

Language Models

CS-585
Natural Language Processing
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Probabilistic Language Models

- Today's goal: assign a probability to a sentence
 - Machine Translation:
 - » $P(\text{"high winds tonite"}) > P(\text{"large winds tonite"})$
 - Spell Correction & Predictive Text Completion
 - » The office is about fifteen minuets from my house
 - » $P(\text{about fifteen minutes from}) > P(\text{about fifteen minuets from})$
 - Speech Recognition
 - » $P(\text{"I saw a van"}) \gg P(\text{"eyes awe of an"})$
 - + Summarization, question-answering, etc., etc.

Probabilistic Language Modeling

- Goal: compute the probability of a sentence or sequence of words:

$$P(W) = P(w_1, w_2, w_3, w_4, w_5 \dots w_n)$$

- Related task: probability of an upcoming word:

$$P(w_5 | w_1, w_2, w_3, w_4)$$

- A model that computes either of these:

$$P(W) \quad \text{or} \quad P(w_n | w_1, w_2 \dots w_{n-1})$$

is called a **language model**.

How to compute $P(W)$

- How to compute this joint probability:
 - $P(\text{its, water, is, so, transparent, that})$
- Intuition: let's rely on the Chain Rule of Probability

Reminder: The Chain Rule

- Recall the definition of conditional probabilities

$$P(B|A) = \frac{P(A, B)}{P(A)}$$

Rewriting:

$$P(A, B) = P(A)P(B|A)$$

- More variables:

$$P(A, B, C, D) = P(A)P(B|A)P(C|A, B)P(D|A, B, C)$$

- The Chain Rule in General

$$P(x_1, x_2, x_3, \dots, x_n) = P(x_1)P(x_2|x_1)P(x_3|x_1, x_2) \dots P(x_n|x_1, \dots, x_{n-1})$$

Joint probability of words in sentence

$$P(w_1 w_2 \dots w_n) = \prod_i P(w_i | w_1 w_2 \dots w_{i-1})$$

$P(\text{"its water is so transparent"}) =$

- $P(\text{its}) \quad \times \quad P(\text{water} | \text{its})$
- $\times \quad P(\text{is} | \text{its water})$
- $\times \quad P(\text{so} | \text{its water is})$
- $\times \quad P(\text{transparent} | \text{its water is so})$

How to estimate these probabilities

- Could we just count and divide?

$P(\text{the lits water is so transparent that}) =$

$\frac{\text{Count}(\text{its water is so transparent that the})}{\text{Count}(\text{its water is so transparent that})}$

- No! Too many possible sentences!
- We'll never see enough data for estimating these

Markov Assumption



Andrei Markov

- Simplifying assumption:

$$P(\text{the lit water is so transparent that}) \approx P(\text{the lit that})$$

- Or maybe

$$P(\text{the lit water is so transparent that}) \approx P(\text{the lit transparent that})$$

Markov Assumption

$$P(w_1 w_2 \dots w_n) \approx \prod_i P(w_i | w_{i-k} \dots w_{i-1})$$

- In other words, we approximate each component in the product

$$P(w_i | w_1 w_2 \dots w_{i-1}) \approx P(w_i | w_{i-k} \dots w_{i-1})$$

Simplest case: Unigram model

$$P(w_1 w_2 \dots w_n) \approx \prod_i P(w_i)$$

Some automatically generated words:

fifth, an, of, future, inflation, most, dollars, quarter, in, is, mass

thrift, did, eighty, said, hard, 'm, july, bullish

that, or, limited, the

Zero-th order

Markov assumption



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Bigram model

- Condition on the previous word:

$$P(w_i | w_1 w_2 \dots w_{i-1}) \approx P(w_i | w_{i-1})$$

texaco, rose, one, in, this, issue, is, pursuing, growth, in,
a, boiler, house, said, mr
control, proposal, without
fifty, five, yen

First order Markov
assumption

outside, new, car, parking, lot, of, the, agreement, reached

this, would, be, a, record, november

N-gram models

- We can extend to trigrams, 4-grams, 5-grams
 - In general this is an insufficient model of language
 - because language has **long-distance dependencies**:
- “The **computer(s)** which I had just put into the machine room on the fifth floor **is (are)** crashing.”
- But we can often get away with N-gram models

