CS 480

Introduction to Artificial Intelligence

October 7th, 2021

Announcements / Reminders

- **Midterm: October 14th!**
  - **Online (NOT Beacon) section: please make arrangements. Contact Mr. Charles Scott (scott@iit.edu) if in doubt**
- **Programming Assignment #01:**
  - **due: October 17th, 11:00 PM CST (will be extended!)**
- **Written Assignment #02:**
  - **due: October 15th, 11:00 PM CST**
- **Re-download the slides for exam preparation**
- **Grading TA assignment:**

  https://docs.google.com/spreadsheets/d/1Cav_GBTGC7fLGzxuBCAUmEuJYPeF-HMLCYvwPbq8Fus/edit?usp=sharing

Plan for Today

- **Predicate / First-Order Logic**

CORRECTION: Laws/Theorems

Equivalence	Law / Theorems
$p \vee q \Leftrightarrow q \vee p$ $p \wedge q \Leftrightarrow q \wedge p$	Commutative laws
$(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$ $(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$	Associative laws
$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$	Distributive laws
$\neg (p \wedge q) \Leftrightarrow \neg q \vee \neg p$ $\neg (p \vee q) \Leftrightarrow \neg q \wedge \neg p$	De Morgan's laws
$p \wedge (p \vee q) \Leftrightarrow p$ $p \vee (p \wedge q) \Leftrightarrow p$	Absorption laws
$\neg (\neg p) \Leftrightarrow p$	Double Negation law (involution)
$p \wedge p \Leftrightarrow p$ $p \vee p \Leftrightarrow p$	Idempotent laws
$p \vee \neg p \Leftrightarrow \mathsf{T}$	Law of Excluded Middle (Negation law)
$p \wedge \neg p \Leftrightarrow \bot$	Contradiction (Negation law)
$p \wedge \mathsf{T} \Leftrightarrow p$ $p \vee \bot \Leftrightarrow p$	Identity laws
$p \wedge \bot \Leftrightarrow \bot$ $p \vee \mathsf{T} \Leftrightarrow \mathsf{T}$	Domination laws
$\neg p \vee q \Leftrightarrow p \Rightarrow q$	Implication law
$p \Rightarrow q \Leftrightarrow \neg q \Rightarrow \neg p$	Contraposition law
$(p \wedge q) \vee (\neg q \wedge \neg p) \Leftrightarrow (p \Leftrightarrow q)$ $(p \Rightarrow q) \wedge (q \Rightarrow p) \Leftrightarrow (p \Leftrightarrow q)$	Equivalence law

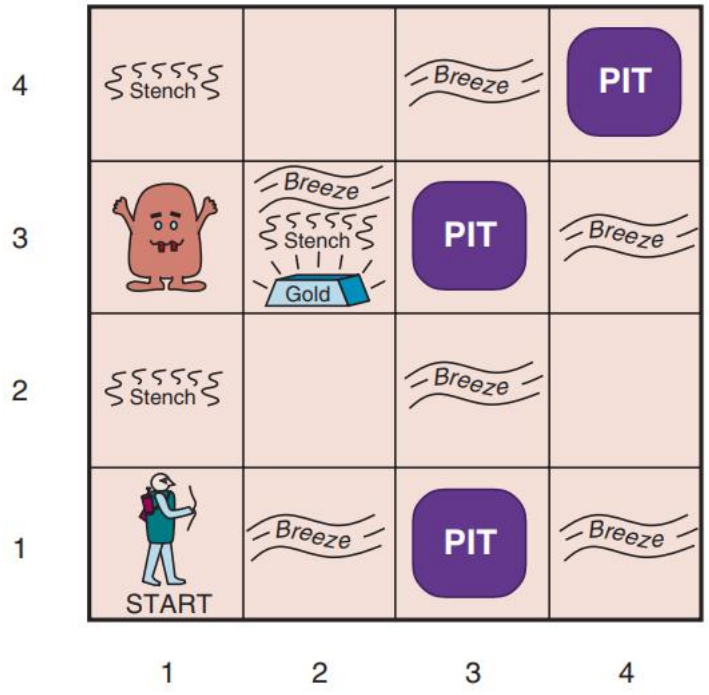
Written Assignment #02: Problem 3.1

Step	Resulting sentence	Applied law / rule
1	$(\neg(p \wedge q)) \Leftrightarrow (\neg p \vee \neg q)$ becomes: $(\neg p \vee \neg q) \Leftrightarrow (\neg p \vee \neg q)$	De Morgan's Law $\neg(a \wedge b) \equiv \neg a \vee \neg b$
2	$(\neg p \vee \neg q) \Leftrightarrow (\neg p \vee \neg q)$ becomes: $((\neg p \vee \neg q) \wedge (\neg p \vee \neg q)) \vee (\neg(\neg p \vee \neg q) \wedge \neg(\neg p \vee \neg q))$	Equivalence Law $(A \wedge B) \vee (\neg B \wedge \neg A) \equiv (A \Leftrightarrow B)$ Assume that $A \equiv \neg p \vee \neg q$, and $B \equiv \neg p \vee \neg q$
3	$((\neg p \vee \neg q) \wedge (\neg p \vee \neg q)) \vee (\neg(\neg p \vee \neg q) \wedge \neg(\neg p \vee \neg q))$ becomes: $((\neg p \vee \neg q) \wedge (\neg p \vee \neg q)) \vee (\neg(\neg p \vee \neg q))$	Idempotent Law $p \wedge p \equiv p$
4	$((\neg p \vee \neg q) \wedge (\neg p \vee \neg q)) \vee (\neg(\neg p \vee \neg q))$ becomes: $(\neg p \vee \neg q) \vee (\neg(\neg p \vee \neg q))$	Idempotent Law $p \wedge p \equiv p$
5	$(\neg p \vee \neg q) \vee (\neg(\neg p \vee \neg q))$ becomes: $(\neg p \vee \neg q) \vee \neg(\neg p \vee \neg q)$	Remove extra parentheses
6	$(\neg p \vee \neg q) \vee \neg(\neg p \vee \neg q)$ becomes: T	Law of Excluded Middle $A \vee \neg A \equiv T$ Assume that $A \equiv \neg p \vee \neg q$
7	So: $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q \equiv T$	We proved that $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$ is a tautology
Add more rows if necessary | Symbols (copy/paste): T⊥∨∧≡⇔¬⇒∴		

