ACCUSTOM

Adaptive Control & Coordination of UAV Swarms for Traffic Offloading in MIMO ecosystems.

$g \in \{1, 2, 3, \ldots, G\}$

$u \in \{1, 2, 3, \ldots, U\}$

1 g x GN Tx ANT m position — h_{mn} — u x UAV Rx ANT n

$p_n \triangleq [x_n, y_n, z_n]$ $n \in \{1, 2, 3, \ldots, A_u\}$

$p_m \triangleq [x_m, y_m, z_m]$

$m \in \{1, 2, 3, \ldots, A_g\}$

$$d_{mn} = \sqrt{(x_m - x_n)^2 + (y_m - y_n)^2 + (z_m - z_n)^2}$$

$$\underbrace{h_{mn}}_{\text{channel gain}} \triangleq \underbrace{\beta_{mn}}_{\text{avg. channel gain at distance of } 1m} - 10\underbrace{\alpha_{mn}}_{\text{pathloss exponent}} \log_{10} \underbrace{d_{mn}}_{\text{3D distance}} + \underbrace{\xi_{mn}}_{\text{shadow fading component}} + \underbrace{\lambda_{mn}}_{\text{small-scale fading component}} \quad (\text{dB}),$$

$$\xi_{mn} \sim \mathcal{N}\left(0, \sigma^2_{SF,mn}\right) \text{ and } \lambda_{mn} \sim \mathcal{CN}\left(\underbrace{\sqrt{\frac{K_{mn}}{K_{mn}+1}}}_{\mu_{SSF,mn}}, \underbrace{\frac{1}{K_{mn}+1}}_{\sigma^2_{SSF,mn}}\right).$$

2 Let $\mathcal{D} \subseteq \mathbb{R}^6$ be the set of all possible GN-UAV position pairs.

Let R be the number of site partitions for our radio map generation, i.e.,

$$\mathcal{D} = D_1 \cup D_2 \cup D_3 \cup \ldots \cup D_R \; ; \; D_i \cap D_j = \phi, \; \forall\, i,j \in \{1, 2, 3, \ldots, R\},\; i \neq j.$$

$$\therefore h_{mn} = h(\underbrace{p_m}_{\text{Tx ANT position}}, \underbrace{p_n}_{\text{Rx ANT position}}) = \sum_{r=1}^{R} \left(\beta_r - 10\alpha_r \log_{10} d(p_m, p_n) + \xi_r + \lambda_r\right) \mathbb{I}\{(p_m, p_n) \in D_r\}.$$

3 Measurements on Wireless InSite :

$$\mathcal{M} \triangleq \{\underbrace{(p_m^{(l)}, p_n^{(l)}, h_{mn}^{(l)})}_{\text{individual measurement + channel gain}} : l = 1, 2, 3, \ldots, L\}$$

at position $(p_m^{(l)}, p_n^{(l)})$ gain

Tx ANT Rx ANT

Assuming a measurement sample $(\mathbf{p}_m, \mathbf{p}_n, h_{mn}) \in \mathcal{M}$ is classified into (or) belongs to partition $r \in \{1, 2, 3, \ldots, R\}$:

$$h_{mn} = \beta_r - 10\alpha_r \log_{10} \underbrace{d_{mn}}_{\|\mathbf{p}_m - \mathbf{p}_n\|_2} + \xi_r + \lambda_r$$

channel gain for this measurement @ this position pair

$\xi_r \rightarrow \mathcal{N}(0, \sigma^2_{SF,r})$

$\lambda_r \rightarrow \mathcal{N}\left(\underbrace{\sqrt{\frac{K_r}{K_r+1}}}_{\mu_{SSF,r}}, \underbrace{\frac{1}{K_r+1}}_{\sigma^2_{SSF,r}}\right)$

Joint PDF Conditioned on belonging to (r):

$$f_r(\mathbf{p}_m, \mathbf{p}_n, h_{mn}) = \frac{1}{\sqrt{2\pi(\sigma^2_{SF,r} + \sigma^2_{SSF,r})}} \exp\left\{-\frac{(h_{mn} - \beta_r + 10\alpha_r \log_{10} d_{mn} - \mu_{SSF,r})^2}{2(\sigma^2_{SF,r} + \sigma^2_{SSF,r})}\right\}$$

General Joint PDF: (Total Prob. Law)

$$\mathbb{P}(P = (\mathbf{p}_m, \mathbf{p}_n), H = h_{mn}, Z) = \sum_{r=1}^{R} \mathbb{P}(P = (\mathbf{p}_m, \mathbf{p}_n), H = h_{mn} \mid Z_r = z_r = 1)\, \mathbb{P}(Z_r = z_r = 1)$$

P: a r.v. for position; H: a r.v. for channel gain; Z: a r.v. for the partition classification vector

e.g., $Z = \underline{z}^{(l)} = [z_1^{(l)}, z_2^{(l)}, z_3^{(l)}, \ldots, z_R^{(l)}]$,

with $z_r^{(l)} \in \{0, 1\}, \forall r \in \{1, 2, 3, \ldots, R\}$,

for any measurement sample $l \in \{1, 2, 3, \ldots, L\}$

Equation (1)

$$f(P = (\mathbf{p}_m, \mathbf{p}_n), H = h_{mn}, Z) = \sum_{r=1}^{R} \underbrace{f_r(\mathbf{p}_m, \mathbf{p}_n, h_{mn})}_{\mathbb{P}(P = (\mathbf{p}_m, \mathbf{p}_n), H = h_{mn} \mid Z_r = z_r = 1)} \cdot \pi_r$$

P: r.v. for e.g., $(\mathbf{p}_m^{(l)}, \mathbf{p}_n^{(l)})$; H: r.v. for e.g., $h_{mn}^{(l)}$; Z: r.v. for e.g., $\underline{z}^{(l)}$; $\pi_r \rightarrow \mathbb{P}(Z_r = z_r = 1)$

$$= \prod_{r=1}^{R} \left(f_r(\mathbf{p}_m, \mathbf{p}_n, h_{mn})\right)^{z_r} \cdot \prod_{r=1}^{R} \pi_r^{z_r},$$

with $\sum_{r=1}^{R} \pi_r = \sum_{r=1}^{R} \mathbb{P}(Z_r = z_r = 1) = 1$.

