

Project 2

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Abstract

This project is to demonstrate the capabilities of functional programming using the tools and techniques - \LaTeX , \AcuTeX , emacs and ML. Each chapter documents the given problems with a structure of:

1. Problem Statement
2. Relevant Code
3. Test Cases
4. Execution Transcripts
5. Explanation of results

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Contents

1	Executive Summary	4
2	Exercise 4.6.3	5
2.1	Problem Statement	5
2.2	Relevant Code	5
2.2.1	4.6.3A	5
2.2.2	4.6.3B	5
2.2.3	4.6.3C	5
2.2.4	4.6.3D	5
2.2.5	4.6.3E	6
2.3	Test Cases	6
2.4	Execution Transcripts	7
2.4.1	Explanation of Results	7
3	Exercise 4.6.4	8
3.1	Problem Statement	8
3.2	Relevant Code	8
3.3	Test Cases	8
3.4	Execution Transcripts	8
3.4.1	Explanation of Results	8
4	Exercise 5.3.4	9
4.1	Problem Statement	9
4.2	Relevant Code	9
4.3	Test Cases	9
4.4	Execution Transcripts	9
4.4.1	Explanation of Results	10
5	Exercise 5.3.5	11
5.1	Problem Statement	11
5.2	Relevant Code	11
5.3	Test Cases	11
5.4	Execution Transcripts	11
5.4.1	Explanation of Results	11
6	Exercise 6.2.1	12
6.1	Problem Statement	12
6.2	Relevant Code	12
6.2.1	6.2.1.1	12
6.2.2	6.2.1.2	12
6.2.3	6.2.1.3	12
6.2.4	6.2.1.4	12
6.2.5	6.2.1.5	12
6.2.6	6.2.1.6	12

6.2.7	6.2.1.7	13
6.3	Execution Transcripts	13
6.3.1	Explanation of Error for 6.2.1.4	13
7	Appendix A: Exercise 4.6.3	14
8	Appendix B: Exercise 4.6.4	17
9	Appendix C: Exercise 5.3.4	18
10	Appendix D: Exercise 5.3.5	19
11	Appendix E: Exercise 6.2.1	20

Executive Summary

All the requirements for this project are statisfied specifically,

Contents

Our report has the following content:

1. Chapter 1: Executive Summary
2. Chapter 2: Exercise 4.6.3
 - (a) Section 2.1 Problem Statement
 - (b) Sectino 2.2 Relevant Code
 - (c) Section 2.3 Test Cases
 - (d) Section 2.4 Execution Transcripts
 - (e) Section 2.4.1 Explanation of Results
3. Chapter 3 Exercise 4.6.4
 - (a) Section 3.1 Problem Statement
 - (b) Section 3.2 Relevant Code
 - (c) Section 3.3 Test Cases
 - (d) Section 3.4 Execution Transcripts
 - (e) Section 3.4.1 Explanation of Results
4. Chapter 4 Exercise 5.3.4
 - (a) Section 4.1 Problem Statement
 - (b) Section 4.2 Relevant Code
 - (c) Section 4.3 Test Cases
 - (d) Section 4.4 Execution Transcripts
 - (e) Section 4.4.1 Explanation of Results
5. Chapter 5 Exercise 5.3.5
 - (a) Section 5.1 Problem Statement
 - (b) Section 5.2 Relevant Code
 - (c) Section 5.3 Test Cases
 - (d) Section 5.4 Execution Transcripts
 - (e) Section 5.4.1 Explanation of Results
6. Chapter 4 Exercise 6.2.1
 - (a) Section 6.1 Problem Statement
 - (b) Section 6.2 Relevant Code
 - (c) Section 6.3 Execution Transcripts
 - (d) Section 6.3.1 Explanation of Error for 6.2.1.4

Reproducibility in ML and \LaTeX

Our ML and \LaTeX source files compile with no errors.

Exercise 4.6.3

2.1 Problem Statement

In this exercise we define five ML functions using `fun` and `val`.