Propositional Logic Weaknesses

Wumpus World is represented by several variables in propositional logic:

- $B_{x,y}$: **there is a breeze in square (x,y)**
- $P_{x,y}$: **there is a pit in square (x,y)**
- $S_{x,y}$: **there is a stench in square (x,y)**
- $W_{x,y}$: **there is a wumpus in square (x,y)**
- $G_{x,y}$: **there is gold in square (x,y)**
- $L_{x,y}$: **agent is located in square (x, y)**



Either one of those can be either true or false. There is 4 x 4 variables for every type.

What if this Wumpus World was a N x N grid and N was a really large number. How many variables we would have?

Propositional Logic Weaknesses

Similarly, consider the following English sentence:

"robot R514 is standing at coordinates (123, 4501)"

It could be represented by a following propositional logic variable:

robot_R514_is_standing_at_coordinates_(123_4501)

and a true **/** false **value. What's the problem?**

Propositional Logic Weaknesses

How about relationships such as this one:

"robot R514 is to the left of robot R89"

It could be represented by a following propositional logic variable:

robot_R514_is_to_the_left_of_R89

and a true **/** false **value. What's the problem?**

Propositional Logic Weaknesses

How about relationships such as this one:

"robot R514 is to the left of robot R89"

It would be much easier to represent it in a form similar to this one:

toTheLeftOf(R514, R89)

Propositional Logic Weaknesses

Propositional logic:

- **CAN represent facts**
- **CANNOT represent objects**
- **CANNOT represent relationships between objects**
- **is overall not very expressive, for example it is impossible to transform this sentence into propositional logic:**

"Some people live in Illinois."

A More Expressive Formal Language

A more expressive formal language could be obtained by augmenting propositional logic with:

- **objects:**
  - **people, houses, humbers, theories, colors, basketball games, wars, etc.**
- **relations:**
  - **unary relations (properties): red, round, bogus, etc.**
  - **n-ary relations: brother of, bigger than, inside, part of, etc.**
- **functions:**
  - **relations in which there is only one "value" for a given "input"**
  - **father of, best friend, one more than, beginning of, etc.**

Predicate / First-Order Logic

Predicate Logic is an extension of Propositional Logic. It is a formal language in which propositions are expressed in terms of predicates, variables, and quantifiers.

