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Second Edition

Pulse and Digital Circuits

A. Anand Kumar

PHI

PULSE AND DIGITAL CIRCUITS

SECOND EDITION

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Contents

Preface

After nearly thirty years of my experience in the classroom, I have strived to develop this comprehensive text on pulse circuitry in order to provide students with a solid grounding in the foundations of analysis and design of pulse and digital circuits. The second edition of this textbook with various new features is suitable for use as one-semester course material by undergraduate students of Electronics and Communication Engineering, Electrical and Electronics Engineering, Electronics and Instrumentation Engineering, and Telecommunication Engineering. Appropriate for self-study, the book will also be useful to AMIE and IETE students.

The text is organized into 10 chapters. The outline of the book is as follows.

When non-sinusoidal signals are transmitted through a linear network, the shape of the waveform undergoes a change. This process called linear wave shaping is discussed in Chapter 1.

Particularly in communication systems, quite often, it is required to remove a part of the waveform above or below some reference level. This process is called clipping. In many pulse systems, quite often a dc level is required to be added to a waveform to fix the top or bottom of the waveform at some reference level. This process is called clamping. Clipping and clamping together is called nonlinear wave shaping. Chapter 2 deals with the various clipping and clamping circuits.

The switching characteristics of junction diodes and transistors as required for a clear understanding of the pulse circuits are covered in Chapter 3.

Memory is the basic requirement of all computers. The basic memory element is a flip-flop, i.e. the bistable multivibrator. The monostable multivibrator is the basic gating circuit. The astable multivibrator is used as a master oscillator, and the Schmitt trigger circuit as a basic voltage comparator. The various types of multivibrators are discussed in Chapter 4.

Time-base generators are essential for display of signals on the screen. Voltage and current time-base generators are presented in Chapter 5.

A large pulse and digital system consists of a number of waveform generators which need to be synchronized with or without frequency division. Synchronization and frequency division of various generators with pulse type as well as symmetrical signals are the topics treated in Chapter 6.

When signals are to be transmitted only for specified intervals of time and are to be blocked during other intervals of time, we require sampling gates. Various types of sampling gates are explained in Chapter 7.

Logic gates are the fundamental building blocks of any digital system. Realization of logic gates using diodes and transistors is discussed in Chapter 8.

Most of the logic gates, flip-flops, counters, shift registers, arithmetic circuits, encoders, decoders, etc. are available in several digital families. The TTL, ECL, IIL, MOS and CMOS class of logic families are discussed in Chapter 9.

When pulses of very large peak power are to be generated, we require blocking oscillators. Several types of monostable and astable blocking oscillators are discussed in Chapter 10.

A large number of design examples have been worked out to help students understand each new concept or analysis method as it is introduced. Extensive short questions and answers and also review questions are included at the end of each chapter to enable the students to prepare for examinations confidently. Fill-in-the-blank type questions, objective type multiple choice questions and numerical problems are provided at the chapter-ends to enable students to build a clear understanding of the subject matter discussed in the text and also to assess their learning. Answers to fill-in-the-blanks, objective type questions and numerical problems are given at the end of the book. Most of the solved and unsolved problems presented in this book have been classroom tested.

I express my profound gratitude to all those individuals without whose assistance and cooperation this book would not have been completed. First of all, I thank Sri. V. Srinivasa Rao, Technician of Adam's Engineering College, Palvancha who typed the entire original manuscript and drew all the figures in this book. I also thank Mr. P. Venkateswara Rao of our college for helping me in the revision of this book.

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Finally, I am indebted to my wife, A. Jhansi, for putting up with my spending countless hours working on the manuscript. Our sons Dr. A. Anil Kumar and Mr. A. Sunil Kumar and daughters-in-law Dr. A. Anureet Kaur and A. Apurupa and granddaughter Khushi supported me with their constant words of encouragement.

The author will gratefully acknowledge constructive criticism from both students and teachers for further improvement of this book.

A. Anand Kumar

Chapter 1

Linear Wave Shaping

A linear network is a network made up of linear elements only. A linear network can be described by linear differential equations. The principle of superposition and the principle of homogeneity hold good for linear networks. In pulse circuitry, there are a number of waveforms, which appear very frequently. The most important of these are sinusoidal, step, pulse, square wave, ramp, and exponential waveforms. The response of *RC*, *RL*, and *RLC* circuits to these signals is described in this chapter. Out of these signals, the sinusoidal signal has a unique characteristic that it preserves its shape when it is transmitted through a linear network, i.e. under steady state, the output will be a precise reproduction of the input sinusoidal signal. There will only be a change in the amplitude of the signal and there may be a phase shift between the input and the output waveforms. The influence of the circuit on the signal may then be completely specified by the ratio of the output to the input amplitude and by the phase angle between the output and the input. No other periodic waveform preserves its shape precisely when transmitted through a linear network, and in many cases the output signal may bear very little resemblance to the input signal.

The process whereby the form of a non-sinusoidal signal is altered by transmission through a linear network is called linear wave shaping.

1.1 THE LOW-PASS *RC* CIRCUIT

Figure 1.1 shows a low-pass *RC* circuit. A low-pass circuit is a circuit, which transmits only low-frequency signals and attenuates or stops high-frequency signals. At zero frequency, the

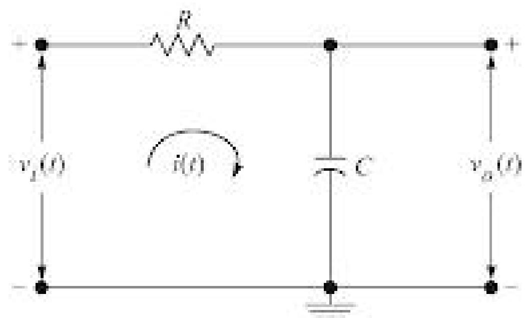

Figure 1.1 The low-pass *RC* circuit.

reactance of the capacitor is infinity (i.e. the capacitor acts as an open circuit) so the entire input appears at the output, i.e. the input is transmitted to the output with zero attenuation. So the output is the same as the input, i.e. the gain is unity. As the frequency increases the capacitive reactance ($X_C = 1/2\pi fC$) decreases and so the output decreases. At very high frequencies the capacitor virtually acts as a short-circuit and the output falls to zero.

1.1.1 Sinusoidal Input

The Laplace transformed low-pass RC circuit is shown in Figure 1.2(a). The gain versus frequency curve of a low-pass circuit excited by a sinusoidal input is shown in Figure 1.2(b). This curve is obtained by keeping the amplitude of the input sinusoidal signal constant and varying its frequency and noting the output at each frequency. At low frequencies the output is equal to the input and hence the gain is unity. As the frequency increases, the output decreases and hence the gain decreases. The frequency at which the gain is $1/\sqrt{2}$ (= 0.707) of its maximum value is called the cut-off frequency. For a low-pass circuit, there is no lower cut-off frequency. It is zero itself. The upper cut-off frequency is the frequency (in the high-frequency range) at which the gain is $1/\sqrt{2}$, i.e. 70.7%, of its maximum value. The bandwidth of the low-pass circuit is equal to the upper cut-off frequency f_2 itself.

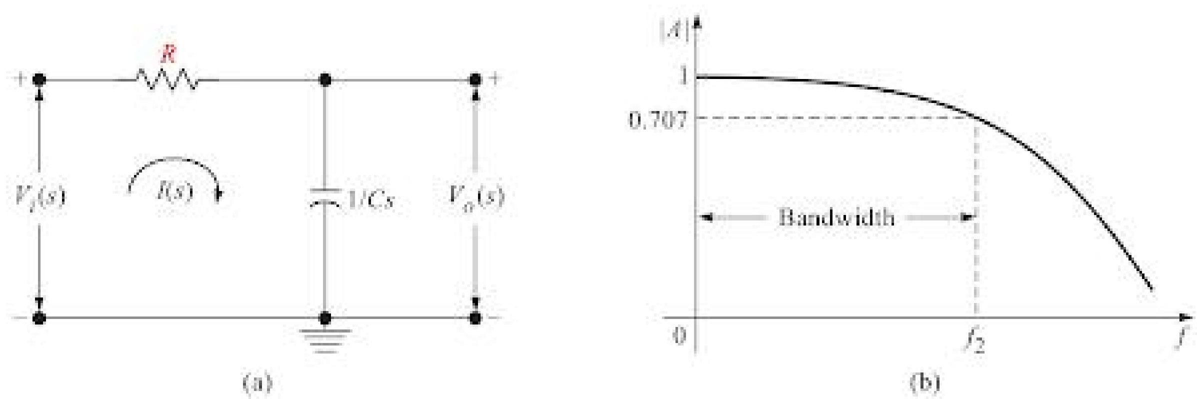

Figure 1.2 (a) Laplace transformed low-pass RC circuit and (b) its frequency response.

For the network shown in Figure 1.2(a), the magnitude of the steady-state gain A is given by

$$A = \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{1}{1 + RCs} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j2\pi fRC}$$

$$\therefore \qquad |A| = \frac{1}{\sqrt{1 + (2\pi fRC)^2}}$$

At the upper cut-off frequency f_2, $|A| = \dfrac{1}{\sqrt{2}}$

$$\therefore \qquad \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + (2\pi f_2 RC)^2}}$$

Squaring both sides and equating the denominators,

$$2 = 1 + (2\pi f_2 RC)^2$$

$\therefore$ The upper cut-off frequency, $f_2 = \dfrac{1}{2\pi RC}$.

So
$$A = \frac{1}{1 + j\dfrac{f}{f_2}} \quad \text{and} \quad |A| = \frac{1}{\sqrt{1 + \left(\dfrac{f}{f_2}\right)^2}}$$

The angle θ by which the output leads the input is given by

$$\theta = \tan^{-1}\frac{f}{f_2}$$

1.1.2 Step-Voltage Input

A step signal is one which maintains the value zero for all times $t < 0$, and maintains the value V for all times $t > 0$. The transition between the two voltage levels takes place at $t = 0$ and is accomplished in an arbitrarily small time interval. Thus, in Figure 1.3(a), $v_i = 0$ immediately before $t = 0$ (to be referred to as time $t = 0^-$) and $v_i = V$, immediately after $t = 0$ (to be referred to as time $t = 0^+$). In the low-pass RC circuit shown in Figure 1.1, if the capacitor is initially uncharged, when a step input is applied, since the voltage across the capacitor cannot change instantaneously, the output will be zero at $t = 0$, and then, as the capacitor charges, the output voltage rises exponentially towards the steady-state value V with a time constant RC as shown in Figure 1.3(b).

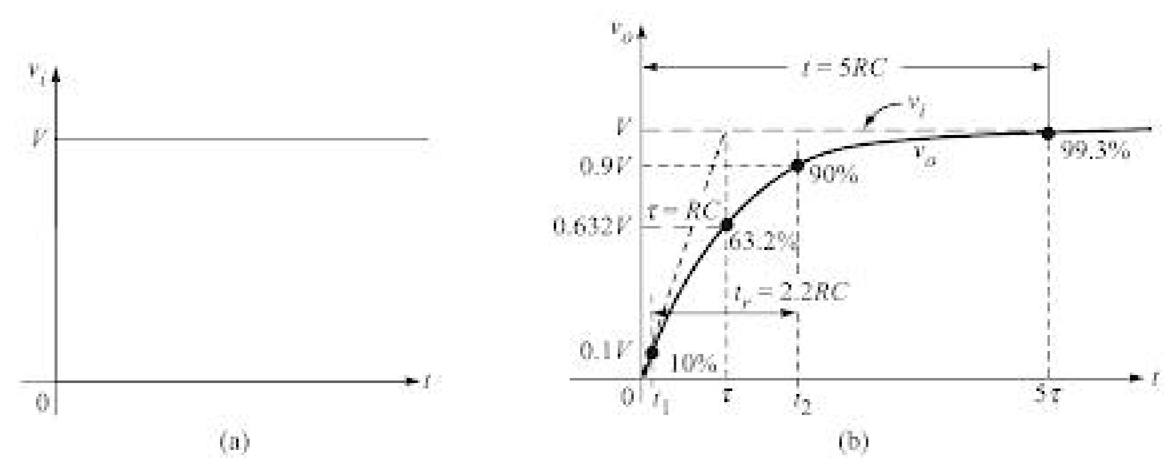

Figure 1.3 (a) Step input and (b) step response of the low-pass RC circuit.

Let V' be the initial voltage across the capacitor.

Writing KVL around the loop in Figure 1.1,

$$v_i(t) = Ri(t) + \frac{1}{C}\int i(t)\, dt$$

Differentiating this equation,

$$\frac{dv_i(t)}{dt} = R\,\frac{di(t)}{dt} + \frac{1}{C}i(t)$$

Since $$v_i(t) = V, \quad \frac{dv_i(t)}{dt} = 0$$

$$\therefore \quad 0 = R\frac{di(t)}{dt} + \frac{1}{C}i(t)$$

Taking the Laplace transform on both sides,

$$0 = R[sI(s) - I(0^+)] + \frac{1}{C}I(s)$$

$$\therefore \quad I(0^+) = I(s)\left(s + \frac{1}{RC}\right)$$

The initial current $I(0^+)$ is given by

$$I(0^+) = \frac{V - V'}{R}$$

$$\therefore \quad I(s) = \frac{I(0^+)}{s + \frac{1}{RC}} = \frac{V - V'}{R\left(s + \frac{1}{RC}\right)}$$

and $$V_o(s) = V_i(s) - I(s)R = \frac{V}{s} - \frac{(V - V')R}{R\left(s + \frac{1}{RC}\right)} = \frac{V}{s} - \frac{V - V'}{s + \frac{1}{RC}}$$

Taking the inverse Laplace transform on both sides,

$$v_o(t) = V - (V - V')e^{-t/RC}$$

where V' is the initial voltage across the capacitor (V_{initial}) and V is the final voltage (V_{final}) to which the capacitor can charge.

So, the expression for the voltage across the capacitor of an RC circuit excited by a step input is given by

$$v_o(t) = V_{\text{final}} - (V_{\text{final}} - V_{\text{initial}})e^{-t/RC}$$

If the capacitor is initially uncharged, then $v_o(t) = V(1 - e^{-t/RC})$

Expression for rise time

When a step signal is applied, the rise time t_r is defined as the time taken by the output voltage waveform to rise from 10% to 90% of its final value. It gives an indication of how fast the circuit can respond to a discontinuity in voltage. Assuming that the capacitor in Figure 1.1 is initially uncharged, the output voltage shown in Figure 1.3(b) at any instant of time is given by

$$v_o(t) = V(1 - e^{-t/RC})$$

At $t = t_1$, $v_o(t) = 10\%$ of $V = 0.1\,\text{V}$

$$\therefore \qquad 0.1V = V(1 - e^{-t_1/RC})$$

$$\therefore \qquad e^{-t_1/RC} = 0.9 \quad \text{or} \quad e^{t_1/RC} = \frac{1}{0.9} = 1.11$$

$$\therefore \qquad t_1 = RC \ln(1.11) = 0.1RC$$

At $t = t_2$, $v_o(t) = 90\%$ of $V = 0.9\,\text{V}$

$$\therefore \qquad 0.9V = V(1 - e^{-t_2/RC})$$

$$\therefore \qquad e^{-t_2/RC} = 0.1 \quad \text{or} \quad e^{t_2/RC} = \frac{1}{0.1} = 10$$

$$\therefore \qquad t_2 = RC \ln 10 = 2.3RC$$

$$\therefore \qquad \text{Rise time, } t_r = t_2 - t_1 = 2.2RC$$

This indicates that the rise time t_r is proportional to the time constant RC of the circuit. The larger the time constant, the slower the capacitor charges, and the smaller the time constant, the faster the capacitor charges.

