

# **CS 480**

## ***Introduction to Artificial Intelligence***

**September 21st, 2021**

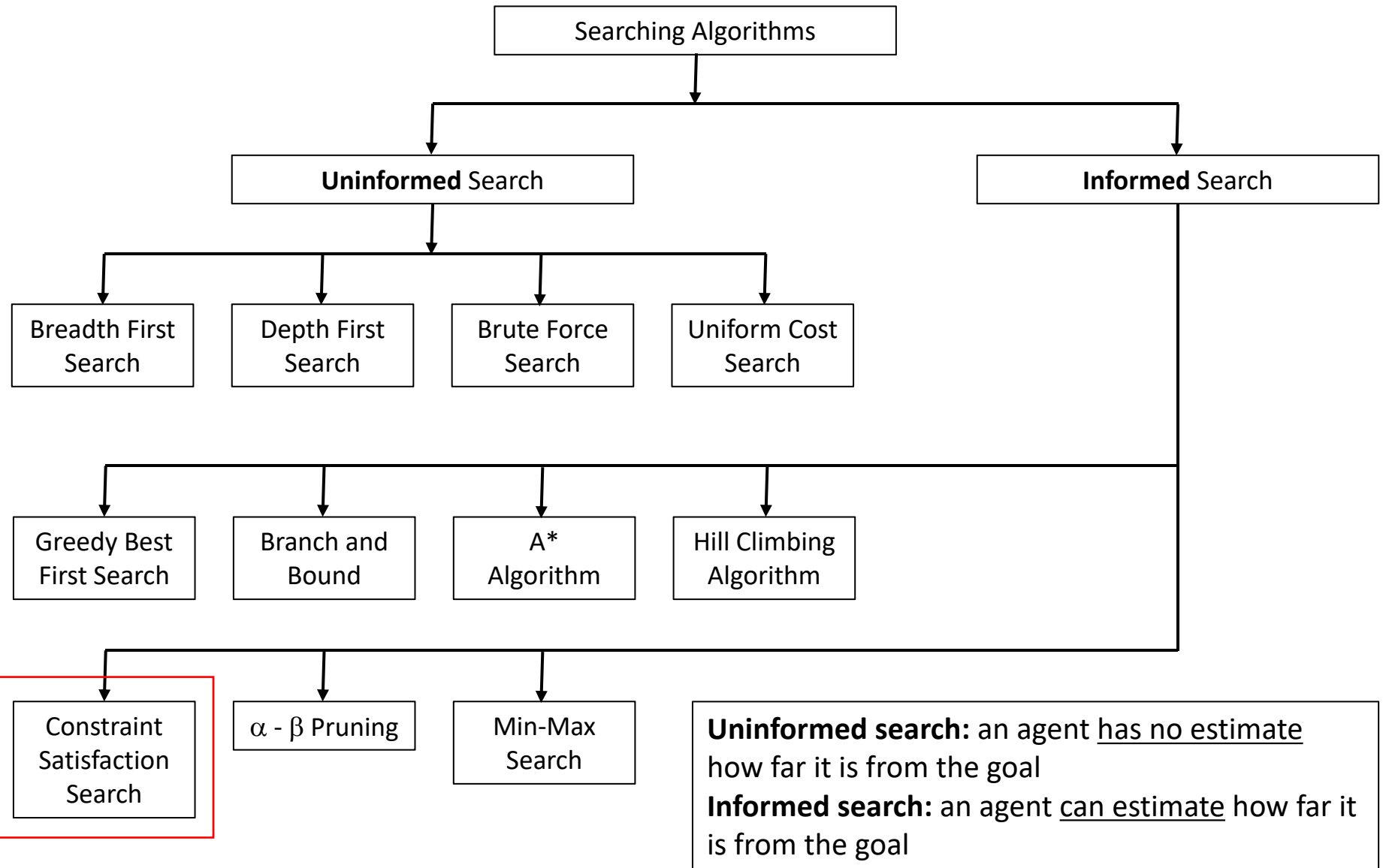
# Announcements / Reminders

- **Written Assignment #01:**
  - **due on Wednesday (09/22/21) at 11:00 PM CST**
- **Programming Assignment #02:**
  - **will be posted this Friday and you will have three (3) weeks to complete it**
- **Contribute to the discussion on Blackboard**
- **Please follow the Week 04 To Do List instructions**
- **My tomorrow's (09/22/21) office hours:**
  - **online only in Blackboard Collaborate Ultra**

# Plan for Today

- **Constraint Satisfaction Problems: Continued**

# Selected Searching Algorithms



# Constraint Satisfaction Problem

**A Constraint Satisfaction Problem (CSP) consists of three components:**

- **a set of variables  $X = \{X_1, \dots, X_n\}$**
- **a set of domains  $D = \{D_1, \dots, D_n\}$**
- **a set of constraints  $C$  that specify allowable combinations of values**
- **A domain  $D_i$  is a set of allowable values  $\{v_1, \dots, v_k\}$  for variable  $X_i$**
- **A constraint  $C_j$  is a  $\langle \text{scope}, \text{relation} \rangle$  pair, for example  $\langle (X_1, X_2), X_1 > X_2 \rangle$**

# Constraint Satisfaction Problem

The goal is to **find an assignment** (variable = value):

$$\{X_1 = v_1, \dots, X_n = v_n\}$$

- If NO constraints violated: **consistent** assignment
- If ALL variables have a value: **complete** assignment
- If SOME variables have NO value: **partial** assignment
- SOLUTION: **consistent** and **complete** assignment
- PARTIAL SOLUTION: **consistent** and **partial** assignment

# CSP: Variable Types

- Domains can be:
  - finite, for example:  $\{1, 2, 3, 5, 8, 20\}$  (simpler)
  - infinite, for example: a set of all integers
- Variables can be:
  - discrete, for example:  $X = \{X_1, \dots, X_n\}$  (simpler)
  - continuous, for example:  $R_+$
- Constraints can be:
  - unary (involve single variable), for example:  $X_1 = 5$
  - binary (involve two variables), for example:  $X_1 = X_2$
  - higher order (involve  $> 2$  variables), for example:  $X_1 = X_2 * X_3$
- Soft constraints (preferences: green over blue) possible

# CSP as a Search Problem

CSP is a variant of a search problem you already know. The problem can be restated / updated with:

- **Initial state**: the empty assignment  $\{ \}$ , in which all variables are unassigned.
- **Successor function**: a value can be assigned to any unassigned variable, provided that it does not conflict with previously assigned variables.
- **Goal test**: the current assignment is complete.
- **Path cost**: a constant cost (e.g., 1) for every step.

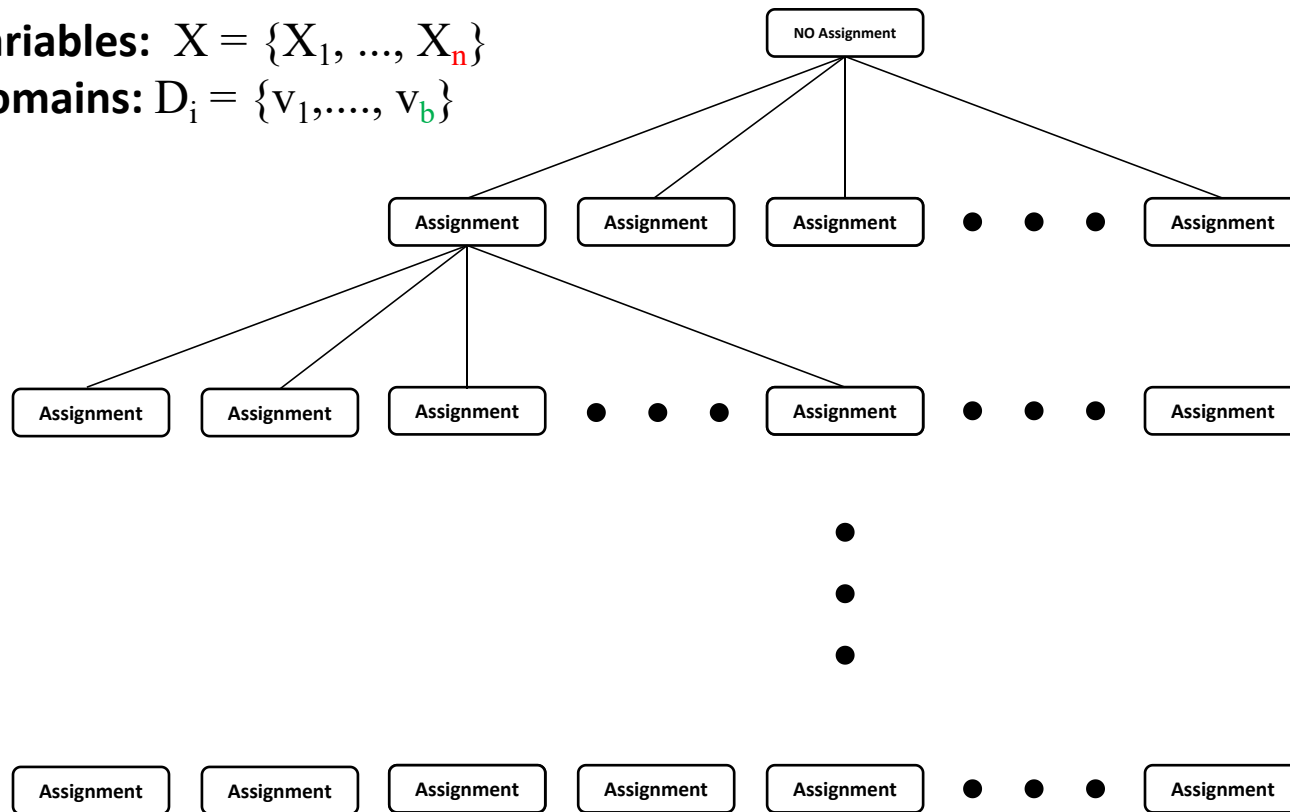


# CSP Search Tree: Idea

**CSP Problem:**

**Variables:**  $X = \{X_1, \dots, X_n\}$

**Domains:**  $D_i = \{v_1, \dots, v_b\}$



0 variable  
assigned

1 variables  
assigned

2 variables  
assigned

•  
•  
•

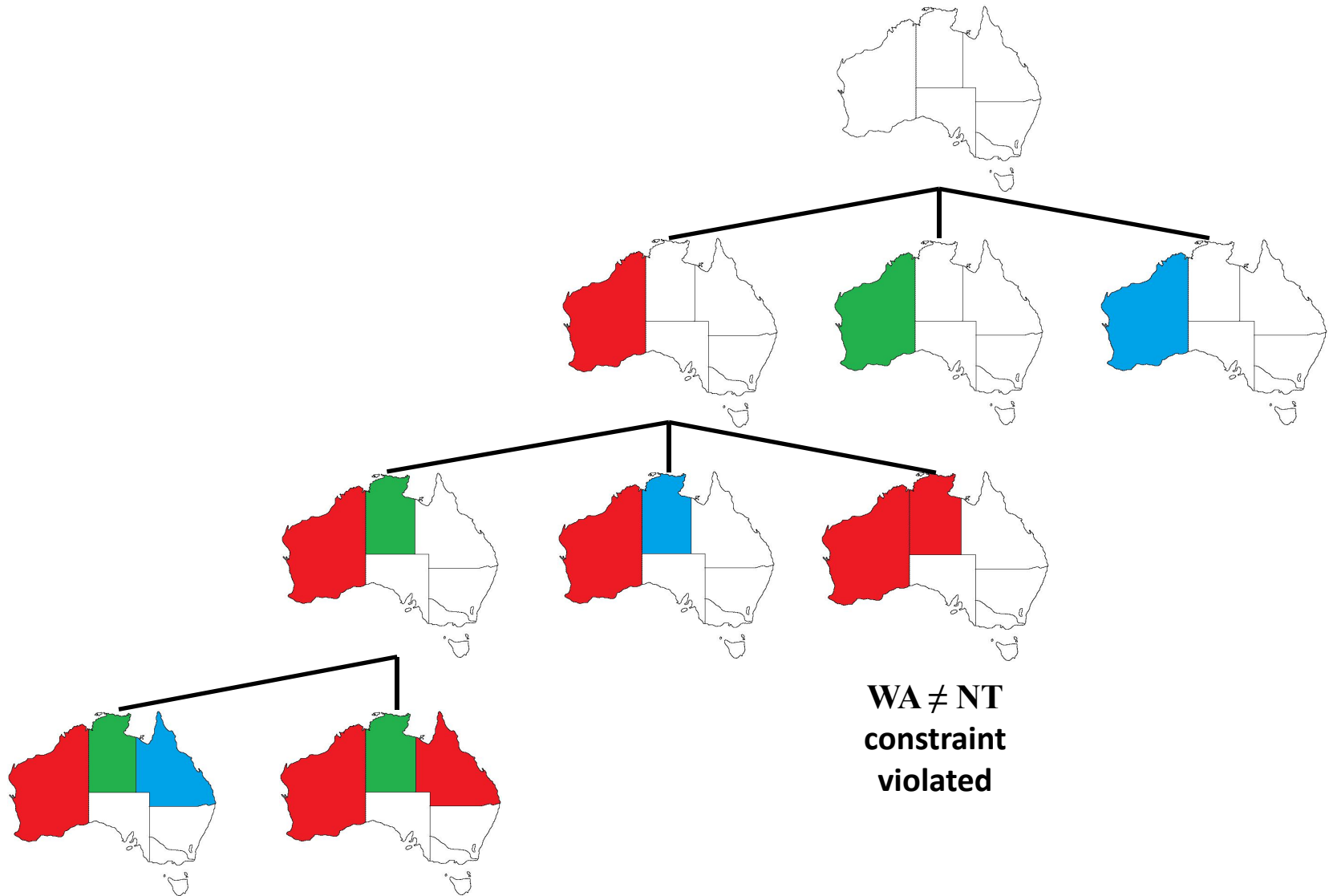
ALL (**n**) variables  
assigned

**Tree leaves are COMPLETE assignments**

The sequence of variable assignments does NOT matter\*

\*(when you disregard performance)

# CSP as a Tree Search Problem

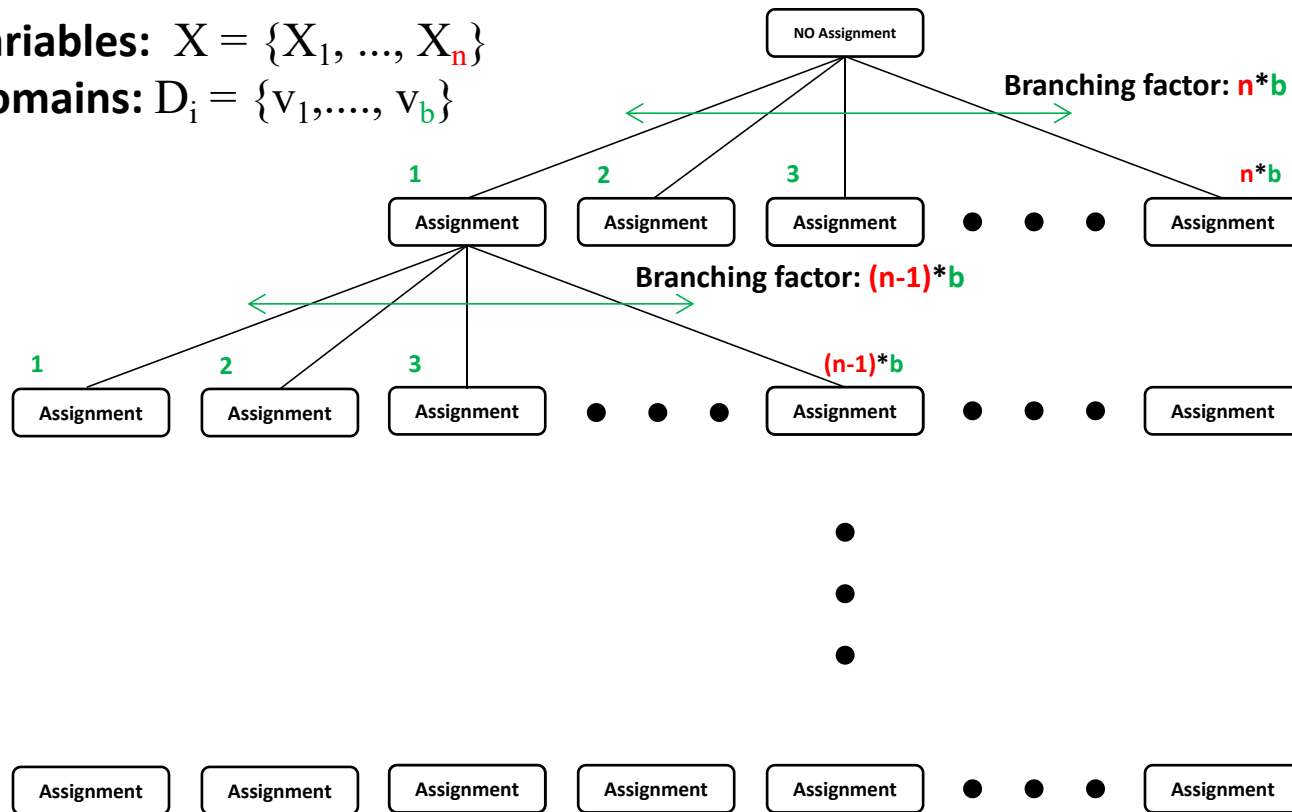


# CSP Search Tree: Size

**CSP Problem:**

**Variables:**  $X = \{X_1, \dots, X_n\}$

**Domains:**  $D_i = \{v_1, \dots, v_b\}$



$$N_0 = 0$$

$$N_1 = n*b$$

$$N_2 = n*b * (n-1)*b = n*(n-1)*b^2$$

•  
•  
•

$$N_n = n! * b^n$$

**Total number of leafnodes / states:**  $n! * b^n$

**(ignores COMMUTATIVITY of CSP assignments:**

**assigning  $X_1 = m$  and then  $X_2 = n$  SAME as assigning  $X_2 = n$  and then  $X_1 = m$  )**

**In reality: there is only  $b^n$  complete assignments**

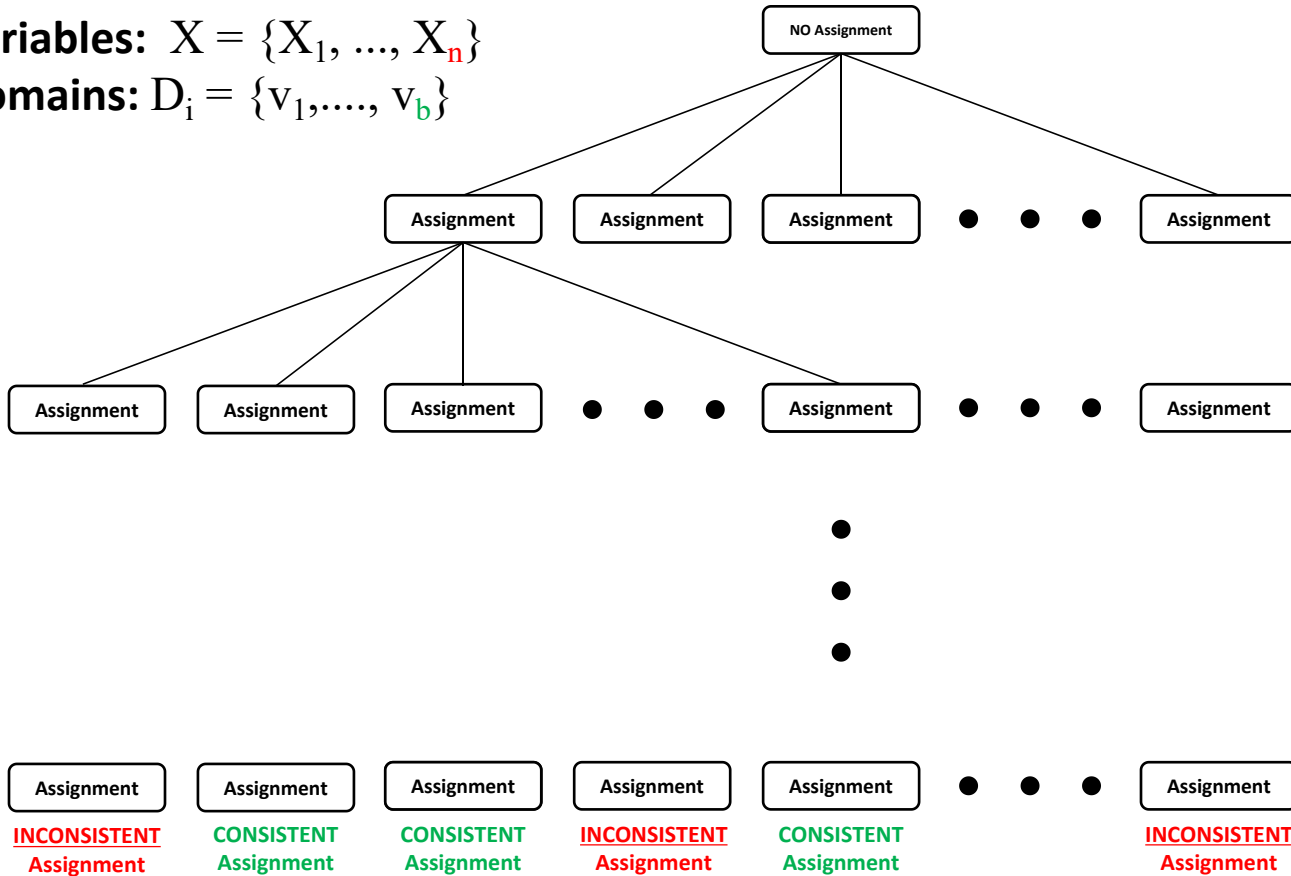
# Can We Do Better?

# CSP Search Tree: Solutions

## CSP Problem:

**Variables:**  $X = \{X_1, \dots, X_n\}$

**Domains:**  $D_i = \{v_1, \dots, v_b\}$

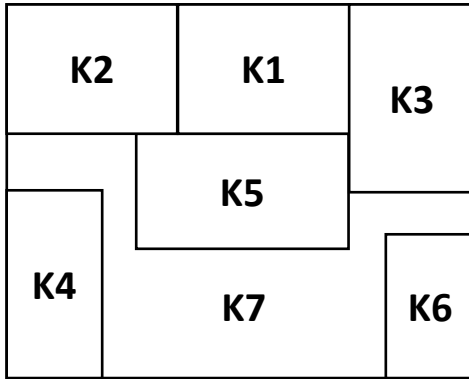


Some nodes / states will be **CONSISTENT**, while others will be **INCONSISTENT**.

Depth first search could possibly visit them all → **WASTEFUL.**

# CSP Example: Map Coloring

Problem:



Variables:

$X = \{K1, K2, K3, K4, K5, K6, K7\}$

Variable Domains:

$D_{K1} = \{\text{RED}, \text{BLUE}, \text{GREEN}\}$   
 $D_{K2} = \{\text{RED}, \text{BLUE}, \text{GREEN}\}$   
 $D_{K3} = \{\text{RED}, \text{BLUE}, \text{GREEN}\}$   
 $D_{K4} = \{\text{RED}, \text{BLUE}, \text{GREEN}\}$   
 $D_{K5} = \{\text{RED}, \text{BLUE}, \text{GREEN}\}$   
 $D_{K6} = \{\text{RED}, \text{BLUE}, \text{GREEN}\}$   
 $D_{K7} = \{\text{RED}, \text{BLUE}, \text{GREEN}\}$

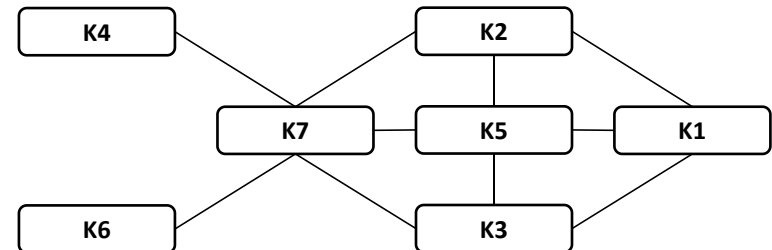
Color this map in a way that no two neighbors have same color

Constraints (Rules):

- Neighboring regions have to have DISTINCT colors:

$\text{CONSTRAINTS} = C = \{K1 \neq K2, K1 \neq K3, K1 \neq K5, K2 \neq K5, K2 \neq K7, K3 \neq K5, K3 \neq K7, K4 \neq K7, K5 \neq K7, K6 \neq K7\}$

Constraint Graph:



# CSP Backtracking: Pseudocode

**function** BACKTRACKING-SEARCH(*csp*) **returns** a solution or *failure*  
    **return** BACKTRACK(*csp*, { })

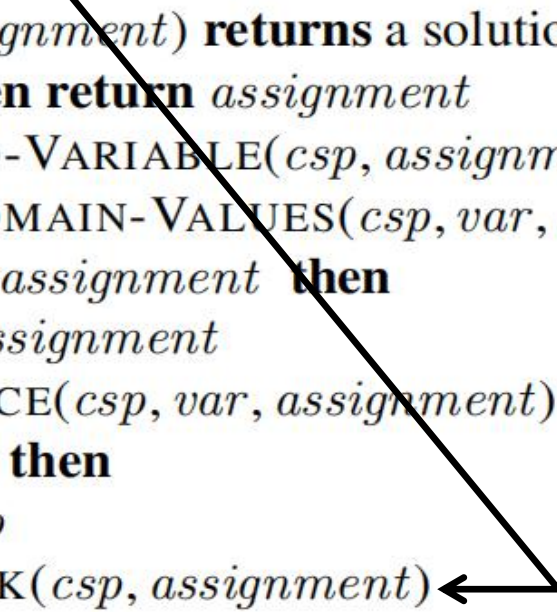
**function** BACKTRACK(*csp*, *assignment*) **returns** a solution or *failure*  
    **if** *assignment* is complete **then return** *assignment*  
    *var*  $\leftarrow$  SELECT-UNASSIGNED-VARIABLE(*csp*, *assignment*)  
    **for each** *value* **in** ORDER-DOMAIN-VALUES(*csp*, *var*, *assignment*) **do**  
        **if** *value* is consistent with *assignment* **then**  
            add {*var* = *value*} to *assignment*  
            *inferences*  $\leftarrow$  INFERENCE(*csp*, *var*, *assignment*)  
            **if** *inferences*  $\neq$  *failure* **then**  
                add *inferences* to *csp*  
                *result*  $\leftarrow$  BACKTRACK(*csp*, *assignment*)  
                **if** *result*  $\neq$  *failure* **then return** *result*  
                remove *inferences* from *csp*  
            remove {*var* = *value*} from *assignment*  
    **return** *failure*



# CSP Backtracking: Pseudocode

**function** BACKTRACKING-SEARCH(*csp*) **returns** a solution or *failure*  
    **return** BACKTRACK(*csp*, { })

**function** BACKTRACK(*csp*, *assignment*) **returns** a solution or *failure*  
    **if** *assignment* is complete **then return** *assignment*  
    *var*  $\leftarrow$  SELECT-UNASSIGNED-VARIABLE(*csp*, *assignment*)  
    **for each** *value* **in** ORDER-DOMAIN-VALUES(*csp*, *var*, *assignment*) **do**  
        **if** *value* is consistent with *assignment* **then**  
            add {*var* = *value*} to *assignment*  
            *inferences*  $\leftarrow$  INFERENCE(*csp*, *var*, *assignment*)  
            **if** *inferences*  $\neq$  *failure* **then**  
                add *inferences* to *csp*  
                *result*  $\leftarrow$  BACKTRACK(*csp*, *assignment*)  
                **if** *result*  $\neq$  *failure* **then return** *result*  
                remove *inferences* from *csp*  
            remove {*var* = *value*} from *assignment*  
    **return** *failure*



RECURSION



## Assignment:

K1: **RED**

K2: ???

K3: ???

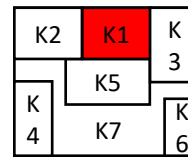
K4: ???

K5: ???

K6: ???

K7: ???

**Initial (NO  
assignment) state  
not shown**



## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$

Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: **RED**, **BLUE**, **GREEN**

## Assignment:

K1: **RED**

K2: **RED**

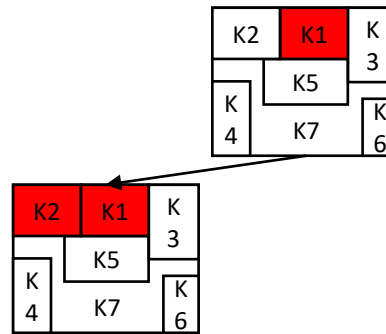
K3: ???

K4: ???

K5: ???

K6: ???

K7: ???



## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$

Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: **RED**, **BLUE**, **GREEN**

## Assignment:

K1: **RED**

K2: **RED**

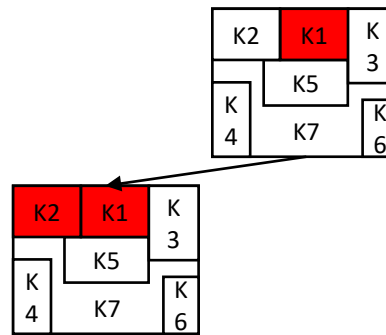
K3: ???

K4: ???

K5: ???

K6: ???

K7: ???



Violates:  
Rule 1

## Constraints:

Rule 1: **K1 ≠ K2**

Rule 2: K1 ≠ K3

Rule 3: K1 ≠ K5

Rule 4: K2 ≠ K5

Rule 5: K2 ≠ K7

Rule 6: K3 ≠ K5

Rule 7: K3 ≠ K7

Rule 8: K4 ≠ K7

Rule 9: K5 ≠ K7

Rule 10: K6 ≠ K7

Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: **RED**, **BLUE**, **GREEN**

## Assignment:

K1: **RED**

K2: ???

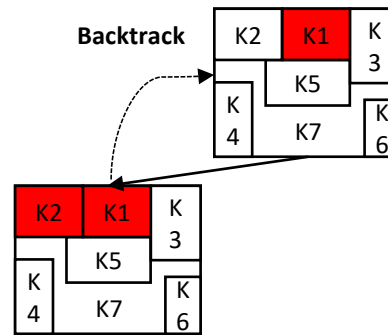
K3: ???

K4: ???

K5: ???

K6: ???

K7: ???



Violates:  
Rule 1

## Constraints:

**Rule 1:  $K1 \neq K2$**

**Rule 2:  $K1 \neq K3$**

**Rule 3:  $K1 \neq K5$**

**Rule 4:  $K2 \neq K5$**

**Rule 5:  $K2 \neq K7$**

**Rule 6:  $K3 \neq K5$**

**Rule 7:  $K3 \neq K7$**

**Rule 8:  $K4 \neq K7$**

**Rule 9:  $K5 \neq K7$**

**Rule 10:  $K6 \neq K7$**

Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: **RED**, **BLUE**, **GREEN**

## Assignment:

K1: **RED**

K2: **BLUE**

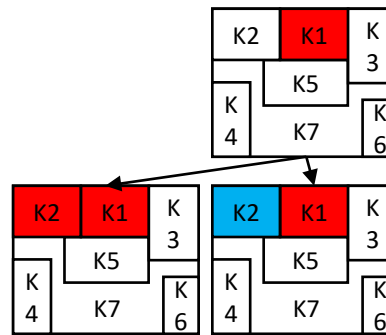
K3: ???

K4: ???

K5: ???

K6: ???

K7: ???



## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$

Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: **RED**, **BLUE**, **GREEN**

## Assignment:

K1: **RED**

K2: **BLUE**

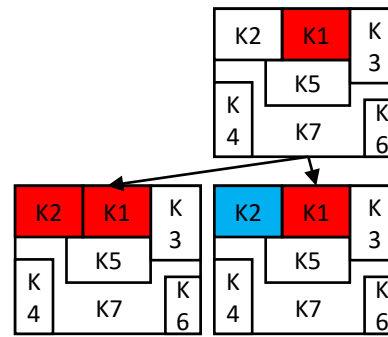
K3: ???

K4: ???

K5: ???

K6: ???

K7: ???



CONSISTENT  
PARTIAL  
Assignment

## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$

Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: **RED**, **BLUE**, **GREEN**

## Assignment:

K1: RED

K2: BLUE

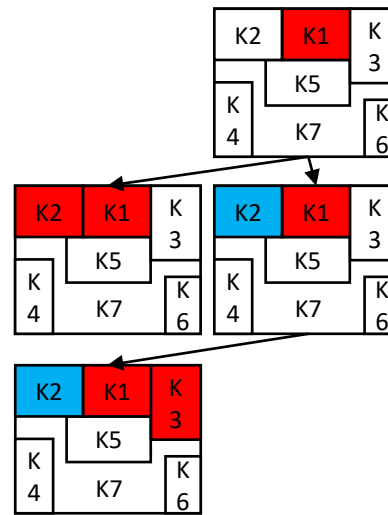
K3: RED

K4: ???

K5: ???

K6: ???

K7: ???



Violates:  
Rule 2

## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$

Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: **RED**

K2: **BLUE**

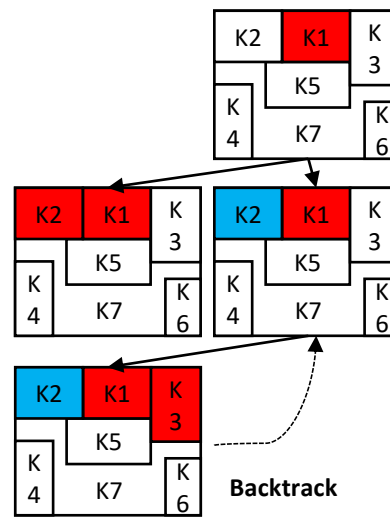
K3: ???

K4: ???

K5: ???

K6: ???

K7: ???



Violates:  
Rule 2

## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$

Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: **RED**, **BLUE**, **GREEN**



## Assignment:

K1: **RED**

K2: **BLUE**

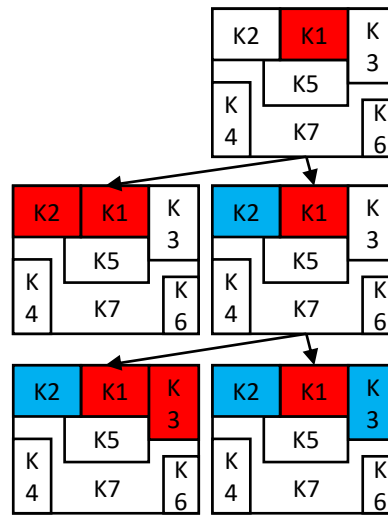
K3: **BLUE**

K4: ???

K5: ???

K6: ???

K7: ???



## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$

Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: **RED**, **BLUE**, **GREEN**

## Assignment:

K1: **RED**

K2: **BLUE**

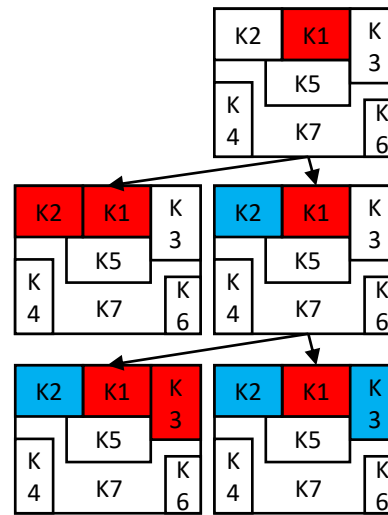
K3: **BLUE**

K4: ???

K5: ???

K6: ???

K7: ???



