CS 480

Introduction to Artificial Intelligence

September 14th, 2021

Announcements / Reminders

- **Written Assignment #01 is posted**
  - **due on Wednesday (09/22/21) at 11:00 PM CST**
- **Contribute to the discussion on Blackboard, please**
- **Please follow the Week 03 To Do List instructions**
- **My tomorrow's (09/15/21) office hours:**
  - **online only in Blackboard Collaborate Ultra**

Plan for Today

- **A* Heuristics revisited**
- **Problem Solving: Adversarial Search**

A* Algorithm: Evaluation Function

Calculate / obtain:

$$f(n) = \mathbf{g(State_n)} + \mathbf{h(State_n)}$$

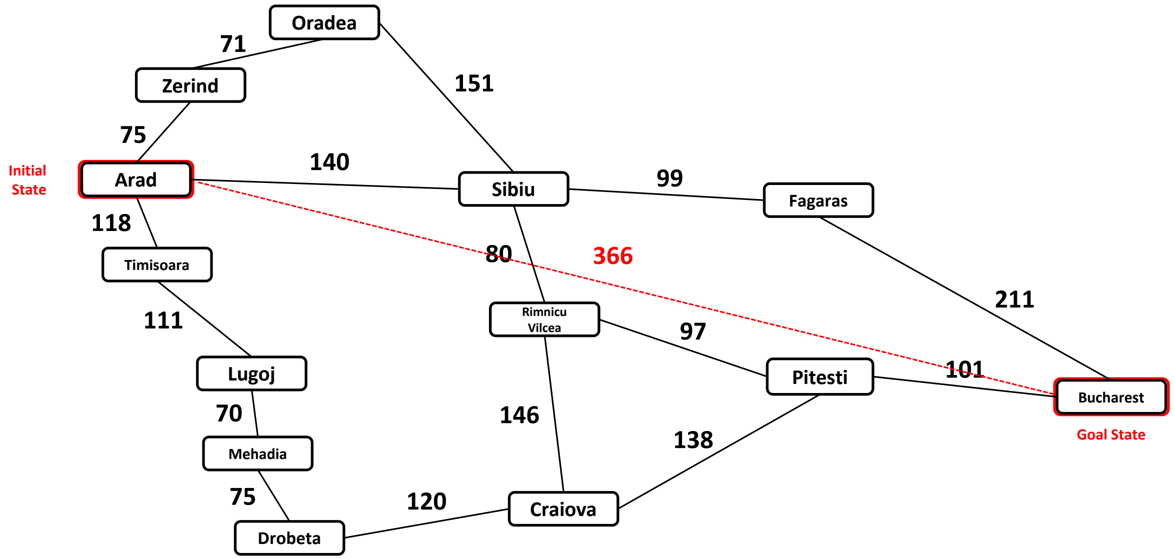
where:

- **g(n) - initial node to node n path cost**
- **h(n) - estimated cost of the best path that continues from node n to a goal node**

A state n with minimum (maximum) f(n) should be chosen for expansion

What Made A* Work Well?

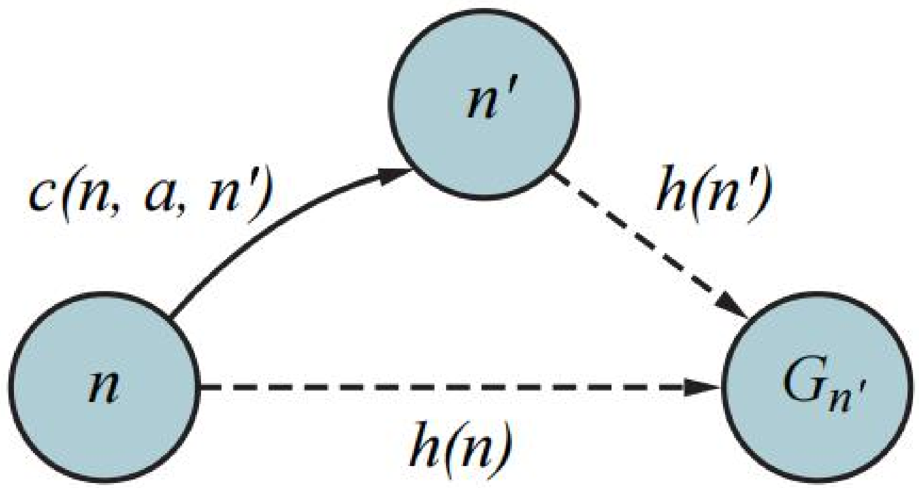
- Straight-line heuristics is admissible: it never overestimates the cost.



- An admissible heuristics is guaranteed to give you the optimal solution

What Made A* Work Well?

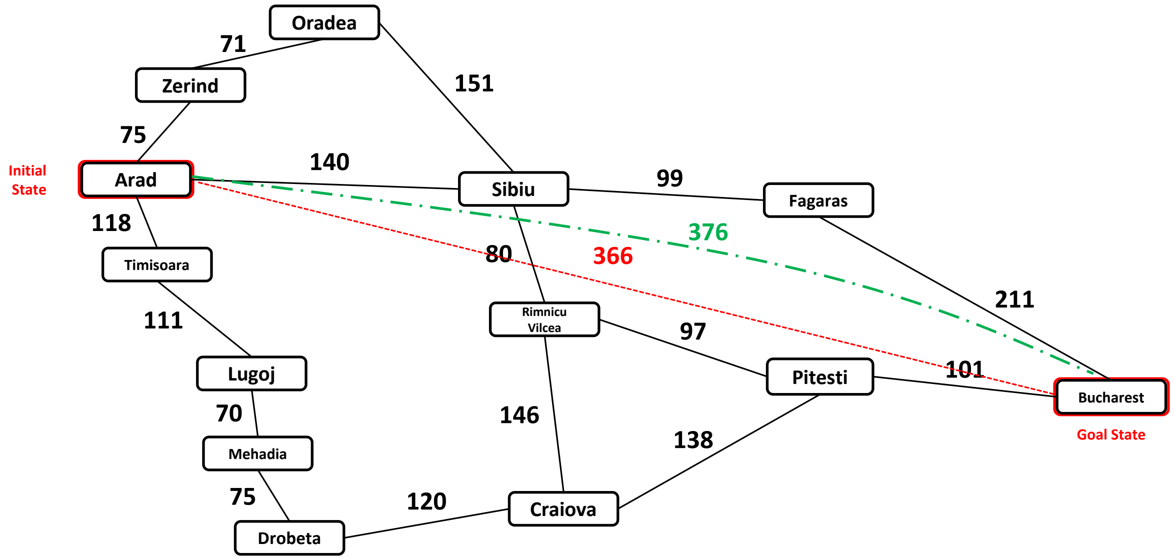
- **Straight-line heuristics is consistent: its estimate is getting better and better as we get closer to the goal**



- **Every consistent heuristics is admissible heuristics, but <u>not the other way around</u>**

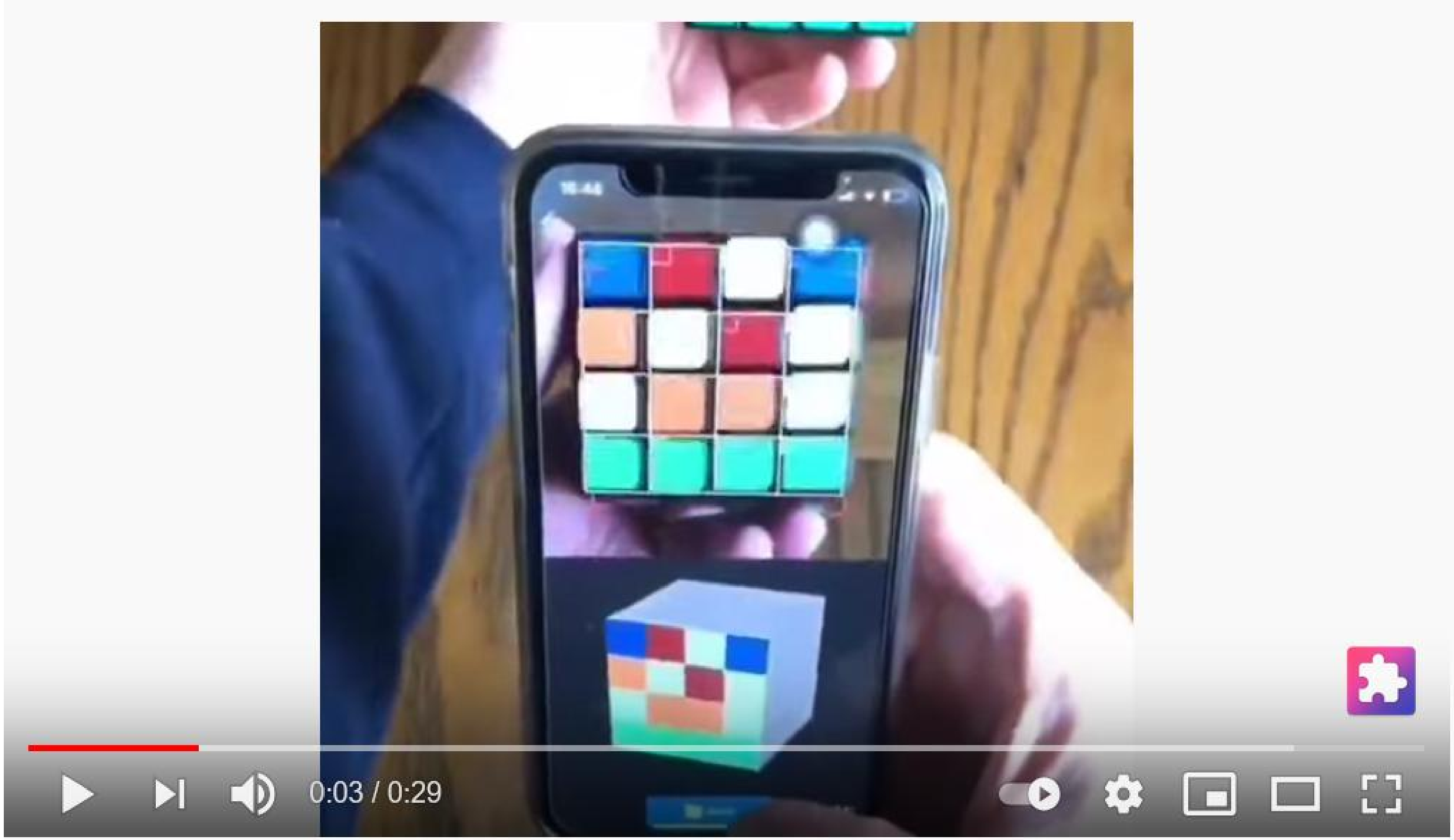
Dominating Heuristics

- We can have more than one available heuristics. For example $h_1(n)$ and $h_2(n)$.



- Heuristics $h_2(n)$ estimate is closer to actual cost than $h_1(n)$. $h_2(n)$ dominates $h_1(n)$. Use $h_2(n)$.

Informed Search: Application Example

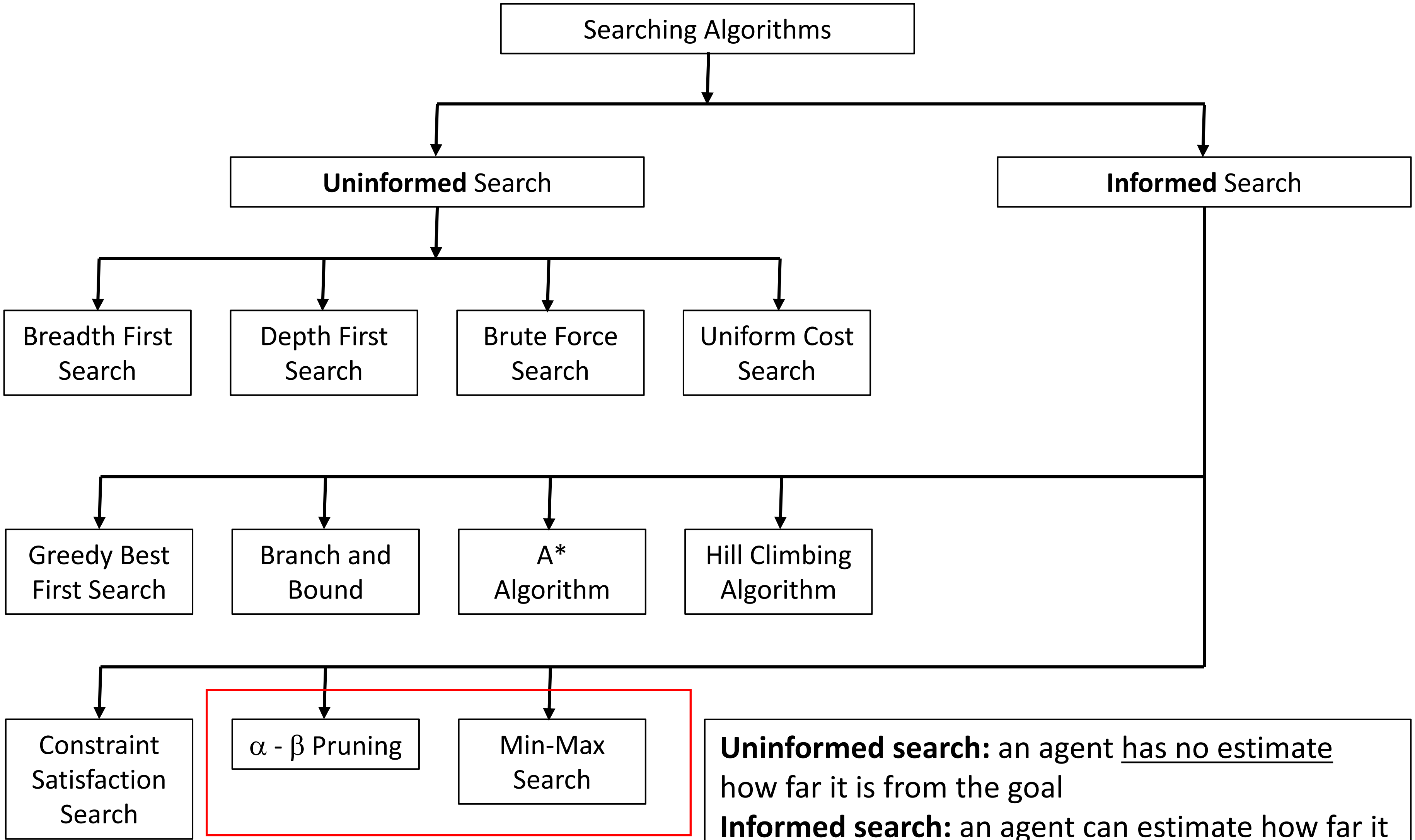


#shorts - Solve Any Rubik Cube Using this Augmented Reality App

2,116 views • Apr 2, 2021 61 5 SHARE SAVE ...

