CS 480

Introduction to Artificial Intelligence

October 28th, 2021

Announcements / Reminders

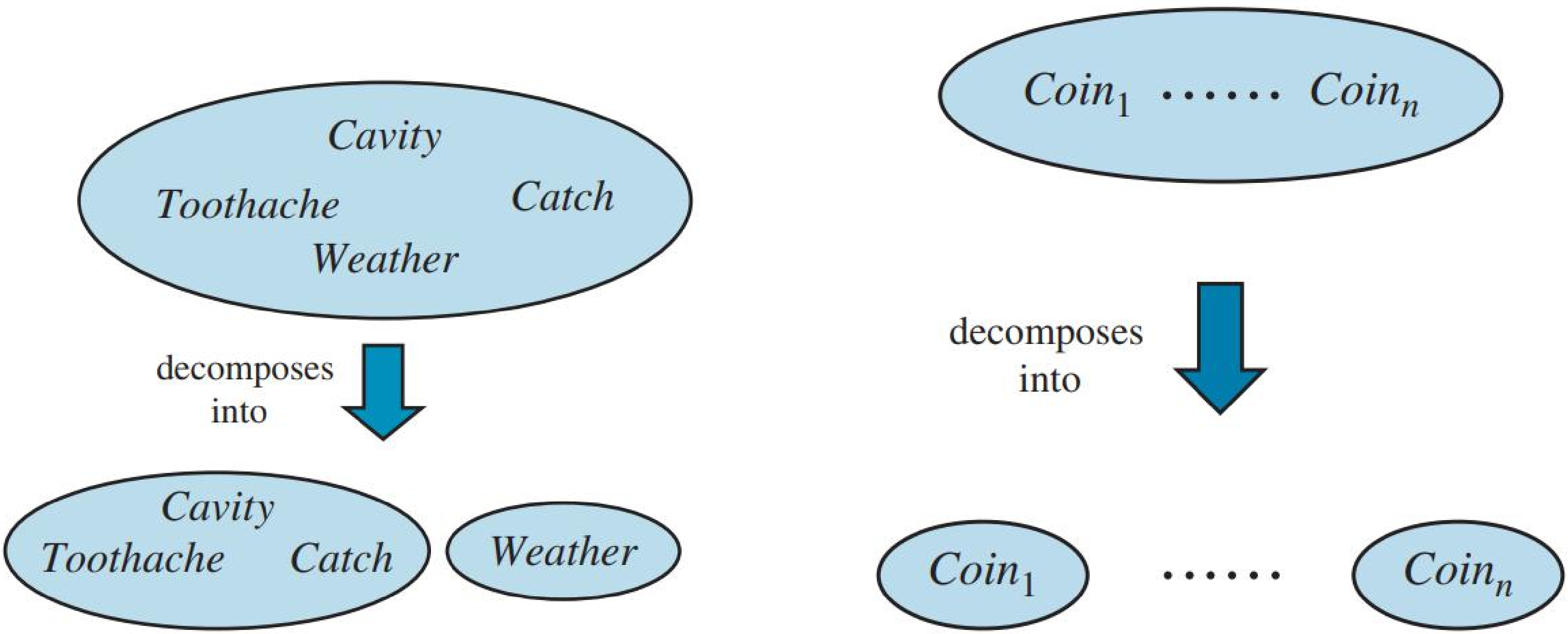
- **Programming Assignment #01:**
  - **due: ~~October 17th~~ ~~October 22th~~ ~~October 24th~~ November 3rd, 11:00 PM CST**
- **Programming Assignment #02:**
  - **Next week**
- **Written Assignment #03:**
  - **Next week**

- **Enjoy your Halloween instead!**

Plan for Today

- **Bayesian Networks**

Factoring / Decomposition



Full Joint Probability Distribution

H: grad	e: female	P(H, e) = P(H ∧ e): P(grad ∧ female)
true	true	$P(H, e) = P(H \wedge e) = P(H) * P(e \mid H) = 18 / 81 * 6 / 18 \approx 0.074$
true	false	$P(H, \neg e) = P(H) * P(\neg e \mid H) = 18 / 81 * 12 / 18 \approx 0.148$
false	true	$P(\neg H, e) = P(\neg H) * P(e \mid \neg H) = 63 / 81 * 7 / 63 \approx 0.086$
false	false	$P(\neg H, \neg e) = P(\neg H) * P(\neg e \mid \neg H) = 63 / 81 * 56 / 63 \approx 0.691$
		SUM = 1

$$P(H \wedge e) = P(H) * P(e \mid parents(e)) = \boxed{P(H)} * \boxed{P(e \mid H)}$$

H: grad	¬H: ¬grad
18 / 81 ≈ 0.22	63 / 81 ≈ 0.78

H: grad	e: female	P(e | H)
true	true	6 / 18 ≈ 0.333
true	false	12 / 18 ≈ 0.667
false	true	7 / 63 ≈ 0.111
false	false	56 / 63 ≈ 0.889

Conditional Probability Table (CPT)

Bayesian (Belief) Network

A Bayesian belief network describes the joint probability distribution for a set of variables.

A Bayesian network is an acyclic, directed graph (DAG), where the nodes are random variables (propositions). There is an edge (arc) from each elements of $\text{parents}(X_i)$ into X_i. Associated with the Bayesian network is a set of conditional probability distributions - the conditional probability of each variable given its parents (which includes the prior probabilities of those variables with no parents).

Consists of:

- **a graph (DAG) with nodes corresponding to random variables**
- **a domain for each random variable**
- **a set of conditional probability distributions $P(X_i \mid \text{parents}(X_i))$**

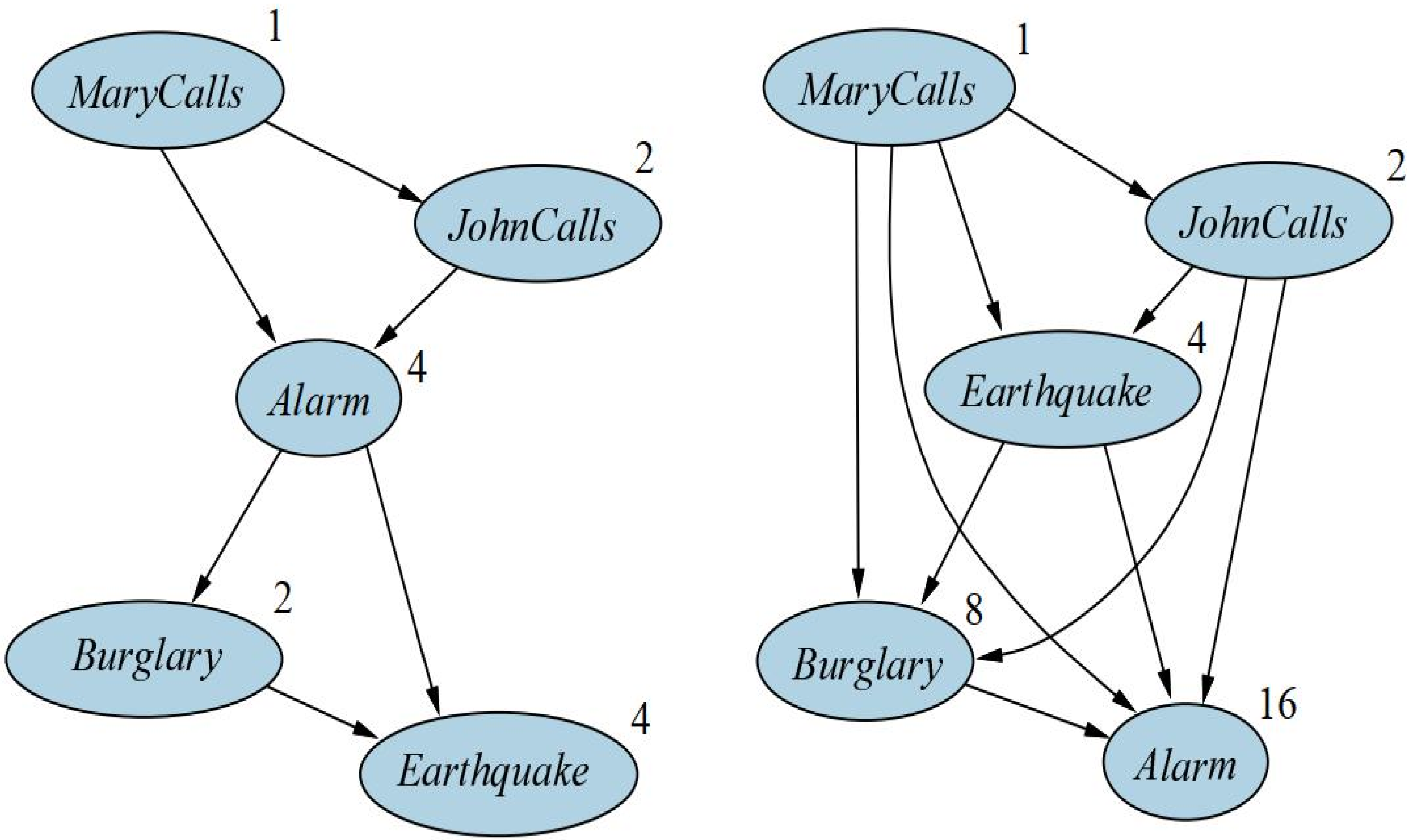
Building Bayesian (Belief) Network

1. **Order Random Variables (ordering matters!)**
2. **Create network nodes for each Random Variable**
3. **Add edges between parent nodes and children nodes**
   - **For every node node X_i:**
     - **choose a minimal set S of parents for X_i**
     - **for each parent node Y in S add an edge from Y to X_i**

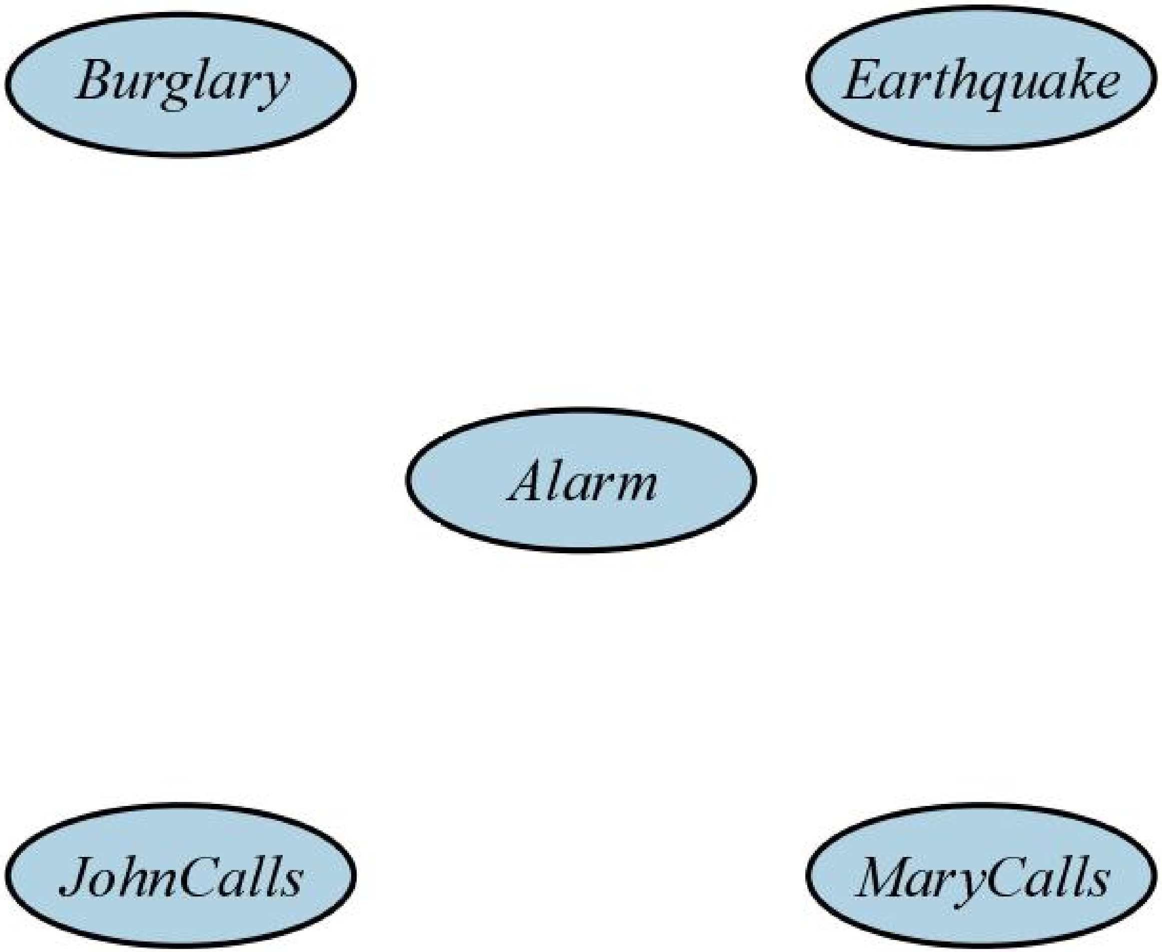
4. Add Conditional Probability Tables

Make it compact / sparse: choose your Random Variable ordering wisely.