**CONSISTENT  
PARTIAL  
Assignment**

## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$

Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: **RED**, **BLUE**, **GREEN**

## Assignment:

K1: **RED**

K2: **BLUE**

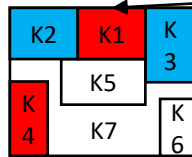
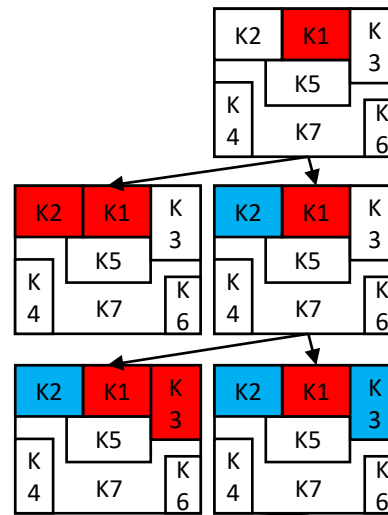
K3: **BLUE**

K4: **RED**

K5: ???

K6: ???

K7: ???



CONSISTENT  
PARTIAL  
Assignment

## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$

Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: **RED**, **BLUE**, **GREEN**

## Assignment:

K1: **RED**

K2: **BLUE**

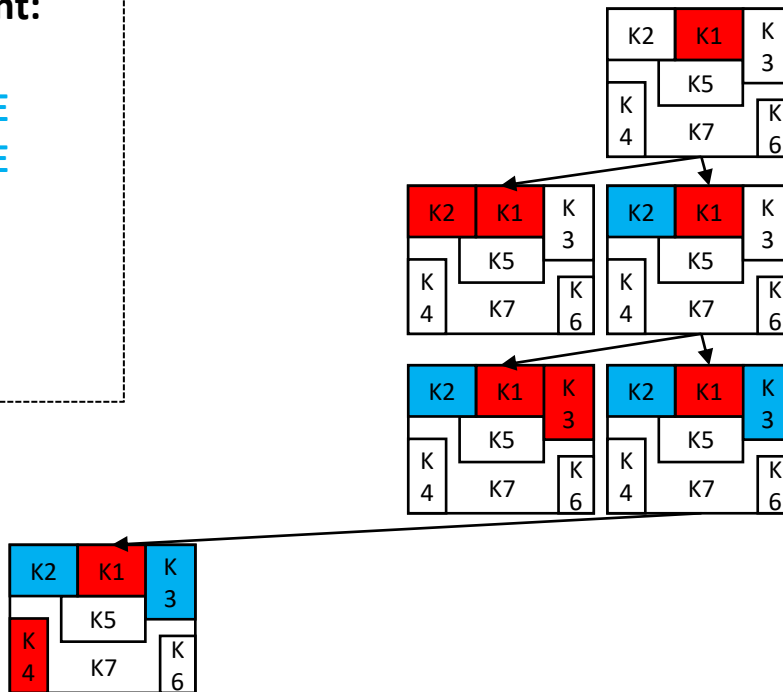
K3: **BLUE**

K4: **RED**

K5: ???

K6: ???

K7: ???



## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$

Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: **RED**, **BLUE**, **GREEN**

## Assignment:

K1: **RED**

K2: **BLUE**

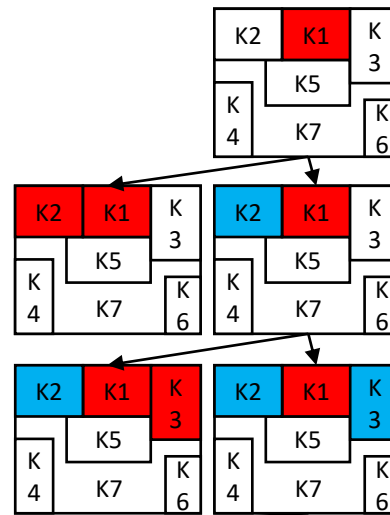
K3: **BLUE**

K4: **RED**

K5: **RED**

K6: ???

K7: ???



## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

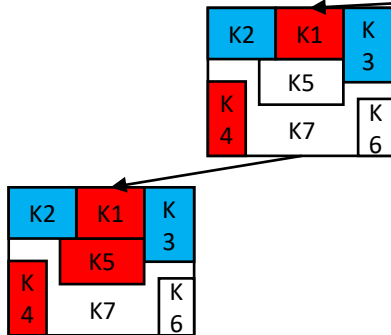
Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$



Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: **RED**, **BLUE**, **GREEN**

## Assignment:

K1: RED

K2: BLUE

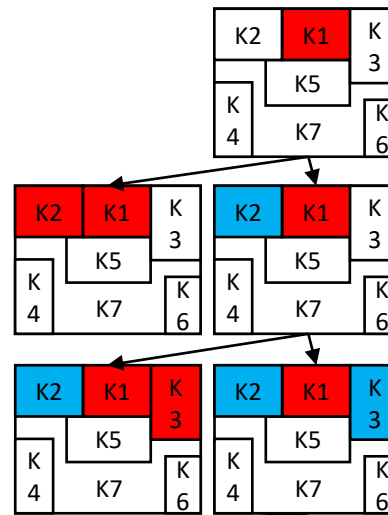
K3: BLUE

K4: RED

K5: RED

K6: ???

K7: ???



## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

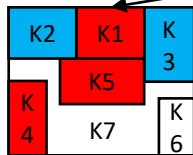
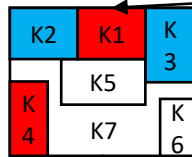
Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$



Violates:  
Rule 3

Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED

K2: BLUE

K3: BLUE

K4: RED

K5: ???

K6: ???

K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

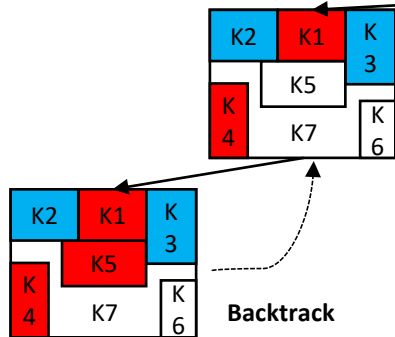
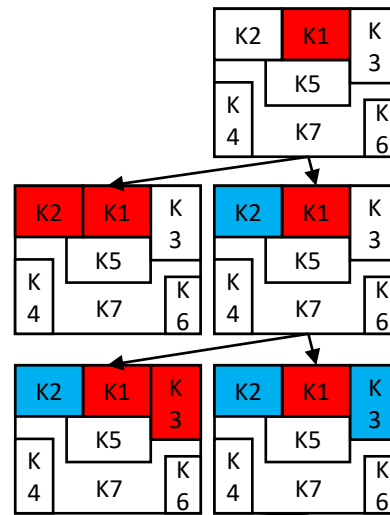
Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

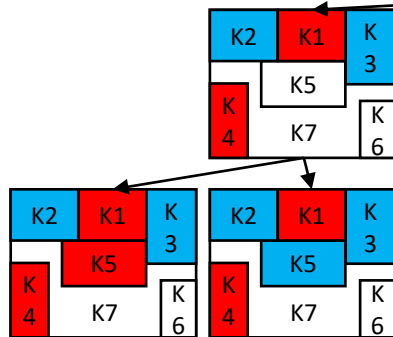
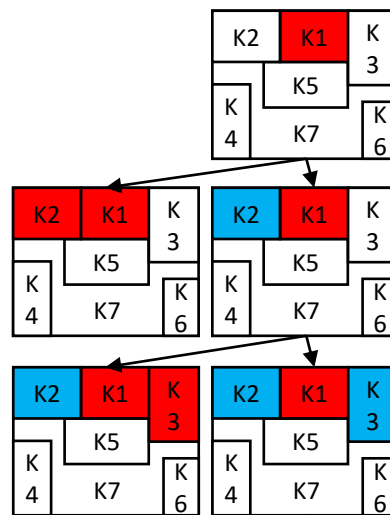
Rule 10:  $K6 \neq K7$



Violates:  
Rule 3

Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

**K7: ???**



**Rule 10:  $K6 \neq K7$**

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## Assignment:

K1: **RED**

K2: **BLUE**

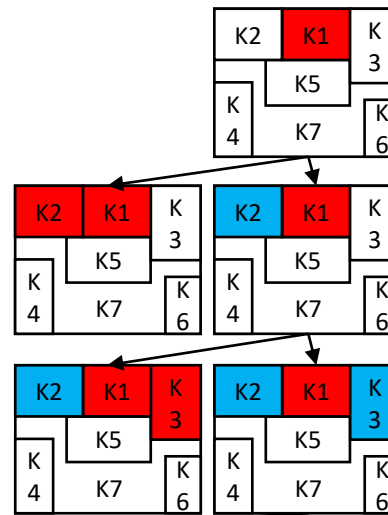
K3: **BLUE**

K4: **RED**

K5: **BLUE**

K6: ???

K7: ???



## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

**Rule 4:  $K2 \neq K5$**

Rule 5:  $K2 \neq K7$

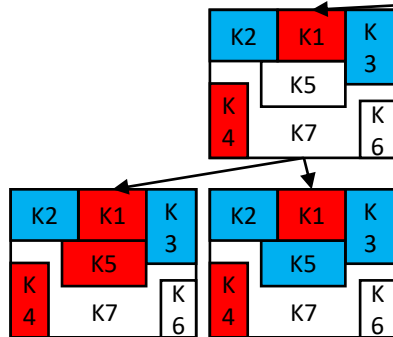
**Rule 6:  $K3 \neq K5$**

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$



**Violates:**

**Rule 4**

**Rule 6**

Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: **RED**, **BLUE**, **GREEN**

## Assignment:

K1: **RED**

K2: **BLUE**

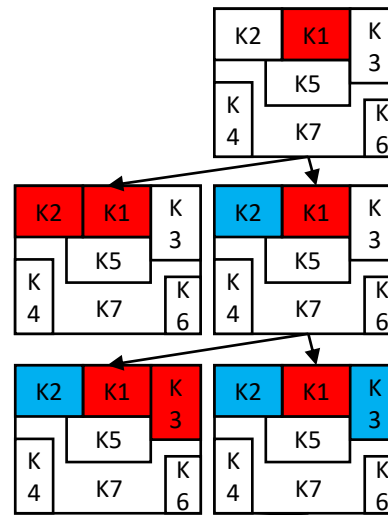
K3: **BLUE**

K4: **RED**

K5: ???

K6: ???

K7: ???



## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

**Rule 4:  $K2 \neq K5$**

Rule 5:  $K2 \neq K7$

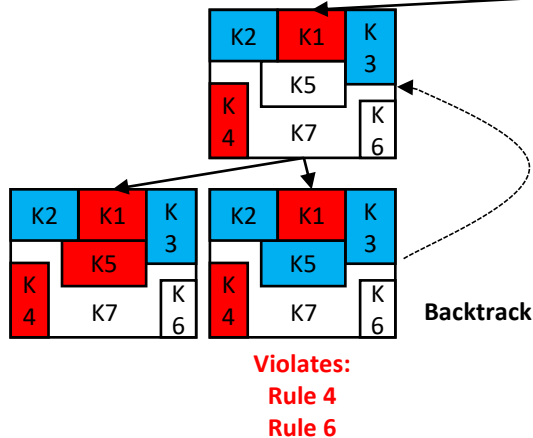
**Rule 6:  $K3 \neq K5$**

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$



Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: **RED**, **BLUE**, **GREEN**

## Assignment:

K1: RED

K2: BLUE

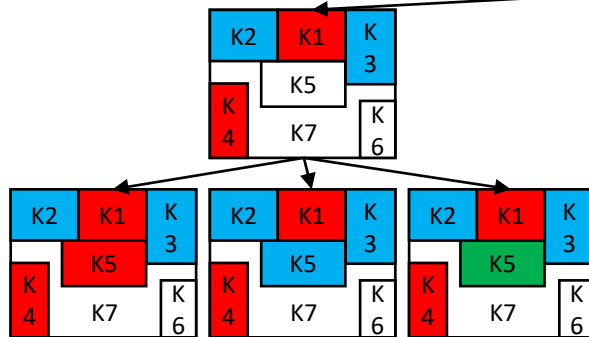
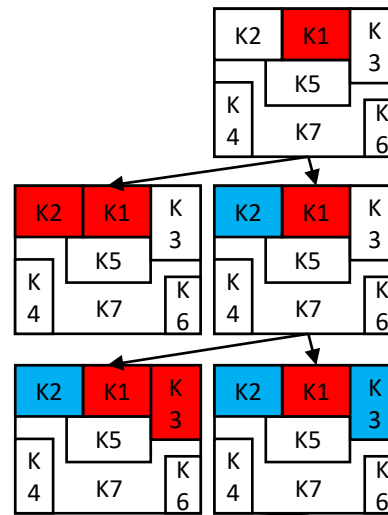
K3: BLUE

K4: RED

K5: GREEN

K6: ???

K7: ???



## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$

Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED

K2: BLUE

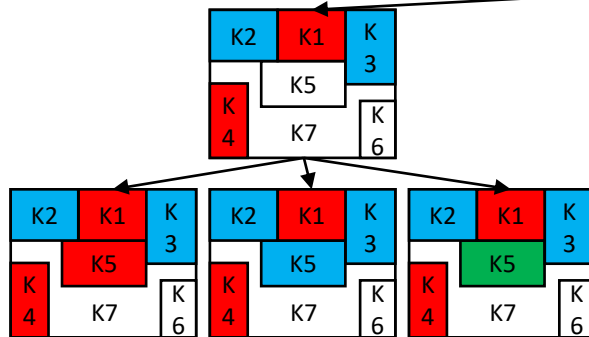
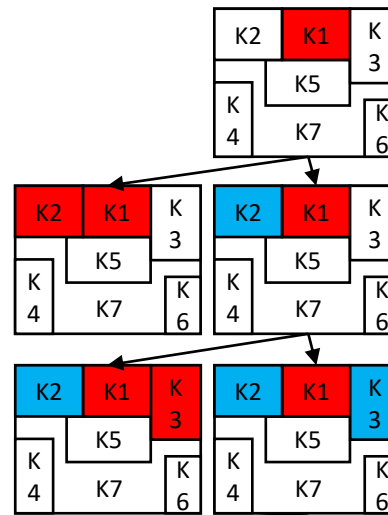
K3: BLUE

K4: RED

K5: GREEN

K6: ???

K7: ???



CONSISTENT  
PARTIAL  
Assignment

## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$

Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED

K2: BLUE

K3: BLUE

K4: RED

K5: GREEN

K6: RED

K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

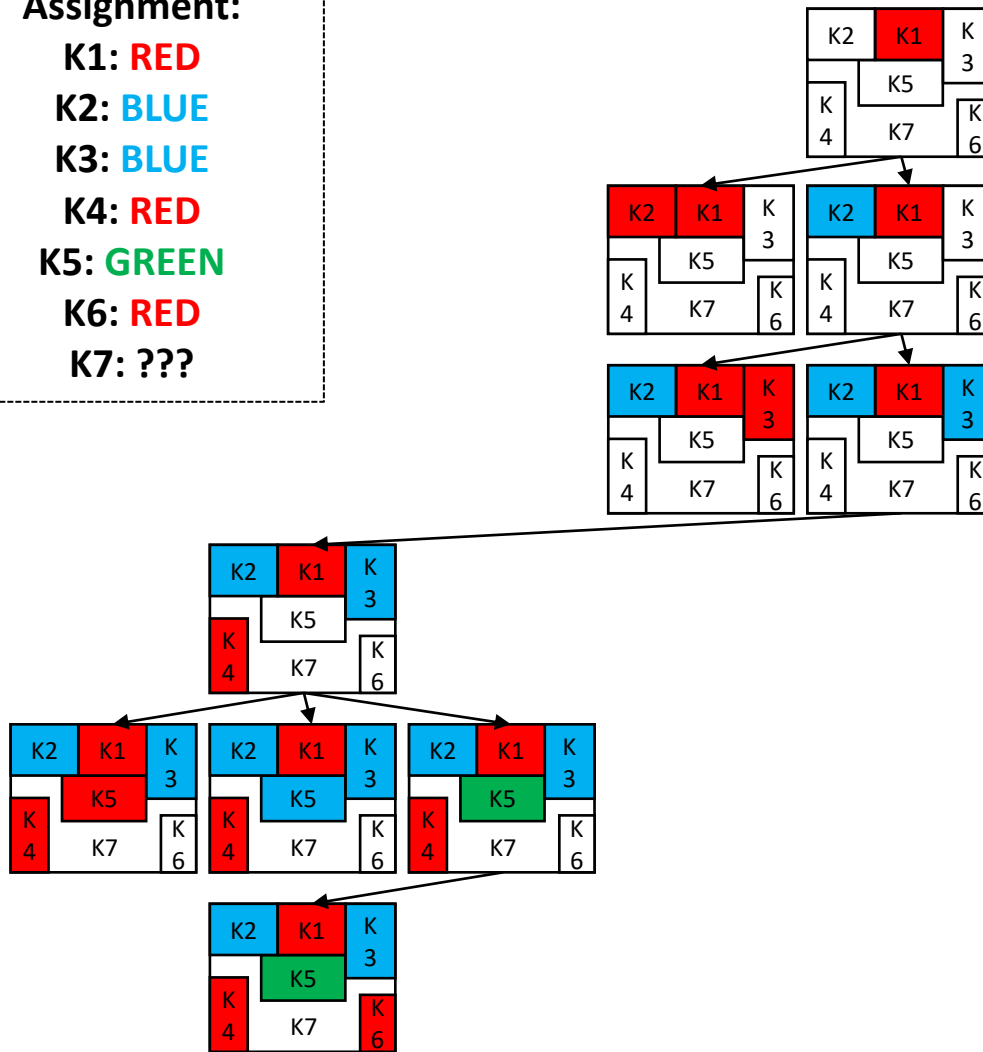
Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$



Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: **RED**

K2: **BLUE**

K3: **BLUE**

K4: **RED**

K5: **GREEN**

K6: **RED**

K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

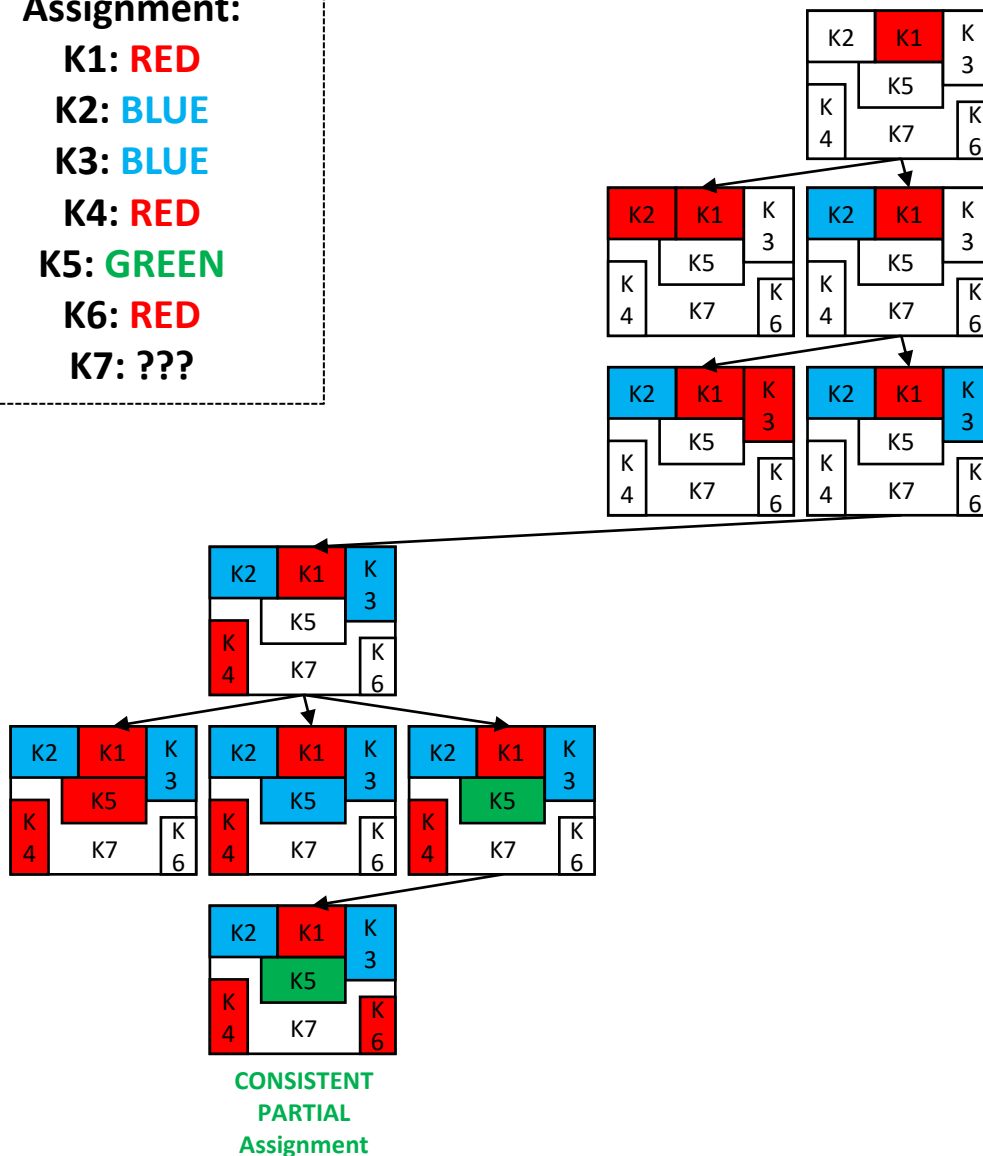
Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$



Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: **RED**, **BLUE**, **GREEN**

## Assignment:

K1: RED

K2: BLUE

K3: BLUE

K4: RED

K5: GREEN

K6: RED

K7: RED

## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

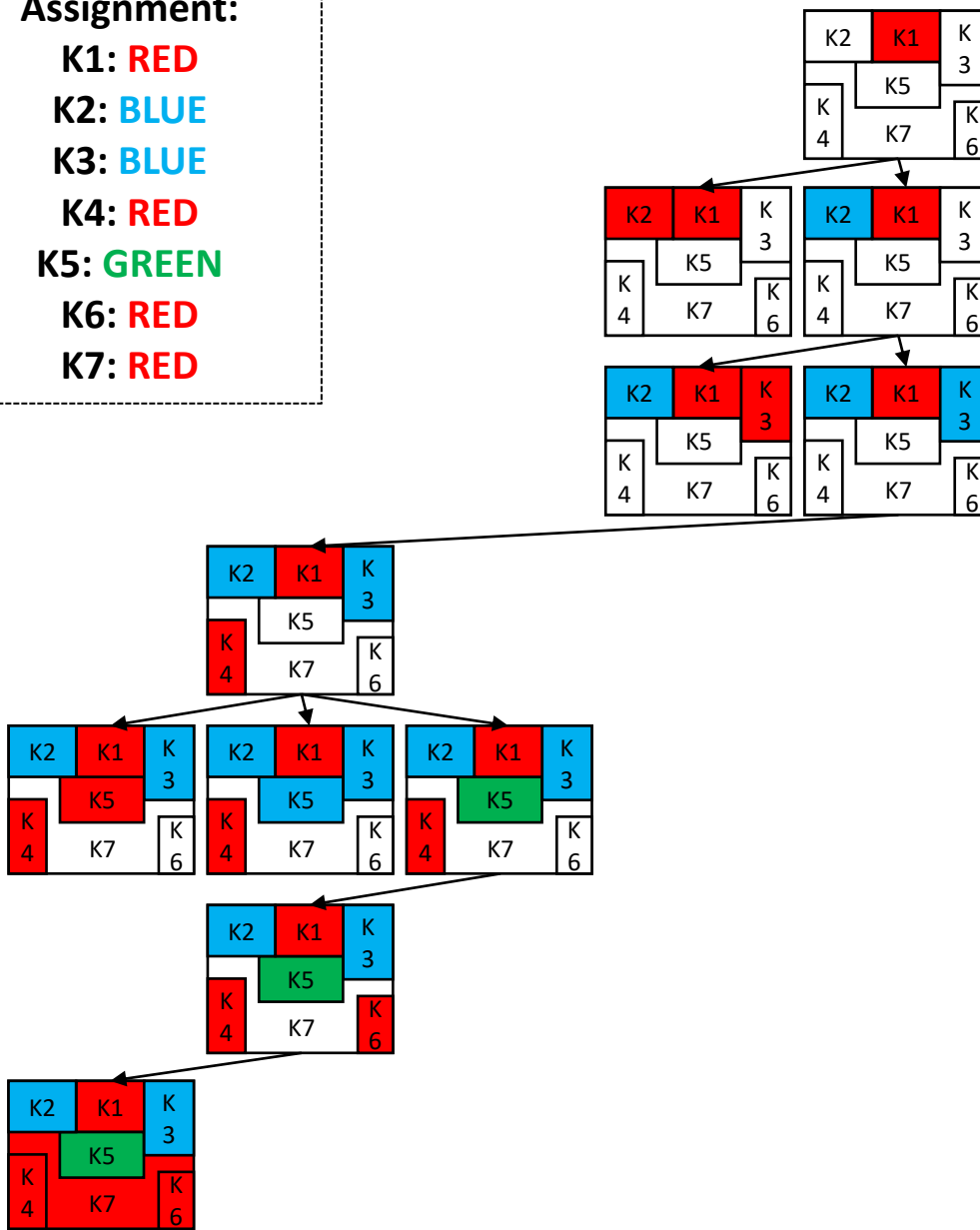
Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$



Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED

K2: BLUE

K3: BLUE

K4: RED

K5: GREEN

K6: RED

K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

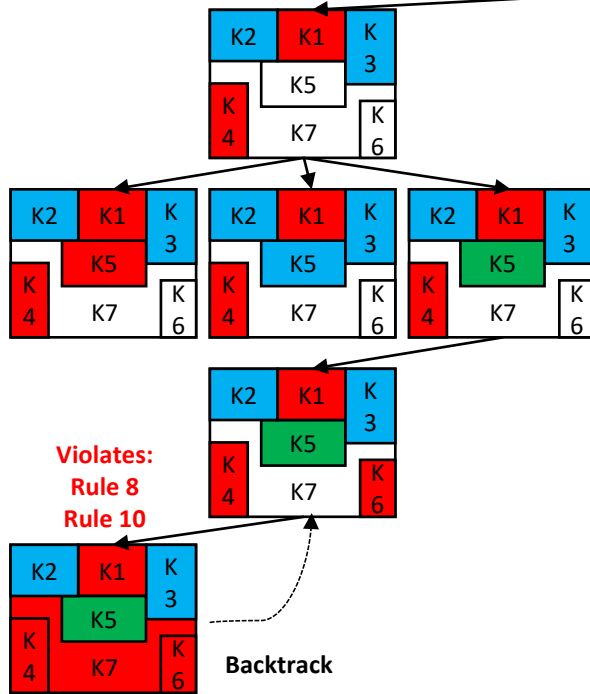
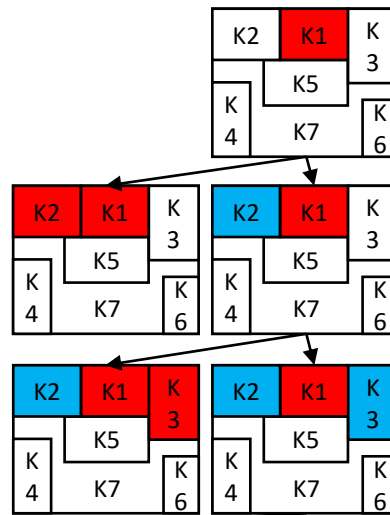
Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

**Rule 8:  $K4 \neq K7$**

**Rule 9:  $K5 \neq K7$**

**Rule 10:  $K6 \neq K7$**



Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

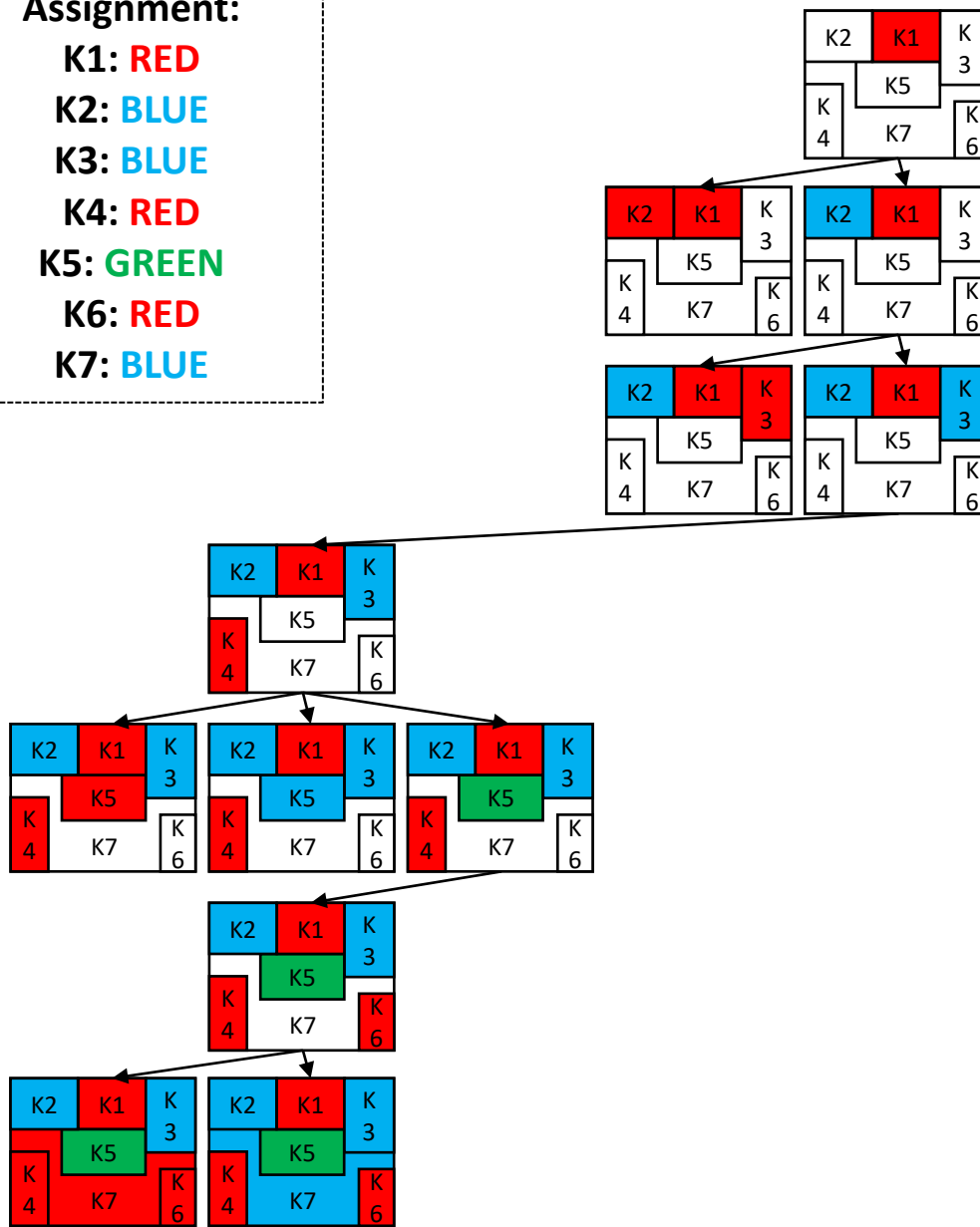


## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: RED  
K5: GREEN  
K6: RED  
K7: BLUE

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



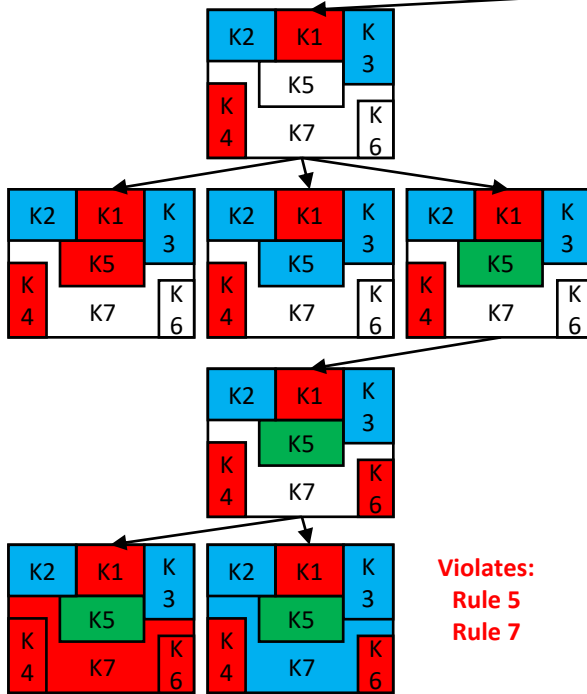
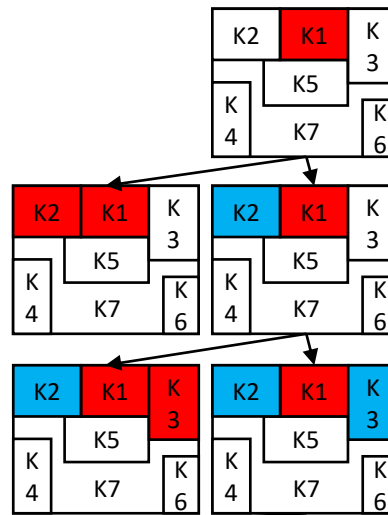
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: RED  
K5: GREEN  
K6: RED  
K7: BLUE

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
**Rule 5:  $K2 \neq K7$**   
Rule 6:  $K3 \neq K5$   
**Rule 7:  $K3 \neq K7$**   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



Violates:  
Rule 5  
Rule 7

Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN



## Assignment:

K1: RED

K2: BLUE

K3: BLUE

K4: RED

K5: GREEN

K6: RED

K7: GREEN

## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

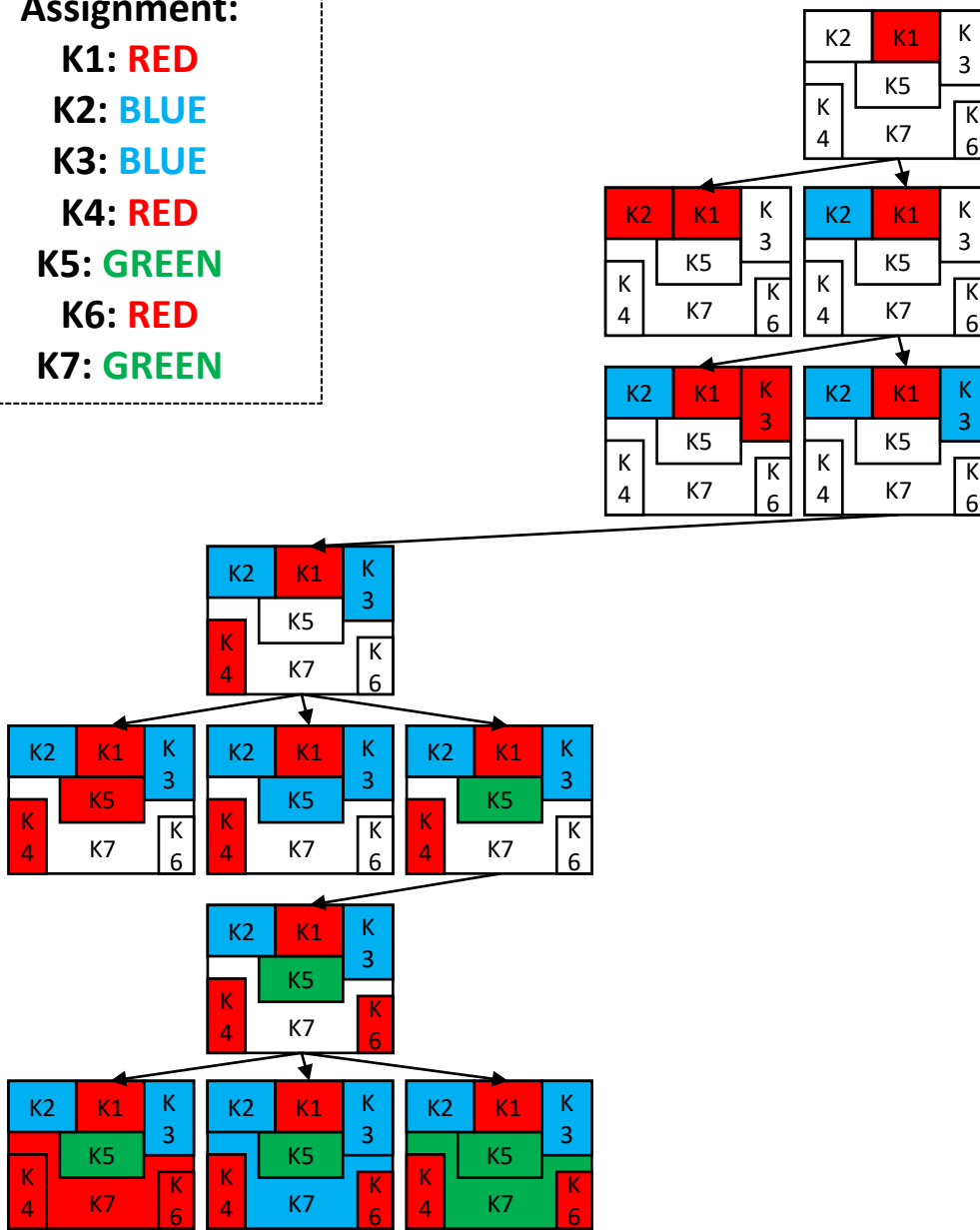
Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$



Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED

K2: BLUE

K3: BLUE

K4: RED

K5: GREEN

K6: RED

K7: GREEN

## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

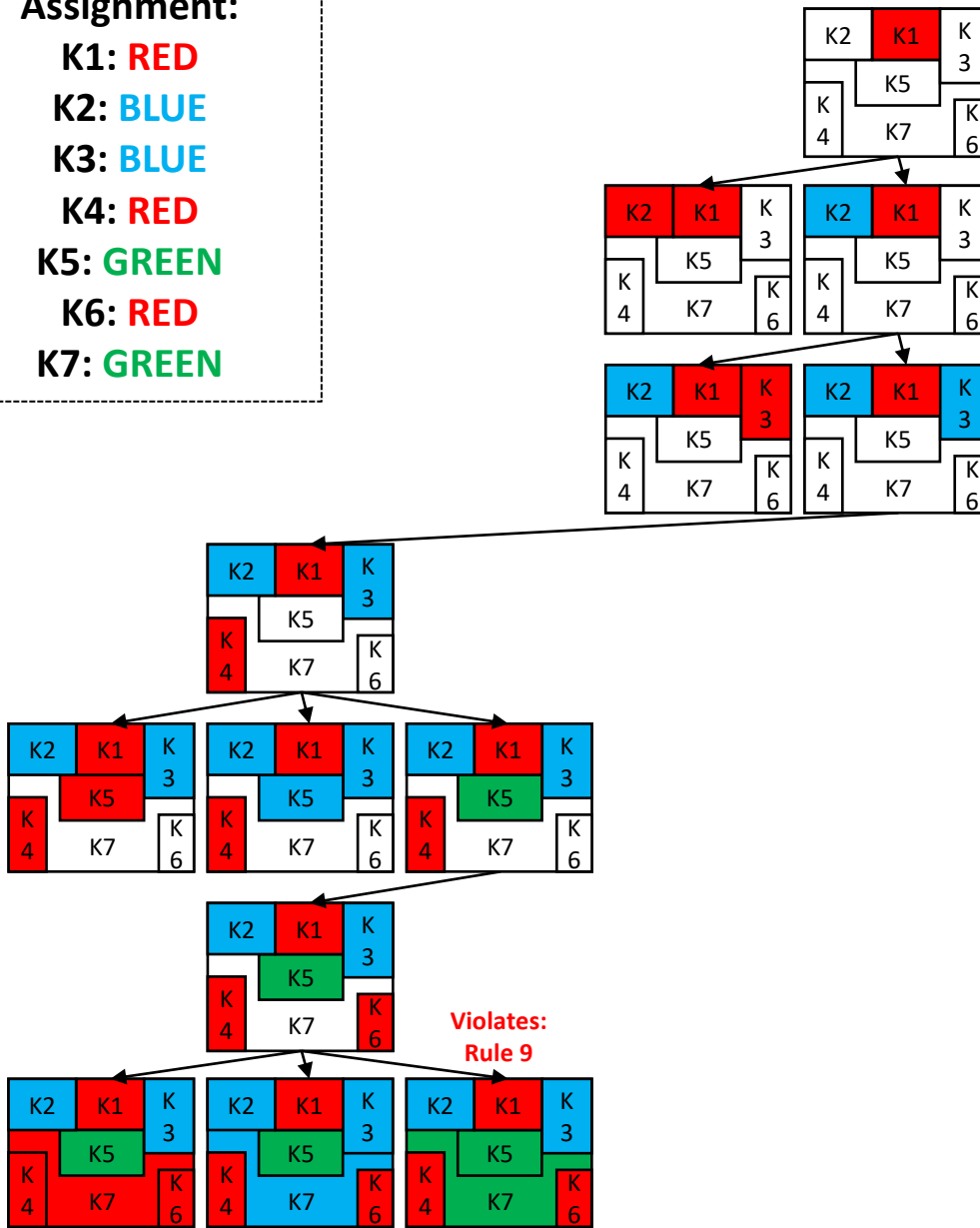
Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$



Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED

K2: BLUE

K3: BLUE

K4: RED

K5: GREEN

K6: RED

K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

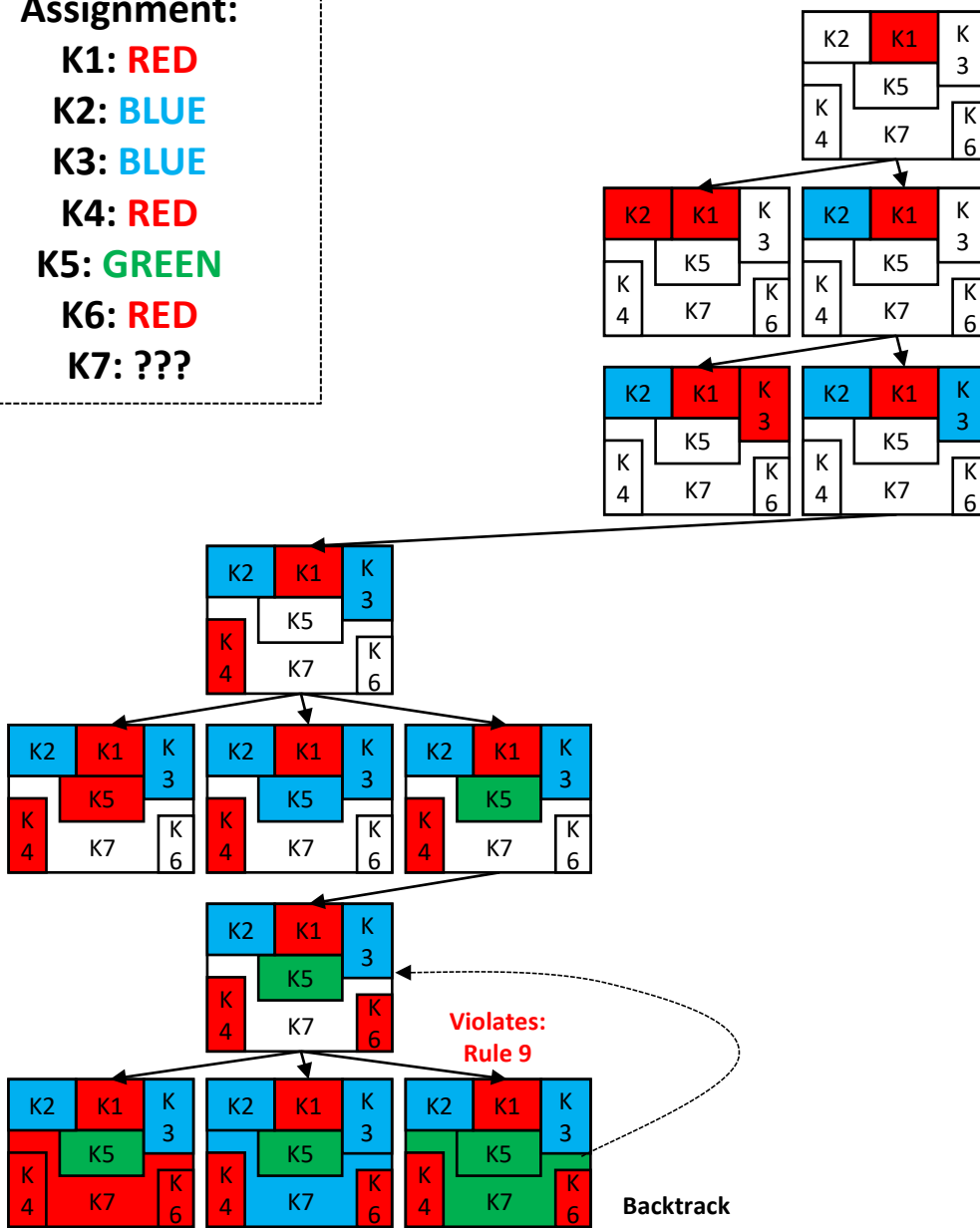
Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$



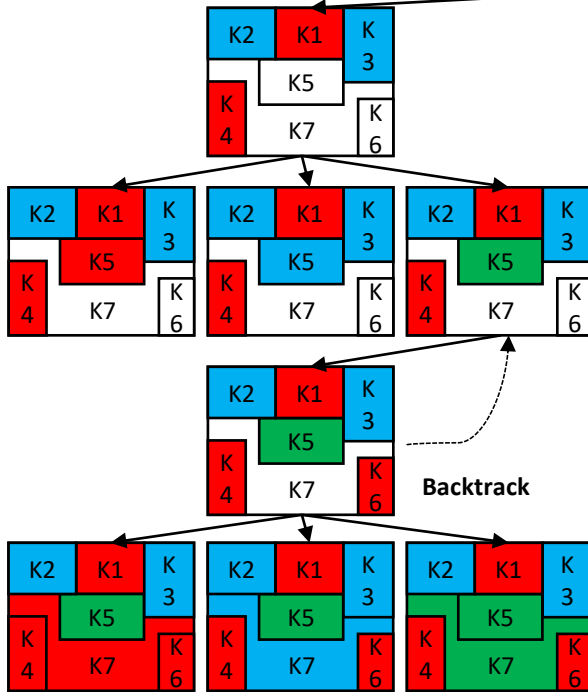
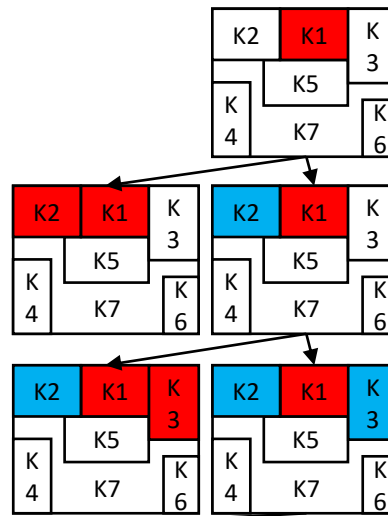
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: RED  
K5: GREEN  
K6: ???  
K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



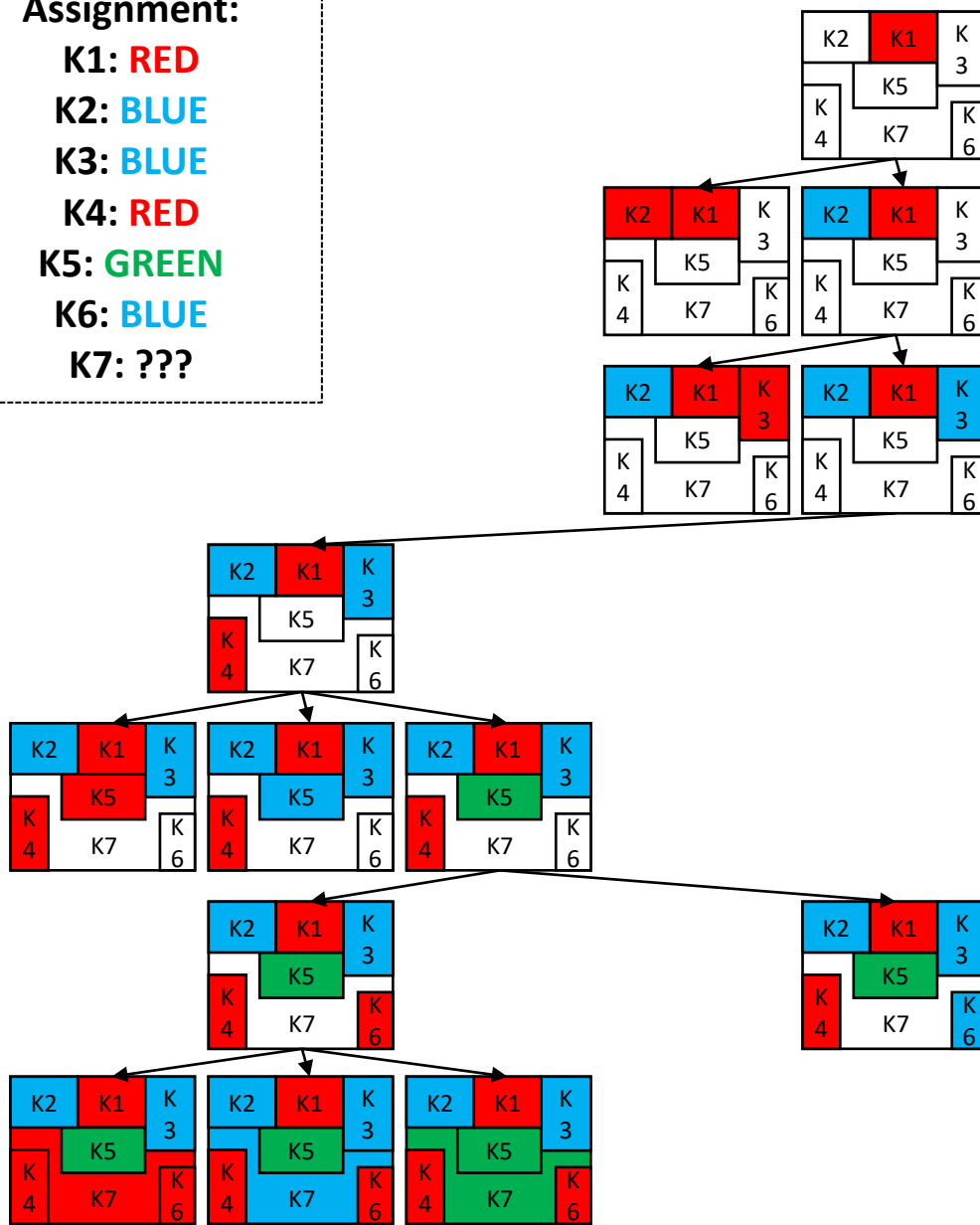
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: RED  
K5: GREEN  
K6: BLUE  
K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

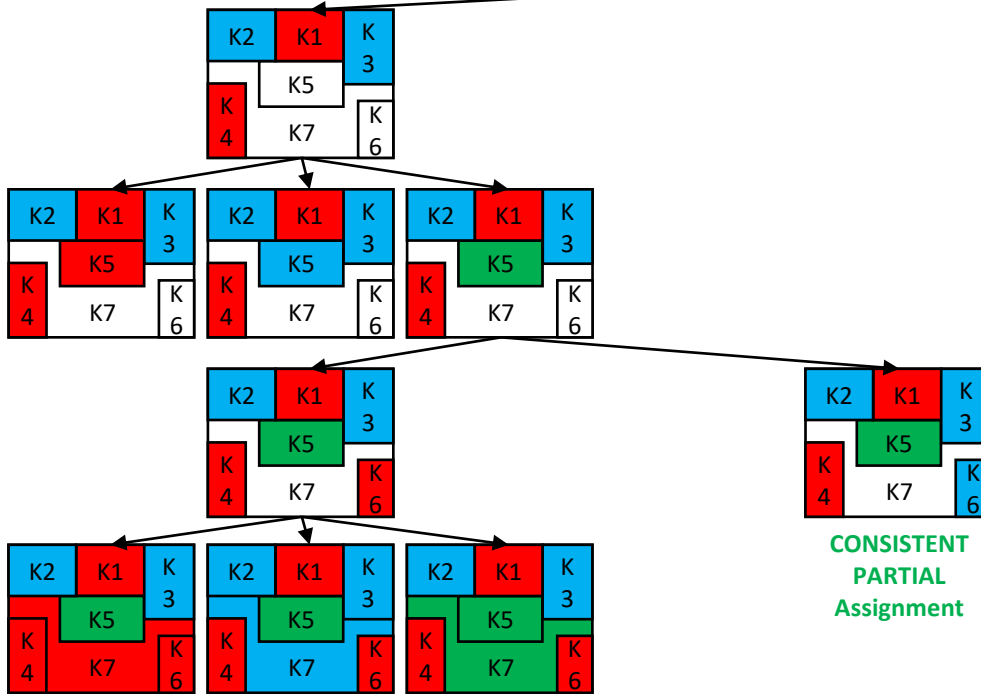
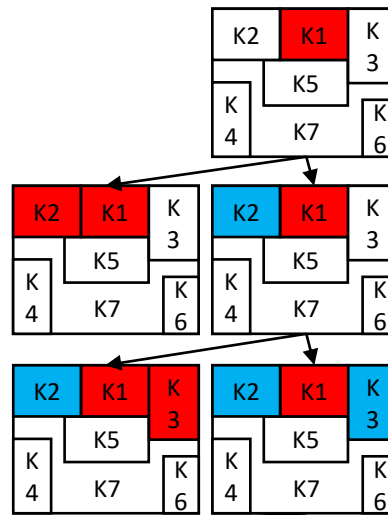


## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: RED  
K5: GREEN  
K6: BLUE  
K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



CONSISTENT  
PARTIAL  
Assignment

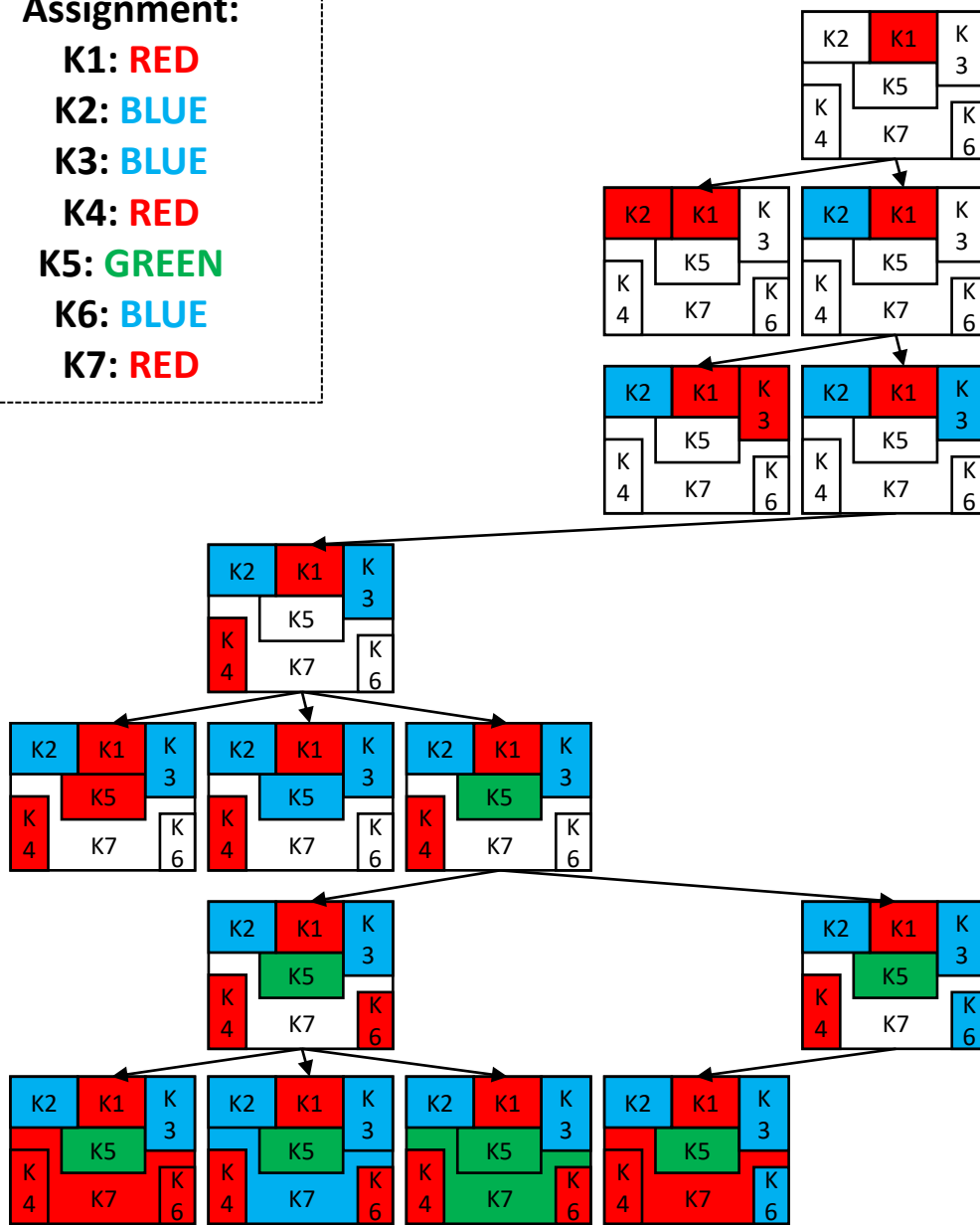
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: RED  
K5: GREEN  
K6: BLUE  
K7: RED

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



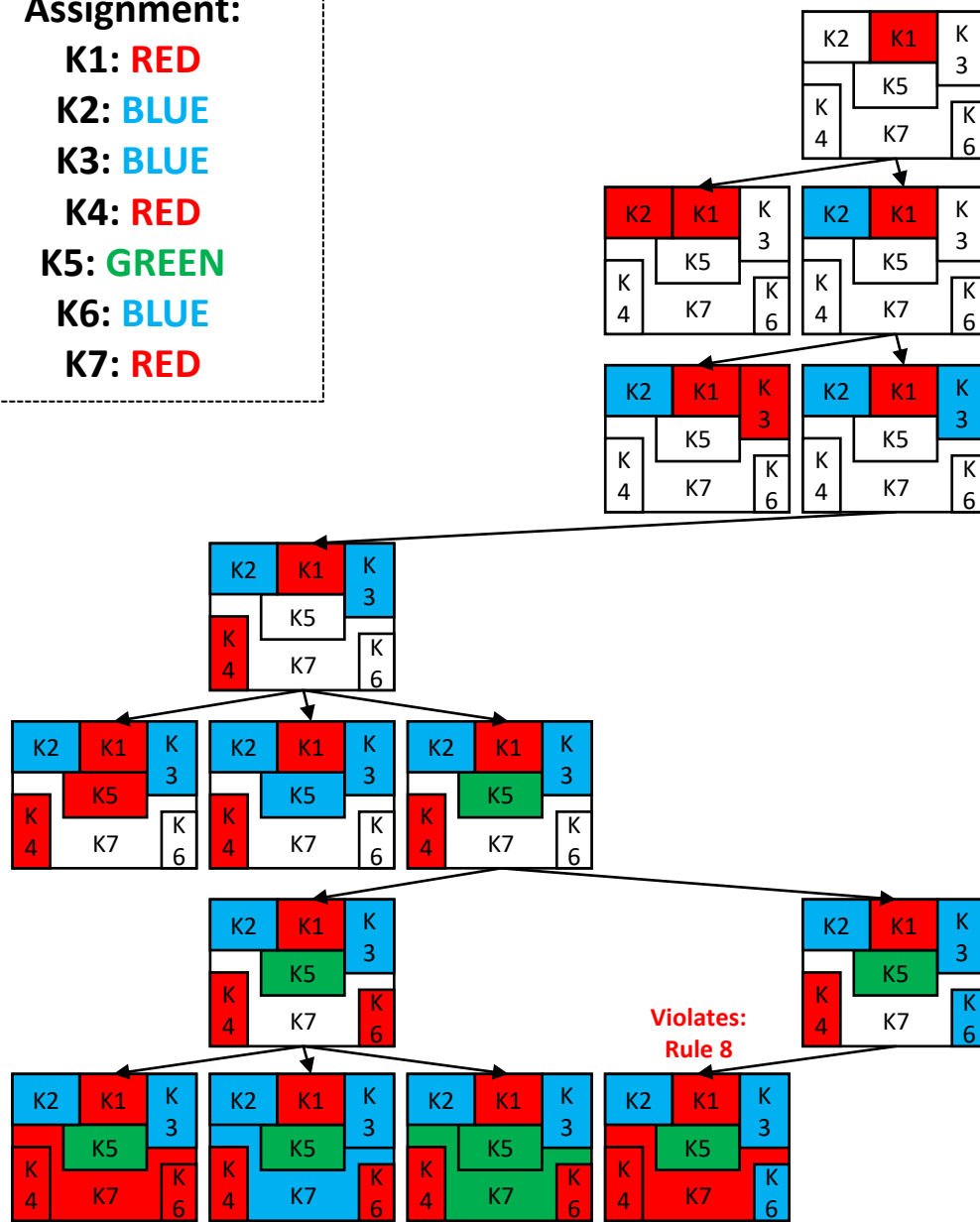
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: RED  
K5: GREEN  
K6: BLUE  
K7: RED

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
**Rule 8:  $K4 \neq K7$**   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

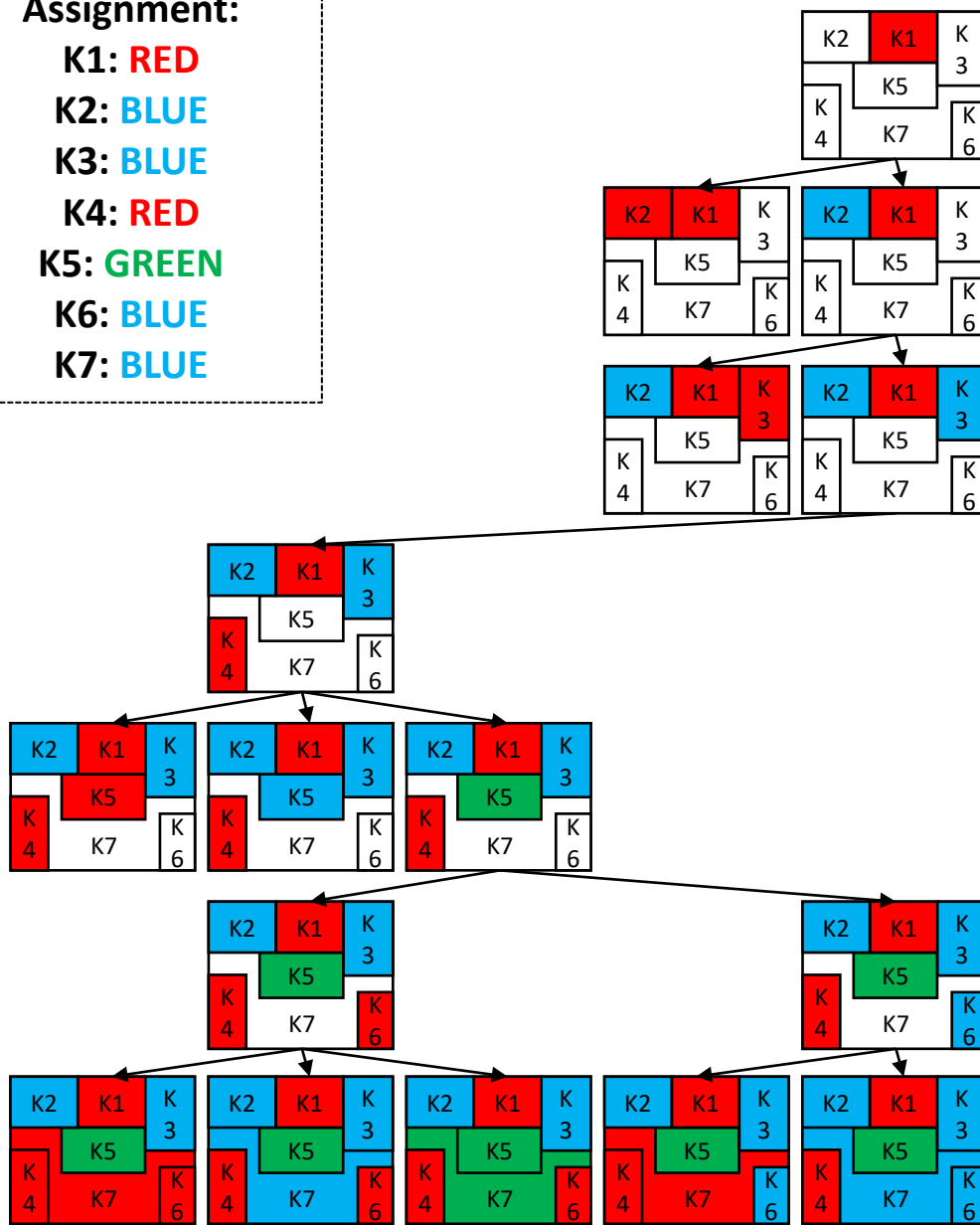


## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: RED  
K5: GREEN  
K6: BLUE  
K7: BLUE

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



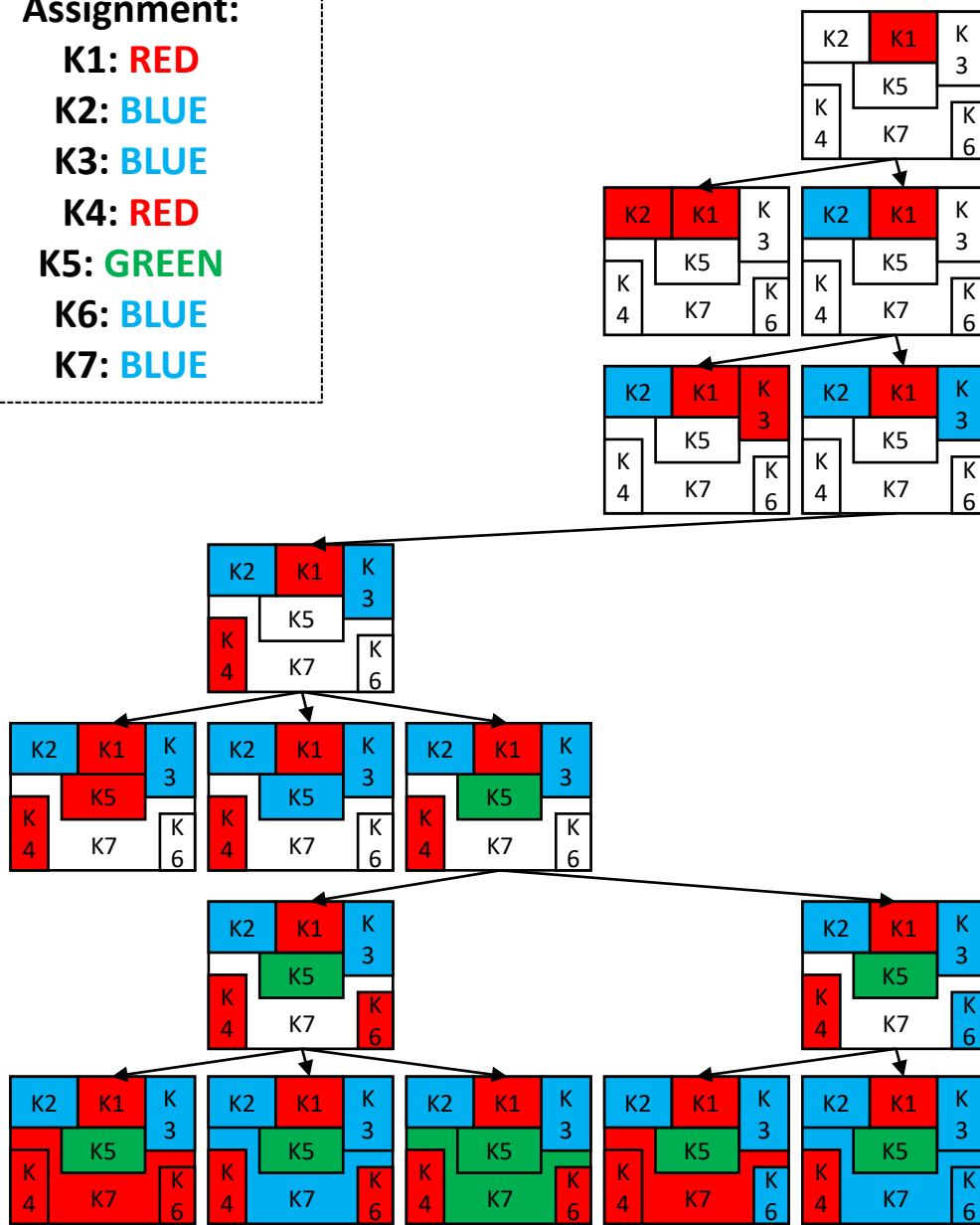
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: RED  
K5: GREEN  
K6: BLUE  
K7: BLUE

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
**Rule 7:  $K3 \neq K7$**   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
**Rule 10:  $K6 \neq K7$**



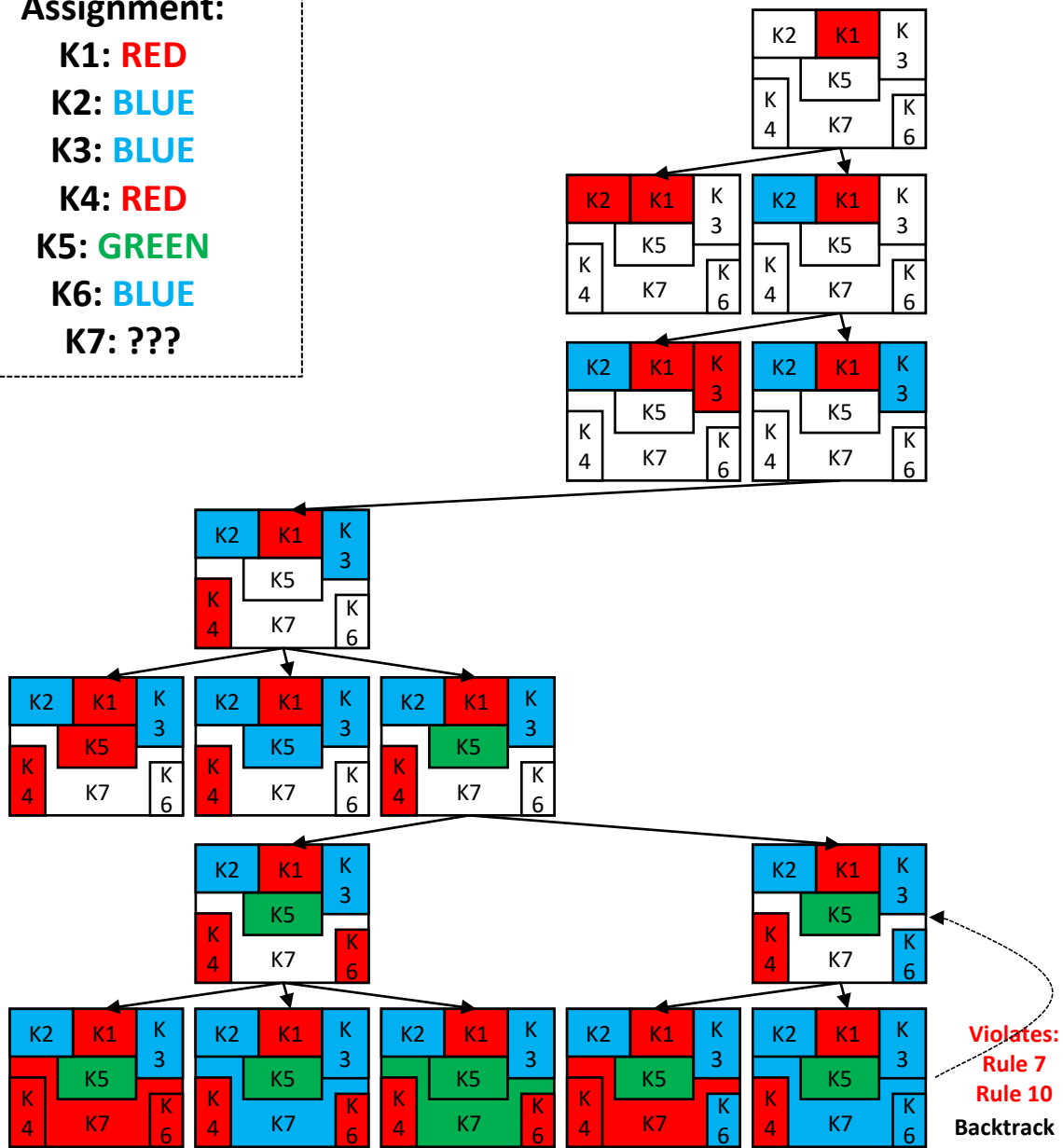
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: RED  
K5: GREEN  
K6: BLUE  
K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
**Rule 7:  $K3 \neq K7$**   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
**Rule 10:  $K6 \neq K7$**

