Neural Word Embeddings

CS-585

Natural Language Processing

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Word vectors: problems

- We saw last time how to build vectors to represent words:
  - One-hot encoding
    - Binary, count, tf*idf representations
  - Hashing trick

$$\text{cat} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \text{dog} = \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \text{mouse} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ 0 \end{bmatrix}, \text{bird} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

- Some problems
  - Large dimensionality of word vectors
  - Lack of meaningful relationships between words

Word vectors: problems

$$\text{cat} = \begin{bmatrix}1\\0\\\vdots\\0\\0\end{bmatrix}, \text{dog} = \begin{bmatrix}0\\1\\\vdots\\0\\0\end{bmatrix}, \text{mouse} = \begin{bmatrix}0\\0\\\vdots\\1\\0\end{bmatrix}, \text{bird} = \begin{bmatrix}0\\0\\\vdots\\0\\1\end{bmatrix}$$

$$CosineSimilarity(cat, dog) = \frac{v_{cat} \cdot v_{dog}}{\|v_{cat}\|\|v_{dog}\|} = 0$$

D_1="the **cat** and the **mouse**" $= \begin{bmatrix}1\\0\\\vdots\\1\\0\end{bmatrix}$, D_2="the **cat** and the **bird**" $= \begin{bmatrix}1\\0\\\vdots\\0\\1\end{bmatrix}$

$$CosineSimilarity(D_1, D_2) = \frac{D_1 \cdot D_2}{\|D_1\|\|D_2\|} = \frac{1}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$$

How to learn word meanings

1. Look in a dictionary/thesaurus
   - "Dictionary-based learning"
2. Ask people
   - Word association and similarity judgement tasks
3. Distributional hypothesis
   - Words that occur in the same contexts tend to be similar in meaning

Distributional hypothesis

You shall know a word by the company it keeps

J.R. Firth, 1957

...if we consider words or morphemes A and B to be more different in meaning than A and C, then we will often find that the distributions of A and B are more different than the distributions of A and C. In other words, difference of meaning correlates with difference of distribution.

Zellig Harris, 1970

Contexts and word meanings

ed to this height .) **Apple** trees grew there also
referred to get their **apple** pies at the local bak
cally this : Wright's **apple** pie ; ; peel , core ,
ard for mom and mom's **apple** pie goes with : Af In
ned by drinking sweet **apple** juice in which pulver
ile as big as a small **apple** . The odor here was m
an upper bough of the **apple** tree . The primary qu
oo small to reach the **apple** tree bough , to twist

ly shaped like a large **pear** , and when properly ri
ncy of a ripe Bartlett **pear** , but oily . The avoca
neither sweet , like a **pear** , nor tart like an ora
side of the house to a **pear** tree , with crowds alr
d to that of `` a rich **pear** '' . Though she did no
C was at the DiGiorgio **pear** orchards in Yuba Count

e strode proudly into **Orange** Square , smiling like
e to a stretch of old **orange** groves , the tree dec
the juice out of the **orange** , now is it '' ? ? ``
robbing wall of fiery **orange** brown haze . Ben Prim
owers squirting great **orange** billows . A wave of f
up into one of those **orange** streetcars , rode awa
r the baby , milk and **orange** juice and vitamins an
n , nine mint , seven **orange** -- around the curve t
e thought . A shot of **orange** juice would make ever

Context may be a word in the neighborhood, a local syntactic relation, or document-level occurrence

USING THE DISTRIBUTIONAL HYPOTHESIS TO LEARN WORD REPRESENTATIONS

Latent Semantic Analysis

- One way to use the distributional hypothesis to learn word vectors is called *latent semantic analysis* (Landauer & Dumais, 1997)
- The idea is to construct a matrix that represents words and their context, and then create a reduced-dimensionality version of that matrix that preserves the most distinctive and important characteristics of words' contextual associations

Term-by-document matrix

- Recall that given a sparse encoding of our words as vectors in $\mathbb{R}^{|V|}$, where V is our vocabulary, we can create a vector for a document by aggregating over the vectors for all the words it contains
  - Binary, count, tf*idf vectorization
- So for $\mathrm{V} = \{the, fox, dog, lazy, jumped, over\}$, using a count vectorizer, we have

"the lazy fox jumped" $\rightarrow [1 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0]$
"the fox jumped over the dog" $\rightarrow [2 \quad 1 \quad 1 \quad 0 \quad 1 \quad 1]$
"the lazy dog" $\rightarrow [1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0]$
"the dog jumped over the dog" $\rightarrow [2 \quad 0 \quad 2 \quad 0 \quad 1 \quad 1]$
"the fox jumped" $\rightarrow [1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0]$

Term-by-document matrix

"the lazy fox jumped" $\rightarrow [1 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0]$

"the fox jumped over the dog" $\rightarrow [2 \quad 1 \quad 1 \quad 0 \quad 1 \quad 1]$

"the lazy dog" $\rightarrow [1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0]$

"the dog jumped over the dog" $\rightarrow [2 \quad 0 \quad 2 \quad 0 \quad 1 \quad 1]$

"the fox jumped" $\rightarrow [1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0]$

D_1	1	1	0	1	1	0
D_2	2	1	1	0	1	1
D_3	1	0	1	1	0	0
D_4	2	0	2	0	1	1
D_5	1	1	0	0	1	0

Term-by-document matrix

	V_1 = the	V_2 = fox	V_3 = dog	V_4 = lazy	V_5 = jumped	V_6 = over
D_1	1	1	0	1	1	0
D_2	2	1	1	0	1	1
D_3	1	0	1	1	0	0
D_4	2	0	2	0	1	1
D_5	1	1	0	0	1	0

Term-by-document matrix

$$v_2 = [1 \quad 1 \quad 0 \quad 0 \quad 1]$$
$$v_4 = [1 \quad 0 \quad 1 \quad 0 \quad 0]$$

	V_1 = the	V_2 = fox	V_3 = dog	V_4 = lazy	V_5 = jumped	V_6 = over
D_1	1	1	0	1	1	0
D_2	2	1	1	0	1	1
D_3	1	0	1	1	0	0
D_4	2	0	2	0	1	1
D_5	1	1	0	0	1	0

Word vectors in term-by-document matrix

$$\begin{aligned}
\text{the} &= v_1 = [1 \quad 2 \quad 1 \quad 2 \quad 1] \\
\text{fox} &= v_2 = [1 \quad 1 \quad 0 \quad 0 \quad 1] \\
\text{dog} &= v_3 = [0 \quad 1 \quad 1 \quad 2 \quad 0] \\
\text{lazy} &= v_4 = [1 \quad 0 \quad 1 \quad 0 \quad 0] \\
\text{jumped} &= v_5 = [1 \quad 1 \quad 0 \quad 1 \quad 1] \\
\text{over} &= v_6 = [0 \quad 1 \quad 0 \quad 1 \quad 0]
\end{aligned}$$

Each word is represented as a vector of values related to that word's occurrence in a document

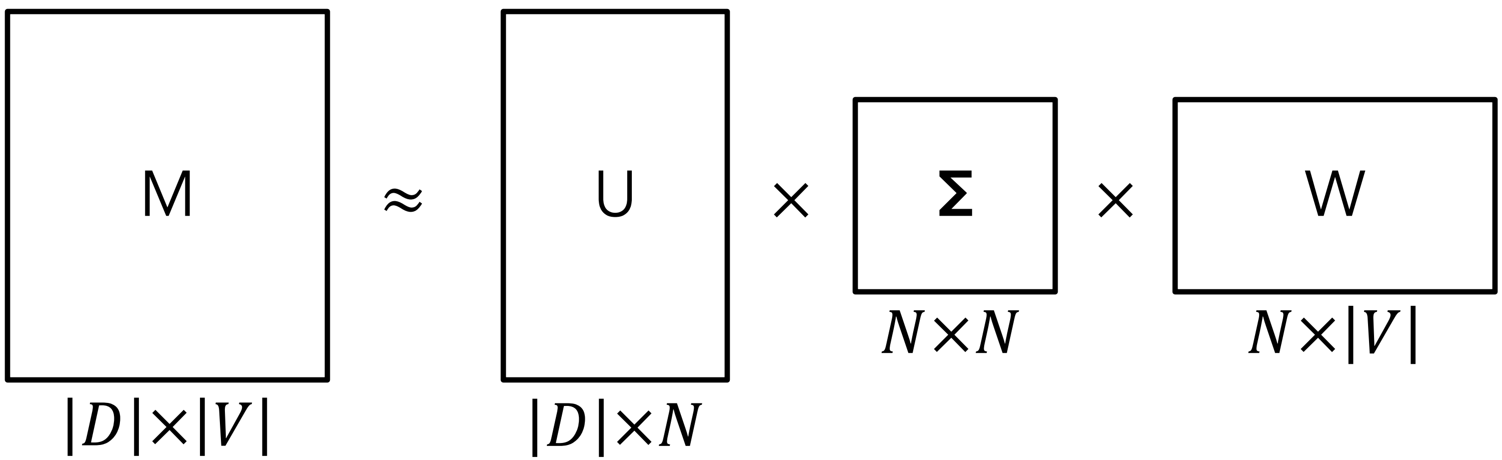
- Instead of $\mathbb{R}^{|V|}$, each vector is now in $\mathbb{R}^{|D|}$. (Vector length = number of documents.)
  - Still pretty high dimension
- Generally, $v_i \cdot v_j \neq 0$! (Vectors are not orthogonal.)

Latent semantic analysis

- The idea is to "compress" the representation of a word, using only $M \ll |D|$ dimensions for each vector
  - Compress for more efficient representation (smaller memory footprint)
  - Compress for *generalization*: retain only most important information, and allow distinctions between similar words to be obscured
- How to do this automatically?