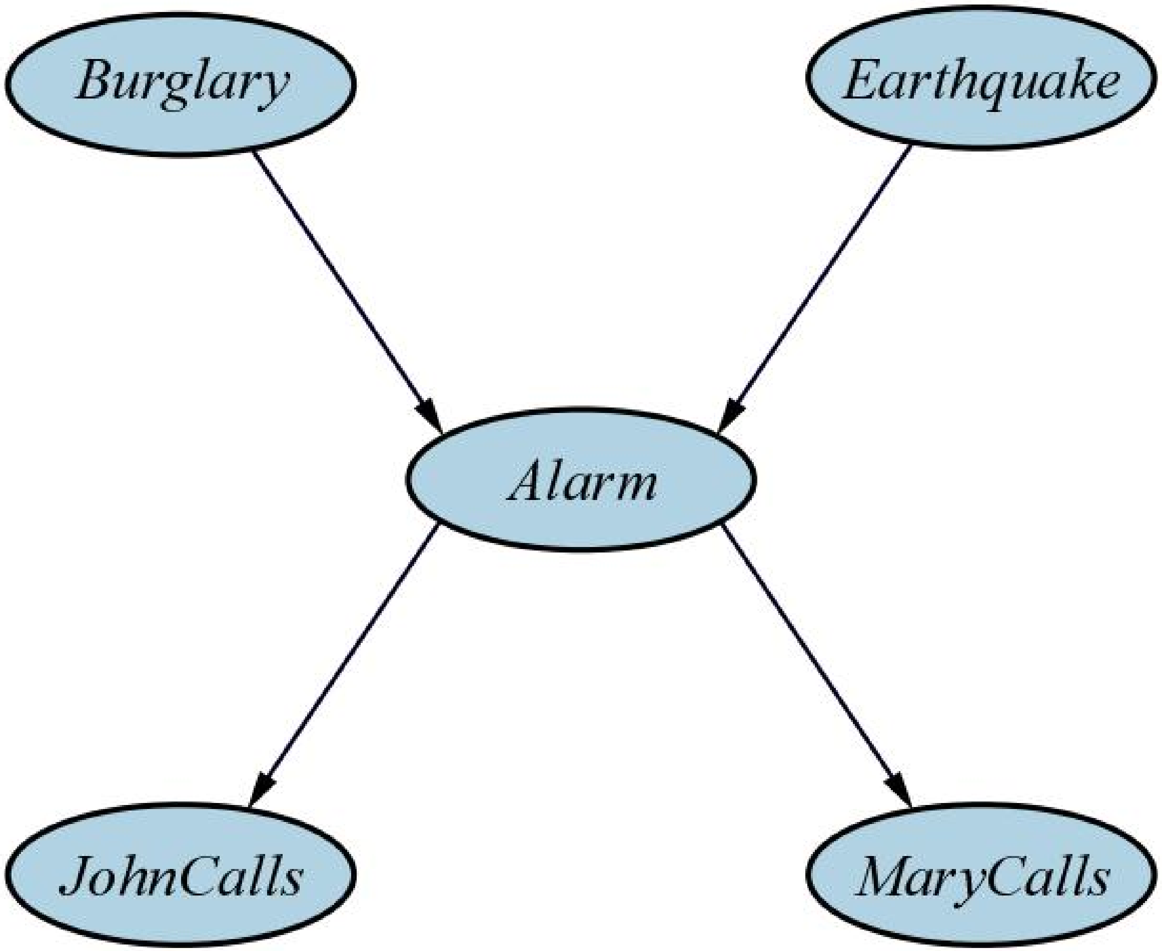
Ordering Matters!



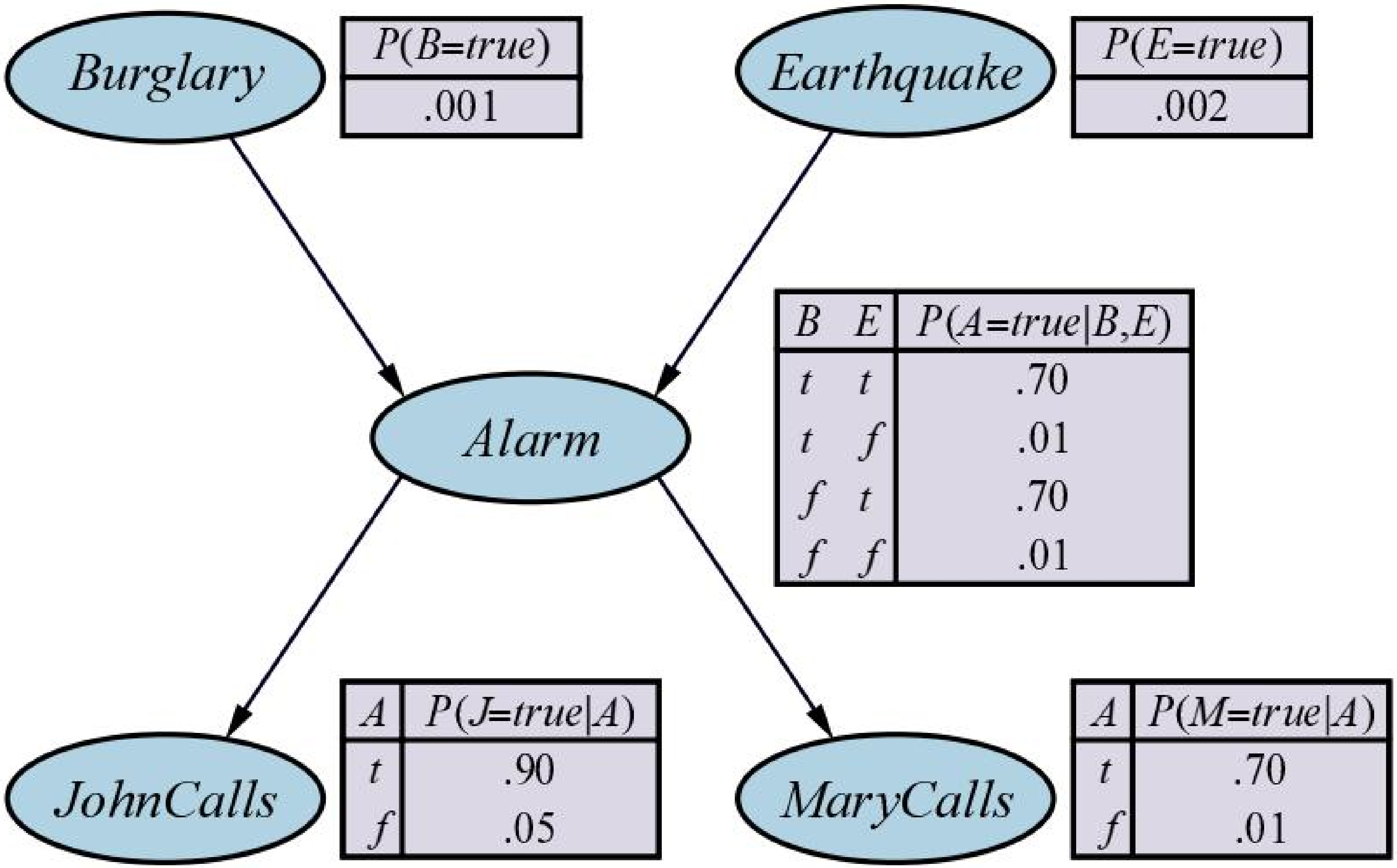
Create Vertices / Node / Random Vars



Add Edges



Add Conditional Probability Tables



$P(B=true)$
.001

$P(E=true)$
.002

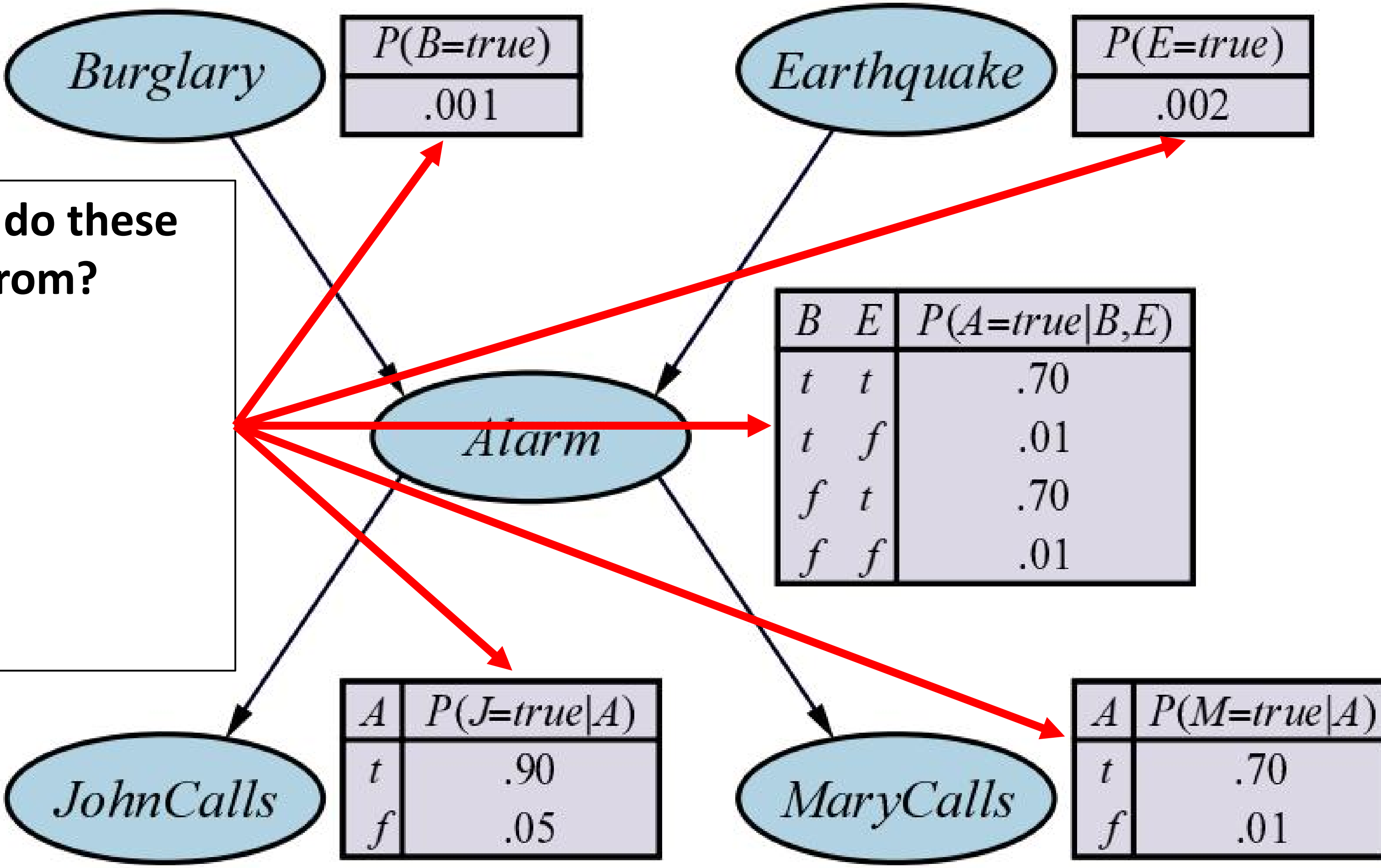
B	E	$P(A=true \mid B,E)$
t	t	.70
t	f	.01
f	t	.70
f	f	.01

A	$P(J=true \mid A)$
t	.90
f	.05

A	$P(M=true \mid A)$
t	.70
f	.01

Add Conditional Probability Tables

Where do these come from?



$P(B{=}true)$
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B	E	$P(A{=}true \mid B,E)$
t	t	.70
t	f	.01
f	t	.70
f	f	.01

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t	.90
f	.05

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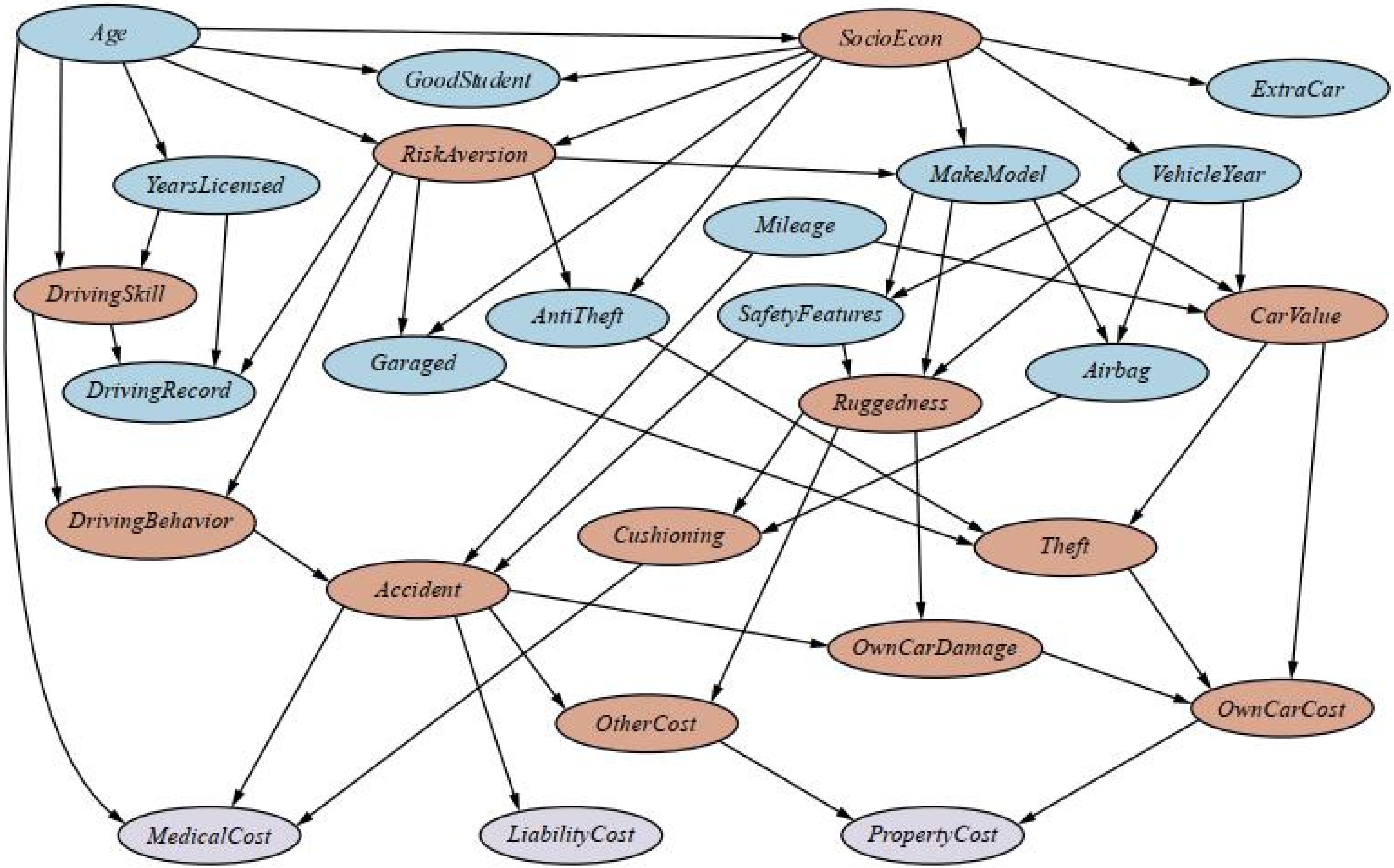
f	.01