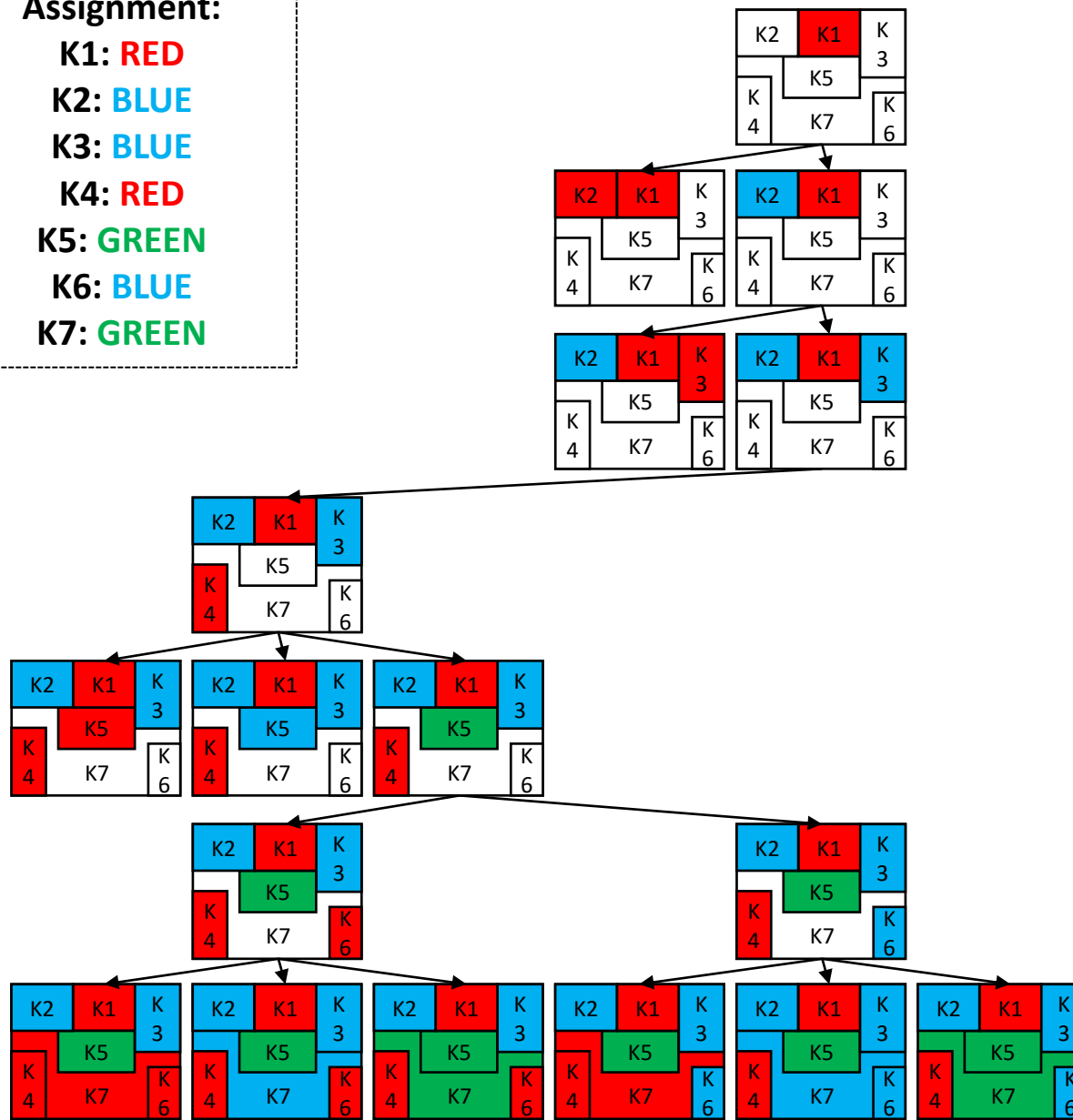


## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: RED  
K5: GREEN  
K6: BLUE  
K7: GREEN

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

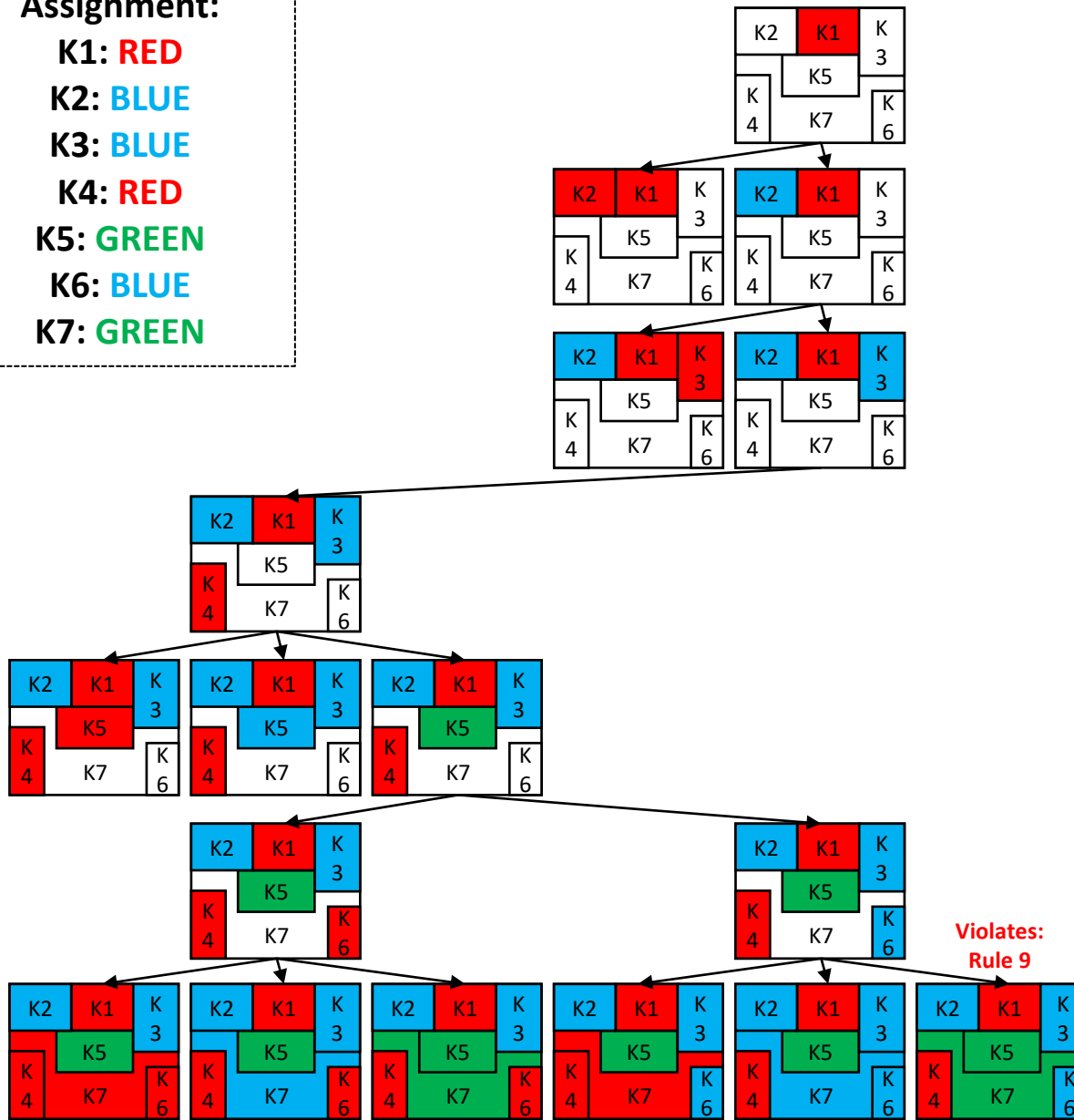


## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: RED  
K5: GREEN  
K6: BLUE  
K7: GREEN

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN



## Assignment:

K1: RED

K2: BLUE

K3: BLUE

K4: RED

K5: GREEN

K6: ???

K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

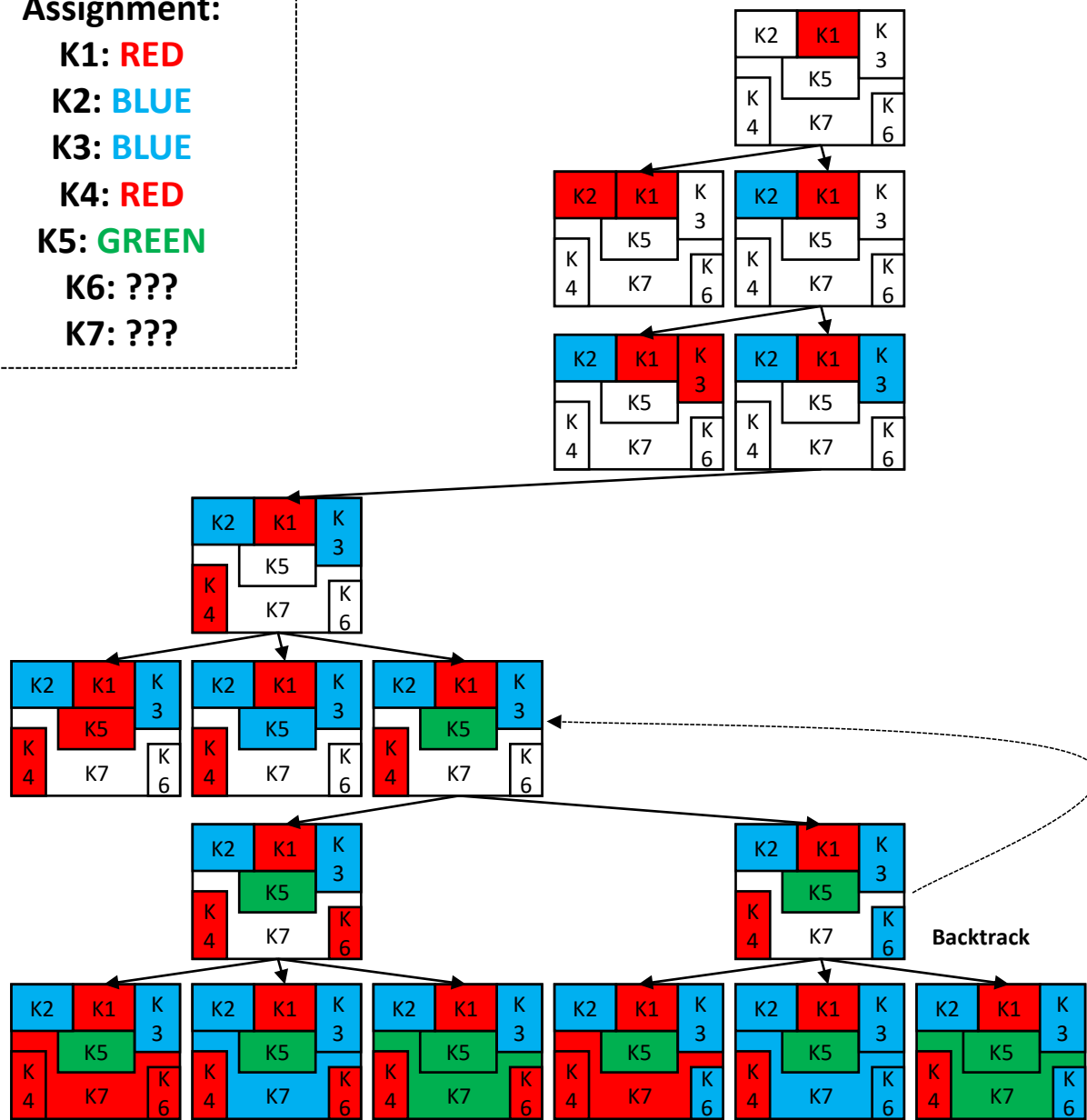
Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$



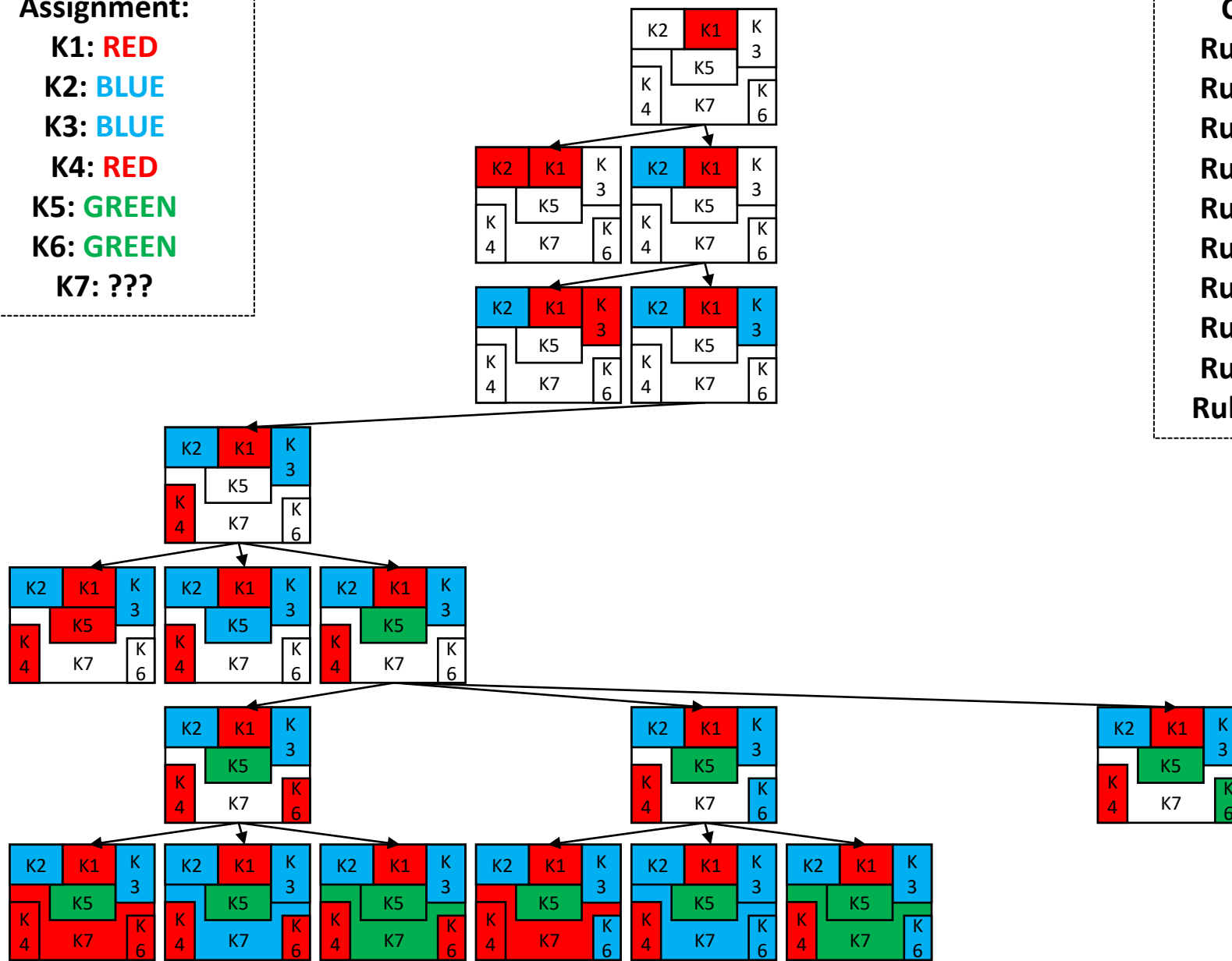
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: **RED**  
K2: **BLUE**  
K3: **BLUE**  
K4: **RED**  
K5: **GREEN**  
K6: **GREEN**  
K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



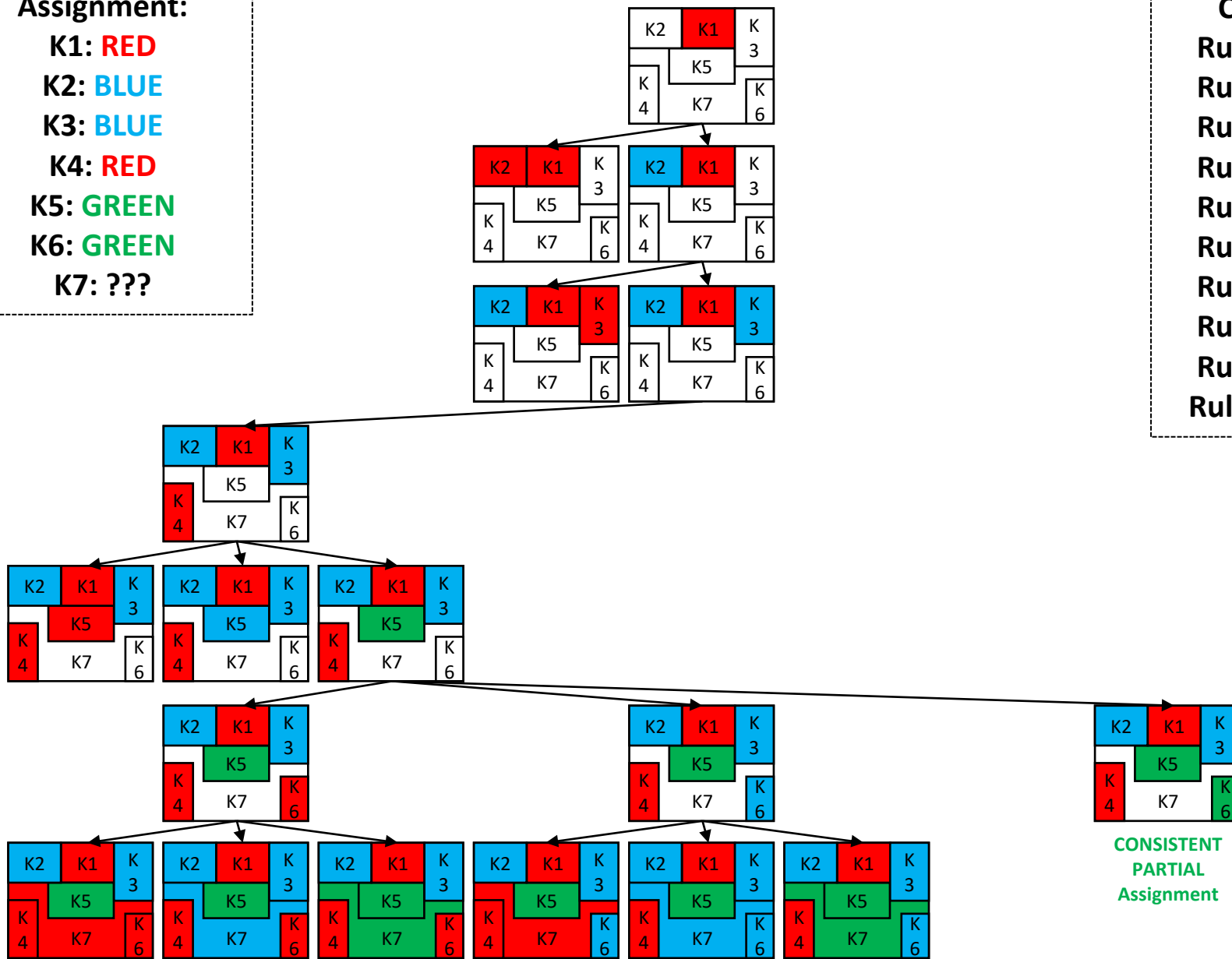
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: **RED**, **BLUE**, **GREEN**

## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: RED  
K5: GREEN  
K6: GREEN  
K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



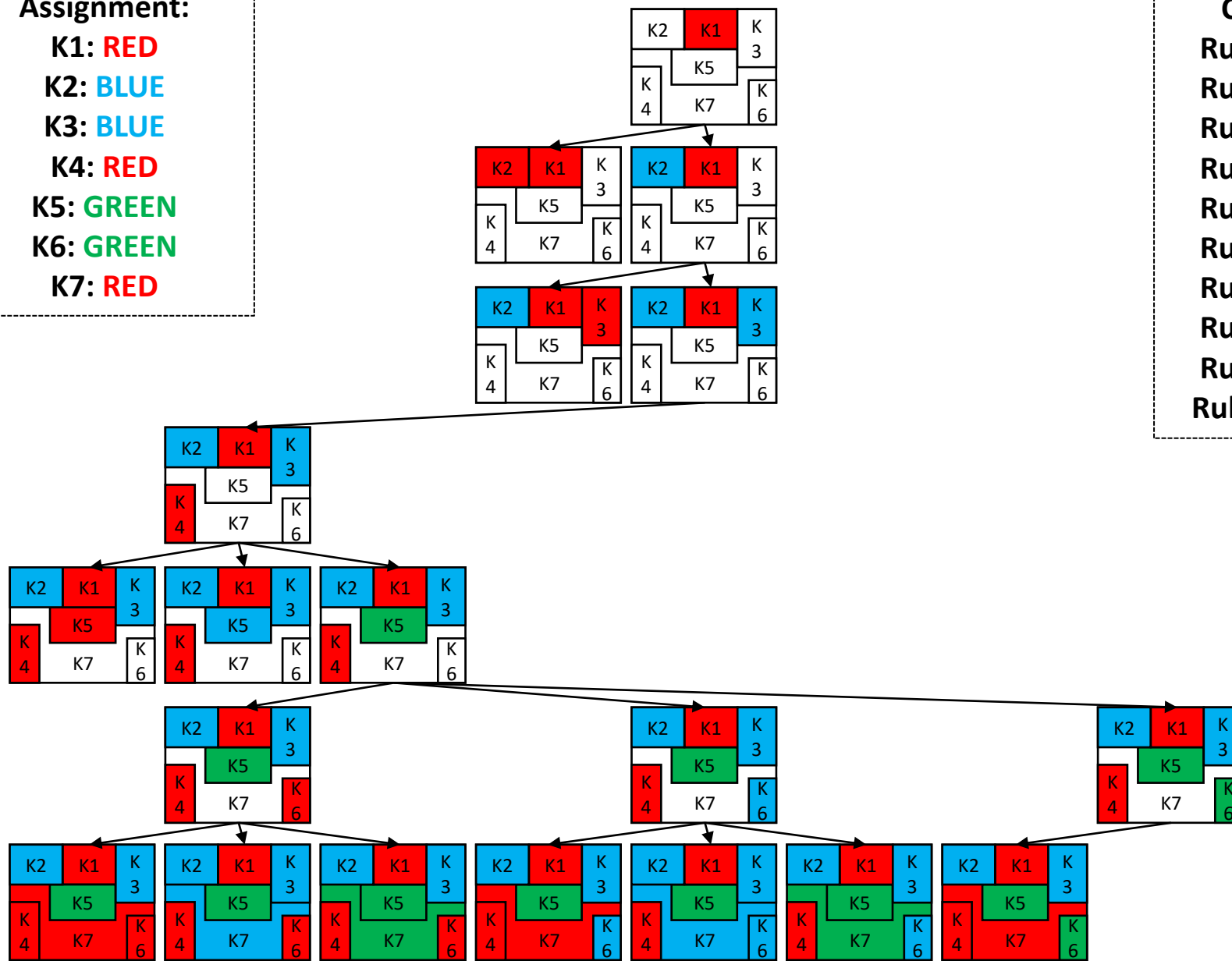
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: RED  
K5: GREEN  
K6: GREEN  
K7: RED

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



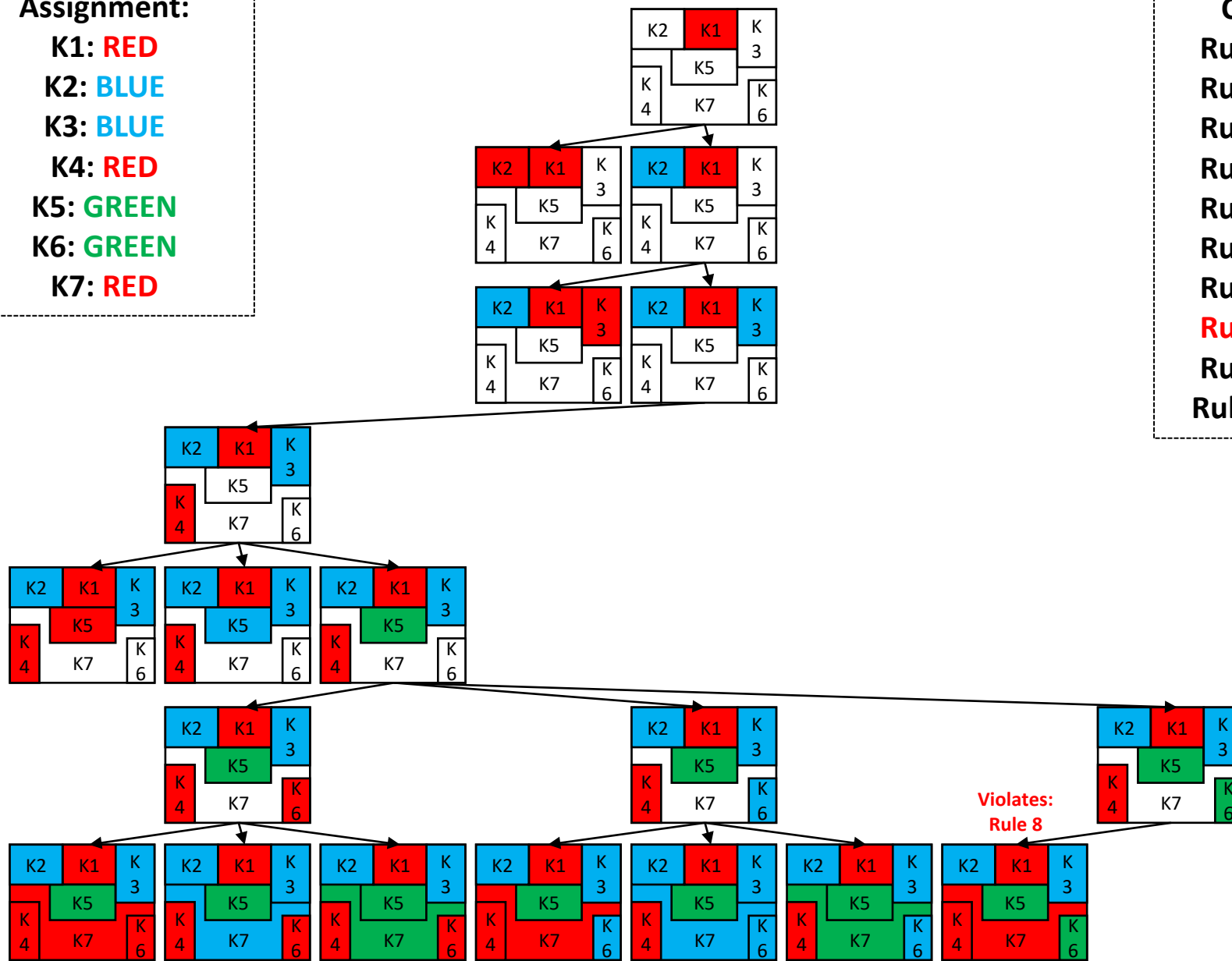
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: RED  
K5: GREEN  
K6: GREEN  
K7: RED

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
**Rule 8:  $K4 \neq K7$**   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



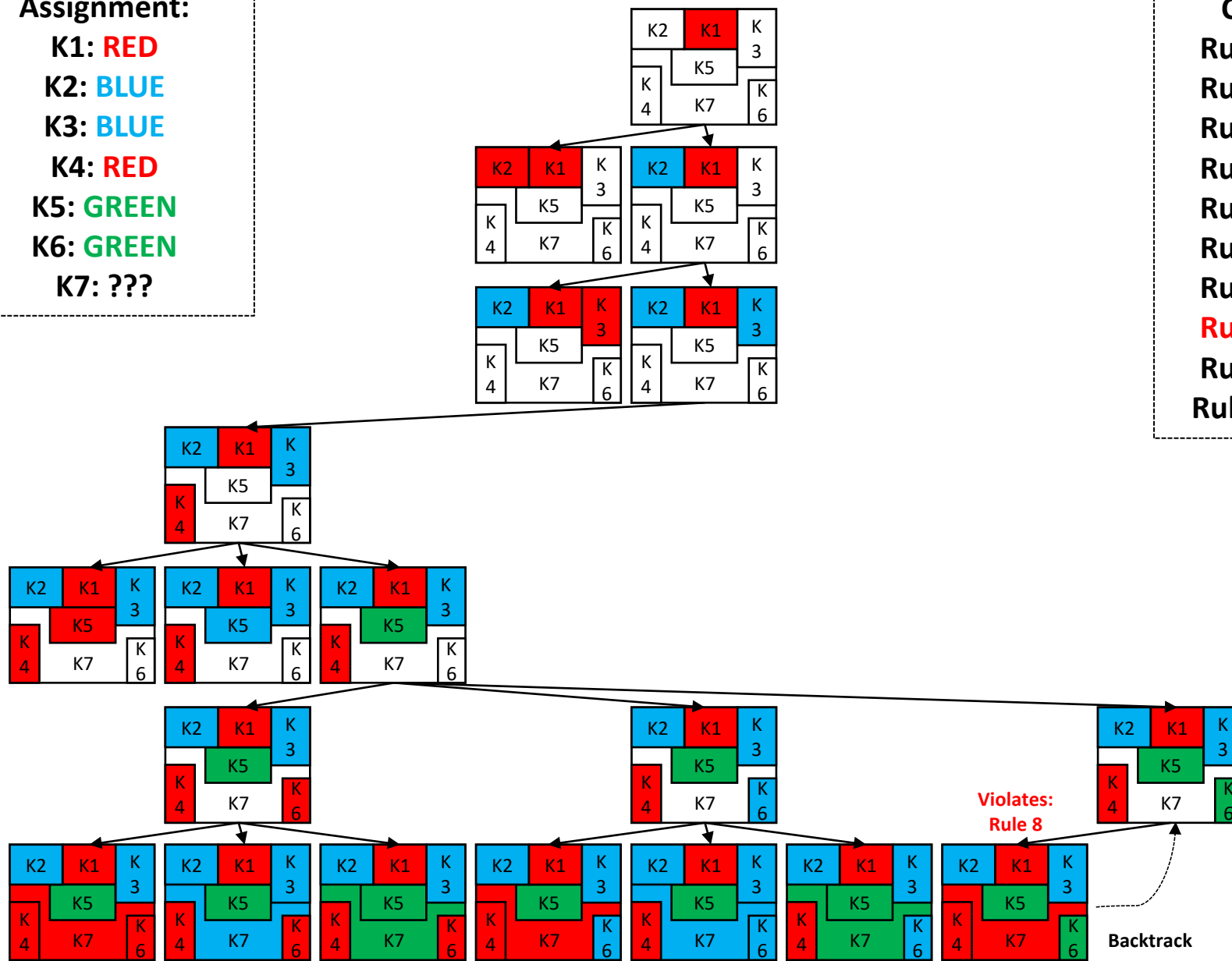
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: RED  
K5: GREEN  
K6: GREEN  
K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
**Rule 8:  $K4 \neq K7$**   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

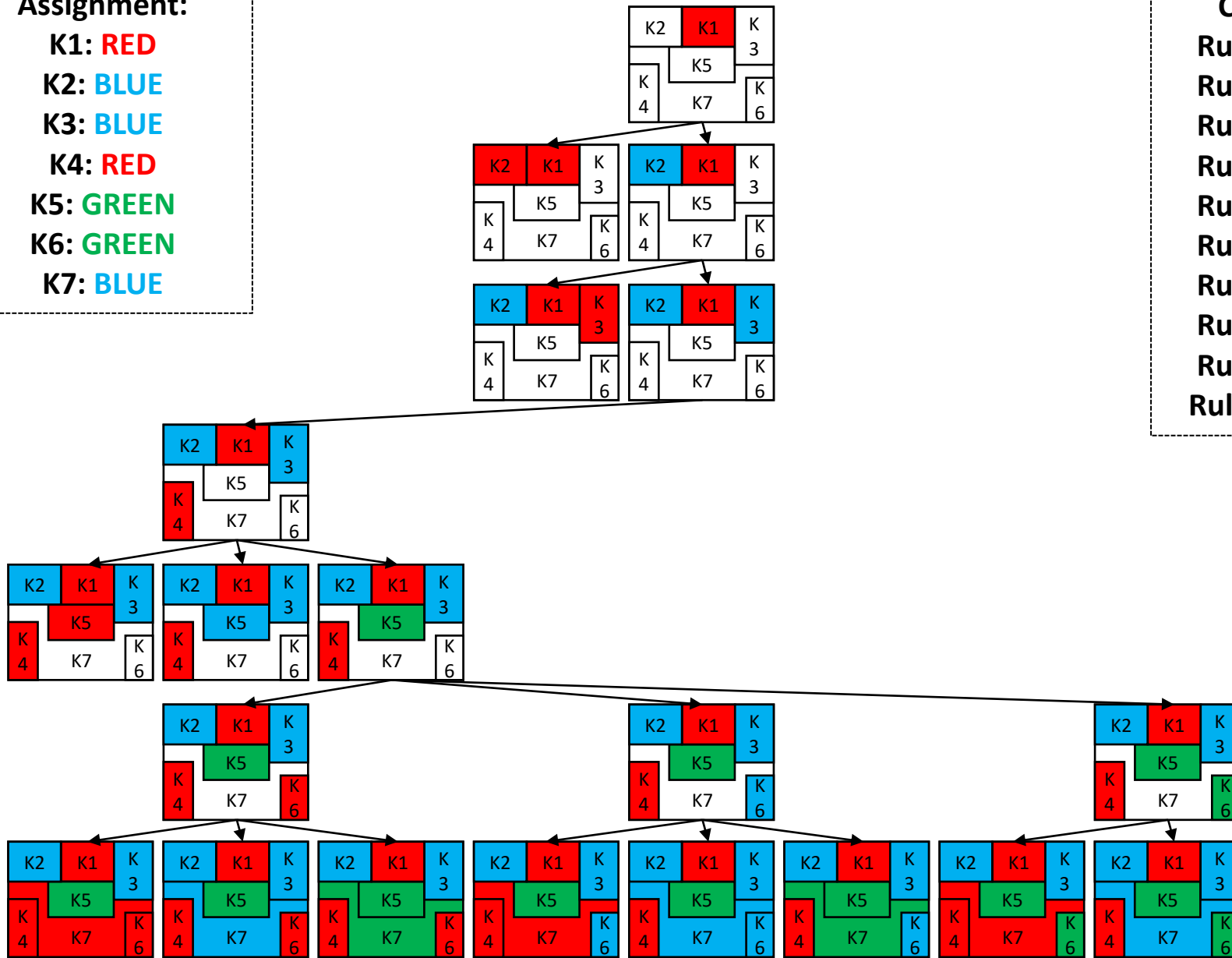


## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: RED  
K5: GREEN  
K6: GREEN  
K7: BLUE