ESTIMATING N-GRAM PROBABILITIES

Estimating bigram probabilities

- The Maximum Likelihood Estimate

$$P(w_i | w_{i-1}) = \frac{\text{count}(w_{i-1}, w_i)}{\text{count}(w_{i-1})}$$

$$P(w_i | w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

An example

$$P(w_i | w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

<s> I am Sam </s>

<s> Sam I am </s>

<s> I do not like green eggs and ham </s>

$$P(\text{I} | \text{<s>}) = \frac{2}{3} = .67 \quad P(\text{Sam} | \text{<s>}) = \frac{1}{3} = .33 \quad P(\text{am} | \text{I}) = \frac{2}{3} = .67$$

$$P(\text{</s>} | \text{Sam}) = \frac{1}{2} = 0.5 \quad P(\text{Sam} | \text{am}) = \frac{1}{2} = .5 \quad P(\text{do} | \text{I}) = \frac{1}{3} = .33$$

More examples: Berkeley Restaurant Project

- can you tell me about any good cantonese restaurants close by
- mid priced thai food is what i'm looking for
- tell me about chez panisse
- can you give me a listing of the kinds of food that are available
- i'm looking for a good place to eat breakfast
- when is caffe venezia open during the day

Raw bigram counts

- Out of 9222 sentences

“I want”: 827

“want I”: 2

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Raw bigram probabilities

- Normalize by unigrams:

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

- Result:

P(want I)	i	want	to	eat	chinese	food	lunch	spend	
P(I want)	i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011	
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087	
eat	0	0	0.0027	0	0.021	0.0027	0.056	0	
chinese	0.0063	0	0	0	0	0.52	0.0063	0	
food	0.014	0	0.014	0	0.00092	0.0037	0	0	
lunch	0.0059	0	0	0	0	0.0029	0	0	
spend	0.0036	0	0.0036	0	0	0	0	0	

Bigram estimates of sentence probabilities

$$P(< \text{s} > \mid \text{I want english food } < / \text{s} >) =$$

$$P(\text{I} \mid < \text{s} >)$$

$$\times P(\text{want} \mid \text{I})$$

$$\times P(\text{English} \mid \text{want})$$

$$\times P(\text{food} \mid \text{english})$$

$$\times P(< / \text{s} > \mid \text{food})$$

$$= .000031$$

What kinds of knowledge?

- $P(\text{english} \mid \text{want}) = .0011$
- $P(\text{chinese} \mid \text{want}) = .0065$
- $P(\text{to} \mid \text{want}) = .66$
- $P(\text{eat} \mid \text{to}) = .28$
- $P(\text{food} \mid \text{to}) = 0$
- $P(\text{want} \mid \text{spend}) = 0$
- $P(\text{i} \mid \langle s \rangle) = .25$

Practical Issues

- We do everything in log space
 - Avoid underflow
 - (also adding is faster than multiplying)

$$\log(p_1 \times p_2 \times p_3 \times p_4) = \log p_1 + \log p_2 + \log p_3 + \log p_4$$