4 Set of channel parameters to be estimated associated with site partitions :

$$\mathcal{B} \triangleq \{(\alpha_r, \beta_r, \sigma_{SF,r}, K_{SSF,r}, \pi_r) : r = 1,2,3,\ldots,R\}$$

For site partition r : α_r — pathloss exponent; β_r — avg. channel gain at dist. of 1m; $\sigma_{SF,r}$ — shadowing std.; $K_{SSF,r}$ — small-scale Rician fading K factor; π_r — $\mathbb{P}(z_r = 1)$ marginal probability that $((p_m, p_n), h_{mn})$ belongs to the r^{th} deployment partition

$r \in \{1,2,3,\ldots,R\}$

5 $$\mathcal{M} = \{(p_m^{(l)}, p_n^{(l)}, h_{mn}^{(l)}, \underbrace{[z_1^{(l)}, z_2^{(l)}, z_3^{(l)}, \ldots, z_R^{(l)}]}_{\underline{z}^{(l)}}) : l = 1,2,3,\ldots,L\}$$

<u>MAXIMUM LIKELIHOOD ESTIMATION (MLE) :</u>

Likelihood function:

$$g(\mathcal{B}) = \prod_{l=1}^{L} \mathbb{P}\left(P = (p_m^{(l)}, p_n^{(l)}), H = h_{mn}^{(l)}, Z = \underline{z}^{(l)} \,\middle|\, \mathcal{B}\right);$$

$$\Downarrow$$

$$\underset{\mathcal{B}, \{\underline{z}^{(l)} : l=1,2,3,\ldots,L\}}{\text{maximize}} \prod_{l=1}^{L} \underbrace{\mathbb{P}\left(P = (p_m^{(l)}, p_n^{(l)}), H = h_{mn}^{(l)}, Z = \underline{z}^{(l)} \,\middle|\, \mathcal{B}\right)}_{f(p_m^{(l)}, p_n^{(l)}, h_{mn}^{(l)}, \underline{z}^{(l)} | \mathcal{B}) \,=\, \prod_{r=1}^{R} \left(f_r(p_m^{(l)}, p_n^{(l)}, h_{mn}^{(l)})\right)^{z_r^{(l)}} \prod_{r=1}^{R} \pi_r^{z_r^{(l)}}};$$

subject to $\sum_{r=1}^{R} \pi_r = 1;$

($\because$ From Equation 1)

$$\Downarrow$$

This is equivalent to maximizing $\log g(\mathcal{B})$, s.t. $\sum_{r=1}^{R} \pi_r = 1;$

marginal prob. that any (p_m, p_n, h_{mn}) belongs to the rth site partition

$$\Downarrow$$

$$\underset{\mathcal{B}, \{\underline{z}^{(l)} : l=1,2,3,\ldots,L\}}{\text{maximize}} \sum_{l=1}^{L} \sum_{r=1}^{R} \left[\log\left(f_r(p_m^{(l)}, p_n^{(l)}, h_{mn}^{(l)})\right)^{z_r^{(l)}} + \log(\pi_r)^{z_r^{(l)}}\right]$$

s.t. $\sum_{r=1}^{R} \pi_r = 1;$

$\Rightarrow$

$$\underset{\mathcal{B},\ \{\underline{z}^{(l)}:\ l=1,2,3,\ldots,L\}}{\text{maximize}} \sum_{l=1}^{L}\sum_{r=1}^{R}\left[z_r^{(l)}\left(\log f_r\left(p_m^{(l)}, p_n^{(l)}, h_{mn}^{(l)}\right) + \log \pi_r\right)\right]$$

(6) Let $\bar{z}_r^{(l)}(\mathcal{B}) \triangleq \mathbb{E}\left[Z_r^{(l)} \,\middle|\, P^{(l)} = (p_m^{(l)}, p_n^{(l)}),\ H^{(l)} = h_{mn}^{(l)},\ \mathcal{B}\right]$

Bernoulli r.v.

$Z_r^{(l)} = z_r^{(l)} \in \{0, 1\}$

$$= \mathbb{P}\Big(\underbrace{Z_r^{(l)} = z_r^{(l)} = 1}_{\substack{\text{let this be temporarily referred}\\ \text{to as } D}} \,\Big|\, \underbrace{P^{(l)} = (p_m^{(l)}, p_n^{(l)})}_{A},\ \underbrace{H^{(l)} = h_{mn}^{(l)}}_{B},\ \underbrace{\mathcal{B}}_{C}\Big).$$

temporary references

Aside:

$$\mathbb{P}(D \mid A, B, C) = \frac{\mathbb{P}(A, B, C \mid D)\,\mathbb{P}(D)}{\mathbb{P}(A, B, C)} \quad (\because \text{BAYES' RULE})$$

$$= \frac{\mathbb{P}(A, B \mid C, D)\,\mathbb{P}(\cancel{C})\,\mathbb{P}(D)}{\mathbb{P}(A, B \mid C)\,\mathbb{P}(\cancel{C})} \quad (\because \text{Definition of CONDITIONAL PROBABILITY})$$

$$= \frac{\mathbb{P}(A, B \mid C, D)\,\mathbb{P}(D)}{\sum_F \mathbb{P}(F)\,\mathbb{P}(A, B \mid C, F)}. \quad (\because \text{TOTAL PROBABILITY})$$