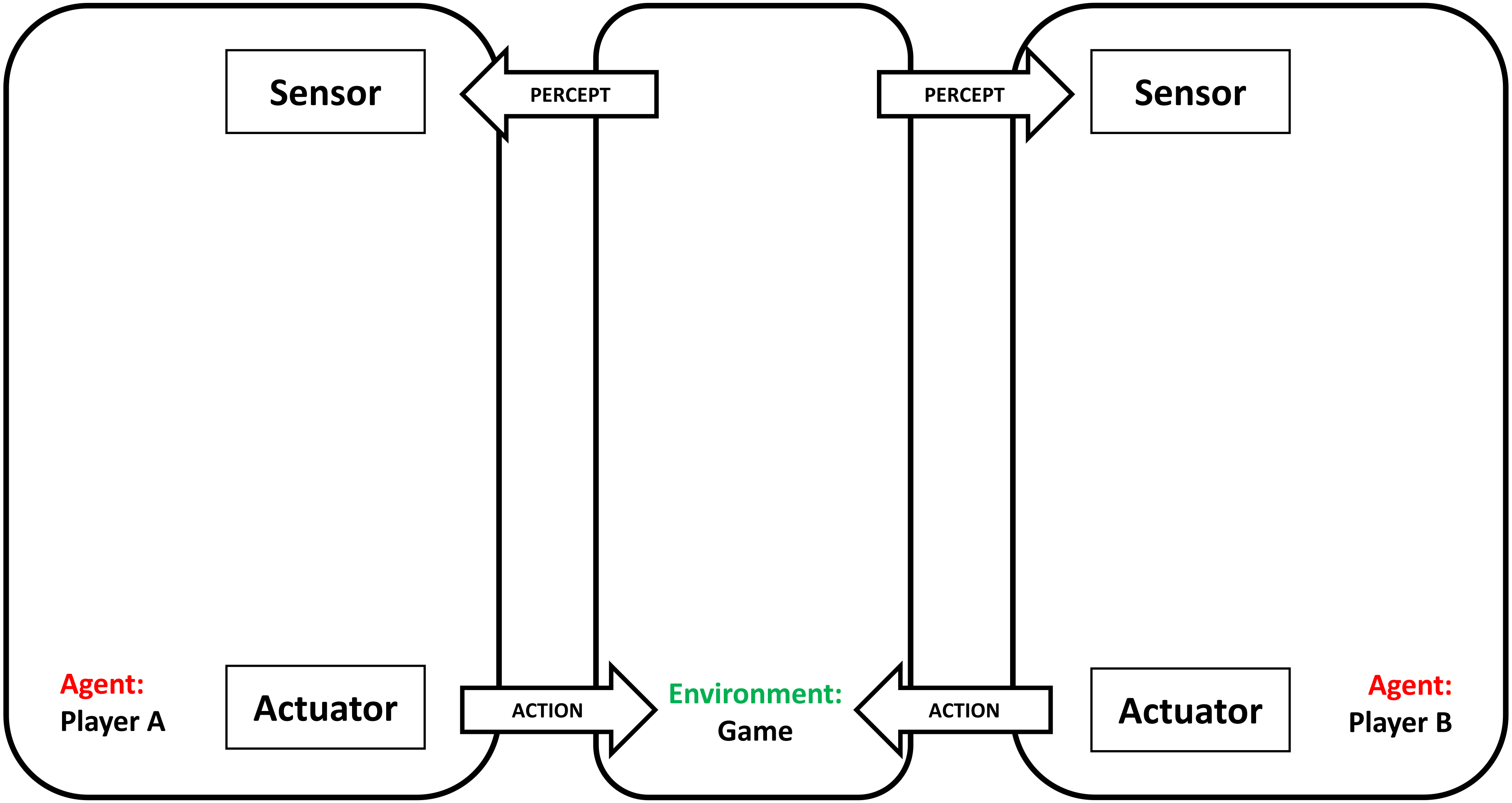
Source: https://www.youtube.com/watch?v=Pxbv2gEhnMk

Selected Searching Algorithms



Uninformed search: an agent has no estimate how far it is from the goal
Informed search: an agent can estimate how far it is from the goal

Two-player Games



Perfect Information Zero Sum Games

- Perfect information = fully observable
- Multiagent: number of players is 2 or more
- Multiagent: agents are competitve
- Zero-sum: “winner takes all”
- Examples:
  - Tic Tac Toe
  - Chess

Two Player Games: Env Assumptions

Works with a "Simple Environment":

- Fully observable
- ~~Single agent~~ Mulitagent (competitive!)
- Deterministic
- Static
- Episodic / sequential
- Discrete
- Known to the agent

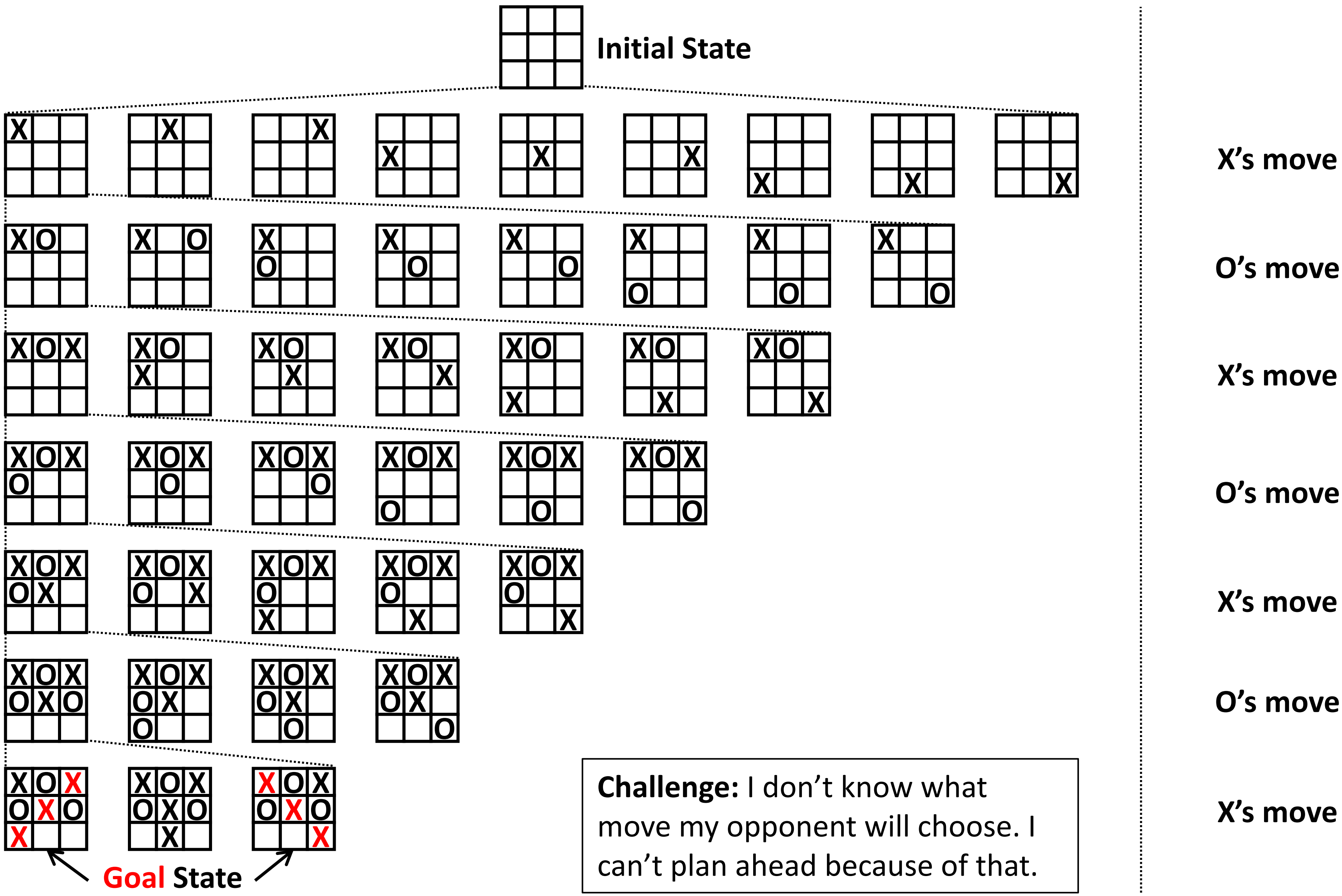
Defining Zero Sum Game Problem

- **Define a set of possible states: State Space**
- **Specify how will you track Whose Move / Turn it is**
- **Specify Initial State**
- **Specify Goal State(s) (there can be multiple)**
- **Define a FINITE set of possible Actions (legal moves) for EACH state in the State Space**
- **Come up with a Transition Model which describes what each action does**
- **Come up with a Terminal Test that verifies if the game is over**
- **Specify the Utility (Payoff / Objective) Function: a function that defines the final numerical value to player *p* when the game ends in terminal state *s***

MinMax Algorithm: the Idea

I don’t know what move my opponent will choose, but I am going to ASSUME that it is going to be the best / optimal option

Tic Tac Toe: Zero Sum Game (2 Players)



Challenge: I don't know what move my opponent will choose. I can't plan ahead because of that.

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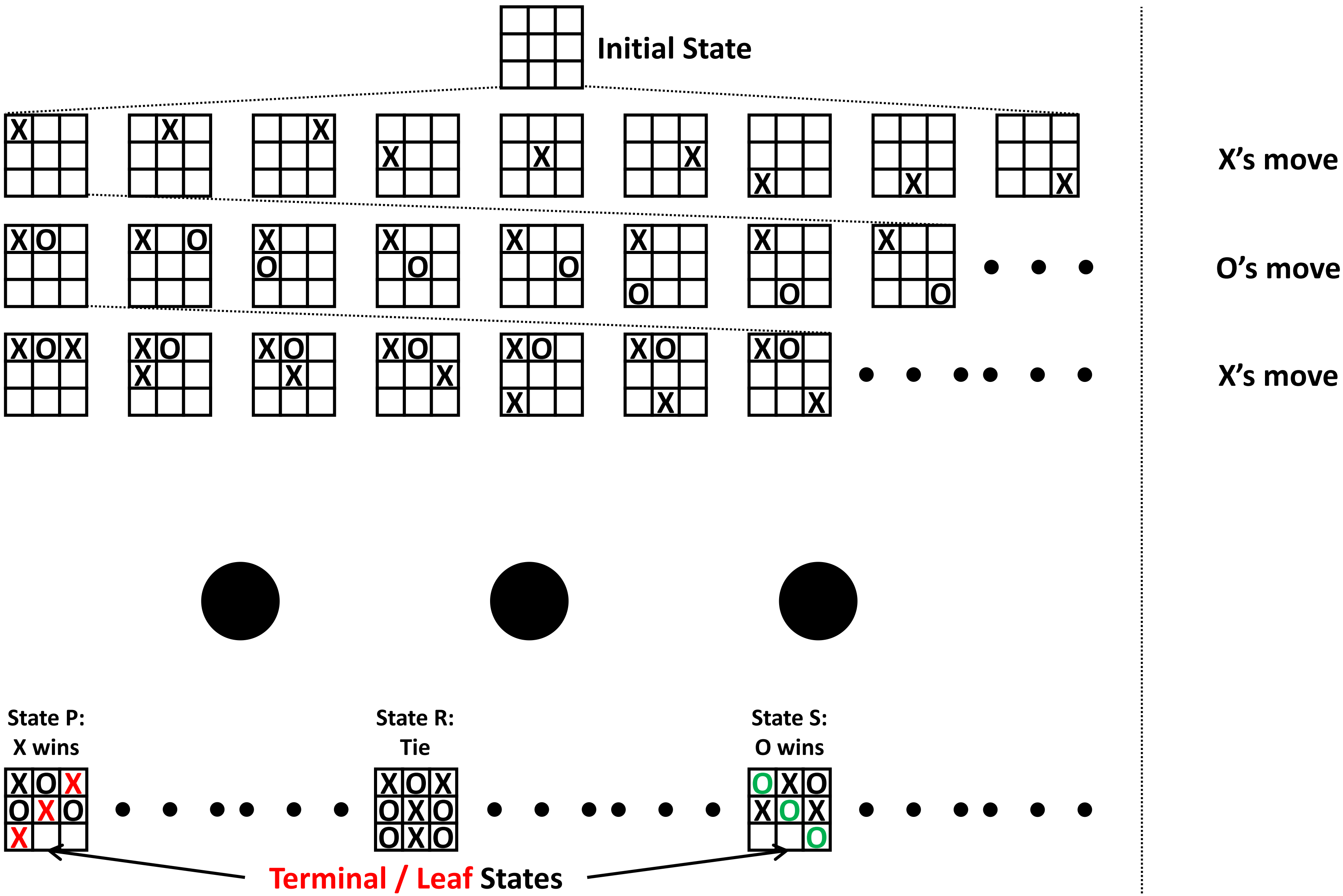