Language Modeling Toolkits

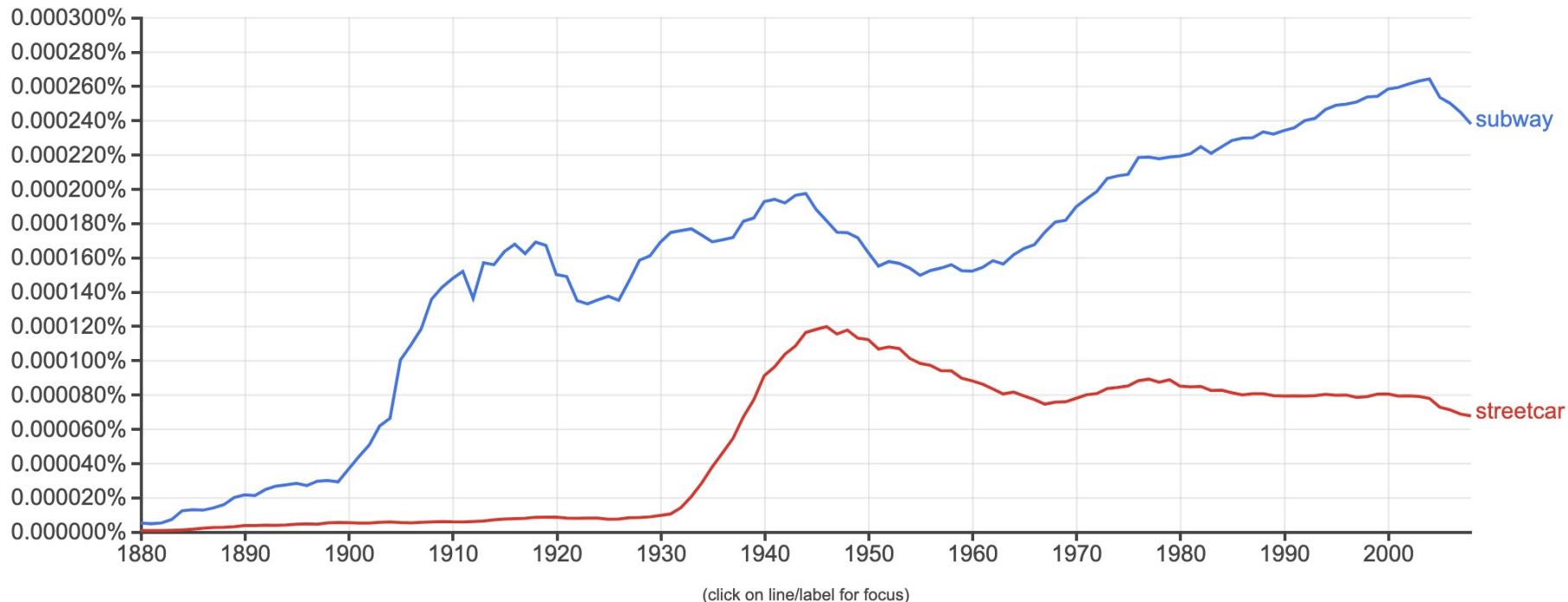
- SRILM
 - <http://www.speech.sri.com/projects/srilm/>
- KenLM
 - <https://kheafield.com/code/kenlm/>
- In nltk:
 - <http://www.nltk.org/api/nltk.model.html>

Google Ngrams

Ngram	Count
serve as the incoming	92
serve as the incubator	99
serve as the independent	794
serve as the index	223
serve as the indication	72
serve as the indication	120
serve as the indicator	45
serve as the indicators	111
serve as the indispensable	40
serve as the individual	234

Google Books Ngrams Viewer

- <https://books.google.com/ngrams>



(click on line/label for focus)

EVALUATION AND PERPLEXITY

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Evaluation: How good is a model?

- Does our language model prefer good sentences to bad ones?
 - Assign higher probability to “real” or “frequently observed” sentences
- Train model parameters on a **training set**.
- Test model performance on data not seen.
 - A **test set** is an unseen dataset that is different from our training set, totally unused.
 - An **evaluation metric** tells us how well our model does on the test set.

Training on the test set

- We can't allow test sentences into the training set
- We will assign it an artificially high probability when we put it in the test set
- “Training on the test set”
 - Bad science!

Extrinsic evaluation

- Best evaluation for comparing models A and B
 - Put each model in a task
 - spelling corrector, speech recognizer, MT system
 - Run the task, get an accuracy for A and for B
 - How many misspelled words corrected properly
 - How many words translated correctly
 - Compare accuracy for A and B

Difficulty of extrinsic evaluation

- Extrinsic evaluation
 - Time-consuming; can take days or weeks
- So
 - Sometimes use **intrinsic** evaluation: **perplexity**
 - Bad approximation
 - unless test data looks just like the training data
 - So generally only useful in pilot experiments
 - But helpful to think about.

Intuition of Perplexity

- The Shannon Game:

- How well can we predict the next word?

