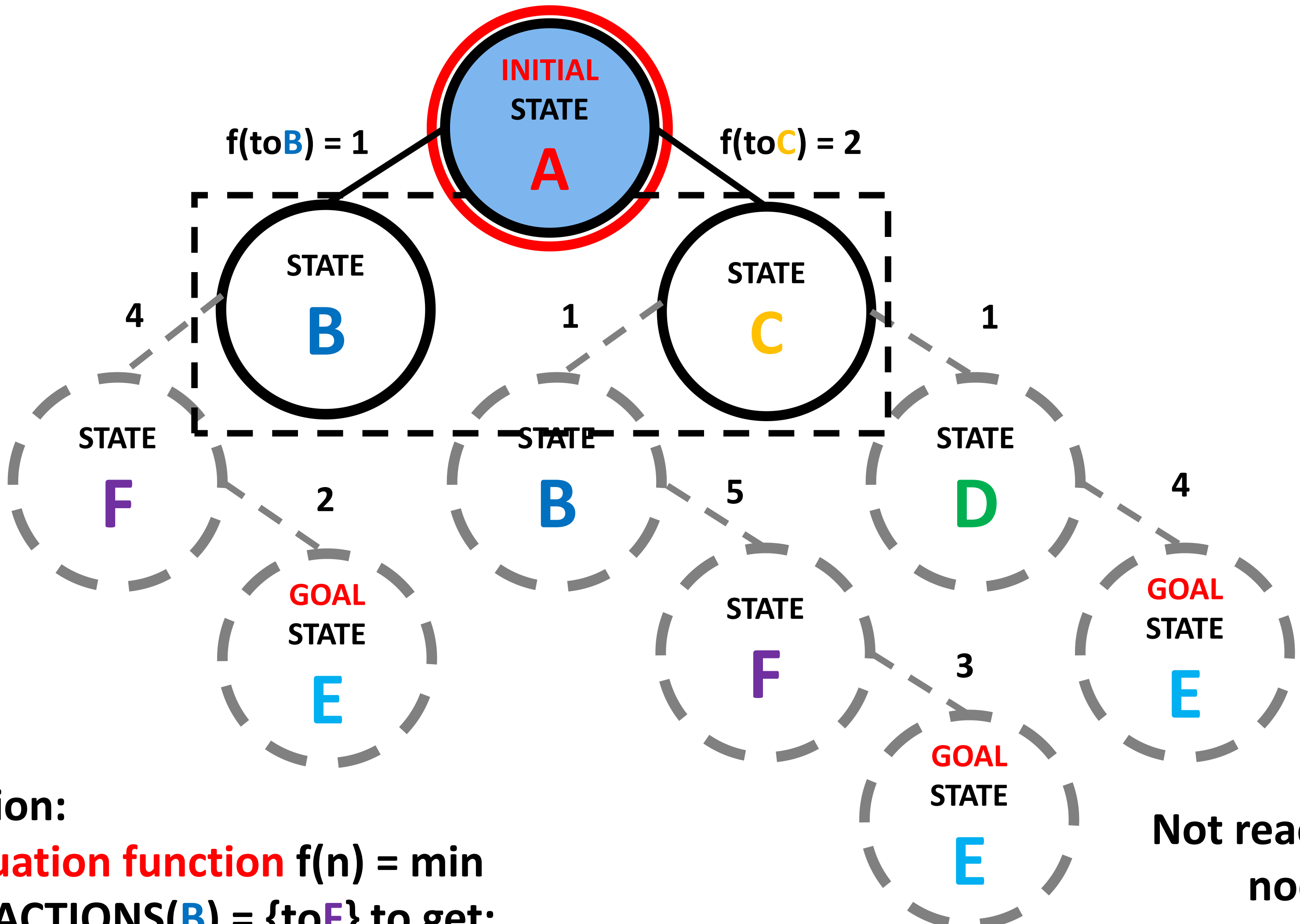
```
function BREADTH-FIRST-SEARCH(problem) returns a solution node or failure
  node ← NODE(problem.INITIAL)
  if problem.IS-GOAL(node.STATE) then return node
  frontier ← a FIFO queue, with node as an element
  reached ← {problem.INITIAL}
  while not IS-EMPTY(frontier) do
    node ← POP(frontier)
    for each child in EXPAND(problem, node) do
      s ← child.STATE
      if problem.IS-GOAL(s) then return child
      if s is not in reached then
        add s to reached
        add child to frontier
  return failure

function UNIFORM-COST-SEARCH(problem) returns a solution node, or failure
  return BEST-FIRST-SEARCH(problem, PATH-COST)
```

