CS 480

Introduction to Artificial Intelligence

October 12th, 2021

Announcements / Reminders

- **Midterm: Thursday! October 14th!**
  - **Online (NOT Beacon) section: please make arrangements. Contact Mr. Charles Scott (scott@iit.edu) if in doubt**
- **Programming Assignment #01:**
  - **due: ~~October 17th~~ October 22th, 11:00 PM CST**
- **Written Assignment #02:**
  - **due: October 15th, 11:00 PM CST**
- **Re-download the slides for exam preparation**
- **Grading TA assignment:**

https://docs.google.com/spreadsheets/d/1Cav_GBTGC7fLGzxuBCAUmEuJYPeF-HMLCYvwPbq8Fus/edit?usp=sharing

Written Assignment #02: Problem 3.1

I was done here, really.

Step	Resulting sentence	Applied law / rule
1	$(\neg(p \land q)) \Leftrightarrow (\neg p \lor \neg q)$ becomes: $\boxed{(\neg p \lor \neg q) \Leftrightarrow (\neg p \lor \neg q)}$	De Morgan's Law $\neg(a \land b) \equiv \neg a \lor \neg b$
2	$(\neg p \lor \neg q) \Leftrightarrow (\neg p \lor \neg q)$ becomes: $((\neg p \lor \neg q) \land (\neg p \lor \neg q)) \lor (\neg(\neg p \lor \neg q) \land \neg(\neg p \lor \neg q))$	Equivalence Law $(A \land B) \lor (\neg B \land \neg A) \equiv (A \Leftrightarrow B)$ Assume that $A \equiv \neg p \lor \neg q$, and $B \equiv \neg p \lor \neg q$
3	$((\neg p \lor \neg q) \land (\neg p \lor \neg q)) \lor (\neg(\neg p \lor \neg q) \land \neg(\neg p \lor \neg q))$ becomes: $((\neg p \lor \neg q) \land (\neg p \lor \neg q)) \lor (\neg(\neg p \lor \neg q))$	Idempotent Law $p \land p \equiv p$
4	$((\neg p \lor \neg q) \land (\neg p \lor \neg q)) \lor (\neg(\neg p \lor \neg q))$ becomes: $(\neg p \lor \neg q) \lor (\neg(\neg p \lor \neg q))$	Idempotent Law $p \land p \equiv p$
5	$(\neg p \lor \neg q) \lor (\neg(\neg p \lor \neg q))$ becomes: $(\neg p \lor \neg q) \lor \neg(\neg p \lor \neg q)$	Remove extra parentheses
6	$(\neg p \lor \neg q) \lor \neg(\neg p \lor \neg q)$ becomes: T	Law of Excluded Middle $A \lor \neg A \equiv$ T Assume that $A \equiv \neg p \lor \neg q$
7	So: $\neg(p \land q) \Leftrightarrow \neg p \lor \neg q \equiv$ T	We proved that $\neg(p \land q) \Leftrightarrow \neg p \lor \neg q$ is a tautology
Add more rows if necessary | Symbols (copy/paste): T⊥∨∧≡⇔¬⇒.∴		

Plan for Today

- **Predicate / First-Order Logic**

Predicate Logic Syntax: Summary

Predicate calculus symbols include:

- **truth symbols: true and false**
- **terms represent specific objects in the world**
  - **constants, variables and functions**
- **predicate symbols refer to a particular relation between objects or represent facts**
- **function symbols refer to objects indirectly (via some relationship)**
- **quantifiers ($\forall$ and $\exists$) and variables refer to collections of objects without explicitly naming each object**

Quantifier Nesting

Quantifiers can be nested to obtain more complex expressions. For example:

$$\forall x\ \forall y\ \mathrm{brother}(x, y) \Rightarrow \mathrm{sibling}(x, y)$$

means "Brothers are siblings". Here

$$\forall x\ \forall y\ \mathrm{sibling}(y, x) \Leftrightarrow \mathrm{sibling}(x, y)$$

a symmetric relationship is expressed.