Relation between rise time and upper 3-dB frequency

We know that the upper 3-dB frequency (same as bandwidth) of a low-pass circuit is

$$f_2 = \frac{1}{2\pi RC} \quad \text{or} \quad RC = \frac{1}{2\pi f_2}$$

$$\therefore \qquad \text{Rise time, } t_r = 2.2RC = \frac{2.2}{2\pi f_2} = \frac{0.35}{f_2} = \frac{0.35}{\text{BW}}$$

Thus, the rise time is inversely proportional to the upper 3-dB frequency.

The *time constant* ($\tau = RC$) of a circuit is defined as the time taken by the output to rise to 63.2% of the amplitude of the input step. It is same as the time taken by the output to rise to 100% of the amplitude of the input step, if the initial slope of rise is maintained. See Figure 1.3(b). The Greek letter τ is also employed as the symbol for the time constant.

1.1.3 Pulse Input

The pulse shown in Figure 1.4(a) is equivalent to a positive step followed by a delayed negative step as shown in Figure 1.4(b). So, the response of the low-pass RC circuit to a pulse for times less than the pulse width t_p is the same as that for a step input and is given by $v_o(t) = V(1 - e^{-t/RC})$. The responses of the low-pass RC circuit for time constant $RC >> t_p$, RC smaller than t_p and RC very small compared to t_p are shown in Figures 1.5(a), 1.5(b), and 1.5(c) respectively.

If the time constant RC of the circuit is very large, at the end of the pulse, the output voltage will be $V_p(t) = V(1 - e^{-t_p/RC})$, and the output will decrease to zero from this value with a time constant RC as shown in Figure 1.5(a). Observe that the pulse waveform is distorted when it is passed through a linear network. The output will always extend beyond

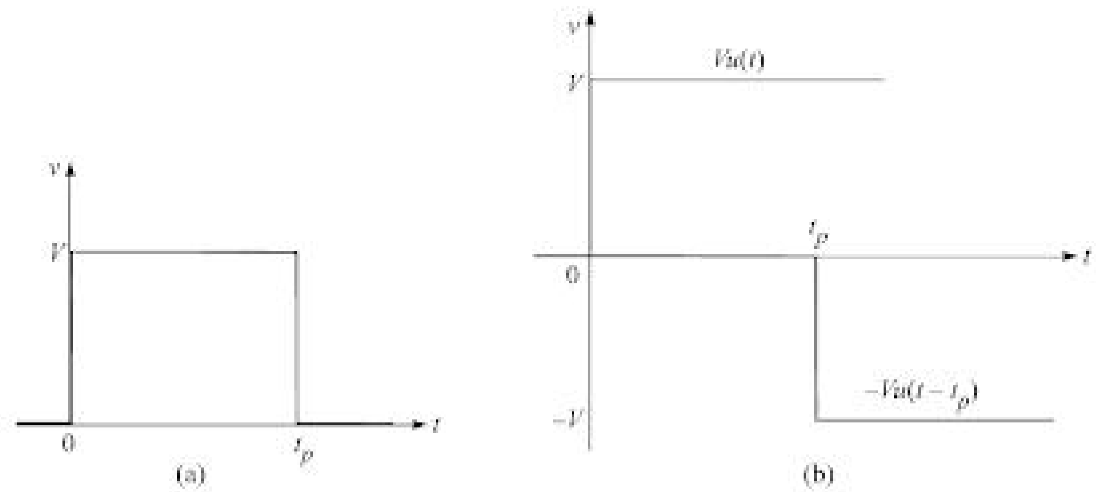

Figure 1.4 (a) A pulse and (b) a pulse in terms of steps.

the pulse width t_p, because whatever charge has accumulated across the capacitor C during the pulse cannot leak off instantaneously.

If the time constant RC of the circuit is very small, the capacitor charges and discharges very quickly and the rise time t_r will be small and so the distortion in the wave shape is small. For minimum distortion (i.e. for preservation of wave shape), the rise time must be small compared to the pulse width t_p. If the upper 3-dB frequency f_2 is chosen equal to the reciprocal of the pulse width t_p, i.e. if $f_2 = 1/t_p$, then $t_r = 0.35t_p$ and the output is as shown in Figure 1.5(b), which for many applications is a reasonable reproduction of the input. As a rule of thumb, we can say:

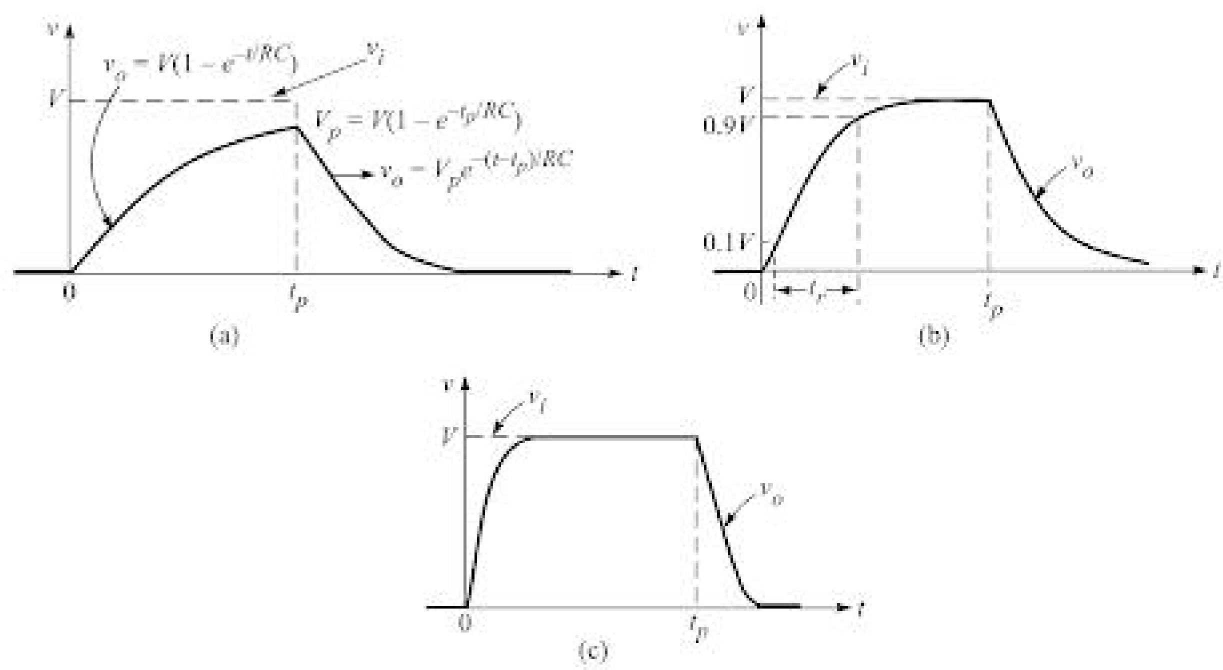

Figure 1.5 Pulse response for (a) $RC >> t_p$, (b) $RC < t_p$, and (c) $RC << t_p$.

A pulse shape will be preserved if the 3-dB frequency is approximately equal to the reciprocal of the pulse width.

Thus to pass a 0.25 μs pulse reasonably well requires a circuit with an upper cut-off frequency of the order of 4 MHz.

1.1.4 Square-Wave Input

A square wave is a periodic waveform which maintains itself at one constant level V' with respect to ground for a time T_1 and then changes abruptly to another level V'', and remains constant at that level for a time T_2, and repeats itself at regular intervals of $T = T_1 + T_2$. A square wave may be treated as a series of positive and negative steps. The shape of the output waveform for a square wave input depends on the time constant of the circuit. If the time constant is very small, the rise time will also be small and a reasonable reproduction of the input may be obtained.

For the square wave shown in Figure 1.6(a), the output waveform will be as shown in Figure 1.6(b) if the time constant RC of the circuit is small compared to the period of the input waveform. In this case, the wave shape is preserved. If the time constant is comparable with the period of the input square wave, the output will be as shown in Figure 1.6(c). The output rises and falls exponentially. If the time constant is very large compared to the period of the input waveform, the output consists of exponential sections, which are essentially linear as indicated in Figure 1.6(d). Since the average voltage across R is zero, the dc voltage at the output is the same as that of the input. This average value is indicated as V_{dc} in all the waveforms of Figure 1.6.

In Figure 1.6(c), the equation for the rising portion is

$$v_{01} = V' - (V' - V_2)e^{-t/RC}$$

where V_2 is the voltage across the capacitor at $t = 0$, and V' is the level to which the capacitor can charge.

The equation for the falling portion is

$$v_{02} = V'' - (V'' - V_1)e^{-(t - T_1)/RC}$$

where V_1 is the voltage across the capacitor at $t = T_1$ and V'' is the level to which the capacitor can discharge.

Setting $v_{01} = V_1$ at $t = T_1$,

$$V_1 = V' - (V' - V_2)e^{-T_1/RC} = V'(1 - e^{-T_1/RC}) + V_2 e^{-T_1/RC}$$

Setting $v_{02} = V_2$ at $t = T_1 + T_2$,

$$V_2 = V'' - (V'' - V_1)e^{-(T_1+T_2-T_1)/RC} = V''(1 - e^{-T_2/RC}) + V_1 e^{-T_2/RC}$$

Substituting this value of V_2 in the expression for V_1,

$$V_1 = V'(1 - e^{-T_1/RC}) + [V''(1 - e^{-T_2/RC}) + V_1 e^{-T_2/RC}]e^{-T_1/RC}$$

i.e.

$$V_1 = \frac{V'(1 - e^{-T_1/RC}) + V''(1 - e^{-T_2/RC})e^{-T_1/RC}}{1 - e^{-(T_1+T_2)/RC}}$$

Similarly substituting the value of V_1 in the expression for V_2,

$$V_2 = \frac{V''(1 - e^{-T_2/RC}) + V'(1 - e^{-T_1/RC})\,e^{-T_2/RC}}{1 - e^{-(T_1+T_2)/RC}}$$

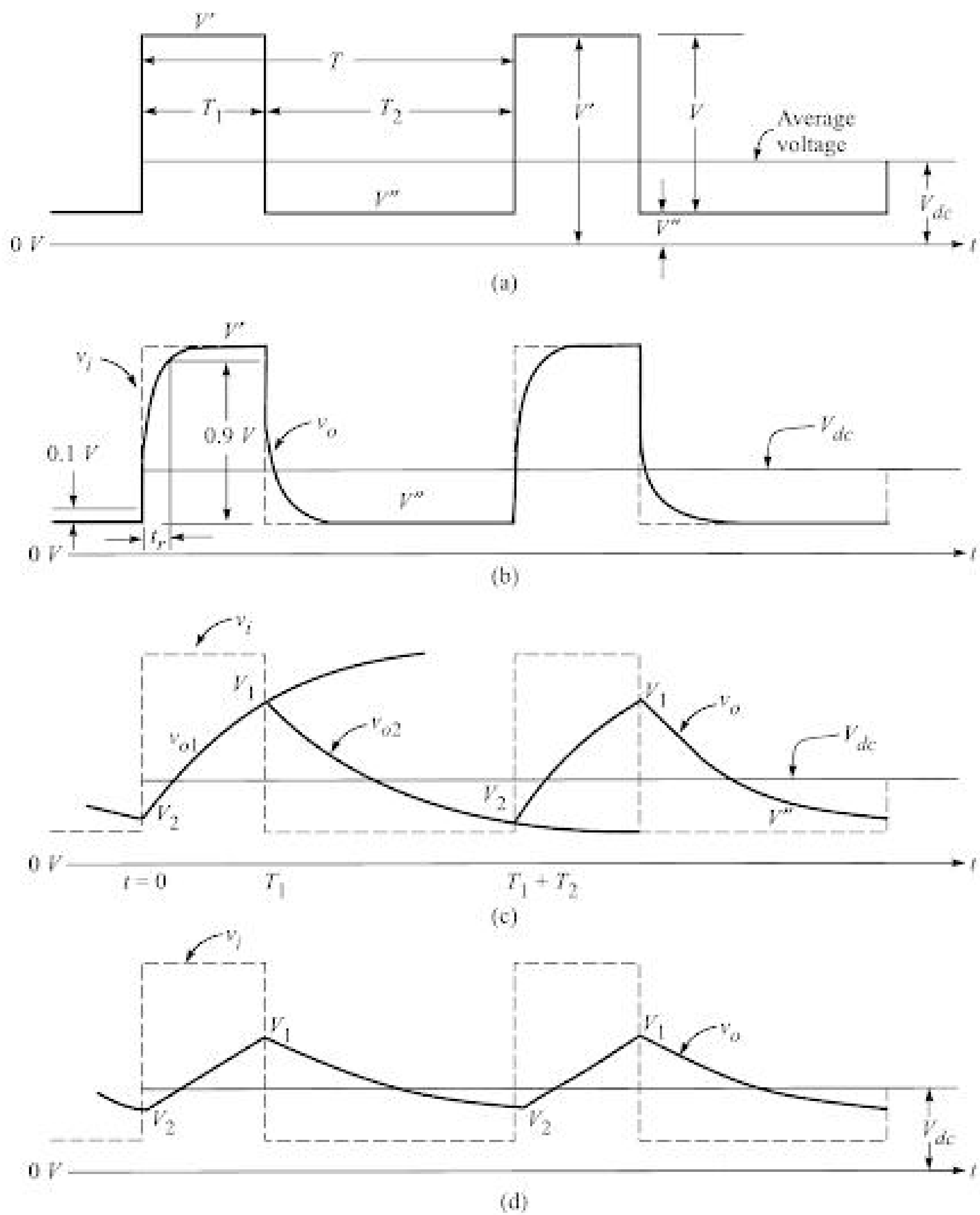

Figure 1.6 Response of a low-pass *RC* circuit to a square wave input: (a) square-wave input wave form, (b) output waveform for $RC << T$, (c) output waveform for $RC = T$, and (d) output waveform for $RC >> T$.