Search Tree: Variable Action Cost



Search Tree: Variable Action Cost



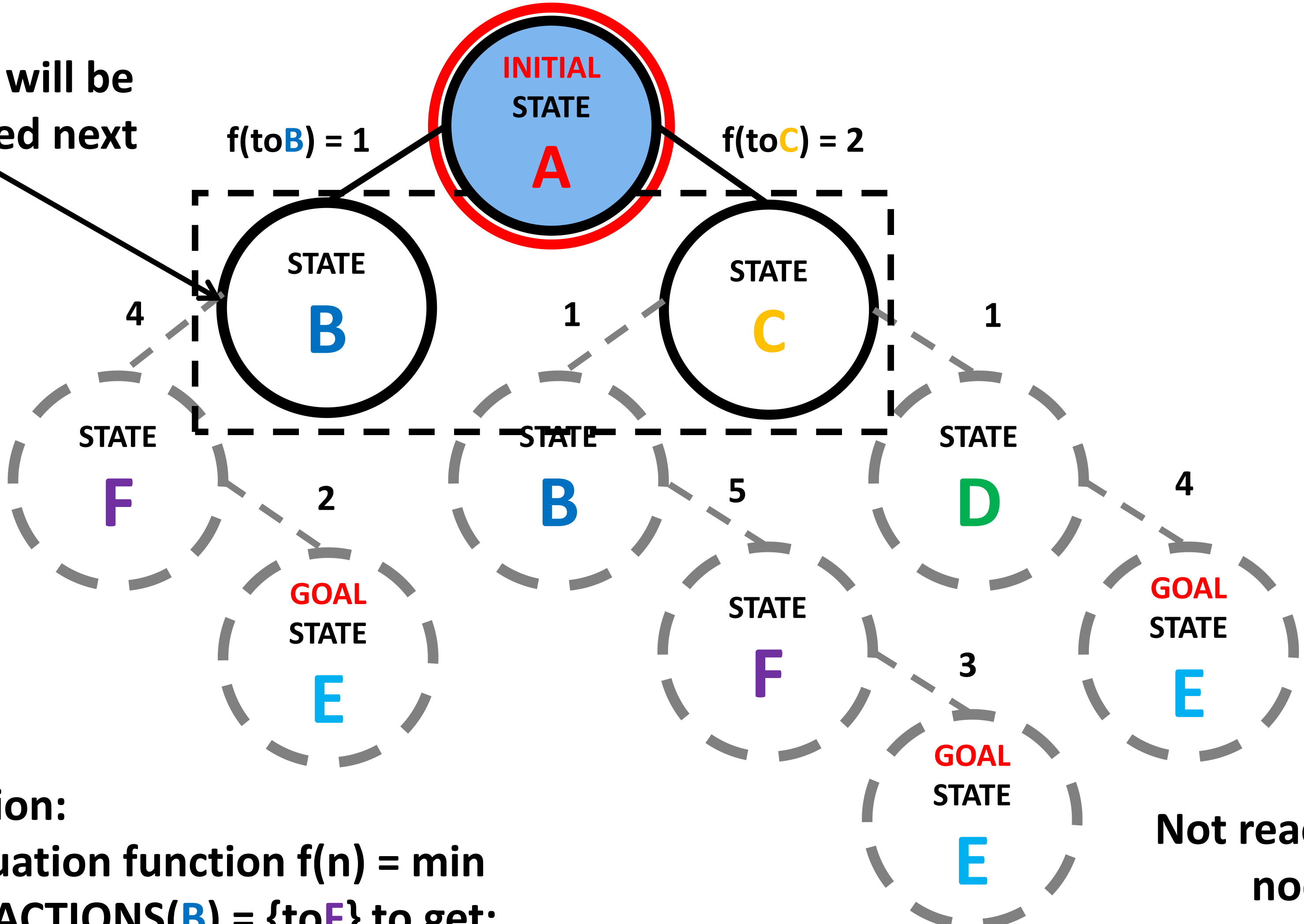
Expansion:

- **Evaluation function f(n) = min**
- **Use ACTIONS(B) = {toF} to get:**
- **RESULT(B, toF) = F**

Not reached yet nodes

Search Tree: Best-First Search

Node B will be expanded next



Not reached yet nodes

Expansion:

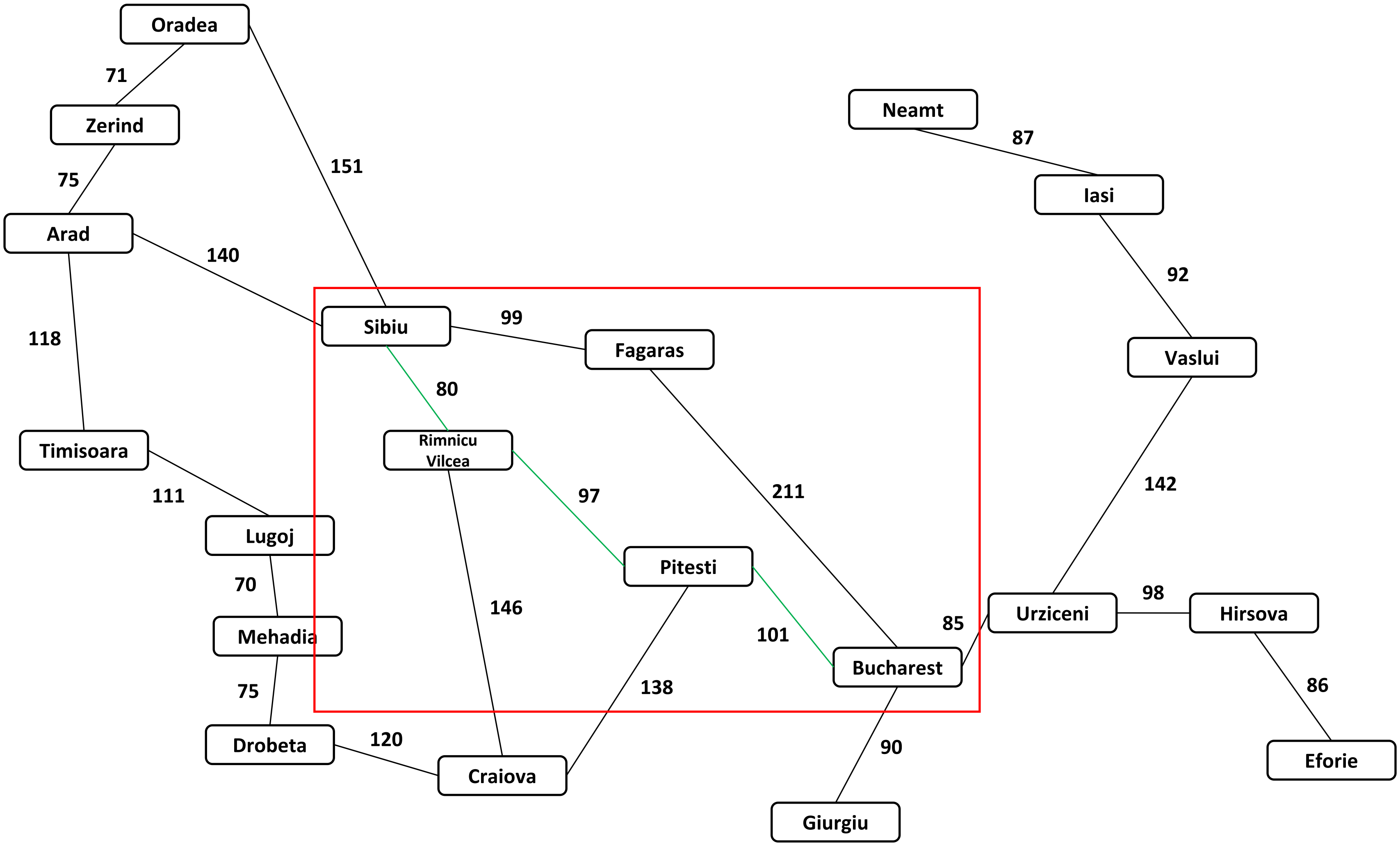
- Evaluation function f(n) = min
- Use ACTIONS(B) = {toF} to get:
- RESULT(B, toF) = F