Quantifier Nesting: Ordering

When quantifiers are nested it is important to pay attention to ordering. For example:

$$\forall x\ \exists y\ loves(x, y)$$

means "Everybody loves somebody". Here

$$\exists x\ \forall y\ loves(x, y)$$

we have "There exists someone who is loved by everyone".

Quantifier Nesting (Use Parentheses)

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Quantifier Nesting (Use Parentheses)

When quantifiers are nested it is important to pay attention to ordering. For example:

$$\forall x \, (\exists y \; loves(x, y))$$

means "Everybody loves somebody". Here

$$\exists x \, (\forall y \; loves(x, y))$$

we have "There exists someone who is loved by everyone".

Quantifier Nesting: Variable Names

Certain quantified sentences may be confusing:

$$\forall x\ (crown(x) \vee (\exists x\ brother(Richard, x)))$$

Variable x **is used twice:**

- **universally quantified** x **in** $\forall x\ (crown(x) \vee$
- **existientially quantified** x **in** $\exists x\ brother(Richard, x)$
- x **and** x **are NOT the same (different "context")**

Rule:

variable belongs to innermost quantifier that mentions it.

Quantifier Nesting: Variable Names

Solution: use different variables if necessary

$$\forall x\ (crown(x) \vee (\exists z\ brother(Richard, z)))$$

Universal Quantifier: Conjuctions

Universal quantifier ("for all") indicates that a sentence is true for all possible values of the variable. For example:

$$\forall x \; likes(x, cake)$$

is true if $likes(x, cake)$ **is** true **for all interpretations of variable** x**. Assuming that**

$$x \in \{x_1, x_2, \ldots, x_n\}$$

we can rewrite $\forall x \; likes(x, cake)$ **as:**

$$likes(x_1, cake) \wedge likes(x_2, cake) \wedge \ldots \wedge likes(x_n, cake)$$

Existential Quantifier: Disjunctions

Existential quantifier ("there exists") indicates that a sentence is true for <u>at least one value</u> of the the variable. For example:

$$\exists x \; likes(x, cake)$$

is true if $likes(x, cake)$ **is** true **for at least one interpretation of variable** x**. Assuming that**

$$x \in \{x_1, x_2, \ldots, x_n\}$$

we can rewrite $\exists x \; likes(x, cake)$ **as:**

$$likes(x_1, cake) \vee likes(x_2, cake) \vee \ldots \vee likes(x_n, cake)$$

Universal/Existential Quantifiers

We assumed that $x \in \{x_1, x_2, \ldots, x_n\}$ **and then we rewrote** $\forall x \; likes(x, cake)$ **as:**

$$likes(x_1, cake) \wedge likes(x_2, cake) \wedge \ldots \wedge likes(x_n, cake)$$

and $\exists x \; likes(x, cake)$ **as:**

$$likes(x_1, cake) \vee likes(x_2, cake) \vee \ldots \vee likes(x_n, cake)$$

From De Morgan's rules we can obtain the following equivalence:

$$\forall x \; likes(x, cake) \equiv \neg \exists x \; \neg likes(x, cake)$$

"Everyone likes cake" ≡ "Nobody dislikes cake"

Universal/Existential Q. Equivalences

Selected equivalences:

$$\forall x\ (P(x) \wedge Q(x)) \equiv \forall x\ (P(x)) \wedge \forall x\ (Q(x))$$

$$\exists x\ (P(x) \vee Q(x)) \equiv \exists x\ (P(x)) \vee \exists x\ (Q(x))$$

$$\neg[\exists x\ (N(x))] \equiv \forall x\ (\neg N(x))$$

$$\neg[\forall x\ (N(x))] \equiv \exists x\ (\neg N(x))$$

Quantifiers: Scope of Quantification

Consider the following sentence:

$$\forall x\ \underbrace{(P(x) \land Q(x))}_{\text{Scope of quantification for variable } x}$$

Variable x is universally quantified in both $P(x)$ and $Q(x)$.