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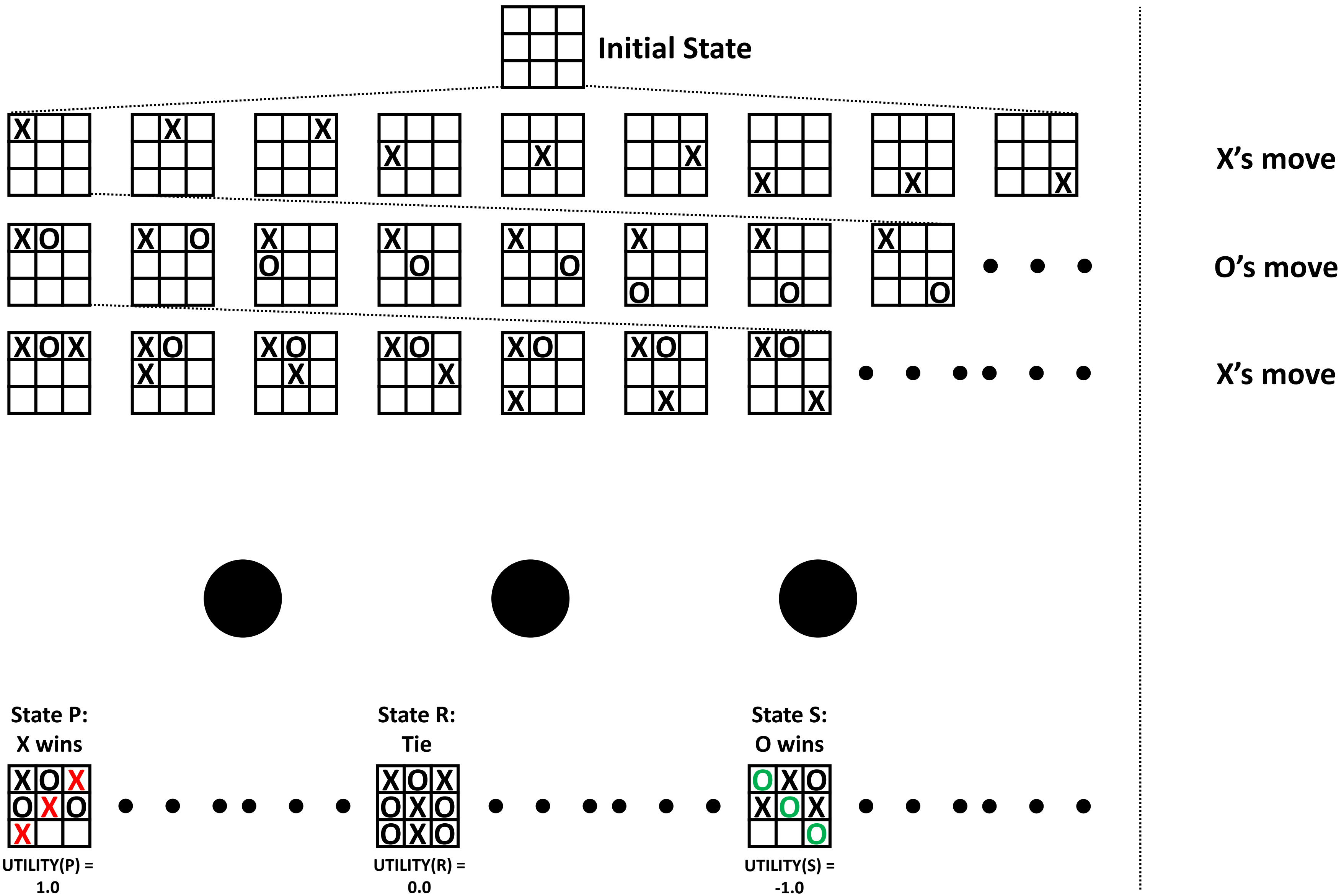
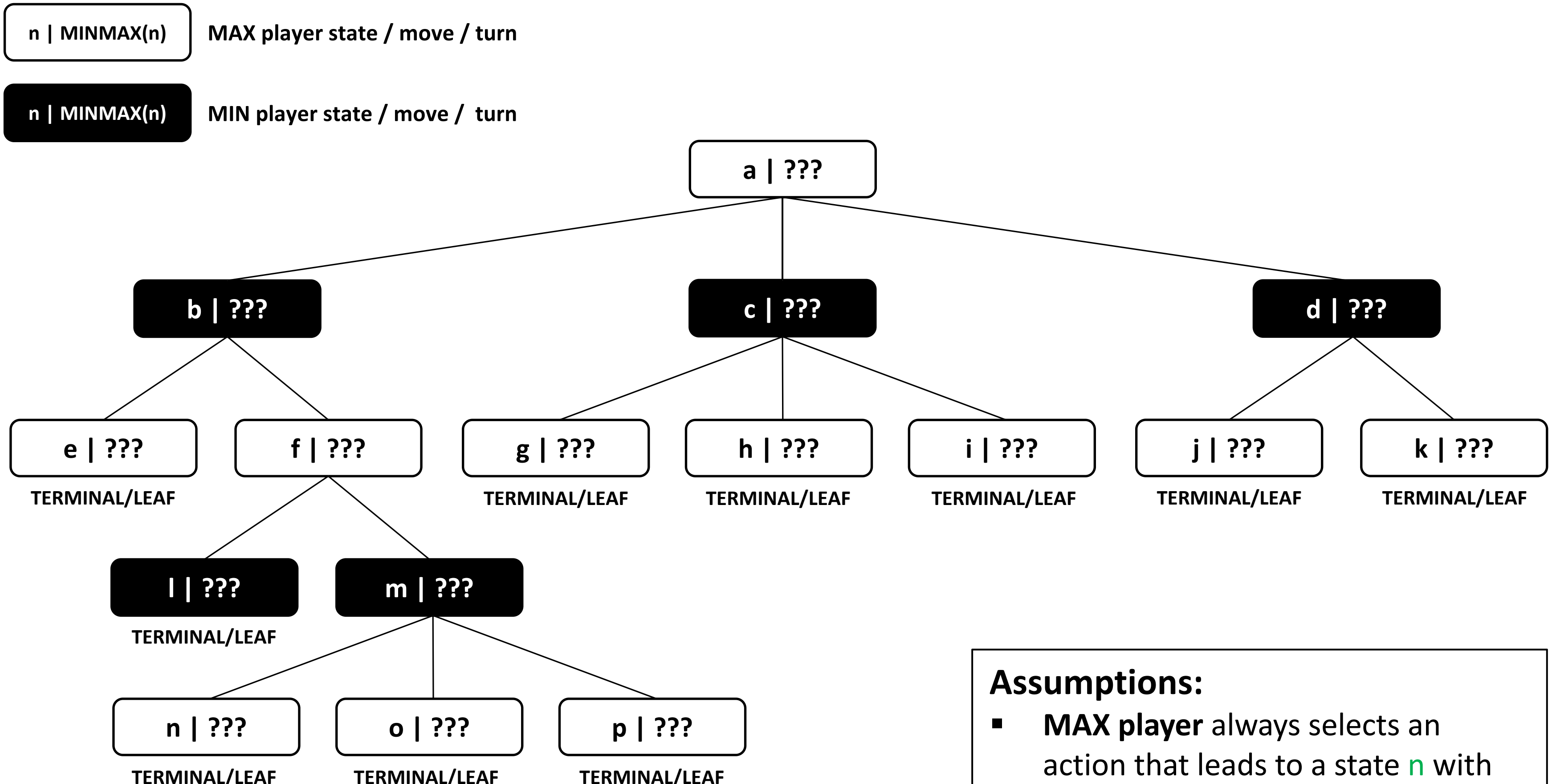


Example MinMax Search Tree



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- **MAX player** always selects an action that leads to a state n with **maximum** MINIMAX(n) value
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- **BOTH players** always play optimally

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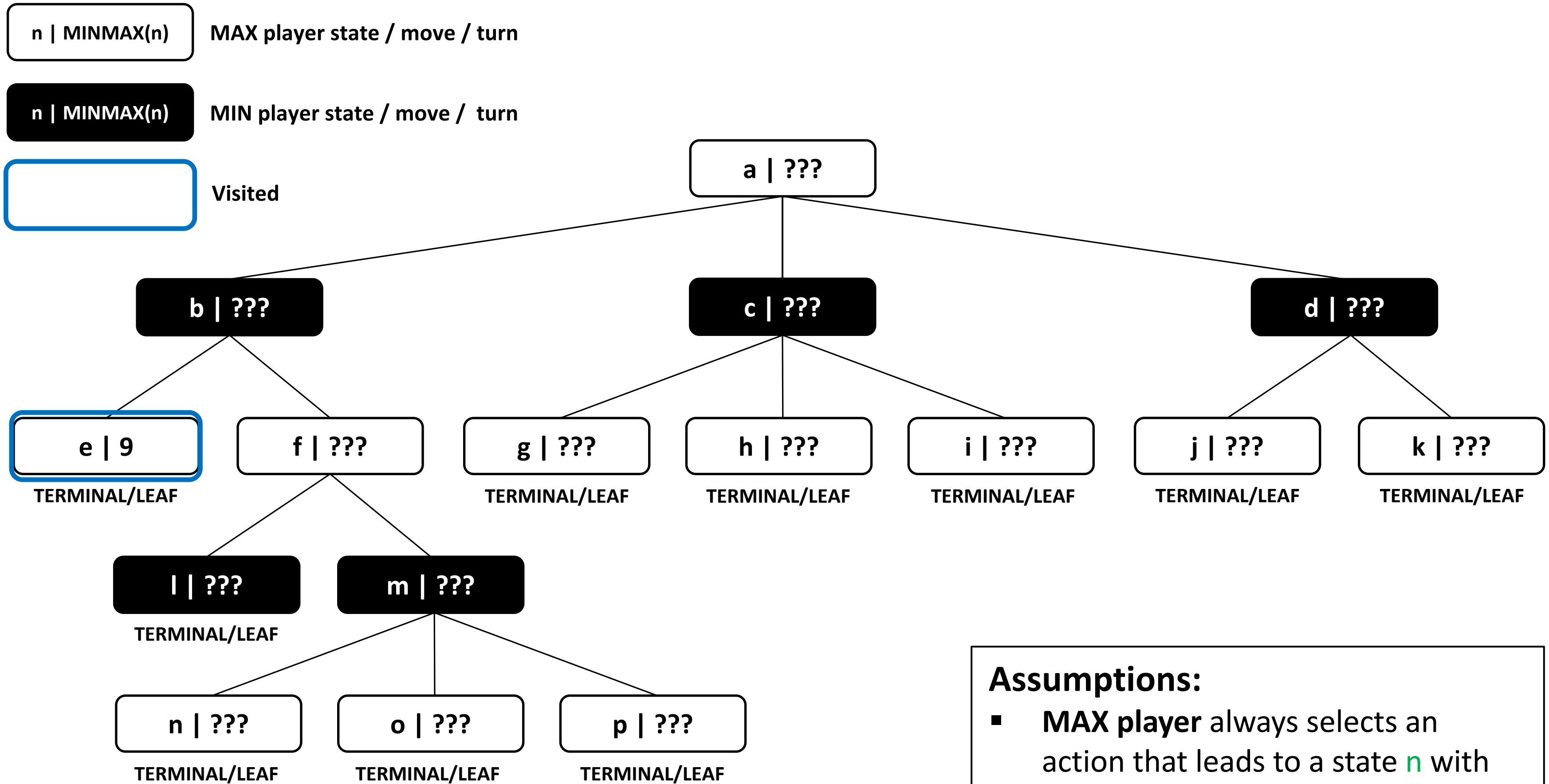