For a symmetrical square wave with zero average value,

$$T_1 = T_2 = \frac{T}{2} \quad \text{and} \quad V' = -V'' = \frac{V}{2}. \text{ So, } V_2 \text{ will be equal to } -V_1$$

$$\therefore \qquad V_1 = \frac{\frac{V}{2}(1 - e^{-T/2RC}) - \frac{V}{2}(1 - e^{-T/2RC})\, e^{-T/2RC}}{1 - e^{-T/RC}}$$

$$= \frac{V}{2}\;\frac{1 - e^{-T/2RC} - e^{-T/2RC} + e^{-T/RC}}{1 - e^{-T/RC}}$$

$$= \frac{V}{2}\;\frac{(1 - e^{-T/2RC})^2}{(1 + e^{-T/2RC})(1 - e^{-T/2RC})}$$

$$= \frac{V}{2}\left(\frac{1 - e^{-T/2RC}}{1 + e^{-T/2RC}}\right)$$

$$= \frac{V}{2}\left(\frac{e^{T/2RC} - 1}{e^{T/2RC} + 1}\right)$$

$$= \frac{V}{2}\left(\frac{e^{2x} - 1}{e^{2x} + 1}\right) = \frac{V}{2}\tanh x$$

where $x = \dfrac{T}{4RC}$ and T is the period of the square wave.

Now, $$V_2 = -V_1 = -\frac{V}{2}\left(\frac{1 - e^{-T/2RC}}{1 + e^{-T/2RC}}\right) = \frac{V}{2}\left(\frac{1 - e^{T/2RC}}{1 + e^{T/2RC}}\right)$$

1.1.5 Ramp Input

When a low-pass RC circuit shown in Figure 1.1 is excited by a ramp input, i.e.

$$v_i(t) = \alpha t, \text{ where } \alpha \text{ is the slope of the ramp}$$

we have, $$V_i(s) = \frac{\alpha}{s^2}$$

From the frequency domain circuit of Figure 1.2(a), the output is given by

$$V_o(s) = V_i(s)\,\frac{\dfrac{1}{Cs}}{R + \dfrac{1}{Cs}} = \frac{\alpha}{s^2}\,\frac{1}{1 + RCs} = \frac{\alpha}{RC}\,\frac{1}{s^2\left(s + \dfrac{1}{RC}\right)}$$

$$= \frac{\alpha}{RC}\left[\frac{-(RC)^2}{s} + \frac{RC}{s^2} + \frac{(RC)^2}{s + \dfrac{1}{RC}}\right]$$

i.e. $$V_o(s) = \frac{-\alpha RC}{s} + \frac{\alpha}{s^2} + \frac{\alpha RC}{s + \dfrac{1}{RC}}$$

Taking the inverse Laplace transform on both sides,

$$v_o(t) = -\alpha RC + \alpha t + \alpha RCe^{-t/RC}$$
$$= \alpha(t - RC) + \alpha RCe^{-t/RC}$$

If the time constant RC is very small, $e^{-t/RC} = 0$

$$\therefore \qquad v_o(t) = \alpha(t - RC)$$

When the time constant is very small relative to the total ramp time T, the ramp will be transmitted with minimum distortion. The output follows the input but is delayed by one time constant RC from the input (except near the origin where there is distortion) as shown in Figure 1.7(a). If the time constant is large compared with the sweep duration, i.e. if $RC/T >> 1$, the output will be highly distorted as shown in Figure 1.7(b).

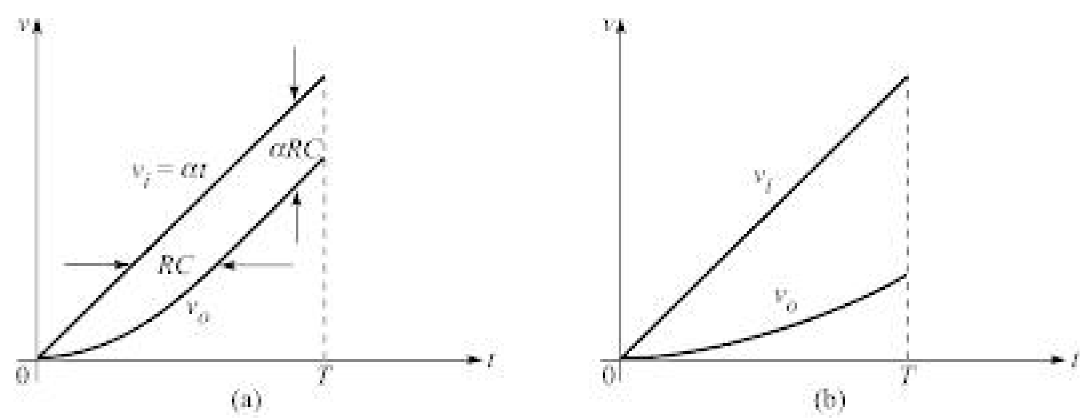

Figure 1.7 Response of a low-pass RC circuit for a ramp input for (a) $RC/T << 1$ and (b) $RC/T >> 1$.

Expanding $e^{-t/RC}$ in to an infinite series in t/RC in the above equation for $v_o(t)$,

$$v_o(t) = \alpha(t - RC) + \alpha RC\left(1 - \frac{t}{RC} + \left(\frac{t}{RC}\right)^2 \frac{1}{2!} - \left(\frac{t}{RC}\right)^3 \frac{1}{3!} + \ldots\right)$$

$$= \alpha t - \alpha RC + \alpha RC - \alpha t + \frac{\alpha t^2}{2RC} - \ldots$$

$$= \frac{\alpha t^2}{2RC} \approx \frac{\alpha}{RC}\left(\frac{t^2}{2}\right)$$

This shows that a quadratic response is obtained for a linear input and hence the circuit acts as an integrator for $RC/T >> 1$.

The transmission error e_t for a ramp input is defined as the difference between the input and the output divided by the input at the end of the ramp, i.e. at $t = T$.

For $RC/T << 1$,

$$e_t = \left.\frac{\alpha t - (\alpha t - \alpha RC)}{\alpha t}\right|_{t=T}$$

$$= \frac{\alpha RC}{\alpha T} = \frac{RC}{T} = \frac{1}{2\pi f_2 T}$$

where f_2 is the upper 3-dB frequency. For example, if we desire to pass a 2 ms pulse with less than 0.1% error, the above equation yields $f_2 > 80$ kHz and $RC < 2$ μs.

1.1.6 Exponential Input

For the low-pass RC circuit shown in Figure 1.1, let the input applied as shown in Figure 1.8 be $v_i(t) = V(1 - e^{-t/\tau})$, where τ is the time constant of the input waveform.

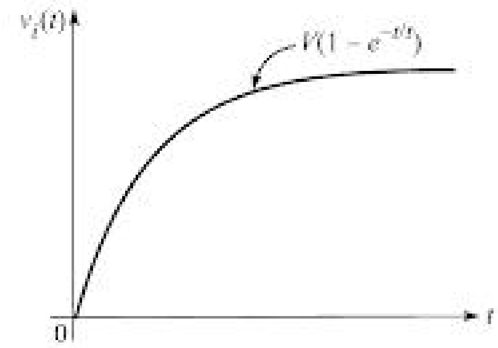

Figure 1.8 Exponential input.

Writing the KVL around the loop,

$$v_i(t) = Ri(t) + v_o(t) = RC\frac{dv_o(t)}{dt} + v_o(t)$$

$$\therefore \qquad V(1 - e^{-t/\tau}) = RC\frac{dv_o(t)}{dt} + v_o(t)$$

Taking the Laplace transform on both sides and neglecting the initial conditions,

$$\frac{V}{s} - \frac{V}{s + \frac{1}{\tau}} = RCsV_o(s) + V_o(s)$$

i.e.
$$V\left[\frac{\frac{1}{\tau}}{s\left(s + \frac{1}{\tau}\right)}\right] = V_o(s)(RCs + 1) = RCV_o(s)\left(s + \frac{1}{RC}\right)$$

$$\therefore \qquad V_o(s) = \frac{V}{RC\tau}\left[\frac{1}{s\left(s + \frac{1}{\tau}\right)\left(s + \frac{1}{RC}\right)}\right]$$

$$= V\left[\frac{1}{s} - \frac{1}{\left(1 - \frac{RC}{\tau}\right)\left(s + \frac{1}{\tau}\right)} + \frac{1}{\left(\frac{\tau}{RC} - 1\right)\left(s + \frac{1}{RC}\right)}\right]$$

Taking the inverse Laplace transform on both sides and letting $RC/\tau = n$,

$$v_o(t) = V\left[1 - \frac{e^{-t/\tau}}{1-n} + \frac{e^{-t/RC}}{\frac{1}{n} - 1}\right]$$

If $t/\tau = x$, then $v_o(t) = V\left[1 - \frac{e^{-x}}{1-n} + \frac{n}{1-n}e^{-x/n}\right]$, if $n \neq 1$

and $\qquad v_o(t) = 1 - (1 + x)e^{-x}, \qquad$ if $n = 1$

These are the expressions for the voltage across the capacitor of a low-pass RC circuit excited by an exponential input of rise time $t_{r1} = 2.2\tau$.

If an exponential of rise time t_{r1} is passed through a low-pass circuit with rise time t_{r2}, the rise time of the output waveform t_r will be given by an empirical relation, $t_r = 1.05\sqrt{t_{r1}^2 + t_{r2}^2}$. This is same as the rise time obtained when a step is applied to a cascade of two circuits of rise times t_{r1} and t_{r2} assuming that the second circuit does not load the first.

1.2 THE LOW-PASS *RC* CIRCUIT AS AN INTEGRATOR

If the time constant of an RC low-pass circuit is very large, the capacitor charges very slowly and so almost all the input voltage appears across the resistor for small values of time. Then, the current in the circuit is $v_i(t)/R$ and the output signal across C is

$$v_o(t) = \frac{1}{C}\int i(t)\,dt = \frac{1}{C}\int \frac{v_i(t)}{R}\,dt = \frac{1}{RC}\int v_i(t)\,dt$$

Hence the output is the integral of the input, i.e. if $v_i(t) = \alpha t$, then

$$v_o(t) = \frac{\alpha t^2}{2RC}$$

As time increases, the voltage drop across C does not remain negligible compared with that across R and the output will not remain the integral of the input. The output will change from a quadratic to a linear function of time.

If the time constant of an RC low-pass circuit is very large in comparison with the time required for the input signal to make an appreciable change, the circuit acts as an integrator.

A criterion for good integration in terms of steady-state analysis is as follows: The low-pass circuit acts as an integrator provided the time constant of the circuit $RC > 15T$, where T is the period of the input sine wave. When $RC > 15T$, the input sinusoid will be shifted at least by 89.4° (instead of the ideal 90° shift required for integration) when it is transmitted through the network.

An RC integrator converts a square wave into a triangular wave.

Integrators are almost invariably preferred over differentiators in analog computer applications for the following reasons:

1. It is easier to stabilize an integrator than a differentiator because the gain of an integrator decreases with frequency whereas the gain of a differentiator increases with frequency.
2. An integrator is less sensitive to noise voltages than a differentiator because of its limited bandwidth.
3. The amplifier of a differentiator may overload if the input waveform changes very rapidly.
4. It is more convenient to introduce initial conditions in an integrator.

EXAMPLE 1.1 A pulse generator with an output resistance $R_S = 500\ \Omega$ is connected to an oscilloscope with an input capacitance of $C_i = 30$ pF. Determine the fastest rise time that can be displayed.

Solution: The circuit works as a low-pass filter shown in Figure 1.1 with a time constant

$$R_S C_i = 500\ \Omega \times 30\ \text{pF} = 15\ \text{ns}$$

$\therefore$ Fastest rise time, $t_r = 2.2RC = 2.2 \times 15\ \text{ns} = 33\ \text{ns}$

EXAMPLE 1.2 A 10 V step is switched on to a 50 kΩ resistor in series with a 500 pF capacitor. Calculate the rise time of the capacitor voltage, the time for the capacitor to charge to 63.2% of its maximum voltage, and the time for the capacitor to be completely charged.

Solution: The circuit acts as a low-pass filter shown in Figure 1.1.

(a) The rise time of the capacitor voltage is

$$t_r = 2.2RC = 2.2 \times 50\ \text{k}\Omega \times 500\ \text{pF} = 55\ \mu\text{s}$$

(b) The time for the capacitor to charge to 63.2% of the maximum voltage is

$$\tau = RC = 50\ \text{k}\Omega \times 500\ \text{pF} = 25\ \mu\text{s}$$

(c) The time for the capacitor to be completely charged (99% value) is

$$5\tau = 5RC = 5 \times 25\ \mu\text{s} = 125\ \mu\text{s}$$

EXAMPLE 1.3 An ideal 1 μs pulse is fed to an amplifier. Calculate and plot the output waveform under the following conditions: The upper 3-dB frequency is:

(a) 10 MHz (b) 1 MHz (c) 0.1 MHz

Solution: The upper 3-dB frequency indicates that the amplifier acts as a low-pass circuit. So the pulse shown in Figure 1.9(a) is applied to the RC low pass circuit shown in Figure 1.9(b).

(a) When the upper 3-dB frequency, $f_2 = 10$ MHz:

Time constant of the circuit, $RC = \dfrac{1}{2\pi f_2} = \dfrac{1}{2\pi \times 10 \times 10^6} = 0.0159\ \mu\text{s}$

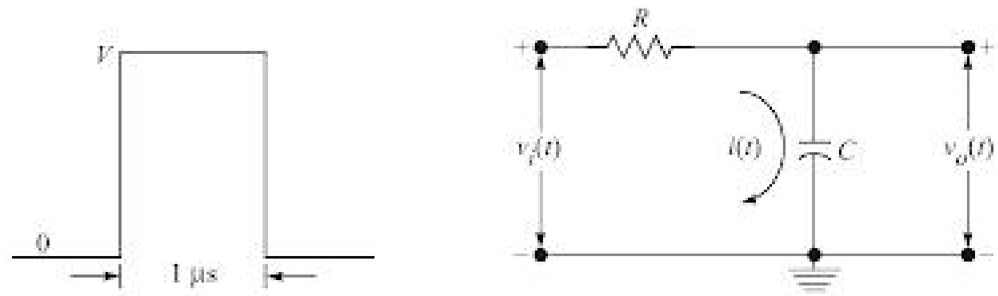

Figure 1.9 Example 1.3: (a) input waveform (b) circuit diagram.

Since, t_p = 1 μs and RC = 0.0159 μs, $RC << t_p$

Since the time constant is very small in comparison with the pulse width, the capacitor C charges rapidly with a rise time,

$$t_r = 2.2 \times 0.0159 = 0.035 \ \mu s$$

The output v_o is given by $v_o = V(1 - e^{-t/RC})$ where V is the amplitude of the pulse.

At $$t = t_p,$$

$$v_o = V(1 - e^{-t_p/RC}) = V(1 - e^{-1/0.0159}) \approx V$$

The output waveform is shown in Figure 1.10(a).

(b) When f_2 = 1 MHz:

$$RC = 1/2\pi f_2 = 1/2\pi \times 10^6 = 0.1591 \ \mu s$$

$$t_p = 1 \mu s \text{ and } RC = 0.1591 \ \mu s$$

$\therefore \quad RC < t_p$.