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



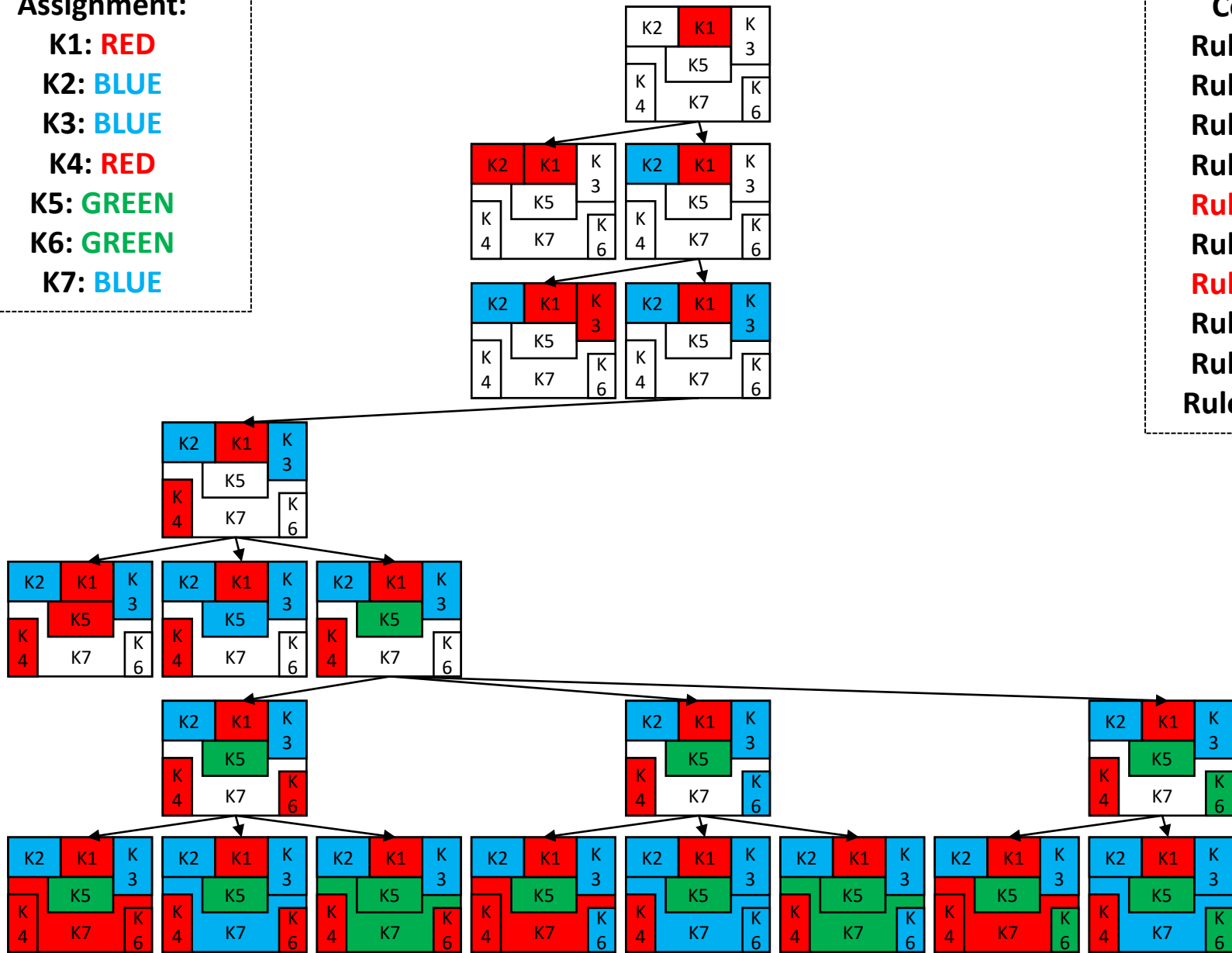
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: RED  
K5: GREEN  
K6: GREEN  
K7: BLUE

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
**Rule 5:  $K2 \neq K7$**   
Rule 6:  $K3 \neq K5$   
**Rule 7:  $K3 \neq K7$**   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

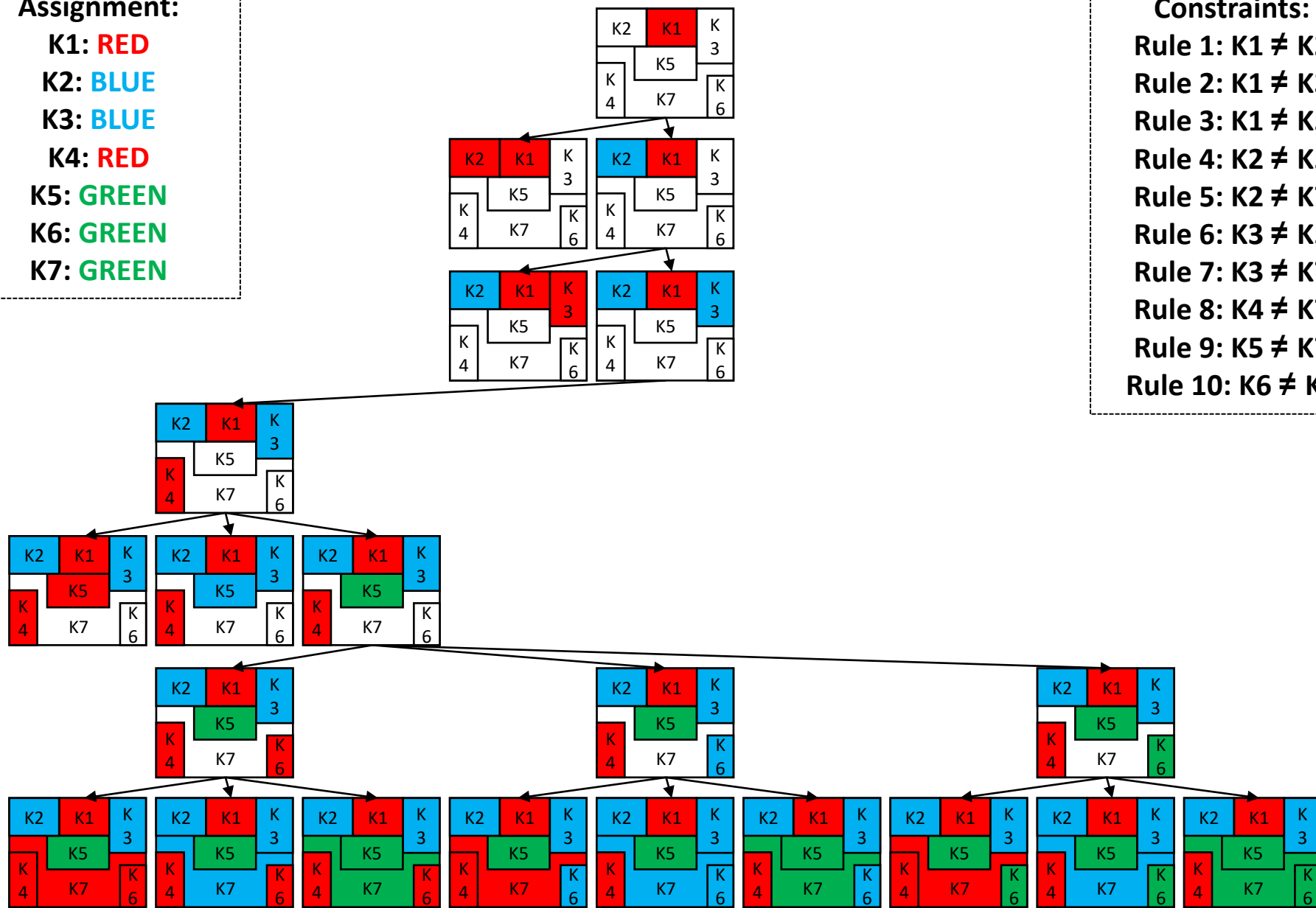


## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: RED  
K5: GREEN  
K6: GREEN  
K7: GREEN

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



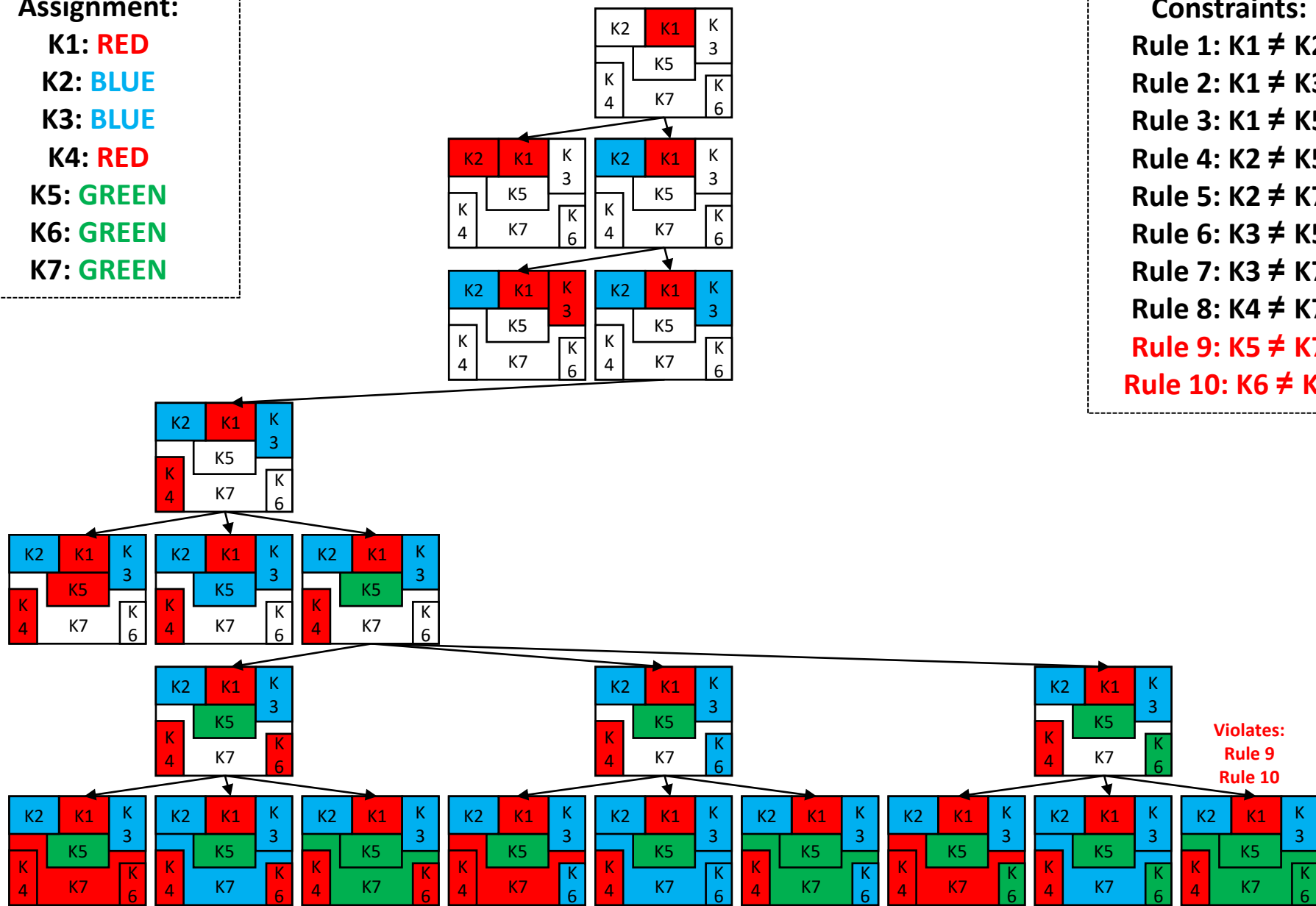
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: RED  
K5: GREEN  
K6: GREEN  
K7: GREEN

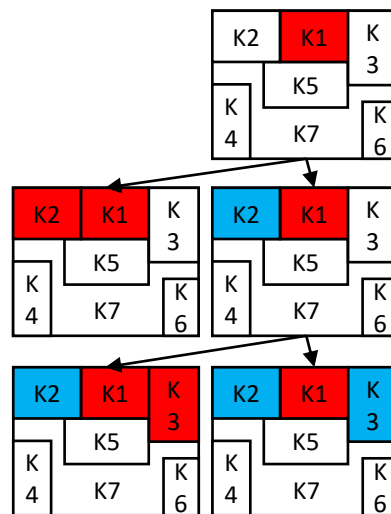
## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$

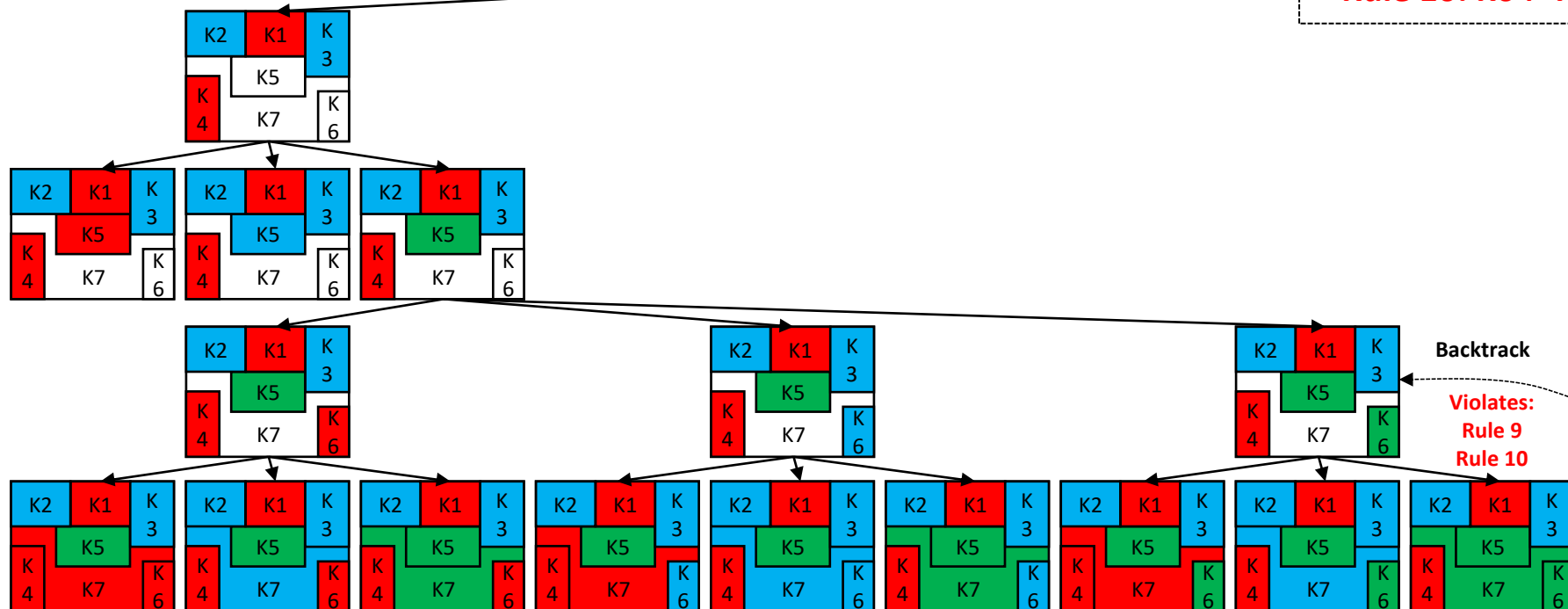


Violates:  
Rule 9  
Rule 10

K1: RED  
K2: BLUE  
K3: BLUE  
K4: RED  
K5: GREEN  
K6: GREEN  
K7: ???



Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



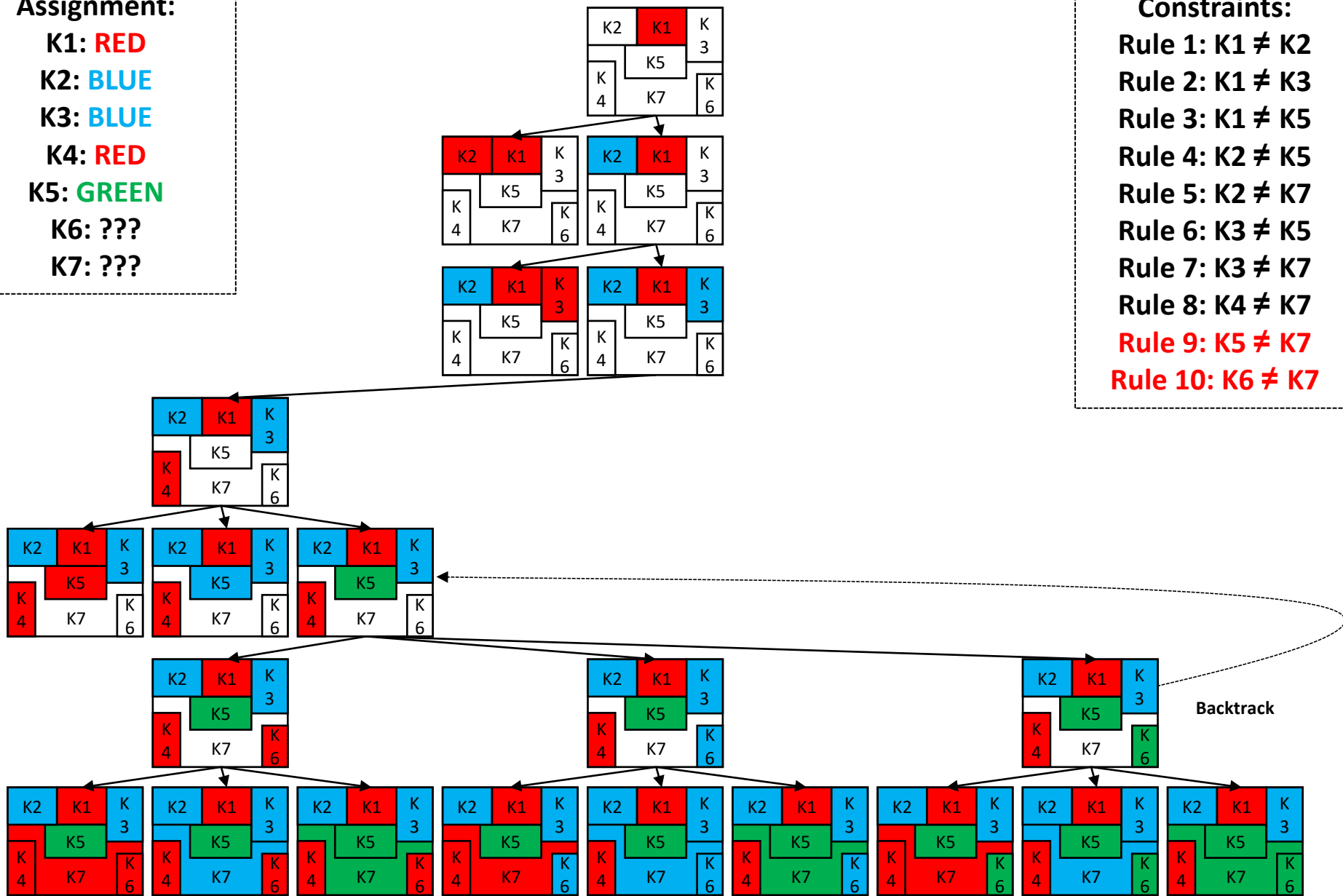
**Variable assignment order:** K1, K2, K3, K4, K5, K6, K7 | **Value assignment order:** RED, BLUE, GREEN

## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: RED  
K5: GREEN  
K6: ???  
K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



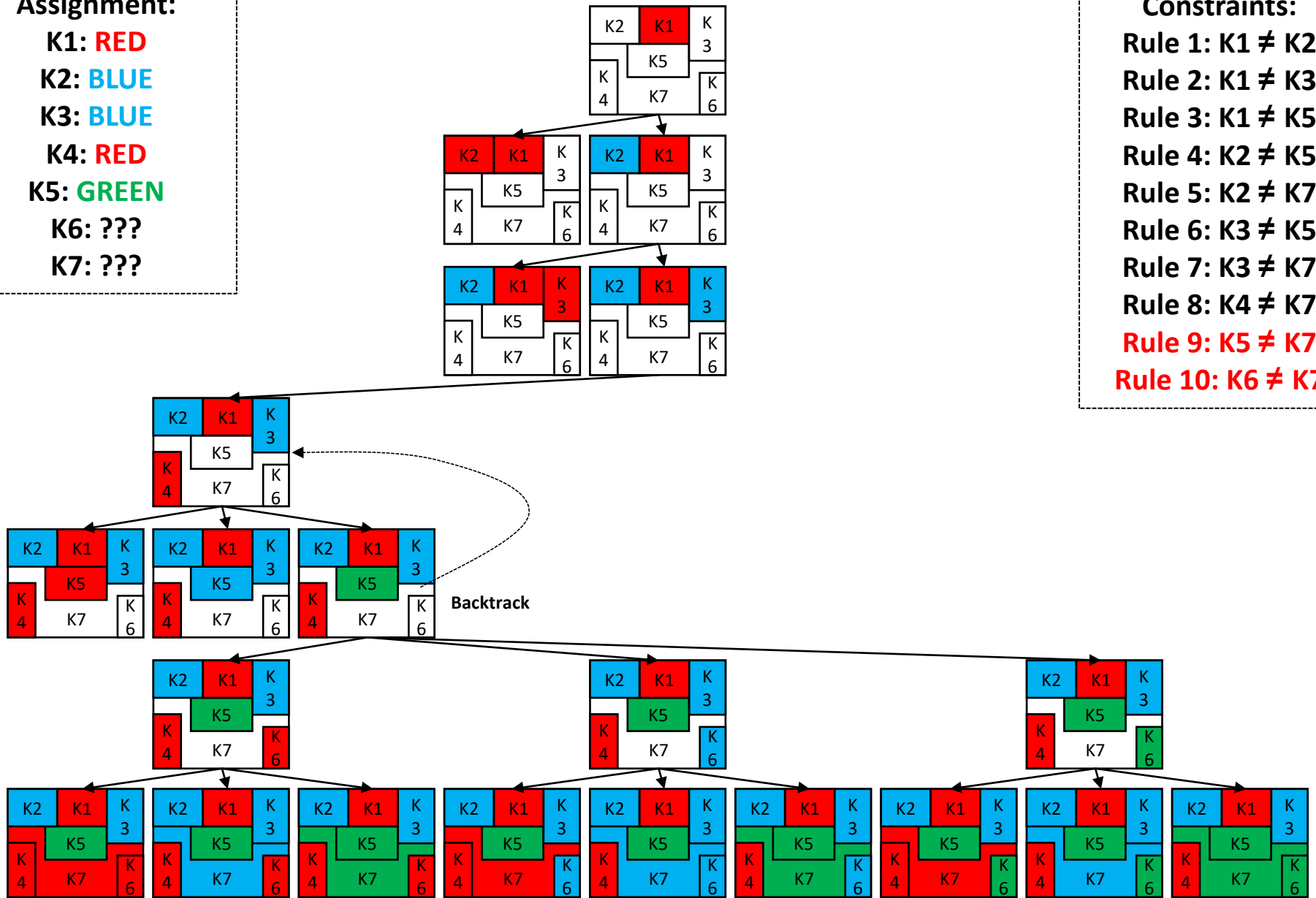
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: RED  
K5: GREEN  
K6: ???  
K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN



## Assignment:

K1: RED

K2: BLUE

K3: BLUE

K4: RED

K5: ???

K6: ???

K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

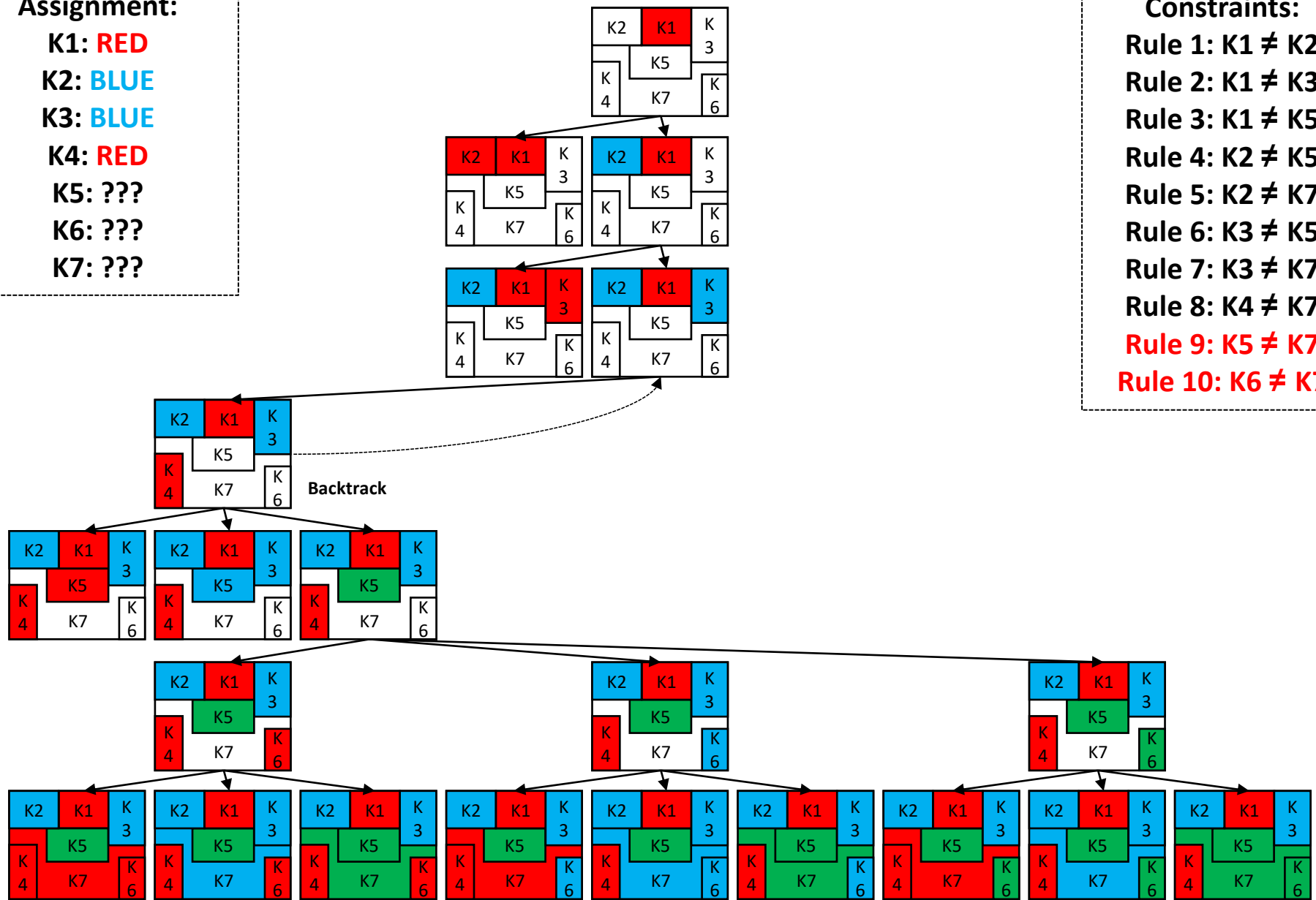
Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$



Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED

K2: BLUE

K3: BLUE

K4: BLUE

K5: ???

K6: ???

K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

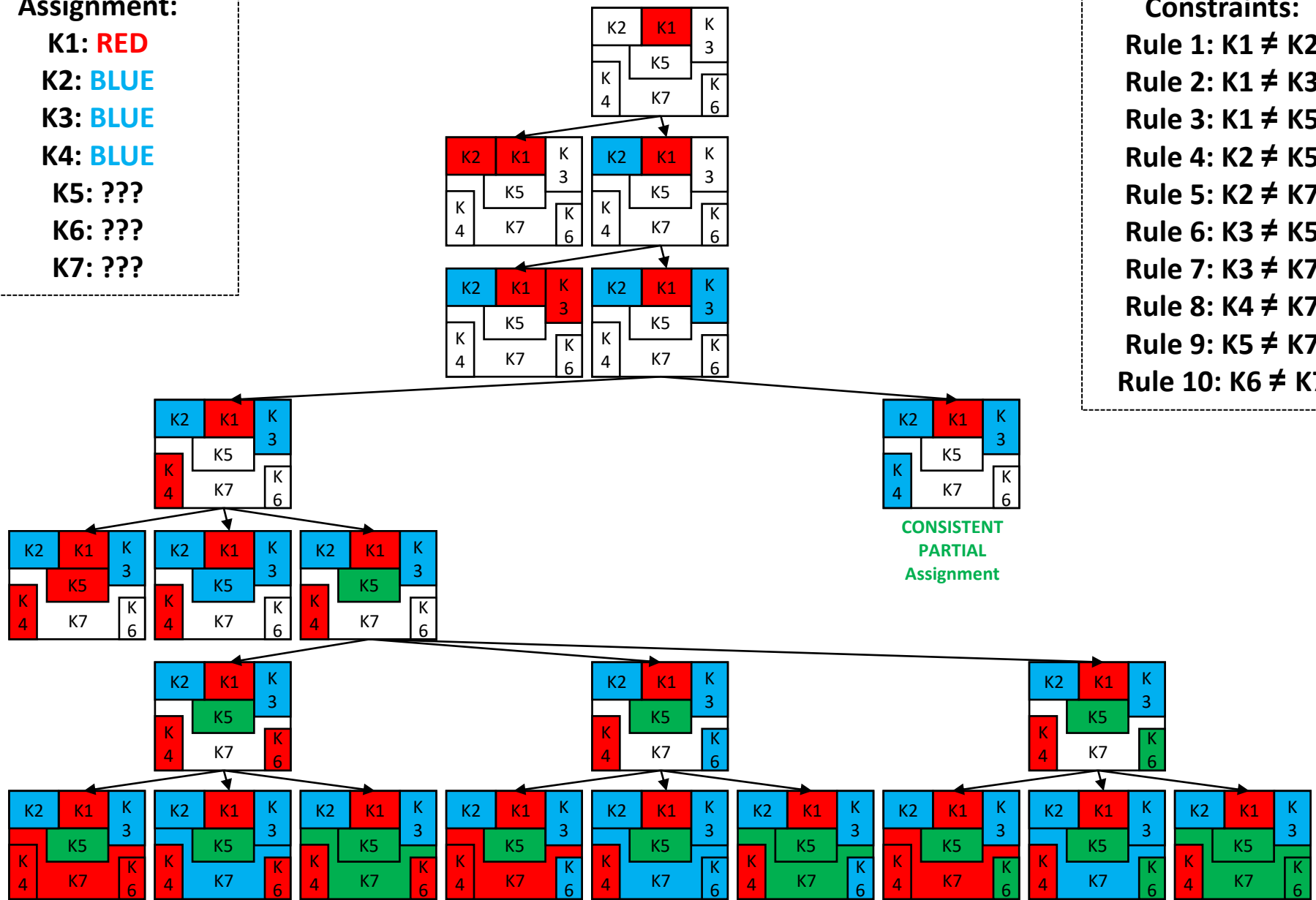
Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

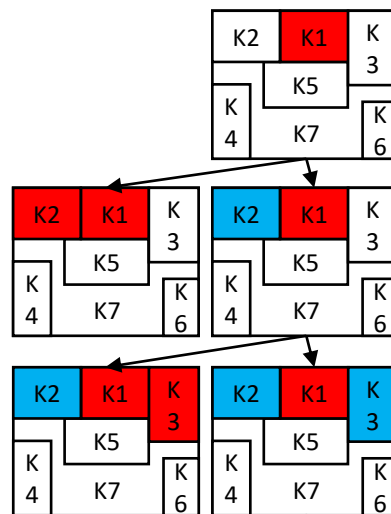
Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$

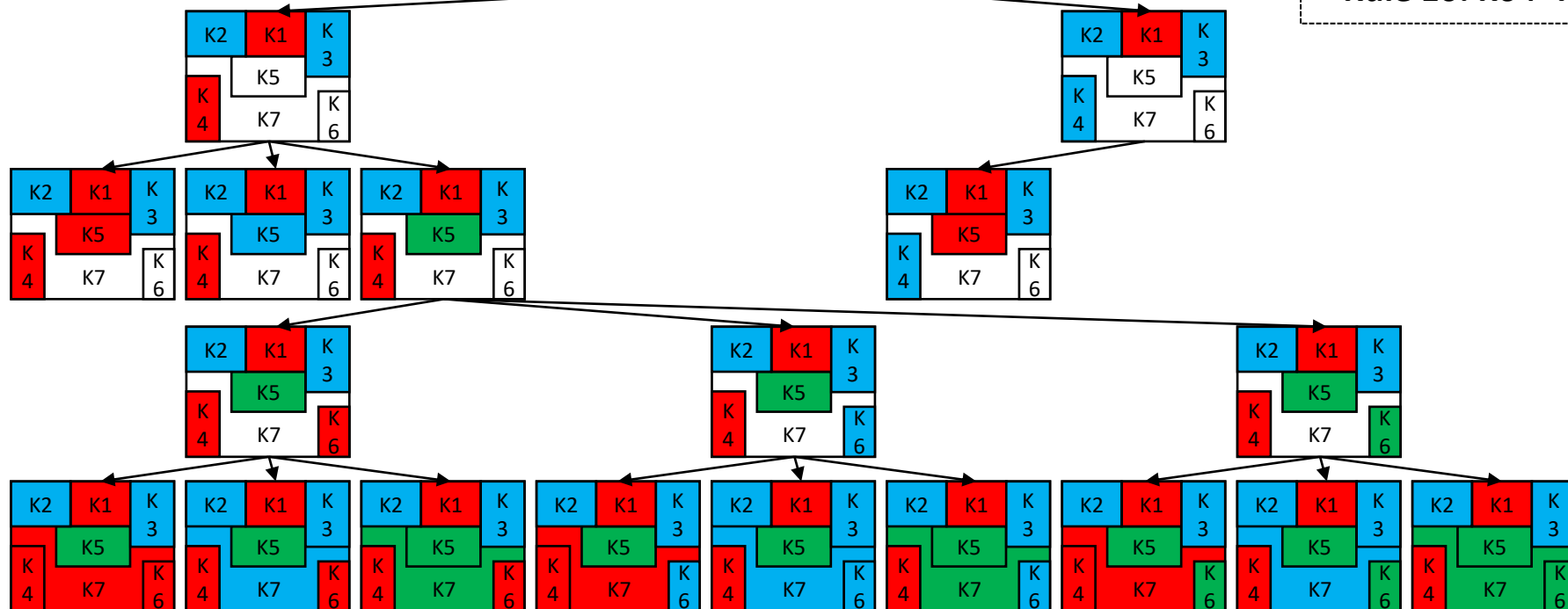


Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

**K7: ???**



**Rule 10:  $K6 \neq K7$**



**Variable assignment order:** K1, K2, K3, K4, K5, K6, K7 | **Value assignment order:** RED, BLUE, GREEN

## Assignment:

K1: RED

K2: BLUE

K3: BLUE

K4: BLUE

K5: RED

K6: ???