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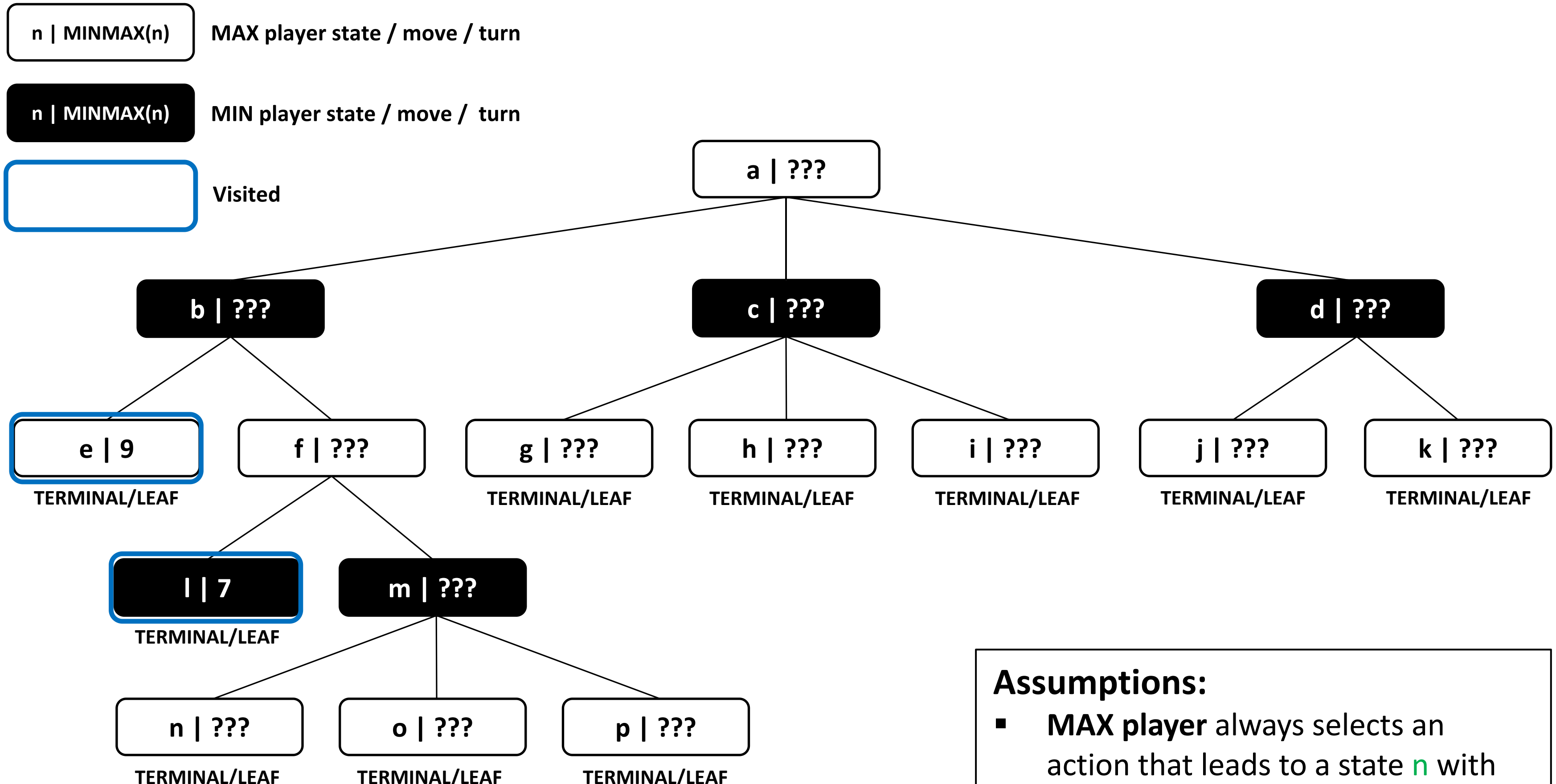

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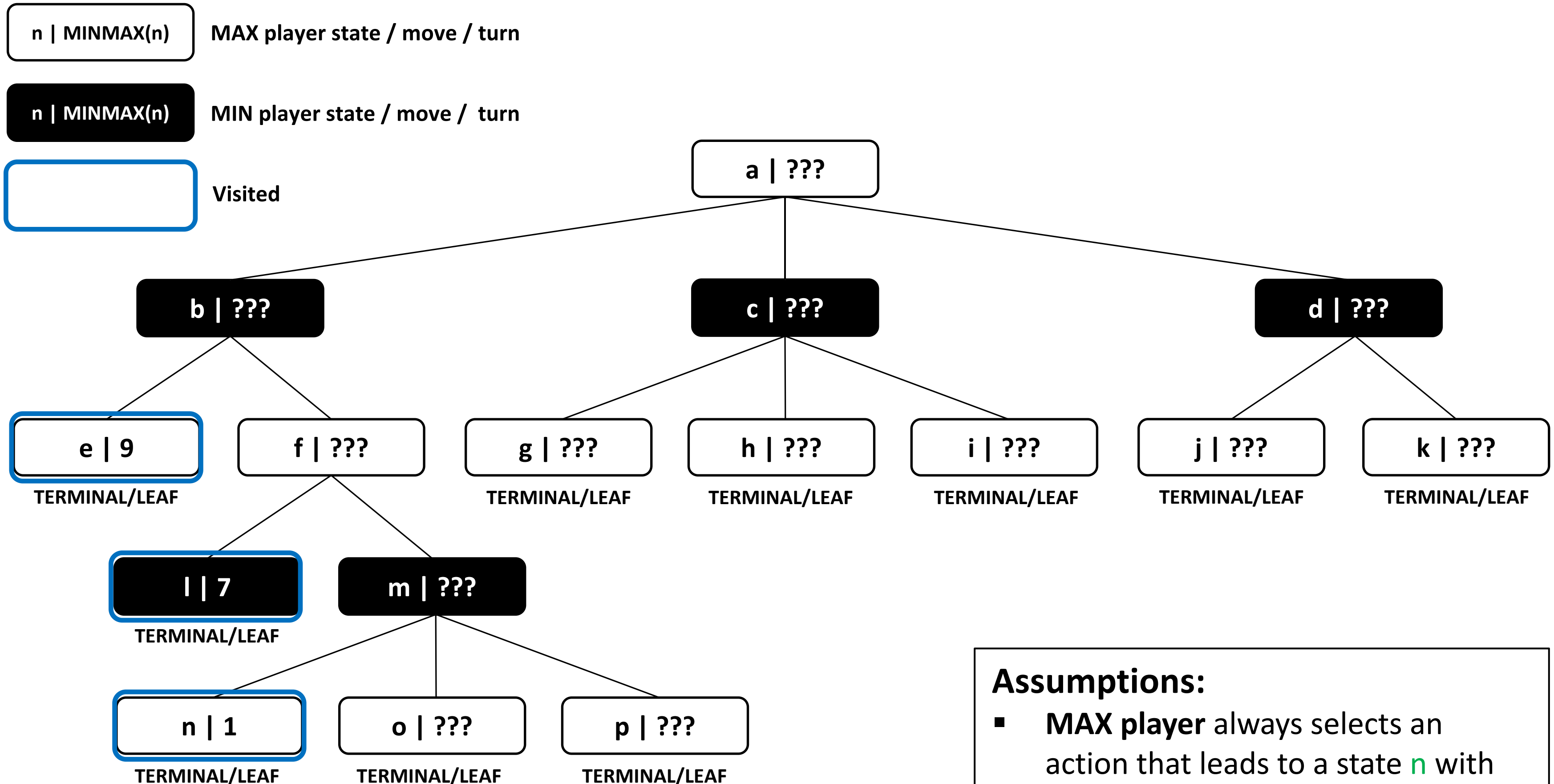

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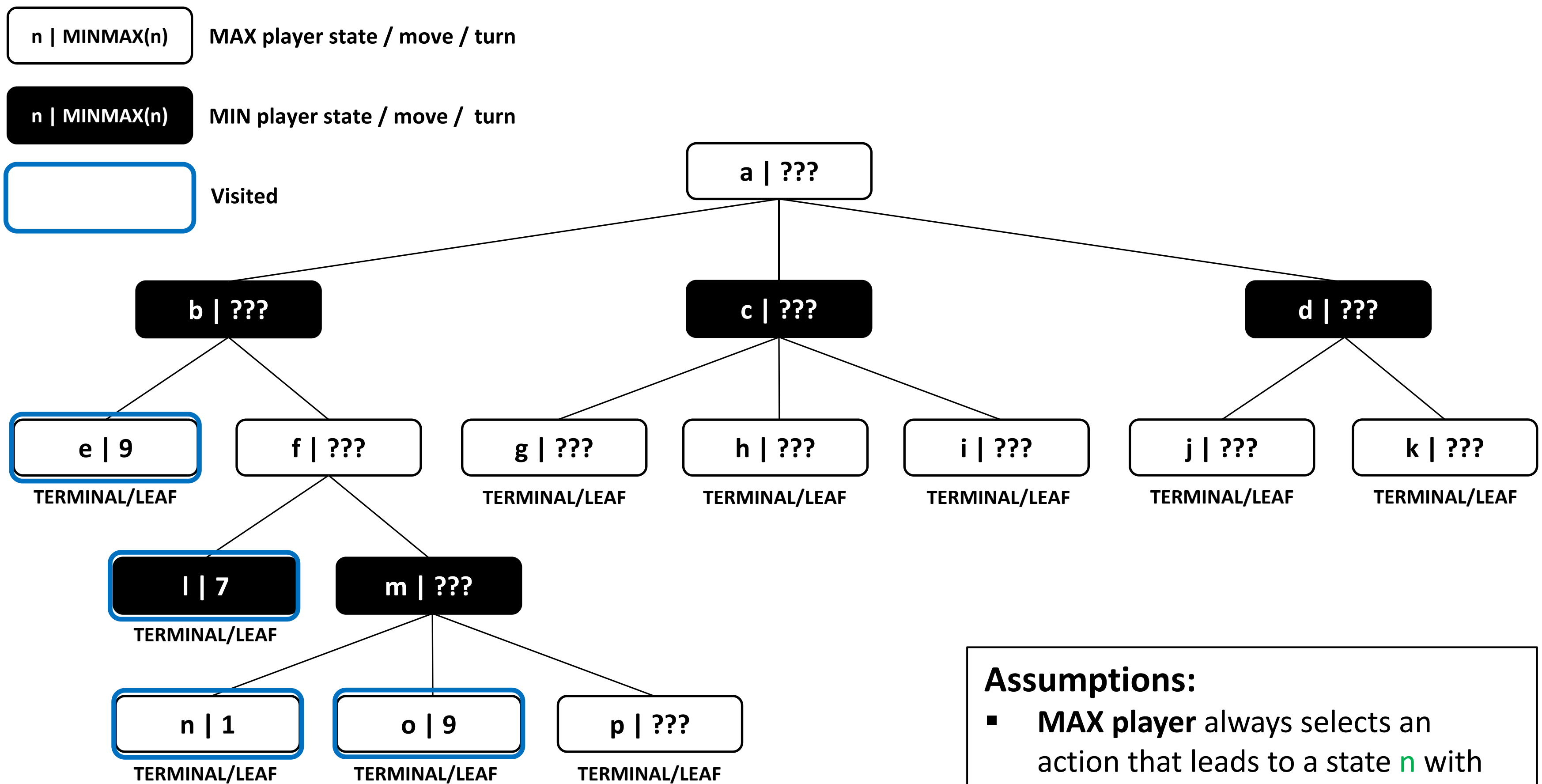