Since the time constant is small, the capacitor charges fast.

Rise time $t_r = 2.2 \ RC = 2.2 \times 0.1591 = 0.35 \ \mu s$

The output is given by $v_o = V(1 - e^{-t/RC})$

At $t = t_p$, $v_o = V(1 - e^{-t_p/RC}) = V(1 - e^{-1/0.1591}) = 0.998 \ V$

The output wave form is shown in Figure 1.10(b).

(c) When f_2 = 0.1 MHz:

$$RC = \frac{1}{2\pi f_2} = \frac{1}{2\pi \times 0.1 \times 10^6} = 1.591 \ \mu s$$

So t_p = 1 μs and RC = 1.591 μs, RC is comparable to t_p.

Since the time constant is comparable to the pulse width, in the interval $0 < t < t_p$, the capacitor charges exponentially according to the equation

$$v_o = V(1 - e^{-t/RC})$$

At $t = t_p$, the output voltage $v_o = V_p$

$$\therefore \qquad V_p = V(1 - e^{-t_p/RC}) = V(1 - e^{-1/1.591}) = 0.4668V$$

For $t > t_p$, v_o decreases according to the equation

$$v_o = V_p e^{-(t-t_p)/RC} = 0.4668 V e^{-(t-1)/1.59}$$

The output waveform is shown in Figure 1.10(c).

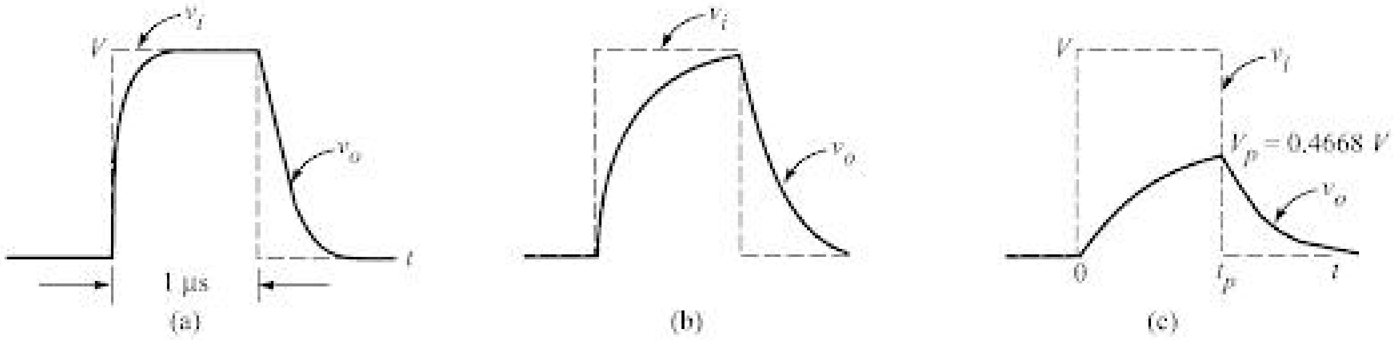

Figure 1.10 Example 1.3: (a), (b) and (c) output waveforms.

EXAMPLE 1.4 A pulse is applied to a low-pass *RC* circuit. Prove by direct integration that the area under the pulse is the same as the area under the output waveform across the capacitor. Explain the result physically.

Solution: A pulse shown by the dotted line in Figure 1.11(b) is applied to the *RC* low-pass circuit shown in Figure 1.11(a). The output waveform is shown by the thick line in Figure 1.11(b).

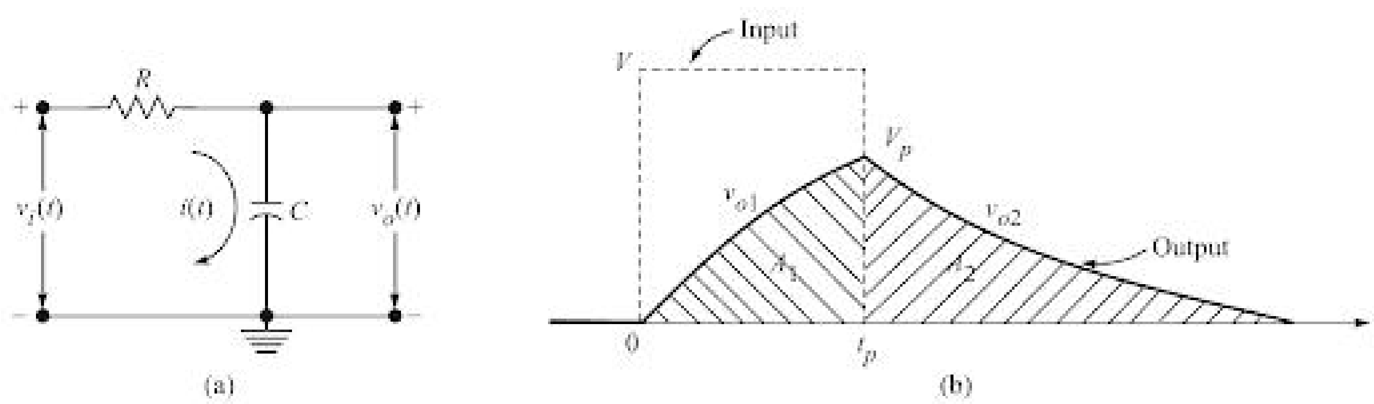

Figure 1.11 Example 1.4: (a) circuit diagram and (b) output waveform.

During the pulse duration $0 < t < t_p$:
Area under the output waveform is

$$A_1 = \int_0^{t_p} v_{o1}(t)\cdot dt = \int_0^{t_p} V(1 - e^{-t/RC})\,dt = V\Big[t + RCe^{-t/RC}\Big]_0^{t_p}$$

$$= Vt_p - VRC(1 - e^{-t_p/RC}) = Vt_p - RCV_p$$

where $\quad V_p = V(1 - e^{-t_p/RC})$

For $t > t_p$, the area under the output waveform is

$$A_2 = \int_{t_p}^{\infty} v_{o2}(t)\,dt = \int_{t_p}^{\infty} V_p e^{-(t-t_p)/RC} \cdot dt$$

$$= (-V_p)(-RC)\left[e^{-(t-t_p)/RC}\right]_{t_p}^{\infty}$$

$$= RCV_p$$

∴ The total area under the output waveform

$$A = A_1 + A_2 = Vt_p - V_pRC + V_pRC = Vt_p$$

= The area under the pulse

Since the average voltage across R is zero, the dc voltage at the output is the same as that at the input. So, the area under the output waveform is the same as that under the pulse.

EXAMPLE 1.5 Sketch the output waveforms for an RC integrating circuit when: (a) $\tau = 10t_p$, (b) $\tau = t_p$, and (c) $\tau = 0.1t_p$.

Solution: The pulse shown by the dotted line in Figure 1.12(b) is applied to the RC integrating circuit shown in Figure 1.12(a).

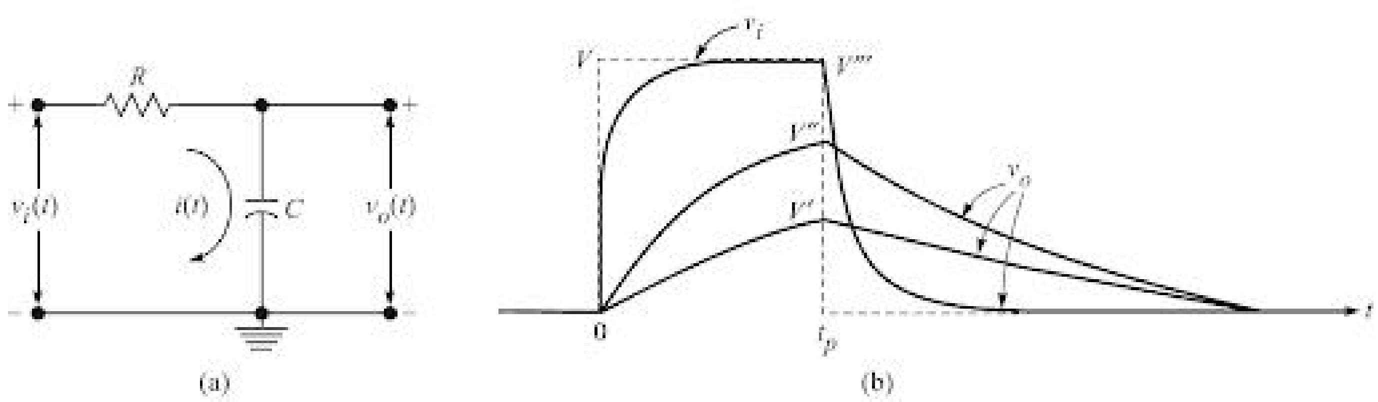

Figure 1.12 Example 1.5: (a) circuit diagram and (b) output waveforms.

(a) $\tau = 10t_p$:

The RC time constant is very large compared to the pulse width and so the capacitor charges slowly to a voltage V' at the end of the pulse given by

$$V' = V(1 - e^{-t_p/\tau}) = V(1 - e^{-t_p/10t_p}) = V(1 - e^{-0.1}) = 0.095\,V$$

or to 9.5% of the pulse amplitude.

(b) $\tau = t_p$:

The RC time constant is comparable to the pulse width and so the capacitor charges gradually to a voltage V'' given by

$$V'' = V(1 - e^{-t_p/t_p}) = V(1 - e^{-1}) = 0.632\,V$$

(c) $\tau = 0.1t_p$:

The RC time constant is very small compared to the pulse width and so the capacitor charges rapidly to a voltage V''' given by

$$V''' = V(1 - e^{-t_p/0.1t_p}) = V(1 - e^{-10}) \approx V$$

with a rise time $t_r = 2.2RC = 0.22t_p$

The output waveforms are sketched in Figure 1.12(b).

EXAMPLE 1.6 A symmetrical square wave of amplitude ± 5 V and frequency 2 kHz is impressed on an RC low-pass circuit. If $R = 5$ kΩ, $C = 0.1$ μF, calculate and plot the steady-state output with respect to time.

Solution: When the input waveform shown by the dotted line in Figure 1.13(b) is applied to the RC low-pass circuit of Figure 1.13(a), the output waveform shown by the thick line in Figure 1.13(b) results.

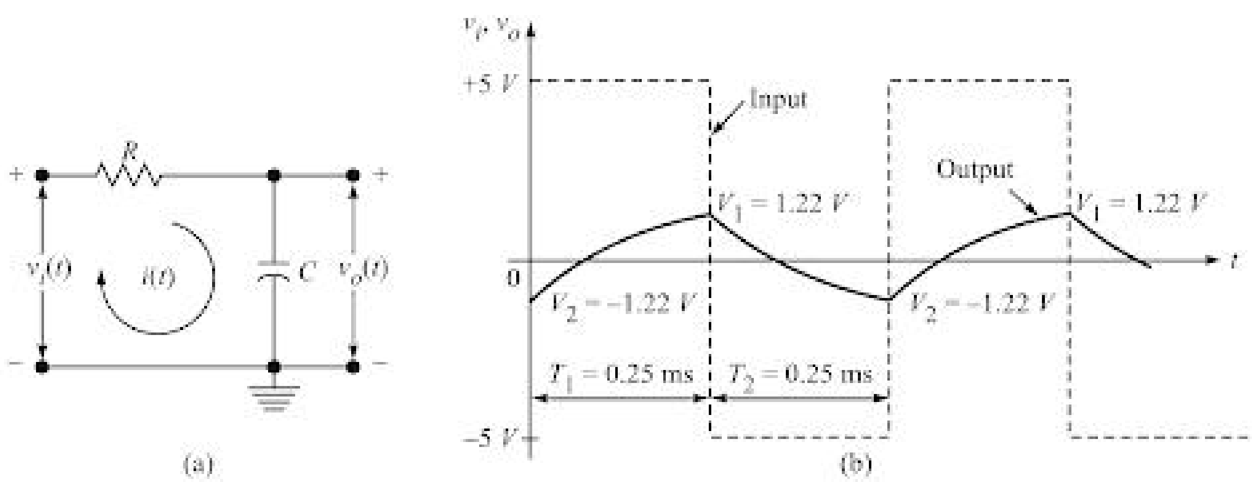

Figure 1.13 Example 1.6: (a) circuit diagram and (b) output waveforms.

Given $\qquad f = 2$ kHz, $T = \dfrac{1}{2\,\text{kHz}} = 0.5$ ms

$$\therefore \qquad T_1 = T_2 = \frac{T}{2} = \frac{0.5\,\text{ms}}{2} = 0.25 \text{ ms}$$

The time constant of the circuit $\tau = RC = 5 \times 10^3 \times 0.1 \times 10^{-6} = 0.5$ ms

Since RC is comparable to $\dfrac{T}{2}$, the capacitor charges and discharges exponentially.

Since the input is a symmetrical square wave

$$V_1 = \frac{V}{2}\left(\frac{1 - e^{-T/2RC}}{1 + e^{-T/2RC}}\right) \quad \text{where } V \text{ is the peak-to-peak value of the input.}$$

$$= \frac{10}{2}\left(\frac{1 - e^{-0.5/1}}{1 + e^{-0.5/1}}\right) = \frac{5 \times 0.393}{1.606} = 1.22 \text{ V}$$

$$V_2 = -V_1 = -1.22 \text{ V}$$

$\therefore$ Peak-to-peak value of output = 1.22 V − (−1.22V) = 2.44 V.

EXAMPLE 1.7 A symmetrical square wave whose average value is zero has a peak-to-peak amplitude of 20 V and a period of 2 μs. This waveform is applied to a circuit whose upper 3 dB frequency is $\frac{1}{2\pi}$ MHz. Calculate and sketch the steady-state output waveform. In particular what is the peak-to-peak output amplitude?

Solution: Since the upper cut-off frequency is specified, the circuit must be a low-pass filter.

$$f_2 = \frac{1}{2\pi RC}$$

$$\therefore \qquad RC = \frac{1}{2\pi f_2} = \frac{1}{2\pi \times \frac{1}{2\pi} \times 10^6} = 1 \text{ μs}$$

When the input waveform shown by dotted lines in Figure 1.14(b) is applied to the RC low-pass circuit of Figure 1.14(a), the output waveform shown by thick lines in Figure 1.14(b) results.

