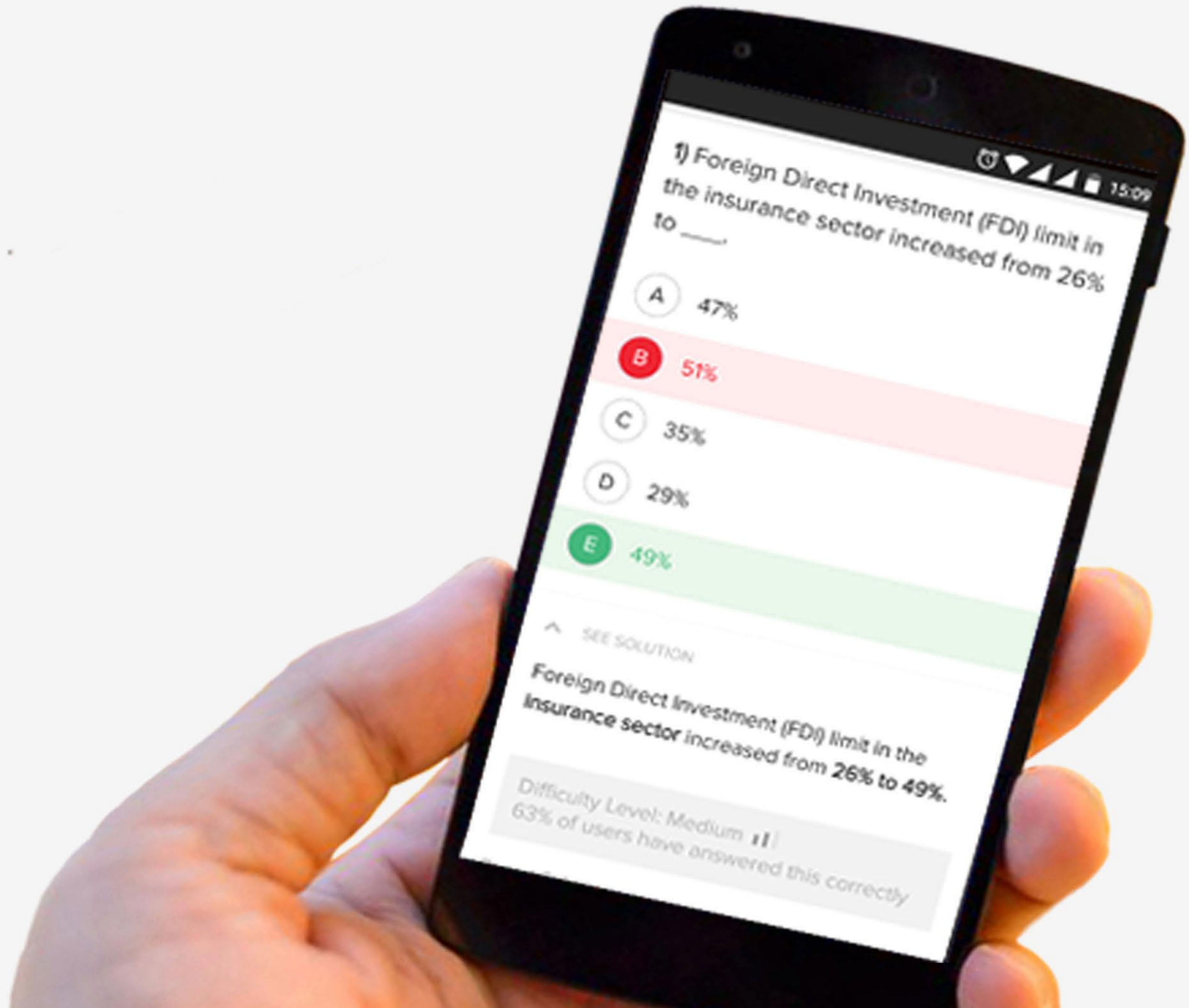
Formulas for Networks

for GATE EC Exam



Network Theory

Current: Electric current is the time rate of change of charge flow.