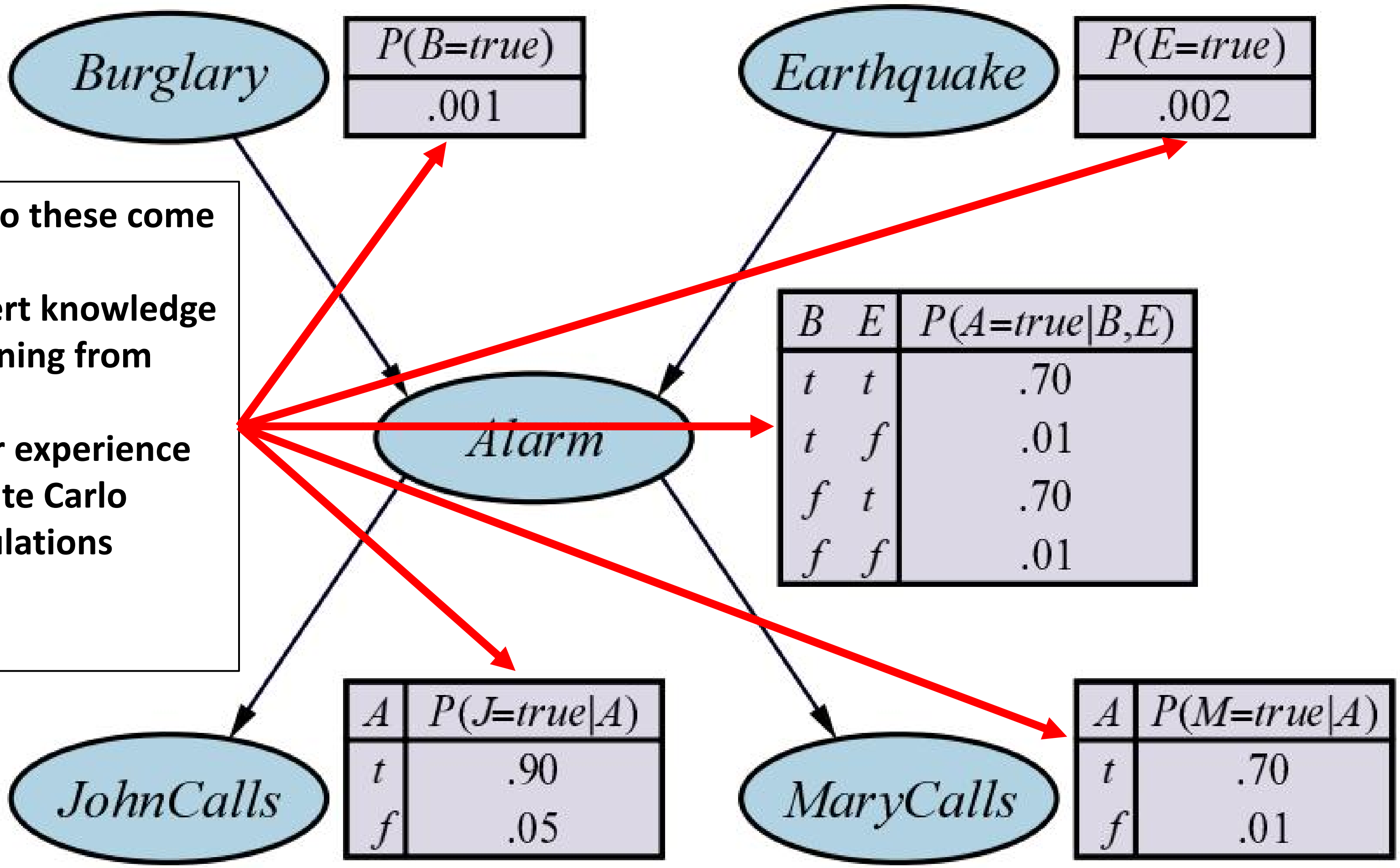
Bayesian Network: Car Insurance



Add Conditional Probability Tables

Where do these come from?

- Expert knowledge
- Learning from data
- Prior experience
- Monte Carlo simulations
- etc.



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t	f	.01
f	t	.70
f	f	.01

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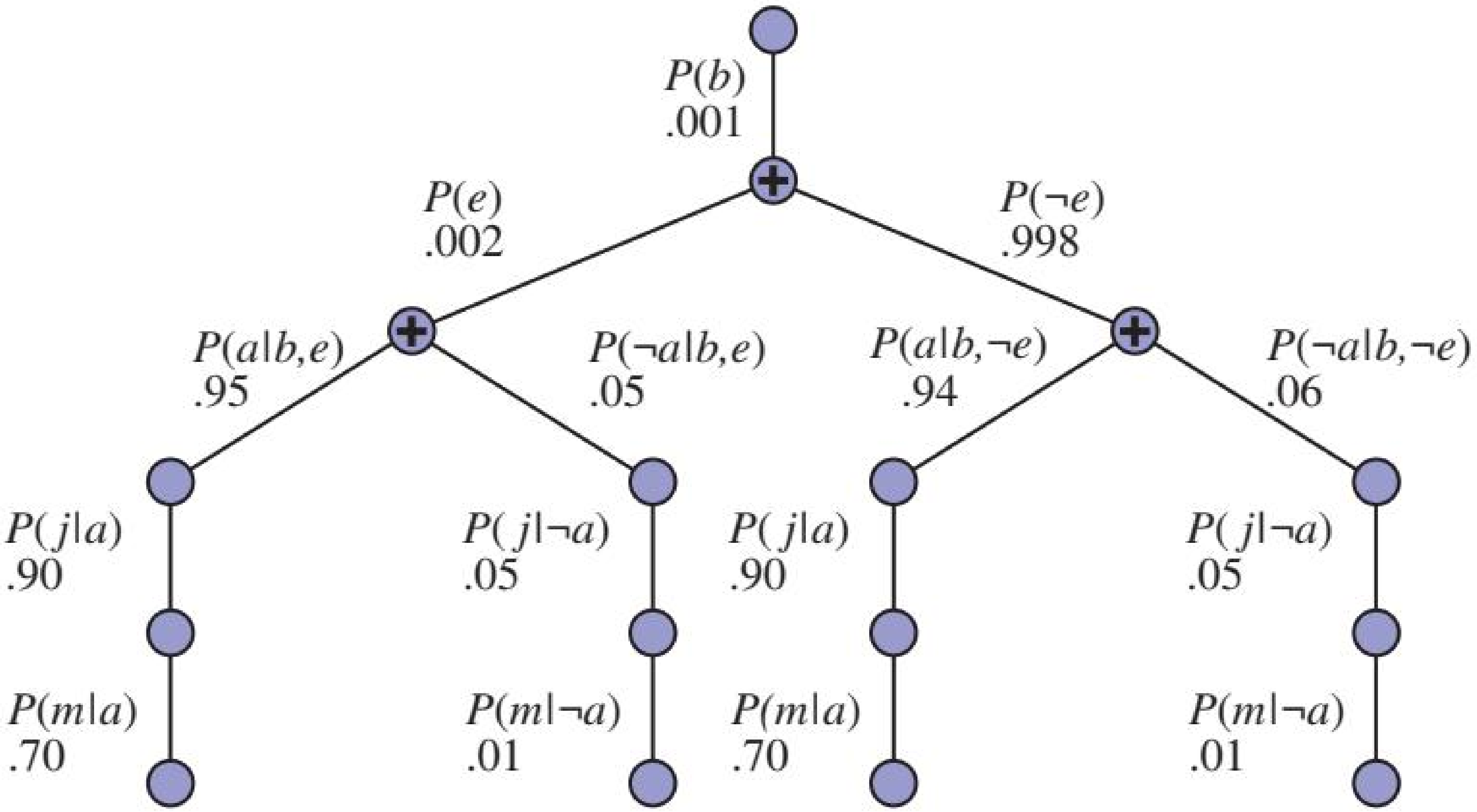
t	.90
f	.05

A	$P(M{=}true \mid A)$
t	.70
f	.01

Inference by Enumeration: Example

Query:

$$\mathbf{P}(Burglary \mid JohnCalls = true \wedge MaryCalls = true)$$



General Inference Procedure

Given:

- **a query involving a single variable X (in our example: Cavity),**
- **a <u>list</u> of evidence variables E (in our example: just Toothache),**
- **a <u>list</u> of observed values e for E,**
- **a list of remaining unobserved variables Y (in our example: just Catch),**

where X, E, and Y together are a COMPLETE set of variables for the domain, the probability P(X | E) can be evaluated as:

$$\boldsymbol{P}(X \mid \boldsymbol{e}) = \alpha * \boldsymbol{P}(X, \boldsymbol{e}) = \alpha * \sum_{\boldsymbol{y}} \boldsymbol{P}(X, \boldsymbol{e}, \boldsymbol{y})$$

where ys are all possible values for Ys, α - normalization constant.

P(X, e, y) is a subset of probabilities from the joint distribution

Inference

Query:

$$P(Burglary \mid JohnCalls = true \wedge MaryCalls = true)$$

Given:

- a query involving a single variable X
- a list of **evidence** variables K,
- a list of **observed** values k for K,
- a list of remaining **unobserved** variables Y

the probability $P(X \mid K)$ can be evaluated as:

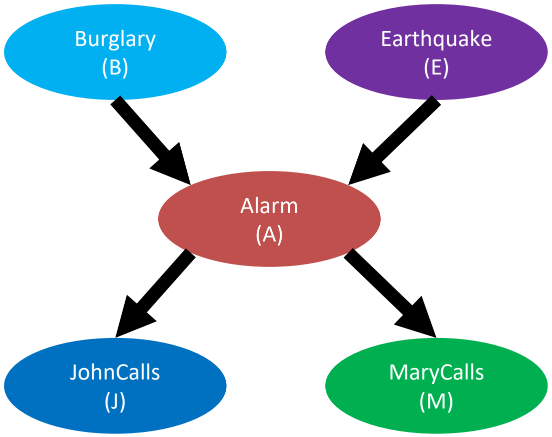
$$\boldsymbol{P}(X \mid \boldsymbol{k}) = \alpha * \boldsymbol{P}(X, \boldsymbol{k})$$

$$= \alpha * \sum_{\boldsymbol{y}} \boldsymbol{P}(X, \boldsymbol{k}, \boldsymbol{y})$$

where ys are all possible values for Ys, α - normalization constant.

P(B)	P(¬B)
0.001	0.999

P(E)	P(¬E)
0.002	0.998



B	E	P(A|B,E)
t	t	0.95
t	f	0.94
f	t	0.29
f	f	0.001

A	P(J|A)
t	0.90
f	0.05

A	P(M|A)
t	0.70
f	0.01

Inference

Query:

$$P(Burglary \mid JohnCalls = true \wedge MaryCalls = true)$$

Given:

- a query involving a single variable X: *Burglary*
- a <u>list</u> of **evidence** variables **K**: *JohnCalls*, *MaryCalls*
- a <u>list</u> of **observed** values **k** for **K**: *johnCalls*, *maryCalls*
- a list of remaining **unobserved** variables **Y**: *Earthquake*, *Alarm*

the probability P(X | **K**) can be evaluated as:

$$\boldsymbol{P}(X \mid \boldsymbol{k}) = \alpha * \sum_{\boldsymbol{y}} \boldsymbol{P}(X, \boldsymbol{k}, \boldsymbol{y})$$

where **y**s are all possible values for **Y**s, α - normalization constant.