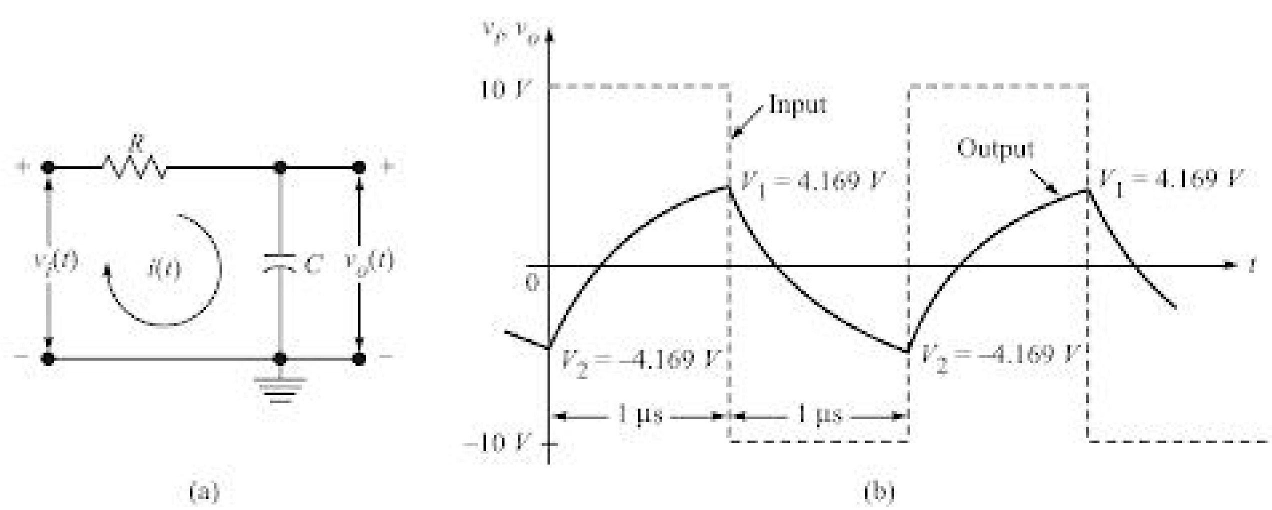

Figure 1.14 Example 1.7: (a) circuit diagram and (b) output waveforms.

Since RC is comparable to $T/2$, the output rises and falls exponentially. Since the input is a symmetrical square wave

$$V_1 = \frac{V}{2}\left(\frac{1 - e^{-T/2RC}}{1 + e^{-T/2RC}}\right)$$

$$= \frac{20}{2}\left(\frac{1 - e^{-1\mu s/1\mu s}}{1 + e^{-1\mu s/1\mu s}}\right)$$

$$= 10\left(\frac{1-0.368}{1+0.368}\right)$$

$$= 4.619 \text{ V}$$

$$V_2 = -V_1 = -4.619 \text{ V}$$

$\therefore$ Peak-to-peak value of output = 4.619 V – (– 4.619 V) = 9.238 V

EXAMPLE 1.8 A symmetrical square wave whose peak-to-peak amplitude is 2 V and whose average value is zero is applied to an *RC* integrating circuit. The time constant of the circuit is equal to half the period of the square wave. Find the peak-to-peak value of the output amplitude.

Solution: The input waveform shown by dotted line in Figure 1.15(b) is applied to the *RC* integrating circuit shown in Figure 1.15(a). Since the time constant *RC* of the circuit is comparable to the period of the input waveform, the output voltage grows considerably. Since the input is a symmetrical square wave with zero dc value, the output must also be symmetrical with respect to the zero level and the positive and negative peaks of the output must be equal in amplitude and opposite in sign. The output waveform under steady-state conditions is shown in Figure 1.15(b).

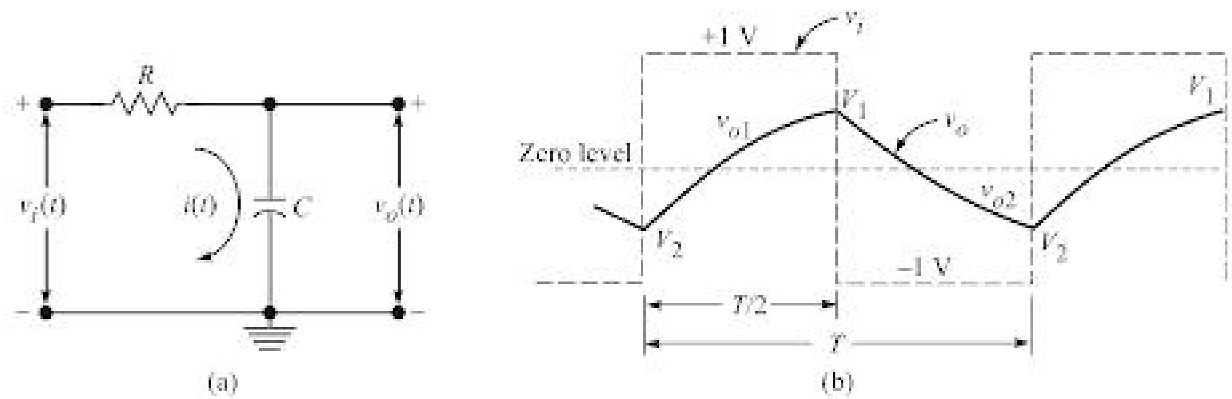

Figure 1.15 Example 1.8: output waveform.

Given $$RC = T/2$$

$$V_1 = \frac{V}{2}\tanh x \text{ where } x = T/4RC$$

or $$V_1 = -V_2 = \frac{V}{2}\left(\frac{1-e^{-T/2RC}}{1+e^{-T/2RC}}\right) = \frac{2}{2}\left(\frac{1-e^{-T/(2T/2)}}{1+e^{-T/(2T/2)}}\right) = \frac{1-e^{-1}}{1+e^{-1}} = 0.4647 \text{ V}$$

$\therefore$ $$V_2 = -0.4647 \text{ V}$$

Peak-to-peak value of the output = $V_1 - V_2 = 2V_1 = 2 \times 0.4647 = 0.9294$ V

EXAMPLE 1.9 A square wave whose peak-to-peak amplitude is 2 V extends ±1 V with respect to ground. The duration of the positive section is 0.1 s and that of the negative section 0.2 s. If this waveform is impressed upon an *RC* integrating circuit whose time constant is 0.2 s, what are the steady-state maximum and minimum values of the output waveform?

Solution: When the input waveform shown by dotted lines in Figure 1.16(b) is applied to the *RC* integrating circuit shown in Figure 1.16(a) under steady-state conditions the output waveform is as shown by the thick line in Figure 1.16.

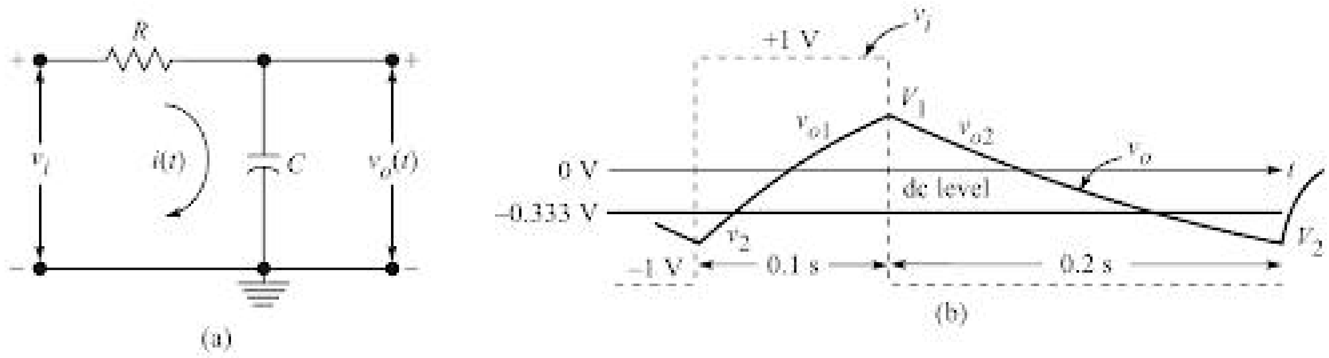

Figure 1.16 Example 1.9: (a) circuit diagram and (b) output waveform.

Given $\quad T_{ON} = 0.1$ s, $T_{OFF} = 0.2$ s and $RC = 0.2$ s

The average value of the input

$$V_{dc} = \frac{(1 \times 0.1) + (-1 \times 0.2)}{0.1 + 0.2} = -0.333 \text{ V}$$

During the positive swing when input is +1 V, the output voltage

$$v_{o1}(t) = 1 - (1 - V_2)e^{-t/RC}$$

At $\quad t = 0.1$ s, $v_{o1}(t) = V_1 = 1 - (1 - V_2)e^{-0.1/0.2} = 1 - 0.606(1 - V_2)$

i.e. $\quad V_1 - 0.606V_2 = 0.394 \qquad$ (i)

During the negative swing when input is −1 V, the output voltage

$$v_{o2} = -1 - (-1 - V_1)e^{-(t-0.1)/RC}$$

At $\quad t = 0.3$ s, $v_{o2} = V_2 = -1 - (-1 - V_1)e^{-0.2/0.2} = -1 + 0.3678(1 + V_1)$

i.e. $\quad 0.3678V_1 - V_2 = 0.6322 \qquad$ (ii)

Solving equations (i) and (ii) $0.3678(0.394 + 0.606\ V_2) - V_2 = 0.6322$

$\therefore \quad V_2 = 0.487/(-0.777) = -0.6271$ V

$V_1 = 0.394 + 0.606\ V_2 = 0.394 + 0.606(-0.6271) = 0.014$ V

we get $\quad V_1 = 0.014$ V and $V_2 = -0.6271$ V

The steady-state maximum value of output $V_1 = 0.014$ V. The steady-state minimum value of output $V_2 = -0.6271$ V.

$\therefore$ Peak-to-peak output voltage $= V_1 - V_2 = 0.014 - (-0.6271) = 0.6411$ V

EXAMPLE 1.10 The periodic waveform shown in Figure 1.17(b) is applied to an *RC* integrating network shown in Figure 1.17(a) whose time constant is 10 μs. Sketch the output waveform. Calculate the maximum and minimum values of the output voltage with respect to ground under steady-state conditions. Also, calculate and plot the output for the first two cycles of the input.

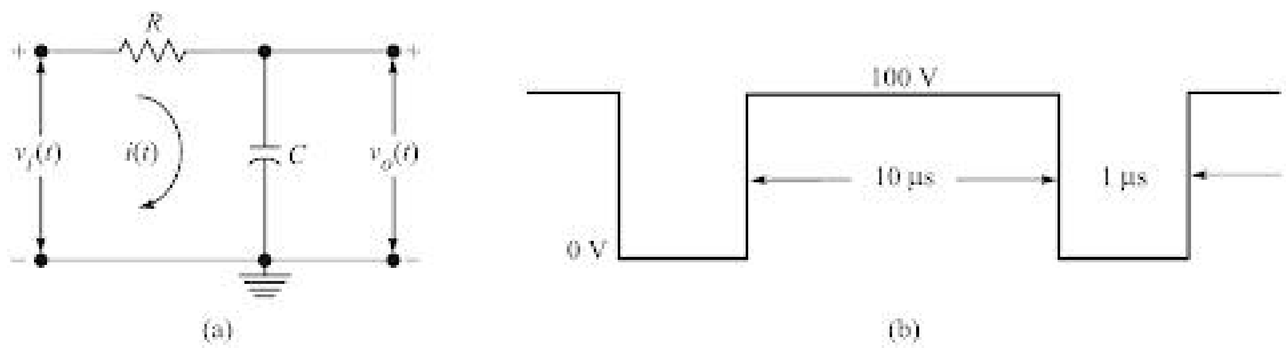

Figure 1.17 Example 1.10: (a) circuit diagram and (b) input waveform.

Solution: The input waveform shown in Figure 1.17(b) is applied to the *RC* integrating circuit shown in Figure 1.17(a).

Steady-state analysis. Under steady-state conditions, the capacitor charges and discharges to the same level in each cycle, i.e. V_1 and V_2 are constants.

Given $$RC = 10\ \mu s,\ T_{ON} = 10\ \mu s,\ T_{OFF} = 1\ \mu s$$

Since the time constant is comparable to the period of the waveform, the capacitor charges and discharges gradually and the output waveform is as shown in Figure 1.18.

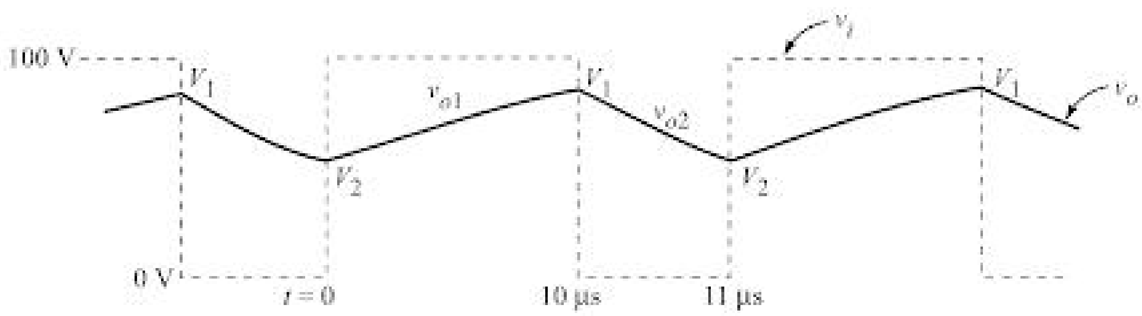

Figure 1.18 Example 1.10: output waveform under steady state.

In the interval $0 < t < 10$ μs, when input = 100 V

$$v_{o1} = 100 - (100 - V_2)e^{-t/10}$$

At $$t = 10\ \mu s,\ v_{o1} = V_1 = 100 - (100 - V_2)e^{-10/10}$$

i.e. $$V_1 - 0.368V_2 = 63.212 \quad \text{(i)}$$

In the interval 10 μs $< t <$ 11 μs, when input = 0 V

$$v_{o2} = 0 - (0 - V_1)e^{-(t-10)/RC}$$

At $$t = 11\ \mu s,\ v_{o2} = V_2 = V_1e^{-1/10} = 0.905V_1 \quad \text{(ii)}$$

Solving Eqs. (i) and (ii),

$$V_1 - 0.368(0.905V_1) = 63.212$$

$$\therefore \qquad V_1 = \frac{63.212}{1 - 0.368 \times 0.905} = 94.91 \text{ V}$$

and

$$V_2 = 0.905V_1 = 0.905 \times 94.91 = 85.07 \text{ V}$$

$\therefore$ The maximum value of output voltage V_1 = 94.91 V

The minimum value of output voltage V_2 = 85.07 V

Transient analysis. The output waveform for the first two cycles of the input waveform is shown in Figure 1.19.