Equation (2)

$$\therefore\ \bar{z}_r^{(l)}(\mathcal{B}) = \frac{\mathbb{P}\left(P^{(l)} = (p_m^{(l)}, p_n^{(l)}),\ H^{(l)} = h_{mn}^{(l)} \,\middle|\, \mathcal{B},\ Z_r^{(l)} = z_r^{(l)} = 1\right)\cdot \mathbb{P}\left(Z_r^{(l)} = z_r^{(l)} = 1\right)}{\sum_{r'=1}^{R} \mathbb{P}\left(Z_{r'}^{(l)} = z_{r'}^{(l)} = 1\right)\mathbb{P}\left(P^{(l)} = (p_m^{(l)}, p_n^{(l)}),\ H^{(l)} = h_{mn}^{(l)} \,\middle|\, \mathcal{B},\ Z_{r'}^{(l)} = z_{r'}^{(l)} = 1\right)}$$

$$= \frac{f_r\left(p_m^{(l)}, p_n^{(l)}, h_{mn}^{(l)} \,\middle|\, \mathcal{B}\right)\pi_r}{\sum_{r'=1}^{R} f_{r'}\left(p_m^{(l)}, p_n^{(l)}, h_{mn}^{(l)} \,\middle|\, \mathcal{B}\right)\pi_{r'}}.$$

(7) $$\mathbb{E}\left[g(\mathcal{B})\right] = \mathbb{E}\left[\sum_{l=1}^{L}\sum_{\gamma=1}^{R}\left\{ z_\gamma^{(l)}\left(\log f_\gamma\left(p_m^{(l)}, p_n^{(l)}, h_{mn}^{(l)}\right) + \log \pi_\gamma\right)\right\}\right]$$

$$= \sum_{l=1}^{L}\sum_{\gamma=1}^{R}\left\{ \underbrace{\mathbb{E}\left[Z_\gamma^{(l)} \,\middle|\, P^{(l)} = \left(p_m^{(l)}, p_n^{(l)}\right), H^{(l)} = h_{mn}^{(l)}, \mathcal{B}\right]}_{\bar{z}_\gamma^{(l)}(\mathcal{B})} \cdot \left(\log f_\gamma\left(p_m^{(l)}, p_n^{(l)}, h_{mn}^{(l)}\right) + \log \pi_\gamma\right)\right\}$$

$$= \sum_{l=1}^{L}\sum_{\gamma=1}^{R}\left[\bar{z}_\gamma^{(l)}(\mathcal{B})\left(\log f_\gamma\left(p_m^{(l)}, p_n^{(l)}, h_{mn}^{(l)}\right) + \log \pi_\gamma\right)\right].$$

(8) Reformulated MLE: $$\underset{\mathcal{B}}{\text{maximize}} \quad \mathbb{E}\left[g(\mathcal{B})\right] \quad \text{subject to}$$

$$\sum_{\gamma=1}^{R} \pi_\gamma = 1.$$

Non-convex : Iterative search to find sub-optimal solution

$\bar{z}_\gamma^{(l)}(\mathcal{B})$ can be found easily using equation (2)

Fix $\bar{z}_\gamma^{(l)}(\mathcal{B})$ and the objective is convex in $\pi_\gamma, \alpha_\gamma, \beta_\gamma, \sigma_{SF,\gamma}$ and $K_{SSF,\gamma}$.

At the i^{th} iteration:

(a) Compute $$\bar{z}_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right) = \frac{f_\gamma\left(p_m^{(l)}, p_n^{(l)}, h_{mn}^{(l)} \,\middle|\, \mathcal{B}^{(i)}\right)\pi_\gamma}{\sum_{\gamma'=1}^{R} f_{\gamma'}\left(p_m^{(l)}, p_n^{(l)}, h_{mn}^{(l)} \,\middle|\, \mathcal{B}^{(i)}\right)\pi_{\gamma'}};$$

(b) Next, $$\mathbb{E}\left[g\left(\mathcal{B}^{(i)}\right)\right] = \sum_{l=1}^{L}\sum_{\gamma=1}^{R}\left[\underbrace{\bar{z}_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right)}_{\text{known from (a)}}\left(\log\left\{\frac{1}{\sqrt{2\pi\left(\sigma_{SF,\gamma}^2+\sigma_{SSF,\gamma}^2\right)}}\exp\left(-\frac{\left(h_{mn}^{(l)}-\beta_\gamma+10\alpha_\gamma\log_{10}d_{mn}^{(l)}-\mu_{SSF,\gamma}\right)^2}{2\left(\sigma_{SF,\gamma}^2+\sigma_{SSF,\gamma}^2\right)}\right)\right\} + \log \pi_\gamma\right)\right]$$

with $\mu_{SSF,\gamma} = \sqrt{\frac{K_{SSF,\gamma}}{K_{SSF,\gamma}+1}}$, $\sigma_{SSF,\gamma}^2 = \frac{1}{K_{SSF,\gamma}+1}$,

$\forall \gamma \in \{1,2,3,\dots,R\}$;

Fix all parameters in $\mathcal{B}^{(i)}$ except for $\left\{(\alpha_\gamma, \beta_\gamma) : \gamma = 1,2,3,\ldots,R\right\}$:

(c)

$$\underset{\{(\alpha_\gamma,\beta_\gamma):\gamma=1,2,3,\ldots,R\}}{\text{maximize}} \; \mathbb{E}\left[g\left(\underline{\alpha}, \underline{\beta}, \underline{\sigma}_{SF}^{(i)}, \underline{K}_{SSF}^{(i)}, \underline{\Pi}^{(i)}\right)\right];$$

OPT VARS: $\{(\alpha_\gamma, \beta_\gamma) : \gamma = 1,2,3,\ldots,R\}$

FIXED VARS: $\sigma_{SF,\gamma} = \sigma_{SF,\gamma}^{(i)}$; $\Pi_\gamma = \Pi_\gamma^{(i)}$, $K_{SSF,\gamma} = K_{SSF,\gamma}^{(i)}$; $\forall \gamma \in \{1,2,3,\ldots R\}$

Get $\mu_{SSF,\gamma}$ and $\sigma_{SSF,\gamma}$ from the K-factor

$[\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_R]$ OPT VARS

$[\beta_1, \beta_2, \beta_3, \ldots, \beta_R]$ OPT VARS

$[\sigma_{SF,1}^{(i)}, \sigma_{SF,2}^{(i)}, \sigma_{SF,3}^{(i)}, \ldots, \sigma_{SF,R}^{(i)}]$ FIXED VARS