Best-First Search: Pseudocode

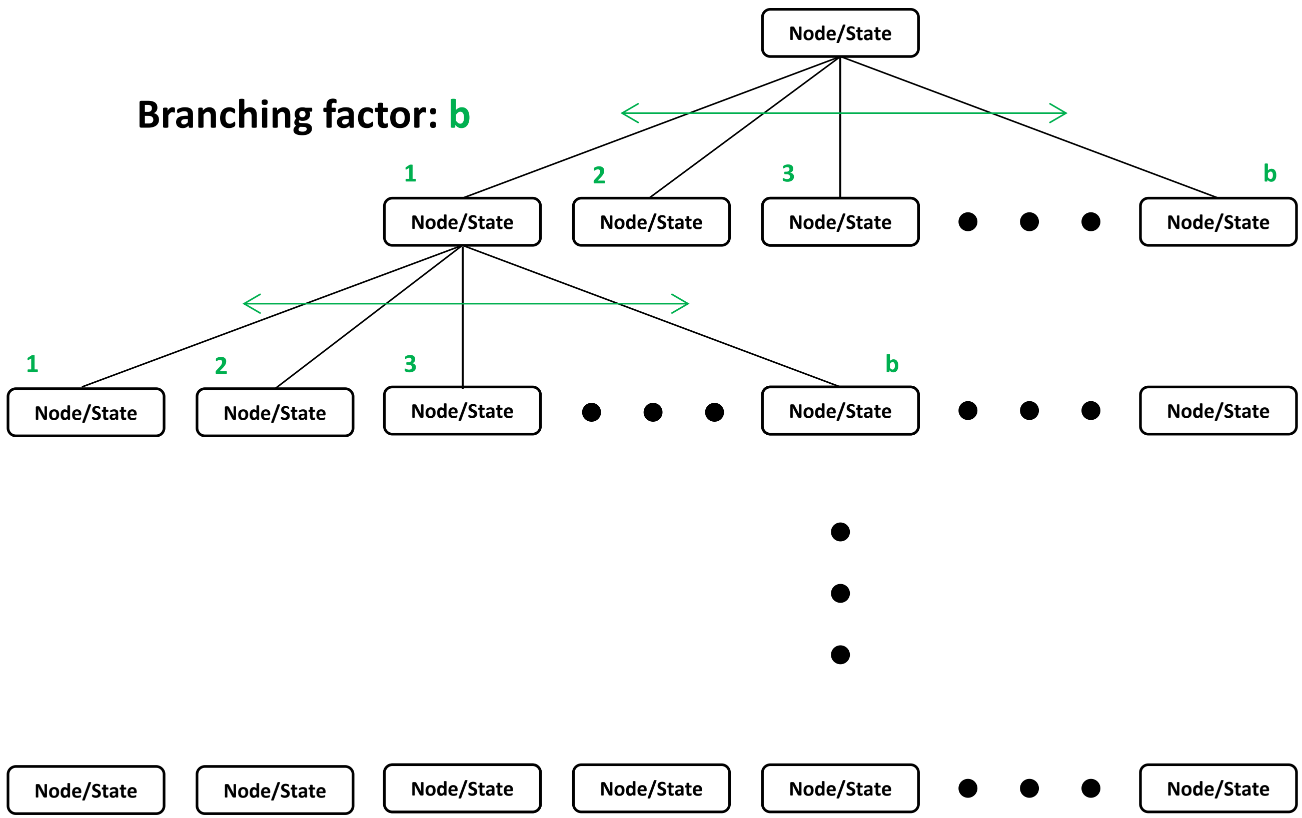
function BEST-FIRST-SEARCH(*problem*, *f*) **returns** a solution node or *failure*
  node ← NODE(STATE=*problem*.INITIAL)
  frontier ← a priority queue ordered by *f*, with *node* as an element
  reached ← a lookup table, with one entry with key *problem*.INITIAL and value *node*
  while not IS-EMPTY(*frontier*) **do**
    node ← POP(*frontier*)
    if *problem*.IS-GOAL(*node*.STATE) **then return** *node*
    for each *child* **in** EXPAND(*problem*, *node*) **do**
      s ← *child*.STATE
      if *s* is not in *reached* **or** *child*.PATH-COST < *reached*[*s*].PATH-COST **then**
        reached[*s*] ← *child*
        add *child* to *frontier*
  return *failure*