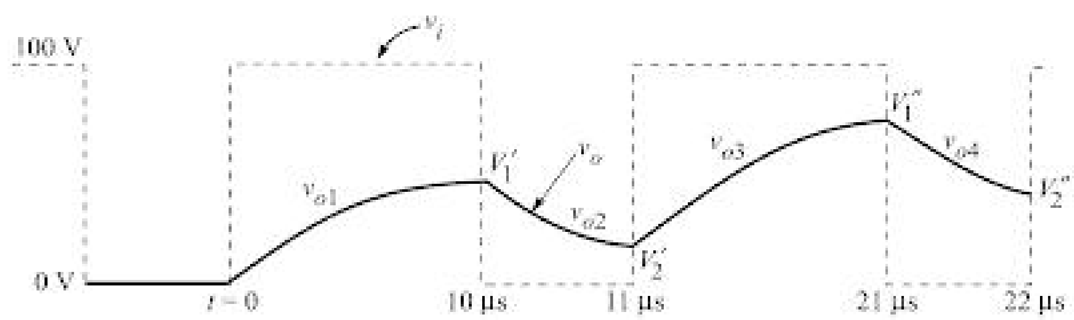

Figure 1.19 Example 1.10: transient response.

Let the capacitor be initially uncharged, i.e. v_o = 0 V at t = 0. Since the voltage across the capacitor cannot change instantaneously, the output voltage rises from 0 V at t = 0, goes exponentially to V_1' at t = 10 μs, to V_2' at t = 11 μs, to V_1'' at t = 21 μs and to V_2'' at t = 22 μs, and so on, as shown in Figure 1.19.

For $0 < t < 10$ μs, $\quad v_{o1} = 100 - (100 - 0)e^{-t/10}$

At t = 10 μs $\quad v_{o1} = V_1' = 100 - (100 - 0)e^{-10/10} = 100 - 36.78 = 63.22$ V

For 10 μs $< t <$ 11 μs, $\quad v_{o2} = 0 - (0 - V_1')e^{-(t-10)/10}$

At t = 11 μs, $\quad v_{o2} = V_2' = 0 - (0 - V_1')e^{-1/10} = 63.22\ e^{-0.1} = 57.203$ V

For 11 μs $< t <$ 21 μs, $\quad v_{o3} = 100 - (100 - V_2')e^{-(t-11)/10}$

At t = 21 μs, $\quad v_{o3} = V_1'' = 100 - (100 - V_2')e^{-10/10} = 84.255$ V

For 21 μs $< t <$ 22 μs, $\quad v_{o4} = 0 - (0 - V_1'')e^{-(t-21)/10}$

At t = 22 μs, $\quad v_{o4} = V_2'' = 0 - (0 - V_1'')e^{-1/10} = 84.255e^{-0.1} = 76.23$ V

EXAMPLE 1.11 For the circuit and the input shown in Figure 1.20:

(a) Determine the level of v_o and I_C at t = 2.5 ms. (b) Sketch the settled waveform of V_C.

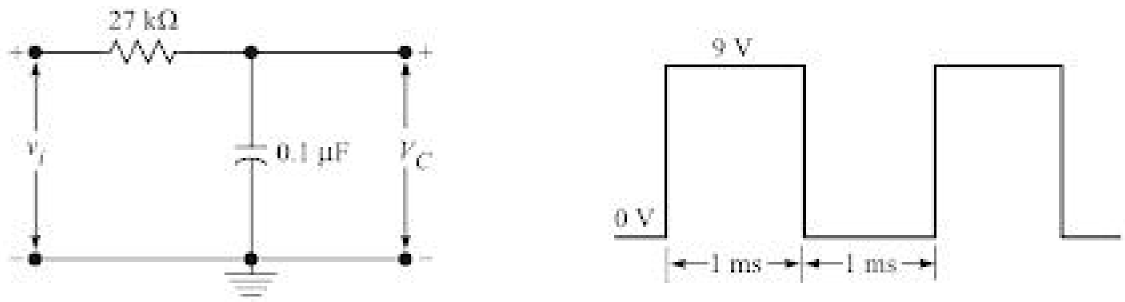

Figure 1.20 Example 1.11: (a) circuit diagram and (b) the input waveform.

Solution: The waveform shown in Figure 1.20(b) is applied to the RC low-pass circuit shown in Figure 1.20(a). Given $T_{ON} = 1$ ms, $T_{OFF} = 1$ ms, $T = 2$ ms. The time constant of the circuit is $RC = 27 \times 10^3 \times 0.1 \times 10^{-6} = 2.7$ ms.

Since the time constant is comparable to the period of the waveform, the output rises gradually.

Transient analysis. The transient response of the circuit is shown in Figure 1.21. Initially the capacitor is uncharged, so the output starts from zero at $t = 0$. The output rises to V' at $t = 1$ ms, falls to V'' at $t = 2$ ms and rises to V''' at $t = 2.5$ ms.

$$V' = 9 - (9 - 0)e^{-1/2.7} = 9(1 - e^{-1/2.7}) = 2.785 \text{ V}$$

$$V'' = 0 - (0 - V')e^{-1/2.7} = 2.785e^{-1/2.7} = 1.923 \text{ V}$$

$$V''' = 9 - (9 - V'')e^{-0.5/2.7} = 9 - (9 - 1.923)e^{-0.5/2.7} = 3.119 \text{ V}$$

$\therefore$ v_o at $t = 2.5$ ms is $V''' = 3.119$ V, and I_C at $t = 2.5$ ms is

$$I_C = \frac{9 - V'''}{R} = \frac{9 - 3.119}{27} = 0.218 \text{ mA}$$

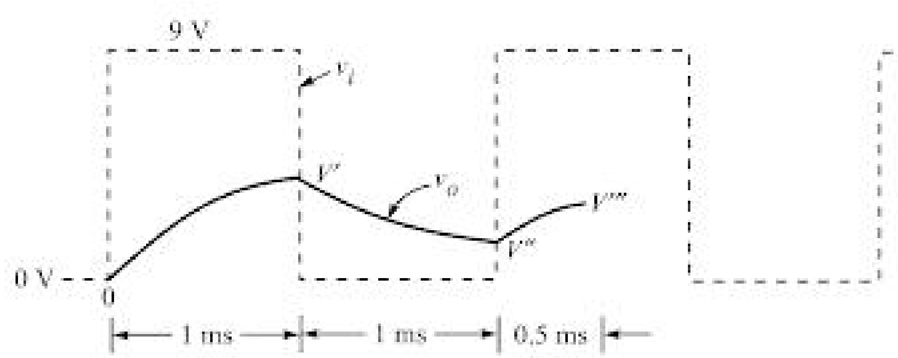

Figure 1.21 Example 1.11: transient response.

Steady-state analysis. Under steady state, the capacitor charges and discharges to the same level in each cycle. So the shape of the output waveform in each cycle is fixed. The steady-state output waveform is shown in Figure 1.22.

For $0 < t < 1$ ms, $\quad v_{o1} = 9 - (9 - V_2)e^{-t/2.7}$

At $t = 1$ ms, $\quad v_{o1} = V_1 = 9 - (9 - V_2)e^{-1/2.7}$

$\therefore \quad V_1 = 2.785 + 0.69V_2$

For 1 ms < t < 2 ms, $\quad v_{o2} = 0 - (0 - V_1)e^{-(t-1)/2.7}$

At t = 2 ms, $\quad v_{o2} = V_2 = V_1 e^{-1/2.7} = 0.69V_1$

$\therefore \quad V_1 = 2.785 + 0.69 \times 0.69V_1$

So, $\quad V_1 = \dfrac{2.785}{0.524} = 5.314 \text{ V}$

and $\quad V_2 = 0.69V_1 = 0.69 \times 5.314 = 3.667 \text{ V}$

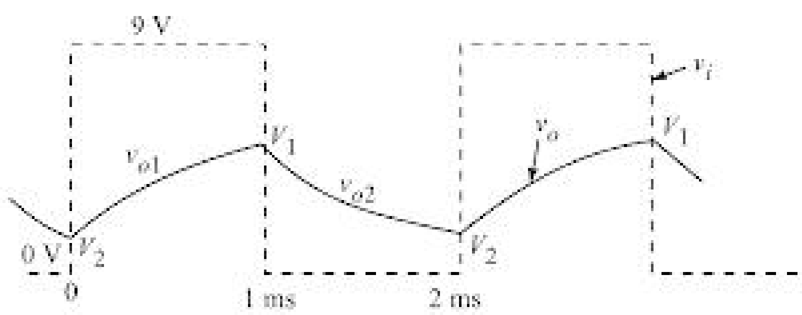

Figure 1.22 Example 1.11: steady-state response.

EXAMPLE 1.12 The square wave shown in Figure 1.23 is fed to an RC integrating circuit. Compute and plot the output waveforms if (a) RC is large, say, $RC = 2.5T$, and (b) RC is small, say $RC = T/2.5$.

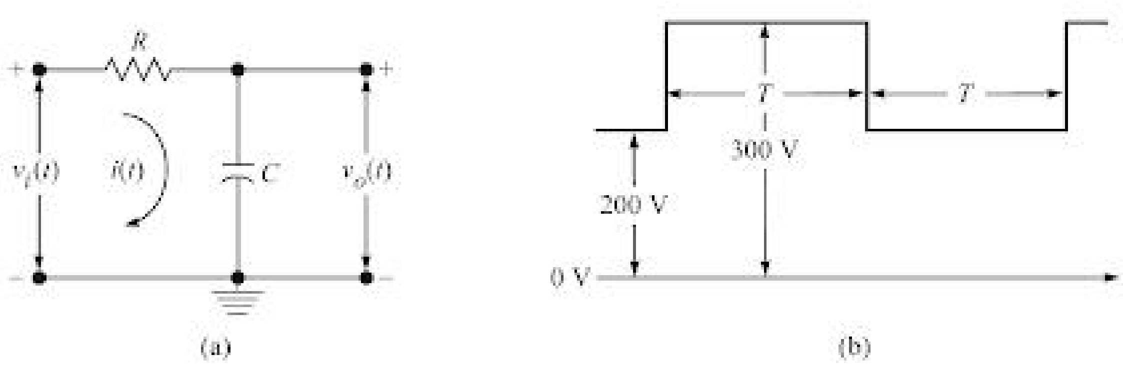

Figure 1.23 Example 1.12: (a) circuit diagram and (b) input waveform.

Solution: The waveform shown in Figure 1.23(b) is applied to the RC integrating circuit shown in Figure 1.23(a).

(a) When $RC = 2.5T$, the capacitor charges slowly. The transient response is as shown in Figure 1.24. Assuming that the capacitor is uncharged initially,

$$V_1 = 300 - (300 - 0)e^{-T/2.5T} = 98.91 \text{ V}$$
$$V_2 = 200 - (200 - 98.91)e^{-T/2.5T} = 132.239 \text{ V}$$
$$V_3 = 300 - (300 - 132.239)e^{-T/2.5T} = 187.55 \text{ V}$$
$$V_4 = 200 - (200 - 187.55)e^{-T/2.5T} = 191.654 \text{ V}$$
$$V_5 = 300 - (300 - 191.654)e^{-T/2.5T} = 227.37 \text{ V}$$

$$V_6 = 200 - (200 - 227.37)e^{-T/2.5T} = 218.35 \text{ V}$$
$$V_7 = 300 - (300 - 218.35)e^{-T/2.5T} = 245.27 \text{ V}$$

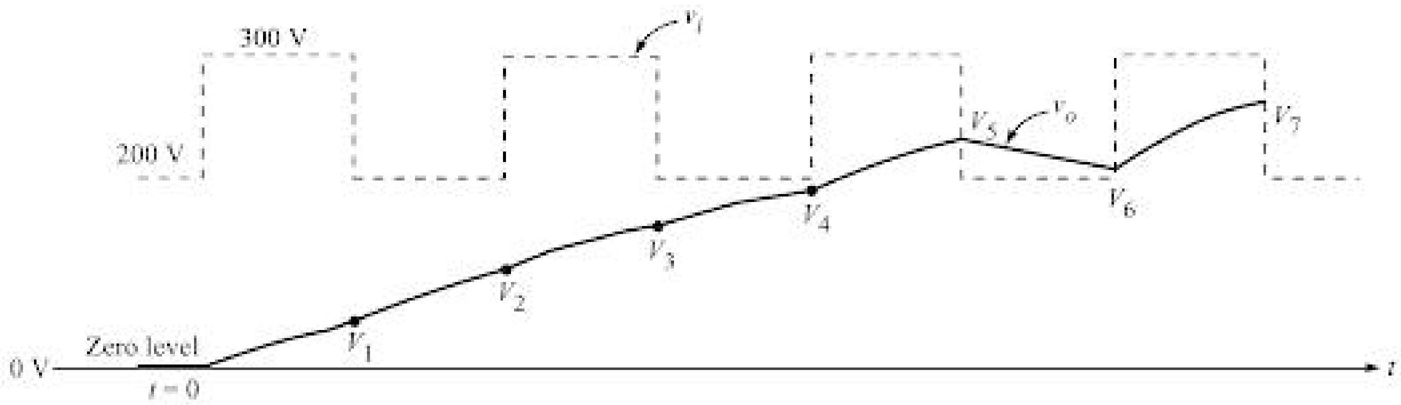

Figure 1.24 Example 1.12: transient response when $RC = 2.5T$.

The steady-state response for $RC = 2.5T$ is shown in Figure 1.25. Under steady-state conditions, the shape of the waveform is fixed because the capacitor charges and discharges to the same levels in each cycle, i.e.

$$V_1 = 300 - (300 - V_2)e^{-T/2.5T} = 300 - (300 - V_2) \times 0.670 = 99 + 0.67V_2$$
$$V_2 = 200 - (200 - V_1)e^{-T/2.5T} = 200 - (200 - V_1) \times 0.670 = 66 + 0.67V_1$$

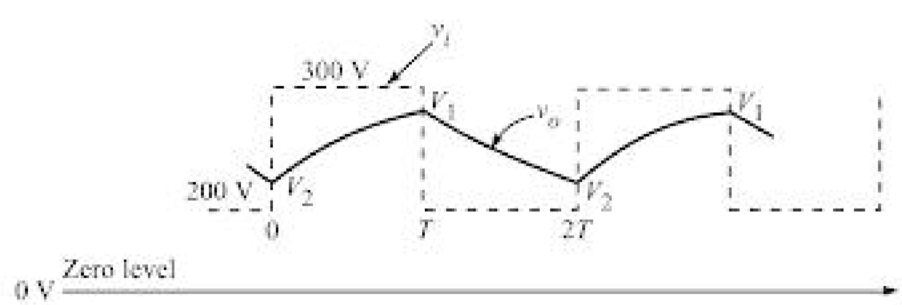

Figure 1.25 Example 1.12: steady-state response when $RC = 2.5T$.