$[K_{SSF,1}^{(i)}, K_{SSF,2}^{(i)}, K_{SSF,3}^{(i)}, \ldots, K_{SSF,R}^{(i)}]$ FIXED VARS

$[\Pi_1^{(i)}, \Pi_2^{(i)}, \Pi_3^{(i)}, \ldots, \Pi_R^{(i)}]$

I

Similarly,

$$\underset{\{\sigma_{SF,\gamma}:\gamma=1,2,3,\ldots,R\}}{\text{maximize}} \; \mathbb{E}\left[g\left(\underline{\alpha}^{(i)}, \underline{\beta}^{(i)}, \underline{\sigma}_{SF}, \underline{K}_{SSF}^{(i)}, \underline{\Pi}^{(i)}\right)\right];$$

OPT VARS: $\{\sigma_{SF,\gamma} : \gamma = 1,2,3,\ldots,R\}$

FIXED VARS: $\alpha_\gamma = \alpha_\gamma^{(i)}$; $\beta_\gamma = \beta_\gamma^{(i)}$; $K_{SSF,\gamma} = K_{SSF,\gamma}^{(i)}$; $\Pi_\gamma = \Pi_\gamma^{(i)}$, $\forall \gamma \in \{1,2,3,\ldots,R\}$

II

Similarly,

$$\underset{\{K_{SSF,\gamma}:\gamma=1,2,3,\ldots,R\}}{\text{maximize}} \; \mathbb{E}\left[g\left(\underline{\alpha}^{(i)}, \underline{\beta}^{(i)}, \underline{\sigma}_{SF}^{(i)}, \underline{K}_{SSF}, \underline{\Pi}^{(i)}\right)\right], \text{ and}$$

OPT VARS: $\{K_{SSF,\gamma} : \gamma = 1,2,3,\ldots,R\}$

FIXED VARS: $\alpha_\gamma = \alpha_\gamma^{(i)}$; $\beta_\gamma = \beta_\gamma^{(i)}$; $\Pi_\gamma = \Pi_\gamma^{(i)}$; $\sigma_{SF,\gamma} = \sigma_{SF,\gamma}^{(i)}$, $\gamma \in \{1,2,3,\ldots,R\}$

III

Finally,

$$\underset{\{\Pi_\gamma:\gamma=1,2,3,\ldots,R\}}{\text{maximize}} \; \mathbb{E}\left[g\left(\underline{\alpha}^{(i)}, \underline{\beta}^{(i)}, \underline{\sigma}_{SF}^{(i)}, \underline{K}_{SSF}^{(i)}, \underline{\Pi}\right)\right]$$

$$\text{s.t. } \sum_{\gamma=1}^{R} \Pi_\gamma = 1.$$

OPT VARS: $\{\Pi_\gamma : \gamma = 1,2,3,\ldots,R\}$

FIXED VARS: $\alpha_\gamma = \alpha_\gamma^{(i)}$; $\beta_\gamma = \beta_\gamma^{(i)}$; $\sigma_{SF,\gamma} = \sigma_{SF,\gamma}^{(i)}$; $K_{SSF,\gamma} = K_{SSF,\gamma}^{(i)}$, $\gamma \in \{1,2,3,\ldots,R\}$

IV

$$\forall \gamma \in \{1, 2, 3, \ldots, R\}$$

Solve I: $\dfrac{\partial}{\partial \alpha_\gamma} \mathbb{E}\left[g\left(\underline{\alpha}, \underline{\beta}, \underline{\sigma}_{SF}^{(i)}, \underline{\kappa}_{SSF}^{(i)}, \underline{\pi}^{(i)}\right)\right] = 0$ and

$$\frac{\partial}{\partial \beta_\gamma} \mathbb{E}\left[g\left(\underline{\alpha}, \underline{\beta}, \underline{\sigma}_{SF}^{(i)}, \underline{\kappa}_{SSF}^{(i)}, \underline{\pi}^{(i)}\right)\right] = 0;$$

$$\sum_{l=1}^{L} \bar{z}_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right)\left[\frac{1}{\cancel{m(\cdot\cdot)}} \cdot \left(\frac{1}{\sqrt{\cdot\cdot}} \exp(\cdot)\right)^{-m(\cdot\cdot)} \left(\frac{-1\cdot}{2(\sigma^2+\sigma^2)} (10\log_{10} d)^2 \right.\right.$$
$$\left. 2(h_{mn} - \beta_\gamma + 10\alpha_\gamma \log d - \mu)\cdot \right] = 0;$$

$$\left\{\left(\alpha_\gamma, \beta_\gamma, \sigma_{SF,\gamma}^{(i)}, \kappa_{SSF,\gamma}^{(i)}, \pi_\gamma^{(i)}\right), \gamma = 1, 2, 3, \ldots, R\right\}$$

$$\sum_{l=1}^{L} \bar{z}_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right) \cdot \frac{-2}{2\left(\sigma_{SF,\gamma}^{(i)2} + \sigma_{SSF,\gamma}^{(i)2}\right)}\left(h_{mn}^{(l)} - \beta_\gamma + 10\alpha_\gamma^{(i+1)} \log_{10} d_{mn}^{(l)} - \mu_{SSF,\gamma}^{(i)}\right) \cdot \left(10\log_{10} d_{mn}^{(l)}\right) = 0;$$

$$\vdots$$

$$\sum_{l=1}^{L} \bar{z}_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right)\left(10\log_{10} d_{mn}^{(l)}\right)\left(h_{mn}^{(l)} - \beta_\gamma^{(i)} + 10\alpha_\gamma^{(i+1)} \log_{10} d_{mn}^{(l)} - \mu_{SSF,\gamma}^{(i)}\right) = 0;$$