2.2 Relevant Code

2.2.1 4.6.3A

In this we define a function that takes a 3-tuple of integers (x, y, z) as input and returns the value corresponding to the sum $x + y + z$.

```
val funA1 = (fn (x,y,z) => x+y+z);  
fun funA2 (x,y,z) = x+y+z;
```

2.2.2 4.6.3B

In this we define a function that takes two integer inputs x and y (where x is supplied first followed by y) and returns the boolean value corresponding to $x \leq y$.

```
val funB1 = (fn x => (fn y => x <= y));  
fun funB2 x y = x <= y;
```

2.2.3 4.6.3C

In this we define a function that takes two strings s_1 and s_2 (where s_1 is supplied first followed by s_2) and concatenates them, where \wedge denotes string concatenation. For example, "Hi" \wedge "there" results in the string "Hi there".

```
val funC1 = (fn s1 => (fn s2 => s1 ^ s2));  
fun funC2 s1 s2 = s1 ^ s2;
```

2.2.4 4.6.3D

In this we define a function that takes two lists $list_1$ and $list_2$ (where $list_1$ comes first) and appends them, where $@$ denotes list append. For example $[true,false] @ [false, false, false]$ results in the list $[true,false,false,false,false]$.

```
val funD1 = (fn l1 => (fn l2 => l1@l2));  
fun funD2 l1 l2 = l1@l2;
```

2.2.5 4.6.3E

In this we define a function that takes a pair of integers (x, y) and returns the larger of the two values. You note that the conditional statement if condition then a else b returns a if condition is true, other-wise it returns b.

```
val funE1 = (fn (x,y) => if (x>y) then x else y);
fun funE2 (x,y) = if (x>y) then x else y;
```

2.3 Test Cases

Below are the test cases to evaluate.

```
(*****)
(* Part A *)
(*****)

val testListA = [(1,2,3),(4,5,6),(7,8,9)]

val outputsA = map funA2 testListA

val testResultA = test463A funA1 funA2 testListA

(*****)
(* Part B *)
(*****)

val testListB = [(0,0),(1,2),(4,3)]

val outputsB = map (f2P funB1) testListB

val testResultB = test463B funB1 funB2 testListB

(*****)
(* Part C *)
(*****)

val testListC = [("Hi","_there!"),("Oh","_no!"),("What","_the_...")]

val outputsC = map (f2P funC1) testListC

val testResultC = test463B funC1 funC2 testListC

(*****)
(* Part D *)
(*****)

val testListD1 = [([0,1],[2,3,4]),([], [0,1])]
val testListD2 = [([true,true],[])]

val outputsD1 = map (f2P funD1) testListD1
val outputsD2 = map (f2P funD2) testListD2
```



```
val testResultD1 = test463B funD1 funD2 testListD1
val testResultD2 = test463B funD1 funD2 testListD2
```

```
(*****)
(* Part E *)
(*****)
```

```
val testListE = [(2,1),(5,5),(5,10)]
```

```
val sampleResultE = map funE1 testListE
```

```
val testResultE = test463A funE1 funE2 testListE
```

2.4 Execution Transcripts

	1
<pre>----- HOL-4 [Kananaskis 11 (stdknl, built Sat Aug 19 09:30:06 2017)] For introductory HOL help, type: help "hol"; To exit type <Control>-D ----- > > > > > # # # # # val test463A = fn: ('a -> 'b) -> ('a -> 'b) -> 'a list -> bool > > > > > # # # # # val f2P = fn: ('a -> 'b -> 'c) -> 'a * 'b -> 'c val test463B = fn: ('a -> 'b -> 'c) -> ('a -> 'b -> 'c) -> ('a * 'b) list -> bool > *** Emacs/HOL command completed *** > val funA1 = fn: int * int * int -> int > val funA2 = fn: int * int * int -> int > # # # # val outputsA = [6, 15, 24]: int list val testListA = [(1, 2, 3), (4, 5, 6), (7, 8, 9)]: (int * int * int) list val testResultA = true: bool > val funB1 = fn: int -> int -> bool > val funB2 = fn: int -> int -> bool > > # # # # val outputsB = [false, true, false]: bool list val testListB = [(0, 0), (1, 2), (4, 3)]: (int * int) list val testResultB = true: bool > val funC1 = fn: string -> string -> string > val funC2 = fn: string -> string -> string > > # # # # val outputsC = ["Hi there!", "Oh no!", "What the ..."]: string list val testListC = [("Hi", " there!"), ("Oh ", "no!"), ("What", " the ...")]: (string * string) list val testResultC = true: bool > val funD1 = fn: 'a list -> 'a list -> 'a list > val funD2 = fn: 'a list -> 'a list -> 'a list > # # # # # val outputsD1 = [[0, 1, 2, 3, 4], [0, 1]]: int list list val outputsD2 = [[true, true]]: bool list list val testListD1 = [(0, 1), (2, 3, 4)], ([], [0, 1]): (int list * int list) list val testListD2 = [[true, true], []]: (bool list * 'a list) list val testResultD1 = true: bool val testResultD2 = true: bool > val funE1 = fn: int * int -> int > val funE2 = fn: int * int -> int > # # # # val sampleResultE = [2, 5, 10]: int list val testListE = [(2, 1), (5, 5), (5, 10)]: (int * int) list val testResultE = true: bool ></pre>	