Solving the above equations for V_1 and V_2,

$$V_1 = 99 + 0.67(66 + 0.67\ V_1)$$
$$\therefore \quad V_1 = 143.22/0.5511 = 259.88 \text{ V}$$
$$V_2 = 66 + 0.67(259.88) = 240.12 \text{ V}$$

The maximum and minimum values of output are

$$V_1 = 259.88 \text{ V} \quad \text{and} \quad V_2 = 240.12 \text{ V}$$

(b) When $RC = T/2.5 = 0.4T$, the capacitor charges and discharges rapidly. The transient response will be as shown in Figure 1.26. Assuming that the capacitor is uncharged initially,

$$V_1 = 300 - (300 - 0)e^{-T/0.4T} = 275.37 \text{ V}$$
$$V_2 = 200 - (200 - 275.37)e^{-T/0.4T} = 206.2 \text{ V}$$

$$V_3 = 300 - (300 - 206.2)e^{-T/0.4T} = 292.3 \text{ V}$$
$$V_4 = 200 - (200 - 292.3)e^{-T/0.4T} = 207.56 \text{ V}$$
$$V_5 = 300 - (300 - 207.56)e^{-T/0.4T} = 292.42 \text{ V}$$
$$V_6 = 200 - (200 - 292.42)e^{-T/0.4T} = 207.57 \text{ V}$$
$$V_7 = 300 - (300 - 207.57)e^{-T/0.4T} = 292.42 \text{ V}$$

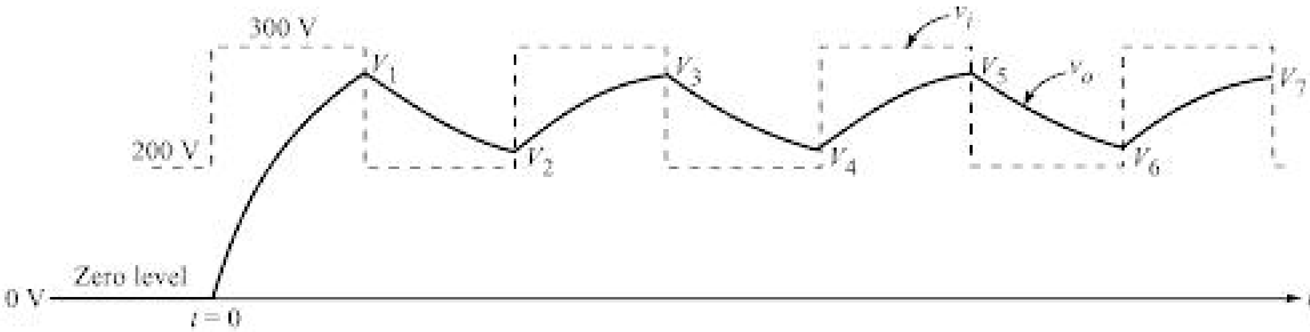

Figure 1.26 Example 1.12: transient response when $RC = T/2.5$.

The steady-state response for $RC = T/2.5$ is shown in Figure 1.27.

$$V_1 = 300 - (300 - V_2)e^{-T/0.4T} = 275.37 + 0.082V_2$$
$$V_2 = 200 - (200 - V_1)e^{-T/0.4T} = 183.6 + 0.082V_1$$

Solving the equations for V_1 and V_2,

$$V_1 = 275.37 + 0.082(183.6 + 0.082\ V_1) = 290.425 + 0.0067\ V_1$$
$$\therefore \quad V_1 = 290.425/0.993 = 292.37 \text{ V}$$
$$V_2 = 183.6 + 0.082 \times 292.37 = 207.5 \text{ V}$$

So the steady-state maximum and minimum values of output are

$$V_1 = 292.37 \text{ V} \quad \text{and} \quad V_2 = 207.5 \text{ V}$$

These results indicate that when the time constant is small, the output reaches its steady values very quickly.

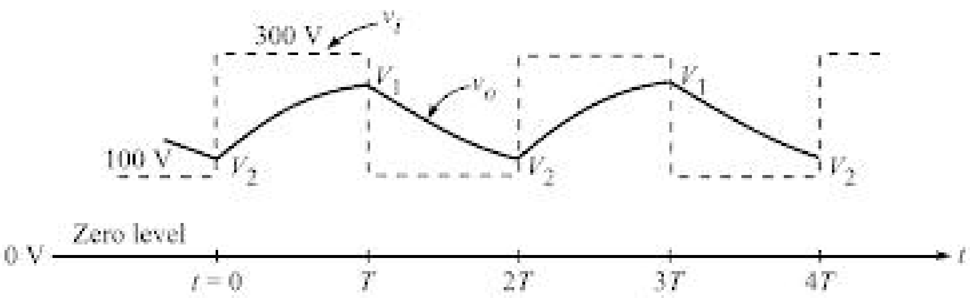

Figure 1.27 Example 1.12: steady-state response for $RC = 0.4T$.

EXAMPLE 1.13 Assuming that the capacitor is initially uncharged, determine the output response of the low-pass RC circuit with time constant 0.05 ms, to the input waveform shown in Figure 1.28(a).

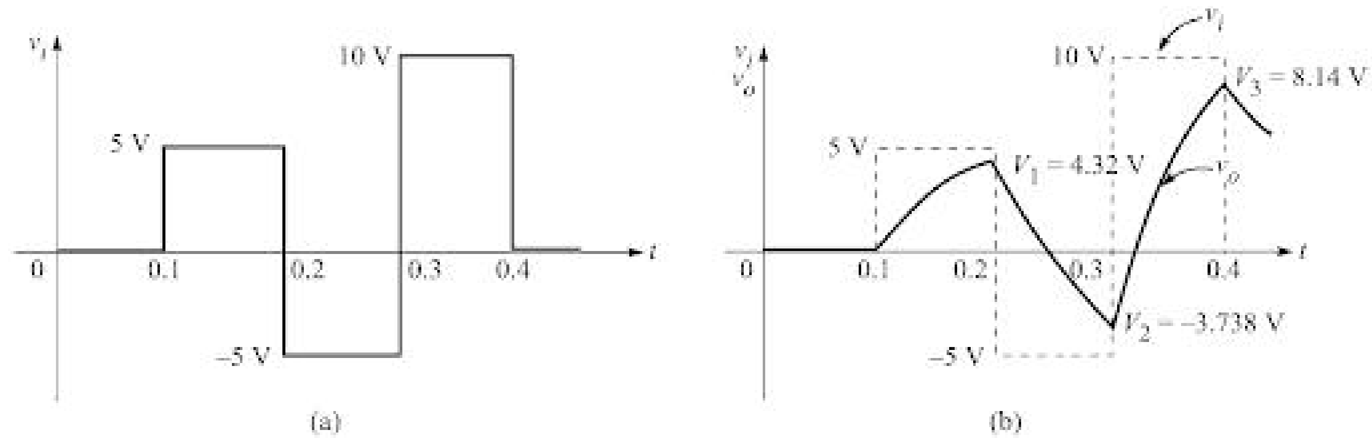

Figure 1.28 Example 1.13: (a) input waveform and (b) output waveform.

Solution: The input waveform shown in Figure 1.28(a) is applied to the *RC* low-pass circuit shown in Figure 1.1. The output waveform is as shown in Figure 1.28(b).

For $0 < t < 0.1$ ms, $\quad v_i = 0 \quad \therefore \; v_o = 0$

For 0.1 ms $< t < 0.2$ ms, $\quad v_o = 5 - (5 - 0)e^{-(t-0.1)/0.05}$

At $t = 0.2$ ms, $\quad v_o = V_1 = 5(1 - e^{-0.1/0.05}) = 4.32$ V

For 0.2 ms $< t < 0.3$ ms, $\quad v_o = -5 - (-5 - 4.32)e^{-(t-0.2)/0.05}$

At $t = 0.3$ ms, $\quad v_o = V_2 = -5 + 9.32e^{-0.1/0.05} = -3.738$ V

For 0.3 ms $< t < 0.4$ ms, $\quad v_o = 10 - (10 - (-3.738))e^{-(t-0.3)/0.05}$

At $t = 0.4$ ms, $\quad v_o = V_3 = 10 - 13.738e^{-0.1/0.05} = 8.14$ V

EXAMPLE 1.14 The periodic ramp voltage shown in Figure 1.29(a) is applied to a low-pass *RC* circuit.

(a) Find the equations from which to determine the steady-state output waveform.
(b) If $T_1 = T_2 = RC$, find the maximum and minimum values of the output voltage and plot the waveform.
(*Note:* The minimum value does not occur at the beginning of interval T_1).

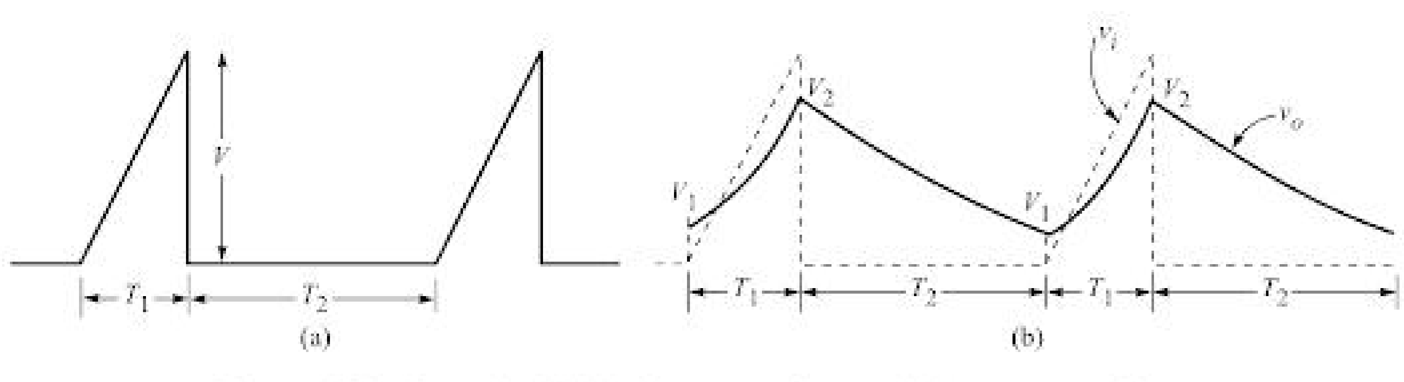

Figure 1.29 Example 1.14: (a) input waveform and (b) output waveform.

Solution: The input waveform shown in Figure 1.29(a) is applied to the *RC* low-pass circuit shown in Figure 1.1. Under steady-state conditions, $V_1 \neq 0$, the capacitor charges from V_1 to V_2 in the interval 0 to T_1. During that interval, $v_i(t) = \alpha t$. Writing KVL for the low-pass RC circuit, we have

$$v_i(t) = \alpha t = Ri(t) + \frac{1}{C}\int i(t)\,dt$$

Differentiating,
$$\frac{dv_i(t)}{dt} = \alpha = R\frac{di(t)}{dt} + \frac{1}{C}i(t)$$

Taking Laplace transform on both sides,

$$\frac{\alpha}{s} = R[sI(s) - I(0^+)] + \frac{1}{C}I(s)$$

$$= RI(s)\left[s + \frac{1}{RC}\right] - RI(0^+), \quad \text{where } I(0^+) = \frac{0 - V_1}{R}$$

$$\therefore \quad I(s) = \frac{\alpha}{Rs\left[s + \frac{1}{RC}\right]} + \frac{I(0^+)}{s + \frac{1}{RC}} = \frac{\alpha}{Rs\left[s + \frac{1}{RC}\right]} - \frac{V_1}{R\left[s + \frac{1}{RC}\right]}$$

Output,
$$V_o(s) = V_i(s) - I(s)R = \frac{\alpha}{s^2} - \frac{\alpha}{s\left[s + \frac{1}{RC}\right]} + \frac{V_1}{s + \frac{1}{RC}}$$

Taking the inverse Laplace transform on both sides,

$$v_o(t) = \alpha t - \alpha RC(1 - e^{-t/RC}) + V_1 e^{-t/RC}$$

When ramp is present, the slope $\alpha = \dfrac{V}{T_1}$

$$\therefore \quad v_o(t) = \frac{V}{T_1}t - \frac{V}{T_1}RC(1 - e^{-t/RC}) + V_1 e^{-t/RC}$$

$v_o(t) = V_2$ at $t = T_1$, i.e. the capacitor charges from V_1 to V_2 in time T_1. Between T_1 and $T_1 + T_2$, i.e. during T_2 when the input is zero, the capacitor discharges from V_2 to V_1 according to the equation

$$v_o(t) = V_2 e^{-t/RC}$$

(b) When $T_1 = T_2 = RC$, the minimum value of output does not occur at $t = 0$, that is, V_1 is not the minimum.

At $t = T_1$, $v_o(t) = V_2$

i.e.
$$V_2 = \frac{V}{T_1} \times T_1 - \frac{V}{T_1} \times T_1(1 - e^{-1}) + V_1 e^{-1} = (V + V_1)\,e^{-1}$$

When the ramp input is reduced to zero,

$$V_1 = V_2 e^{-T_2/RC} = V_2 e^{-1}$$

i.e. $$V_1 = (V + V_1)e^{-2} \quad \text{or} \quad V_1 = \frac{Ve^{-2}}{1 - e^{-2}} = 0.1563V$$

$\therefore$ $$V_2 = (V_2 e^{-1} + V)e^{-1} = V_2 e^{-2} + Ve^{-1}$$

$\therefore$ $$V_2 = \frac{Ve^{-1}}{1 - e^{-2}} = 0.425V$$

When the minimum value of output occurs, $\dfrac{dv_o(t)}{dt} = 0$

Since $$v_o(t) = \alpha t + V_1 e^{-t/RC} - \alpha RC(1 - e^{-t/RC})$$

$\therefore$ $$\frac{dv_o(t)}{dt} = \alpha - \frac{V_1}{RC} e^{-t/RC} - \frac{\alpha RC}{RC} e^{-t/RC} = 0$$

i.e. $$\frac{V}{T_1} - \frac{V_1}{T_1} e^{-t/RC} - \frac{V}{T_1} e^{-t/RC} = 0$$

or $$\frac{V}{T_1} - e^{-t/RC}\left(\frac{V_1}{T_1} + \frac{V}{T_1}\right) = 0$$

or $$(V_1 + V)e^{-t/RC} = V$$

i.e. $$e^{-t/RC} = \frac{V}{V + V_1} = \frac{V}{V + 0.1563V} = 0.865$$

$\therefore$ $$\frac{t}{RC} = 0.145$$

Substituting this value of $e^{-t/RC}$ in the expression for $v_o(t)$,

$$V_o(\text{min}) = V \times 0.145 - V(1 - 0.865) + 0.1563V \times 0.865$$

i.e. $$V_o(\text{min}) = 0.145V$$

The output waveform is shown in Figure 1.29(b).

EXAMPLE 1.15 Prove that an RC low-pass circuit behaves as a reasonably good integrator if $RC > 15\ T$, where T is the period of the input sinusoid $E_m \sin \omega t$.

Solution: Consider the RC low-pass circuit shown in Figure 1.1.