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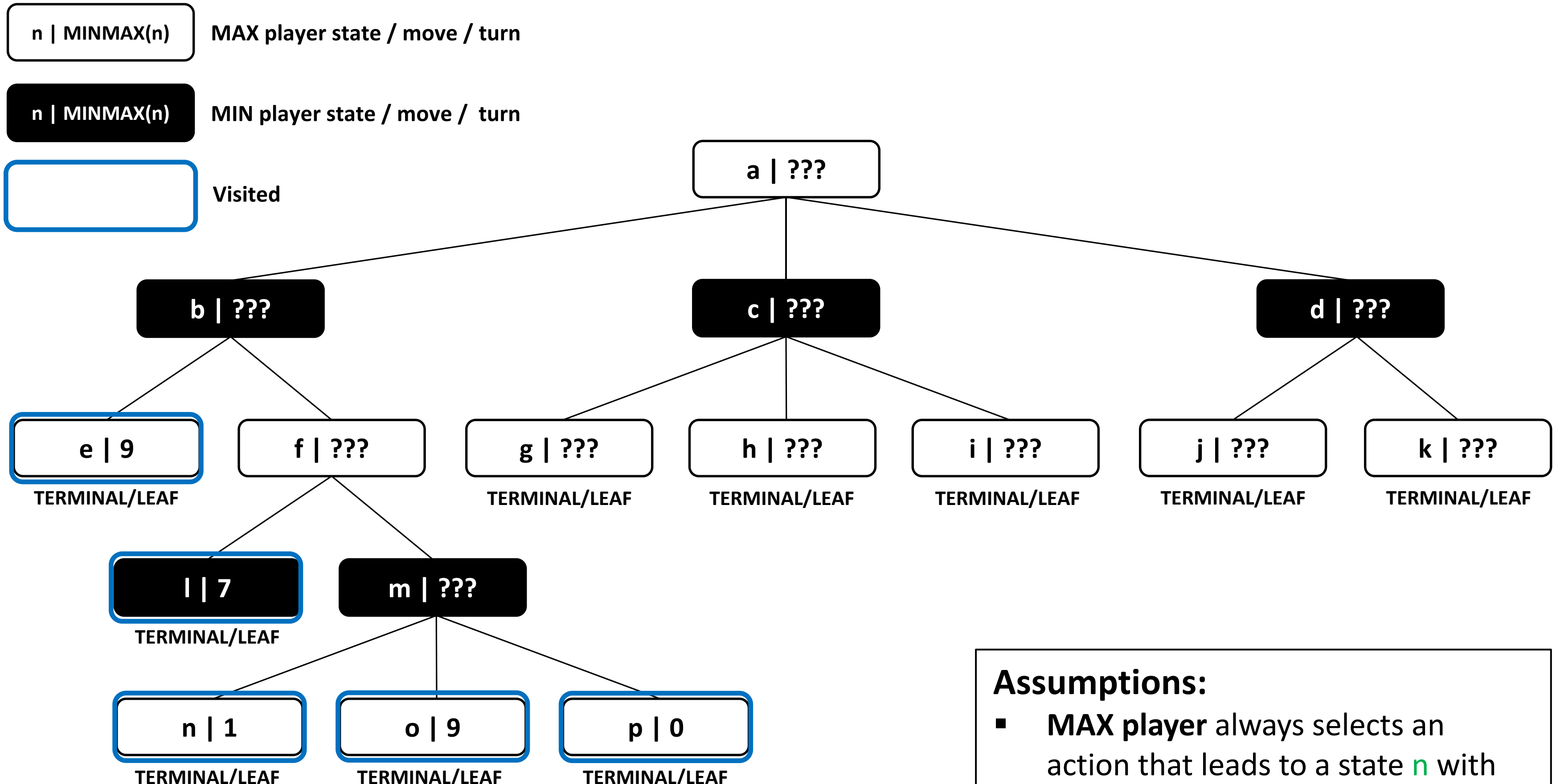

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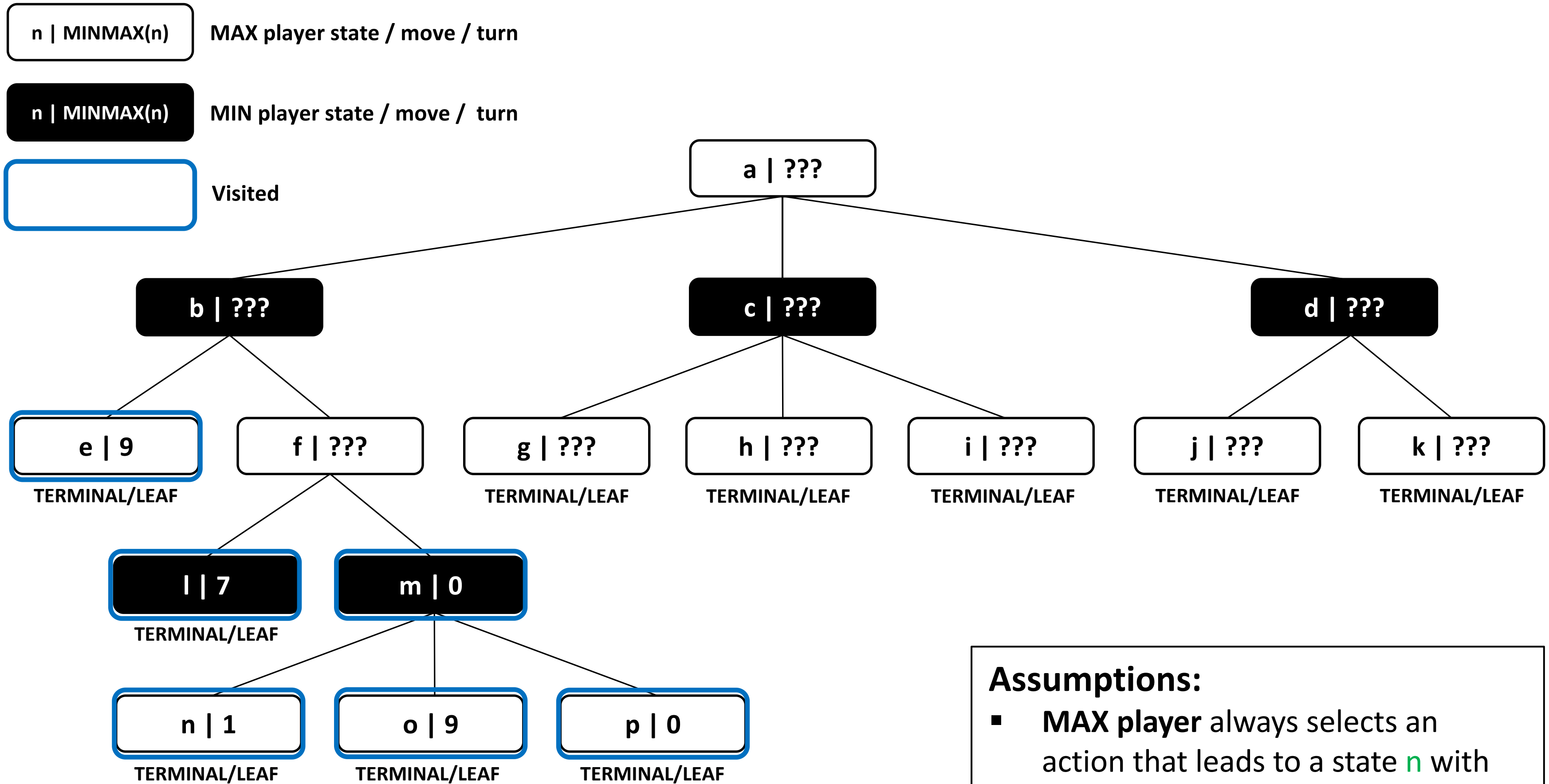

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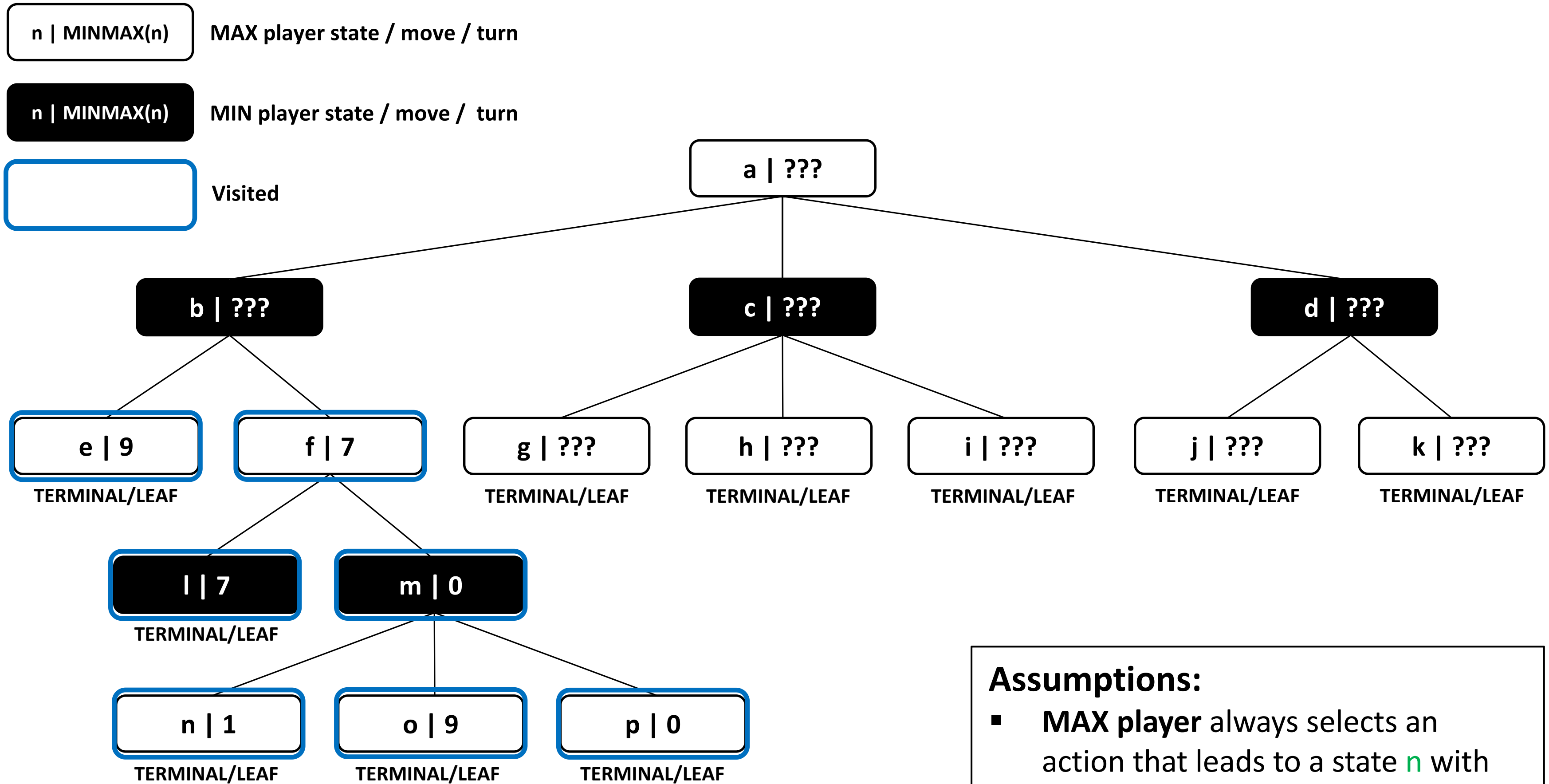

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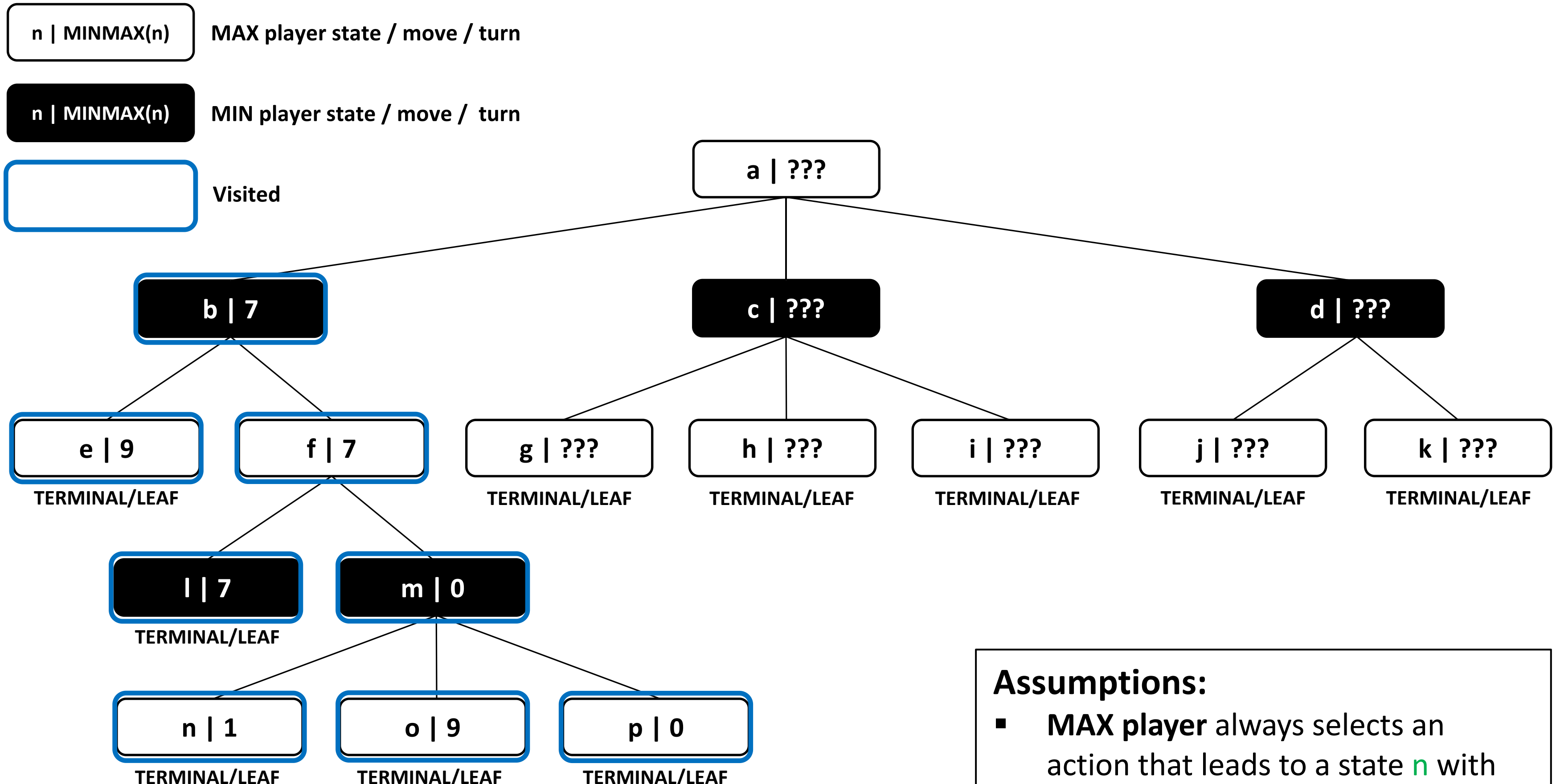