2.4.1 Explanation of Results

All the results in the test cases shows they are passed against the given test function.

Exercise 4.6.4

3.1 Problem Statement

In this exercise we need to solve the list concatenation as stated below:

In ML, define a function `listSquares` that when applied to the empty list of integers returns the empty list, and when applied to a non-empty list of integers returns a list where each element is squared. For example, `listSquares [2,3,4]` returns `[4,9,16]`. Define the function using a `let` expression in ML. A function that takes two lists `list 1` and `list 2` (where `list 1` comes first) and appends them, where `'@'` denotes list append. For example `[true,false] @ [false, false, false]` results in the list `[true,false,false,false,false]`.

3.2 Relevant Code

```
fun listSquares list =  
let  
  fun squareNum x = x*x  
  in  
    map squareNum list  
  end;
```

3.3 Test Cases

The required test cases are:

```
val testList = [1,2,3,4,5]
```

3.4 Execution Transcripts

1

```
-----  
HOL-4 [Kananaskis 11 (stdknl, built Sat Aug 19 09:30:06 2017)]  
  
For introductory HOL help, type: help "hol";  
To exit type <Control>-D  
-----  
> > > # # # # val listSquares = fn: int list -> int list  
> val testList = [1, 2, 3, 4, 5]: int list  
> val testResults = [1, 4, 9, 16, 25]: int list  
>
```

3.4.1 Explanation of Results

The above transcript shows the given tests has been passed.

Exercise 5.3.4

4.1 Problem Statement

In this exercise we need to define a function `Filter` in ML, whose behavior is identical to `filter`. Note: you cannot use `filter` in the definition of `Filter`. However, you can adapt the definition of `filter` and use it in your definition. Show test cases of your function returning the expected results by comparing the outputs of both `Filter` and `filter`.

4.2 Relevant Code

```
fun Filter l list=
let
  fun fnA l []=[]
    | fnA l xs=map l xs

  fun fnB [] fail=[]
    | fnB fail []=[]
    | fnB (b::bs) (x::xs)=if b then x::(fnB bs xs) else fnB bs xs
in
  fnB (fnA l list) list
end;
```

4.3 Test Cases

The required test cases are:

```
val testResults = Filter (fn x => x < 5) [1,2,3,4,5,6,7,8,9]
val testResults2 = Filter (fn x => x < 5) [4,6]
```

4.4 Execution Transcripts

1

```
-----
HOL-4 [Kananaskis 11 (stdknl, built Sat Aug 19 09:30:06 2017)]

For introductory HOL help, type: help "hol";
To exit type <Control>-D
-----
> > > ##### val Filter = fn: ('a -> bool) -> 'a list -> 'a list
> > val testResults = [1, 2, 3, 4]: int list
> val testResults2 = [4]: int list
>
```

4.4.1 Explanation of Results

The above transcript shows the given tests has been passed.

Exercise 5.3.5

5.1 Problem Statement

In this exercise we need to define a ML function `addPairsGreaterThan n list`, whose behavior is defined as follows: (1) given an integer `n`, and (2) given a list of pairs of integers `list`, `addPairsGreaterThan n list` will return a list of integers where each element is the sum of integer pairs in `list` where both elements of the pairs are greater than `n`.