It has been shown earlier that for a large time constant the output and input voltages of an RC low-pass circuit are related as

$$v_o = \frac{1}{RC} \int v_i dt$$

Given $v_i = E_m \sin \omega t$

$$v_o = \frac{1}{RC}\int E_m \sin \omega t \, dt$$

$$= -\left(\frac{E_m}{RC\omega}\right) \cos \omega t$$

$$= -\left(\frac{E_m}{RC\omega}\right) \sin \left(\omega t - \frac{\pi}{2}\right)$$

From the circuit diagram we have

$$v_i = i(R - jX_C) \text{ and } v_o = -ijX_C$$

$$\therefore \qquad \frac{v_o}{v_i} = \frac{-jX_C}{R - jX_C} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

$$\therefore \qquad v_o = \left(\frac{1}{1 + j\omega RC}\right) v_i = \frac{E_m \sin \omega t}{1 + j\omega RC}$$

$$= \frac{E_m}{\sqrt{1 + \omega^2 R^2 C^2}} \sin (\omega t - \theta) \text{ where } \theta = \tan^{-1} \omega RC$$

We have $\omega = 2\pi f = \frac{2\pi}{T}$, since $f = \frac{1}{T}$

$$\therefore \qquad v_o = \frac{E_m}{\sqrt{1 + \left(\frac{2\pi}{T}\right)^2 R^2 C^2}} \sin \left(\frac{2\pi}{T} t - \theta\right)$$

Comparison of the equations for v_o reveals that for the RC low-pass circuit to act as an integrator, it is essential that the angle θ must be equal to $\frac{\pi}{2}$ radians, i.e. 90°.

If $RC = 15T$, we have

$$\theta = \tan^{-1} \omega RC$$

$$= \tan^{-1}\left(\frac{2\pi}{T} \times 15T\right)$$

$$= \tan^{-1} 30\pi = 89.4°$$

That means if $RC > 15\,T$, angle θ becomes almost 90°. So this is the condition to be satisfied for the RC low-pass circuit to behave like a reasonably good integrator.

1.3 THE HIGH-PASS *RC* CIRCUIT

Figure 1.30 shows a high-pass *RC* circuit. At zero frequency the reactance of the capacitor is infinity and so it blocks the input and hence the output is zero. Hence, this capacitor is called the *blocking capacitor* and this circuit, also called the *capacitive coupling circuit*, is used to provide dc isolation between the input and the output. As the frequency increases, the reactance of the capacitor decreases and hence the output and gain increase. At very high frequencies, the capacitive reactance is very small so a very small voltage appears across C and, so the output is almost equal to the input and the gain is equal to 1. Since this circuit attenuates low-frequency signals and allows transmission of high-frequency signals with little or no attenuation, it is called a high-pass circuit.

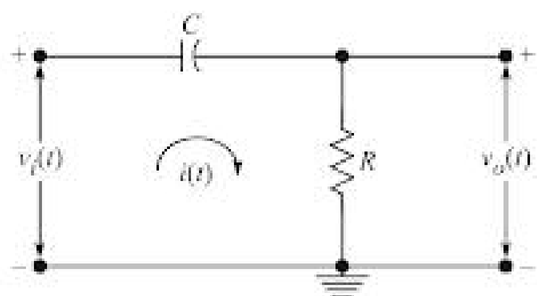

Figure 1.30 The high-pass *RC* circuit.

1.3.1 Sinusoidal Input

Figure 1.31(a) shows the Laplace transformed high-pass *RC* circuit. The gain versus frequency curve of a high-pass circuit excited by a sinusoidal input is shown in Figure 1.31(b). For a sinusoidal input v_i, the output signal v_o increases in amplitude with increasing frequency. The frequency at which the gain is $1/\sqrt{2}$ of its maximum value is called the lower cut-off or lower 3-dB frequency. For a high-pass circuit, there is no upper cut-off frequency because all high frequency signals are transmitted with zero attenuation. Therefore, $f_2 = \infty$. Hence bandwidth $= f_2 - f_1 = \infty$.

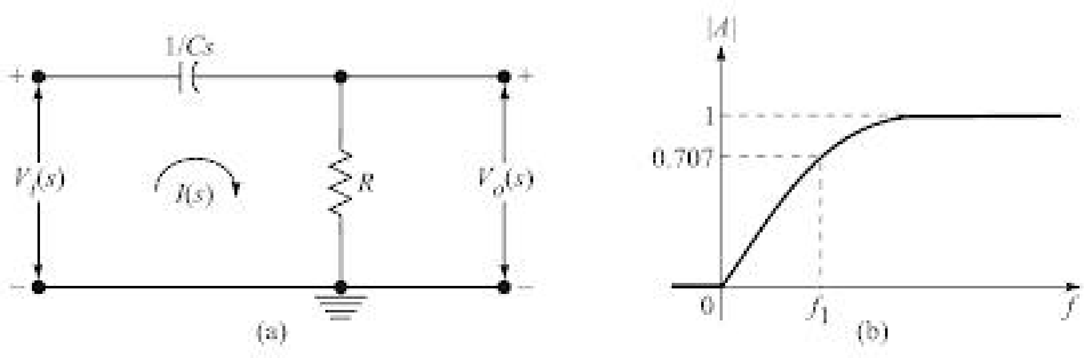

Figure 1.31 (a) Laplace transformed high-pass circuit and (b) gain versus frequency plot.

Expression for the lower cut-off frequency

For the high-pass *RC* circuit shown in Figure 1.31(a), the magnitude of the steady-state gain A, and the angle θ by which the output leads the input are given by

$$A = \frac{V_o(s)}{V_i(s)} = \frac{R}{R + \dfrac{1}{Cs}} = \frac{1}{1 + \dfrac{1}{RCs}}$$

Putting
$$s = j\omega,\ A = \frac{1}{1 - j\dfrac{1}{\omega RC}} = \frac{1}{1 - j\dfrac{1}{2\pi fRC}}$$

$$\therefore \qquad |A| = \frac{1}{\sqrt{1 + \left(\dfrac{1}{2\pi fRC}\right)^2}} \quad \text{and} \quad \theta = -\tan^{-1}\frac{1}{2\pi fRC}$$

At the lower cut-off frequency f_1, $|A| = 1/\sqrt{2}$

$$\therefore \qquad \frac{1}{\sqrt{1 + \left(\dfrac{1}{2\pi f_1 RC}\right)^2}} = \frac{1}{\sqrt{2}}$$

Squaring and equating the denominators,

$$\frac{1}{2\pi f_1 RC} = 1 \quad \text{i.e.} \quad f_1 = \frac{1}{2\pi RC}$$

This is the expression for the lower cut-off frequency of a high-pass circuit.

Relation between f_1 and tilt

The lower cut-off frequency of a high-pass circuit is $f_1 = 1/2\pi RC$. The lower cut-off frequency produces a tilt.

For a 10% change in capacitor voltage, the time or pulse width involved is

$$t = 0.1RC = \text{PW}$$

$$\therefore \qquad \frac{\text{PW}}{RC} = 0.1 = \text{Fractional tilt}$$

$$\therefore \qquad \text{Fractional tilt} = \frac{\text{PW}}{RC} = 2\pi f_1 \cdot \text{PW}$$

This equation applies only when the tilt is 10% or less. When the tilt exceeds 10%, the voltage should be treated as exponential instead of linear and the equation

$$V_o = V_f - (V_f - V_i)e^{-t/RC}$$

should be applied.

1.3.2 Step Input

When a step signal of amplitude V volts shown in Figure 1.32(a) is applied to the high-pass RC circuit of Figure 1.30, since the voltage across the capacitor cannot change instantaneously the output will be just equal to the input at $t = 0$ (for $t < 0$, $v_i = 0$ and $v_o = 0$). Later when the capacitor charges exponentially, the output reduces exponentially with the same time constant RC. The expression for the output voltage for $t > 0$ is given by

$$v_o(t) = V_f - (V_f - V_{in})e^{-t/RC} = 0 - (0 - V)e^{-t/RC} = Ve^{-t/RC}$$

Figure 1.32(b) shows the response of the circuit for large, small, and very small time constants.

For $t > 5\tau$, the output will reach more than 99% of its final value. Hence although the steady state is approached asymptotically, for most applications we may assume that the final value has been reached after 5τ. If the initial slope of the exponential is maintained, the output falls to zero in a time $t = \tau$.

The voltage across a capacitor can change instantaneously only when an infinite current passes through it, because for any finite current $i(t)$ through the capacitor, the instantaneous change in voltage across the capacitor is given by $\frac{1}{C}\int_0^0 i(t)\,dt = 0$.

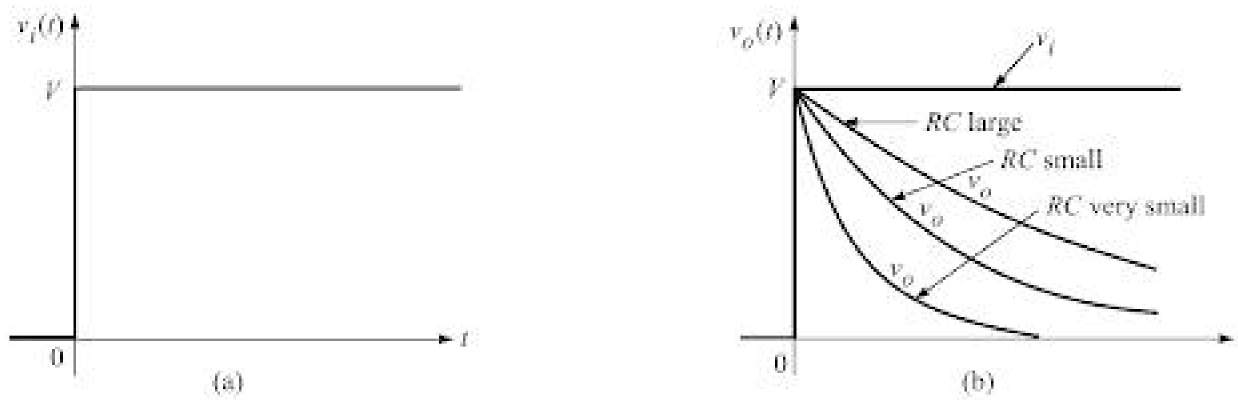

Figure 1.32 (a) Step input and (b) step response for different time constants.

1.3.3 Pulse Input

A pulse of amplitude V and duration t_p shown in Figure 1.4(a) is nothing but the sum of a positive step of amplitude V starting at $t = 0$ and a negative step of amplitude V starting at t_p as shown in Figure 1.4(b). So, the response of the circuit for $0 < t < t_p$ for the pulse input is the same as that for a step input and is given by $v_o(t) = Ve^{-t/RC}$. At $t = t_p$, $v_o(t) = V_p = Ve^{-t_p/RC}$. At $t = t_p$, since the input falls by V volts suddenly and since the voltage across the capacitor cannot change instantaneously, the output also falls suddenly by V volts to $V_p - V$. Hence at $t = t_{p+}$, $v_o(t) = Ve^{-t_p/RC} - V$. Since $V_p < V$, $V_p - V$ is negative. So there is an undershoot at $t = t_p$ and hence for $t > t_p$, the output is negative. For $t > t_p$, the output rises exponentially towards zero with a time constant RC according to the expression $(Ve^{-t_p/RC} - V)e^{-(t-t_p)/RC}$.

The output waveforms for $RC >> t_p$, RC comparable to t_p and $RC << t_p$ are shown in Figures 1.33(a), (b), and (c) respectively. There is distortion in the outputs and the distortion is the least when the time constant is very large. Observe that there is positive area and negative area in the output waveforms. The negative area will always be equal to the positive area. So if the time constant is very large the tilt (the almost linear decrease in the output voltage) will be small and hence the undershoot will be very small, and for $t > t_p$, the output rises towards the zero level very very slowly. If the time constant is very small compared to the pulse width (i.e. $RC/t_p << 1$), the output consists of a positive spike or pip of amplitude V volts at the beginning of the pulse and a negative spike of the same amplitude at the end of the pulse. Hence a high-pass circuit with a very small time constant is called a *peaking circuit* and this process of converting pulses into pips by means of a circuit of short time constant is called peaking.

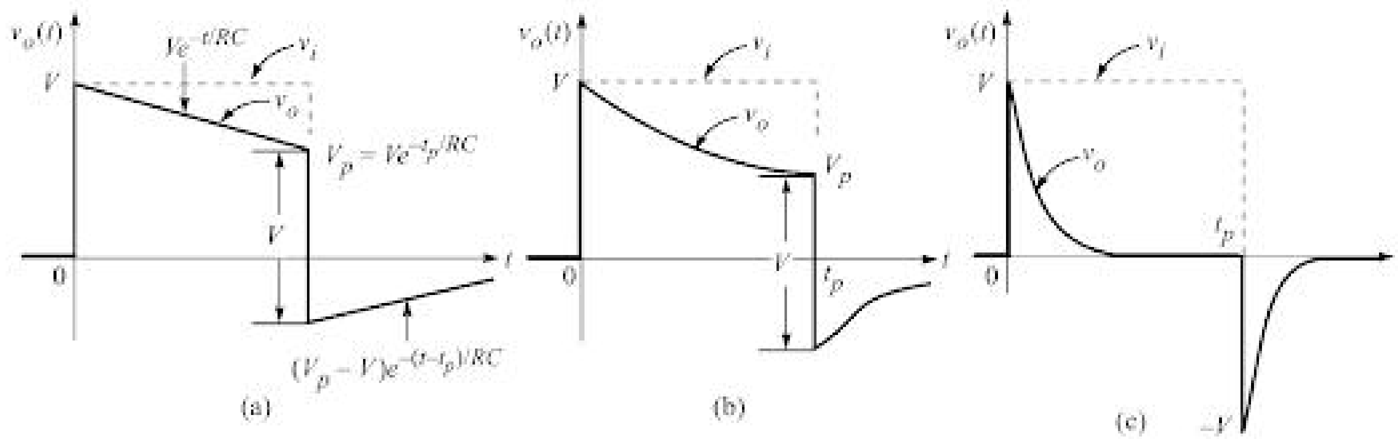

Figure 1.33 Pulse response for (a) $RC >> t_p$, (b) RC comparable to t_p, and (c) $RC << t_p$.

1.3.4 Square-Wave Input

A square wave shown in Figure 1.34(a) is a periodic waveform, which maintains itself at one constant level V' with respect to ground for a time T_1 and then changes abruptly to another level V'' and remains constant at that level for a time T_2, and then repeats itself at regular intervals of $T = T_1 + T_2$. A square wave may be treated as a series of positive and negative steps. The shape of the output depends on the time constant of the circuit. Figures 1.34(b), 1.34(c), 1.34(d), and 1.34(e) show the output waveforms of the high-pass RC circuit under steady-state conditions for the cases (a) $RC >> T$, (b) $RC > T$, (c) $RC = T$, and (d) $RC << T$ respectively.

When the time constant is arbitrarily large (i.e. RC/T_1 and RC/T_2 are very very large in comparison to unity) the output is same as the input but with zero dc level. When $RC > T$, the output is in the form of a tilt. When RC is comparable to T, the output rises and falls exponentially. When $RC << T$ (i.e. RC/T_1 and RC/T_2 are very very small in comparison to unity), the output consists of alternate positive and negative spikes. In this case the peak-to-peak amplitude of the output is twice the peak-to-peak value of the input.

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