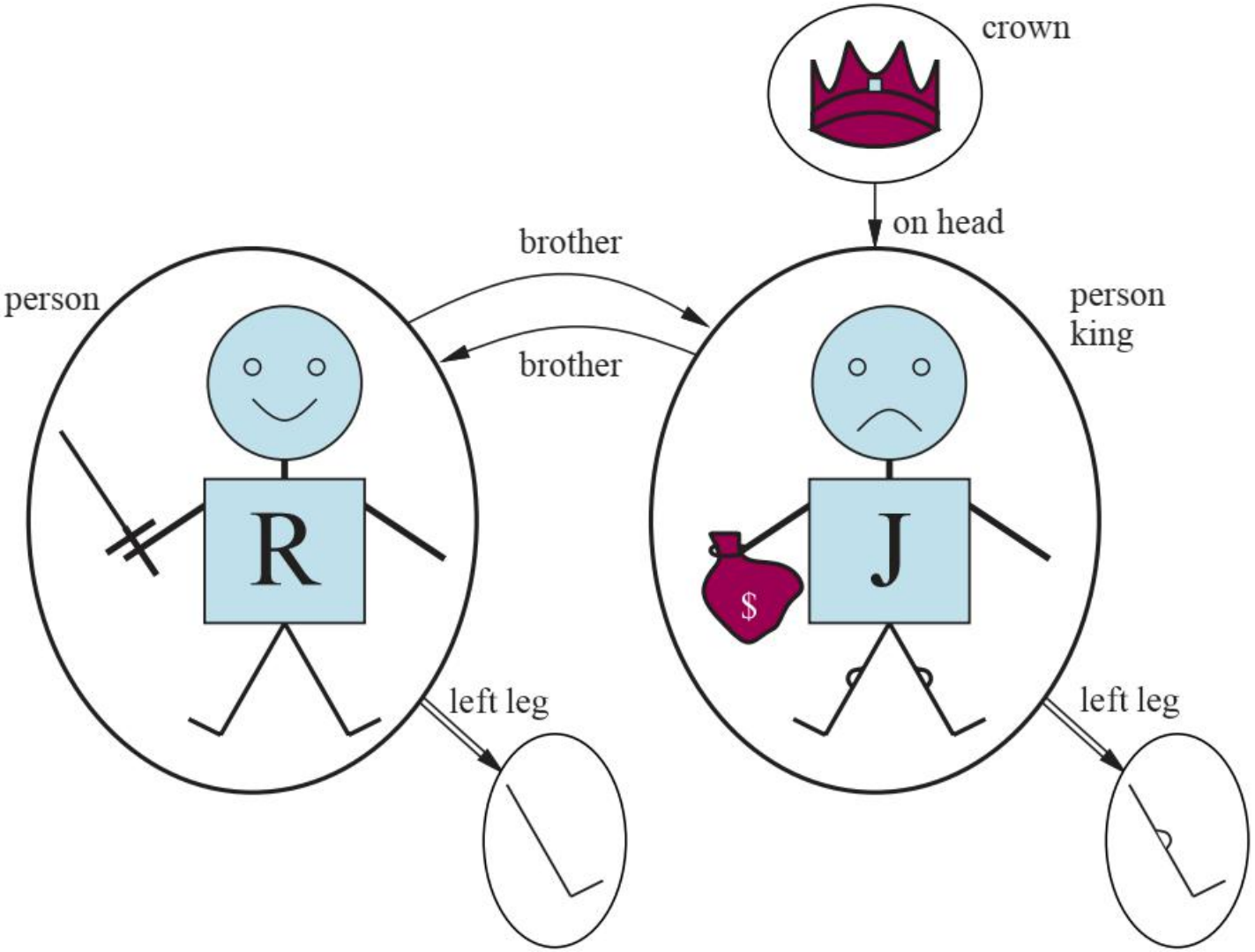
Predicates

In logic, a predicate is a symbol which represents a property or a relation. For example:

- **in the sentence $\mathrm{P(a)}$, the symbol P is a predicate which applies to the individual constant a,**
- **in the formula $\mathrm{R(a,b)}$ the symbol R is a predicate which applies to the individual constants a and b.**

In the semantics of logic, predicates are interpreted as relations.

Predicate Logic: Model of the World



Predicate Logic: Model and Objects

The domain of a model $\mathbb{D}$ is a nonempty set of domain elements it contains. Objects may be related.

Symbols: Refresher

Symbol	Name	Alternative symbols*	Should be read
$\neg$	Negation	$\sim$, !	not
$\wedge$	(Logical) conjunction	$\bullet$, &	and
$\vee$	(Logical) disjunction	+, $\|\|$	or
$\Rightarrow$	(Material) implication	$\rightarrow$, $\supset$	implies
$\Leftrightarrow$	(Material) equivalence	$\leftrightarrow$, $\equiv$, iff	if and only if
$\top$	Tautology	T, 1, $\blacksquare$	truth
$\bot$	Contradiction	F, 0, $\square$	falsum empty clause
$\forall$	Universal quantification		for all; for any; for each
$\exists$	Existential quantification		there exist

* you can encounter it elsewhere in literature

Predicate Logic Syntax: Symbols

Predicate calculus symbols include:

- **truth symbols: true and false**
- **constant symbols**
- **variable symbols**
- **predicate symbols**
- **function symbols**

Syntax: Constant

A constant c (c - symbol) represent objects

- **for example:**
  - lightSaber
  - kingJohn
  - lukeSkywalker
  - box
  - table
  - crown

Predicate Logic: Model and Objects

Domain D

lukeSkywalker

genghisKhan

wyattEarp

gun

sword

lightSaber

Existing constants in this representation are

D = {lukeSkywalker, genghisKhan, wyattEarp, gun, sword, lightSaber}

Syntax: Variable

A variable v (v - symbol) can be used to represent classes of objects or properties in the world.

- **for example:**
  - weapon
  - character
  - person
  - height
  - distance

Predicate Logic: Model and Objects

Domain D

lukeSkywalker

genghisKhan

wyattEarp

gun

sword

lightSaber

Possible variables for this model: person **and** weapon

D_{person} = {lukeSkywalker, genghisKhan, wyattEarp} D_{weapon} = {gun, sword, lightSaber}

Syntax: Predicate Symbols

A predicate p (p - symbol) **maps object(s) to a boolean value.**

p(argument$_1$, argument$_2$, ..., argument$_N$) **or just** p

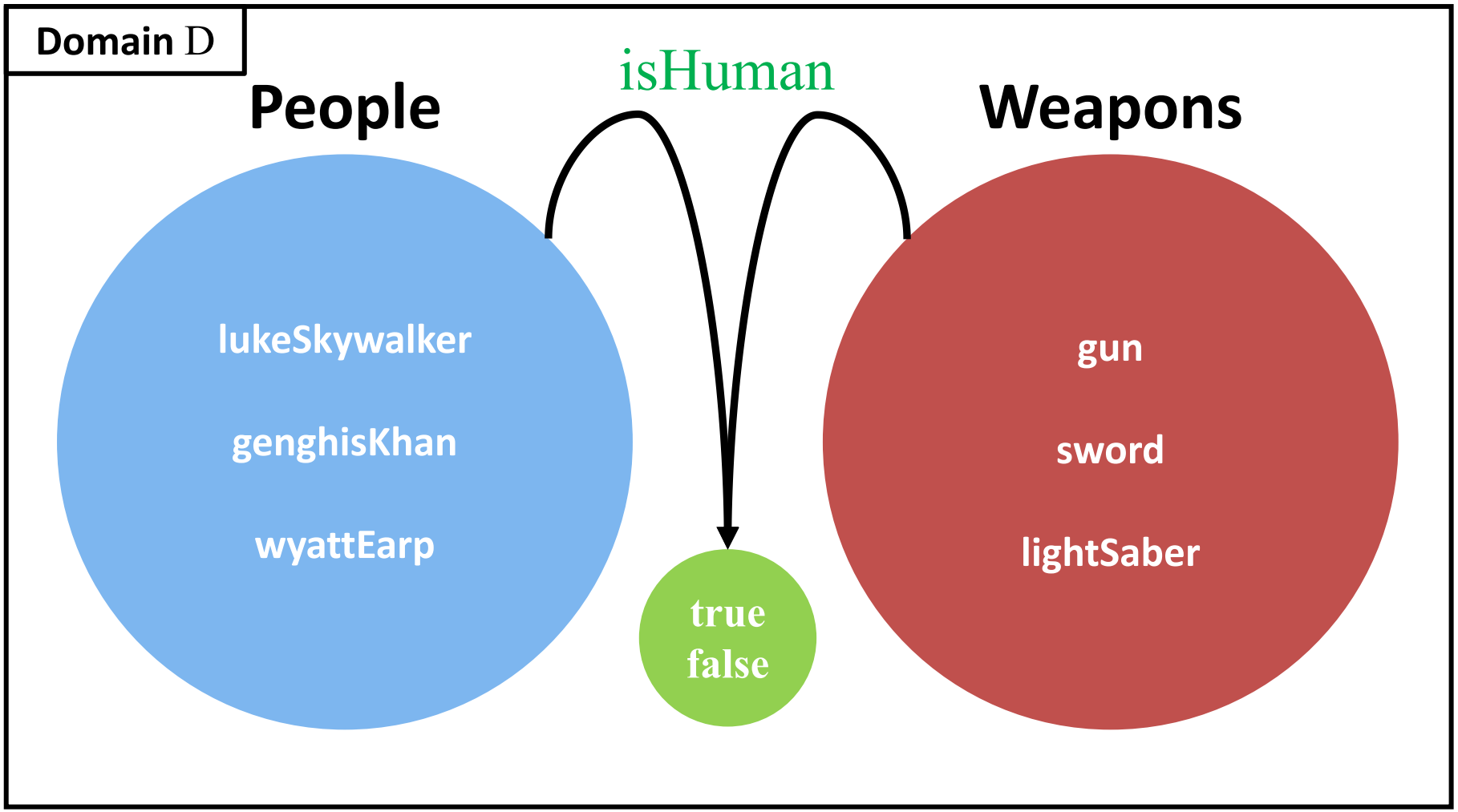
Predicates:

- **take objects as arguments**
- **output boolean values** true / false
- **for example:**

happy(student) **would output value** false **or** true

wasRaining(today) **would output value** true

Predicates: Example



isHuman(lukeSkywalker) = true

isHuman(sword) = false

Syntax: Function Symbols

A function f (f - symbol) maps object(s) to an object.

$f(argument_1, argument_2, ..., argument_N)$

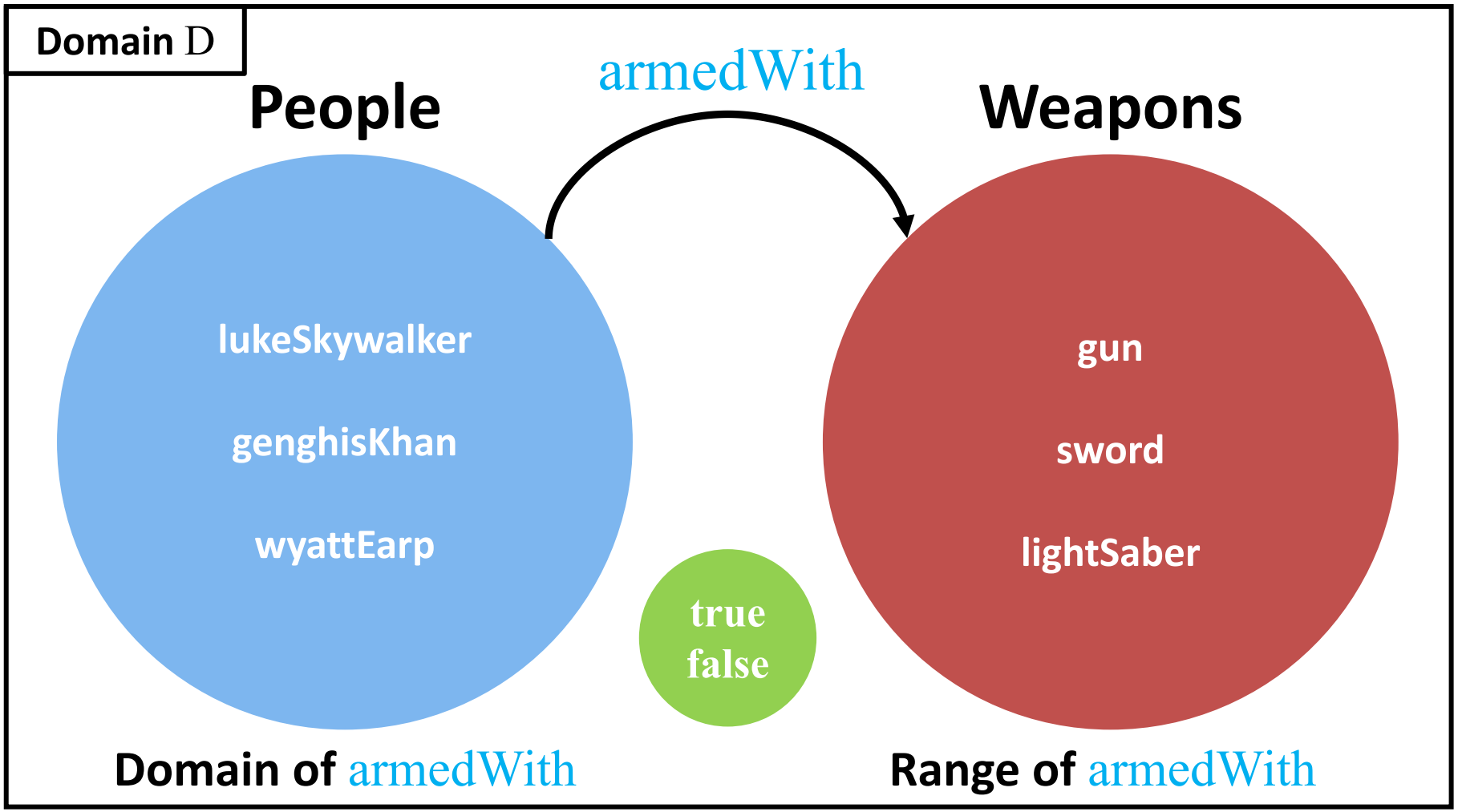
Unlike predicates, functions:

- **take objects as arguments**
- **output objects**
- **for example:**

currentCourse(student) **would output object** cs480

instructor(cs480) **would output object** Jacek

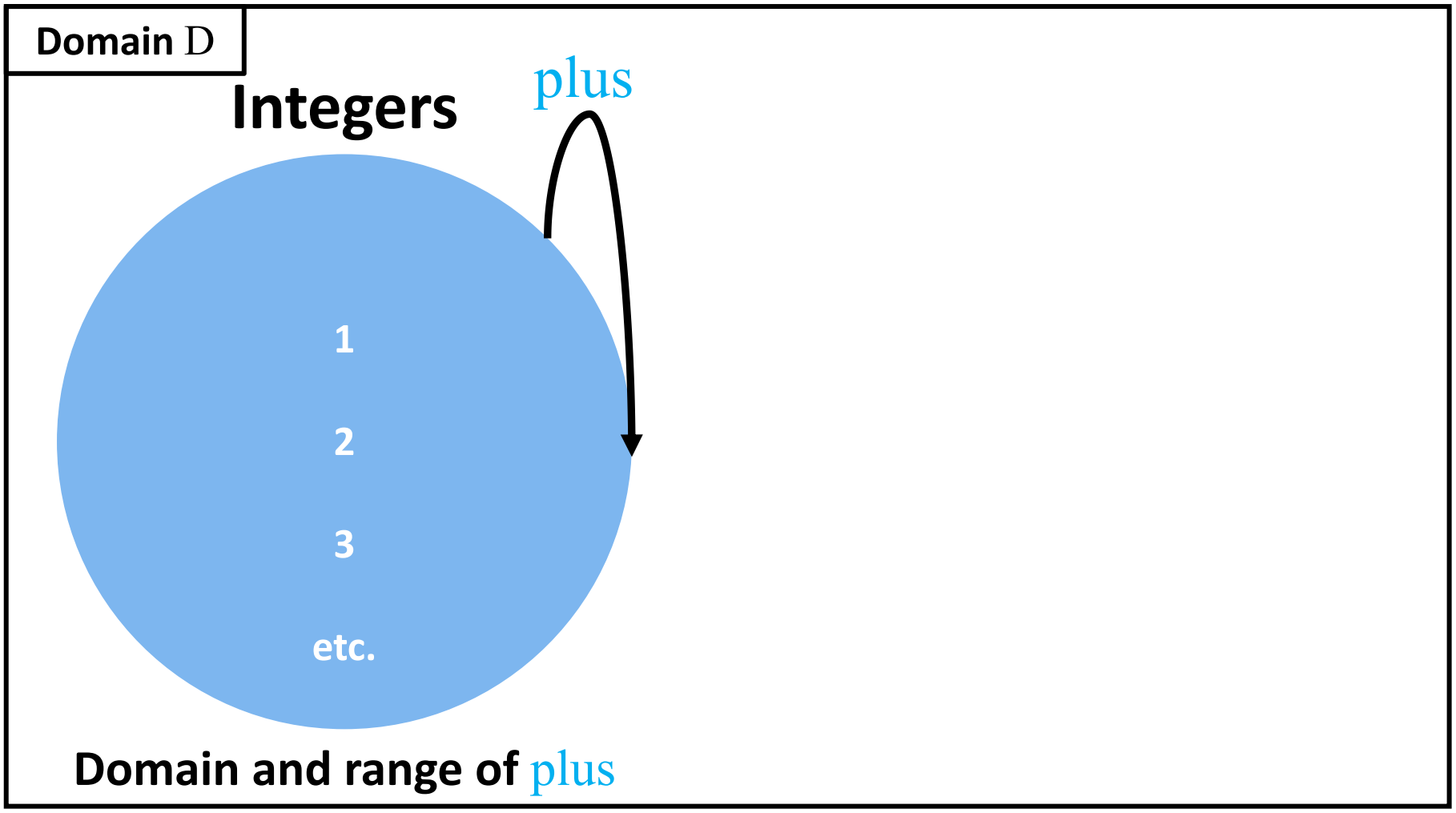
Functions: Example



armedWith(lukeSkywalker) = lightSaber

armedWith(genghisKhan) = sword

Functions: Example



$$\text{plus}(2,3) = 5$$

Syntax: Terms

A term is a logical expression that refers to an object. Terms can be:

- **constants:** lukeSkywalker
- **variables:** son
- **complex term - a function symbol followed by a parenthesized list of terms as arguments:**

father(lukeSkywalker) **or**

father(brother(princessLeia))

Both output an OBJECT: darthVader

Syntax: Complex Sentences

Complex sentences can be constructed using logical connectives (just like in propositional logic). For example:

$$\neg \text{brother}(\text{leftLeg}(\text{Richard}), \text{John})$$

$$\text{brother}(\text{Richard}, \text{John}) \wedge \text{brother}(\text{John}, \text{Richard})$$

$$\text{king}(\text{Richard}) \vee \text{king}(\text{John})$$

$$\neg \text{king}(\text{Richard}) \Rightarrow \text{king}(\text{John})$$

Syntax: Universal Quantifier

Quantifiers allow expressing properties of collections of objects.

Universal quantifier ("for all") indicates that a sentence is true for all possible values of the variable. For example:

$$\forall x \; likes(x, cake)$$

- $\forall$ **is the universal quantifier symbol**
- x **is a variable**
- $likes(x, cake)$ **is a sentence**

Syntax: Existential Quantifier

Quantifiers allow expressing properties of collections of objects.

Existential quantifier ("there exists") indicates that a sentence is true for at least one value of the the variable. For example:

$$\exists x \; likes(x, cake)$$

- $\exists$ **is the existential quantifier symbol**
- x **is a variable**
- $likes(x, cake)$ **is a sentence**

Syntax: Equality

In predicate logic we can ise the equality symbol to indicate that two terms refer to the same object.