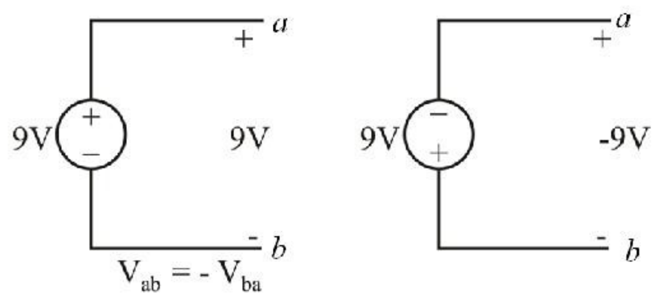
$$i = \frac{dq}{dt} \quad \text{(Ampere)}$$

- Charge transferred between time t_o and t

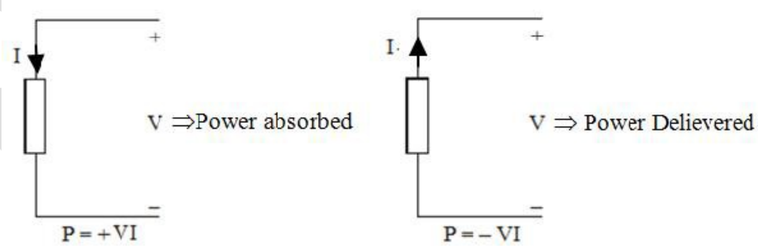
$$q = \int_{t_o}^{t} i\,dt$$

Sign Convention: A negative current of –5A flowing in one direction is same as a current of +5A in opposite direction.

Voltage: Voltage or potential difference is the energy required to move a unit charge through an element, measured in volts.



Power: It is time rate of expending or absorbing energy.



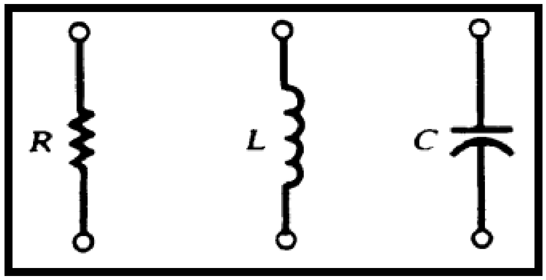
- Law of conservation of energy must be obeyed in any electric circuit.
- Algebraic sum of power in a circuit, at any instant of time, must be zero.

i.e. $\Sigma P = 0$

Circuit Elements:

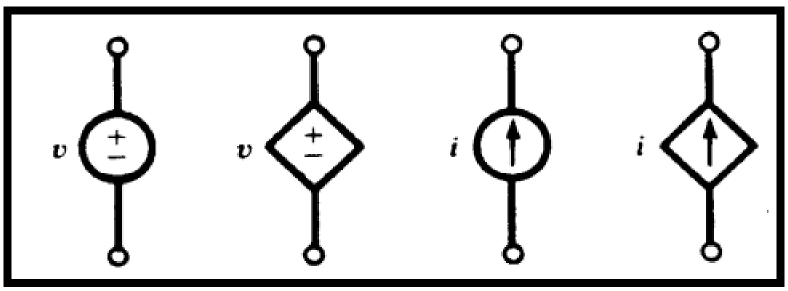
Passive element: If it is not capable of delivering energy, then it is passive element. Example: Resistor, Inductor, and Capacitor

Passive Elements



Active element: If an element is capable of delivering energy independently, then it is called active element. Example: Voltage source, and Current source.

Actie Elements



Linear and Non linear elements: If voltage and current across an element are related to each other through a constant coefficient then the element is called as linear element otherwise it is called as non-linear.

Unidirectional and Bidirectional: When elements characteristics are independent of direction of current then element is called bi-directional element otherwise it is called as unidirectional.

- R, L & C are bidirectional
- Diode is a unidirectional element.
- Voltage and current sources are also unidirectional elements.
- Every linear element should obey the bi-directional property but vice versa as is not necessary.

Resistor: Linear and bilateral (conduct from both direction)

- In time domain V(t) = I(t)R
- In s domain: V(s) = RI(s)

$$\boxed{R=\frac{\rho l}{A} \text{ ohm}}$$

- l = length of conductor, ρ= resistivity, A = area of cross section

- Extension of wire to n times results in increase in resistance:

$$\boxed{R' = n^2 R}$$

- Compression of wire results in decrease in resistance:

$$\boxed{R' = \frac{R}{n^2}}$$

Capacitor: All capacitors are linear and bilateral, except electrolytic capacitor which is unilateral.