Singular value decomposition

For a term-by-document matrix M, based on documents D and vocabulary V, we approximate



U is an orthogonal matrix with one row per document

W is an orthogonal matrix with one column per word

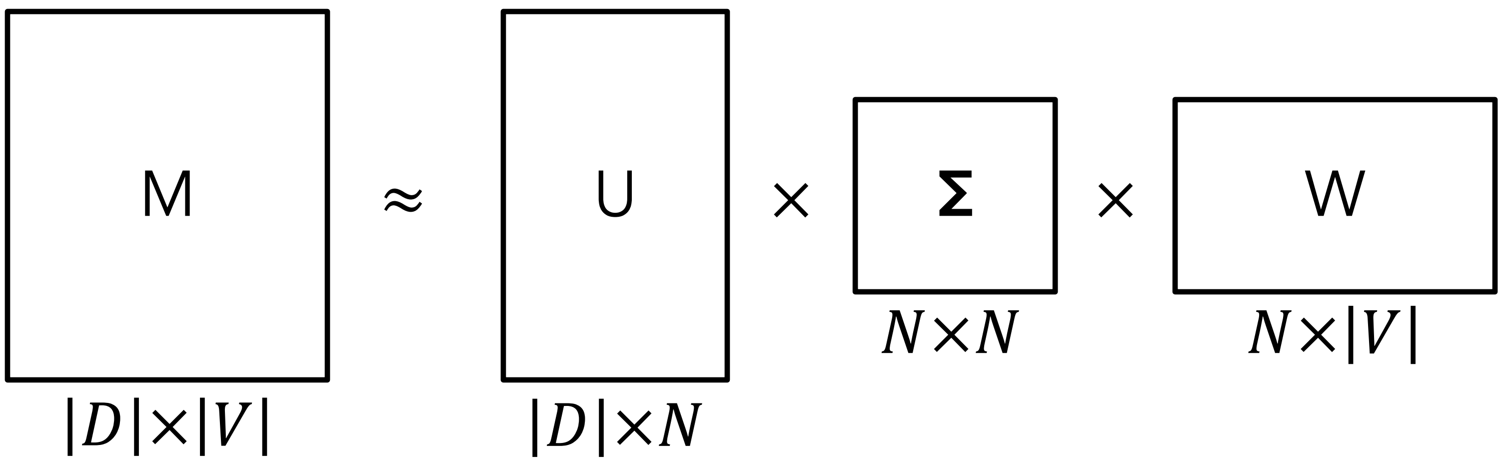
Σ is a diagonal matrix of "singular values"

Singular value decomposition

$$\underbrace{M}_{|D|\times|V|} \approx \underbrace{U}_{|D|\times N} \times \underbrace{\boldsymbol{\Sigma}}_{N\times N} \times \underbrace{W}_{N\times|V|}$$

Cannot be factored exactly, because $N \ll \min(|D|, |V|)$

Singular value decomposition



Approximate factorization can be done with the objective of minimizing $\sum_{i=1}^{|D|} \sum_{j=1}^{|V|} \left(M_{ij} - \widehat{M}_{ij}\right)^2$, where $\widehat{M} = U \times \Sigma \times W$

LSA: example

- [Notebook]

Related approaches

- pLSA: Word vectors based on document-based co-occurrence, but in graphical modeling framework
- Non-negative matrix factorization (NNMF): Constrain matrices in decomposition to include no negative values (for interpretability)

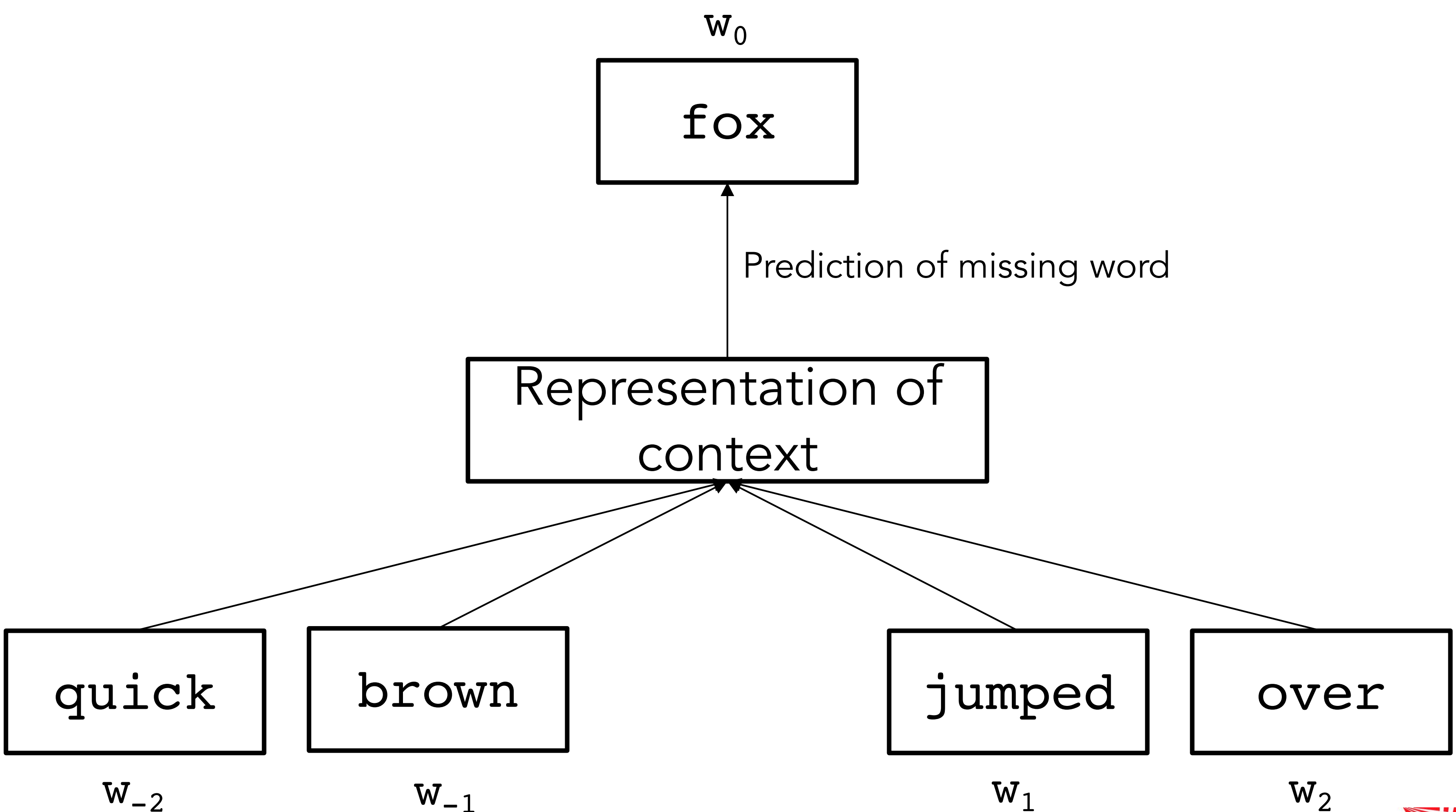
Neural Word Embeddings: `word2vec`

- Efficient and widely-used method for getting word embeddings (vectors) using a neural network framework
- What is a neural network?
  - Uses vector/matrix/tensor representations
  - Applies a sequence of algebraic operations (matrix multiplication, etc.)
  - Trained using some variant of gradient descent

`word2vec` model

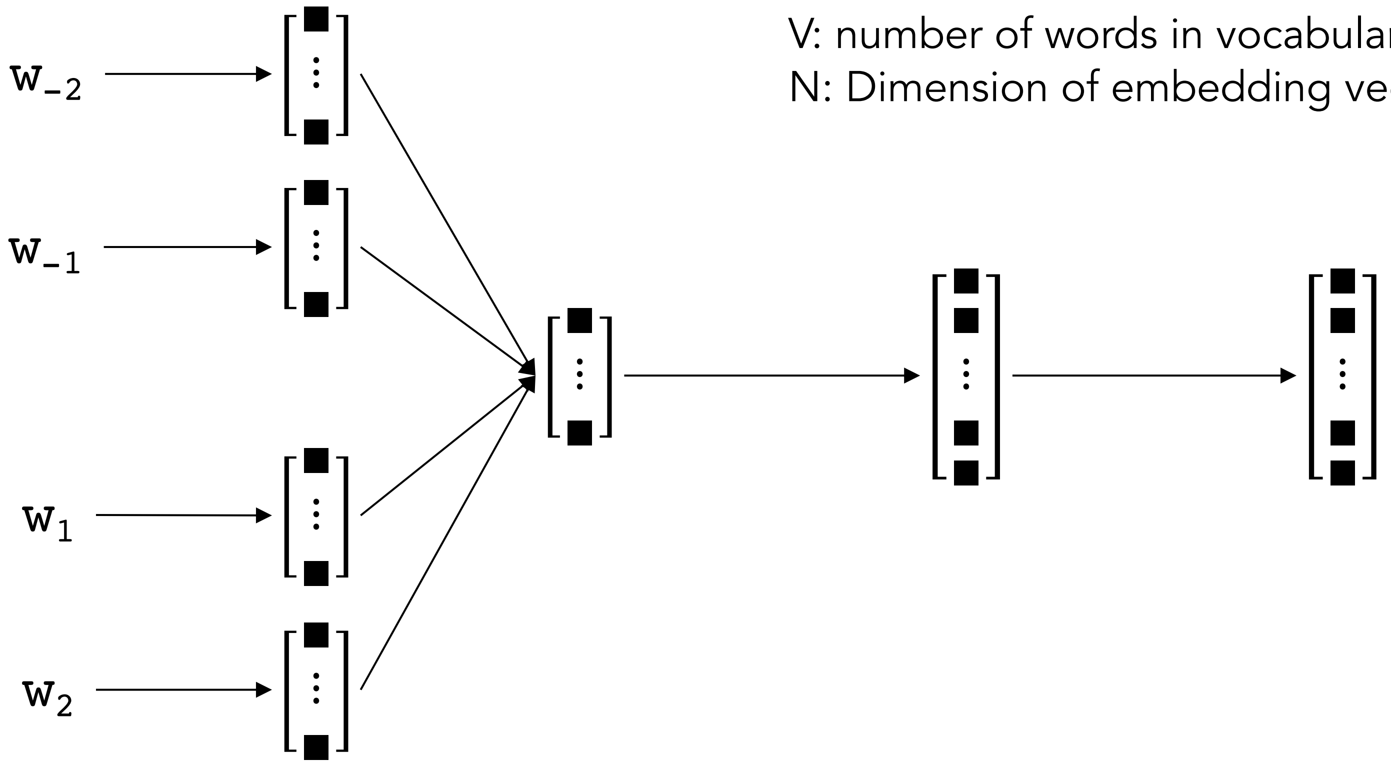
- The idea is that we will build a model that tries to predict the word that will show up in a given context
  - Alternatively, predict the context that is appropriate for a given word
- In the course of trying to do this task accurately, the model will be forced to select which information about a word/context is most important to represent
- *Representation learning* – automatically learning useful features for a task, rather than manually creating them
  - In this case, we want to learn a vector of useful attributes of words, rather than recording things like “is a noun”, “refers to a person”, “female gender”, etc.