For example:

$$\text{father}(\text{John}) = \text{Henry}$$

This sentence means that the output of function father(John) **is the same as object (constant)** Henry**.**

Syntax: Summary

$$
\begin{aligned}
\textit{Sentence} &\rightarrow \textit{AtomicSentence} \mid \textit{ComplexSentence} \\
\textit{AtomicSentence} &\rightarrow \textit{Predicate} \mid \textit{Predicate}(\textit{Term}, \ldots) \mid \textit{Term} = \textit{Term} \\
\textit{ComplexSentence} &\rightarrow (\ \textit{Sentence}\ ) \\
&\mid \neg\ \textit{Sentence} \\
&\mid \textit{Sentence} \wedge \textit{Sentence} \\
&\mid \textit{Sentence} \vee \textit{Sentence} \\
&\mid \textit{Sentence} \Rightarrow \textit{Sentence} \\
&\mid \textit{Sentence} \Leftrightarrow \textit{Sentence} \\
&\mid \textit{Quantifier}\ \textit{Variable}, \ldots\ \textit{Sentence} \\
\\
\textit{Term} &\rightarrow \textit{Function}(\textit{Term}, \ldots) \\
&\mid \textit{Constant} \\
&\mid \textit{Variable} \\
\\
\textit{Quantifier} &\rightarrow \forall \mid \exists \\
\textit{Constant} &\rightarrow A \mid X_1 \mid \textit{John} \mid \cdots \\
\textit{Variable} &\rightarrow a \mid x \mid s \mid \cdots \\
\textit{Predicate} &\rightarrow \textit{True} \mid \textit{False} \mid \textit{After} \mid \textit{Loves} \mid \textit{Raining} \mid \cdots \\
\textit{Function} &\rightarrow \textit{Mother} \mid \textit{LeftLeg} \mid \cdots \\
\\
\textsc{Operator Precedence} &: \neg, =, \wedge, \vee, \Rightarrow, \Leftrightarrow
\end{aligned}
$$

Predicate Calculus: Interpretation

Let the domain D be a non-empty set. An *interpretation* I over D is an assignment of the entities in D to each of the constant, variable, predicate, and function symbols of a predicate calculus expression, such that:

- **each constant is assigned an element of D,**
- **each variable is assigned to non-empty subset of D; these are the allowable substitutions for that variable,**
- **each function f of arity m is defined on m arguments of D and defines mapping from D^m into D,**
- **each predicate p of arity n is defined on n arguments from D and defines a mapping from D^n into $\{true, false\}$.**

Predicate Calculus: Sentences

Predicate calculus sentences are created according to following rules:

- **every atomic sentence** s **is a sentence**
- **if** s **a sentence, so it its negation** $\neg s$**,**
- **if** s_1 **and** s_2 **are sentences, then so is their conjunction** $s_1 \wedge s_2$**,**
- **if** s_1 **and** s_2 **are sentences, then so is their disjunction** $s_1 \vee s_2$**,**
- **if** s_1 **and** s_2 **are sentences, then so is their implication** $s_1 \Rightarrow s_2$**,**
- **if** s_1 **and** s_2 **are sentences, then so is their equivalence** $s_1 \Rightarrow s_2$**,**
- **if** x **is a variable** s **a sentence, then** $\forall x$ **is a sentence,**
- **if** x **is a variable** s **a sentence, then** $\exists x$ **is a sentence.**

Predicate Calculus: Evaluation

Assume a sentence E and an interpretation I for E over a non-empty domain D. The truth value for E is determined by:

- **the value of each constant is the element of D it is assigned to by I,**
- **the value of a variable is the set of elements of D it is assigned to by I,**
- **the value of a function expression is that element of D obtained by evaluating the function for the parameter values I assigned ,**
- **the value of "true" is true and "false" is false,**
- **The value of an atomic sentence is either true or false, as determined by the interpretation I.**

Predicate Calculus: Evaluation

- **the value of negation of a sentence is** true **if the value of the sentence is** false **(is false if the value of the sentence is** true**),**
- **the value of the conjunction of two sentences is** true **if the value of both sentences is** true **and is** false **otherwise**
- **similarly, the truth value of expressions using** $\vee$, $\Rightarrow$, **and** $\Leftrightarrow$ **is determined using related truth tables,**

- **The value of** $\forall x\ s$ **is** true **if** s **is** true **for all assignments to** x **under** I**, and it is** false **otherwise,**
- **The value of** x s **is** true **if** s **is** true **if there is an assignment to** x **under** I**, and it is** false **otherwise,**