I always order pizza with cheese and _____

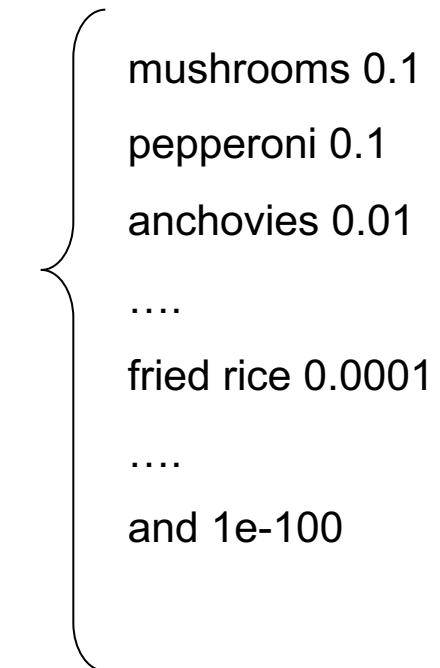
The 33rd President of the US was _____

I saw a _____

- Unigrams are terrible at this game. (Why?)

- A better model of a text

- is one which assigns a higher probability to the word that **actually occurs**



Perplexity

IDEA:

The best language model is one that best predicts an unseen test set...

- Gives the highest $P(\text{sentence})$

Perplexity

Per word perplexity = inverse probability of the test set, normalized by the number of words:

$$\begin{aligned} PP(W) &= P(w_1 w_2 \dots w_N)^{-\frac{1}{N}} \\ &= \sqrt[N]{\frac{1}{P(w_1 w_2 \dots w_N)}} \end{aligned}$$

Chain rule:

$$PP(W) = \sqrt[N]{\prod_{i=1}^N \frac{1}{P(w_i | w_1 \dots w_{i-1})}}$$

For bigrams:

$$PP(W) = \sqrt[N]{\prod_{i=1}^N \frac{1}{P(w_i | w_{i-1})}}$$

Minimizing perplexity is the same as maximizing probability

Perplexity as branching factor

- Let's suppose a sentence consisting of random digits
- What is the per-word perplexity of this sentence according to a model that assigns $P=1/10$ to each digit?

$$\begin{aligned}\text{PP}(W) &= P(w_1 w_2 \dots w_N)^{-\frac{1}{N}} \\ &= \left(\frac{1}{10}^N\right)^{-\frac{1}{N}} \\ &= \frac{1}{10}^{-1} \\ &= 10\end{aligned}$$

Lower perplexity = better model

- Training 38 million words, test 1.5 million words, WSJ

N-gram Order	Unigram	Bigram	Trigram
Per-word perplexity	962	170	109

GENERALIZATION AND ZEROS

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The Shannon Visualization Method

- Choose a random bigram ($< s >$, w) according to its probability
- Now choose a random bigram (w, x) according to its probability
- And so on until we choose $< /s >$
- Then string the words together

```
<s> I  
I want  
want to  
to eat  
eat Chinese  
Chinese food  
food </s>  
  
I want to eat Chinese food
```

Approximating Shakespeare

1
gram

–To him swallowed confess hear both. Which. Of save on trail for are ay device and
rote life have

–Hill he late speaks; or! a more to leg less first you enter

2
gram

–Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live
king. Follow.

–What means, sir. I confess she? then all sorts, he is trim, captain.

3
gram

–Fly, and will rid me these news of price. Therefore the sadness of parting, as they say,
'tis done.

–This shall forbid it should be branded, if renown made it empty.

4
gram

–King Henry. What! I will go seek the traitor Gloucester. Exeunt some of the watch. A
great banquet serv'd in;

–It cannot be but so.

Shakespeare as corpus

- $N=884,647$ tokens, $V=29,066$
- Shakespeare produced 300,000 bigram types out of $V^2= 844$ million possible bigrams.
 - So 99.96% of the possible bigrams were never seen (have zero entries in the table)
- Quadrigrams worse: What's coming out looks like Shakespeare because it *is* Shakespeare

The Wall Street Journal is not Shakespeare (no offense)

1
gram

Months the my and issue of year foreign new exchange's september were recession exchange new endorsed a acquire to six executives

2
gram

Last December through the way to preserve the Hudson corporation N. B. E. C. Taylor would seem to complete the major central planners one point five percent of U. S. E. has already old M. X. corporation of living on information such as more frequently fishing to keep her

3
gram

They also point to ninety nine point six billion dollars from two hundred four oh six three percent of the rates of interest stores as Mexico and Brazil on market conditions

Can you guess the “author” of this random 3-gram sentence?