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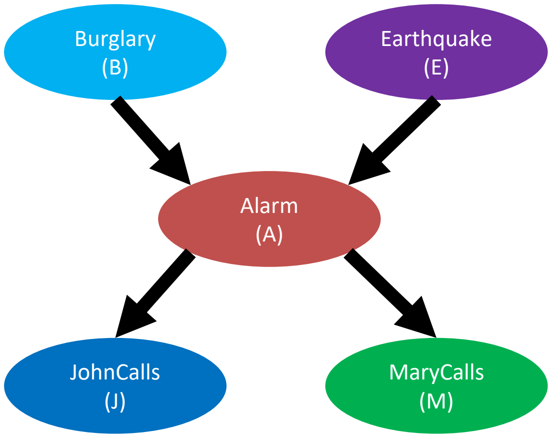

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A	P(M|A)
t	0.70
f	0.01

Inference

Query:

$$P(Burglary \mid JohnCalls = true \wedge MaryCalls = true)$$

Given:

- a query involving a single variable X: B
- a <u>list</u> of **evidence** variables $\mathbf{K}$: J, M
- a <u>list</u> of **observed** values $\mathbf{k}$ for $\mathbf{K}$: j, m
- a list of remaining **unobserved** variables $\mathbf{Y}$: E, A

the probability $\mathrm{P}(\mathrm{X} \mid \mathbf{K})$ can be evaluated as:

$$\boldsymbol{P}(X \mid \boldsymbol{k}) = \alpha * \sum_{\boldsymbol{y}} \boldsymbol{P}(X, \boldsymbol{k}, \boldsymbol{y})$$

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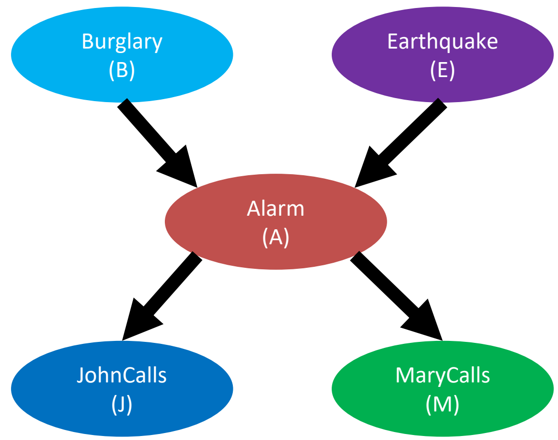

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Inference

Query:

$$P(Burglary \mid JohnCalls = true \wedge MaryCalls = true)$$

Given:

- **a query involving a single variable** B
- **a <u>list</u> of evidence variables** $\mathbf{K}$**:** J, M
- **a <u>list</u> of observed values** $\mathbf{k}$ **for** $\mathbf{K}$**:** j, m
- **a list of remaining unobserved variables** $\mathbf{Y}$**:** E, A

the probability $P(B \mid J, M)$ **can be evaluated as:**

$$P(B \mid j, m) = \alpha * \sum_{e}\sum_{a} P(B, j, m, e, a)$$

where ys are all possible values for Ys, α **- normalization constant.**

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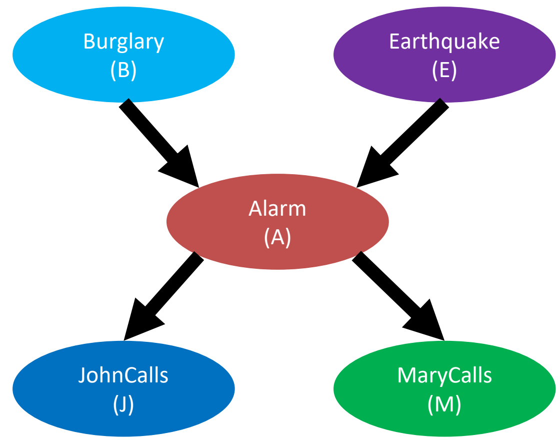

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t	0.70
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Inference

Query (let's change it a bit for simplicity):

$$\boldsymbol{P}(Burglary\ =\ true\,|\,JohnCalls\ =\ true \wedge MaryCalls\ =\ true)$$

Given:

- **a query involving a single variable** B
- **a <u>list</u> of evidence variables** $\mathbf{K}$**:** J, M
- **a <u>list</u> of observed values** $\mathbf{k}$ **for** $\mathbf{K}$**:** j, m
- **a list of remaining unobserved variables** $\mathbf{Y}$**:** E, A

the query can be evaluated as:

$$P(b\,|\,j, m)\ =\ \alpha * \sum_{e} \sum_{a} P(b,\ j, m,\ e,\ a)$$

By Chain rule:

$$P(b,\ j, m,\ e,\ a)$$
$$=\ P(b) * P(e) * P(a|b,e) * P(j|a) * P(m|a)$$

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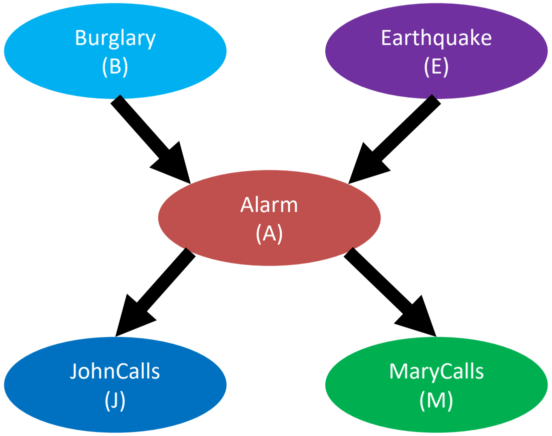

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Inference

Query (let's change it a bit for simplicity):

$P(Burglary = true \mid JohnCalls = true \land MaryCalls = true)$

Given:

- **a query involving a single variable** B
- **a list of evidence variables K:** J, M
- **a list of observed values k for K:** j, m
- **a list of remaining unobserved variables Y:** E, A

the query can be evaluated as:

$$P(b \mid j, m) = \alpha * \sum_e \sum_a P(b) * P(e) * P(a|b,e) * P(j|a) * P(m|a)$$

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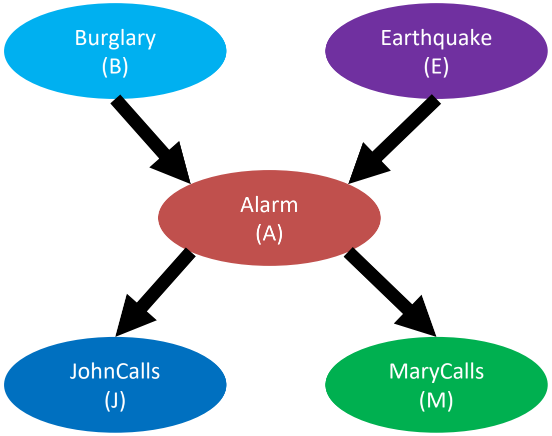

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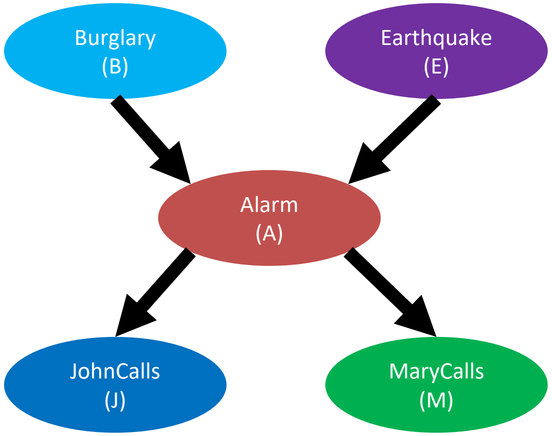

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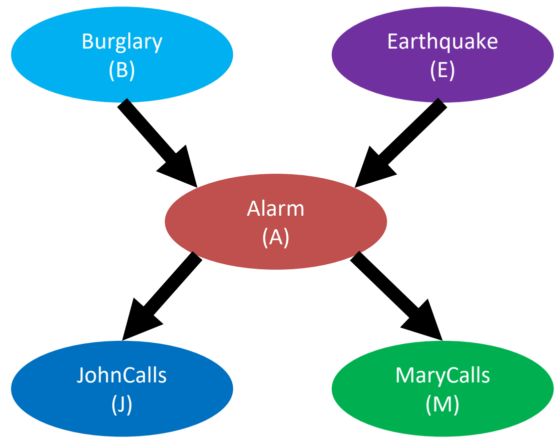

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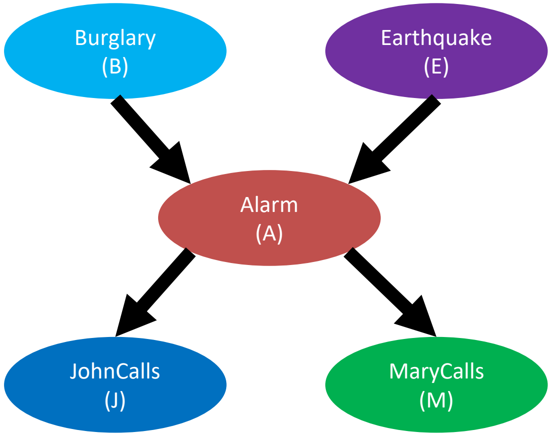

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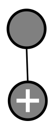
$P(b)$



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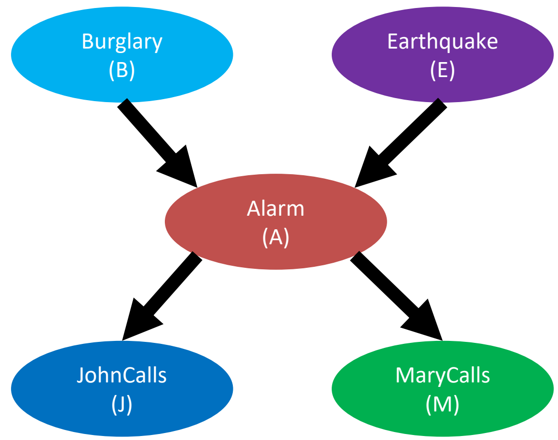

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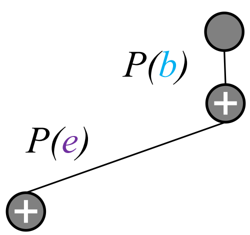

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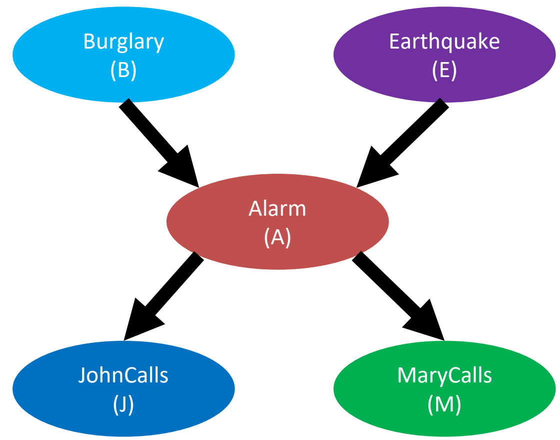

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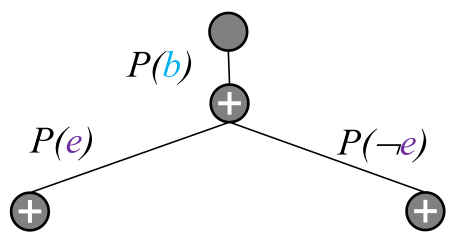

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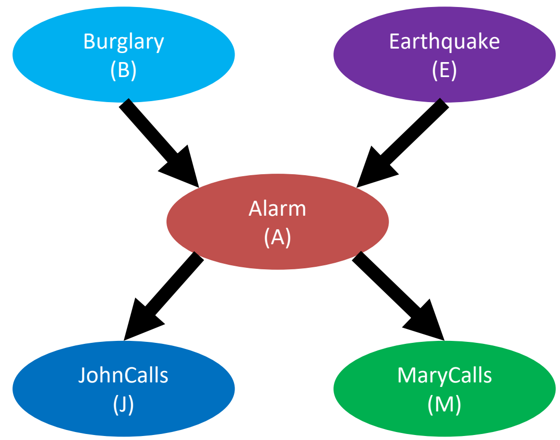

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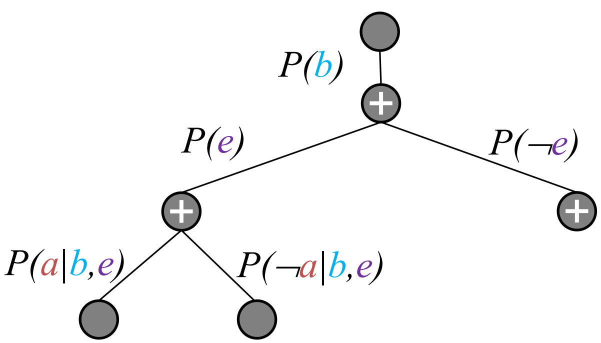
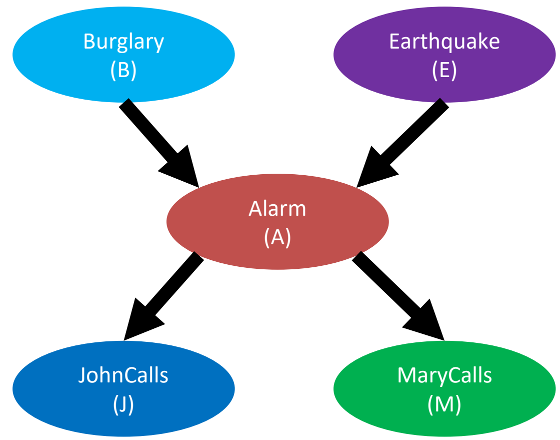
$$= \alpha * P(b) * \sum_e P(e) * \boxed{\sum_a \boxed{P(a|b,e)} * P(j|a) * P(m|a)}$$