5.2 Relevant Code

```
filter ;

fun addPairsGreaterThan n list =
let
fun sumList [] = []
  | sumList ((x,y) :: xs) = (x+y) :: (sumList xs)

fun fil n (x,y) = (x>n andalso y>n)
in
sumList (filter (fil n) list)
end;
```

5.3 Test Cases

The required test cases are:

```
addPairsGreaterThan 0 [(0,1),(2,0),(2,3),(4,5)];
```

5.4 Execution Transcripts

1

```
-----
HOL-4 [Kananaskis 11 (stdknl, built Sat Aug 19 09:30:06 2017)]

For introductory HOL help, type: help "hol";
To exit type <Control>-D
-----
> > > val it = fn: ('a -> bool) -> 'a list -> 'a list
> ##### val addPairsGreaterThan = fn: int -> (int * int) list -> int list
> > val it = [5, 9]: int list
>
```

5.4.1 Explanation of Results

The above transcript shows the given tests has been passed.

Exercise 6.2.1

6.1 Problem Statement

In this exercise we have to show the HOL equivalent code for the given sub-problems.

6.2 Relevant Code

6.2.1 6.2.1.1

HOL equivalent of $P(x) \supset Q(y)$:

```
‘‘P x  $\implies$  Q y ‘‘;
```

6.2.2 6.2.1.2

$P(x)$ supset $Q(y)$ with x constrain to HOL type `:num` and y to Hol type `:bool`

```
‘‘(P:num  $\rightarrow$  bool) (x:num)  $\implies$  (Q:bool $\rightarrow$ bool) (y:bool) ‘‘;
```

6.2.3 6.2.1.3

forall $x\ y\ P(x)$ supset $Q(y)$ without specifying types

```
‘‘!x y.(P x)  $\implies$  (Q y) ‘‘;
```

6.2.4 6.2.1.4

for some $(x : \text{num}). R(x : \text{.})$.

```
‘‘?(x :num).(R (x :‘a)) ‘‘;
```

6.2.5 6.2.1.5

$x.P(x) \supset Q(x) = x.P(x) \supset Q(x)$

```
‘‘(~!x.(P x)\/(Q x))=(?x.(~(P x))/\~(Q x)) ‘‘;
```

6.2.6 6.2.1.6

All people are mortal, where $P(x)$ represents x is a person and $M(x)$ represents x is mortal.

```
‘‘!x.(P x)  $\implies$  (M x) ‘‘;
```

6.2.7 6.2.1.7

Some people are funny, where $\text{Funny}(x)$ denotes x is funny.

$\text{‘‘?}x.(P\ x) \implies (\text{Funny}\ x)\text{‘‘};$

6.3 Execution Transcripts

1

```

-----
HOL-4 [Kananaskis 11 (stdknl, built Sat Aug 19 09:30:06 2017)]

For introductory HOL help, type: help "hol";
To exit type <Control>-D
-----
> > > ##### ** types trace now on
> ##### ** Unicode trace now off
> <<HOL message: inventing new type variable names: 'a, 'b>>
val it =
  '(P : 'a -> bool) (x : 'a) ==> (Q : 'b -> bool) (y : 'b)'' :
    term
> val it =
  '(P : num -> bool) (x : num) ==> (Q : bool -> bool) (y : bool)'' :
    term
> <<HOL message: inventing new type variable names: 'a, 'b>>
val it =
  '!(x : 'a) (y : 'b). (P : 'a -> bool) x ==> (Q : 'b -> bool) y'' :
    term
> <<HOL message: inventing new type variable names: 'a>>

Type inference failure: unable to infer a type for the application of

(x : num)

at line 22, character 16

to

(: ' : 'a)

on line 22, characters 18-22

unification failure message: Attempt to unify different type operators: num$num and min$fun
Exception-
  HOL_ERR
  {message =
    "on line 22, characters 18-22:\n\ntype inference failure: unable to infer a type for the application of\n\n(x : num)
\n\nat line 22, character 16\n\nto\n\n(: ' : 'a)\n\non line 22, characters 18-22\n\nunification failure message:
  Attempt to unify different type operators: num$num and min$fun\n",
    origin_function = "type-analysis", origin_structure = "Preterm"} raised
> <<HOL message: inventing new type variable names: 'a>>
val it =
  ' ~(!(x : 'a). (P : 'a -> bool) x \ / (Q : 'a -> bool) x) <=>
  ?(x : 'a). ~P x /\ ~Q x'' :
    term
> <<HOL message: inventing new type variable names: 'a>>
val it =
  ' !(x : 'a). (P : 'a -> bool) x ==> (M : 'a -> bool) x'' :
    term
> <<HOL message: inventing new type variable names: 'a>>
val it =
  ' ?(x : 'a). (P : 'a -> bool) x ==> (Funny : 'a -> bool) x'' :
    term
>