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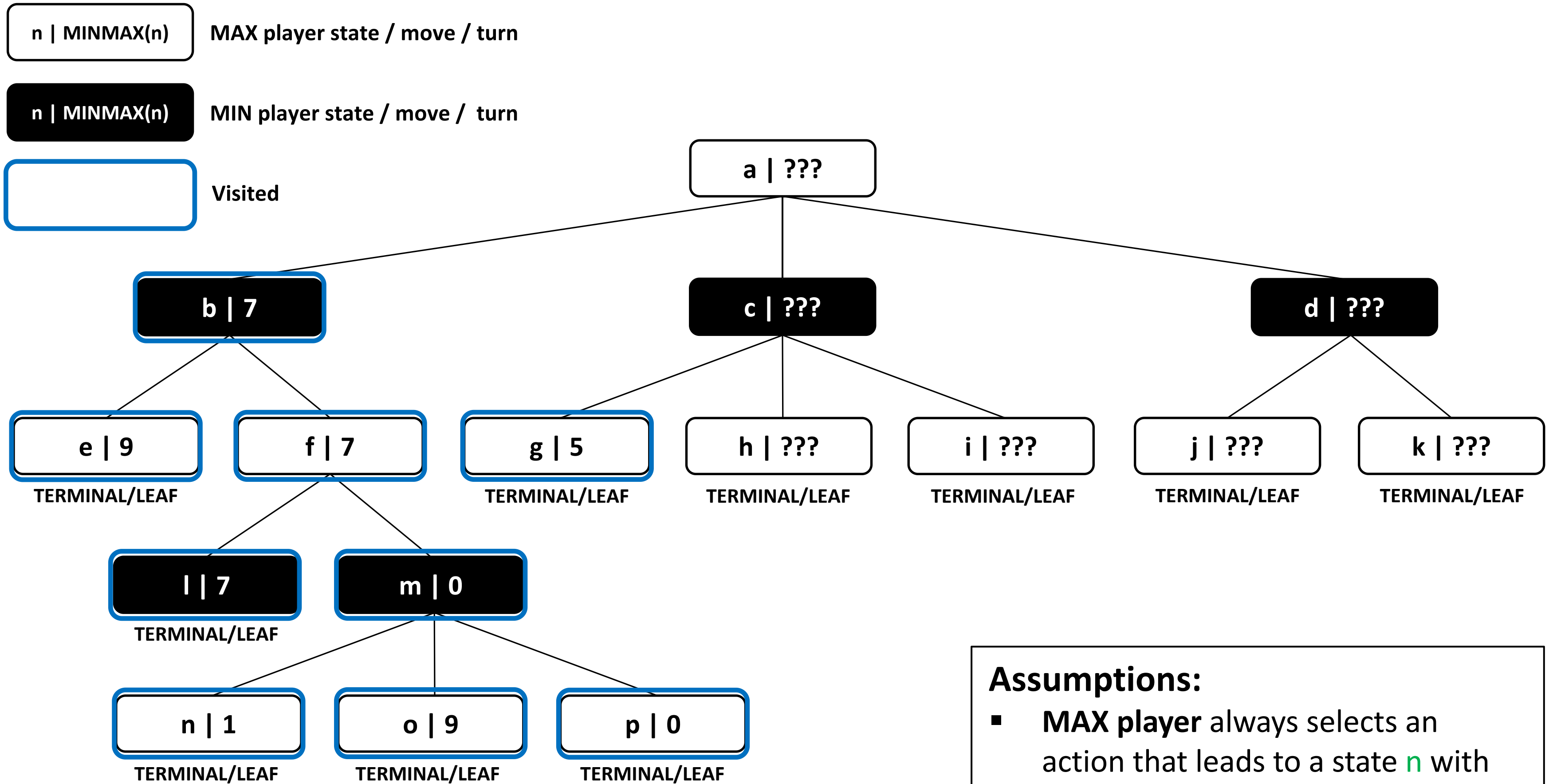

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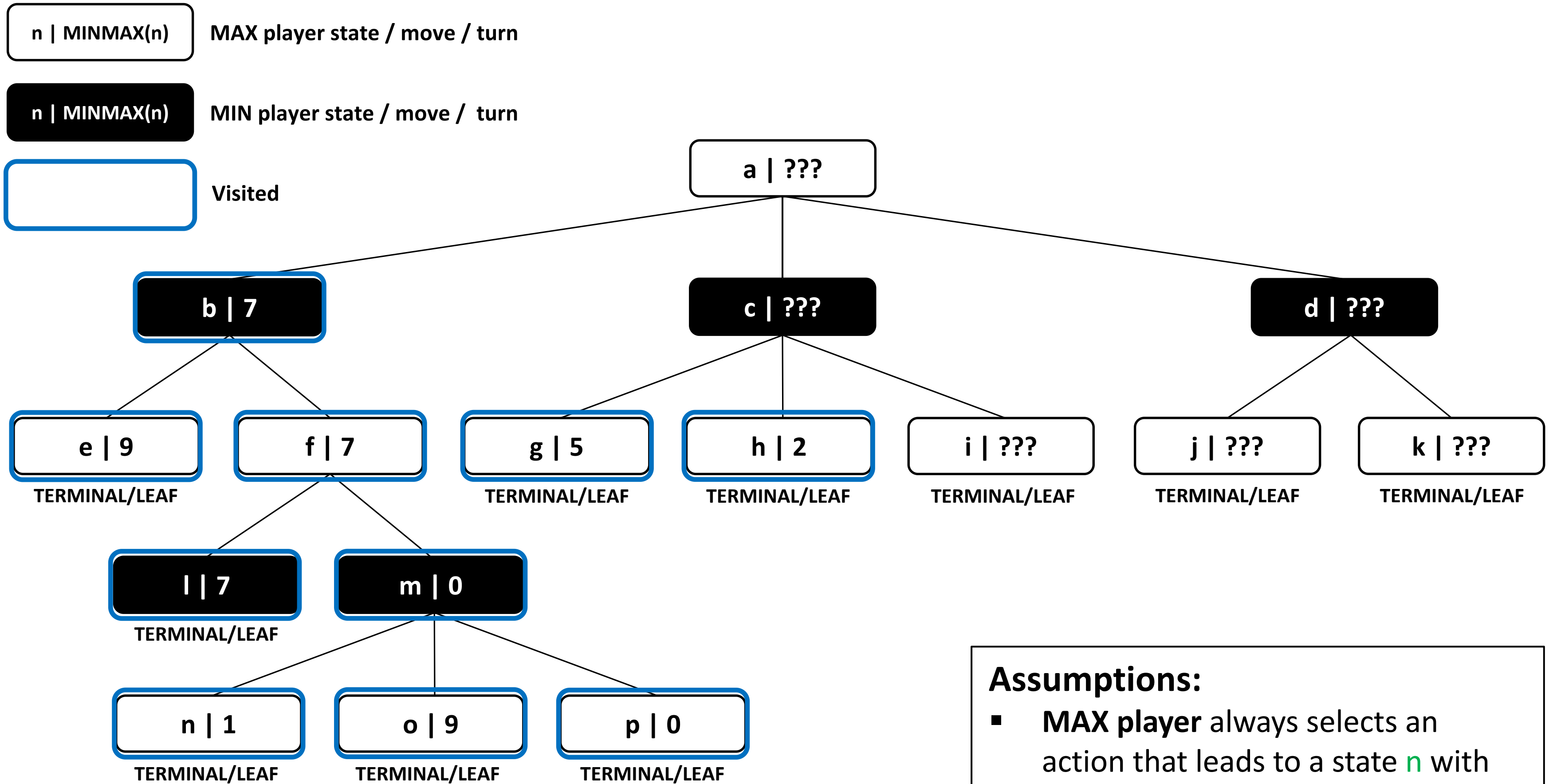

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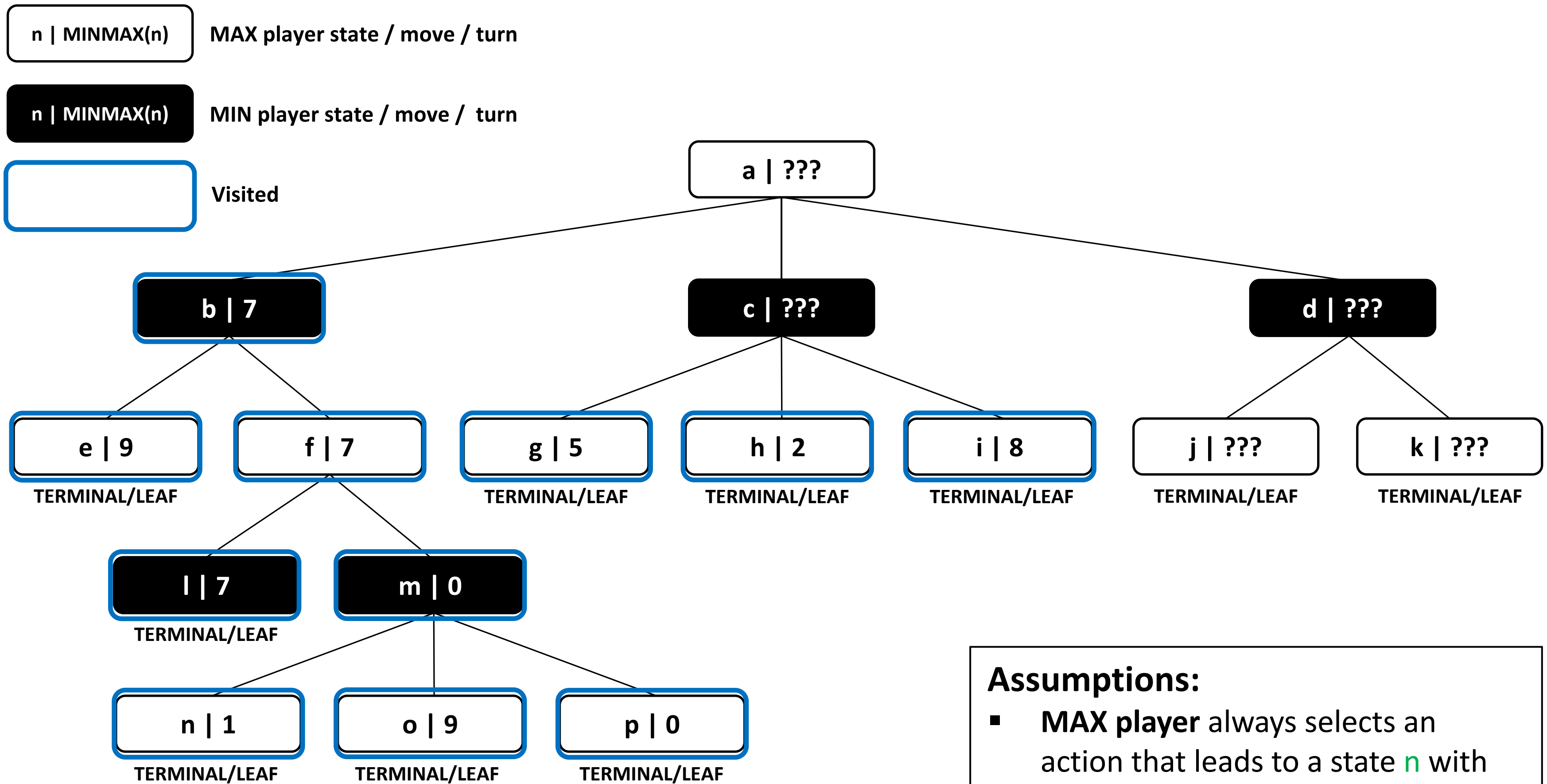