P(B)	P(¬B)
0.001	0.999

P(E)	P(¬E)
0.002	0.998



B	E	P(A|B,E)
t	t	0.95
t	f	0.94
f	t	0.29
f	f	0.001

A	P(J|A)
t	0.90
f	0.05

A	P(M|A)
t	0.70
f	0.01

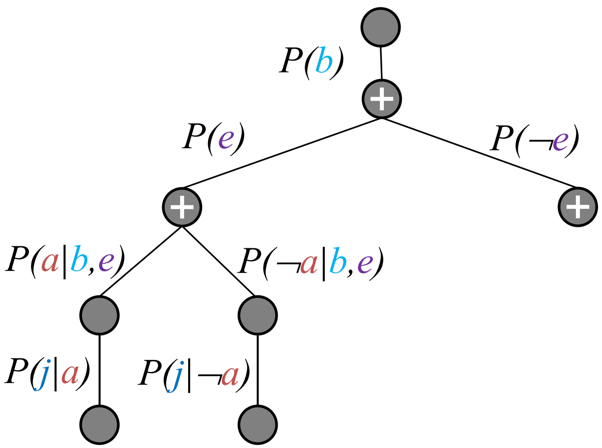
Inference

Query (let's change it a bit for simplicity):

$P(Burglary = true \mid JohnCalls = true \wedge MaryCalls = true)$

Query rewritten:

$$P(b \mid j, m)$$
$$= \alpha * \sum_e \sum_a P(b) * P(e) * P(a|b,e) * P(j|a) * P(m|a)$$
$$= \alpha * P(b) * \sum_e P(e) * \boxed{\sum_a P(a|b,e) * \boxed{P(j|a)} * P(m|a)}$$



P(B)	P(¬B)
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P(E)	P(¬E)
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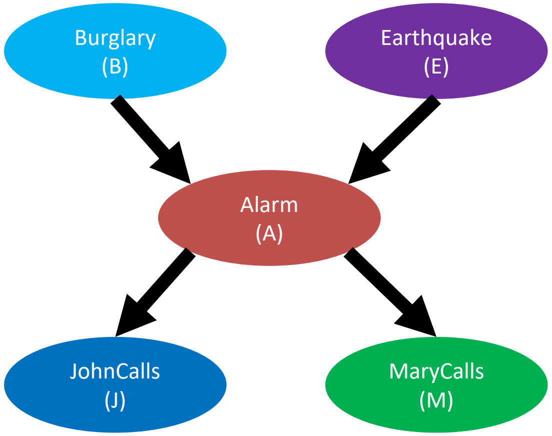

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t	0.90
f	0.05

A	P(M|A)
t	0.70
f	0.01

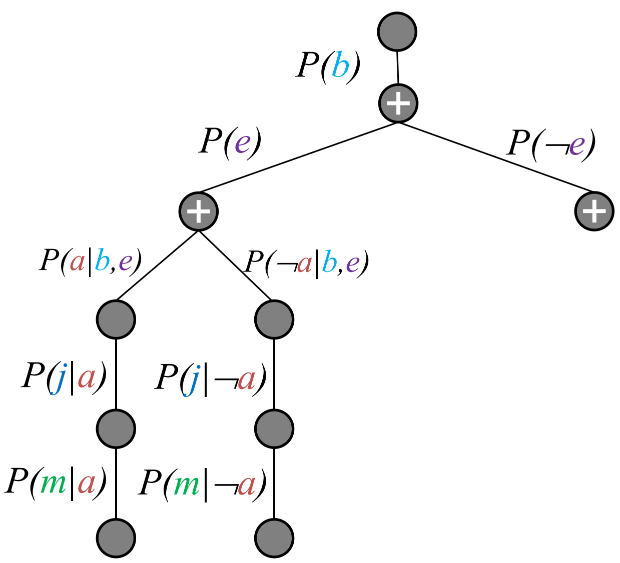
Inference

Query (let's change it a bit for simplicity):

$P(Burglary = true \mid JohnCalls = true \wedge MaryCalls = true)$

Query rewritten:

$$
\begin{aligned}
&P(b \mid j, m) \\
&= \alpha * \sum_{e} \sum_{a} P(b) * P(e) * P(a|b,e) * P(j|a) * P(m|a) \\
&= \alpha * P(b) * \sum_{e} P(e) * \boxed{\sum_{a} P(a|b,e) * P(j|a) * \boxed{P(m|a)}}
\end{aligned}
$$



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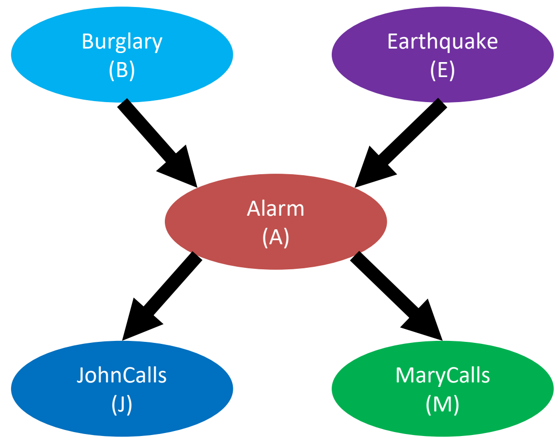

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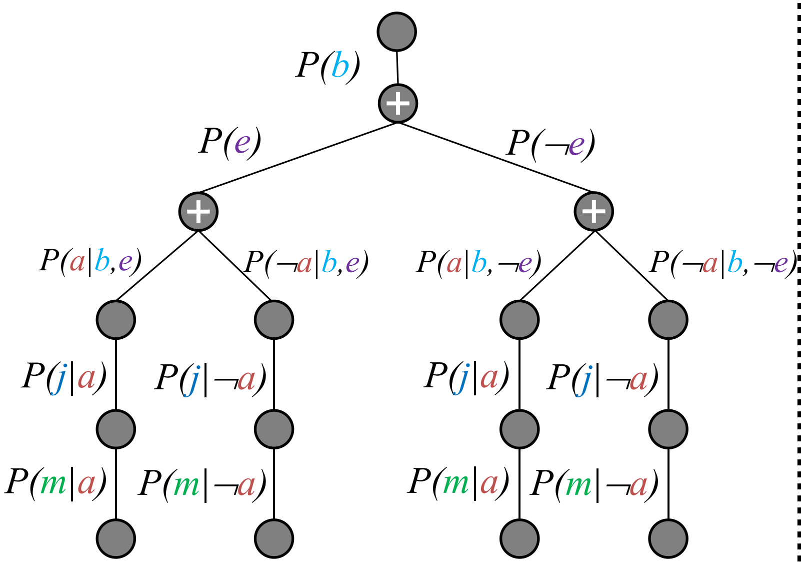
Inference

Query (let's change it a bit for simplicity):

$P(Burglary = true \mid JohnCalls = true \wedge MaryCalls = true)$

Query rewritten:

$$P(b \mid j, m)$$
$$= \alpha * \sum_{e} \sum_{a} P(b) * P(e) * P(a|b,e) * P(j|a) * P(m|a)$$
$$= \alpha * P(b) * \sum_{e} P(e) * \boxed{\sum_{a} P(a|b,e) * P(j|a) * P(m|a)}$$



P(B)	P(¬B)
0.001	0.999

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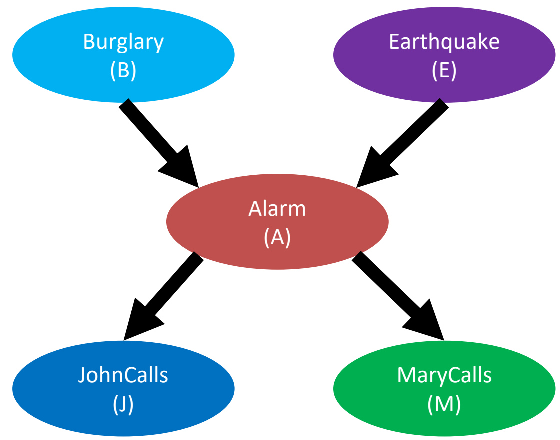

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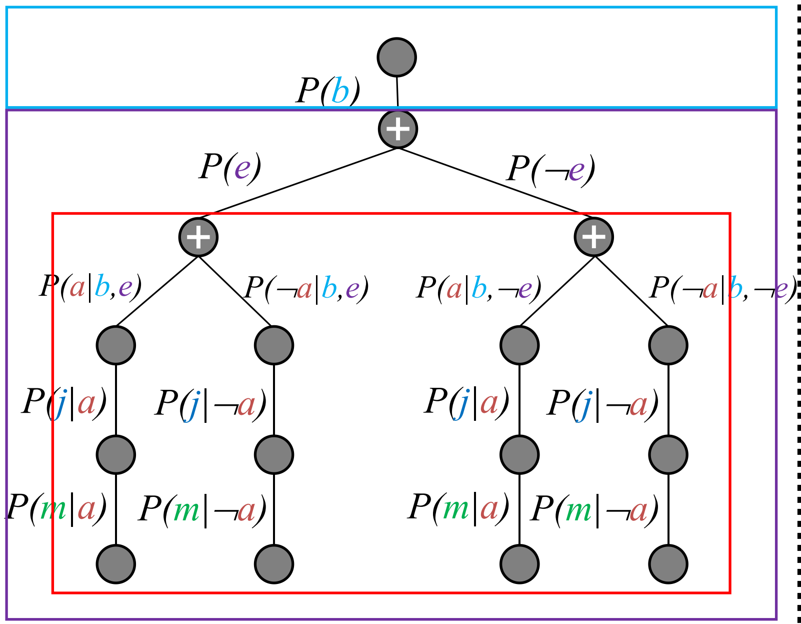
Inference

Query (let's change it a bit for simplicity):

$P(Burglary = true \mid JohnCalls = true \wedge MaryCalls = true)$

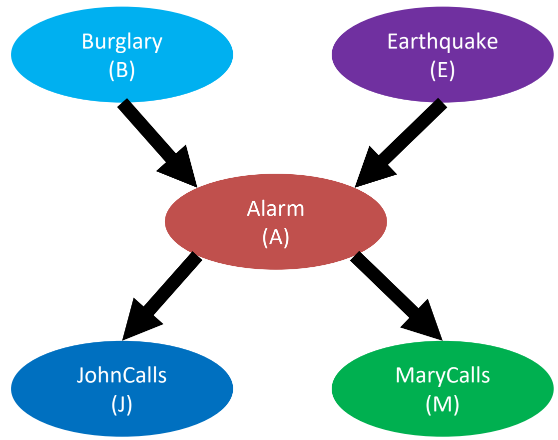
Query rewritten:

$$P(b \mid j, m)$$
$$= \alpha * \sum_e \sum_a P(b) * P(e) * P(a|b,e) * P(j|a) * P(m|a)$$
$$= \alpha * \boxed{P(b)} * \boxed{\sum_e P(e) * \boxed{\sum_a P(a|b,e) * P(j|a) * P(m|a)}}$$



P(B)	P(¬B)
0.001	0.999

P(E)	P(¬E)
0.002	0.998



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A	P(J|A)
t	0.90
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A	P(M|A)
t	0.70
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Inference

Query (let's change it a bit for simplicity):

$P(Burglary = true \mid JohnCalls = true \wedge MaryCalls = true)$

Query rewritten:

$$P(b \mid j, m)$$
$$= \alpha * \sum_e \sum_a P(b) * P(e) * P(a|b,e) * P(j|a) * P(m|a)$$
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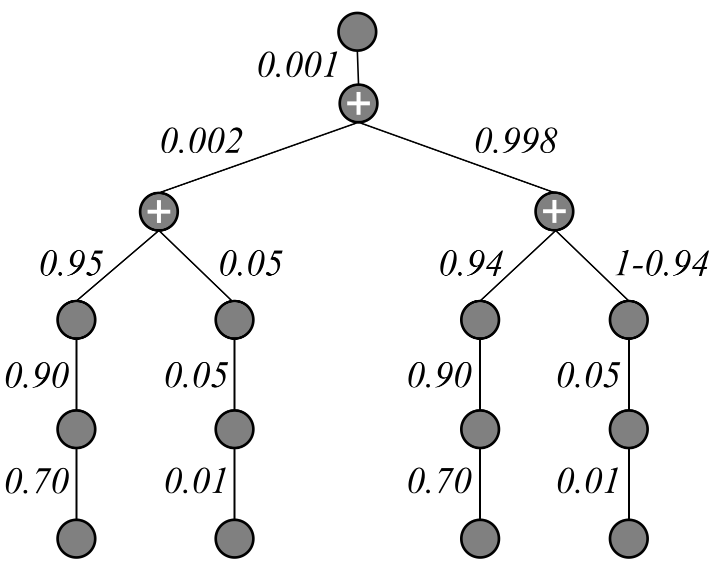

P(B)	P(¬B)
0.001	0.999

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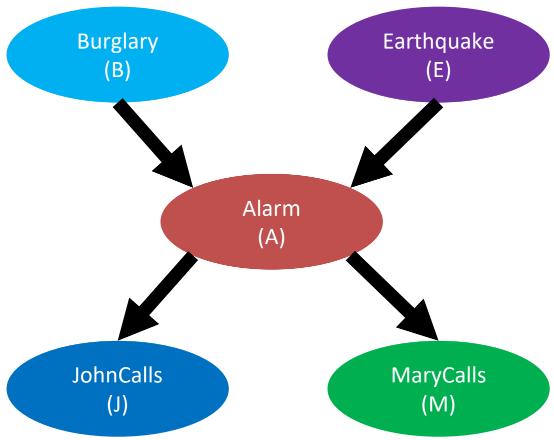
0.002	0.998