$$-\sum_{l=1}^{L} \bar{z}_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right)\left(10\log_{10} d_{mn}^{(l)}\right) h_{mn}^{(l)} + \sum_{l=1}^{L} \bar{z}_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right)\left(10\log_{10} d_{mn}^{(l)}\right)\beta_\gamma^{(i)}$$
$$+ \sum_{l=1}^{L} \bar{z}_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right)\left(10\log_{10} d_{mn}^{(l)}\right)\mu_{SSF,\gamma}^{(i)}$$
$$= \sum_{l=1}^{L} \bar{z}_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right)\left(10\log_{10} d_{mn}^{(l)}\right)\left(10\alpha_\gamma^{(i+1)} \log_{10} d_{mn}^{(l)}\right);$$

$$\alpha_\gamma^{(i+1)}\left(\sum_{l=1}^{L} \bar{z}_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right)\left(10\log_{10} d_{mn}^{(l)}\right)^2\right)$$

$$\therefore \alpha_\gamma^{(i+1)} = \frac{1}{\sum_{l=1}^{L} \bar{z}_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right)\left(10\log_{10} d_{mn}^{(l)}\right)^2}\left\{\sum_{l=1}^{L} \bar{z}_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right)\left(10\log_{10} d_{mn}^{(l)}\right)\beta_\gamma^{(i)} + \sum_{l=1}^{L} \bar{z}_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right)\left(10\log_{10} d_{mn}^{(l)}\right)\mu_{SSF,\gamma}^{(i)} - \sum_{l=1}^{L} \bar{z}_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right)\left(10\log_{10} d_{mn}^{(l)}\right) h_{mn}^{(l)}\right\}$$ and

Similarly,

$$\therefore \beta_\gamma^{(i+1)} = \frac{1}{\sum_{l=1}^{L} \bar{z}_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right)\left(10\log_{10} d_{mn}^{(l)}\right)}\left\{\sum_{l=1}^{L} \bar{z}_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right)\left(10\log_{10} d_{mn}^{(l)}\right)^2 \alpha_\gamma^{(i)} - \sum_{l=1}^{L} \bar{z}_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right)\left(10\log_{10} d_{mn}^{(l)}\right)\mu_{SSF,\gamma}^{(i)} + \sum_{l=1}^{L} \bar{z}_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right)\left(10\log_{10} d_{mn}^{(l)}\right) h_{mn}^{(l)}\right\}.$$

$$\frac{1}{x} \qquad \frac{x(0)-x(i)}{\Delta t}$$

Solve II $\quad \dfrac{\partial}{\partial \sigma_{SF,\gamma}} \mathbb{E}\left[g\left(\underline{\alpha}^{(i)}, \underline{\beta}^{(i)}, \underline{\sigma}_{SF}, \underline{K}^{(i)}_{SSF}, \underline{\Pi}^{(i)} \right) \right] = 0;$

$$\sum_{l=1}^{L} \bar{z}_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right) \left(\frac{1}{m(\dots)} \cdot \left(\frac{1}{\sqrt{2\pi\left(\sigma_{SF,\gamma}^2+\sigma_{SSF,\gamma}^{(i)2}\right)}} \cdot \exp(\dots) \cdot \cancel{\frac{1}{2}} \cdot \frac{\left(\cancel{\sigma_{SF,\gamma}^2+\sigma_{SSF,\gamma}^{(i)2}}\right)(0) - (\cdot)^2 (2\sigma_{SF,\gamma})}{\left(\sigma_{SF,\gamma}^2+\sigma_{SSF,\gamma}^{(i)2}\right)^2} + \exp(-\dots)\frac{1}{\sqrt{2\pi}} \left(\frac{\sqrt{\sigma^2+\sigma^2}(0) - 1 \frac{1}{2\sqrt{\sigma^2+\sigma^2}} \cdot 2\sigma_{SF,\gamma}}{\left(\sigma_{SF,\gamma}^2+\sigma_{SSF,\gamma}^{(i)2}\right)} \right) \right) \right) = 0$$

$$\left\{ \left(\alpha_\gamma^{(i)}, \beta_\gamma^{(i)}, \sigma_{SF,\gamma}, K_{SSF,\gamma}^{(i)}, \pi_\gamma^{(i)} \right) : \gamma = 1,2,3,\dots,R \right\}$$

$$\sum_{l=1}^{L} \bar{z}_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right) \left(\frac{1}{\cancel{m(\dots)}} \left(\cancel{m(\dots)} \cdot \left(\left(h_{mn}^{(l)} - \beta_\gamma^{(i)} + 10\alpha_\gamma^{(i)} \log_{10} d_{mn}^{(l)} - \mu_{SSF,\gamma}^{(i)} \right)^2 \cdot \sigma_{SF,\gamma}^{(i+1)} \right) + \frac{1}{\cancel{\sqrt{2\pi\left(\sigma_{SF,\gamma}^2+\sigma_{SSF,\gamma}^2\right)}}} \cancel{\exp(\dots)} \cdot \left(\frac{-\sigma_{SF,\gamma}^{(i+1)}}{\left(\sqrt{\sigma_{SF,\gamma}^{(i+1)2}+\sigma_{SSF,\gamma}^{(i)2}}\right)^2} \right) \right) \right) = 0$$

$m(\dots)$

$$\sum_{l=1}^{L} \bar{z}_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right) \left(\sigma_{SF,\gamma}^{(i+1)} \left(h_{mn}^{(l)} - \beta_\gamma^{(i)} + 10\alpha_\gamma^{(i)} \log_{10} d_{mn}^{(l)} - \mu_{SSF,\gamma}^{(i)} \right)^2 - \frac{\sigma_{SF,\gamma}^{(i+1)}}{\sigma_{SF,\gamma}^{2(i+1)} + \sigma_{SSF,\gamma}^{(i)2}} \right) = 0$$

$$\cancel{\sigma_{SF,\gamma}^{(i+1)}} \sum_{l=1}^{L} \bar{z}_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right) \underbrace{\left(h_{mn}^{(l)} - \beta_\gamma^{(i)} + 10\alpha_\gamma^{(i)} \log_{10} d_{mn}^{(l)} - \mu_{SSF,\gamma}^{(i)} \right)^2}_{func(\mathcal{B}^{(i)})} = \frac{\cancel{\sigma_{SF,\gamma}^{(i+1)}}}{\sigma_{SF,\gamma}^{2(i+1)} + \sigma_{SSF,\gamma}^{2(i)}} \sum_{l=1}^{L} \bar{z}_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right)$$

$$\sigma_{SF,\gamma}^{2(i+1)} = \frac{\sum_{l=1}^{L} \bar{z}_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right)}{\sum_{l=1}^{L} \bar{z}_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right)\left(func\left(\mathcal{B}^{(i)}\right)\right)^2} - \sigma_{SSF,\gamma}^{2(i)}$$