```

6.3.1 Explanation of Error for 6.2.1.4

This cannot be evaluated, because x is specified to num then specify to α , So there is a type error

Appendix A: Exercise 4.6.3

The following code is from the file ex-4-6-3Tests.sml.

```
(*****  
(* Exercise 4.6.3 *)  
(* Author: Shiu-Kai Chin *)  
(* Modified - Added function code: Bharath Karumudi *)  
(* Date: Jul 19, 2019 *)  
(*****)  
  
(*****  
(* Test functions you will need. *)  
(* *)  
(* *)  
(*****)  
  
fun test463A f1 f2 inList =  
  let  
    val list1 = map f1 inList  
    val list2 = map f2 inList  
  in  
    foldr  
      (fn (x,y) => (x andalso y))  
      true  
      (ListPair.map (fn (x,y) => x = y) (list1 , list2))  
  end;  
  
fun f2P f (x,y) = f x y  
  
fun test463B f1 f2 inList =  
  let  
    val list1 = map (f2P f1) inList  
    val list2 = map (f2P f2) inList  
  in  
    foldr  
      (fn (x,y) => (x andalso y))  
      true  
      (ListPair.map (fn (x,y) => x = y) (list1 , list2))  
  end;
```

```

(*****)
(* Part A *)
(*****)

(* ===== *)

(* function A1, A2 *)

val funA1 = (fn (x,y,z) => x+y+z);
fun funA2 (x,y,z) = x+y+z;

(* ===== *)

val testListA = [(1,2,3),(4,5,6),(7,8,9)]

val outputsA = map funA2 testListA

val testResultA = test463A funA1 funA2 testListA

(*****)
(* Part B *)
(*****)

(* ===== *)
(* function B1, B2 *)

val funB1 = (fn x => (fn y => x < y));
fun funB2 x y = x < y;

(* ===== *)

val testListB = [(0,0),(1,2),(4,3)]

val outputsB = map (f2P funB1) testListB

val testResultB = test463B funB1 funB2 testListB

(*****)
(* Part C *)
(*****)

(* ===== *)
(* function C1, C2 *)

val funC1 = (fn s1 => (fn s2 => s1 ^ s2));
fun funC2 s1 s2 = s1 ^ s2;

(* ===== *)

val testListC = [("Hi", "there!"), ("Oh", "no!"), ("What", "the...")]

val outputsC = map (f2P funC1) testListC

```

```
val testResultC = test463B funC1 funC2 testListC
```

```
(*****)
(* Part D *)
(*****)
```

```
(* ===== *)
(* function D1, D2 *)
```

```
val funD1 = (fn l1 => (fn l2 => l1@l2));
fun funD2 l1 l2 = l1@l2;
```

```
(* ===== *)
```

```
val testListD1 = [( [0,1], [2,3,4] ), ([], [0,1])]
val testListD2 = [( [true,true], [] )]
```

```
val outputsD1 = map (f2P funD1) testListD1
val outputsD2 = map (f2P funD2) testListD2
```

```
val testResultD1 = test463B funD1 funD2 testListD1
val testResultD2 = test463B funD1 funD2 testListD2
```

```
(*****)
(* Part E *)
(*****)
```

```
(* ===== *)
(* function E1, E2 *)
```

```
val funE1 = (fn (x,y) => if (x>y) then x else y);
fun funE2 (x,y) = if (x>y) then x else y;
```

```
(* ===== *)
```

```
val testListE = [(2,1), (5,5), (5,10)]
```

```
val sampleResultE = map funE1 testListE
```

```
val testResultE = test463A funE1 funE2 testListE
```