“You are uniformly charming!” cried he,
with a smile of associating and now and
then I bowed and they perceived a chaise
and four to wish for.

The perils of overfitting

- N-grams only work well for word prediction if the test corpus looks like the training corpus
 - In real life, it often doesn't
 - We need to train robust models that generalize!
- One kind of generalization issue: Zeros!
 - Things that don't ever occur in the training set
 - ...but occur in the test set

...what will happen?

Zeros

Training set:

- ... denied the allegations
- ... denied the reports
- ... denied the claims
- ... denied the request

Test set

- ... denied the offer
- ... denied the loan

$$P(\text{"offer"} \mid \text{denied}) = 0$$

Zero probability bigrams

- Bigrams with zero probability
 - mean that we will assign **0** probability to the test set!
- And hence we cannot compute perplexity (can't divide by 0)!

SMOOTHING: ADD-ONE (LAPLACE) SMOOTHING

The intuition of smoothing (from Dan Klein)

- When we have sparse statistics:

$P(w \mid \text{denied the})$

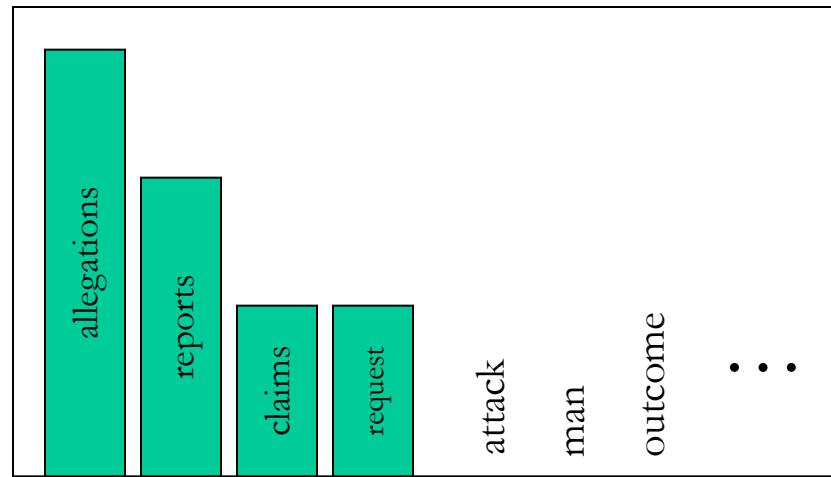
3 allegations

2 reports

1 claims

1 request

7 total



- Steal probability mass to generalize better

$P(w \mid \text{denied the})$

2.5 allegations

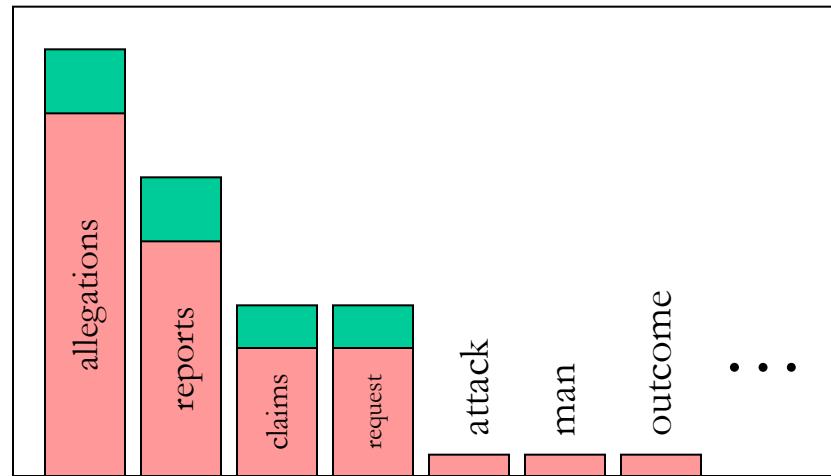
1.5 reports

0.5 claims

0.5 request

2 other

7 total



Add-one estimation

- Also called Laplace smoothing
- Pretend we saw each word one more time than we did
- Just add one to all the counts!
- MLE estimate:
- Add-1 estimate:

$$P_{MLE}(w_i | w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

$$P_{Add-1}(w_i | w_{i-1}) = \frac{c(w_{i-1}, w_i) + 1}{c(w_{i-1}) + V}$$



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Maximum Likelihood Estimates