K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

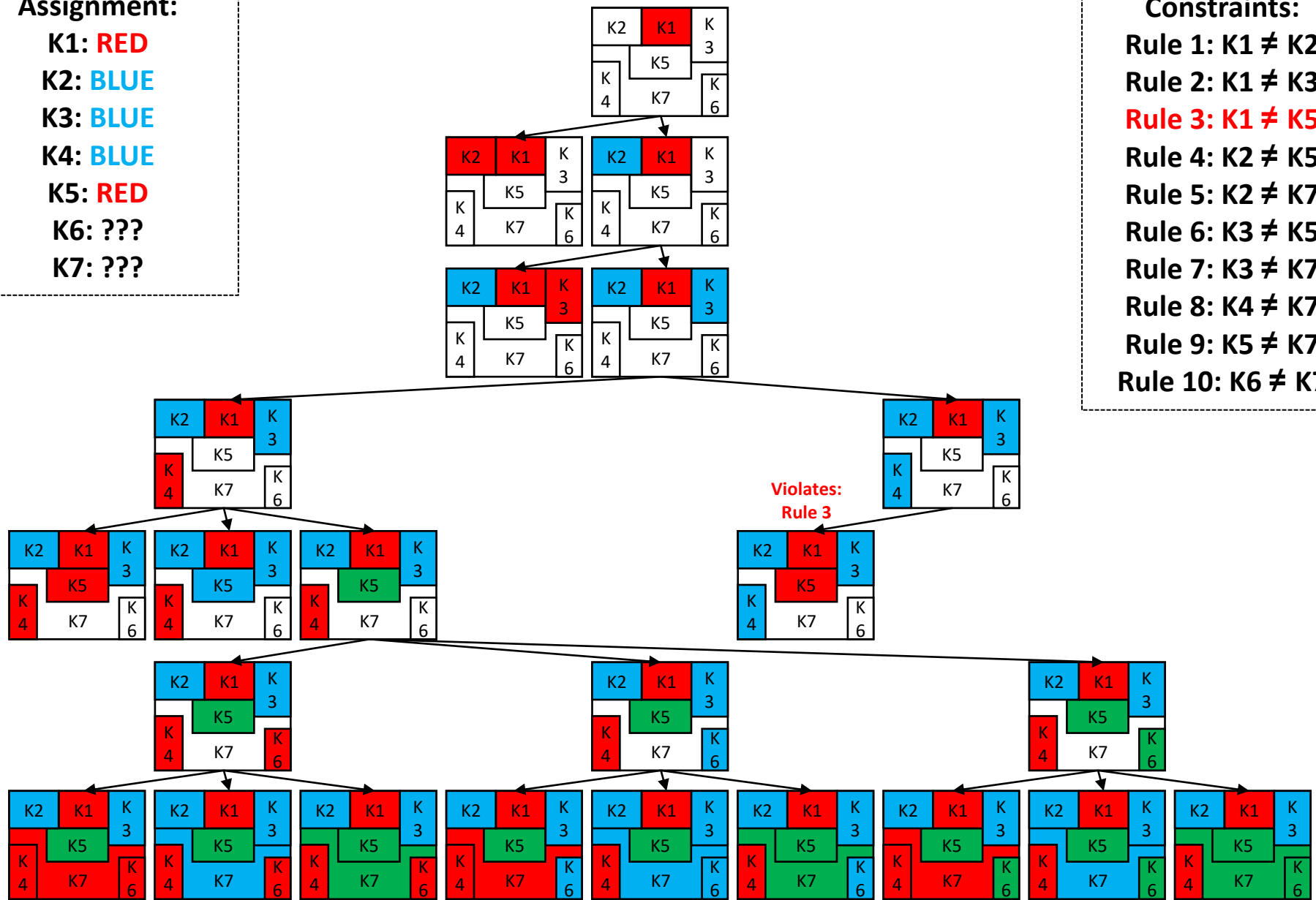
Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$



Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED

K2: BLUE

K3: BLUE

K4: BLUE

K5: ???

K6: ???

K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

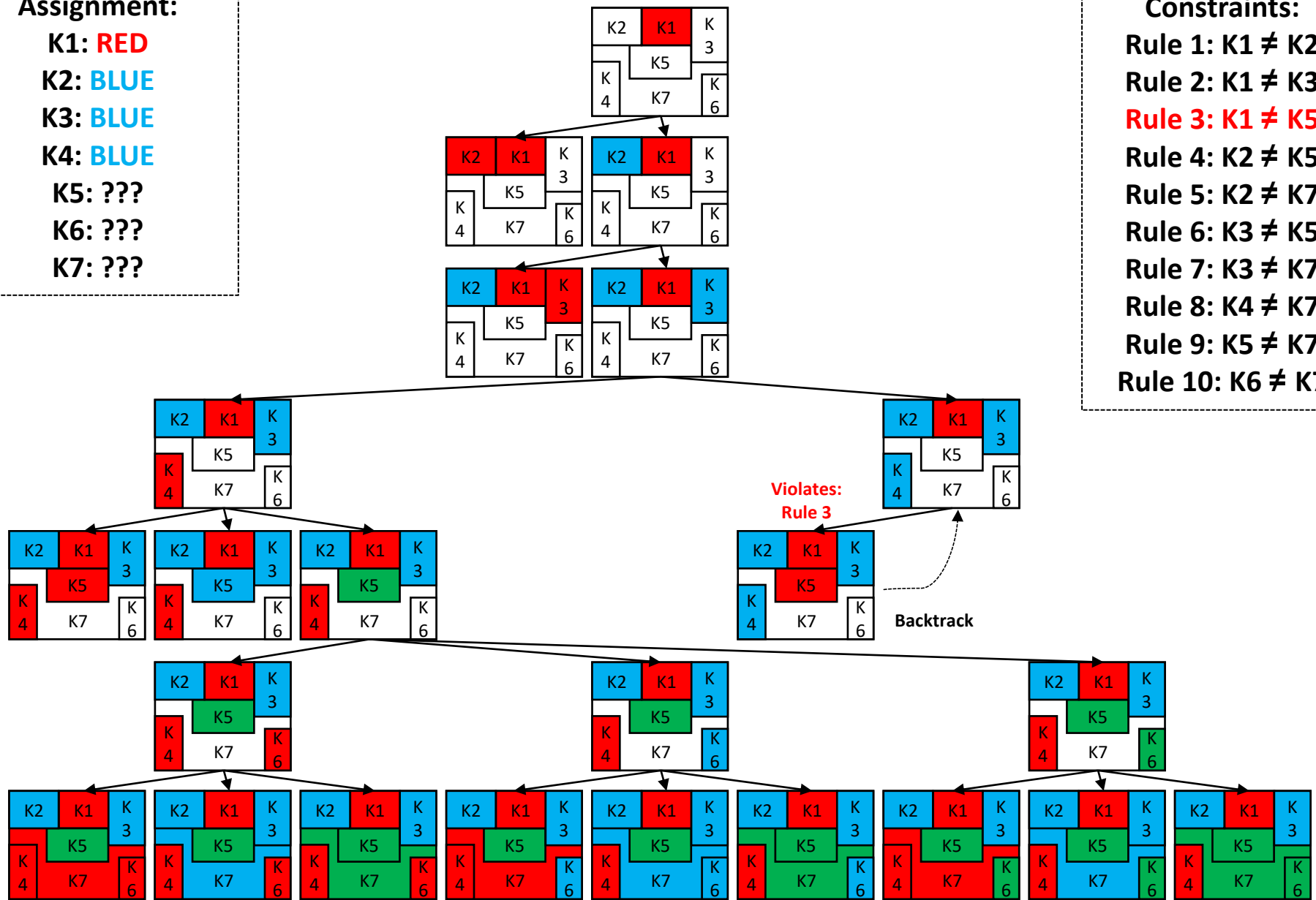
Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$



Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED

K2: BLUE

K3: BLUE

K4: BLUE

K5: BLUE

K6: ???

K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

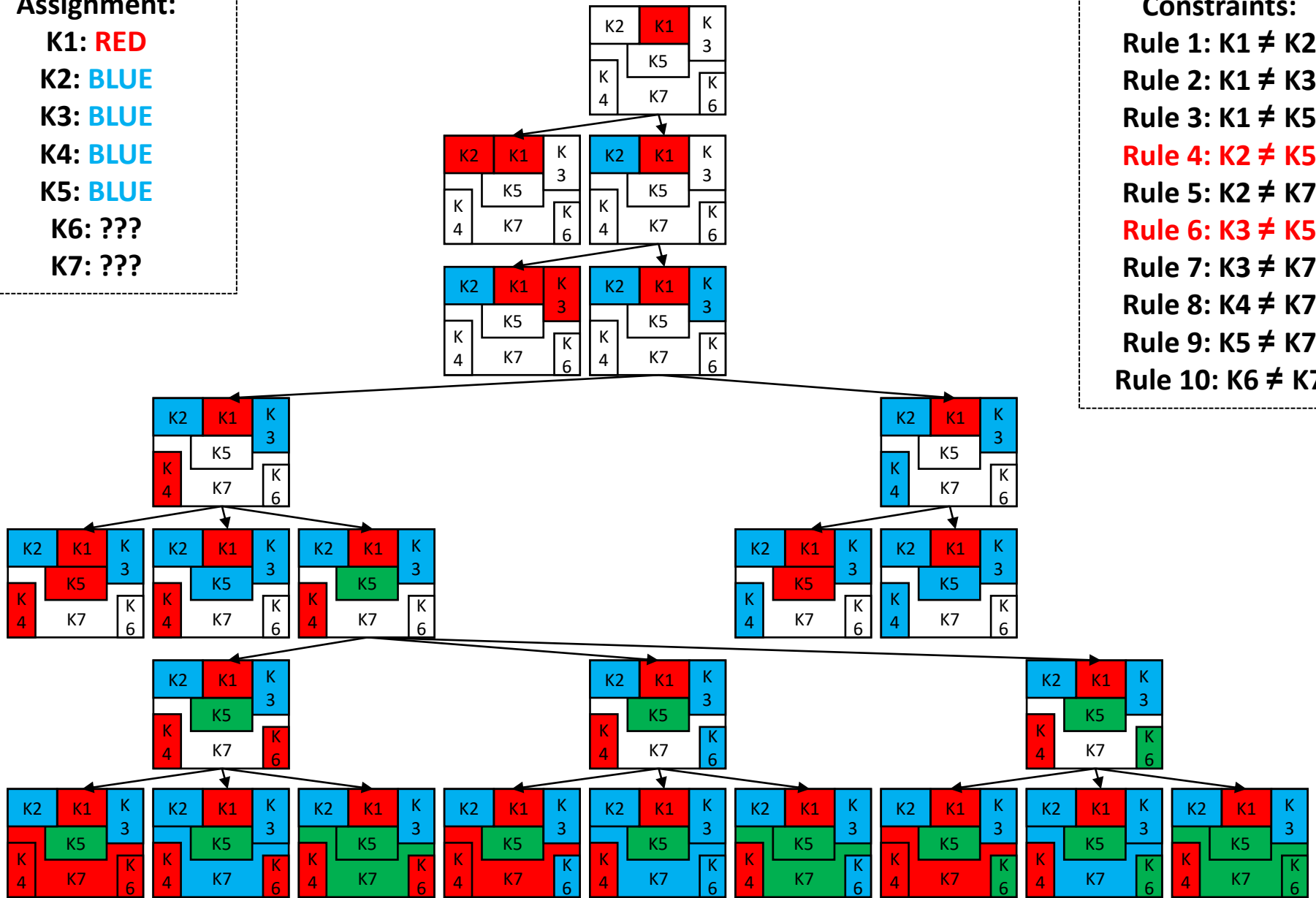
Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$



Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN



## Assignment:

K1: RED

K2: BLUE

K3: BLUE

K4: BLUE

K5: ???

K6: ???

K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$

Rule 2:  $K1 \neq K3$

Rule 3:  $K1 \neq K5$

Rule 4:  $K2 \neq K5$

Rule 5:  $K2 \neq K7$

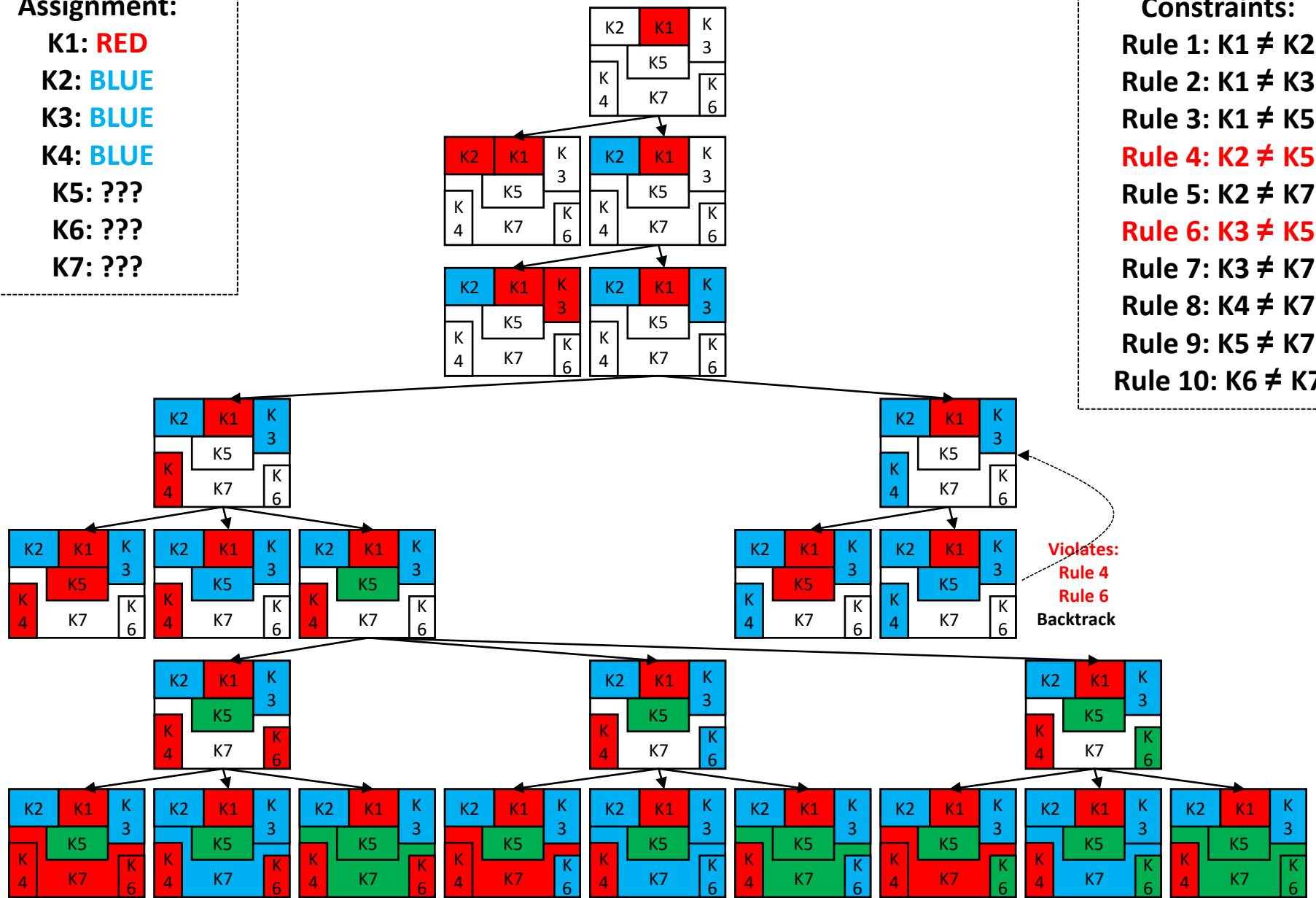
Rule 6:  $K3 \neq K5$

Rule 7:  $K3 \neq K7$

Rule 8:  $K4 \neq K7$

Rule 9:  $K5 \neq K7$

Rule 10:  $K6 \neq K7$



Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

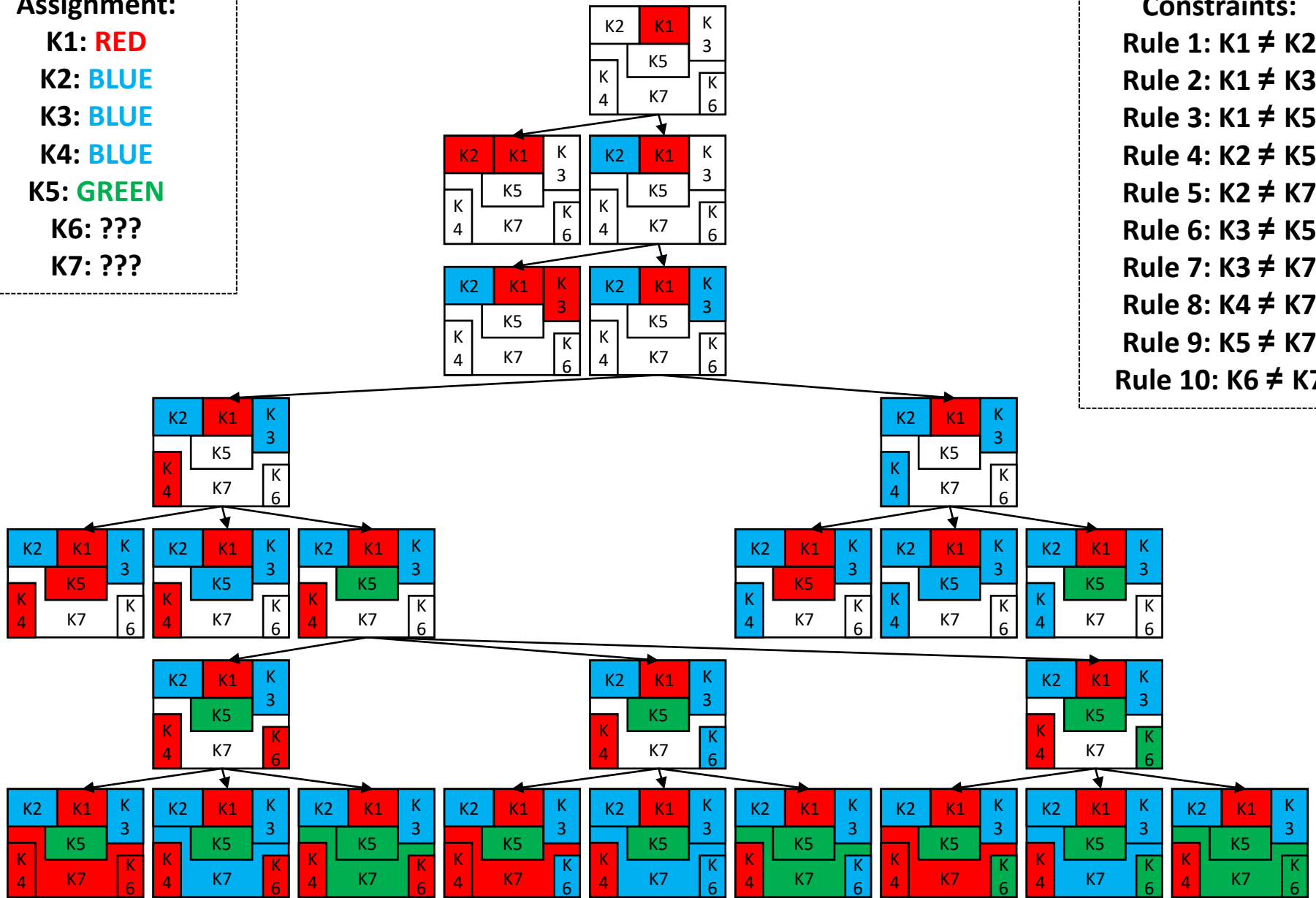


## Assignment:

K1: **RED**  
K2: **BLUE**  
K3: **BLUE**  
K4: **BLUE**  
K5: **GREEN**  
K6: ???  
K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



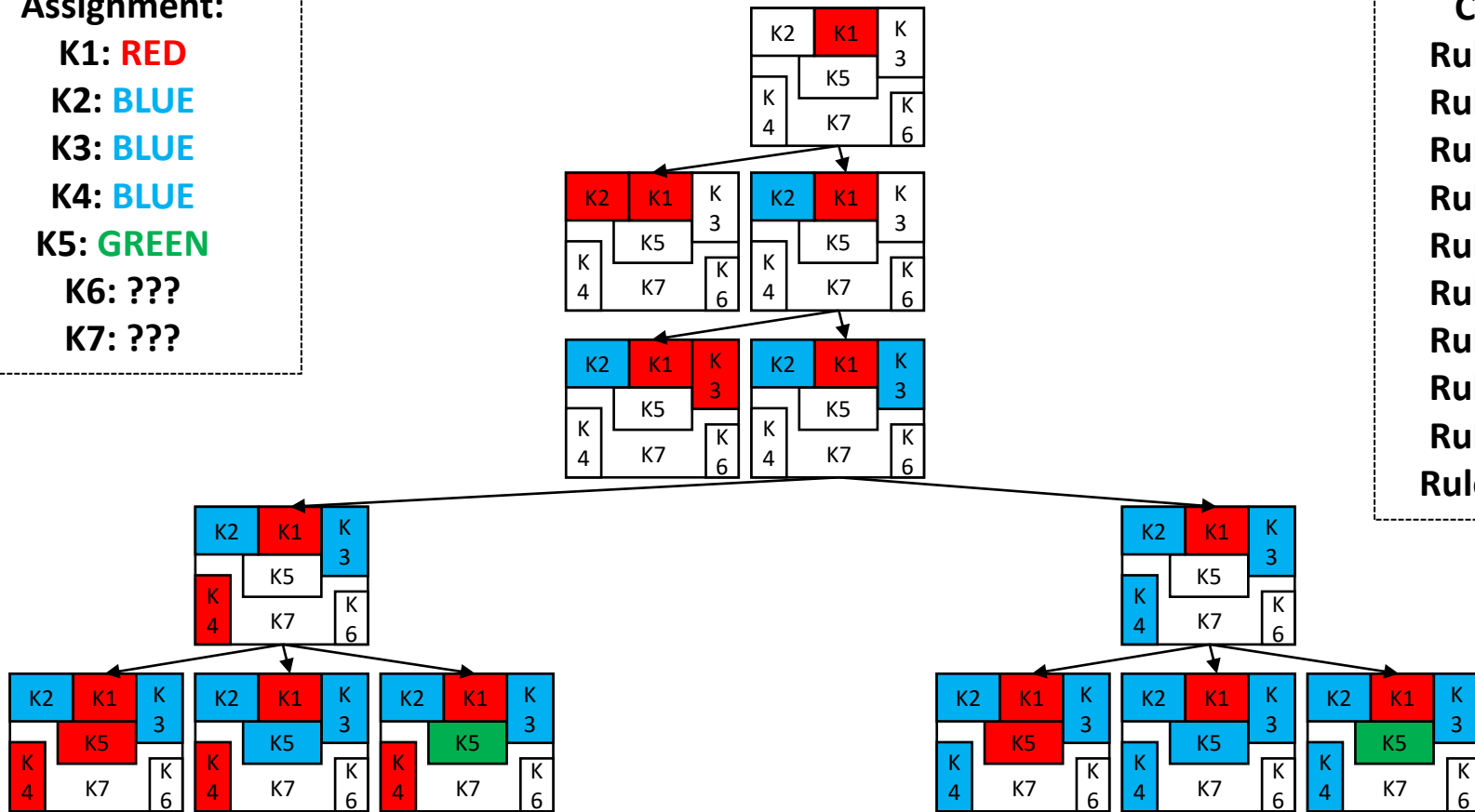
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: **RED**, **BLUE**, **GREEN**

## Assignment:

K1: **RED**  
K2: **BLUE**  
K3: **BLUE**  
K4: **BLUE**  
K5: **GREEN**  
K6: ???  
K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



Visited / dead ends

**Complete, but inconsistent** assignments

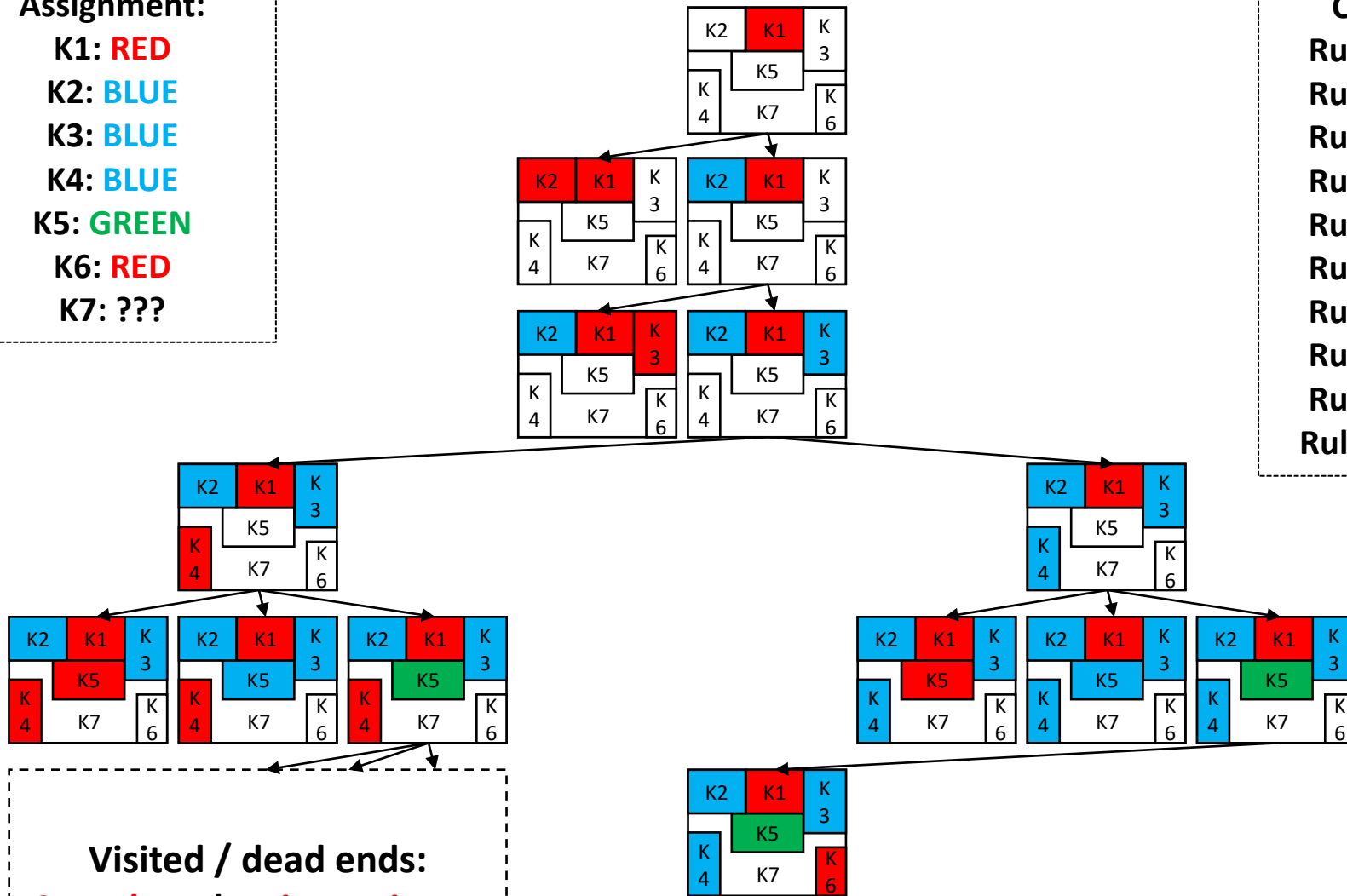
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: **RED**, **BLUE**, **GREEN**

## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: BLUE  
K5: GREEN  
K6: RED  
K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



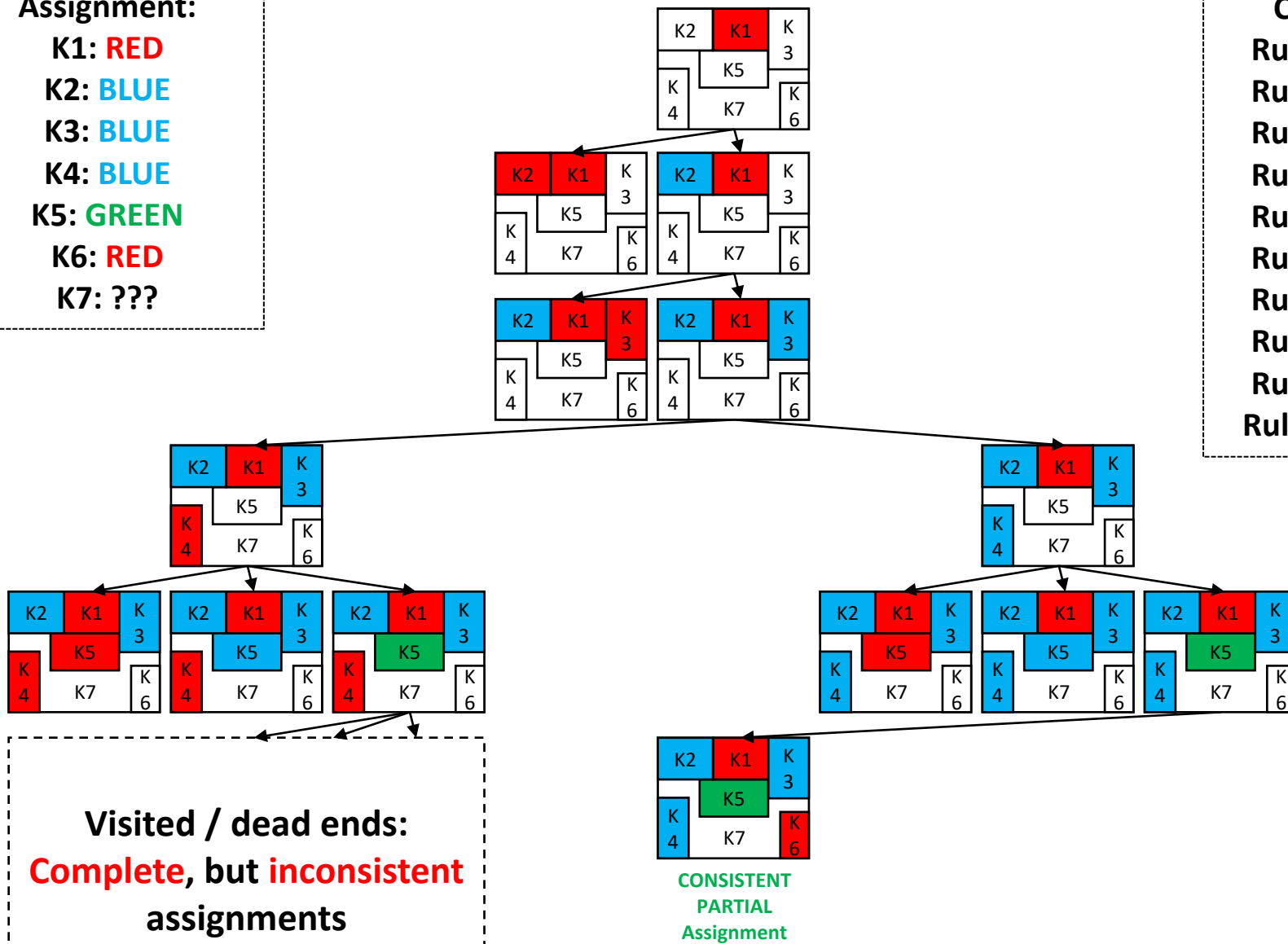
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: BLUE  
K5: GREEN  
K6: RED  
K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



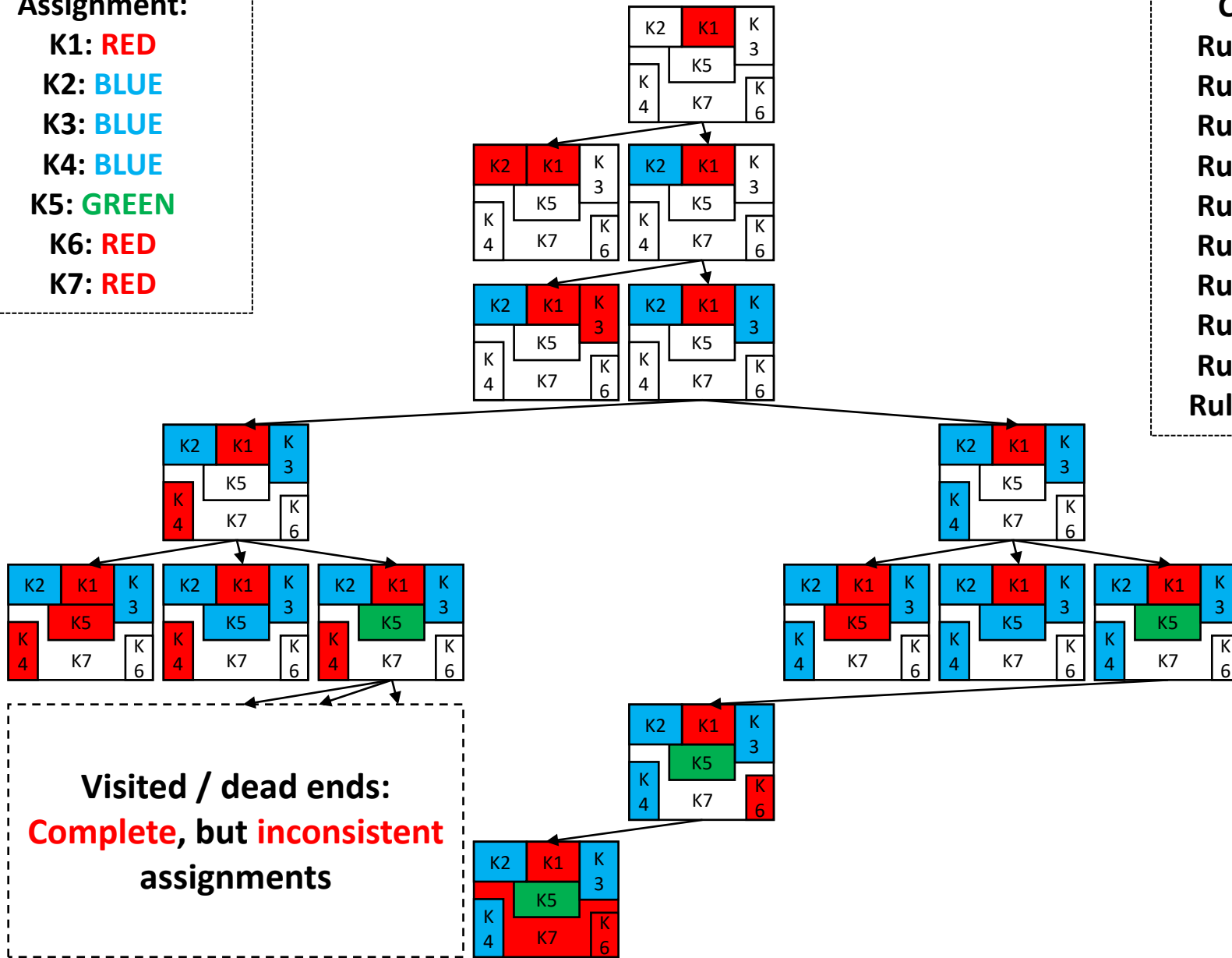
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: BLUE  
K5: GREEN  
K6: RED  
K7: RED