word2vec
Contextual Bag of Words (CBOW)



word2vec
Contextual Bag of Words (CBOW)

V: number of words in vocabulary
N: Dimension of embedding vectors



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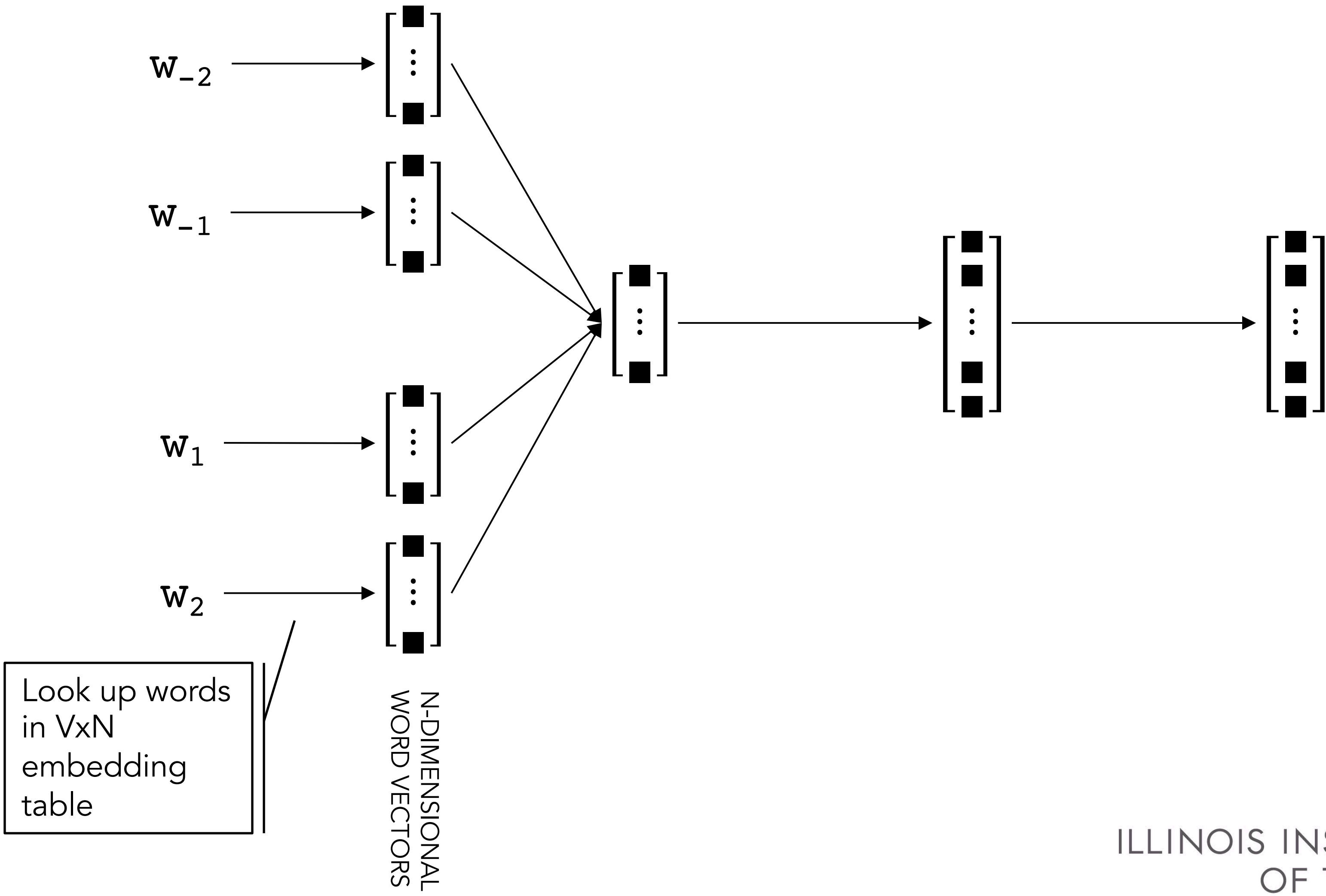

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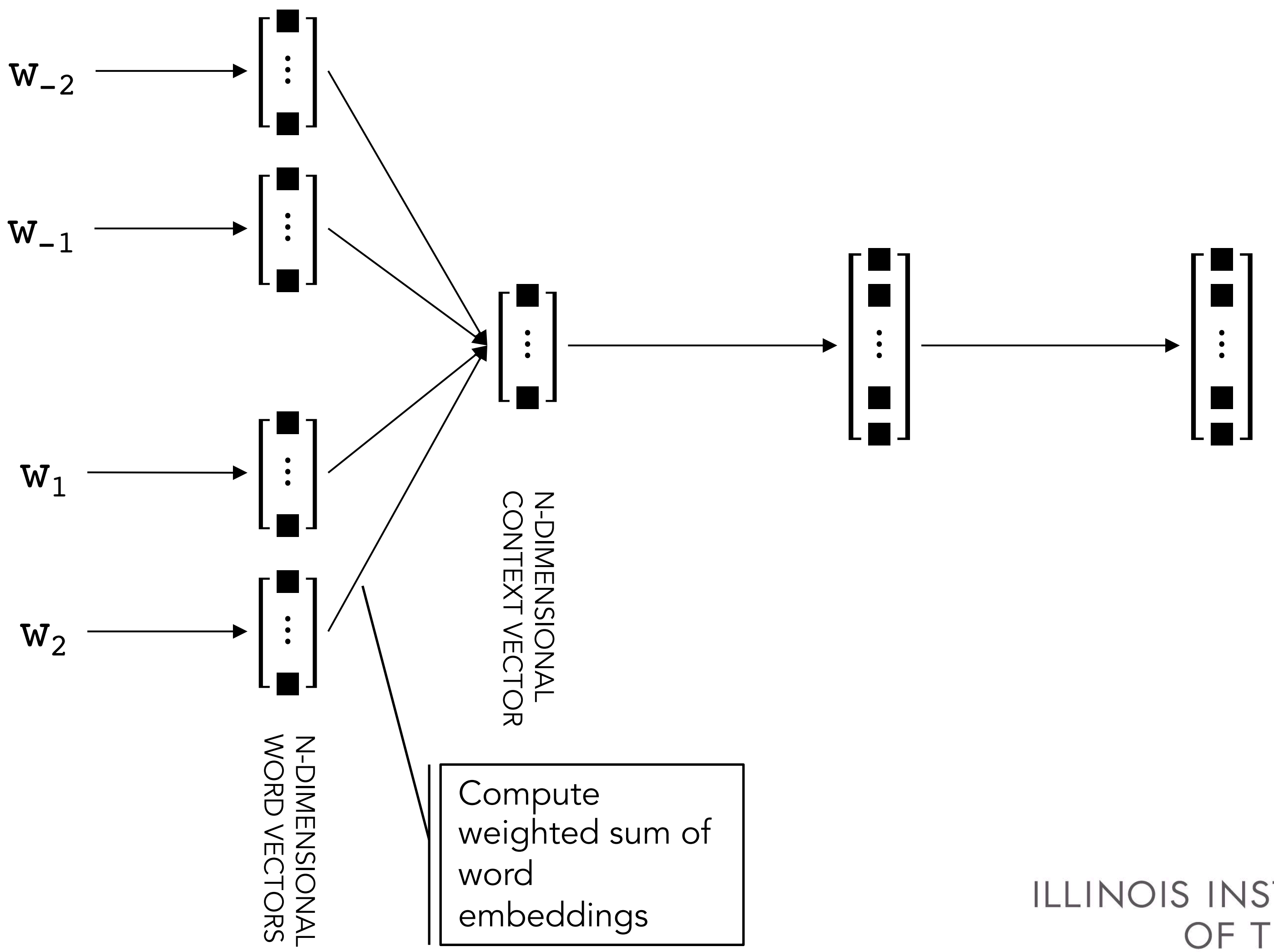