First-Order Logic Sentences: Examples

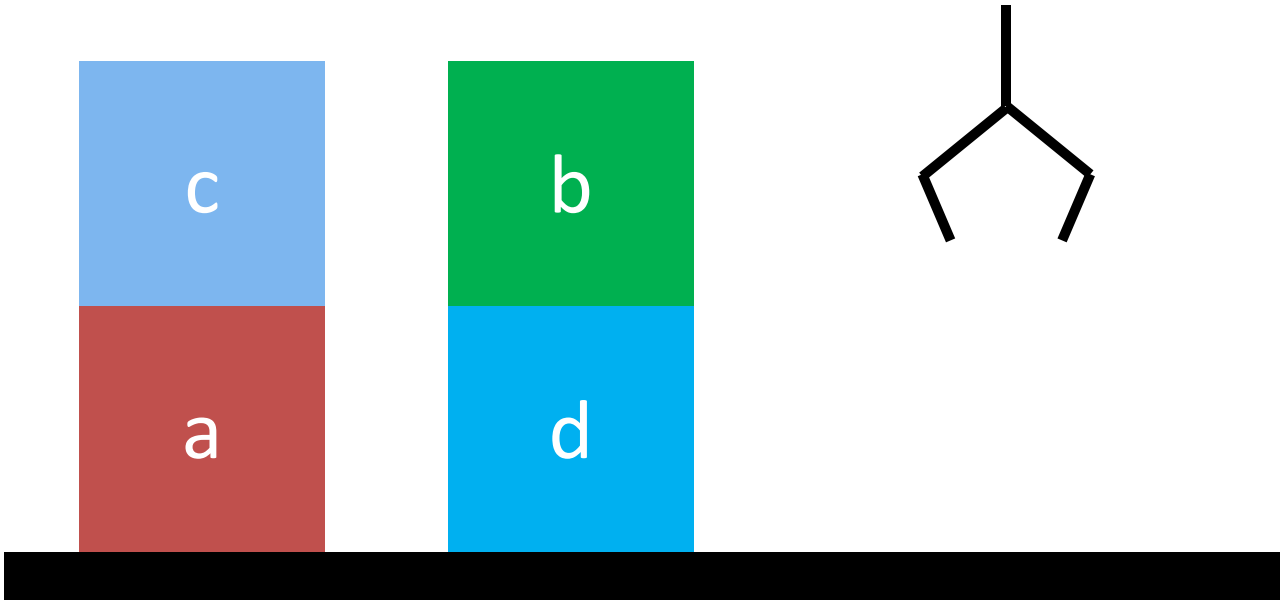
Sentence in First-Order Logic	Sentence in English
$\forall x\ frog(x) \Rightarrow green(x)$	All frogs are green. (For all xs being a frog implies being green)
$\forall x\ frog(x) \wedge brown(x) \Rightarrow big(x)$	All brown frogs are big. (For all xs being a frog and brown implies being big)
$\forall x\ likes(x, cake)$	Everyone likes cake. (All xs like cake.)
$\neg\forall x\ likes(x, cake)$	Not everyone likes cake. (Not all xs like cake.)
$\exists x\ \forall y\ likes(y, x)$	There is something that everyone likes. (There exists something x that all ys like.)
$\exists x\ \forall y\ likes(x, y)$	There is someone that likes everyone. (There exists an x that is likes all ys.)
$\forall x\ \exists y\ likes(y, x)$	Everything is liked by someone. (For all xs there exists a y that likes them.)
$\forall x\ \exists y\ likes(x, y)$	Everyone likes something. (For all xs there exists a y that is being liked by x.)

First-Order Logic Sentences: Examples

Sentence in First-Order Logic	Sentence in English
$\forall x\ customer(x) \Rightarrow likes(bob, x)$	Bob likes every customer. (For all xs being a customer implies being liked by Bob.)
$\exists x\ customer(x) \wedge likes(x, bob)$	There is a customer who likes Bob. (There exist an x who is a customer and likes Bob.)
$\exists x\ baker(x) \wedge \forall y\ customer(y) \Rightarrow likes(x, y)$	There is a baker who likes all customers. (There exist an x who is a baker such that for all ys being a customer, implies that y is liked by x.)
$\forall x\ older(mother(x), x)$ *	Every mother is older then her child. (For all xs, mother of x is older than x.)
$\forall x\ older(mother(mother(x)), x)$ *	Every grandmother is older then her daugther's child. (For all xs, mother of mother of x is older than x.)
$brother(x, bob) \Rightarrow \exists y\ parent(y, x) \wedge parent(y, bob)$	If x is Bob's brother, x and Bob must have the same parent.
$\forall x \forall y \forall z\ f(x, y) \wedge f(y, z) \Rightarrow f(x, z)$	f is a transitive relation.

* mother is function symbol here

Predicate Calculus: Model Example

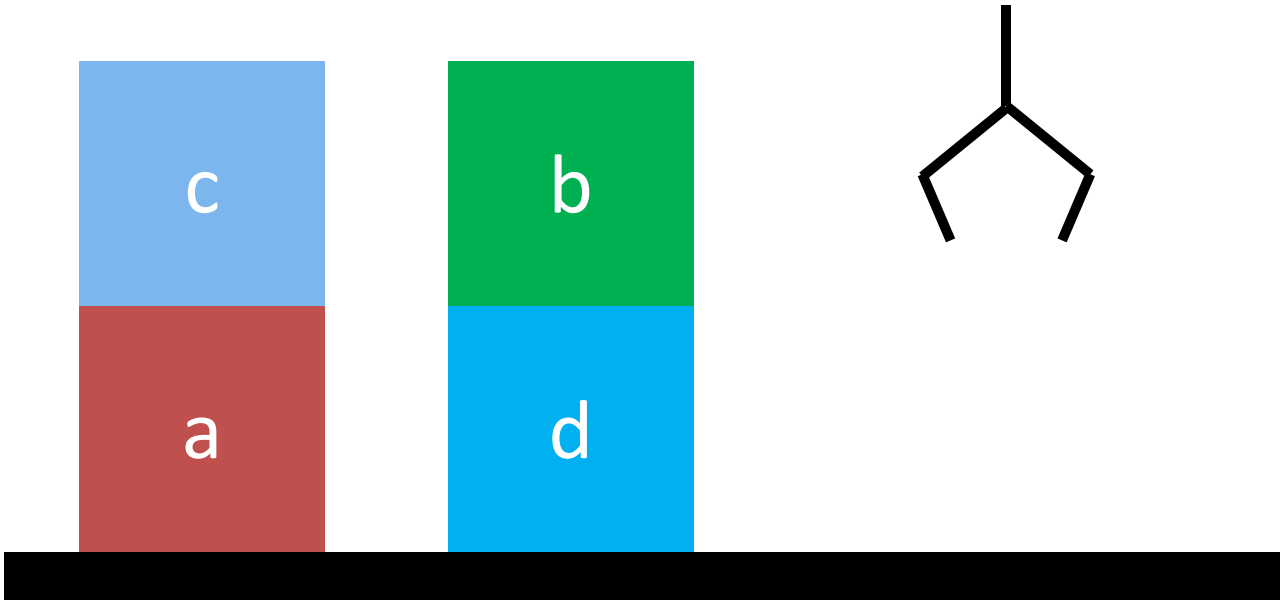


Predicate calculus description of the world:

on(c, a)
on(b, d)
onTable(a)
onTable(d)

clear(b)
clear(c)
manipulatorEmpty

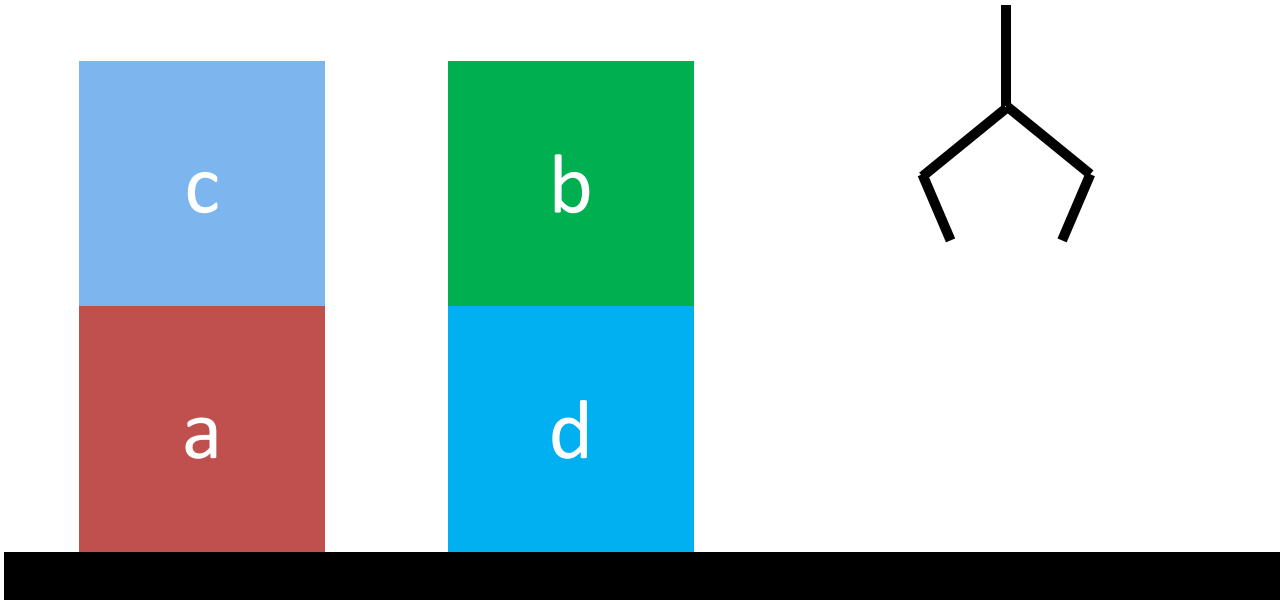
Predicate Calculus: Objects



Objects:

table
blocks a, b, c, d
manipulator

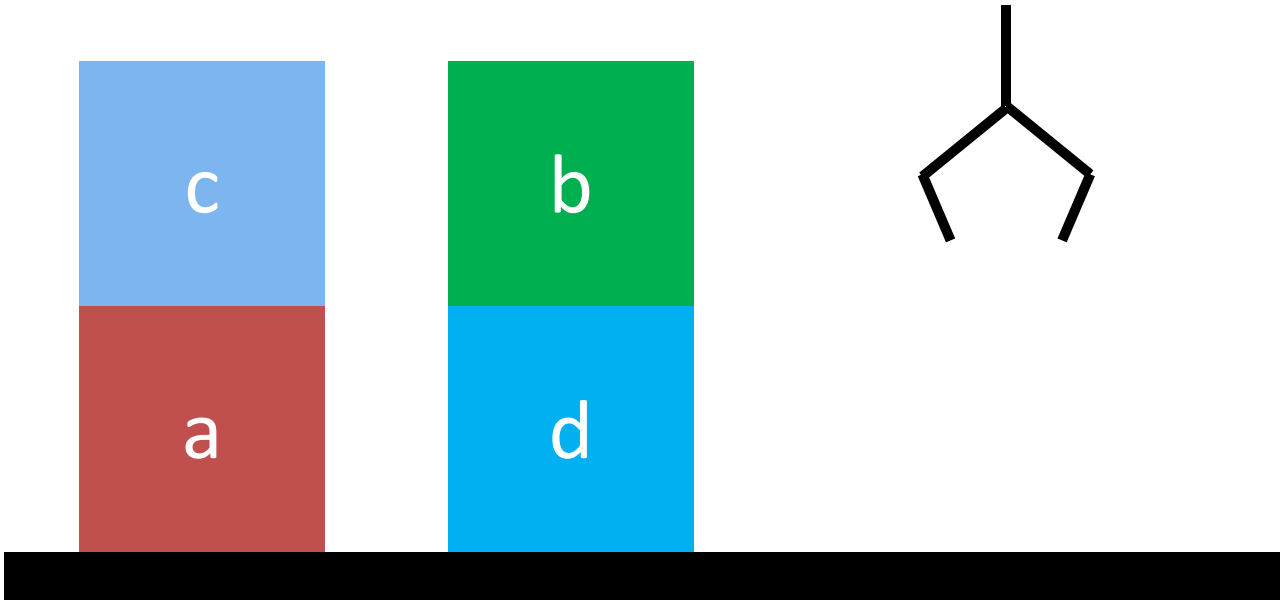
Predicate Calculus: Predicates



Predicates:

on(x, y)	**asserts that object x is on top of y**
onTable(x)	**asserts that object x is on the table**
clear(x)	**asserts that no objects is on top of x**
manipulatorEmpty	**asserts the status of the manipulator**

Model of the World



Predicate calculus description of the world (model):

on(c, a)
on(b, d)
onTable(a)
onTable(d)

clear(b)
clear(c)
manipulatorEmpty