- Time Domain:

$$i(t) = \frac{Cdv(t)}{dt} \qquad v(t) = \frac{1}{C}\int_{-\infty}^{t} i(t)dt$$

- In s-domain:

$$I(s) = sCV(s) \qquad V(s) = \frac{1}{sC} I(s)$$

- Capacitor doesn’t allow sudden change of voltage, until impulse of current is applied.
- It stores energy in the form of electric field and power dissipation in ideal capacitor is zero.

Impedance:

$$Z_c = -jX_c\Omega \quad \& \quad X_c = \frac{1}{\omega C}; X_c \rightarrow \text{Capacitive reactance} ; \omega = 2\pi f$$

Inductor: Linear and bilinear element

Time Domain: $v(t) = L\frac{di(t)}{dt}$ $\qquad i(t) = \frac{1}{L}\int_{\infty}^{t} v(t)dt$

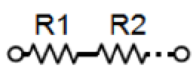
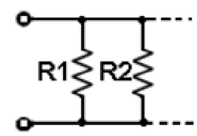
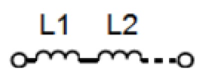
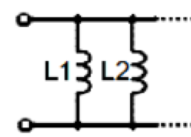
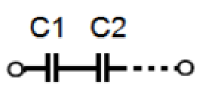
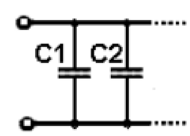
Impedance $Z_L = jX_L\,\Omega$ $\quad \&$ $\quad X_L = \omega L\ \Omega$

In s-domain $V(s) = sL\ I(s)$ $\qquad I(s) = \frac{1}{sL} V(s)$

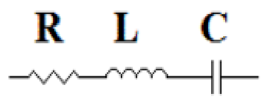
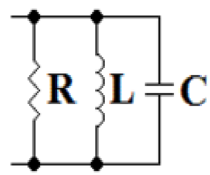
- Inductor doesn’t allowed sudden change of current, until impulse of voltage is applied. It stores energy in the form of magnetic field.
- Power dissipation in ideal inductor is zero.

FORMULAS FOR THE BASIC CIRCUIT COMPONENTS						
CIRCUIT ELEMENT	IMPEDANCE		VOLT-AMP EQUATIONS		ENERGY (dissipated on R or stored in L, C)
	absolute value	complex form	instantaneous values	RMS values for sinusoidal signals	
RESISTANCE	R	R	v=i×R	Vrms=Irms×R	$E=Irms^2R\times t$
INDUCTANCE	2πfL	jωL	v=L×di/dt	Vrms=Irms×2πfL	$E=Li^2/2$
CAPACITANCE	1/(2πfC)	1/jωC	i=C×dv/dt	Vrms=Irms/(2πfC)	$E=Cv^2/2$

Notes:
R- resistance in ohms, L- inductance in henrys, C- capacitance in farads, f - frequency in hertz, t- time in seconds, π≈3.14159;
ω=2πf - angular frequency;
j - imaginary unit ($j^2=-1$)
Euler's formula: $e^{jx}=\cos x+j\sin x$

EQUATIONS FOR SERIES AND PARALLEL CONNECTIONS				
CIRCUIT ELEMENT	SERIES CONNECTION		PARALLEL CONNECTION	
RESISTORS		Rseries= R1+R2+...		Rparallel= 1/ (1/R1+1/R2+...)
INDUCTORS		Lseries= L1+L2+...		Lparallel= 1/(1/L1+1/L2+...)
CAPACITORS		Cseries= 1/ (1/C1+1/C2+...)		Cparallel= C1+C2+...