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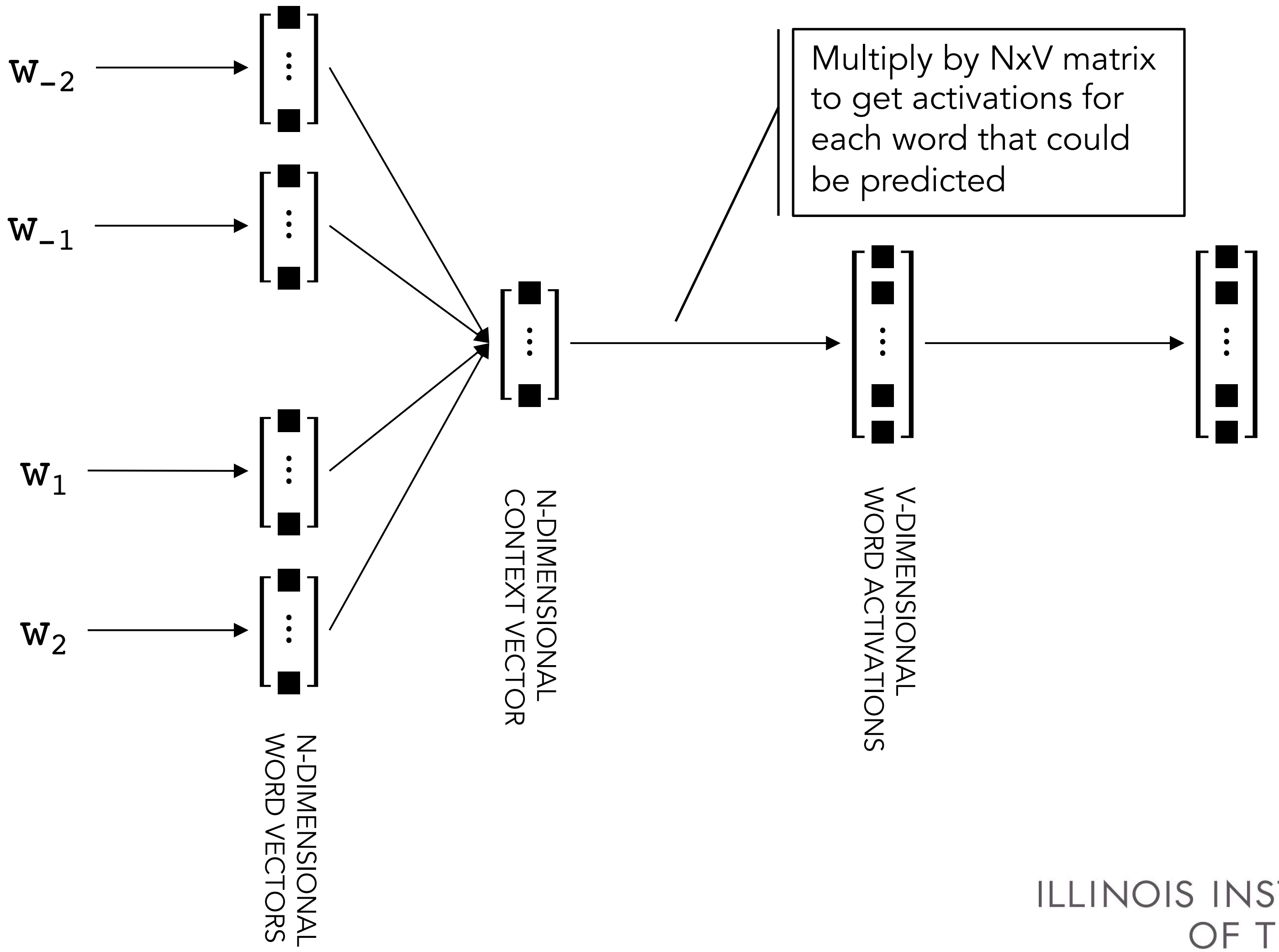

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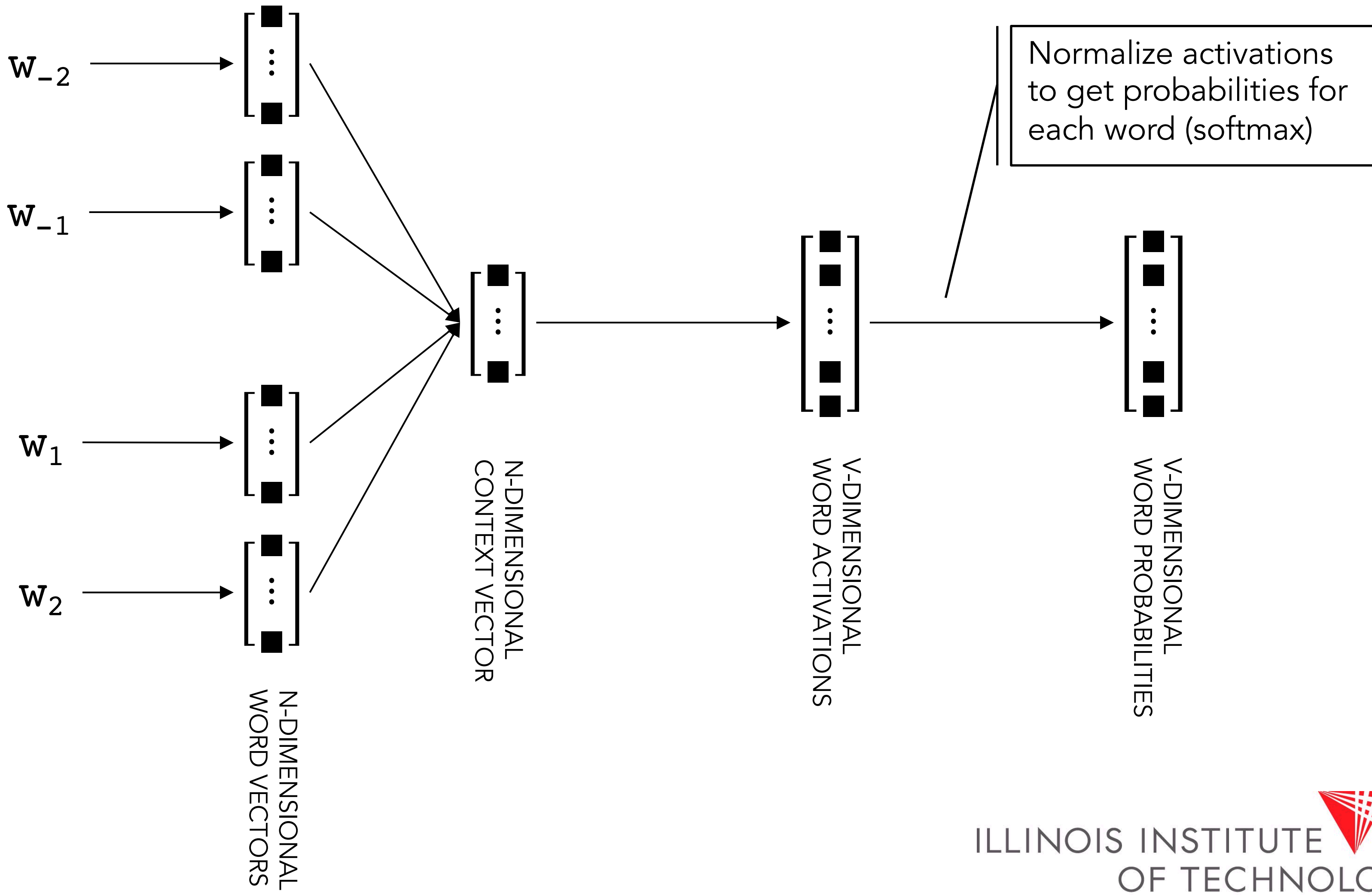

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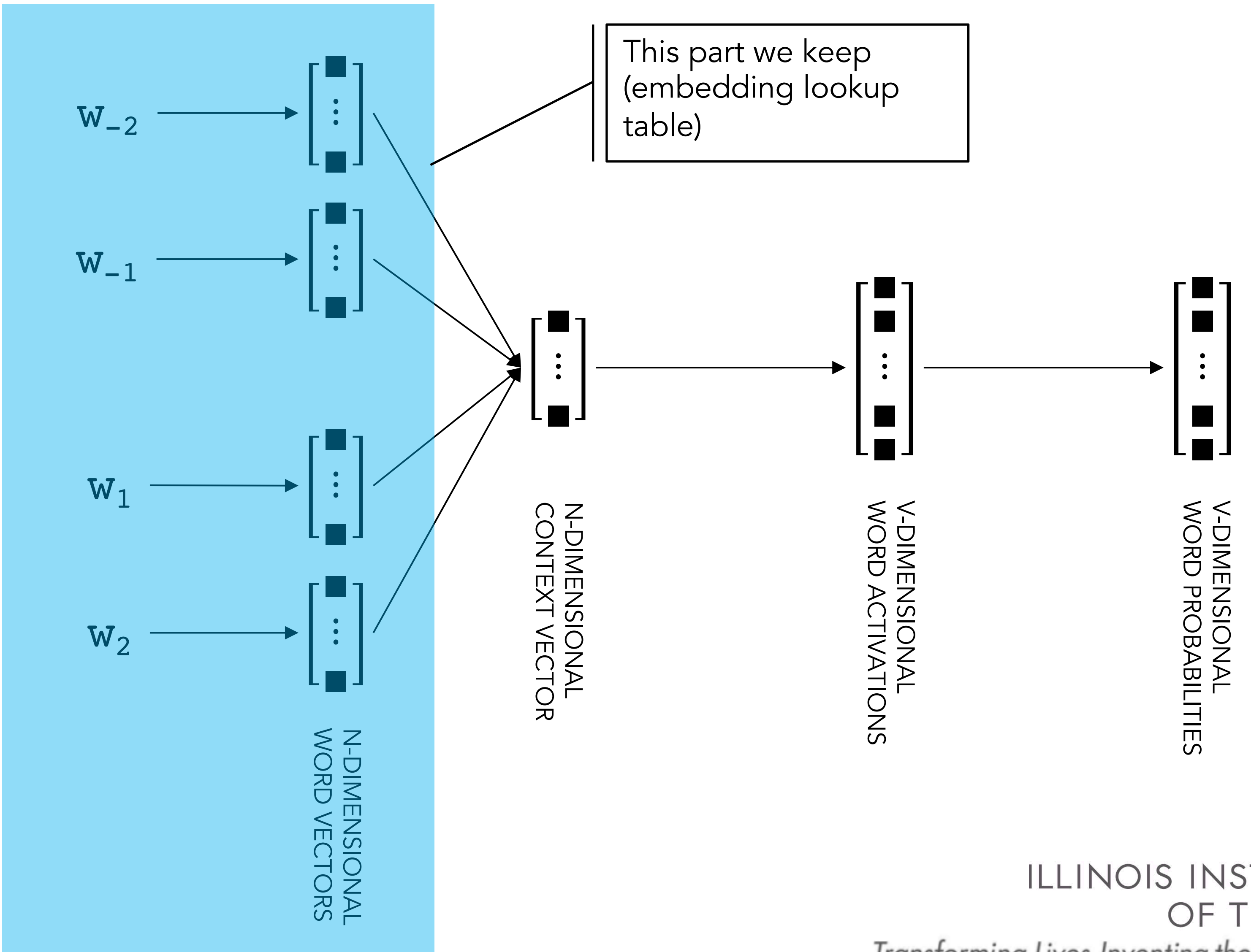
Contextual Bag of Words (CBOW)