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



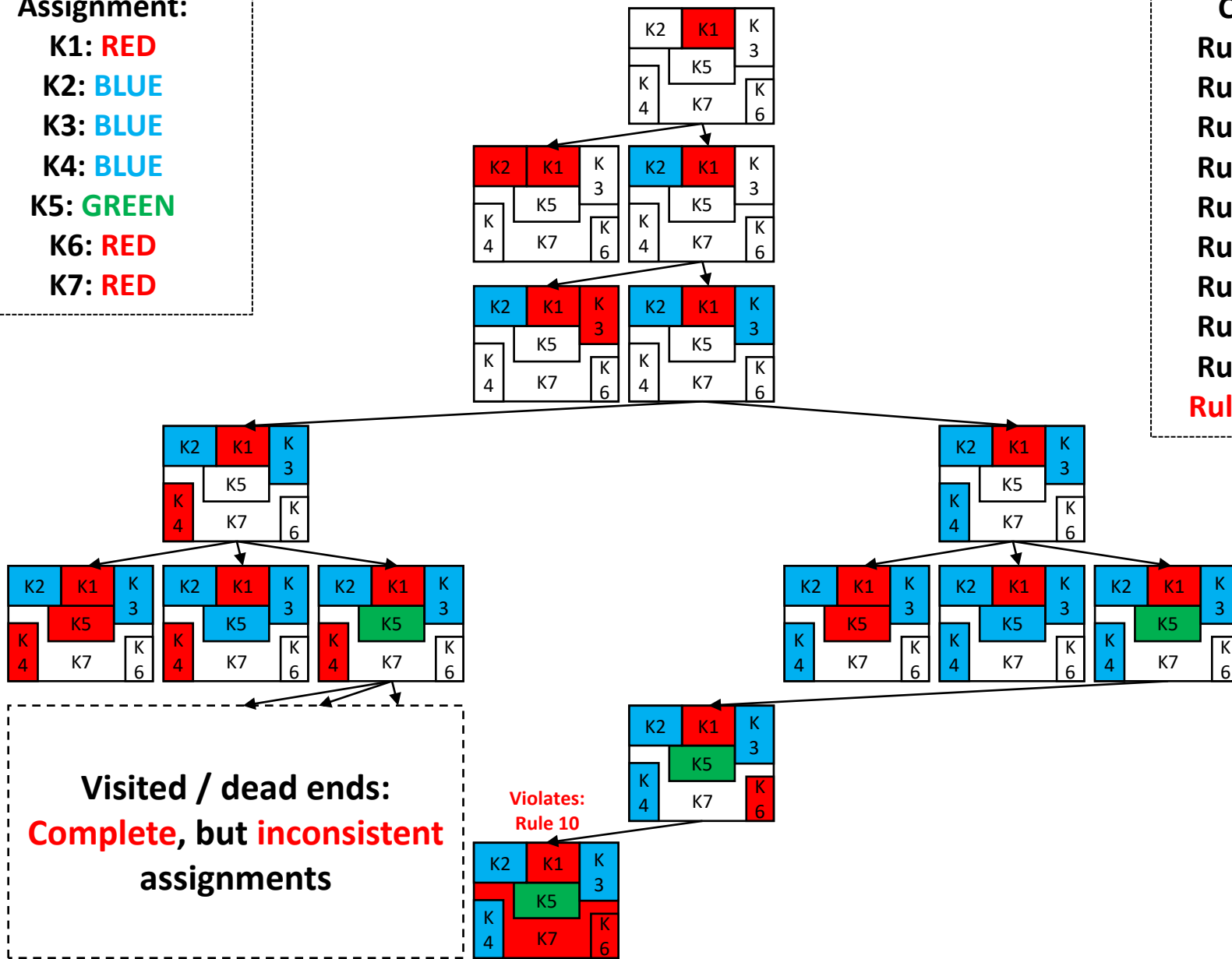
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: BLUE  
K5: GREEN  
K6: RED  
K7: RED

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
**Rule 10:  $K6 \neq K7$**



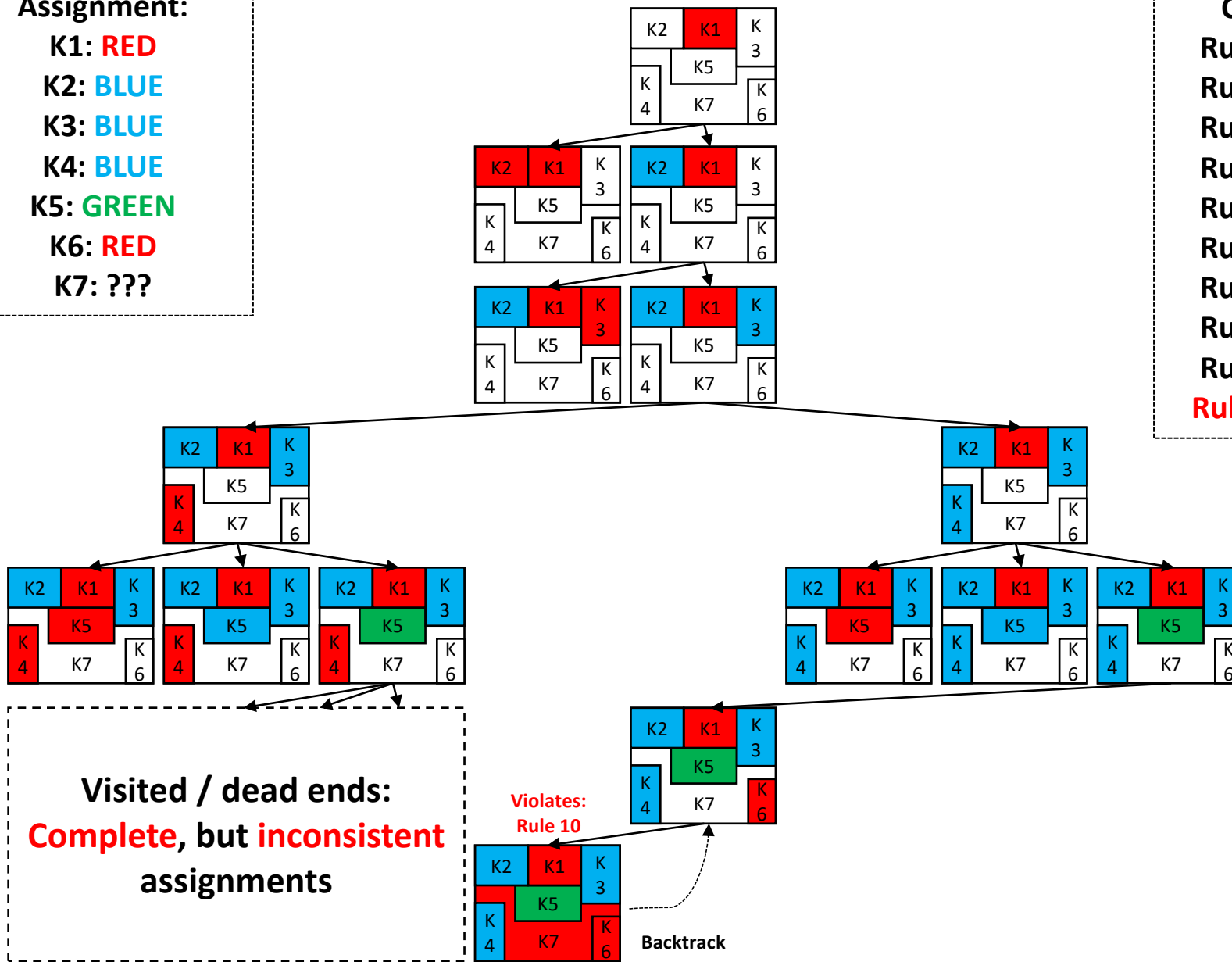
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: BLUE  
K5: GREEN  
K6: RED  
K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
**Rule 10:  $K6 \neq K7$**



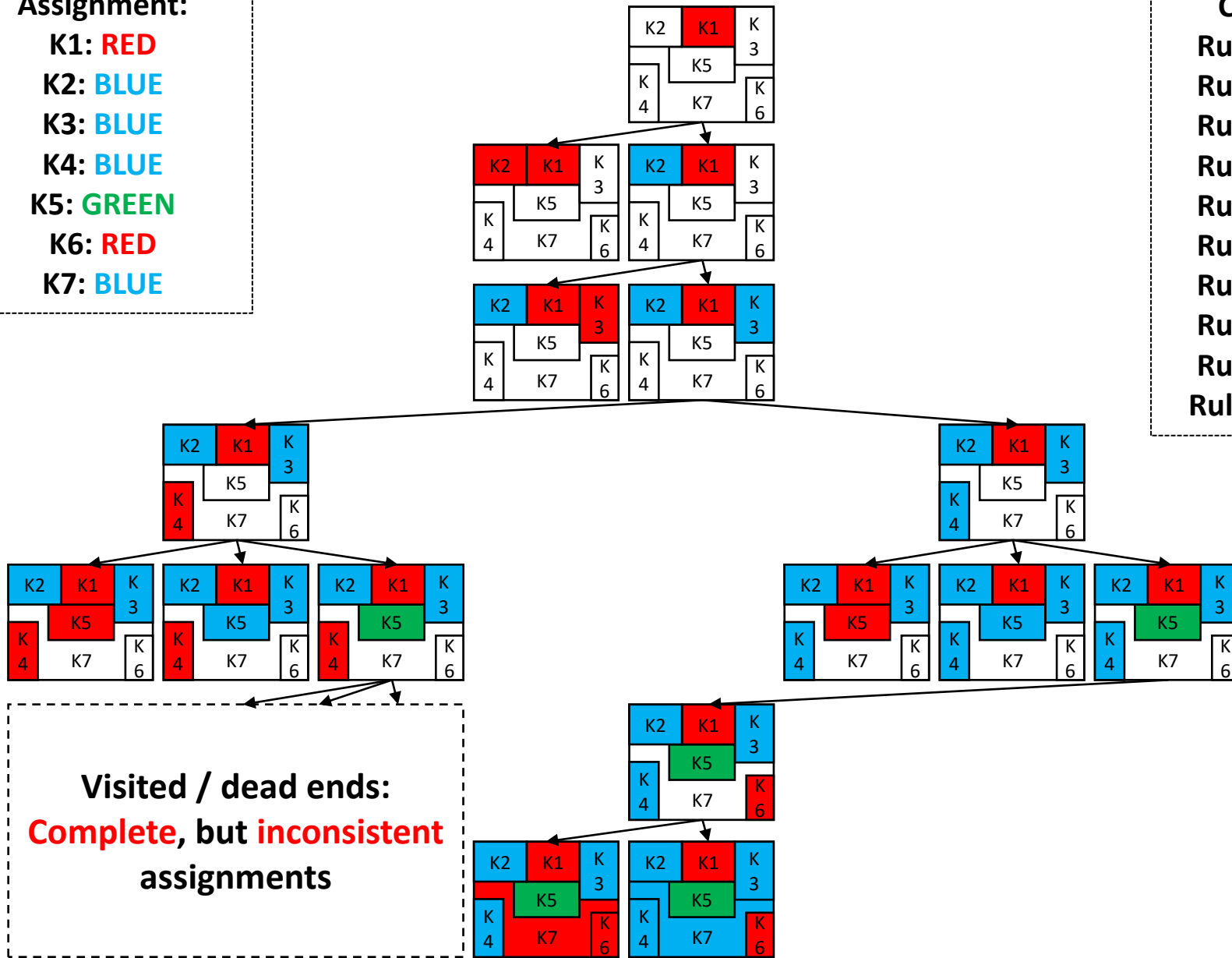
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: BLUE  
K5: GREEN  
K6: RED  
K7: BLUE

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN



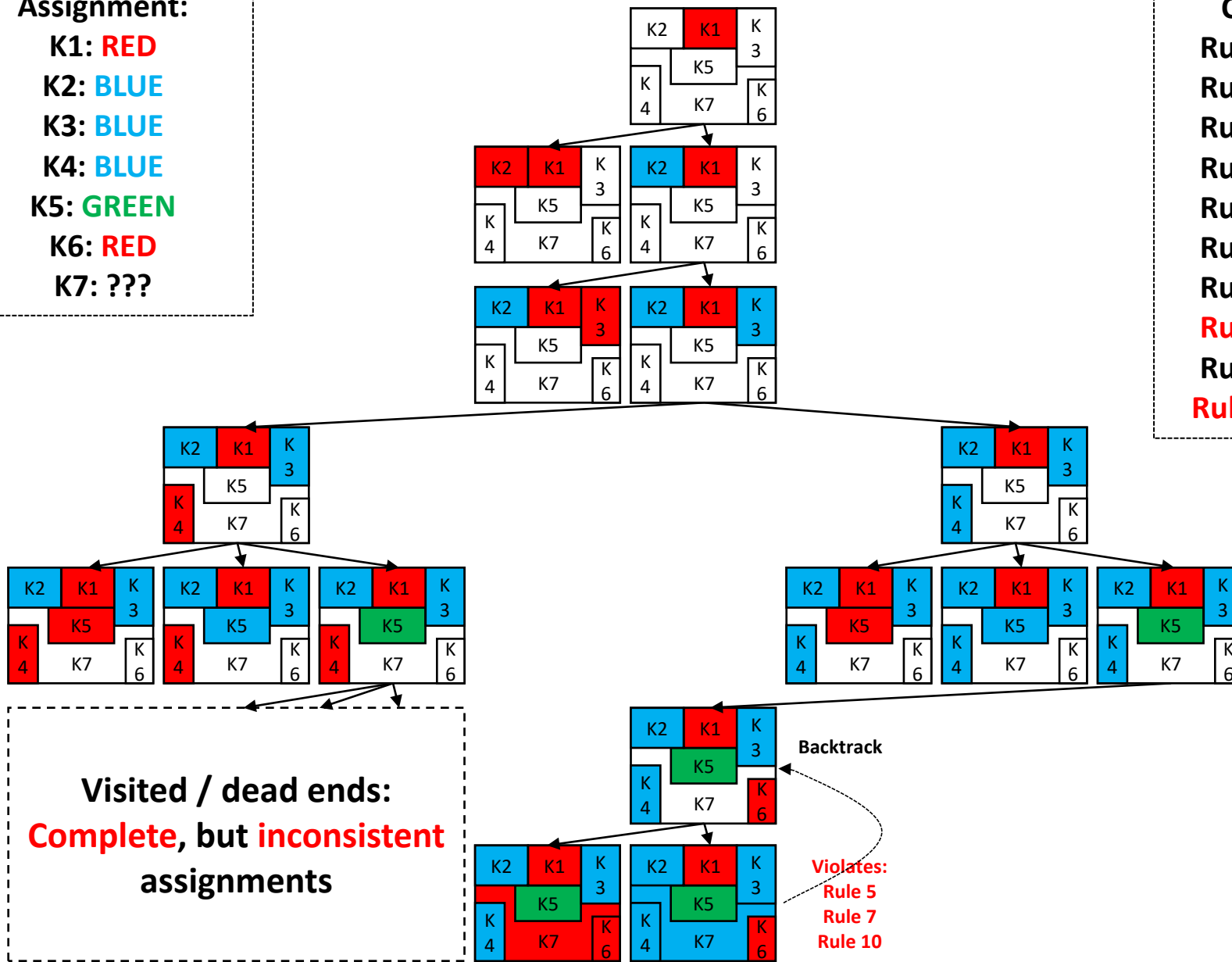


## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: BLUE  
K5: GREEN  
K6: RED  
K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



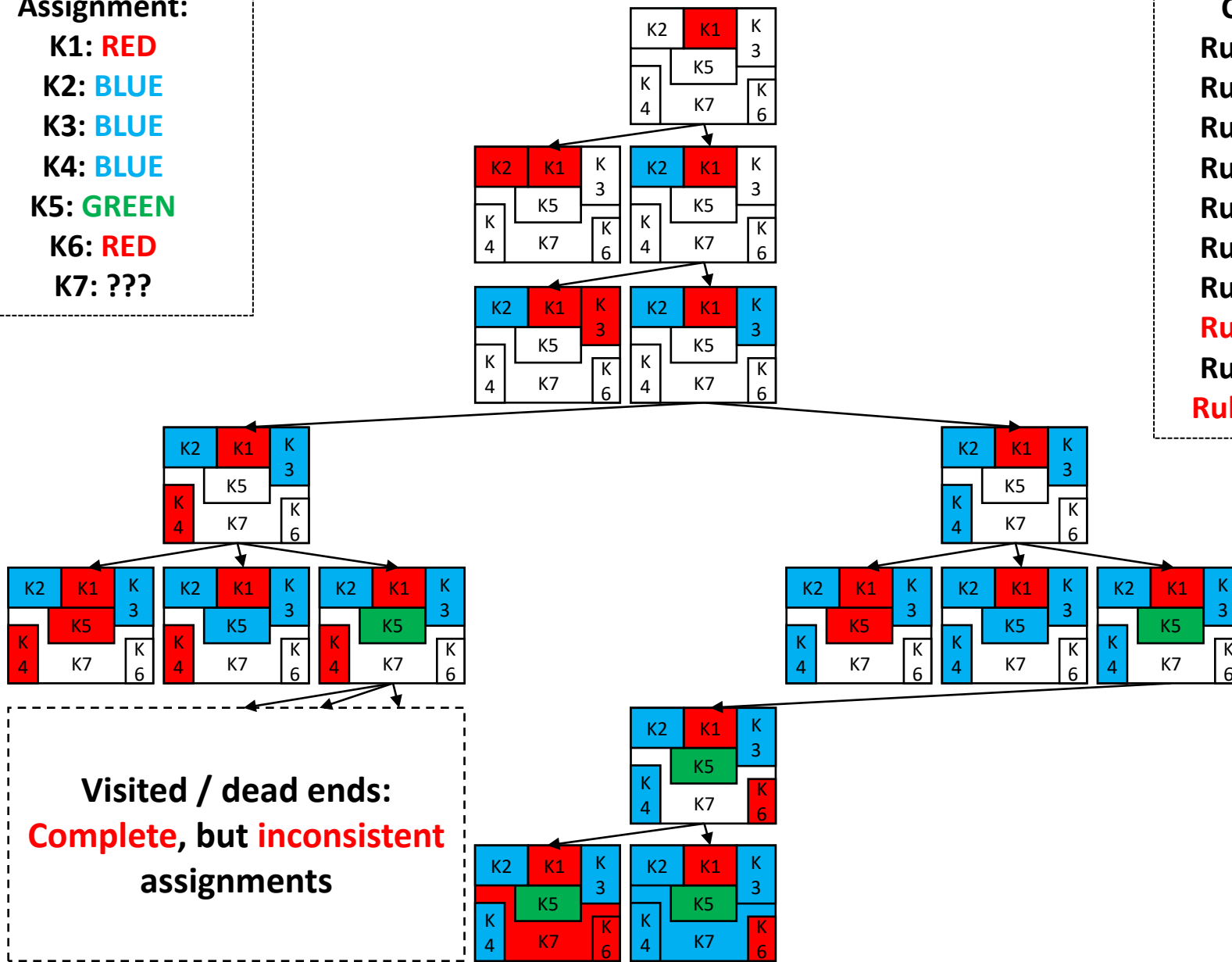
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: BLUE  
K5: GREEN  
K6: RED  
K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



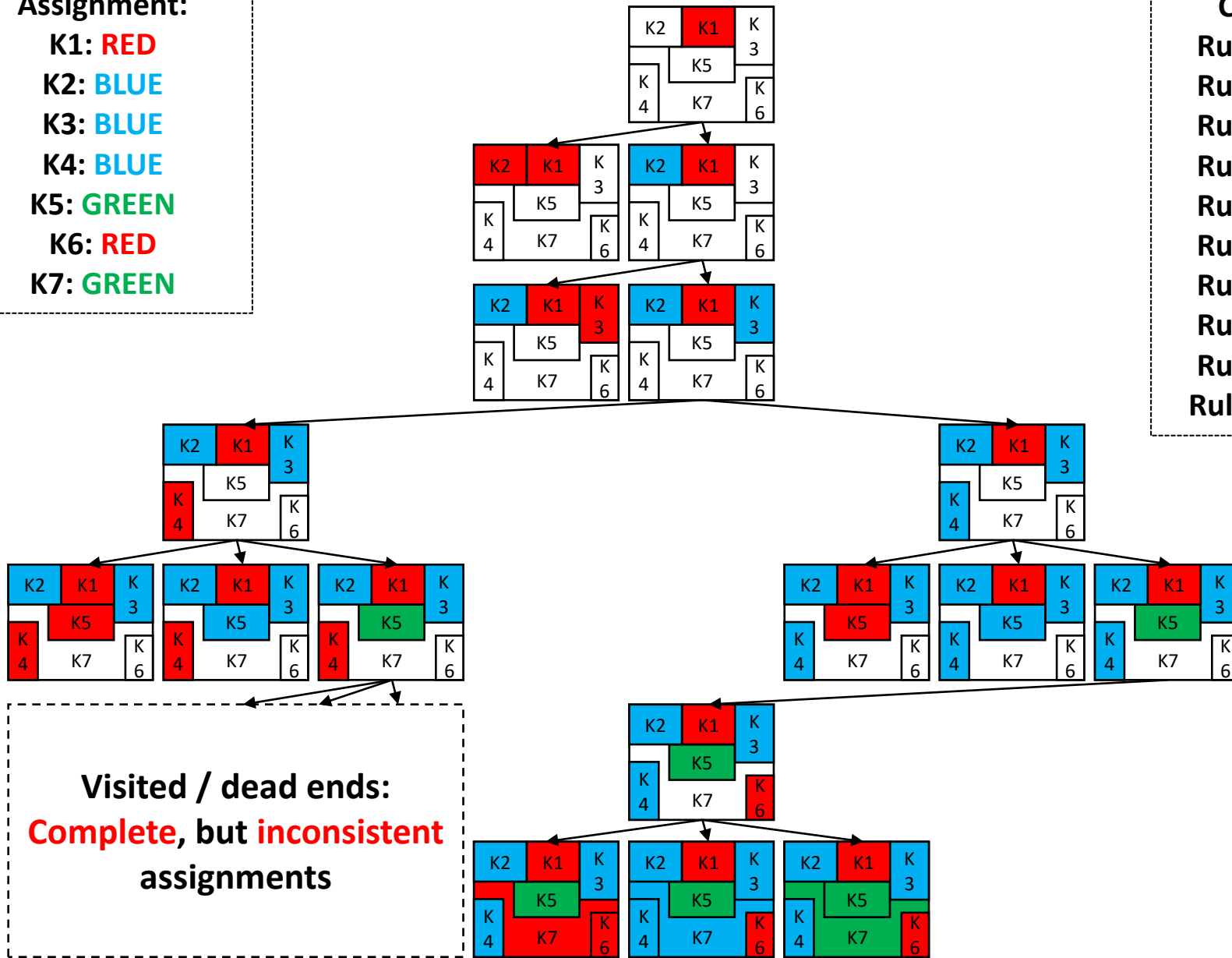
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: BLUE  
K5: GREEN  
K6: RED  
K7: GREEN

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



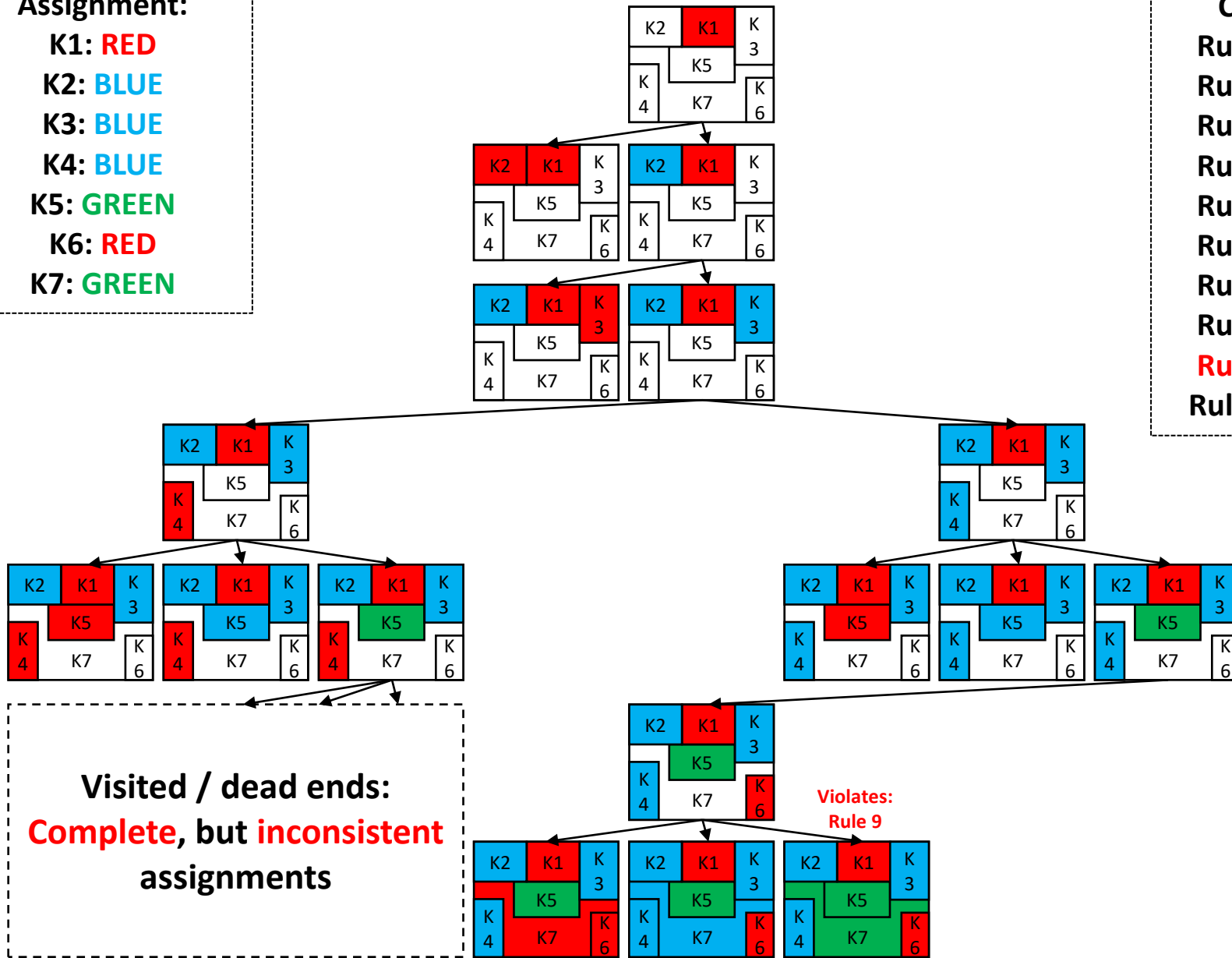
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: BLUE  
K5: GREEN  
K6: RED  
K7: GREEN

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



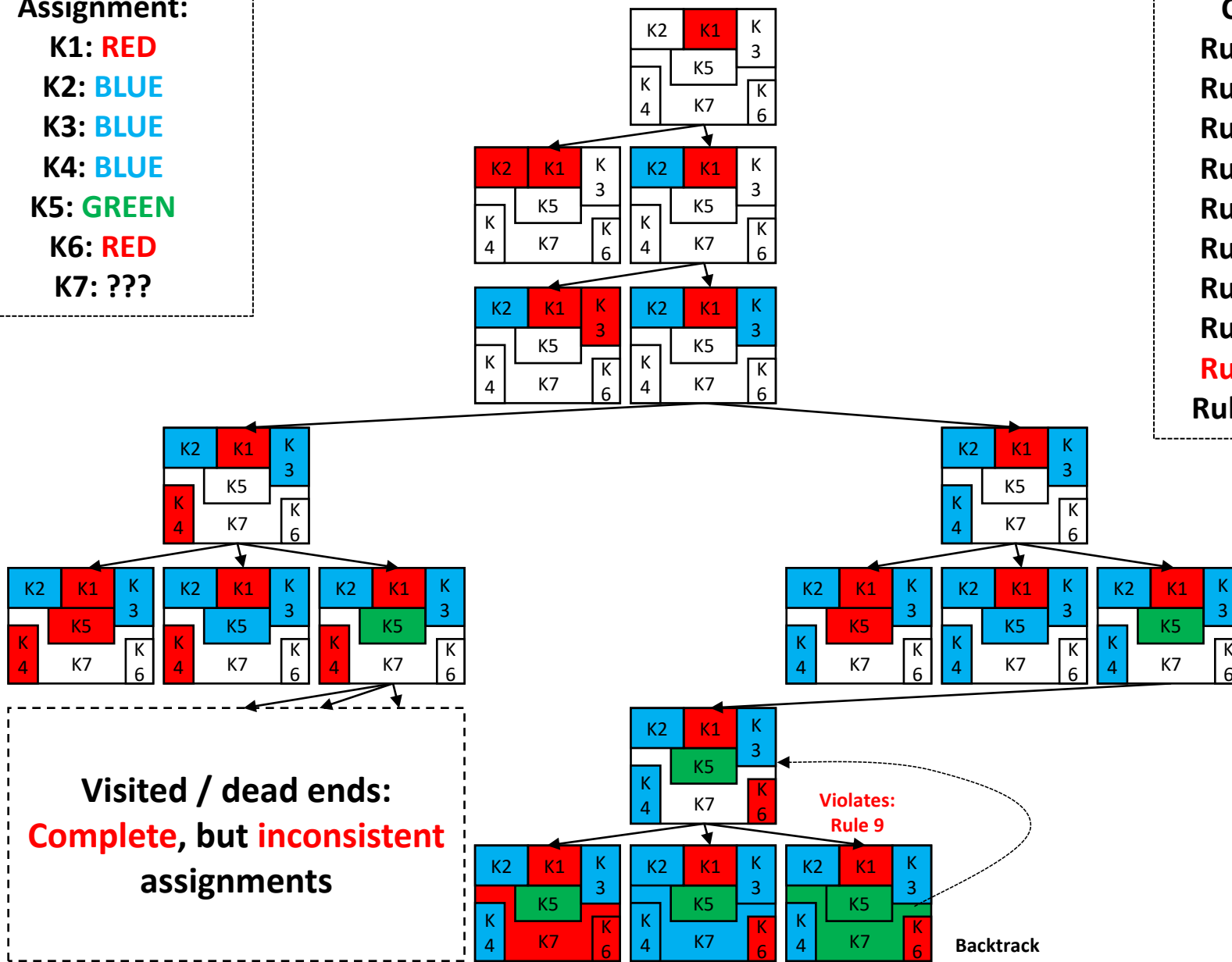
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: BLUE  
K5: GREEN  
K6: RED  
K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



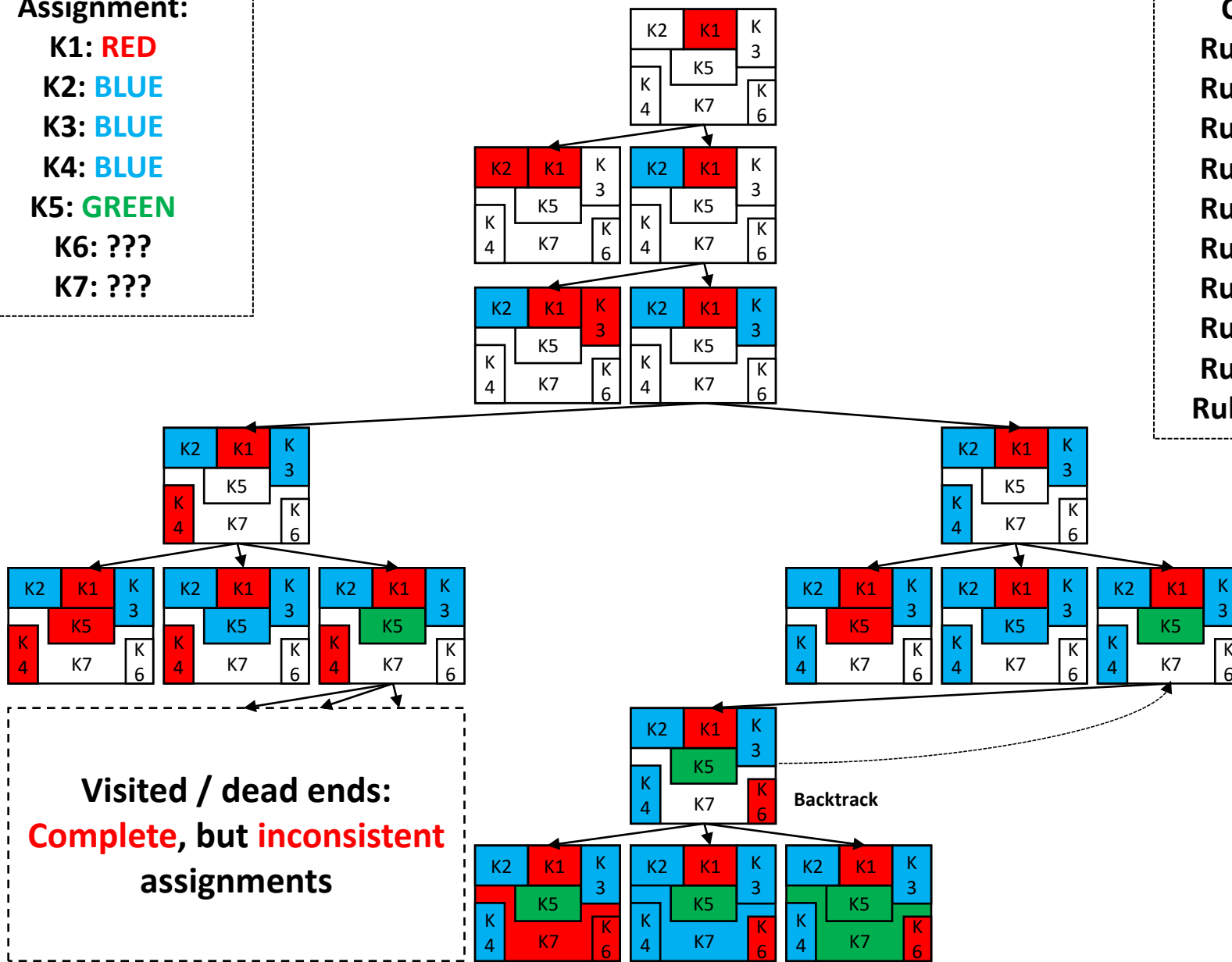
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

## Assignment:

K1: **RED**  
K2: **BLUE**  
K3: **BLUE**  
K4: **BLUE**  
K5: **GREEN**  
K6: ???  
K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



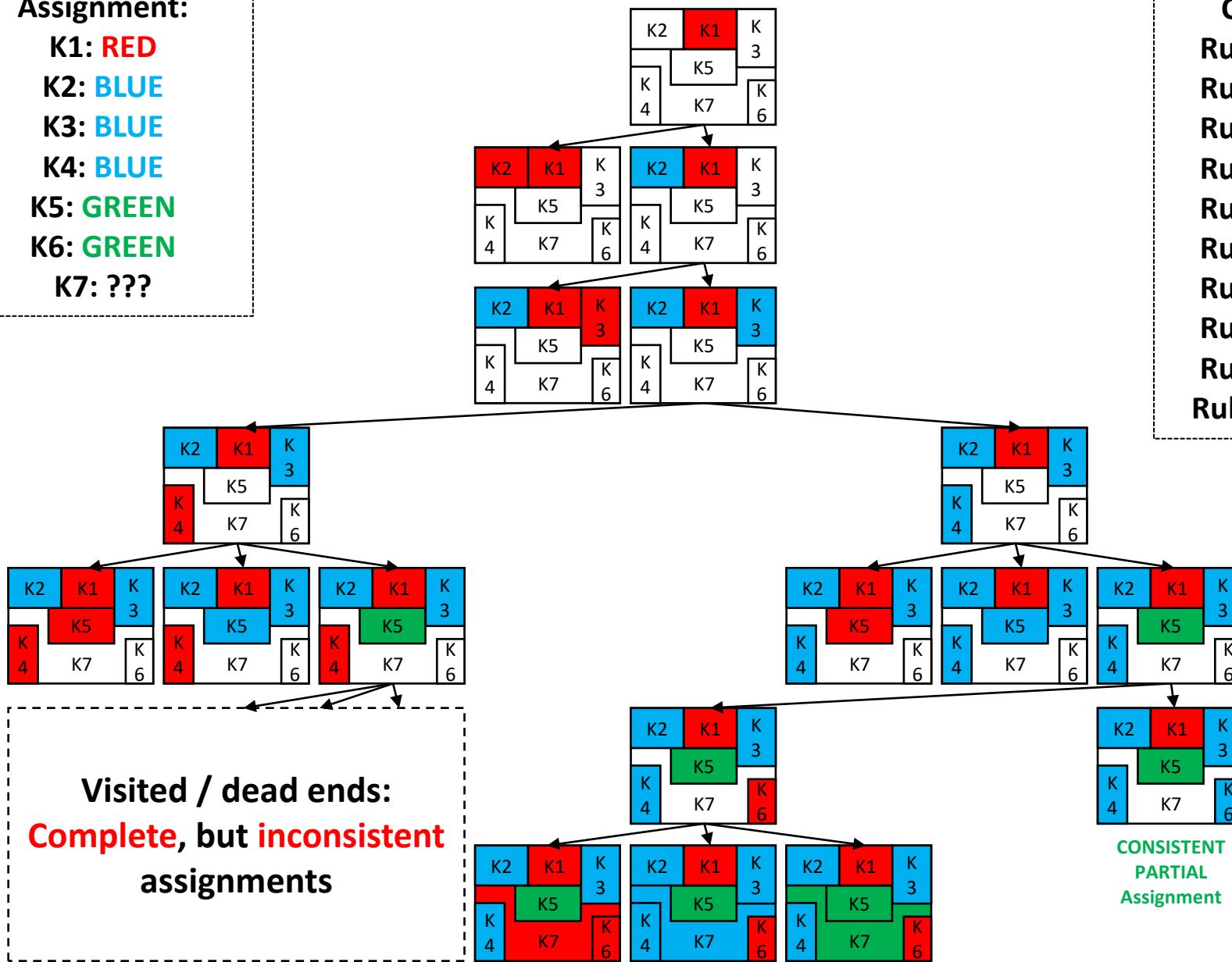
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: **RED**, **BLUE**, **GREEN**

## Assignment:

K1: **RED**  
K2: **BLUE**  
K3: **BLUE**  
K4: **BLUE**  
K5: **GREEN**  
K6: **GREEN**  
K7: ???