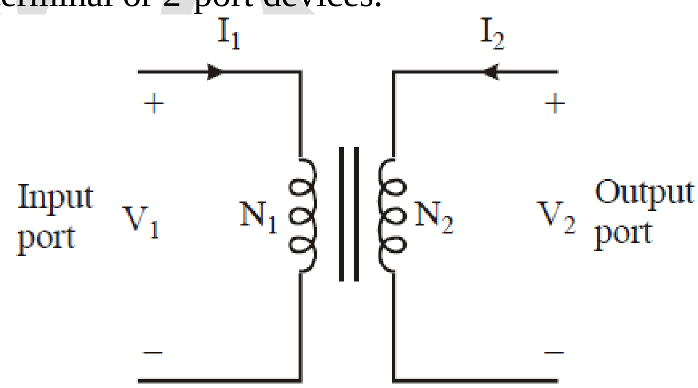
Rules of series	Rules of Parallel
$V_{eq} = V_1 + V_2 + V_3$ $R_{eq} = R_1 + R_2 + R_3$ $C_{eq} = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}}$	$i_{eq} = i_1 + i_2 + i_3$ $R_{eq} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$ $C_{eq} = C_1 + C_2 + C_3$

CALCULATIONS OF EQUIVALENT RLC IMPEDANCES		
CIRCUIT CONNECTION	COMPLEX FORM	ABSOLUTE VALUE
 Series	Z=R+jωL+1/jωC	$Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$
 Parallel	Z= 1/(1/R+1/jωL+jωC)	$Z = \frac{1}{\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}}$

Mesh Analysis:

- Path - A set of element that may be traversed in order, without passing thru the same node twice
- Loop - a closed path
- Mesh - A loop that does not contain any other loop within it
- Planar Circuit - A circuit that may be drawn on a plane surface in such a way that there are no branch crossovers
- Non-Planar Circuit - A circuit that is not planar, i.e, some branch(es) pass over some other branch(es) (Can not use Mesh Analysis)

Transformer: 4 terminal or 2-port devices.



- $N_1 > N_2$: Step down transformer

$$\boxed{\frac{V_1}{V_2} = \frac{N_1}{N_2}}$$

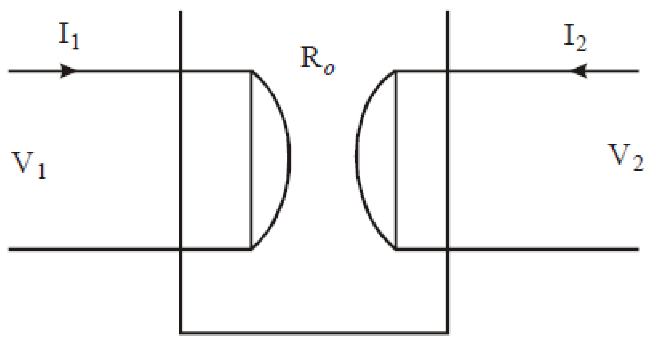
- $N_2 > N_1$: Step up transformer

$$\boxed{\frac{I_1}{I_2} = \frac{N_2}{N_1}}$$

$$\text{Where } \frac{N_1}{N_2} = K \rightarrow \textit{Turns ratio.}$$

- Transformer doesn't work as amplifier because current decreases in same amount power remain constant.

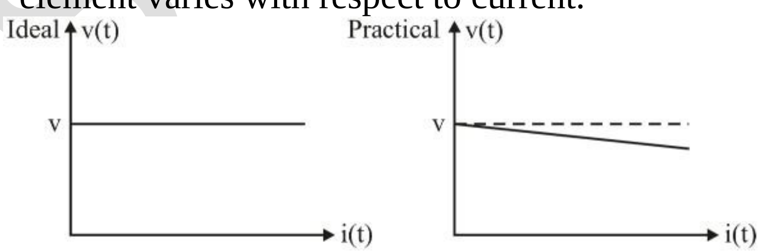
Gyrator:



- R_0 = Coefficient of Gyrator
- $V_1 = R_0 I_2$
- $V_2 = -R_0 I_1$
- If load is capacitive then input impedance will be inductive and vice versa.
- If load is inductive then input impedance will be capacitive.
- It is used for simulation of equivalent value of inductance.

Voltage Source:

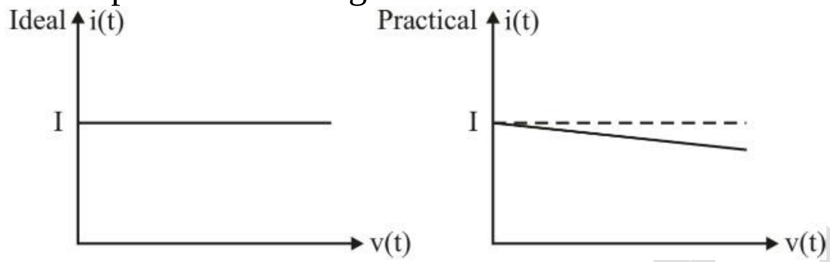
- In practical voltage source, there is small internal resistance, so voltage across the element varies with respect to current.