In this sentence:

$$\exists x\ \underbrace{(P(x) \lor Q(y) \Rightarrow R(x))}_{\text{Scope of quantification for variable } x}$$

Variable x is existentionally quantified in both $P(x)$ and $R(x)$.

Proof by Resolution

The process of proving by resolution is as follows:

A. **Formalize the problem: "English to Predicate Logic"**

B. **Derive** $KB \wedge \neg Q$

C. **Convert** $KB \wedge \neg Q$ **into CNF ("standardized") form**

D. **Apply resolution rule to resulting clauses. New clauses will be generated (add them to the set if not already present)**

E. **Repeat (C) until:**

   a. **no new clause can be added (**KB **does NOT entail** Q**)**

   b. **last two clauses resolve to yield the empty clause (**KB **entails** Q**)**

Proof by Resolution

The process of proving by resolution is as follows:

A. **Formalize the problem: “English to Predicate Logic”**
B. **Negate the input statement/claim C to obtain $\neg C$**
C. **Convert $\neg C$ into CNF (“standardized”) form**
D. **Apply resolution rule to resulting clauses. New clauses will be generated (add them to the set if not already present)**
E. **Repeat (C) until:**
    a. **no new clause can be added (C is false)**
    b. **last two clauses resolve to yield the empty clause (C is true)**

Problem: Argument Mismatch

The goal of the resolution process is to find a contradiction. When two contradicting literals “cancel” each other out, we can end up with an empty clause:

$$\frac{(w), (\neg w)}{()}$$

It is not so easy in predicate logic. This

$$\frac{(setting(sun)), (\neg setting(sun))}{()}$$

will work (predicate arguments match). This

$$\frac{(beautiful(day)), (\neg beautiful(night))}{???????}$$

will not, because predicate arguments don’t match.

Predicate (First-Order) Logic to CNF

Variables and quantifiers are a challenge:

1. Eliminate all equivalences $\Leftrightarrow$ and implications $\Rightarrow$
2. Reduce the scope of all $\neg$ to single term (De Morgan)
3. Make all variable names unique (standardize apart)
4. Move quantifiers left (convert to prenex normal form)
5. Eliminate Existential quantifiers (skolemization)
6. Eliminate Universal quantifiers
7. Convert to conjunction of disjuncts
8. Create separate clause for each conjunct

1. Remove Equivalences/Implications

Use propositional logic laws to do it where possible.

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2. Reduce the Scope of All $\neg$

Consider a predicate $N(x)$ asserting the fact that x is non-vegetarian. Now, let's create the following sentence:

$$\forall x\ (\neg N(x))$$

Which roughly translates to "No one is a non-vegetarian." Let's try a slightly different sentence:

$$\neg[\forall x\ (N(x))]$$

Which roughly translates to "It is not true that everyone is a non-vegetarian". This also means "At least one person is a non-vegetarian" and we could rewrite it as:

$$\exists x\ (\neg N(x)),\ \textbf{so}\ \neg[\forall x\ (N(x))] \equiv \exists x\ (\neg N(x))$$

Using the following logic, we can establish this equivalence:

$$\forall x\ (\neg N(x)) \equiv \neg[\exists x\ (\neg N(x))]$$

2. Reduce the Scope of All $\neg$

Recall that for a domain of objects $\{a, b, c\}$, the sentence

$\forall x\ (N(x))$ **is equivalent to** $N(a) \wedge N(b) \wedge N(c)$

Similarly, the sentence equivalence:

$\exists x\ (N(x))$ **is equivalent to** $N(a) \vee N(b) \vee N(c)$

Now, if we apply De Morgan's rules, we get the following:

$$\neg[N(a) \vee N(b) \vee N(c)] \equiv [\neg N(a) \wedge \neg N(b) \wedge \neg N(c)]$$

and:

$$\neg[N(a) \wedge N(b) \wedge N(c)] \equiv [\neg N(a) \vee \neg N(b) \vee \neg N(c)]$$