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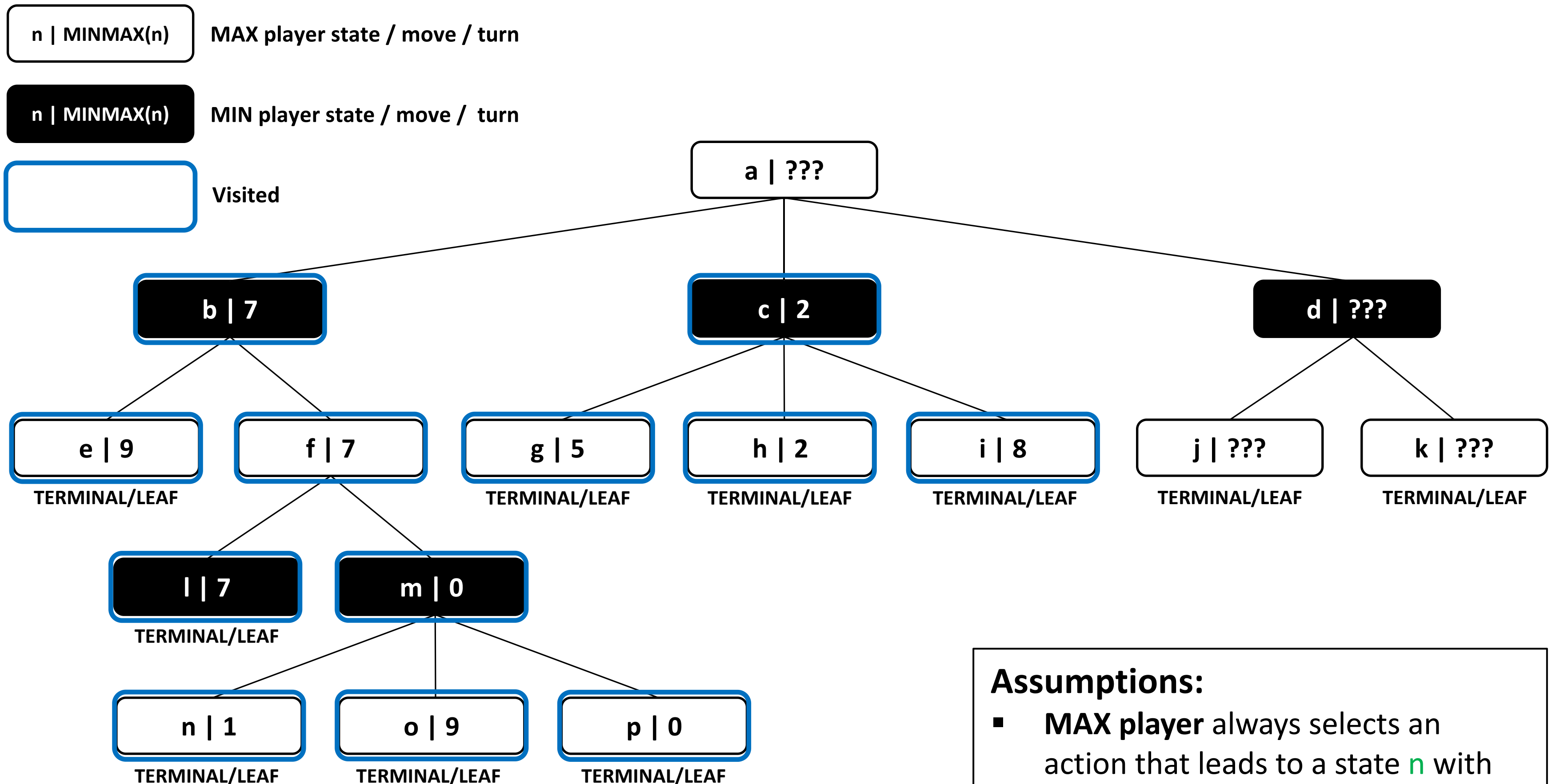

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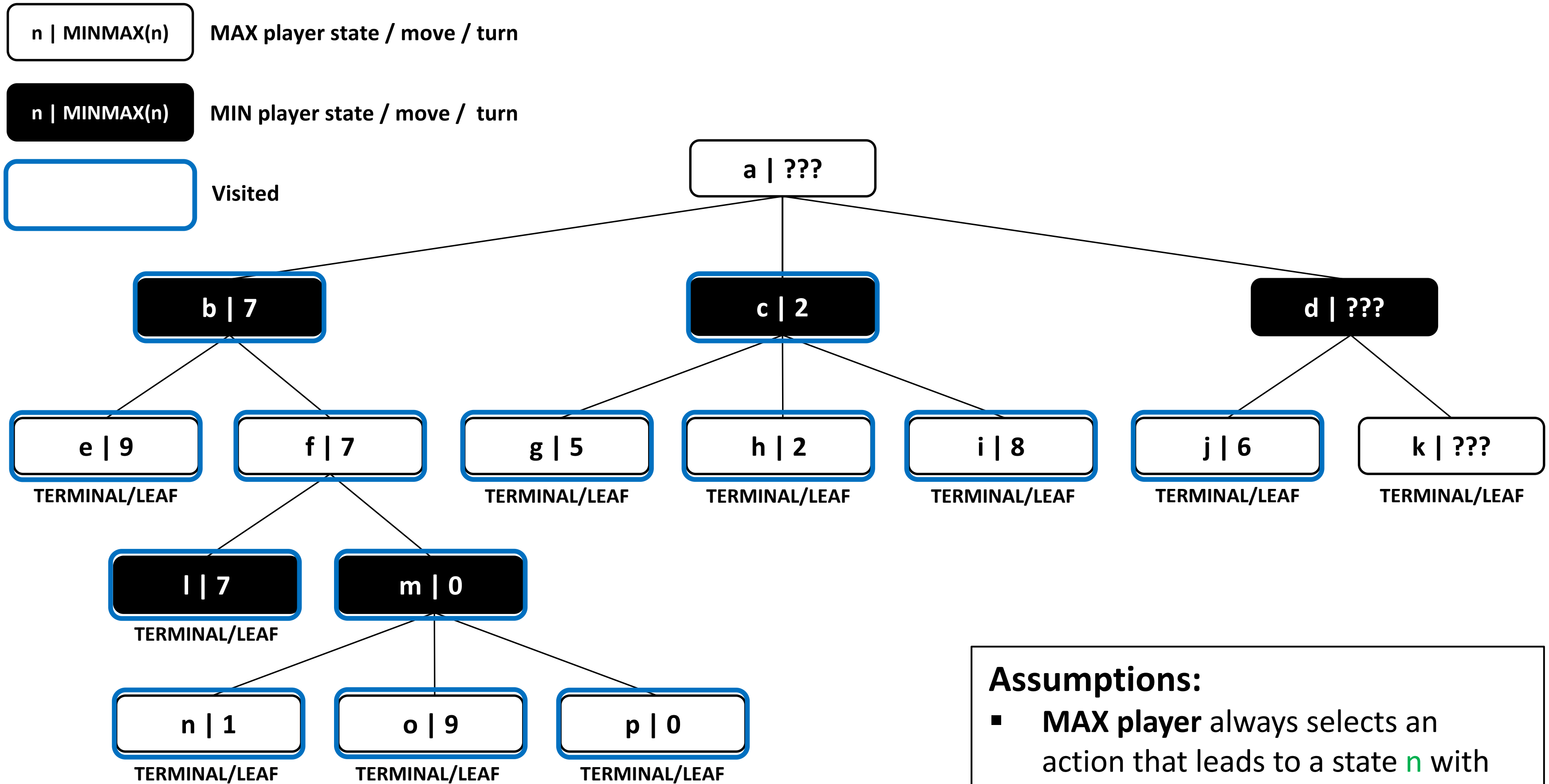

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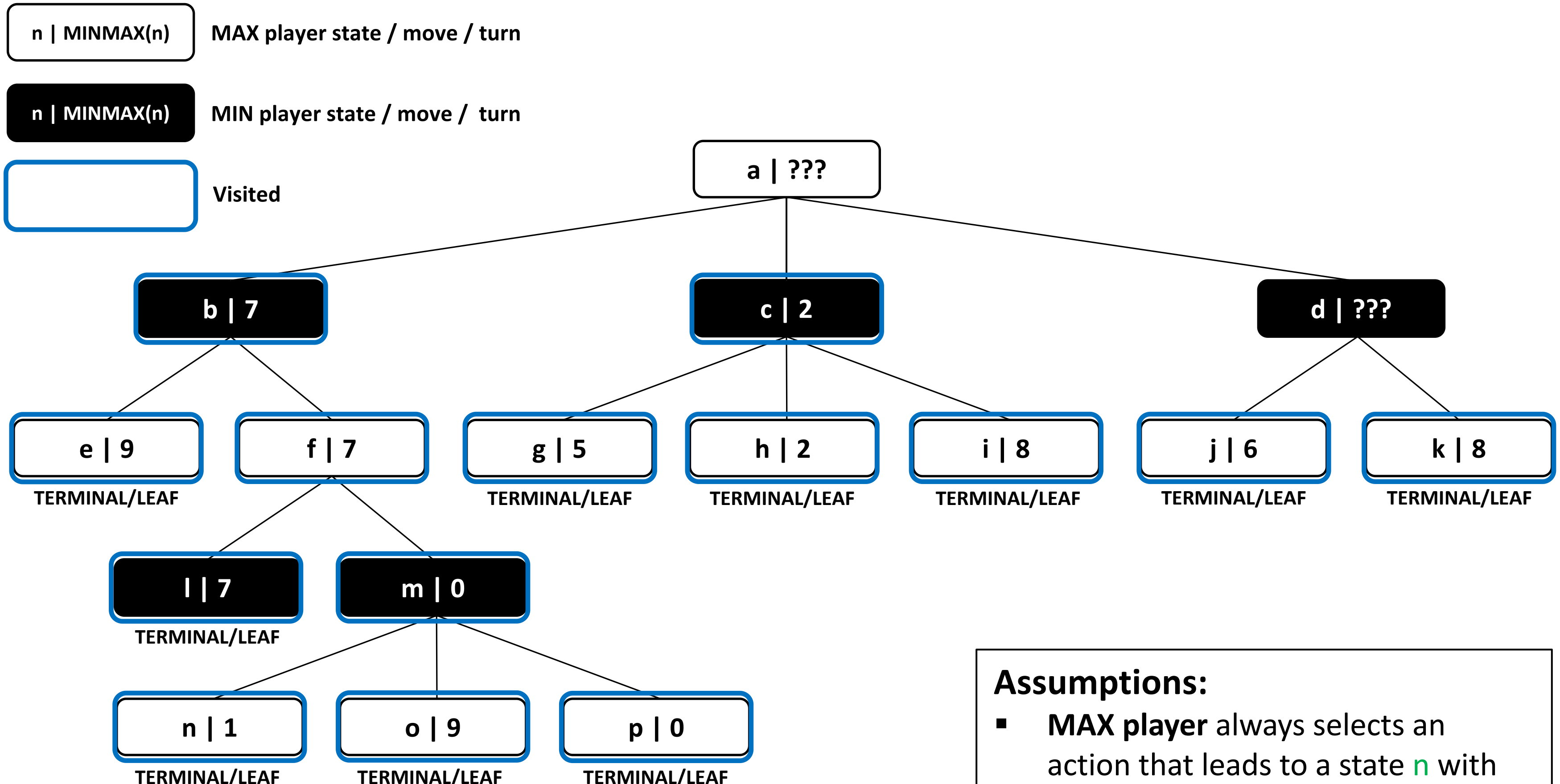

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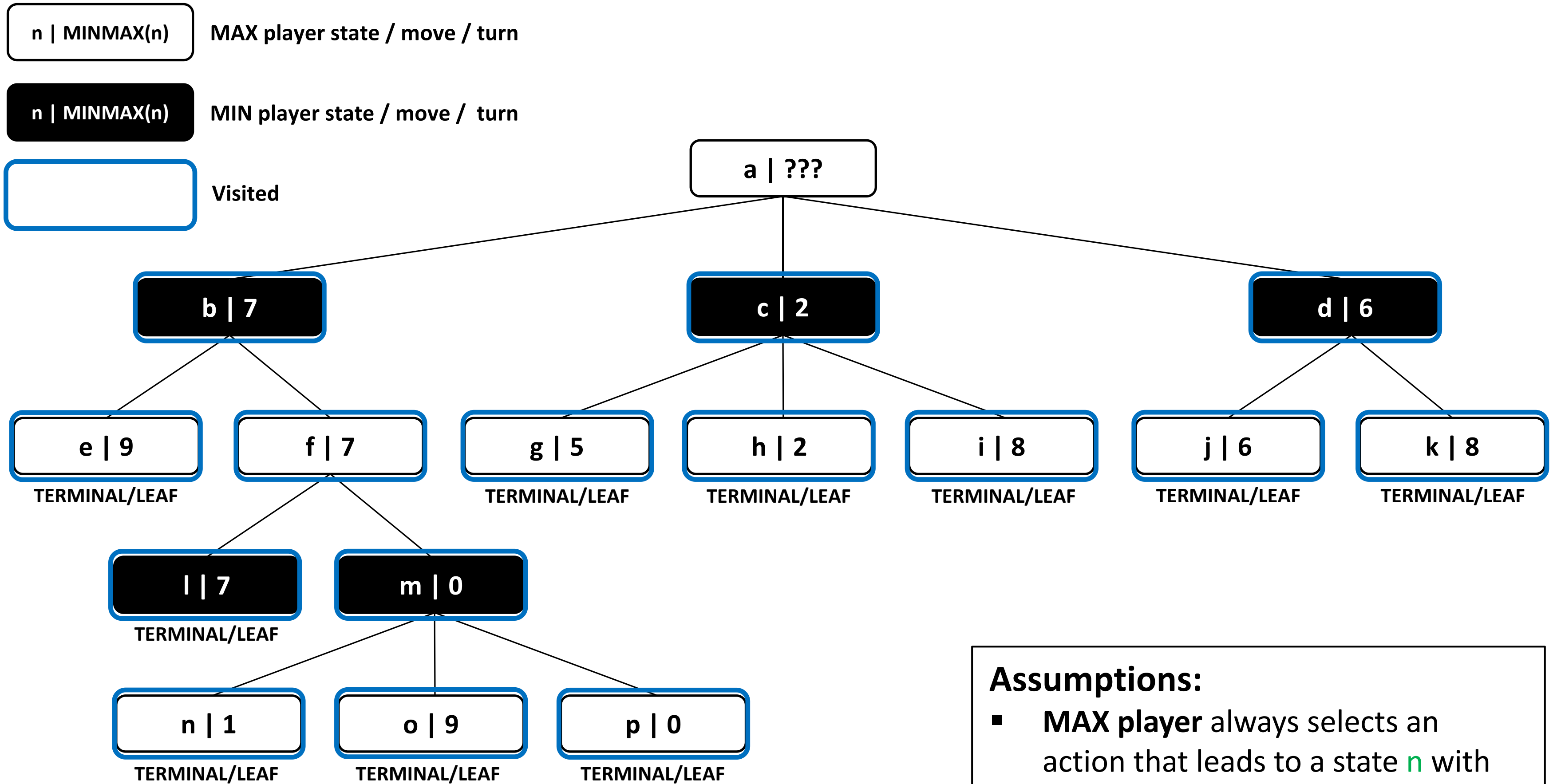