function EXPAND(*problem*, *node*) **yields** nodes
  s ← *node*.STATE
  for each *action* **in** *problem*.ACTIONS(*s*) **do**
    s' ← *problem*.RESULT(*s*, *action*)
    cost ← *node*.PATH-COST + *problem*.ACTION-COST(*s*, *action*, s')
    yield NODE(STATE=s', PARENT=*node*, ACTION=*action*, PATH-COST=*cost*)

Best First Search: Issue



Let’s Go Back to Depth First Search



Depth: 0 | $N_0 = 1$

Depth: 1 | $N_1 = b$

Depth: 2 | $N_2 = b^2$

Depth: d | $N_d = b^d$

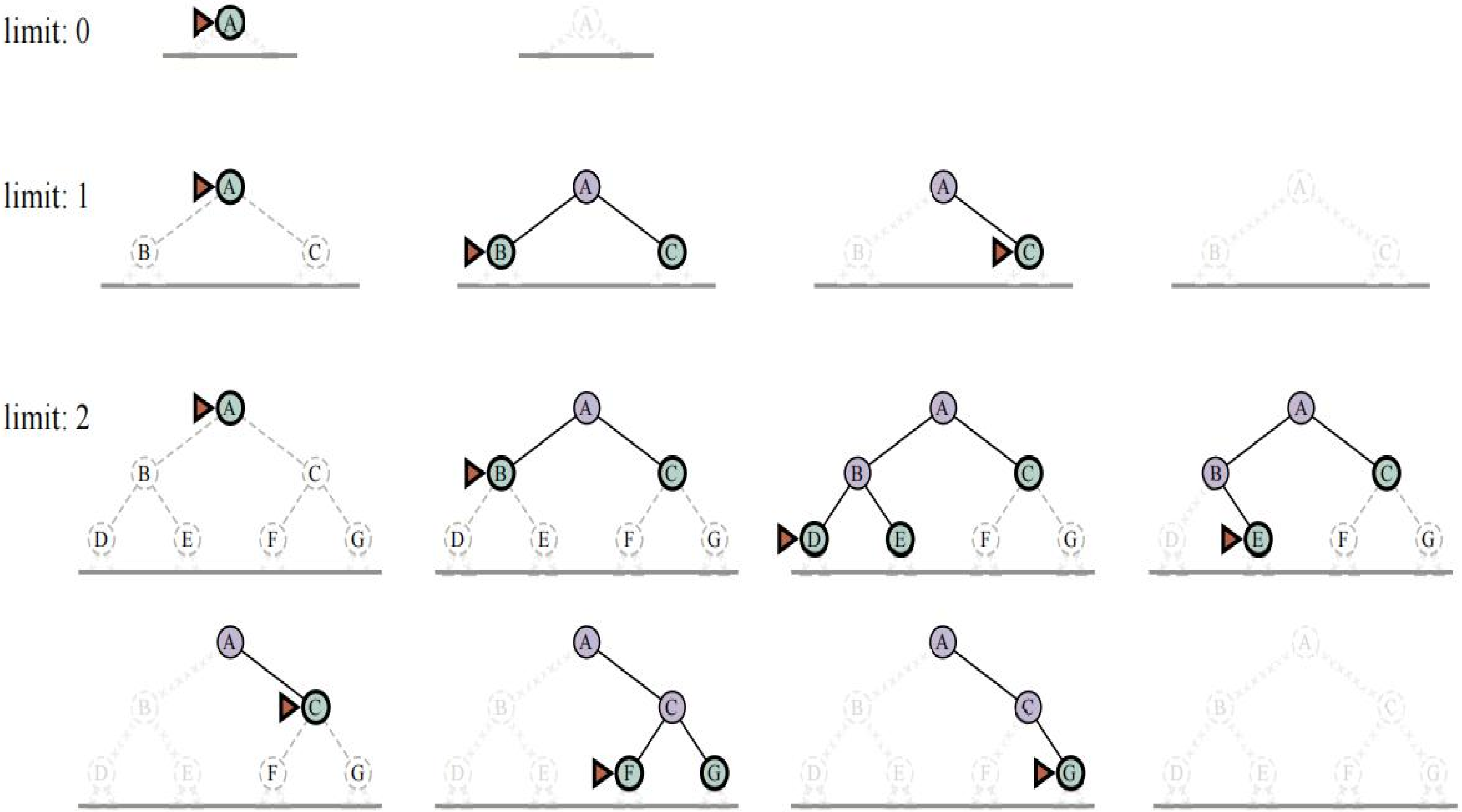
Tree depth is an issue!