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: **RED**, **BLUE**, **GREEN**

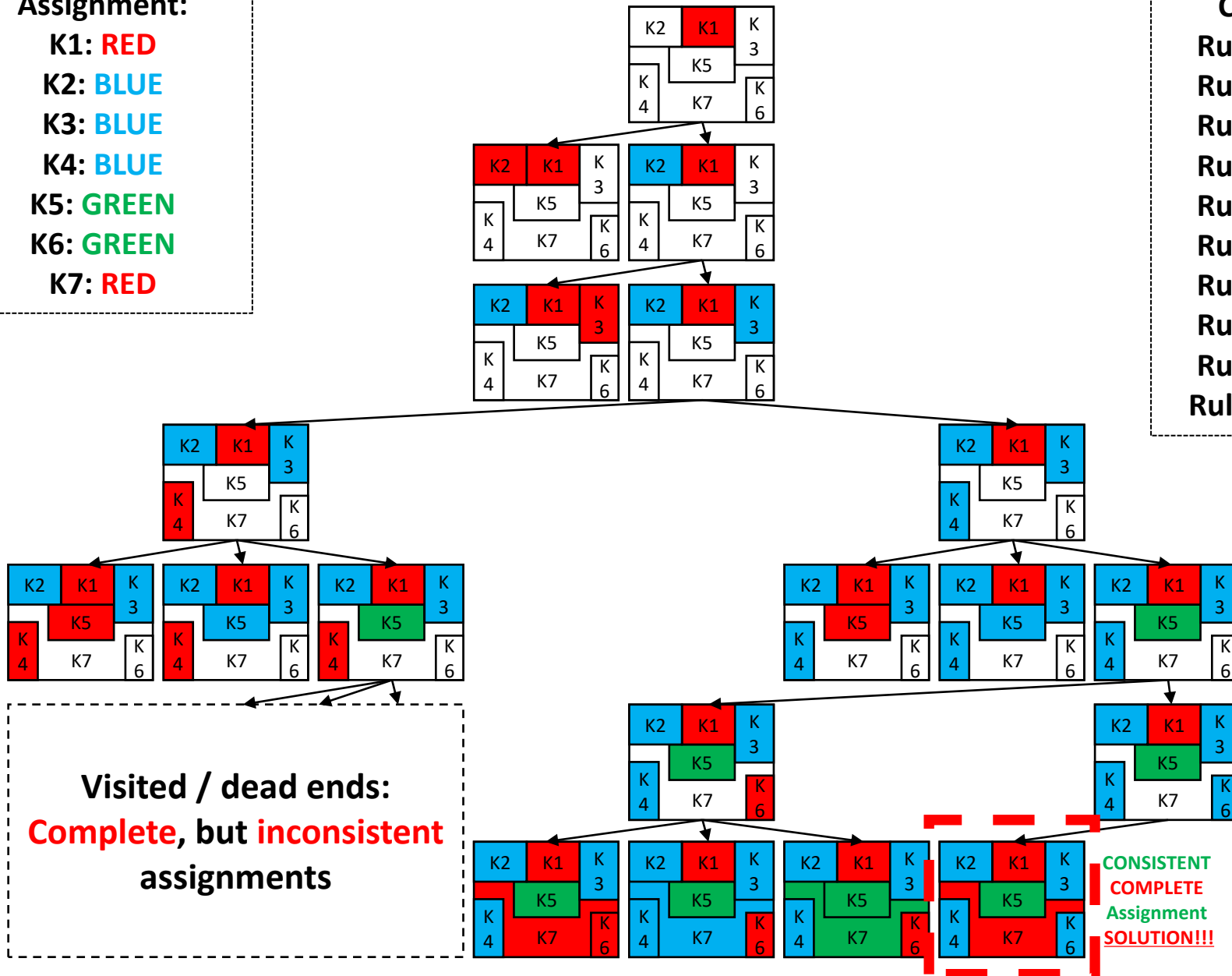


## Assignment:

K1: RED  
K2: BLUE  
K3: BLUE  
K4: BLUE  
K5: GREEN  
K6: GREEN  
K7: RED

## Constraints:

Rule 1:  $K1 \neq K2$   
Rule 2:  $K1 \neq K3$   
Rule 3:  $K1 \neq K5$   
Rule 4:  $K2 \neq K5$   
Rule 5:  $K2 \neq K7$   
Rule 6:  $K3 \neq K5$   
Rule 7:  $K3 \neq K7$   
Rule 8:  $K4 \neq K7$   
Rule 9:  $K5 \neq K7$   
Rule 10:  $K6 \neq K7$



Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN

# Can We Do Better?

# CSP Backtracking: Pseudocode

**function** BACKTRACKING-SEARCH(*csp*) **returns** a solution or *failure*  
    **return** BACKTRACK(*csp*, { })

**function** BACKTRACK(*csp*, *assignment*) **returns** a solution or *failure*  
    **if** *assignment* is complete **then return** *assignment*

*var*  $\leftarrow$  SELECT-UNASSIGNED-VARIABLE(*csp*, *assignment*)

**for each** *value* **in** ORDER-DOMAIN-VALUES(*csp*, *var*, *assignment*) **do**

**if** *value* is consistent with *assignment* **then**

            add {*var* = *value*} to *assignment*

*inferences*  $\leftarrow$  INFERENCE(*csp*, *var*, *assignment*)

**if** *inferences*  $\neq$  *failure* **then**

                add *inferences* to *csp*

*result*  $\leftarrow$  BACKTRACK(*csp*, *assignment*)

**if** *result*  $\neq$  *failure* **then return** *result*

                remove *inferences* from *csp*

            remove {*var* = *value*} from *assignment*

**return** *failure*

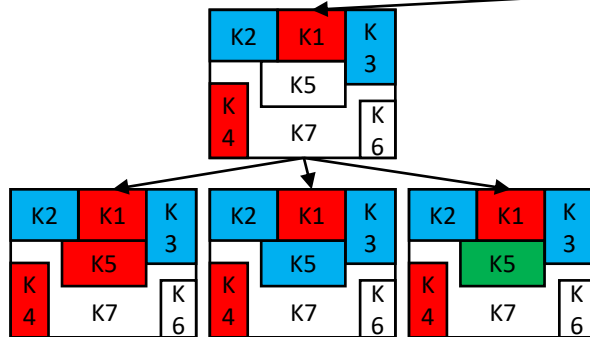
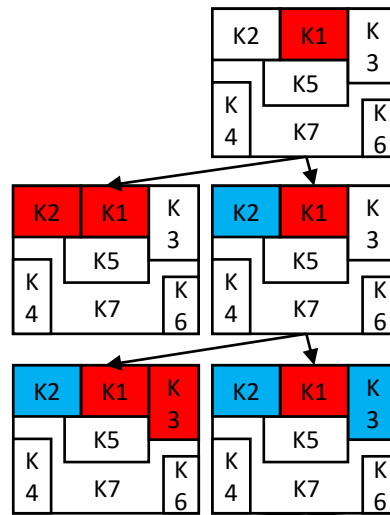
Which variable  
should we choose to  
assign a value to next?

Does it matter?

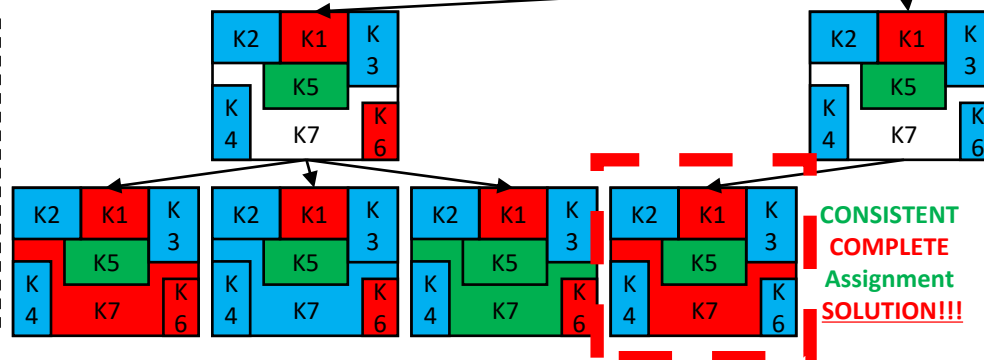
**Variable assignment  
order:**

K1, K2, K3, K4, K5, K6,  
K7

(but do we have to  
keep that order?)



**Visited / dead ends:**  
**Complete, but inconsistent**  
**assignments**



Variable assignment order: K1, K2, K3, K4, K5, K6, K7

K1 = ???

K2 = ???

K3 = ???

K4 = ???

K5 = ???

K6 = ???

K7 = ???

**CONSISTENT  
COMPLETE  
Assignment  
SOLUTION!!!**

# Variable Ordering: Alternatives

**function** BACKTRACKING-SEARCH(*csp*) **returns** a solution or *failure*  
**return** BACKTRACK(*csp*, { })

**function** BACKTRACK(*csp*, *assignment*) **returns** a solution or *failure*  
**if** *assignment* is complete **then return** *assignment*

*var*  $\leftarrow$  SELECT-UNASSIGNED-VARIABLE(*csp*, *assignment*)

**for each** *value* **in** ORDER-DOMAIN-VALUES(*csp*, *var*, *assignment*) **do**

**if** *value* is consistent with *assignment* **then**

add {*var* = *value*} to *assignment*

*inferences*  $\leftarrow$  INFERENCE(*csp*, *var*, *assignment*)

**if** *inferences*  $\neq$  *failure* **then**

add *inferences* to *csp*

*result*  $\leftarrow$  BACKTRACK(*csp*, *assignment*)

**if** *result*  $\neq$  *failure* **then return** *result*

remove *inferences* from *csp*

remove {*var* = *value*} from *assignment*

**return** *failure*

You can modify this function to **change the variable ordering** and potentially improve performance



# Variable Ordering: Alternatives

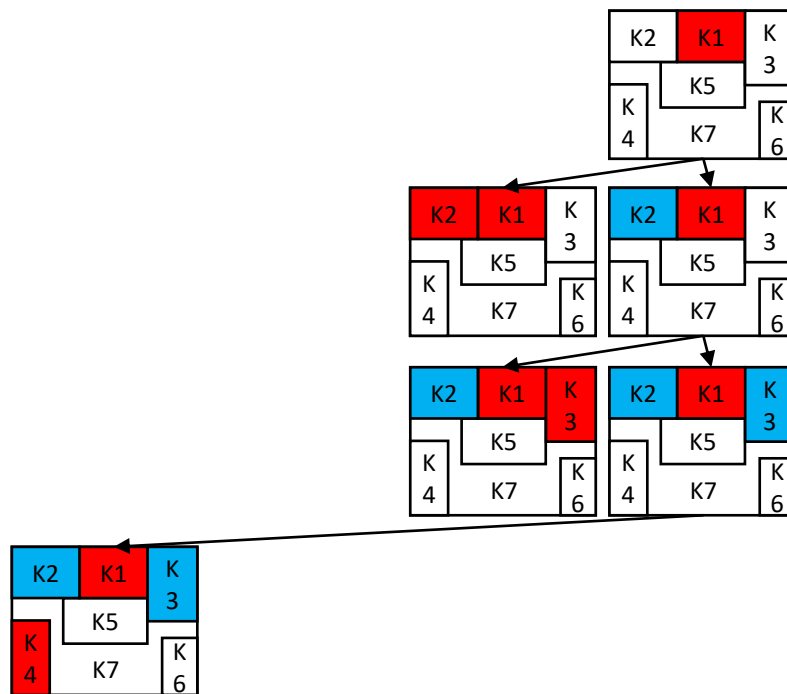
**CSP Backtracking algorithm can use a number of variable ordering strategies:**

- **Static: choose the variables in order (we did that)**
- **Random: order variables in random sequence**
- **Minimum-remaining-values (MRV) heuristic:**
  - **choose the variable with the “fewest” legal values**
- **Degree heuristic:**
  - **choose the variable involved in the largest amount of constraints on other unassigned variables**
  - **choose the variable with highest node degree on a constraint graph**

# Variable Ordering: MRV Heuristic

As CSP Backtracking algorithm progresses, the number of possible value assignments for each variable will shrink (due to constraints):

- MRV uses “fail-first” heuristics (also called “most constrained variable” heuristics)
- MRV picks a variable with lowest value assignment options “left”
  - expecting to limit exploration depth
  - likely to find a failure assignment faster
- Usually better than static and random orderings on average



K1 = ???

K2 = ???

K3 = ???

K4 = ???

Which variable to explore next (ignore the EXPECTED sequence on the right)?

Available options:

K5: {GREEN}

K6: {RED, BLUE, GREEN}

K7: {GREEN}

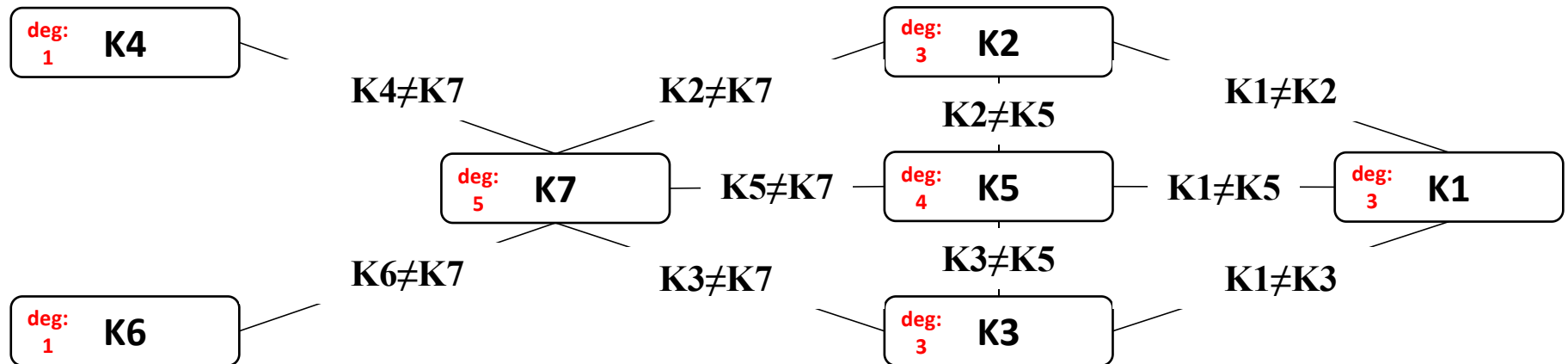
MRV should pick K5 or K7  
("fail first" variable).

Tie needs to be resolved.



# Variable Ordering: Degree Heuristics

Consider the following constraint graph representation of the problem we analyzed:



- degree heuristics is considered less effective than MRV
- degree heuristics can be used as a tie-breaker (two variables with the same “potential” according to MRV)
- attempts to reduce the branching factor on future choices

# Value Ordering: Alternatives

**function** BACKTRACKING-SEARCH(*csp*) **returns** a solution or *failure*  
    **return** BACKTRACK(*csp*, { })

**function** BACKTRACK(*csp*, *assignment*) **returns** a solution or *failure*  
    **if** *assignment* is complete **then return** *assignment*

*var*  $\leftarrow$  SELECT-UNASSIGNED-VARIABLE(*csp*, *assignment*)

**for each** *value* **in** ORDER-DOMAIN-VALUES(*csp*, *var*, *assignment*) **do**

**if** *value* is consistent with *assignment* **then**

            add {*var* = *value*} to *assignment*

*inferences*  $\leftarrow$  INFERENCE(*csp*, *var*, *assignment*)

**if** *inferences*  $\neq$  *failure* **then**

            add *inferences* to *csp*

*result*  $\leftarrow$  BACKTRACK(*csp*, *assignment*)

**if** *result*  $\neq$  *failure* **then return** *result*

            remove *inferences* from *csp*

        remove {*var* = *value*} from *assignment*

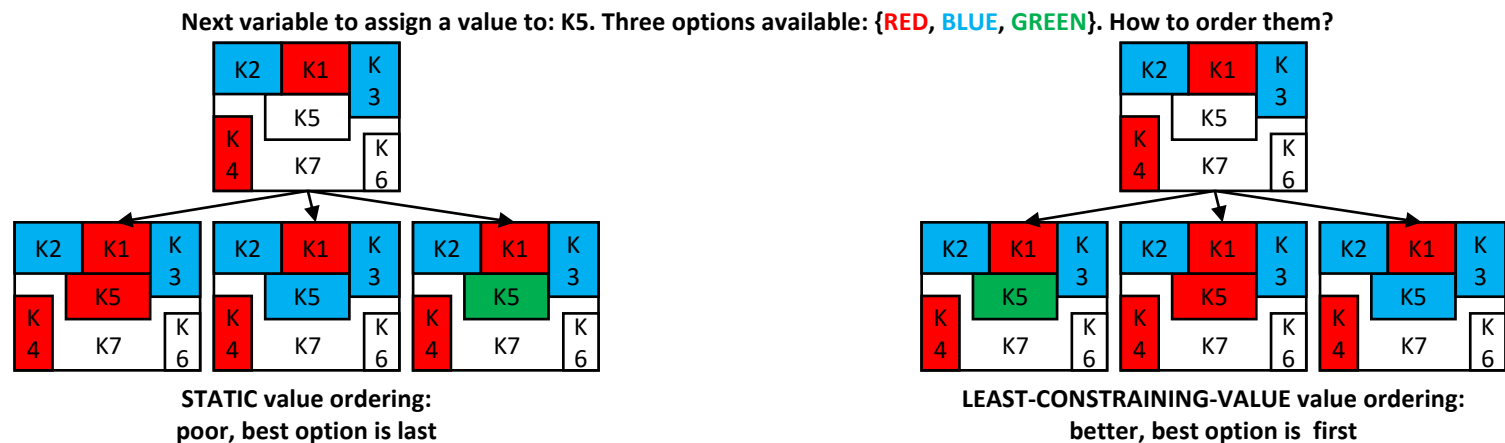
**return** *failure*

You can modify this order to **change the value assignment ordering** and potentially improve performance

# Least-Constraining-Value Heuristics

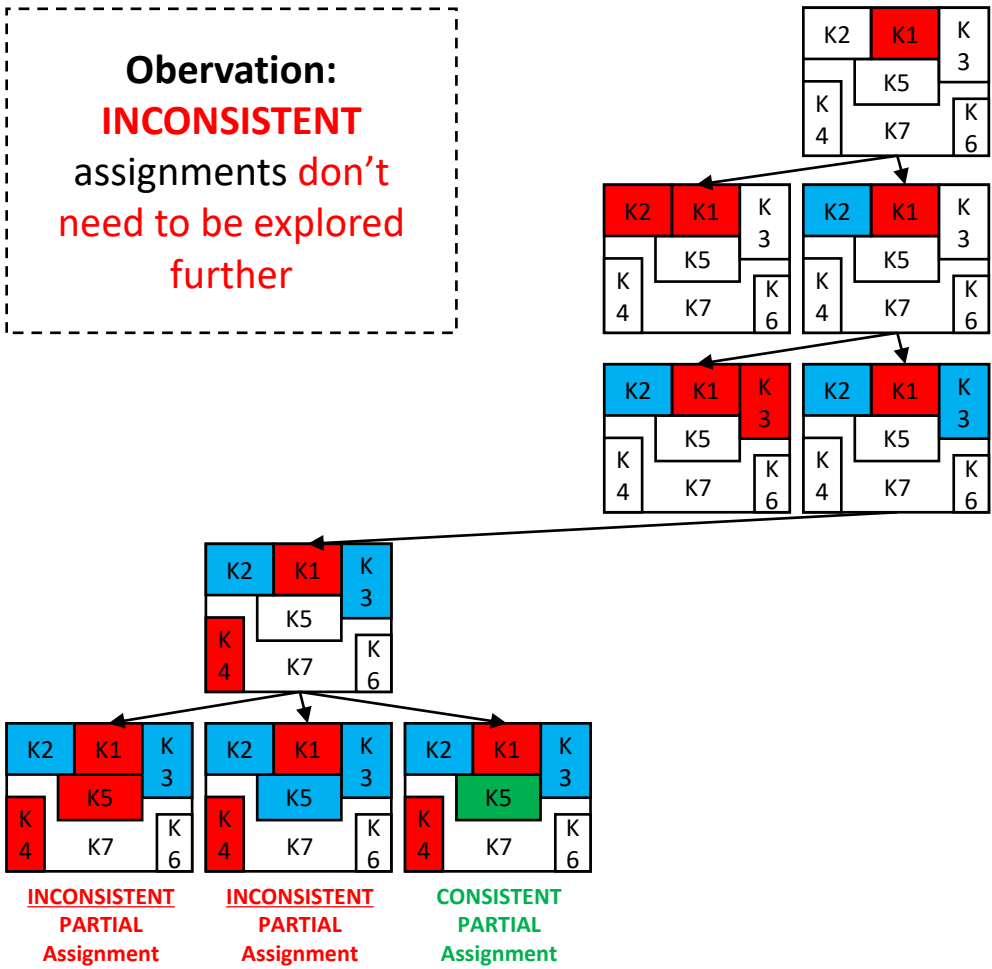
We picked (SELECT-UNASSIGNED-VARIABLE) the next variable to assign a value to and we have a number of values to choose from. What next?

- use the least-constraining-value heuristic
  - picks a value that **rules out the fewest choices for neighboring variables in the constraining graph** (increase **flexibility for FUTURE** assignments)
  - ORDER-DOMAIN-VALUES is the function that orders values here



# Can We Do Better?

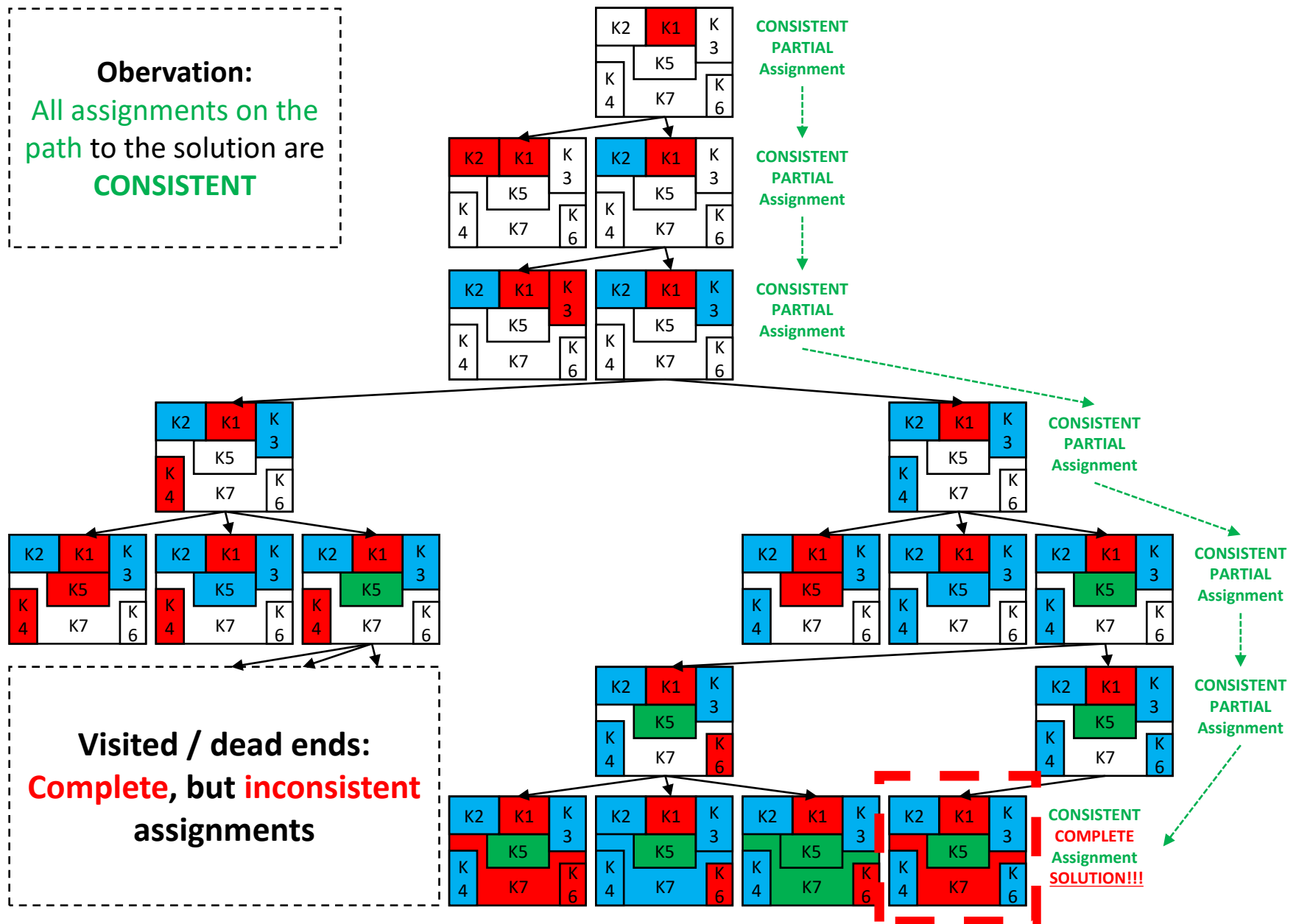
**Obervation:**  
**INCONSISTENT**  
 assignments **don't**  
 need to be explored  
 further



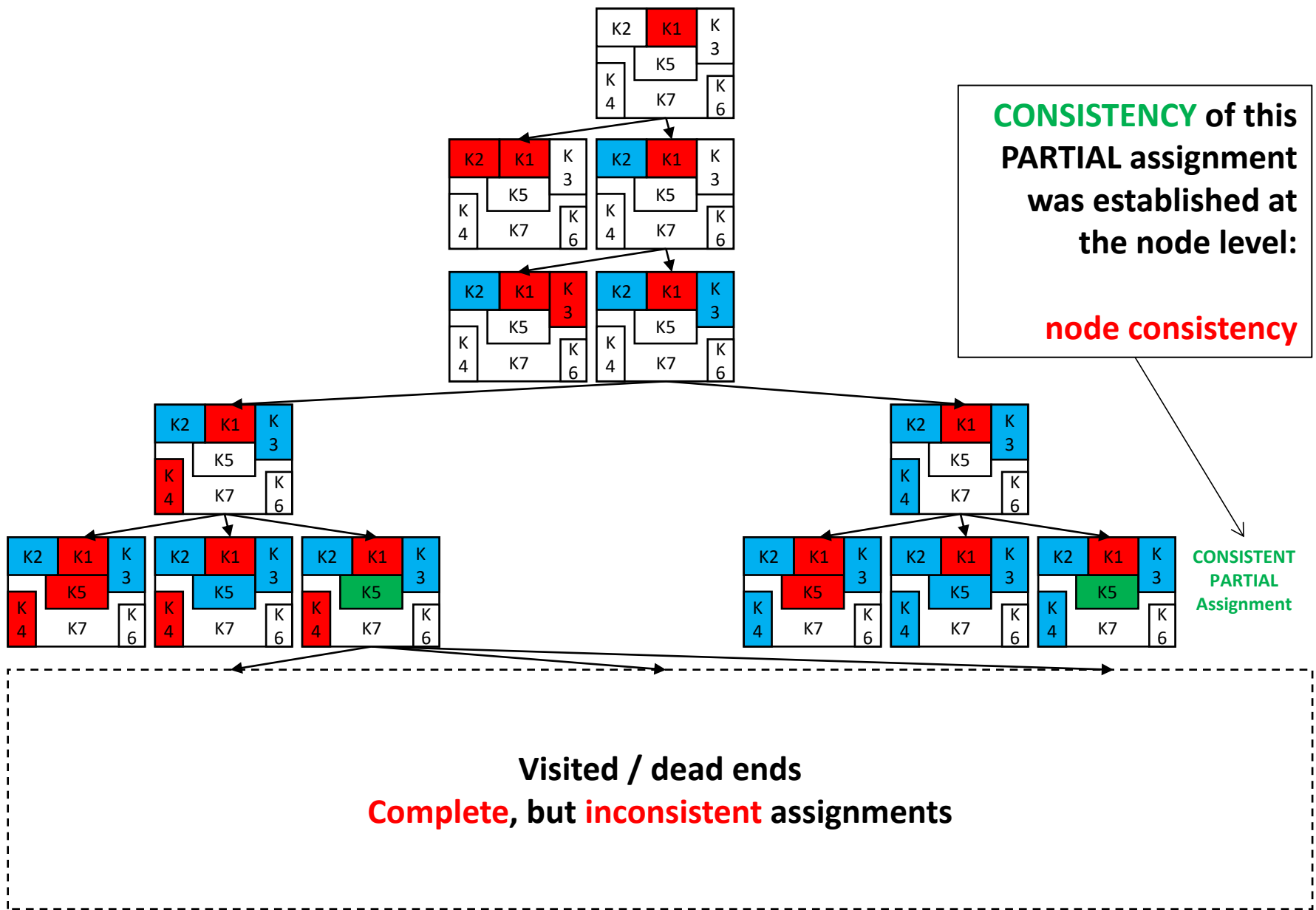
Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: **RED**, **BLUE**, **GREEN**

## Observation:

All assignments on the  
path to the solution are  
**CONSISTENT**

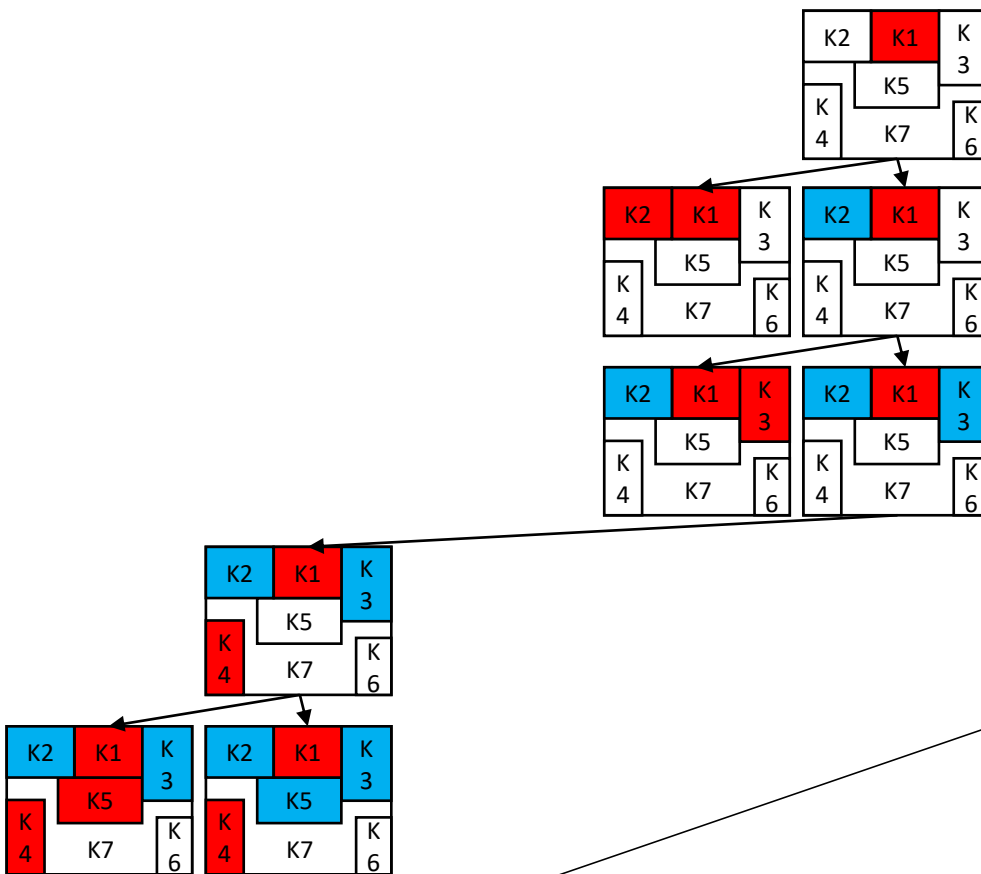


Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: RED, BLUE, GREEN



Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: **RED**, **BLUE**, **GREEN**





Violates:  
Rule 4  
Rule 6

**INCONSISTENCY** of  
this **PARTIAL**  
assignment was  
established at the  
node level:

**node consistency**

Can we detect /  
“predict”  
**INCONSISTENCIES**  
earlier and prune  
useless branches?  
**YES!**

Variable assignment order: K1, K2, K3, K4, K5, K6, K7 | Value assignment order: **RED**, **BLUE**, **GREEN**



# CSP: More Pruning with Inference

**function** BACKTRACKING-SEARCH(*csp*) **returns** a solution or *failure*  
    **return** BACKTRACK(*csp*, { })

**function** BACKTRACK(*csp*, *assignment*) **returns** a solution or *failure*  
    **if** *assignment* is complete **then return** *assignment*  
    *var*  $\leftarrow$  SELECT-UNASSIGNED-VARIABLE(*csp*, *assignment*)  
    **for each** *value* **in** ORDER-DOMAIN-VALUES(*csp*, *var*, *assignment*) **do**  
        **if** *value* is consistent with *assignment* **then**  
            add {*var* = *value*} to *assignment*  
            inferences  $\leftarrow$  INFERENCE(*csp*, *var*, *assignment*)  
            **if** inferences  $\neq$  *failure* **then**  
                add inferences to *csp*  
                *result*  $\leftarrow$  BACKTRACK(*csp*, *assignment*)  
                **if** *result*  $\neq$  *failure* **then return** *result*  
                remove inferences from *csp*  
            remove {*var* = *value*} from *assignment*  
    **return** *failure*

With the information available to you, you can INFER that a particular branch is going to be INCONSISTENT