`word2vec` Contextual Bag of Words (CBOW)

$$P(w_0|w_{-2},w_{-1},w_1,w_2) = Softmax\left(\left(\sum_{i\in\{-2,-1,1,2\}} a_i E_{w_i}\right)\times P\right)$$

- E is the VxN matrix of word embeddings
- P is the NxV matrix mapping contextual representations to word predictions
- a_i is the weighting of a word at location i in the contextual representation (words closer to the target position are weighted more highly)

Softmax

- A convenient way to get a proper probability distribution out of an unconstrained vector of values
- Maps a vector with values in $[-\infty, \infty]$ to a vector with values in $[0,1]$ that sum to one
- Also conveniently differentiable (see *backpropagation*)

$$Softmax(\vec{x}) = \left[\frac{e^{\vec{x}_i}}{\sum_{\forall j} e^{\vec{x}_j}} \right]_{\forall i}$$

Softmax

$$Softmax\left(\begin{bmatrix}-1\\0\\1.5\end{bmatrix}\right) = Normalize\left(\begin{bmatrix}e^{-1}\\e^{0}\\e^{1.5}\end{bmatrix}\right)$$

$$= Normalize\left(\begin{bmatrix}0.37\\1\\4.48\end{bmatrix}\right)$$

$$= \begin{bmatrix}\dfrac{.37}{.37+1+4.48}\\\dfrac{1}{.37+1+4.48}\\\dfrac{4.48}{37+1+4.48}\end{bmatrix} = \begin{bmatrix}0.06\\0.17\\0.77\end{bmatrix}$$

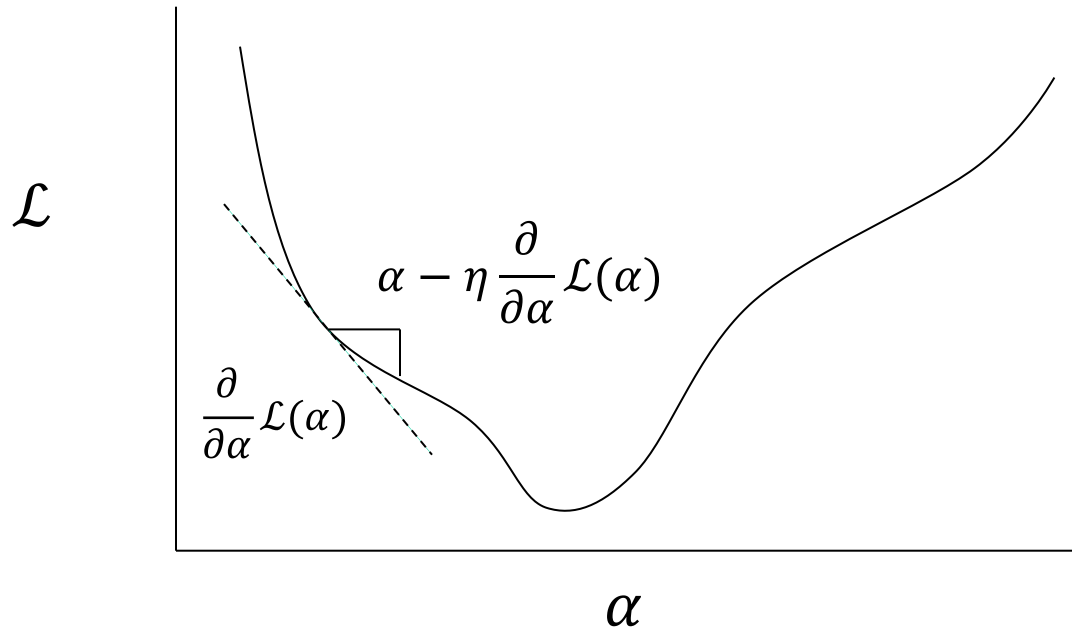
Backpropagation and gradient descent

Error backpropagation is a general framework for learning the parameters of a model (especially, neural network)

1. Define a loss function $\mathcal{L}$ to be minimized
   – Must be differentiable
2. Calculate the derivatives of the loss with respect to its inputs
   – Vector of derivatives derived from vector of inputs: gradient ($\nabla\mathcal{L}$)
3. Use the chain rule of differentiation to get the derivatives of the loss for all parameters in the model
   – $\frac{\partial}{\partial x}(u(v(x))) = \frac{\partial}{\partial v}(u(v))\,\frac{\partial}{\partial x}(v(x))$
4. Update all parameters in the direction of the negative gradient
   – "gradient descent"

Gradient descent

- Skiing downhill
  - start with a given parameter value
  - Find the slope of the loss function
  - Take a step in the downward direction



Gradient descent for CBOW

1. Loss function $\mathcal{L}$ is the cross-entropy:

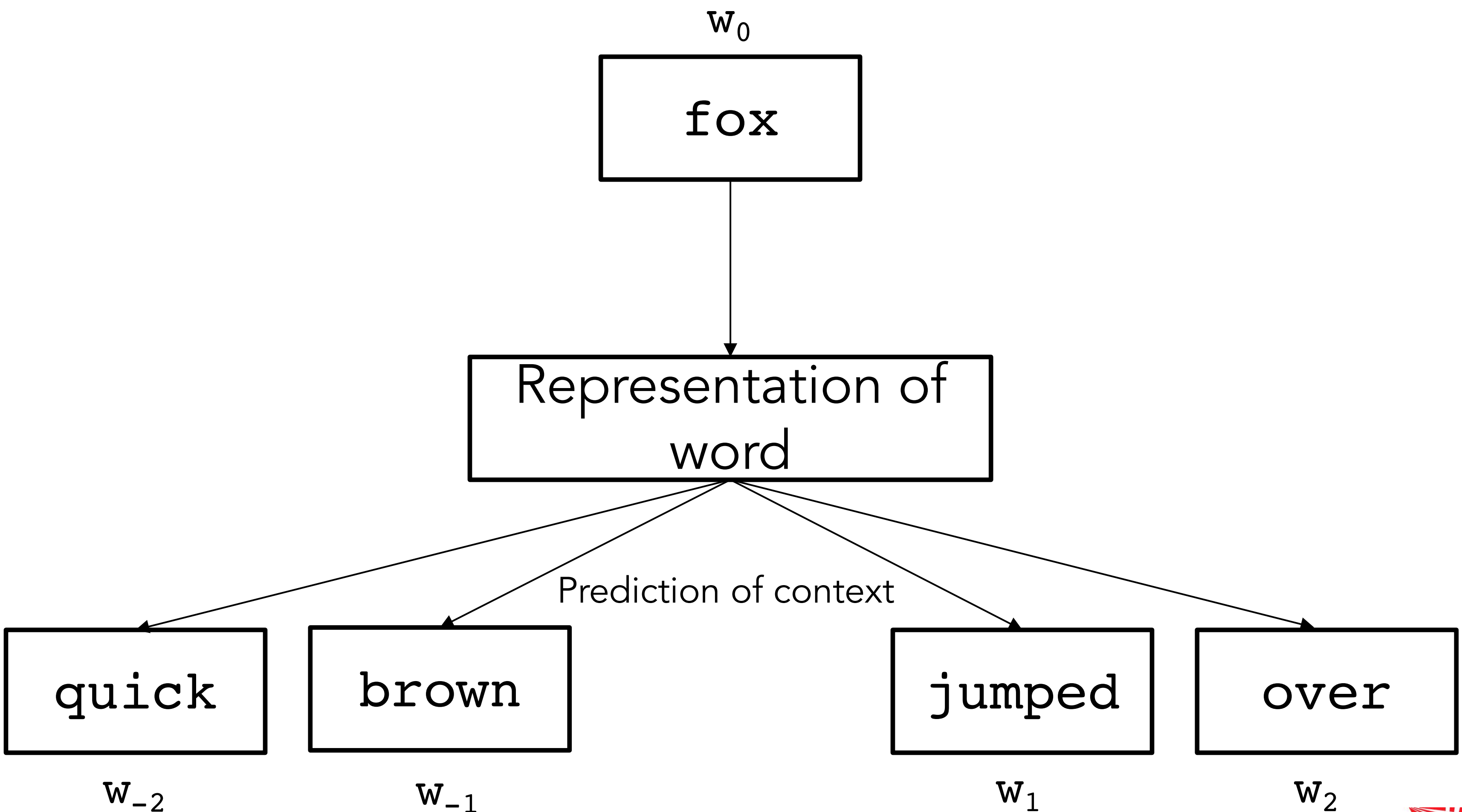
$$\mathcal{L} = -\sum_{w \in V} y_w \log \hat{p}(w)$$