“Controlled” DFS: Pseudocode

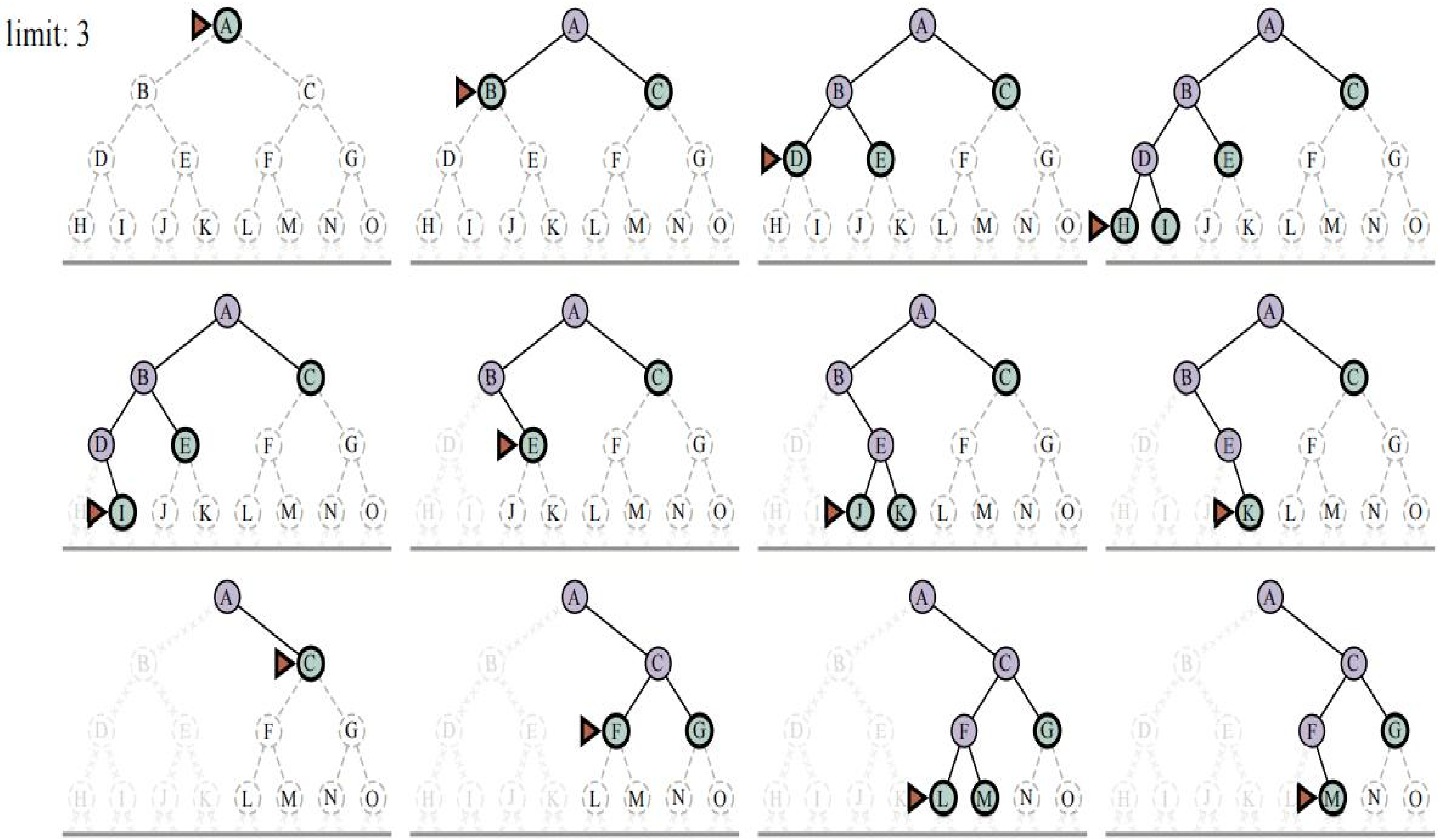
```
function ITERATIVE-DEEPENING-SEARCH(problem) returns a solution node or failure
  for depth = 0 to ∞ do
    result ← DEPTH-LIMITED-SEARCH(problem, depth)
    if result ≠ cutoff then return result

function DEPTH-LIMITED-SEARCH(problem, ℓ) returns a node or failure or cutoff
  frontier ← a LIFO queue (stack) with NODE(problem.INITIAL) as an element
  result ← failure
  while not IS-EMPTY(frontier) do
    node ← POP(frontier)
    if problem.IS-GOAL(node.STATE) then return node
    if DEPTH(node) > ℓ then
      result ← cutoff
    else if not IS-CYCLE(node) do
      for each child in EXPAND(problem, node) do
        add child to frontier
  return result
```