2. Reduce the Scope of All $\neg$

Recall that for a domain of objects $\{a, b, c\}$**, the sentence**

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Similarly, the sentence equivalence:

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Now, if we apply De Morgan's rules, we get the following:

$$\neg[\underbrace{N(a) \vee N(b) \vee N(c)}_{\neg[\exists x\ (N(x))]}] \equiv [\underbrace{\neg N(a) \wedge \neg N(b) \wedge \neg N(c)}_{\forall x\ (\neg N(x))}]$$

and:

$$\neg[\underbrace{N(a) \wedge N(b) \wedge N(c)}_{\neg[\forall x\ (N(x))]}] \equiv [\underbrace{\neg N(a) \vee \neg N(b) \vee \neg N(c)}_{\exists x\ (\neg N(x))}]$$

Predicate (First-Order) Logic to CNF

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3. Make All Variable Names Unique

Given a qualified sentence:

$$\forall x\ (\mathrm{crown}(x) \lor (\exists x\ \mathrm{brother}(\mathrm{Richard}, x))$$

change variables to avoid duplicates:

$$\forall x\ (\mathrm{crown}(x) \lor (\exists z\ \mathrm{brother}(\mathrm{Richard}, z))$$

3. Make All Variable Names Unique

Given a qualified sentence:

$$\forall x\ (P(x) \Rightarrow Q(x)) \wedge \exists x\ (Q(x)) \wedge \exists z\ (P(z)) \wedge \exists z\ (Q(z) \Rightarrow R(z))$$

change variables to avoid duplicates:

$$\forall y\ (P(y) \Rightarrow Q(y)) \wedge \exists u\ (Q(u)) \wedge \exists w\ (P(w)) \wedge \exists z\ (Q(z) \Rightarrow R(z))$$

Also called: "standardizing variables apart"

Predicate (First-Order) Logic to CNF

Variables and quantifiers are a challenge:

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4. Move Quantifiers Left

Consider the following sentence in predicate logic:

$$\exists x\ (A(x) \vee \forall x\ (B(x)))$$

The two occurences of x **(**x **and** x**) do not refer to the same variable. Let's make all variables unique (standardize) first:**

$$\exists x\ (A(x) \vee \forall y\ (B(y)))$$

Because variable y **bound by** $\forall y$ **does not interact with the variable** x **bound by** $\exists x$**, we can extend the scope of the universal quantifier** $\forall y$ **to entire sentence:**

$$\exists x\ \forall y\ (A(x) \wedge (B(y)))$$

Now, $A(x) \wedge (B(y)$ **is almost a propositional logic sentence.**

4. Move Quantifiers Left | PNF

A predicate logic formula φ is in prenex normal form (PNF) if it holds that:

- $\varphi = Q_1 x_1 \ldots Q_n x_n \psi$
- ψ **is a quantifierless sentence**
- $Q_i \in \{\forall, \exists\}$ **for** $i = 1, \ldots, n$

For example this sentence is NOT in PNF:

$$\exists x\, (A(x) \vee \forall y\, (B(y)))$$

This sentence is in PNF:

$$\exists x\, \forall y\, (A(x) \wedge (B(y)))$$

4. Move Quantifiers Left | PNF

Every predicate logic sentence can be transformed into an equivalent sentence in prenex normal form.

Predicate (First-Order) Logic to CNF

Variables and quantifiers are a challenge:

1. Eliminate all equivalences $\Leftrightarrow$ and implications $\Rightarrow$
2. Reduce the scope of all $\neg$ to single term (De Morgan)
3. Make all variable names unique (standardize apart)
4. Move quantifiers left (convert to prenex normal form)
5. $\boxed{\text{Eliminate Existential quantifiers (skolemization)}}$
6. Eliminate Universal quantifiers
7. Convert to conjunction of disjuncts
8. Create separate clause for each conjunct

5. Eliminating Existential Quantifiers

In order to convert a predicate logic to a propositional logic CNF form we need to remove quantifiers.