   - y_w is an indicator variable with value 1 when w is the correct word w^*, and 0 otherwise
   - $\hat{p}(w)$ is the model's estimated probability for word w
   - So the loss function simplifies to $\mathcal{L} = -\log \hat{p}(w^*)$
2. Calculate the derivatives of the loss
   - In terms of the pre-softmax activations, $\nabla_{a_w}\mathcal{L} = \hat{p}(w) - y_w$
3. Use the chain rule of differentiation…
4. Update all parameters
   - For all word embeddings, model weights ϕ: $\phi \coloneqq \phi - \eta \nabla \mathcal{L}$

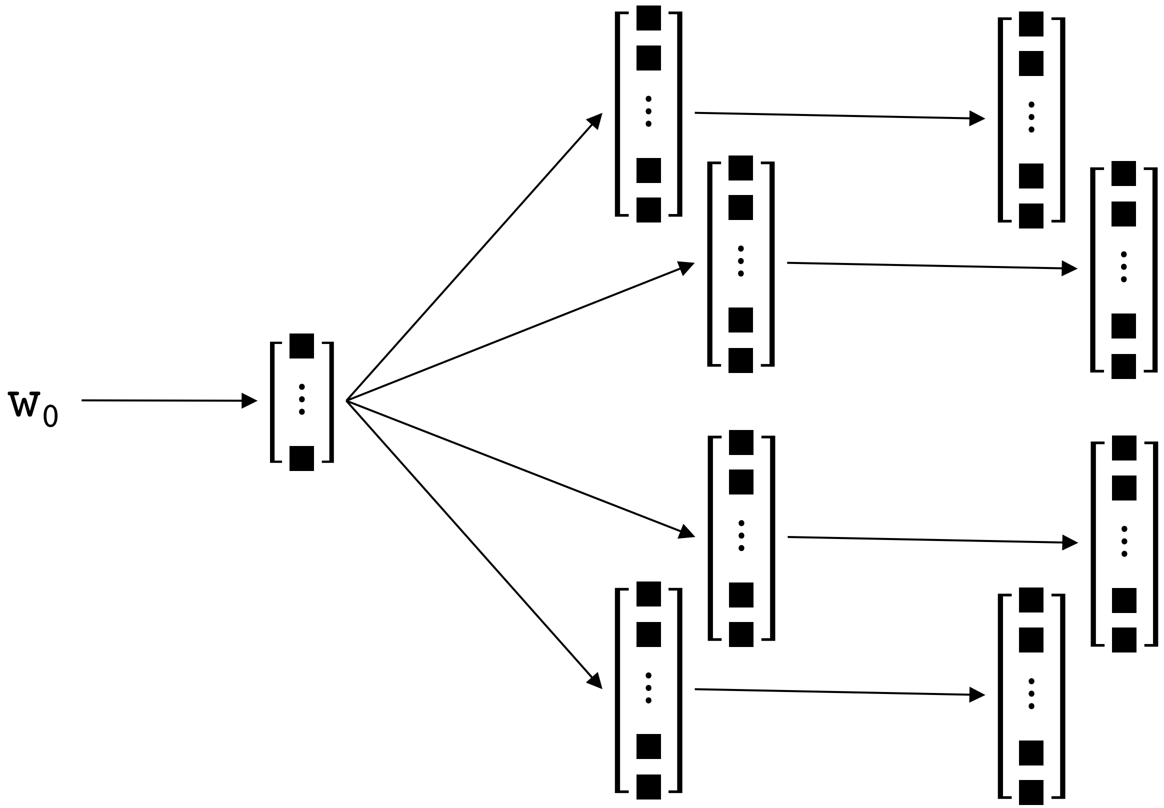
Computational considerations

- Conveniently, calculating the cross-entropy loss only requires $\hat{p}(w^*)$, not $\hat{p}(w)$ for all values of w
- Inconveniently, the gradients still require all values of $\hat{p}(w)$
  - Final matrix multiplication in model is $O(V \times D)$
- Solution 1: *hierarchical softmax*
  - Instead of a single $O(V \times D)$ matrix multiplication to get word activations from embedding layer, break operation down into sequential binary predictions. Complexity reduced to $O(\log V \times D)$
- Solution 2: *negative sampling*
  - Update weights for only a small sample of "negative words" $w \neq w^*$ per iteration

word2vec
Skip-gram



word2vec
Skip-gram



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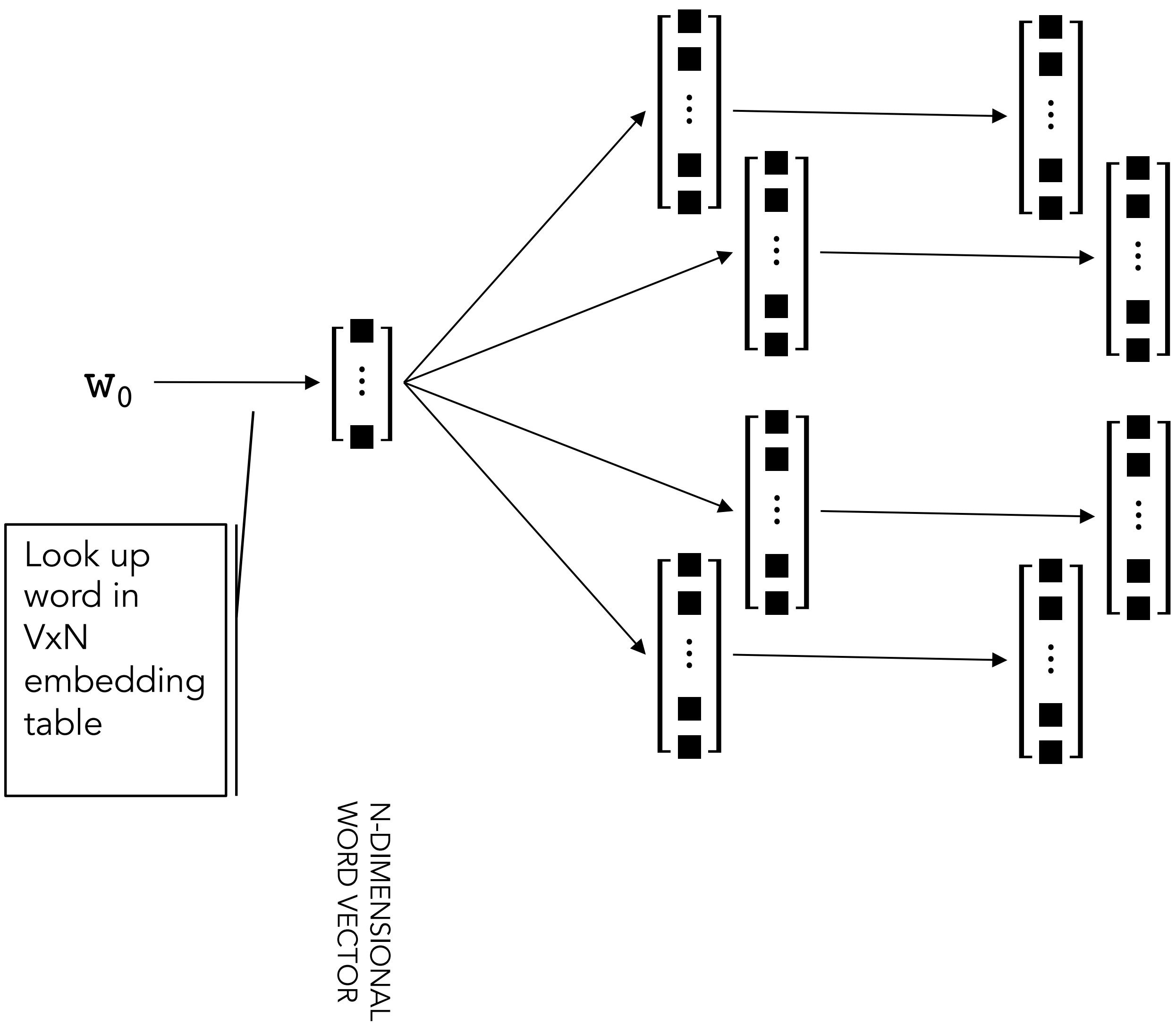
Skip-gram