Solve III $\quad \mu_{SSF,\gamma}$ and $\sigma_{SSF,\gamma}$, $\forall \gamma \in \{1,2,3,\dots,R\}$

$$\sigma_{SSF,\gamma}^{2(i+1)} = \frac{\sum_{l=1}^{L} \bar{z}_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right)}{\sum_{l=1}^{L} \bar{z}_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right)\left(func\left(\mathcal{B}^{(i)}\right)\right)^2} - \sigma_{SF,\gamma}^{2(i)} \quad \text{and}$$

$$\frac{\partial}{\partial \sigma_{SSF,\gamma}} \mathbb{E}\left[g\left(\underline{\alpha}^{(i)}, \underline{\beta}^{(i)}, \underline{\sigma}_{SSF}^{(i)}, \underline{\mu}_{SSF}^{(i)}, \underline{\sigma}_{SSF}^2, \underline{\Pi}^{(i)} \right) \right] = 0$$

$$\frac{\partial}{\partial \mu_{SSF,\gamma}} \mathbb{E}\left[g\left(\underline{\alpha}^{(i)}, \underline{\beta}^{(i)}, \underline{\sigma}_{SF}, \underline{\mu}_{SSF}, \underline{\sigma}_{SSF}, \underline{\Pi}^{(i)} \right) \right] = 0 ;$$

$$\sum_{l=1}^{L} z_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right) \left(\cancel{\frac{1}{m(\ldots)}} \cdot \cancel{m(\ldots)} \cdot \frac{1}{\cancel{2}\left(\sigma_{SF,\gamma}^{(i)2} + \sigma_{SSF,\gamma}^{(i)2}\right)} \cdot \cancel{2}\left(h_{mn}^{(l)} - \beta_\gamma^{(i)} + 10\alpha_\gamma^{(i)} \log_{10} d_{mn}^{(l)} - \mu_{SSF,\gamma}^{(i+1)} \right) \cdot \cancel{-1} \right) = 0 ;$$

$$\left\{ \left(\alpha_\gamma^{(i)}, \beta_\gamma^{(i)}, \sigma_{SF,\gamma}^{(i)}, \mu_{SSF,\gamma}, \sigma_{SSF,\gamma}^{(i)}, \Pi_\gamma^{(i)} \right) : \gamma = 1, 2, 3, \ldots, R \right\}$$

$$\sum_{l=1}^{L} z_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right) \left(h_{mn}^{(l)} - \beta_\gamma^{(i)} + 10\alpha_\gamma^{(i)} \log_{10} d_{mn}^{(l)} - \mu_{SSF,\gamma}^{(i+1)} \right) = 0 ;$$

$$\therefore \mu_{SSF,\gamma}^{(i+1)} = \frac{1}{\sum_{l=1}^{L} z_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right)} \left(\sum_{l=1}^{L} z_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right) \left(h_{mn}^{(l)} + 10\alpha_\gamma^{(i)} \log_{10} d_{mn}^{(l)} \right) \right) - \beta_\gamma^{(i)} .$$

Solve IV

$$\mathcal{L}\left(\underline{\Pi}, \lambda \,\middle|\, \mathcal{B}^{(i)} \right) = \mathbb{E}\left[g\left(\mathcal{B}^{(i)}\right) \right] + \lambda \left(\sum_{\gamma=1}^{R} \Pi_\gamma - 1 \right) ;$$

$$\left\{ \left(\alpha_\gamma^{(i)}, \beta_\gamma^{(i)}, \sigma_{SF,\gamma}^{(i)}, K_{SSF,\gamma}^{(i)}, \Pi_\gamma \right) : \gamma = 1, 2, 3, \ldots, R \right\}$$

KKT

$$\underbrace{\frac{\partial}{\partial \Pi_\gamma} \mathcal{L}\left(\underline{\Pi}, \lambda \,\middle|\, \mathcal{B}^{(i)} \right) = 0}_{\text{primal feasibility}} ; \quad \underbrace{\lambda \left(\sum_{\gamma=1}^{R} \Pi_\gamma - 1 \right) = 0}_{CS} ; \quad \underbrace{\lambda \geq 0}_{\text{dual feasibility}}$$

$$\sum_{l=1}^{L} z_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right) \frac{1}{\Pi_\gamma} + \lambda = 0 ;$$

With $\sum_{\gamma=1}^{R} \Pi_\gamma = 1$, $\lambda = 0$ $\nearrow$ $\therefore \Pi_\gamma^{(i+1)} = \frac{1}{L} \sum_{l=1}^{L} z_\gamma^{(l)}\left(\mathcal{B}^{(i)}\right) .$

At iteration (i):

$$\therefore \Pi_{\gamma}^{(i+1)} = \frac{1}{L}\left[\sum_{l=1}^{L} \Xi_{\gamma}^{(l)}\left(\mathcal{B}^{(i)}\right)\right], \forall \gamma \in \{1,2,3,\ldots,R\},$$