Appendix B: Exercise 4.6.4

The following code is from the file ex-4-6-4Tests.sml.

```
(*****)  
(* Exercise 4.6.4 *)  
(* Author: Shiu-Kai Chin *)  
(* Modified — Added function code: Bharath Karumudi *)  
(* Date: Jul 19 2019 *)  
(*****)  
  
(* ===== *)  
(* function listSquares *)  
  
fun listSquares list =  
  let  
    fun squareNum x = x*x  
    in  
      map squareNum list  
    end;  
  
(* ===== *)  
  
val testList = [1,2,3,4,5]  
  
val testResults = listSquares testList
```

Appendix C: Exercise 5.3.4

The following code is from the file ex-5-3-4Tests.sml.

```
(***** *)
(* Exercise 5.3.4 *)
(* Author: Shiu-Kai Chin *)
(* Modified - Added function code: Bharath Karumudi *)
(* Date: 20 September 2015 *)
(***** *)

(* ===== *)
(* function Filter *)

fun Filter l list=
let

fun fnA l []=[]
  | fnA l xs=map l xs

fun fnB [] fail=[]
  | fnB fail []=[]
  | fnB (b::bs) (x::xs)=if b then x::(fnB bs xs) else fnB bs xs
in
fnB (fnA l list) list
end;

(* ===== *)

val testResults = Filter (fn x => x < 5) [1,2,3,4,5,6,7,8,9]

(* specified test cases *)
val testResults2 = Filter (fn x => x < 5)[4,6]
```

Appendix D: Exercise 5.3.5

The following code is from the file ex-5-3-5Tests.sml.

```
(***** *)
(* Exercise 5.3.5 *)
(* Author: Shiu-Kai Chin *)
(* Modified - Added function Code: Bharath Karumudi *)
(* Date: Jul 20, 2019 *)
(***** *)

(* ===== *)
(* function addPairsGreaterThan *)

filter;

fun addPairsGreaterThan n list =
let
fun sumList [] = []
  | sumList ((x,y) :: xs) = (x+y) :: (sumList xs)

fun fil n (x,y) = (x>n andalso y>n)
in
sumList (filter (fil n) list)
end;

(* ===== *)

addPairsGreaterThan 0 [(0,1),(2,0),(2,3),(4,5)];
```

Appendix E: Exercise 6.2.1

The following code is from the file ex-6-2-1.sml

```
(*****
(* Exercise 6.2
(* Author: Bharath Karumudi
(* Date: Jul 20, 2019
(* *****)

(* 1.  $P(x)$  supset  $Q(y)$  *)

‘‘P x  $\implies$  Q y’’;

(* 2.  $P(x)$  supset  $Q(y)$  with  $x$  constrain to HOL type :num *)
(* and  $y$  to Hol type :bool *)

‘‘(P:num  $\rightarrow$  bool) (x:num)  $\implies$  (Q:bool $\rightarrow$ bool) (y:bool)’’;

(* 3. forall  $x y$   $P(x)$  supset  $Q(y)$  without specifying types *)
‘‘!x y.(P x)  $\implies$  (Q y)’’;

(* 4. *)
‘‘?(x :num).(R (x :a))’’;

(** Error: This cannot be evaluated, because x is specified to num then specify to *)
(* alpha, So there is a type error **)

(* 5. *)
‘‘(¬!x.(P x)  $\wedge$  (Q x)) = (?x.(¬(P x))  $\wedge$  ¬(Q x))’’;

(* 6. All people are mortal, where  $P(x)$  represents  $x$  is a person and *)
(*  $M(x)$  represents  $x$  is mortal.**)
‘‘!x.(P x)  $\implies$  (M x)’’;

(* 7. Some people are funny, where  $Funny(x)$  denotes  $x$  is funny. *)
‘‘?x.(P x)  $\implies$  (Funny x)’’;
```