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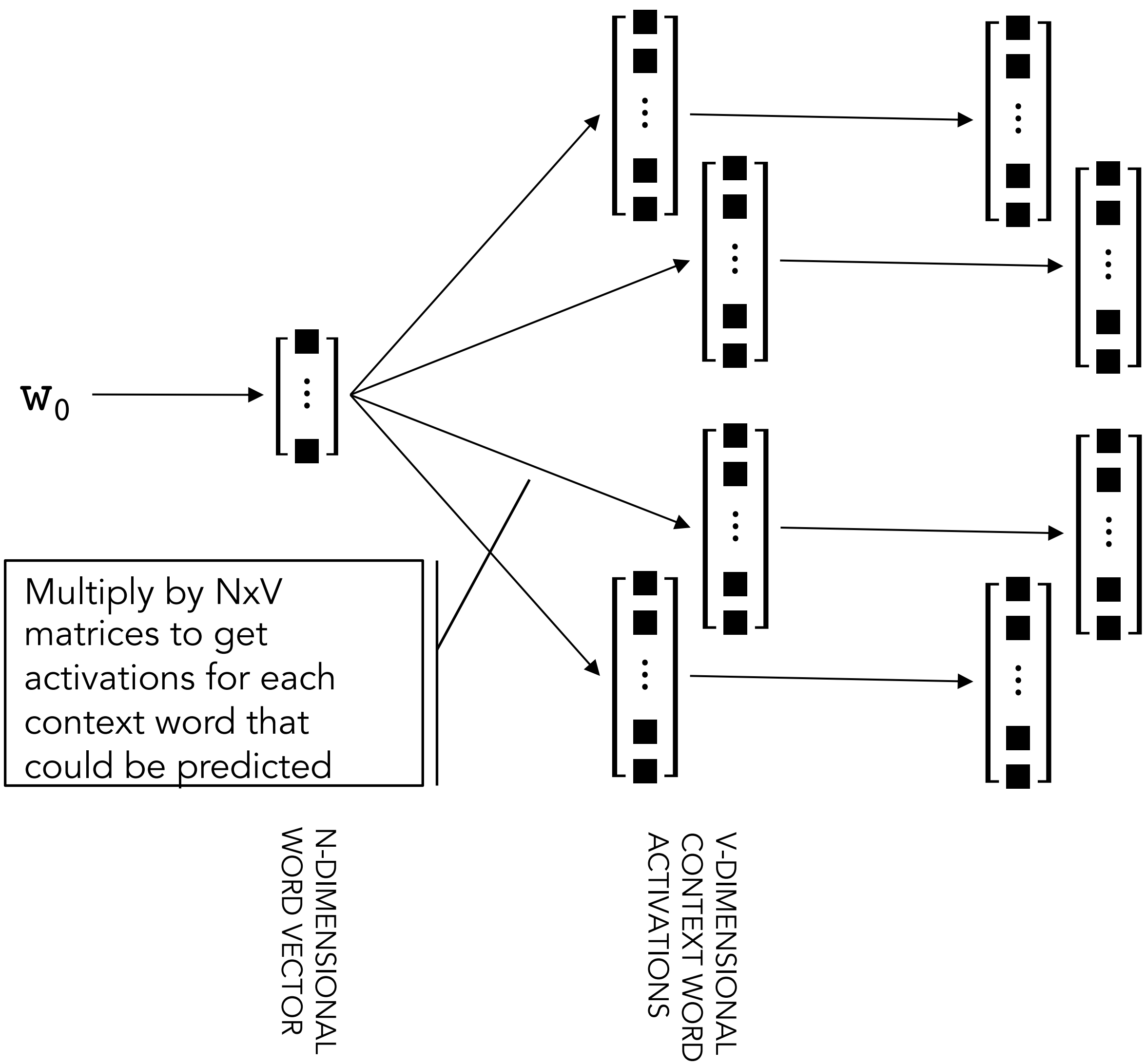
Skip-gram



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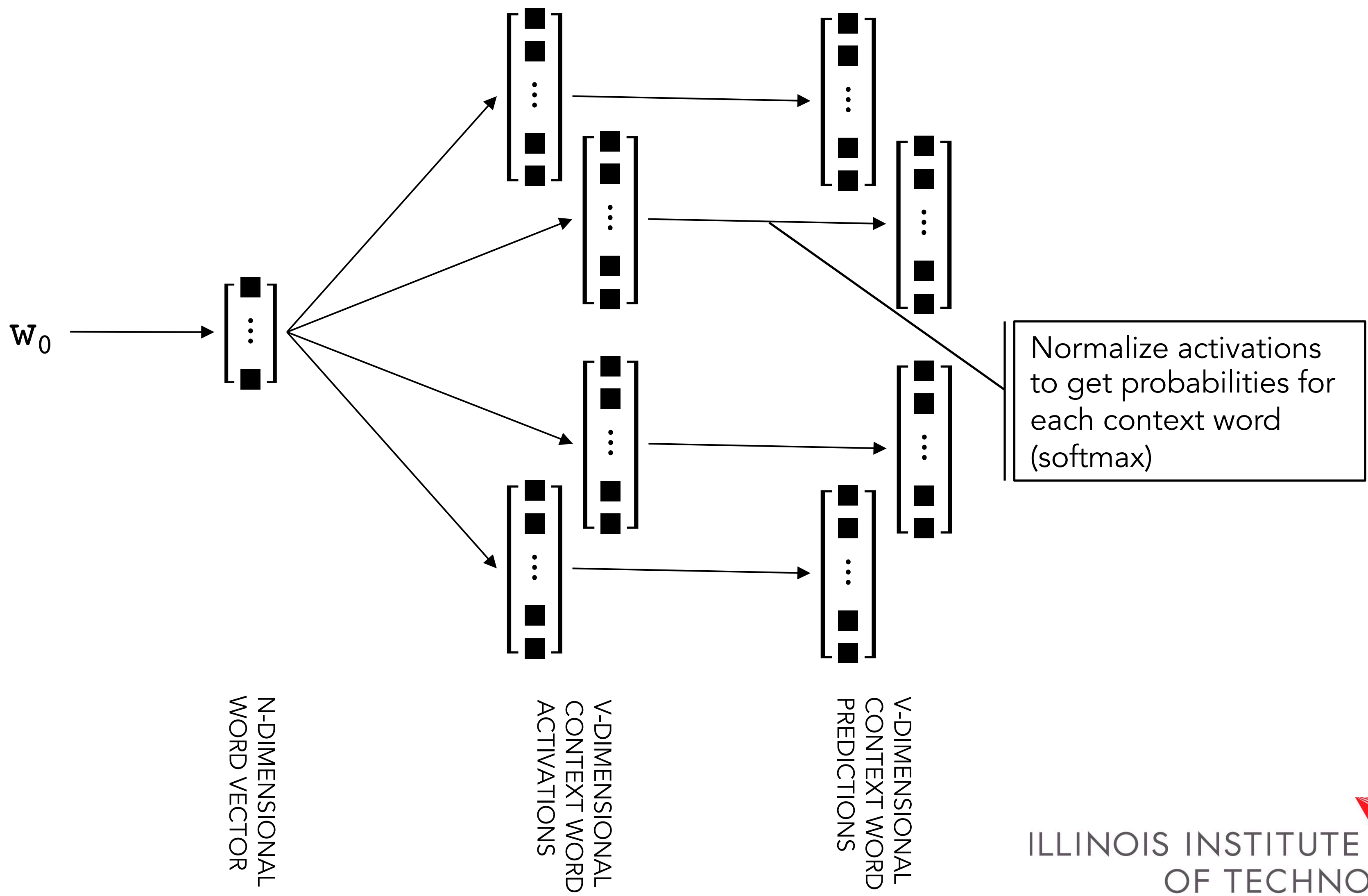
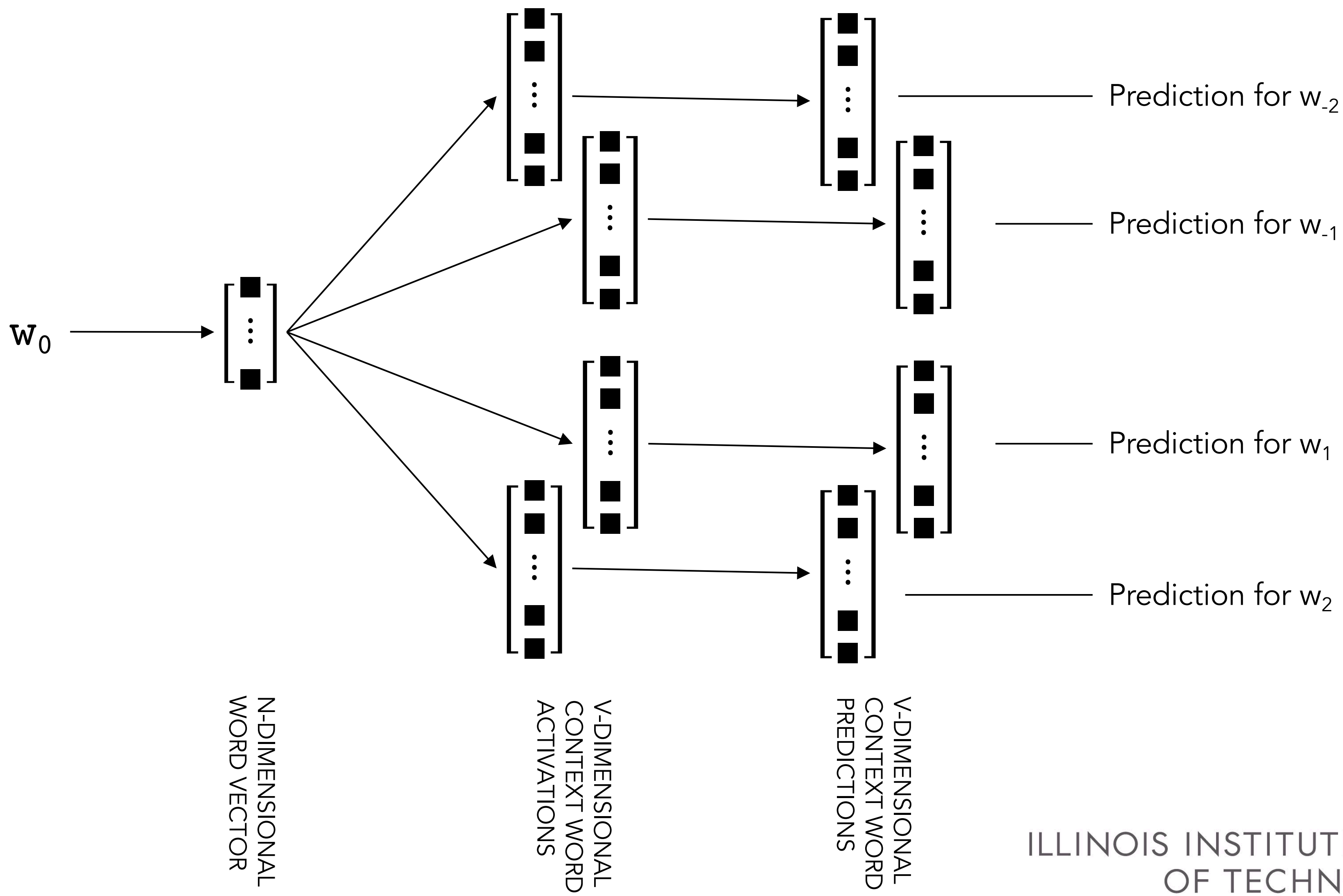
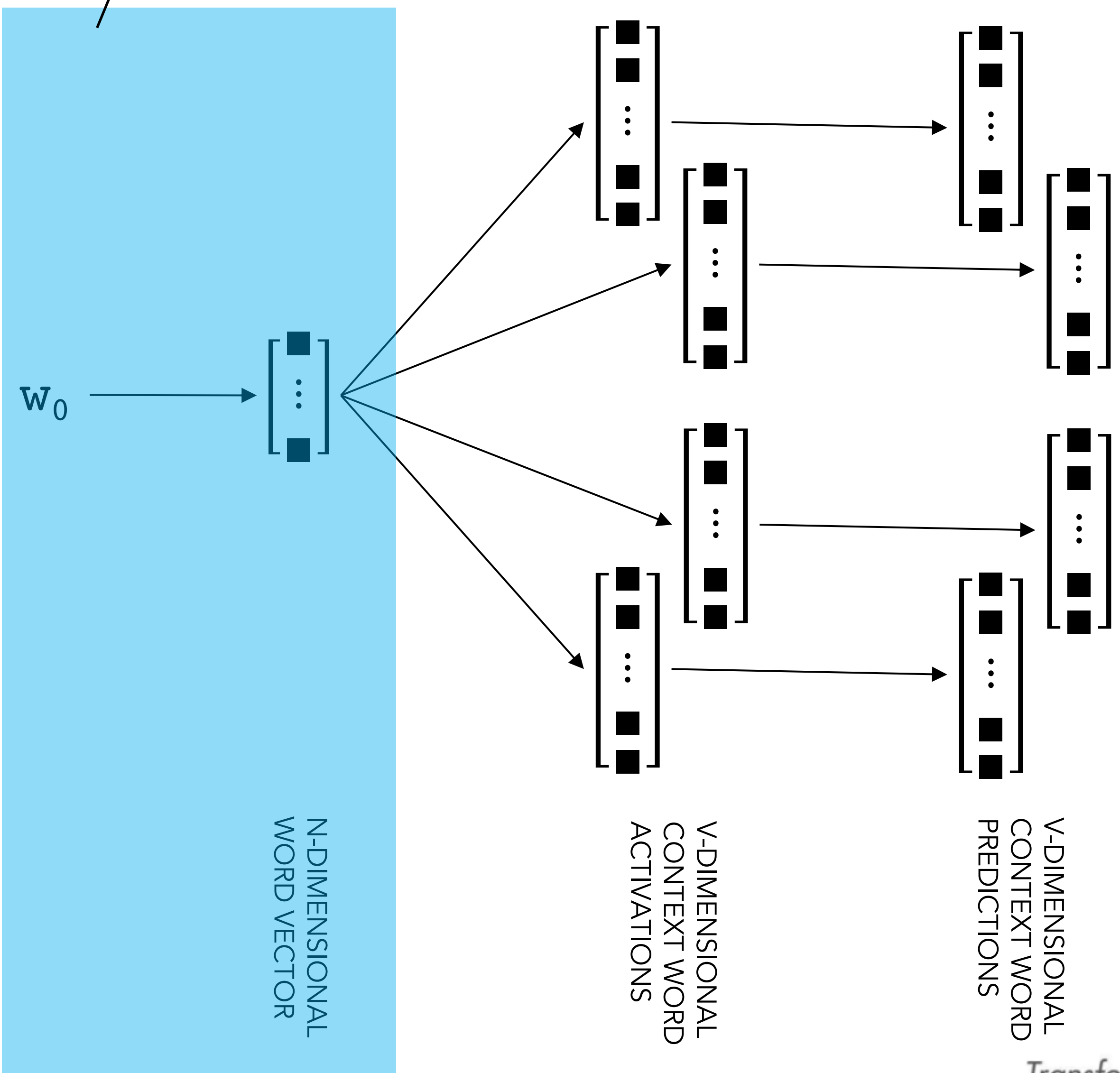