Iterative Deepening DFS: Illustration



Iterative Deepening DFS: Illustration

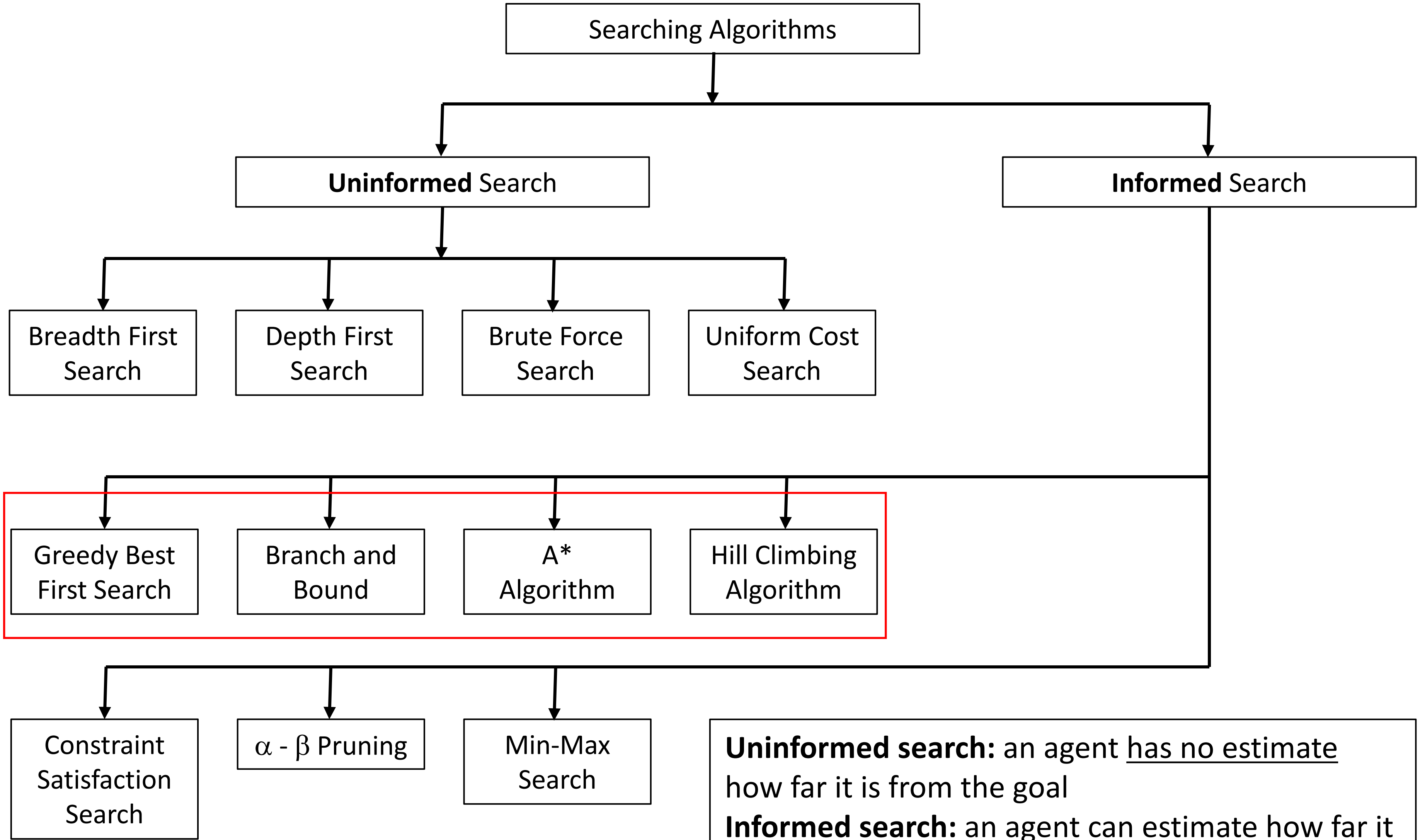


Uninformed Search Algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Complete?	Yes[1]	Yes[1,2]	No	No	Yes[1]	Yes[1,4]
Optimal cost?	Yes[3]	Yes	No	No	Yes[3]	Yes[3,4]
Time	$O(b^d)$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(b^m)$	$O(b^\ell)$	$O(b^d)$	$O(b^{d/2})$
Space	$O(b^d)$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(bm)$	$O(b\ell)$	$O(bd)$	$O(b^{d/2})$

Figure 3.15 Evaluation of search algorithms. b is the branching factor; m is the maximum depth of the search tree; d is the depth of the shallowest solution, or is m when there is no solution; ℓ is the depth limit. Superscript caveats are as follows: [1] complete if b is finite, and the state space either has a solution or is finite. [2] complete if all action costs are $\geq \epsilon > 0$; [3] cost-optimal if action costs are all identical; [4] if both directions are breadth-first or uniform-cost.

Selected Searching Algorithms



Uninformed search: an agent has no estimate how far it is from the goal
Informed search: an agent can estimate how far it is from the goal

Informed Search and Heuristics

Informed search relies on domain-specific knowledge / hints that help locate the goal state.

$$h(n) = h(\text{State } n)$$

$$h(n) = n(\text{relevant information about State } n)$$

h(n) : heuristic function - estimated cost of the cheapest path from State n to the goal state