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f	f	0.001

A	P(J|A)
t	0.90
f	0.05

A	P(M|A)
t	0.70
f	0.01

Inference

Query (let's change it a bit for simplicity):

$P(Burglary = true \mid JohnCalls = true \wedge MaryCalls = true)$

We can now calculate:

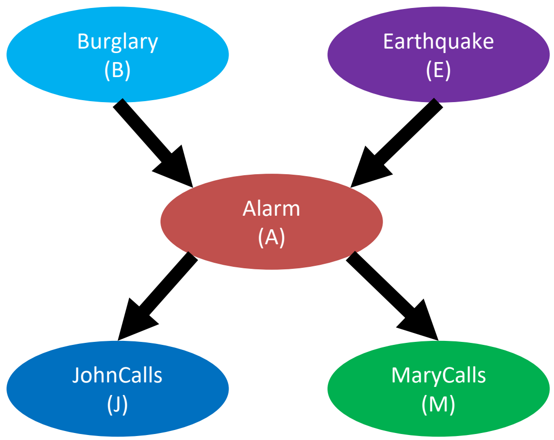
$$P(b \mid j, m) = \alpha * 0.00059224$$

And through similar process (not shown):

$$P(\neg b \mid j, m) = \alpha * 0.0014919$$

P(B)	P(¬B)
0.001	0.999

P(E)	P(¬E)
0.002	0.998



B	E	P(A|B,E)
t	t	0.95
t	f	0.94
f	t	0.29
f	f	0.001

A	P(J|A)
t	0.90
f	0.05

A	P(M|A)
t	0.70
f	0.01

Inference

Query (now we can get joint distribution):

$$\boldsymbol{P}(Burglary \mid JohnCalls = true \wedge MaryCalls = true)$$

We can now calculate:

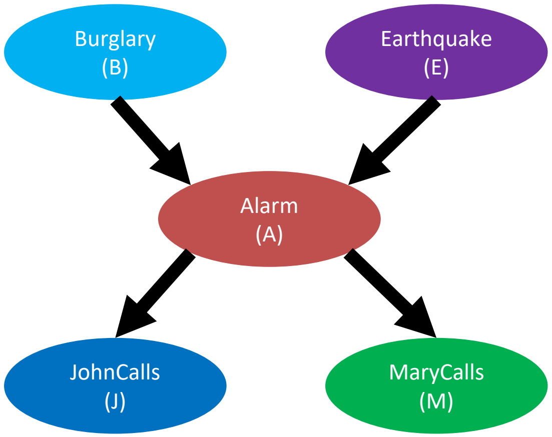
$$\boldsymbol{P}(B \mid j, m) = \alpha * \ < 0.00059224, 0.0014919 >$$

And after normalization:

$$\boldsymbol{P}(B \mid j, m) \approx < 0.284, 0.716 >$$

P(B)	P(¬B)
0.001	0.999

P(E)	P(¬E)
0.002	0.998



B	E	P(A|B,E)
t	t	0.95
t	f	0.94
f	t	0.29
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A	P(J|A)
t	0.90
f	0.05

A	P(M|A)
t	0.70
f	0.01

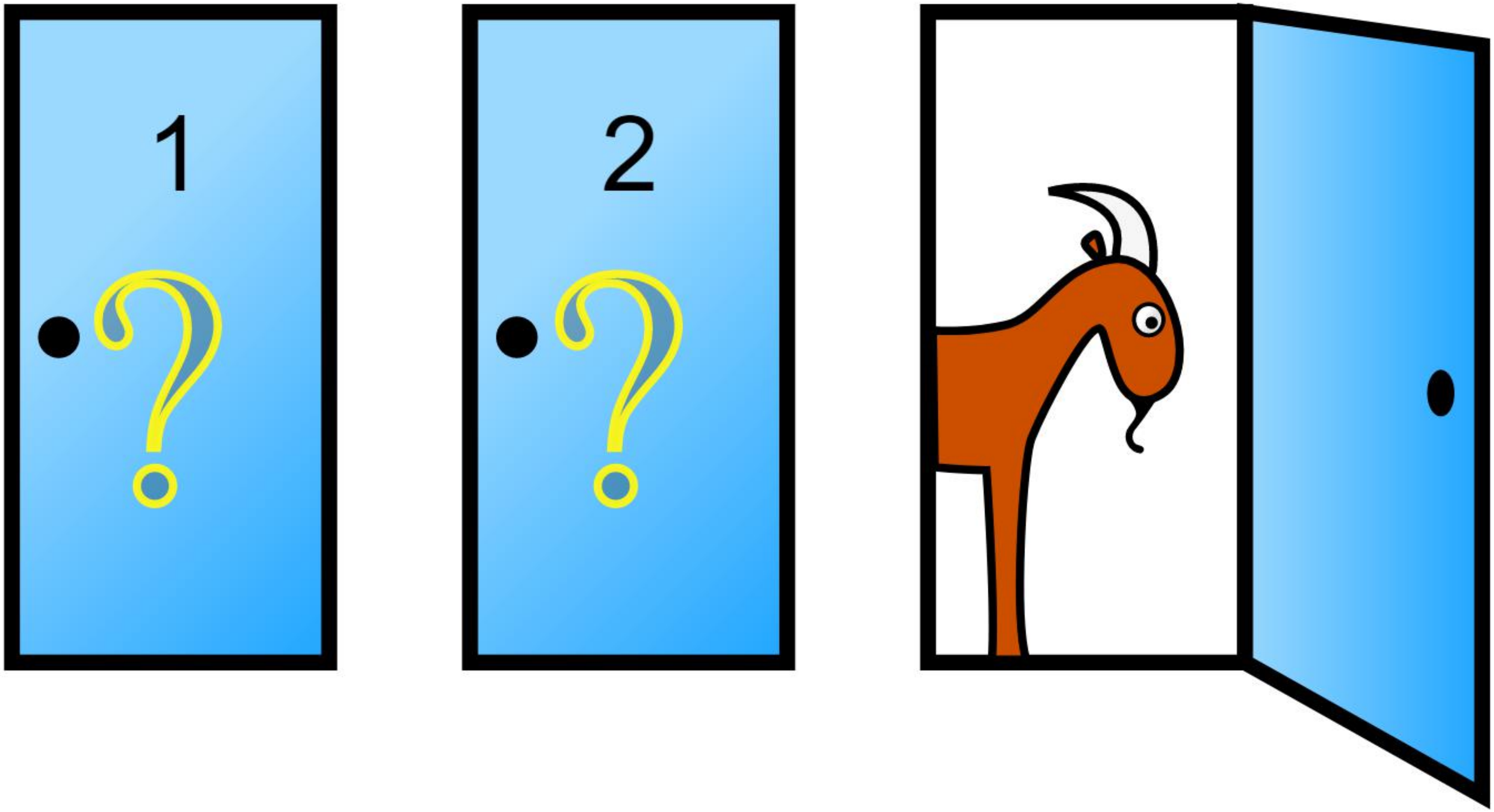
Inference by Enumeration:Pseudocode

function ENUMERATION-ASK(X, **e**, bn) **returns** a distribution over X
  inputs: X, the query variable
    e, observed values for variables **E**
    bn, a Bayes net with variables $vars$

  $\mathbf{Q}(X) \leftarrow$ a distribution over X, initially empty
  for each value x_i of X **do**
    $\mathbf{Q}(x_i) \leftarrow$ ENUMERATE-ALL($vars$, $\mathbf{e}_{x_i}$)
      where $\mathbf{e}_{x_i}$ is **e** extended with $X = x_i$
  return NORMALIZE($\mathbf{Q}(X)$)

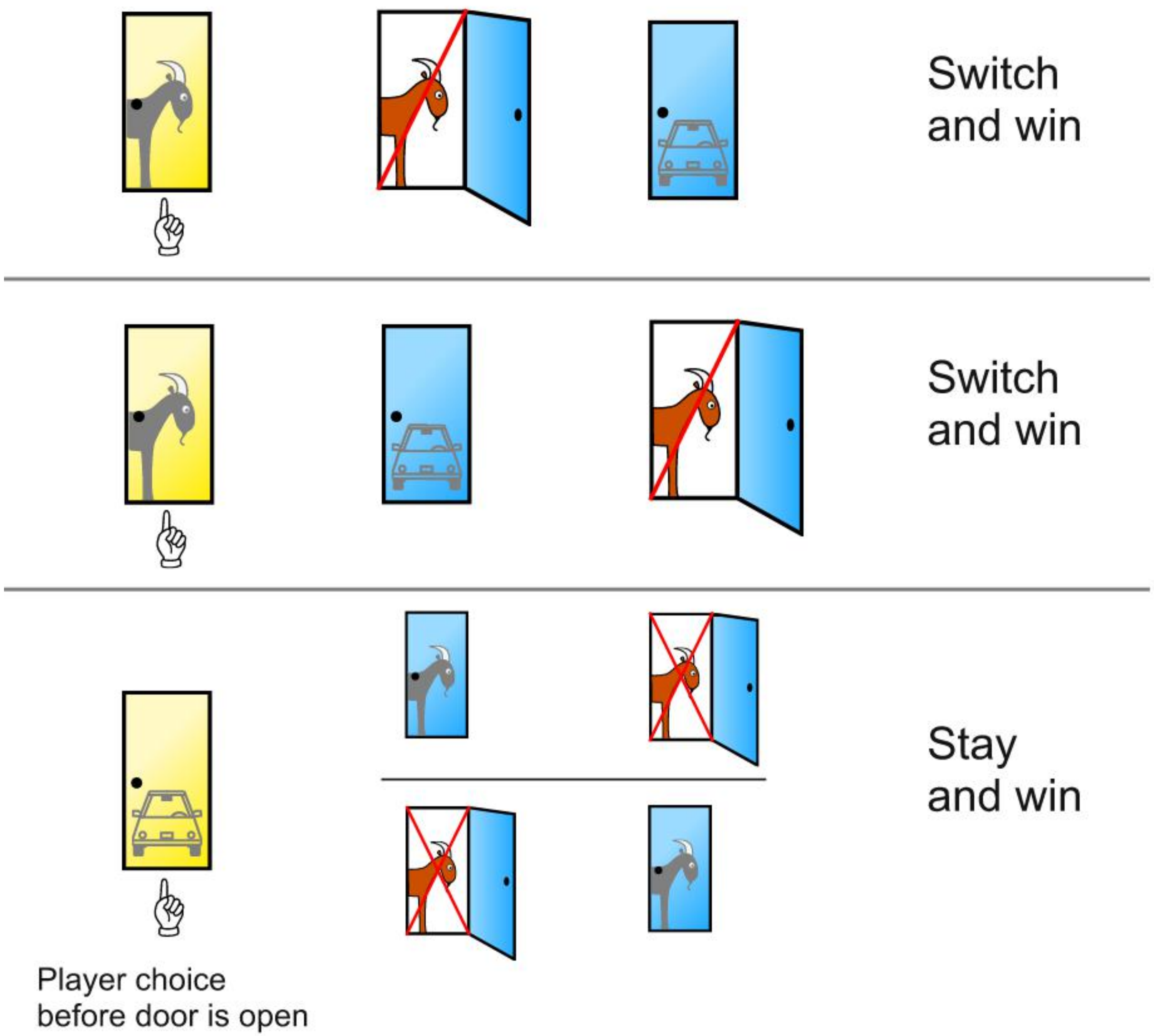
function ENUMERATE-ALL($vars$, **e**) **returns** a real number
  if EMPTY?($vars$) **then return** 1.0
  $V \leftarrow$ FIRST($vars$)
  if V is an evidence variable with value v in **e**
    then return $P(v \mid parents(V)) \times$ ENUMERATE-ALL(REST($vars$), **e**)
    else return $\sum_v P(v \mid parents(V)) \times$ ENUMERATE-ALL(REST($vars$), $\mathbf{e}_v$)
      where $\mathbf{e}_v$ is **e** extended with $V = v$