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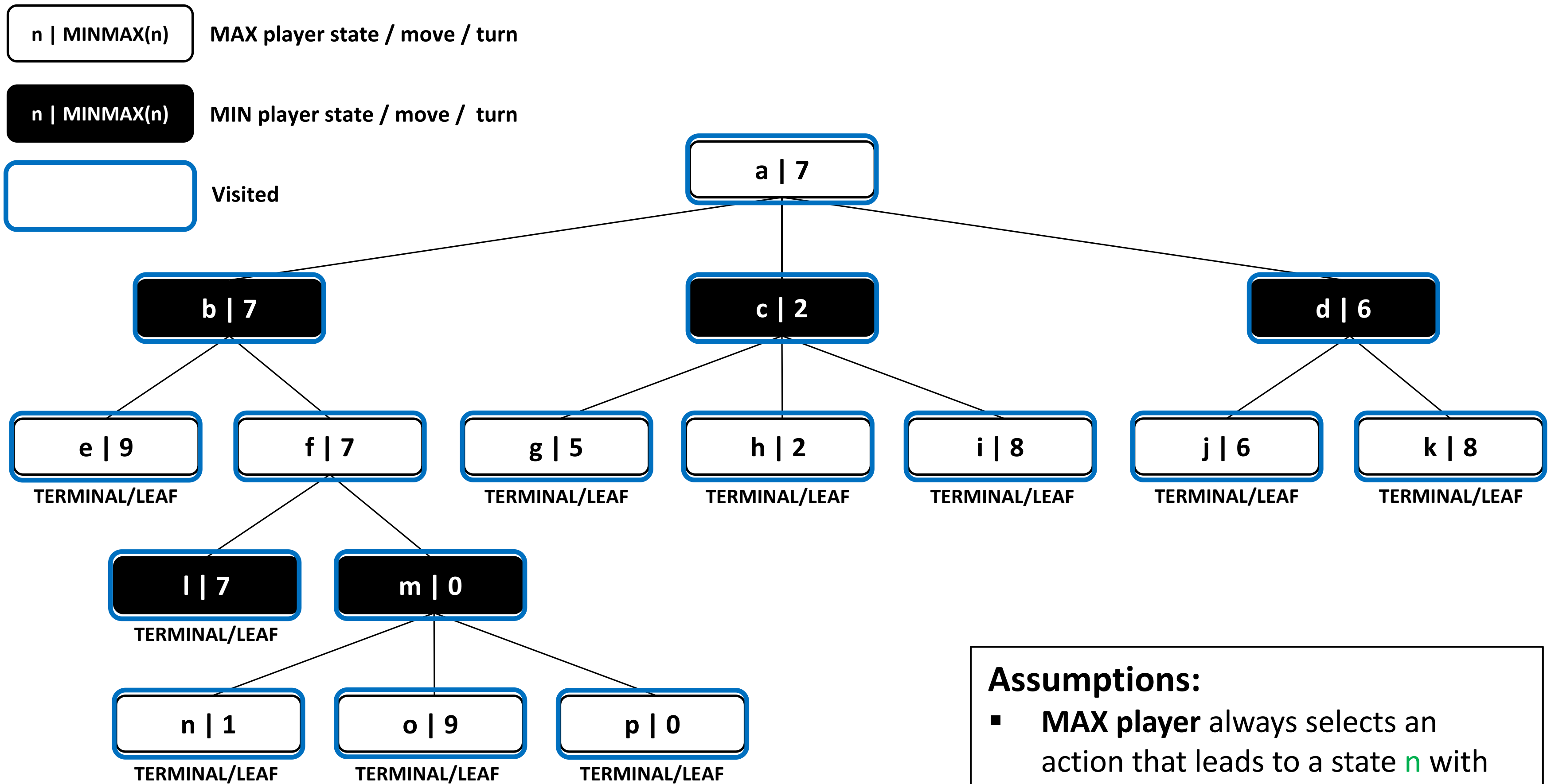

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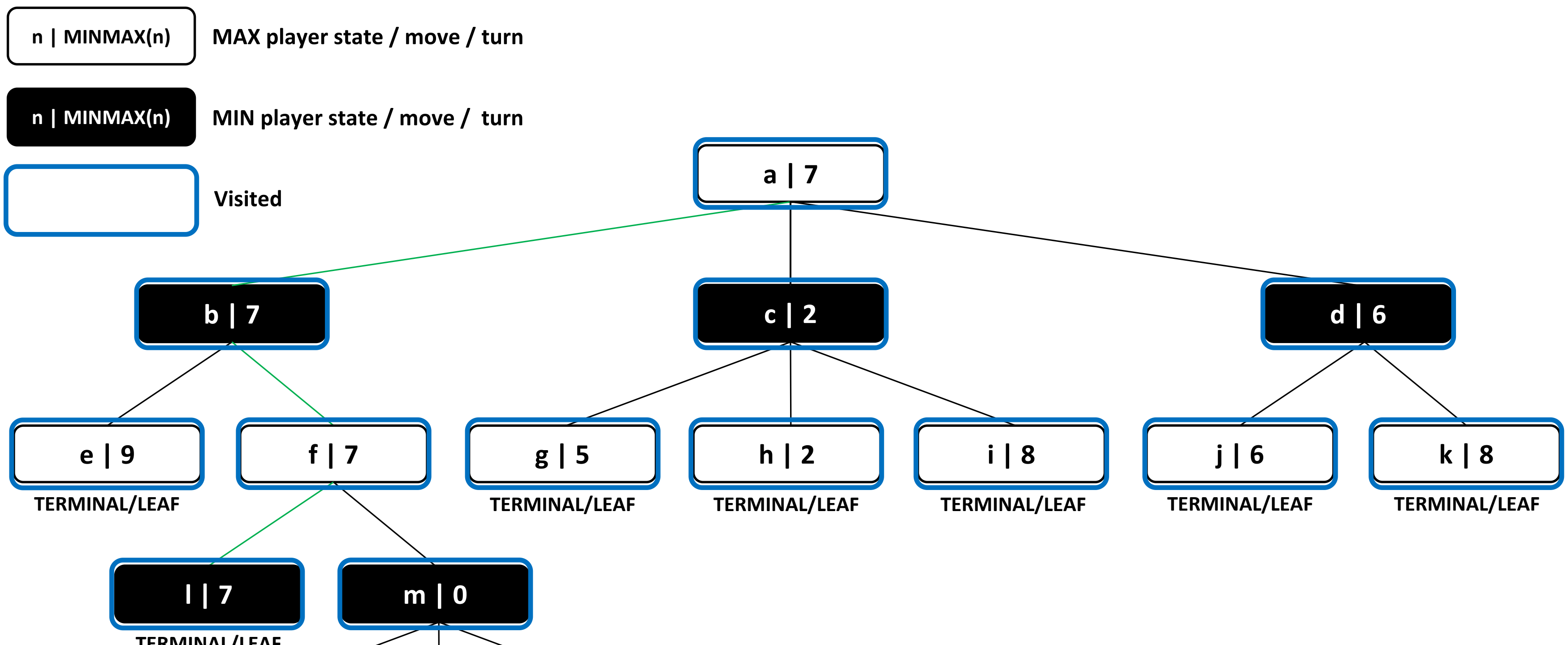

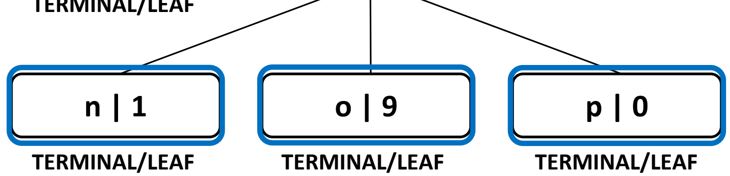

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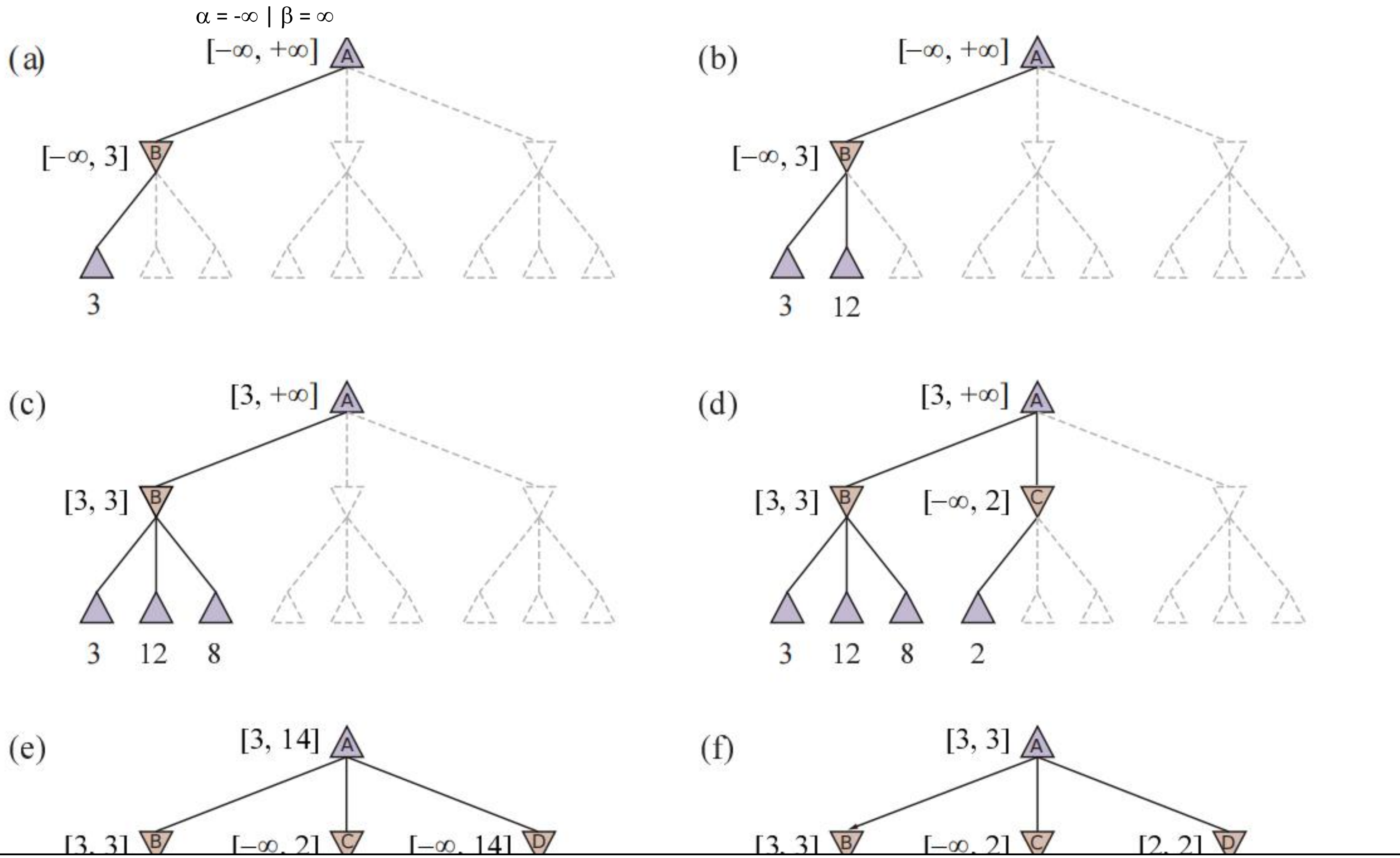
MinMax: What is the Challenge?

Example MinMax with α-β Pruning



α: the value of the best (highest-value) choice we have found so far at any choice point along the path for MAX player ("at least")
β: the value of the best (lowest-value) choice we have found so far at any choice point along the path for MIN player ("at most")

Example MinMax with α-β Pruning