- Ideal voltmeter, R_V is infinite (Internal resistance)

Current Source:

- In practical current source, there is small internal resistance, so current varies with respect to the voltage across element



- Ideal Ammeter, R_a is 0 (Internal resistance)
- Internal resistance of voltage source is in series with the source.
- Internal resistance of ideal voltage source is zero.
- Internal resistance of current source is in parallel with the source.
- Internal resistance of ideal current source is infinite.

Independent Source: Voltage or current source whose values doesn't depend on any other parameters.

- Example: Generator

Dependent Source: Voltage or current source whose values depend upon other parameters like current, voltage.

Lumped Network: A network in which all network elements are physically separable is known as lumped network.

Distributed network: A network in which the circuit elements like resistance, inductance etc, are not physically separate for analysis purpose, is called distributed network. Example: Transmission line.

Thevenin's Theorem: Any linear network can be replaced by an independent voltage sources in series with an impedance such that the current voltage relation at the terminals is unchanged.

Norton's Theorem: Identical to thevenin's statement except that the equivalent circuit is an independent current source in parallel with an impedance. ($Z_S = R_{Th}$)

Average Power

$$P_{Avg} = \frac{V_m}{\sqrt{2}} \frac{I_m}{\backslash\text{sqrt}} \cos(\theta)$$

$$or \quad = V_{rms} I_{rms} \cos(\theta)$$

Power Factor

$$PF = \cos(\theta)$$

$$\theta = \theta_V - \theta_i$$

Reactive power

$$Q = V_{rms} I_{rm} \sin(\theta)$$

Measured $VARs$

Volt Amperes Reactive

$$P^2 + Q^2 = (V_{rms} I_{rms})^2$$

Apparent Power (S)

$$S = V_{rms} I_{rms} = Va$$

$$S = P + j\bar{Q}$$

$$S = I^2 Z$$

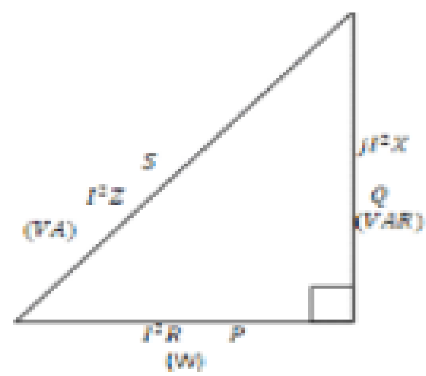
Maxpower Transfer

$$Z_L = R_L + jX_L$$

$$= R_{Th} - jX_{Th} = Z_{Th}$$

True Power

$$P = I^2 R$$

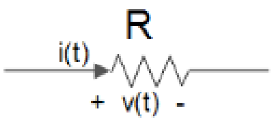
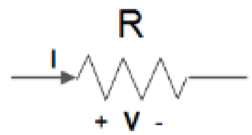
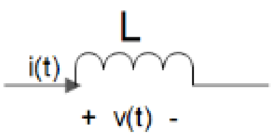
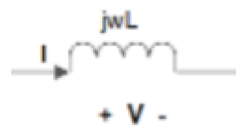
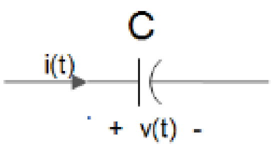


Root Mean Square

Average of a signal that is symmetric about the horizontal Axis

$$V_{rms} = \frac{V_m}{\sqrt{2}} \qquad I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$V_{eff} = V_{rms} \qquad I_{eff} = I_{rms}$$

Time	Domain	Frequency	domain
	$v = Ri$	$\boldsymbol{V} = R\boldsymbol{I}$	
	$v = L\dfrac{di}{dt}$	$\boldsymbol{V} = j\omega L\mathbf{I}$	
	$v = \dfrac{1}{C}\displaystyle\int i\,dt$	$\boldsymbol{V} = \dfrac{1}{jwC}\boldsymbol{I}$