Monty Hall Problem



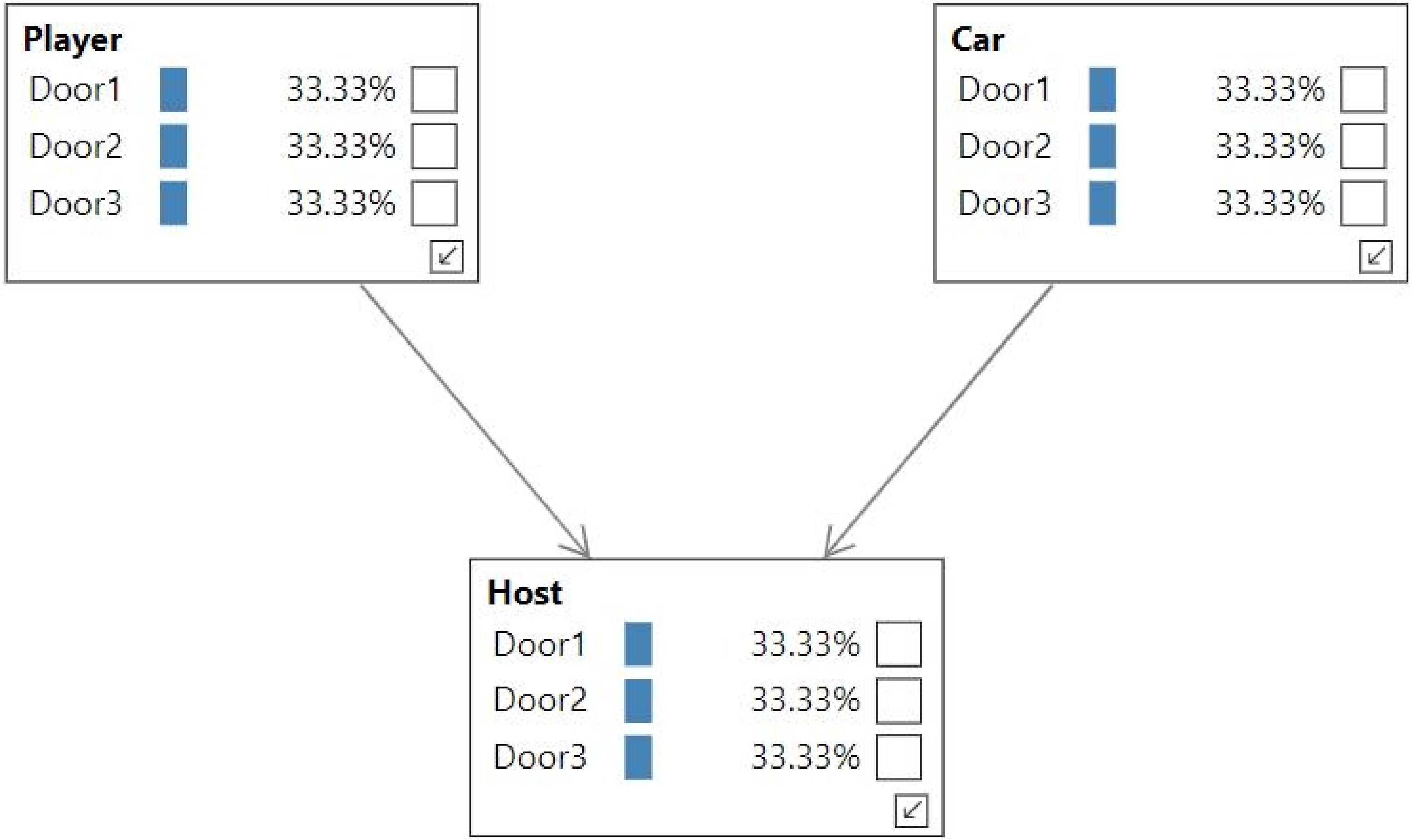
Source: https://en.wikipedia.org/wiki/Monty_Hall_problem

Monty Hall Problem



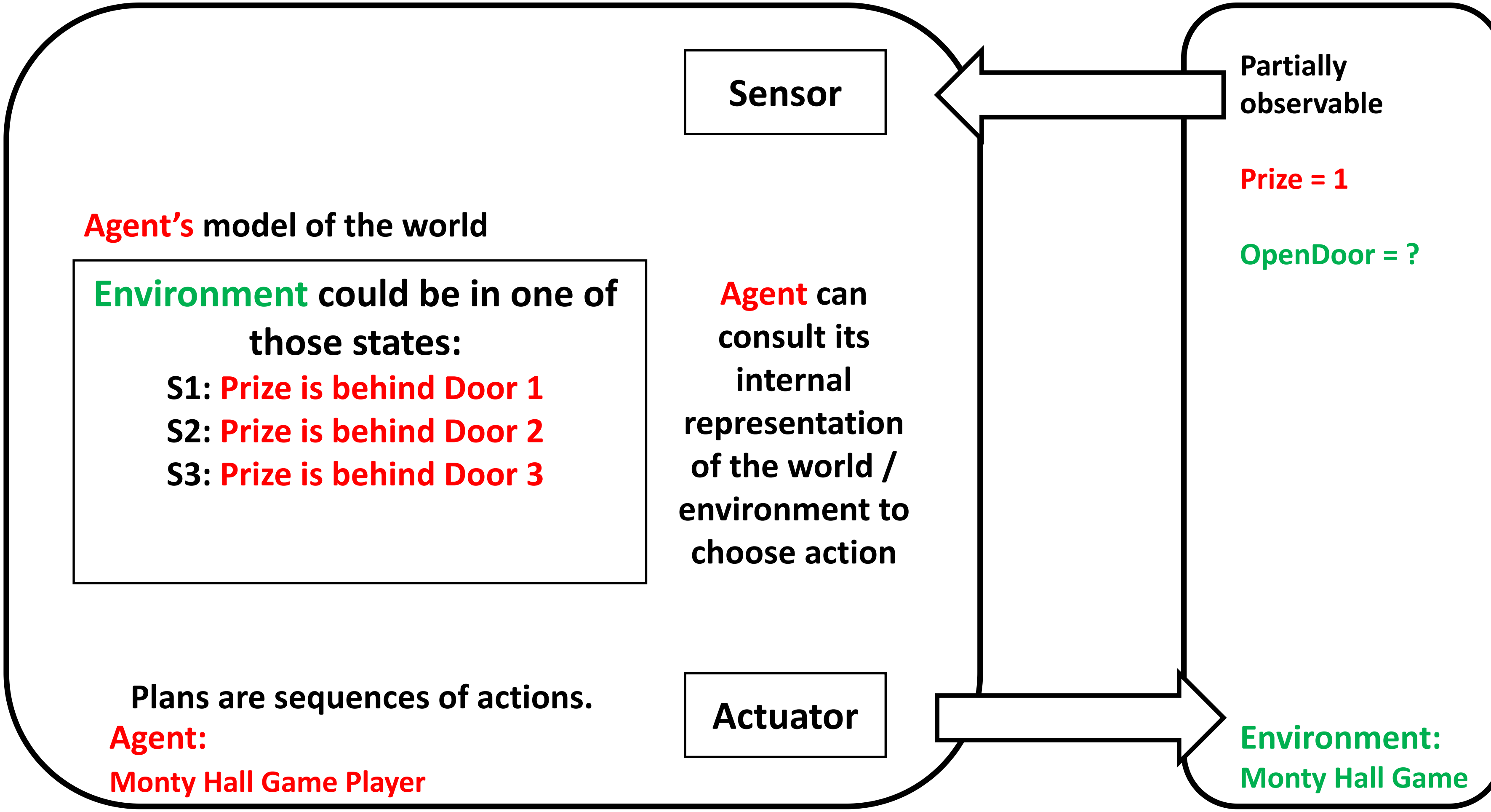
Source: https://en.wikipedia.org/wiki/Monty_Hall_problem

Monty Hall Game: Bayesian Network



Try it yourself: https://www.bayesserver.com/examples/networks/monty-hall

Monty Hall Player Agent



Assume: $D_{Prize} = \{1,2,3\}$ and $D_{OpenDoor} = \{1,2,3\}$

Monty Hall Player Agent

Sensor

Partially observable

Prize = 1

OpenDoor = ?

Agent's model of the world

Environment could be in one of those states:

S1: Prize = 1 → Belief: P(S1) = 1/3
S2: Prize = 2 → Belief: P(S2) = 1/3
S3: Prize = 3 → Belief: P(S1) = 1/3

Agent can consult its internal representation of the world / environment to choose action

Plans are sequences of actions.

Agent:
Monty Hall Game Player

Actuator

Environment:
Monty Hall Game

Assume: D_{Prize} = {1,2,3} and $D_{OpenDoor}$ = {1,2,3}

Monty Hall Player Agent

Sensor

Partially observable

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S3: Prize = 3 → Belief: P(S1) = 1/3

Agent can consult its internal representation of the world / environment to choose action

Plans are sequences of actions.

Agent:
Monty Hall Game Player

Actuator

Choice = 2

Environment:
Monty Hall Game

Assume: $D_{Prize} = \{1,2,3\}$ and $D_{OpenDoor} = \{1,2,3\}$

Monty Hall Player Agent

Sensor

Partially observable

Prize = 1

OpenDoor = 3

Agent's model of the world

Environment could be in one of those states:

S1: Prize = 1 → Belief: P(S1) = 1/3
S2: Prize = 2 → Belief: P(S2) = 1/3
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Agent can consult its internal representation of the world / environment to choose action

Plans are sequences of actions.

Agent:
Monty Hall Game Player

Actuator

Environment:
Monty Hall Game

Assume: $D_{Prize} = \{1,2,3\}$ and $D_{OpenDoor} = \{1,2,3\}$

Monty Hall Player Agent

Sensor

OpenDoor = 3

Partially observable

Prize = 1

OpenDoor = 3

Agent's model of the world

Environment could be in one of those states:

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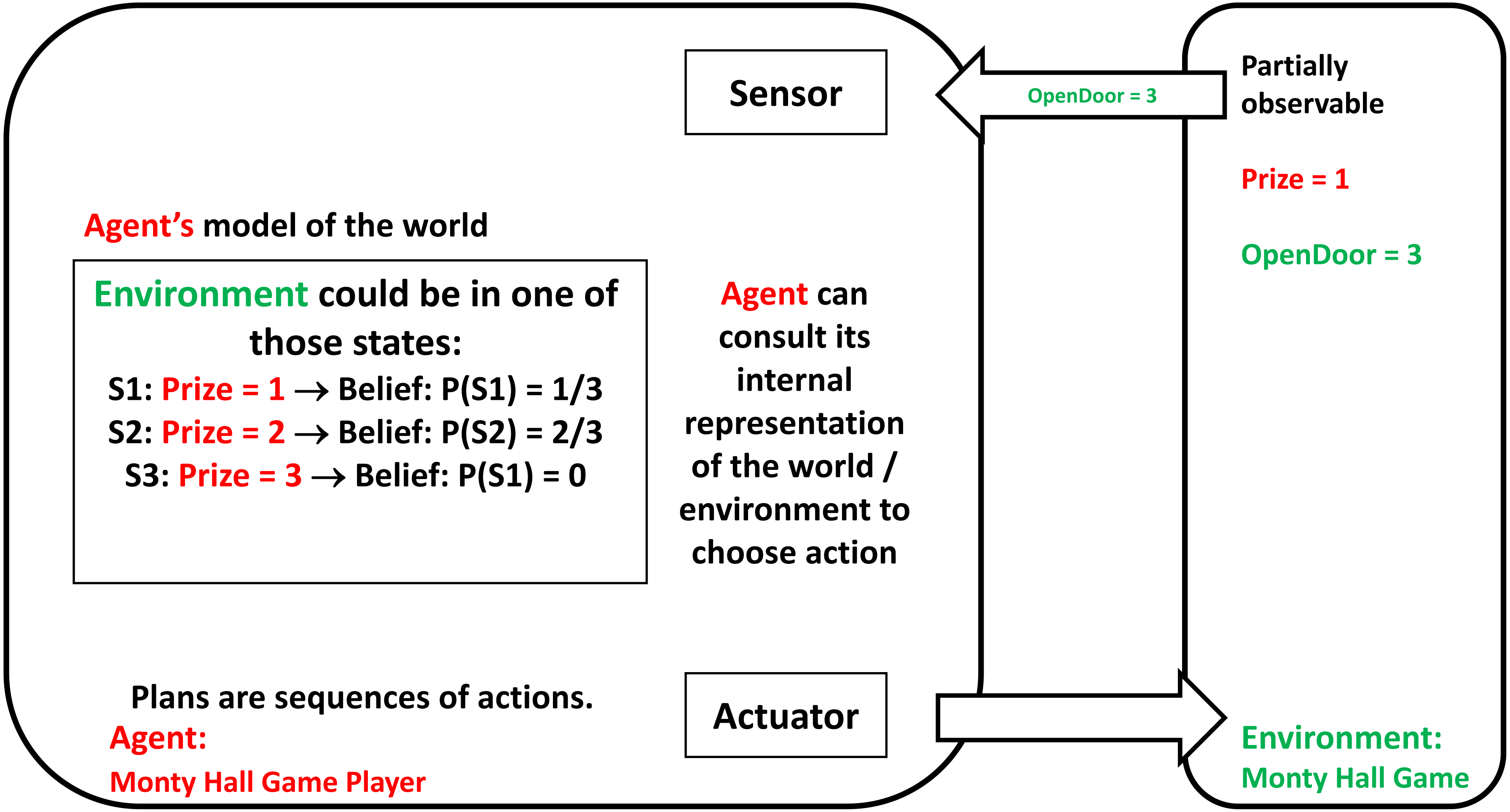
Monty Hall Player Agent