- The maximum likelihood estimate
 - of some parameter of a model M from a training set T
 - maximizes the likelihood of the training set T given the model M
- Suppose the word “bagel” occurs 400 times in a corpus of a million words
- What is the probability that a random word from some other text will be “bagel”?
- MLE estimate is $400/1,000,000 = .0004$
- This may be a bad estimate for some other corpus
 - But it is the **estimate** that makes it **most likely** that “bagel” will occur 400 times in a million word corpus.

Berkeley Restaurant Corpus: Laplace smoothed bigram counts

	i	want	to	eat	chinese	food	lunch	spend
i	6	828	1	10	1	1	1	3
want	3	1	609	2	7	7	6	2
to	3	1	5	687	3	1	7	212
eat	1	1	3	1	17	3	43	1
chinese	2	1	1	1	1	83	2	1
food	16	1	16	1	2	5	1	1
lunch	3	1	1	1	1	2	1	1
spend	2	1	2	1	1	1	1	1

Laplace-smoothed bigrams

$$P^*(w_n|w_{n-1}) = \frac{C(w_{n-1}w_n) + 1}{C(w_{n-1}) + V}$$

	i	want	to	eat	chinese	food	lunch	spend
i	0.0015	0.21	0.00025	0.0025	0.00025	0.00025	0.00025	0.00075
want	0.0013	0.00042	0.26	0.00084	0.0029	0.0029	0.0025	0.00084
to	0.00078	0.00026	0.0013	0.18	0.00078	0.00026	0.0018	0.055
eat	0.00046	0.00046	0.0014	0.00046	0.0078	0.0014	0.02	0.00046
chinese	0.0012	0.00062	0.00062	0.00062	0.00062	0.052	0.0012	0.00062
food	0.0063	0.00039	0.0063	0.00039	0.00079	0.002	0.00039	0.00039
lunch	0.0017	0.00056	0.00056	0.00056	0.00056	0.0011	0.00056	0.00056
spend	0.0012	0.00058	0.0012	0.00058	0.00058	0.00058	0.00058	0.00058

Reconstituted counts

$$c^*(w_{n-1}w_n) = \frac{[C(w_{n-1}w_n) + 1] \times C(w_{n-1})}{C(w_{n-1}) + V}$$

	i	want	to	eat	chinese	food	lunch	spend
i	3.8	527	0.64	6.4	0.64	0.64	0.64	1.9
want	1.2	0.39	238	0.78	2.7	2.7	2.3	0.78
to	1.9	0.63	3.1	430	1.9	0.63	4.4	133
eat	0.34	0.34	1	0.34	5.8	1	15	0.34
chinese	0.2	0.098	0.098	0.098	0.098	8.2	0.2	0.098
food	6.9	0.43	6.9	0.43	0.86	2.2	0.43	0.43
lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16

Compare with raw bigram counts

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

	i	want	to	eat	chinese	food	lunch	spend
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eat	0.34	0.34	1	0.34	5.8	1	15	0.34
chinese	0.2	0.098	0.098	0.098	0.098	8.2	0.2	0.098
food	6.9	0.43	6.9	0.43	0.86	2.2	0.43	0.43
lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16

Add-1 estimation is a blunt instrument

- So add-1 isn't used for N-grams:
 - We'll see better methods
- But add-1 is used to smooth other NLP models
 - For text classification
 - In domains where the number of zeros isn't so huge.