This first term looks inverted in the ref paper DOUBLE CHECK

$$\therefore \sigma_{SF,\gamma}^{2\,(i+1)} = \frac{\sum_{l=1}^{L} \Xi_{\gamma}^{(l)}\left(\mathcal{B}^{(i)}\right)}{\sum_{l=1}^{L} \Xi_{\gamma}^{(l)}\left(\mathcal{B}^{(i)}\right)\left(h_{mn}^{(l)} - \beta_{\gamma}^{(i)} + 10\alpha_{\gamma}^{(i)}\log_{10} d_{mn}^{(l)} - \mu_{SSF,\gamma}^{(i)}\right)^2} - \sigma_{SSF,\gamma}^{(i)\,2},$$

$\forall \gamma \in \{1,2,3,\ldots,R\}$;

Again, DOUBLE CHECK (inverted!)

$$\therefore \sigma_{SSF,\gamma}^{2\,(i+1)} = \frac{\sum_{l=1}^{L} \Xi_{\gamma}^{(l)}\left(\mathcal{B}^{(i)}\right)}{\sum_{l=1}^{L} \Xi_{\gamma}^{(l)}\left(\mathcal{B}^{(i)}\right)\left(h_{mn}^{(l)} - \beta_{\gamma}^{(i)} + 10\alpha_{\gamma}^{(i)}\log_{10} d_{mn}^{(l)} - \mu_{SSF,\gamma}^{(i)}\right)^2} - \sigma_{SF,\gamma}^{(i)\,2},$$

$\forall \gamma \in \{1,2,3,\ldots,R\}$;

$$\therefore \mu_{SSF,\gamma}^{(i+1)} = \frac{1}{\sum_{l=1}^{L} \Xi_{\gamma}^{(l)}\left(\mathcal{B}^{(i)}\right)}\left[\sum_{l=1}^{L} \Xi_{\gamma}^{(l)}\left(\mathcal{B}^{(i)}\right)\left(h_{mn}^{(l)} - \beta_{\gamma}^{(i)} + 10\alpha_{\gamma}^{(i)}\log_{10} d_{mn}^{(l)}\right)\right],$$

$\forall \gamma \in \{1,2,3,\ldots,R\}$;

DOUBLE CHECK

$$\therefore \alpha_{\gamma}^{(i+1)} = \frac{1}{\sum_{l=1}^{L} \Xi_{\gamma}^{(l)}\left(\mathcal{B}^{(i)}\right)\left(10\log_{10} d_{mn}^{(l)}\right)^2}\left[\sum_{l=1}^{L} \Xi_{\gamma}^{(l)}\left(\mathcal{B}^{(i)}\right)\left(10\log_{10} d_{mn}^{(l)}\right)\left(\beta_{\gamma}^{(i)} + \mu_{SSF,\gamma}^{(i)} - h_{mn}^{(l)}\right)\right],$$

$\forall \gamma \in \{1,2,3,\ldots,R\}$, and

$$\therefore \beta_{\gamma}^{(i+1)} = \frac{1}{\sum_{l=1}^{L} \Xi_{\gamma}^{(l)}\left(\mathcal{B}^{(i)}\right)\left(10\log_{10} d_{mn}^{(l)}\right)}\left[\sum_{l=1}^{L} \Xi_{\gamma}^{(l)}\left(\mathcal{B}^{(i)}\right)\left(10\log_{10} d_{mn}^{(l)}\right)\left(h_{mn}^{(l)} + 10\alpha_{\gamma}^{(i)}\log_{10} d_{mn}^{(l)} - \mu_{SSF,\gamma}^{(i)}\right)\right].$$

Summary

- R partitions of the deployment site.
- Each partition represented by : $(\alpha_r, \beta_r, \sigma_{SF,r}, \underbrace{K_{SSF,r}}_{\mu_{SSF,r},\ \sigma_{SSF,r}}, \pi_r)$, $\forall r \in \{1,2,3,\ldots,R\}$

partition index

Additionally, each partition is represented by a subset of measurements from $\mathcal{M}$ whose $z_r^{(l)} = 1$, for any $l \in \{1,2,3,\ldots,L\}$.

So, $$\mathcal{R} = \left\{ \left(\alpha_r, \beta_r, \sigma_{SF,r}, K_{SSF,r}, \pi_r, \left\{ p_m^{(l)}, p_n^{(l)}, h_{mn}^{(l)}, \left[z_1^{(l)}, z_2^{(l)}, z_3^{(l)}, \ldots, z_R^{(l)} \right] : z_r^{(l)} = 1,\ z_{r'}^{(l)} = 0, \forall r \neq r' \right\} \right), r = \{1,2,3,\ldots,R\} \right\}.$$

- What the MLE procedure does is to obtain this set $\mathcal{R}$, i.e., the channel parameters for each partition and the representative set of measurements that belongs to each partition, from the labeled measurements (in terms of channel gain).

ALGORITHM

Initialization (I1): Partition $\mathcal{M} = \{(p_m, p_n, h_{mn}, \underline{z}^{(l)}) : l = 1,2,3,\ldots,L\}$ into R partitions using the K-means clustering algorithm, with the channel gains used as the clustering variable.

$$\underset{\{\mathcal{R}_1, \mathcal{R}_2, \mathcal{R}_3, \ldots, \mathcal{R}_R\}}{\operatorname{argmin}} \sum_{r=1}^{R} \sum_{h_{mn} \in \mathcal{R}_r} \left\| h_{mn} - \bar{h}_{r,mn} \right\|_2^2 \quad \text{s.t.}$$

$$R \ll L \text{ and } |\mathcal{R}_1| + |\mathcal{R}_2| + |\mathcal{R}_3| + \ldots + |\mathcal{R}_R| = L,$$

where $\bar{h}_{r,mn} = \dfrac{1}{|\mathcal{R}_r|} \sum_{h_{mn} \in \mathcal{R}_r} h_{mn}$, $\forall r \in \{1,2,3,\ldots,R\}$.

K-means clustering

Fuzzy Initialization : Randomly choose R measurements from $\mathcal{M}$ and treat them as the centroids of the R partitions.