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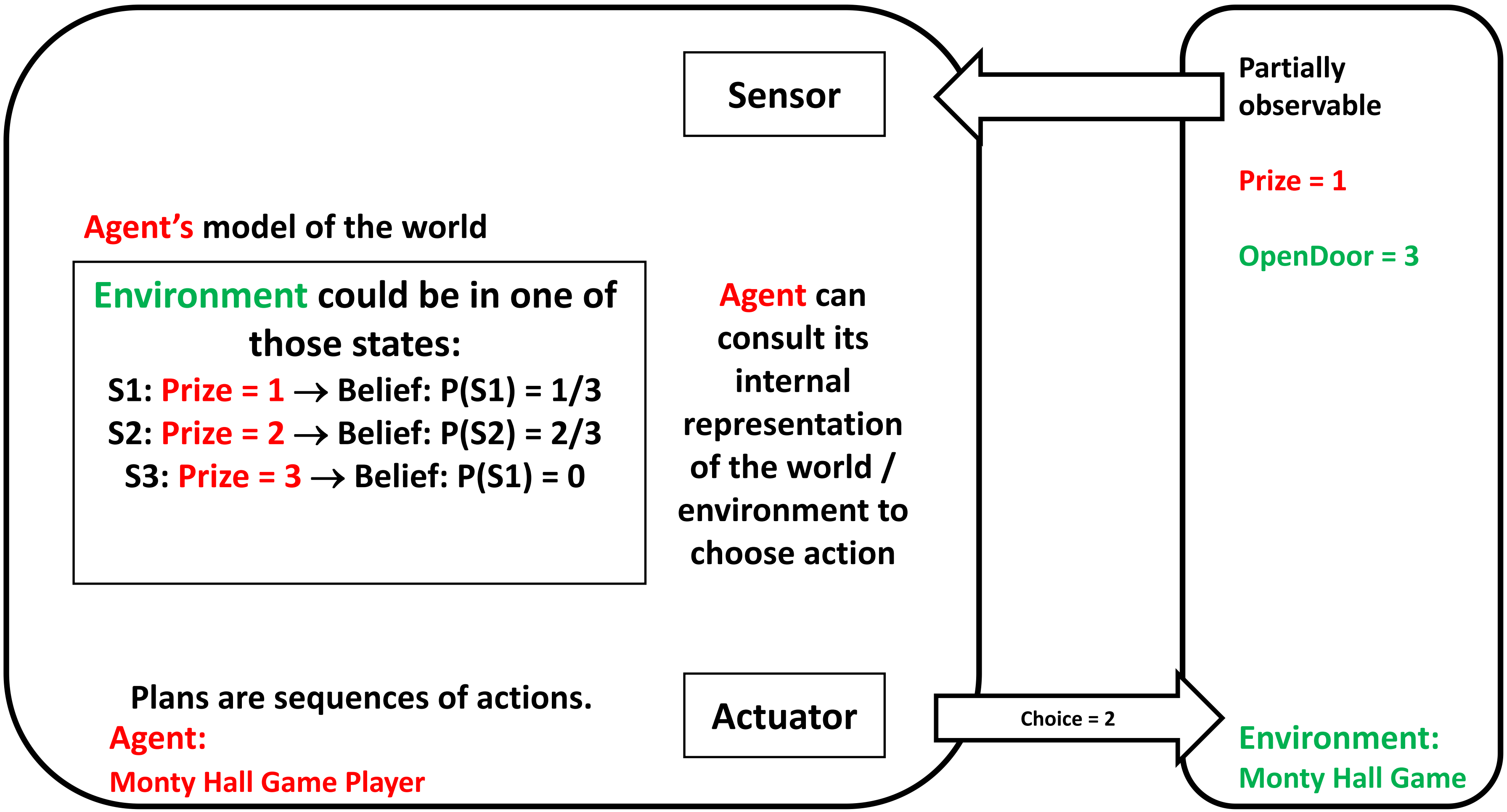
Monty Hall Player Agent



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Monty Hall Player Agent

Sensor

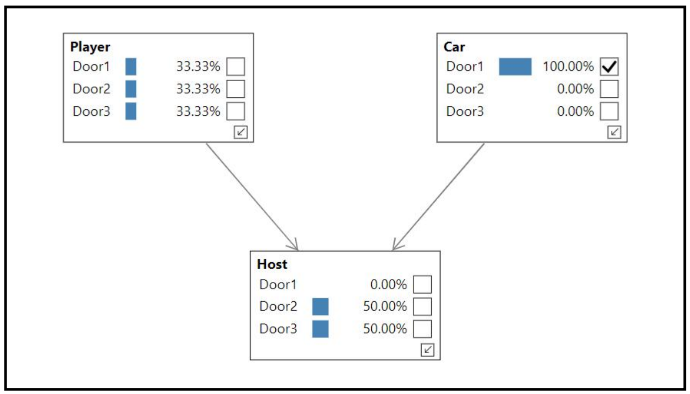
OpenDoor = 3

Partially observable

Prize = 1

OpenDoor = 3

Agent's model of the world



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Plans are sequences of actions.

Agent:

Monty Hall Game Player

Actuator

Environment:

Monty Hall Game

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Monty Hall Player Agent

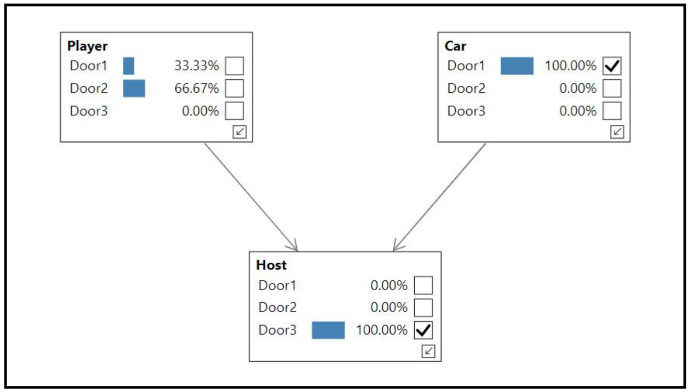
Sensor

Partially observable

Prize = 1

OpenDoor = 3

Agent's model of the world



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Monty Hall Game Player

Actuator

Choice = 2

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Monty Hall Game

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Monty Hall Player Agent

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Partially observable

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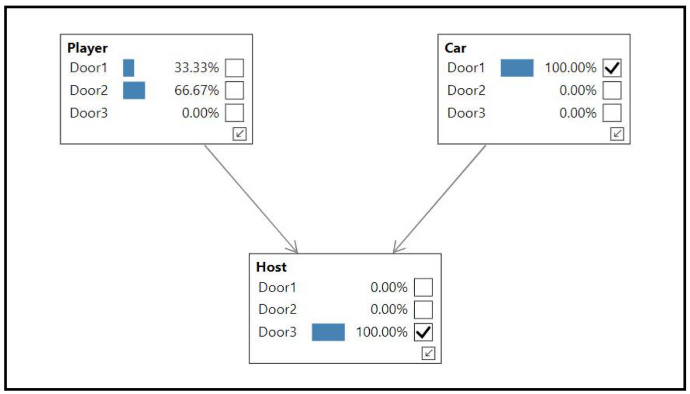
Agent's model of the world



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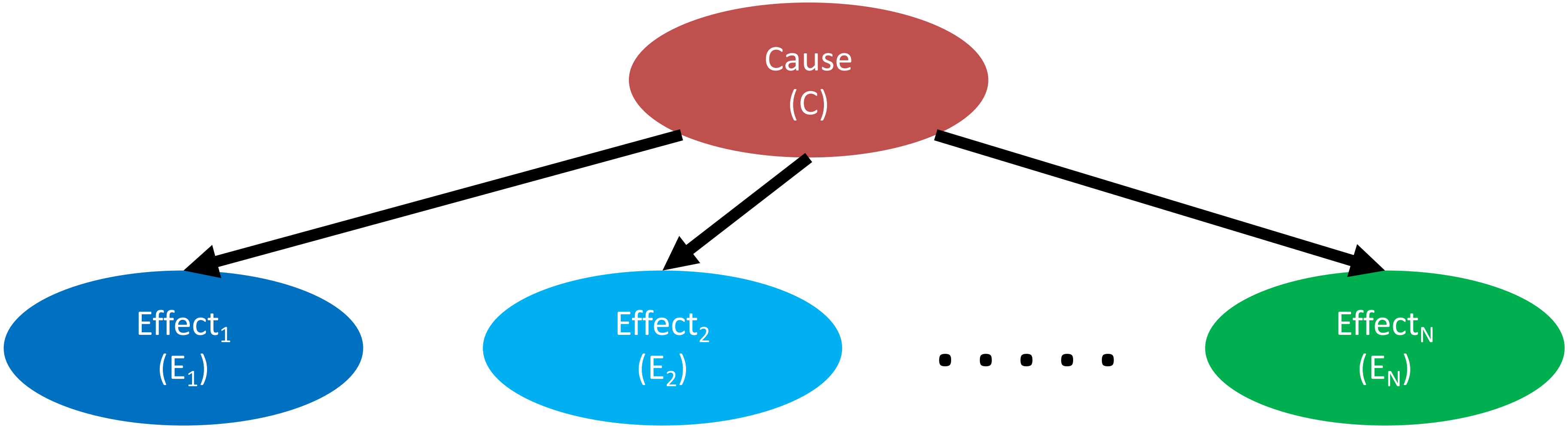
Actuator

Environment:
Monty Hall Game

Assume: $D_{Prize} = \{1,2,3\}$ and $D_{OpenDoor} = \{1,2,3\}$

Naive Bayes Models



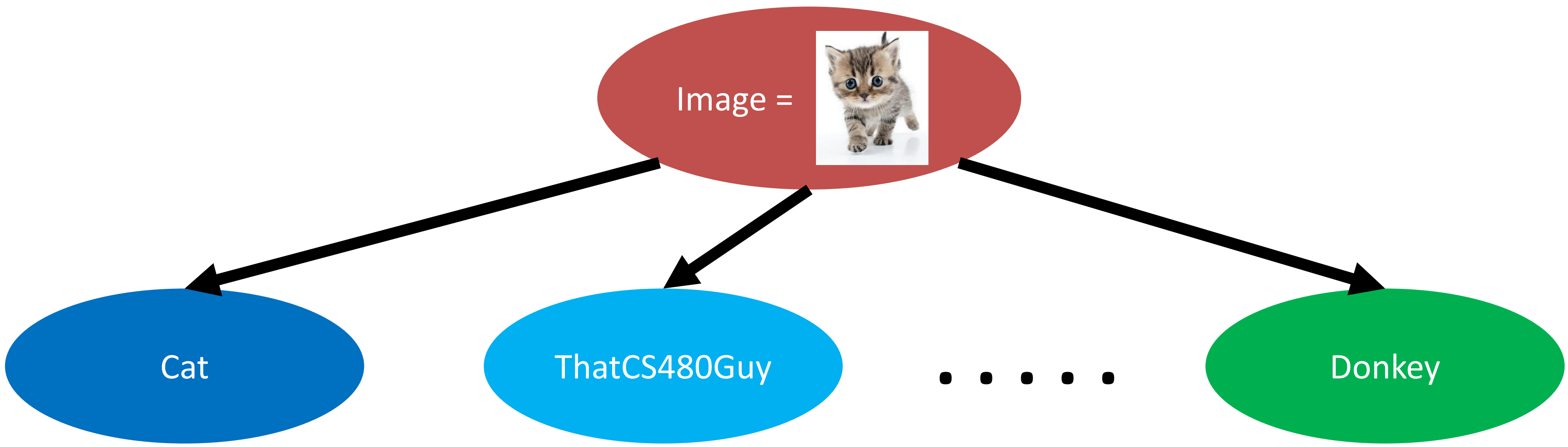
Consider a situation where all effects E_1, E_2, ..., E_N are conditionally independent given the cause. If that's true we can express full joint probability with:

$$\boldsymbol{P}(Cause, Effect_1, \ldots, Effect_N) = \boldsymbol{P}(Cause) * \prod_{\boldsymbol{i}} \boldsymbol{P}(Effect_i \mid Cause)$$

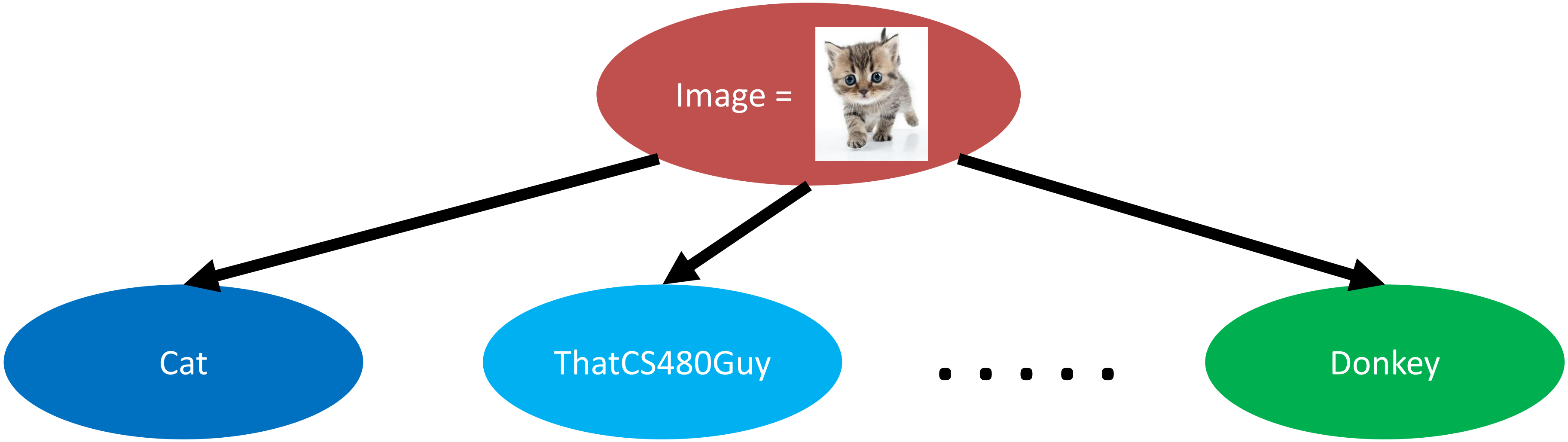
and from that:

$$\boldsymbol{P}(Cause|\boldsymbol{e}) = \alpha * \boldsymbol{P}(Cause) * \prod_{\boldsymbol{j}} \boldsymbol{P}(e_j \mid Cause)$$

Naive Bayes "Classifier"



Naive Bayes "Classifier"



P(Image | Cat) = 0.9

P(Image | ThatCS480Guy) = 0.1

...

P(Image | Donkey) = 0.3