INTERPOLATION, BACKOFF, AND WEB-SCALE LMS

Backoff and Interpolation

- Sometimes it helps to use **less context**
 - Condition on less context for contexts you haven't learned much about
- **Backoff:**
 - use trigram if you have good evidence,
 - otherwise bigram, otherwise unigram
- **Interpolation:**
 - mix unigram, bigram, trigram
- Interpolation works better

Linear Interpolation

- Simple interpolation

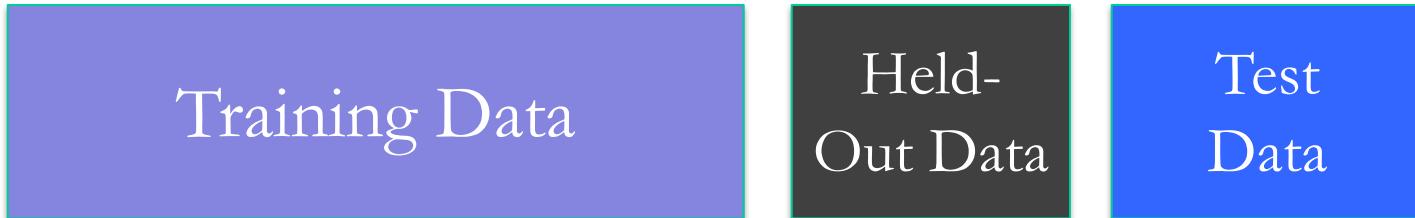
$$\hat{P}(w_n | w_{n-2} w_{n-1}) = \lambda_1 P(w_n | w_{n-2} w_{n-1}) + \lambda_2 P(w_n | w_{n-1}) + \lambda_3 P(w_n)$$
$$\sum_i \lambda_i = 1$$

- Lambdas conditional on context:

$$\hat{P}(w_n | w_{n-2} w_{n-1}) = \lambda_1(w_{n-2}^{n-1}) P(w_n | w_{n-2} w_{n-1}) + \lambda_2(w_{n-2}^{n-1}) P(w_n | w_{n-1}) + \lambda_3(w_{n-2}^{n-1}) P(w_n)$$

How to set the lambdas?

- Use a **held-out** corpus



- Choose λ s to maximize the probability of held-out data:
 - Fix the N-gram probabilities (on the training data)
 - Then search for λ s that give largest probability to held-out set:

$$\log P(w_1 \dots w_n | M(\lambda_1 \dots \lambda_k)) = \sum_i \log P_{M(\lambda_1 \dots \lambda_k)}(w_i | w_{i-1})$$

Unknown words: Open versus closed vocabulary tasks

- If we know all the words in advance, so the vocabulary V is fixed
 - Closed vocabulary task
- Often we don't know this
 - Out Of Vocabulary = OOV words
 - Open vocabulary task
- Instead: create an unknown word token <UNK>
 - Training of <UNK> probabilities
 - Create a fixed lexicon L of size V
 - At text normalization phase, any training word not in L changed to <UNK>
 - Now we train its probabilities like a normal word
 - At decoding time
 - If text input: Use UNK probabilities for any word not in training

Huge web-scale n-grams

- How to deal with, e.g., Google N-gram corpus
- Pruning
 - Only store N-grams with count > threshold.
 - Entropy-based pruning
- Efficiency
 - Efficient data structures like tries
 - Bloom filters: approximate language models
 - Store words as indexes, not strings
 - Use Huffman coding to fit large numbers of words into two bytes
 - Quantize probabilities (4-8 bits instead of 8-byte float)

Smoothing for Web-scale N-grams

- “Stupid backoff” (Brants *et al.* 2007)
- No discounting, just use relative frequencies

$$S(w_i | w_{i-k+1}^{i-1}) = \begin{cases} \frac{\text{count}(w_{i-k+1}^i)}{\text{count}(w_{i-k+1}^{i-1})} & \text{if } \text{count}(w_{i-k+1}^i) > 0 \\ 0.4S(w_i | w_{i-k+2}^{i-1}) & \text{otherwise} \end{cases}$$

$$S(w_i) = \frac{\text{count}(w_i)}{N}$$

N-gram Smoothing Summary

- Add-1 smoothing:
 - OK for text categorization, not for language modeling
- The most commonly used method:
 - Extended Interpolated Kneser-Ney
- For very large N-grams like the Web:
 - Stupid backoff

Advanced Language Modeling

- Discriminative models:
 - choose n-gram weights to improve a task, not to fit the training set
- Parsing-based models
- Caching Models
 - Recently used words are more likely to appear

$$P_{CACHE}(w | history) = \lambda P(w_i | w_{i-2} w_{i-1}) + (1 - \lambda) \frac{c(w \in history)}{|history|}$$