Existential quantifiers can appear in sentences:

- **in isolation ($\exists$ is OUTSIDE the scope of some $\forall$):**

$$\exists x\ (A(x))$$

This can be resolved using Skolem constant(s).

- **in relation ($\exists$ is INSIDE the scope of some $\forall$)**

$$\forall y\ (\exists x\ (A(x,\ y)))$$

This can be resolved using Skolem function(s).

The process is known as skolemization.

5. Eliminating Existential Quantifiers

Any object variable that is existentially quantified outside of the scope of a universal quantifier, such as

$$\exists x\ (A(x))$$

can be replaced by a single new constant expression $A(t)$, where t is a Skolem constant:

$$\exists x\ (A(t))$$

and the existential quantifier can be dropped to obtain

$$A(t)$$

5. Eliminating Existential Quantifiers

With multiple variables that are existentially quantified outside of the scope of a universal quantifier, such as

$$\exists x, y\ (A(x, y))$$

multiple corresponding Skolem constants (t, u) will be needed to create a new constant expression $A(t, u)$,

$$\exists x, y\ (A(t, u))$$

and the existential quantifier can be dropped to obtain

$$A(t, u)$$

5. Eliminating Existential Quantifiers

With multiple variables bound to <u>different</u> existential quantifiers <u>outside of</u> the scope of a <u>universal</u> quantifier:

$$[\exists x\ (B(x))] \lor [\exists y\ (C(y))]$$

multiple Skolem constants t, u will be needed to create new constant expressions $B(t)$ and $C(u)$:

$$[\exists x\ (B(t))] \lor [\exists y\ (C(u))]$$

and existential quantifiers can be dropped to obtain

$$[B(t)] \lor [C(u)]$$

5. Eliminating Existential Quantifiers

Any object variable that is existentially quantified <u>outside</u> <u>of</u> the scope of a <u>universal</u> quantifier, such as

$$[\forall x\ (B(x))] \vee [\exists y\ (C(y))]$$

can be replaced by a single new constant expression $C(t)$, where t is a Skolem constant:

$$[\forall x\ (B(x))] \vee [\exists y\ (C(t))]$$

and the existential quantifier can be dropped to obtain

$$[\forall x\ (B(x))] \vee [(C(t))]$$

5. Eliminating Existential Quantifiers

We can say that following predicate logic sentences:

$$\exists x\ (A(x)) \equiv_I A(t)$$

$$\exists x, y\ (A(x, y)) \equiv_I A(t, u)$$

$$[\exists x\ (B(x))] \vee [\exists y\ (C(y))] \equiv_I [B(t)] \vee [C(u)]$$

$$[\forall x\ (B(x))] \vee [\exists y\ (C(y))] \equiv_I [\forall x\ (B(x))] \vee [(C(t))]$$

are <u>inferentially</u> equivalent ($\equiv_I$). Skolemization leads to sentences that are not completely equivalent, but this is good enough for proofs and inference.

5. Eliminating Existential Quantifiers

Inferentially equivalent sentences are not completely equivalent, but this is good enough for proofs. Why?

Consider following two predicate logic sentences:

$\exists x\, (studies(x))$: **there exist at least one** x **who studies**

$studies(t)$: t **studies (just one, specific object** t**)**

Constant t **is assumed to be a possible value for variable** x**. If for some object** t, $studies(t)$ **is true, then** $\exists x\, (studies(x))$ **also must be true (**t **and possibly other objects** $study$**).**

5. Eliminating Existential Quantifiers

Note: when choosing a Skolem constant for a existentially quantified expressions such as:

$$\exists x\ (A(x))$$

DON'T choose EXISTING constants as Skolem constants to create a new constant expression $A(t)$, where t is a Skolem constant:

So: $\exists x\ (A(t))$ **YES,**

but $\exists x\ (A(lukeSkywalker))$ **NO**

Assuming that lukeSkywalker **is an existing object.**

5. Eliminating Existential Quantifiers

Any object variable that is existentially quantified inside of the scope of a universal quantifier, such as:

$$\forall y\ (\exists x\ (A(x, y)))$$

can be replaced by with a Skolem function of the universal variable $f(y)$:

$$\forall y\ (\exists x\ (A(f(y), y)))$$

and the existential quantifier can be dropped to obtain

$$\forall y\ (A(f(y), y))$$

5. Eliminating Existential Quantifiers

An existential quantifier inside of the scope of MORE THAN ONE universal quantifier, such as:

$$\forall y \ \forall z \ (\exists x \ (B(x, y, z)))$$

can be replaced by with a multivariable Skolem function of $g(y, z)$:

$$\forall y \ \forall z \ (\exists x \ (B(g(y, z), y, z)))$$

and the existential quantifier can be dropped to obtain

$$\forall y \ \forall z \ (B(g(y, z), y, z))$$

5. Eliminating Existential Quantifiers

Consider the following example:

$$\forall x\ [\exists y\ (A(x) \Rightarrow B(y)) \lor \forall w\ (\exists z\ (D(x) \land E(w) \land F(z) \Rightarrow C(z))]$$

can be modified using skolemization to obtain:

$$\forall x\ [(A(x) \Rightarrow B(\underline{f(x)})) \lor \forall w\ ((D(x) \land E(w) \land F(\underline{g(x, w)}) \Rightarrow C(\underline{g(x, w)})))]$$

Skolem functions $f()$ and $g()$.

Variable x is inside the scope of $\forall x$, hence: $f(x)$

Variable z is inside the scope of $\forall x$ and $\forall w$, hence: $g(x, w)$

5. Eliminating Existential Quantifiers

In general: existential quantifiers can also be eliminated through the use of Existential Instantiation.

For any sentence S, variable x, and constant symbol k (that <u>does not appear anywhere in the knowledge base</u>), we can use the following rule:

$$\frac{\exists x\ S}{\mathrm{SUBST}(\{x / k\}, S)}$$

Where is a result of applying substitution $\{x / k\}$ to the sentence S.

5. Eliminating Existential Quantifiers

For example, from the sentence:

$$\exists x\ (crown(x) \land onHead(x, John))$$

we can infer the sentence

$$crown(C_1) \land onHead(C_1, John)$$

using the substitution $\{x / C_1\}$ **as long as** C_1 **does not exist in the knowledge base.**

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5. Eliminate Existential quantifiers (skolemization)
6. Eliminate Universal quantifiers
7. Convert to conjunction of disjuncts
8. Create separate clause for each conjunct

6. Eliminating Universal Quantifiers

In general universal quantifiers can also be eliminated through the use of Universal Instantiation.

For any sentence S, variable x, and constant symbol g (that does not appear anywhere in the knowledge base), we can use the following rule:

$$\frac{\forall x\ S}{\text{SUBST}(\{x / g\}, S)}$$

Where is a result of applying substitution $\{x / g\}$ to the sentence S.

6. Eliminating Universal Quantifiers

For example, from the sentence:

$$\forall x\ (king(x) \wedge greedy(x) \Rightarrow evil(x))$$

we can infer the sentence

$$king(John) \wedge greedy(John) \Rightarrow evil(x)$$

using the substitution $\{x\ /\ John\}$**.**

Problem: Argument Mismatch

The goal of the resolution process is to find a contradiction. When two contradicting literals "cancel" each other out, we can end up with an empty clause:

$$\frac{(w), (\neg w)}{()}$$

It is not so easy in predicate logic. This

$$\frac{(setting(sun)), (\neg setting(sun))}{()}$$

will work (predicate arguments match). This

$$\frac{(beautiful(day)), (\neg beautiful(night))}{???????}$$

will not, because predicate arguments don't match.

Unification: Next Lecture