`word2vec`
Skip-gram



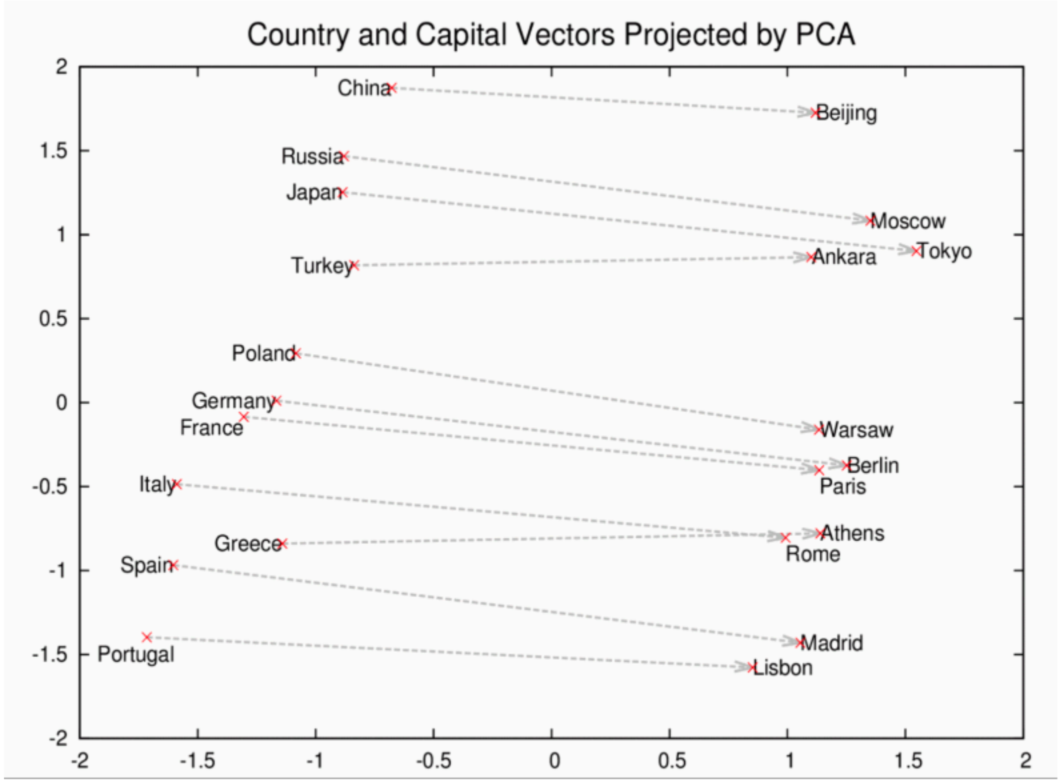
word2vec Skip-gram

This part we keep (embedding lookup table)



`word2vec` examples

- Vectors derived from word2vec are useful, but there is structure to the space, as well
- Semantic relations between words often correspond to translation operations (a consistent direction and distance in vector space)



Mikolov et al., *Distributed Representations of Words and Phrases and their Compositionality*

Analogies using `word2vec`

- Analogies represent consistent semantic relationships; we can rephrase them in terms of vector translation operations

king : man :: queen : woman

$$\overrightarrow{king} - \overrightarrow{man} \approx \overrightarrow{queen} - \overrightarrow{woman}$$

$$\overrightarrow{queen} \approx \overrightarrow{king} - \overrightarrow{man} + \overrightarrow{woman}$$

`man`	`:`	`king`	`::`	`woman`	`:`	`queen`
`China`	`:`	`Beijing`	`::`	`Russia`	`:`	`Moscow`
`knee`	`:`	`leg`	`::`	`elbow`	`:`	`arm`
`building`	`:`	`architect`	`::`	`software`	`:`	`programmer`

Representation learning and fairness/bias

- Learning from data is powerful
  - Eliminates need for manual knowledge engineering
  - Produces models with high accuracy
- But there may be statistical relationships in the data that we prefer the model *not* leverage
  - And it may not be easy to determine when the model is using it

man	:	king	::	woman	: queen
		doctor			nurse
		computer programmer			homemaker
		carpentry			sewing
		chuckle			giggle
		superstar			diva

Bolukbasi et al., *Man is to Computer Programmer as Woman is to Homemaker? Debiasing Word Embeddings*

Other Neural Word Embeddings

- GloVe (Stanford) – Trained on word-word co-occurrence statistics rather than CBOW/skip-gram
- Fasttext (Facebook AI) – Word vectors derived from vectors for smaller units (character ngrams)
- ELMO (Allen AI) – Word vector representation differs based on context of use
- Embeddings learned through end-to-end task-specific training

Advantages of word embeddings

- More efficient, concise representation → faster code (?)
- "Deeper" language features related to meaning, rather than specific words
- Transfer learning: build representations using large, general-purpose data set; fine-tune model based on smaller task-specific data