ADVANCED: KNESER-NEY SMOOTHING

Absolute discounting: just subtract a little from each count

- Suppose we wanted to subtract a little from a count of 4 to save probability mass for the zeros
- How much to subtract ?
- Church and Gale (1991)'s clever idea
- Divide up 22 million words of AP Newswire
 - Training and held-out set
 - for each bigram in the training set
 - see the actual count in the held-out set!
- It sure looks like $c^* = (c - .75)$

Bigram count in training	Bigram count in heldout set
0	.0000270
1	0.448
2	1.25
3	2.24
4	3.23
5	4.21
6	5.23
7	6.21
8	7.21
9	8.26

Absolute Discounting Interpolation

- Save ourselves some time and just subtract 0.75 (or some d)!

$$P_{\text{AbsoluteDiscounting}}(w_i | w_{i-1}) = \frac{c(w_{i-1}, w_i) - d}{c(w_{i-1})} + \lambda(w_{i-1}) P(w)$$

discounted bigram
 Interpolation weight
 unigram

- (Maybe keeping a couple extra values of d for counts 1 and 2)
 - But should we really just use the regular unigram $P(w)$?

Kneser-Ney Smoothing I

- Better estimate for probabilities of lower-order unigrams!
 - Shannon game: *I can't see without my reading Francisco glasses*?
 - “Francisco” is more common than “glasses”
 - ... but “Francisco” always follows “San”
- The unigram is useful just when we **haven't** seen this bigram!
- Instead of $P(w)$: “How likely is w ”
- $P_{\text{continuation}}(w)$: “How likely is w to appear as a novel continuation?”
 - For each word, count the number of bigram types it completes
 - Every bigram type was a novel continuation when first seen

$$P_{\text{CONTINUATION}}(w) \propto |\{w_{i-1} : c(w_{i-1}, w) > 0\}|$$

Kneser-Ney Smoothing II

- How many times does w appear as a novel continuation:

$$P_{CONTINUATION}(w) \propto |\{w_{i-1} : c(w_{i-1}, w) > 0\}|$$

- Normalized by the total number of word bigram types

$$\left| \{(w_{j-1}, w_j) : c(w_{j-1}, w_j) > 0\} \right|$$

$$P_{CONTINUATION}(w) = \frac{\left| \{w_{i-1} : c(w_{i-1}, w) > 0\} \right|}{\left| \{(w_{j-1}, w_j) : c(w_{j-1}, w_j) > 0\} \right|}$$

Kneser-Ney Smoothing III

- Alternative metaphor: The number of # of word types seen to precede w

$$|\{w_{i-1} : c(w_{i-1}, w) > 0\}|$$

- normalized by the # of words preceding all words:

$$P_{CONTINUATION}(w) = \frac{|\{w_{i-1} : c(w_{i-1}, w) > 0\}|}{\sum_{w'} |\{w'_{i-1} : c(w'_{i-1}, w') > 0\}|}$$

- A frequent word (Francisco) occurring in only one context (San) will have a low continuation probability

Kneser-Ney Smoothing IV

$$P_{KN}(w_i | w_{i-1}) = \frac{\max(c(w_{i-1}, w_i) - d, 0)}{c(w_{i-1})} + \lambda(w_{i-1}) P_{CONTINUATION}(w_i)$$

λ is a normalizing constant; the probability mass we've discounted

$$\lambda(w_{i-1}) = \frac{d}{c(w_{i-1})} |\{w : c(w_{i-1}, w) > 0\}|$$

the normalized discount

The number of word types that can follow w_{i-1}
= # of word types we discounted
= # of times we applied normalized discount

Kneser-Ney Smoothing: Recursive formulation

$$P_{KN}(w_i | w_{i-n+1}^{i-1}) = \frac{\max(c_{KN}(w_{i-n+1}^i) - d, 0)}{c_{KN}(w_{i-n+1}^{i-1})} + \lambda(w_{i-n+1}^{i-1}) P_{KN}(w_i | w_{i-n+2}^{i-1})$$

$$c_{KN}(\bullet) = \begin{cases} count(\bullet) & \text{for the highest order} \\ continuationcount(\bullet) & \text{for lower order} \end{cases}$$

Continuation count = Number of unique single word contexts for \bullet