Iteration

Assignment : $$\mathcal{R}_r^{(i)} = \left\{ h_{mn}^{(l)}, l = \{1,2,3,\ldots,L\} : \left\| h_{mn}^{(l)} - \bar{h}_{r,mn}^{(i)} \right\|_2^2 \leq \left\| h_{mn}^{(l)} - \bar{h}_{r',mn}^{(i)} \right\|_2^2, \forall r' \in \{1,2,3,\ldots,R\}, r \neq r' \right\}.$$ $\forall r \in \{1,2,3,\ldots,R\}$

Update : $$\bar{h}_{r,mn}^{(i)} = \frac{1}{|\mathcal{R}_r^{(i)}|} \sum_{h_{mn} \in \mathcal{R}_r^{(i)}} h_{mn}.$$ Iteration index (i) for K-means clustering

Termination : Assignments no longer change.

(I2) Then, $\bar{z}_r^{(l)}(\mathcal{B}^{(0)}) = 1$, if $h_{mn}^{(l)} \in \mathcal{R}_r$, i.e.,

$(f_m^{(l)}, f_n^{(l)}, h_{mn}^{(l)}) \in \mathcal{M}$ belongs to $\mathcal{R}_r$,

$\forall r \in \{1,2,3,\ldots,R\}$;

Else $\bar{z}_r^{(l)}(\mathcal{B}^{(0)}) = 0$.

Iteration (A) : Using these estimates of $\bar{z}_r^{(l)}(\mathcal{B}^{(0)})$, $\forall l \in \{1,2,3,\ldots,L\}$, $\forall r \in \{1,2,3,\ldots,R\}$,

$\forall r \in \{1,2,3,\ldots R\}$ find $\alpha_r^{(i)}, \beta_r^{(i)}, \sigma_{SF,r}^{(i)}, \underbrace{\mu_{SSF,r}^{(i)}, \sigma_{SSF,r}^{(i)}}_{K_{SSF,r}^{(i)}}, \pi_r^{(i)}$ using (I)-(IV),

$\alpha_r^{(i)}, \beta_r^{(i)}, \sigma_{SF,r}^{(i)}, \mu_{SSF,r}^{(i)}, \sigma_{SSF,r}^{(i)}, \pi_r^{(i)}$

Termination (A) : until these values do not change above a preset threshold.

Thus, we now have

$$\mathcal{R} = \{\mathcal{R}_1, \mathcal{R}_2, \mathcal{R}_3, \ldots, \mathcal{R}_R : r = 1, 2, 3, \ldots, R\}, \text{ where}$$

$$\mathcal{R}_r = \Big\{ \Big(\alpha_r, \beta_r, \sigma_{SF,r}, K_{SSF,r}, \Pi_r, \{ p_m^{(l)}, p_n^{(l)}, h_{mn}^{(l)}, [z_1^{(l)}, z_2^{(l)}, z_3^{(l)}, \ldots, z_R^{(l)}] : z_r^{(l)} = 1,\ z_{r'}^{(l)} = 0,\ \forall r \neq r',\ l = 1, 2, 3, \ldots, L \} \Big), r = 1, 2, 3, \ldots, R \Big\}.$$

Next, we need to dynamically classify each positional pair $(p_m, p_n) \notin \mathcal{M}$ but in the wider deployment site as a whole using KNN classification.

Classification (A) Define the index set of S-nearest neighbors of $(p_m, p_n) \in \mathcal{D}$ as

$$\mathcal{S}(p_m, p_n) = \underset{\substack{\tilde{\mathcal{S}} \subseteq \{1,2,3,\ldots L\} \\ |\tilde{\mathcal{S}}| = S}}{\operatorname{argmin}} \sum_{l \in \tilde{\mathcal{S}}} \left[\| p_m - p_m^{(l)} \|_2 + \| p_n - p_n^{(l)} \|_2 \right]$$

s.t. $R << L$ and $S << L$.

Then, the partition to which $(p_m, p_n) \in \mathcal{D}$ belongs to is given by

$$\begin{bmatrix} \hat{z}_1 \\ \hat{z}_2 \\ \hat{z}_3 \\ \vdots \\ \hat{z}_R \end{bmatrix}$$

$$\hat{\underline{z}}(p_m, p_n) = \mu \sum_{l \in \mathcal{S}(p_m, p_n)} \mathcal{K}(p_m, p_n, p_m^{(l)}, p_n^{(l)}) \, \underline{\bar{z}}^{(l)}, \text{ where}$$

$\bar{z}_{r(l)}^{(l)}$ known from alg (A)

$$\mathcal{K}(p_m, p_n, p_m^{(l)}, p_n^{(l)}) \triangleq \exp\left\{ - \frac{\left(\| p_m - p_m^{(l)} \|_2 + \| p_n - p_n^{(l)} \|_2 \right)^2}{s} \right\},$$

kernel function

s is the kernel parameter $(s > 0)$ and $\mu > 0$ is chosen such that

$$\sum_{r=1}^{R} \hat{z}_r(p_m, p_n) = 1.$$

classifying the unknown positional pair into 1 of R partitions (probabilistically speaking...), then, we have the channel parameters for this positional pair.

Finally, to get the channel gain at this positional pair :

$$(\mathbf{p}_m, \mathbf{p}_n)$$

GN Tx ANT position $\mathbf{p}_m \in \mathbb{R}^3$

UAV Rx ANT position $\mathbf{p}_n \in \mathbb{R}^3$

$$\therefore \quad \hat{h}_{mn}(\mathbf{p}_m, \mathbf{p}_n) = \sum_{r=1}^{R} \left(\beta_r - 10\,\alpha_r \log_{10} d_{mn} + \xi_r + \lambda_r \right) \hat{i}_r(\mathbf{p}_m, \mathbf{p}_n)$$

$$\|\mathbf{p}_m - \mathbf{p}_n\|_2$$

where $\xi_r \sim \mathcal{N}\left(0, \sigma^2_{SF,r}\right)$ and $\lambda_r \sim \mathcal{CN}\left(\sqrt{\frac{K_{SSF,r}}{K_{SSF,r}+1}}, \frac{1}{K_{SSF,r}+1}\right)$