Iterative Gradient Descent (IGD) & Iterative Brute Force (IBF) ALGORITHMS

10/30/2023

- Designs two distributed algorithms to achieve the highest degree of freedom LoS MIMO channel, while minimizing the distance traveled.

BS N_T antennas $\longrightarrow$ N_R UAVs, each UAV has one antenna

$$N_R \geq N_T$$

Location of m^{th} Tx antenna p_m, $m \in \{1, 2, \ldots, N_T\}$, $p_m \in \mathbb{R}^3$.

Location of n^{th} UAV q_n, $n \in \{1, 2, \ldots, N_R\}$, $q_n \in \mathbb{R}^3$.

- MIMO channel $\mathbf{H} \in \mathbb{C}^{N_R \times N_T}$ between the m^{th} transmitter and the n^{th} receiver is

$$\{\mathbf{H}\}_{m,n} \triangleq h_{m,n} = \gamma_{m,n} \exp\left(-j\left(\frac{2\pi}{\lambda}\right)\|p_m - q_n\|_2\right),$$

where $\gamma_{m,n} = \dfrac{\lambda}{4\pi(\|p_m - q_n\|_2)}$ is the pathloss coefficient and λ is the wavelength.

$$\mathbf{H} = \begin{bmatrix} | & | & & | \\ h_1 & h_2 & \cdots & h_{N_T} \\ | & | & & | \end{bmatrix}$$

- Approximate $\gamma_{m,n} \approx \dfrac{\lambda}{4\pi R}$, where R is the distance between the BS and the UAV swarm.

SU-MIMO

4 antennas 4 streams

16 antennas 16 UAVs

Maximum channel capacity (and hence minimum GW service latency in Hermes) can be achieved when

$$\sigma_1 = \sigma_2 = --- = \sigma_{N_T}$$

singular values of $\mathbf{H}$

$$\Downarrow$$

columns of H have to be made orthogonal

$$h_l^* h_k = 0, \quad \forall (l,k) \in \mathcal{K}$$

Hermitian transpose

$$\mathcal{K} \equiv \{(l,k) \in \{1,2,-- N_T\} \times \{1,2,- N_R\} \text{ s.t. } l \neq k\}.$$

$$\underset{\text{Inverse Condition Number}}{\varepsilon(\mathbf{H})} = \frac{1}{\underset{\text{condition number}}{\kappa(\mathbf{H})}} = \frac{\sigma_{N_T}}{\sigma_1}$$

Since $\varepsilon(\mathbf{H}) = 1$, when $\sigma_1 = \sigma_2 = --- = \sigma_{N_T}$ (i.e, max capacity solution), we can use ICN as the metric of orthogonality.

$$\underset{q_1, q_2, \ldots, q_{N_R}}{\text{minimize}} \quad \sum_{n=1}^{N_R} \|\bar{q}_n - q_n\|_2^2,$$

$$\text{s.t.} \quad \varepsilon(\mathbf{H}) \geq \alpha. \qquad 0 \leq \alpha \leq 1$$

minimizing the total distance traveled by each UAV s.t. a orthogonality of $\mathbf{H}$ requirement.

α choice enforces how strict of an orthogonality constraint we want.

There is no precoding at the Tx.

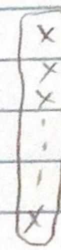
- Single-stream SINR is used as the measure of performance, which is then used to determine MIMO capacity.

SINR of the stream between Tx antenna 1 and the UAV

$$\mathrm{SINR} \triangleq \frac{|\underline{w}^* \underline{h}_1|^2}{\underbrace{\sum_{m=2}^{N_T} |\underline{w}^* \underline{h}_m|^2}_{\text{interference from Tx antenna } 2-N_T \text{ streams}} + \underbrace{\|\underline{w}\|^2 \sigma_n^2}_{\text{noise at the UAV}}}$$

σ_n^2 → noise variance

$\underline{w}$ → combining vector



Treat them as one

16 UAVs serving as a receiver / planar array

ZF combining → $\underline{w}$ is orthogonal to the range space of $\{\underline{h}_2, \ldots, \underline{h}_{N_T}\}$ so that we can perfectly recover the symbol in stream 1 from Tx antenna 1.

Matched Filtering (MF) → $\underline{w} = \underline{h}_1$

If H is orthogonal, ZF = MF.

$\min_{q \in \text{Cluster domain}} \sum_{m=1}^{AU} \| q - q_m \|_2^2 \quad q_1^*, \ldots, q_{AU}^*$

$\cancel{\sum_{q \in \text{Cluster domain}} \| q}$

Hermes: Given $q_1^*, q_2^*, \ldots, q_{N_R}^*$, find the centroid as use it as the position.

Original Problem Formulation

$$\underset{q_1, q_2, \ldots, q_{N_R}}{\text{minimize}} \sum_{m=1}^{N_R} \| \bar{q}_m - q_m \|_2^2$$

initial position of the m^{th} UAV

Reformulate

m^{th} UAV
i^{th} iteration:

$$\underset{\underline{r}_m^{(i)}}{\text{minimize}} \; f\left(q_m^{(i)} + \underline{r}_m^{(i)}\right)$$

position of the m^{th} UAV in the i^{th} iteration

where $q_m^{(i+1)} = q_m^{(i)} + \underline{r}_m^{(i)}$,

$$f\left(q_m\right) = \underbrace{\sum_{(l,k) \in \mathcal{K}} \left| \underline{h}_l^*(q_m) \, \underline{h}_k(q_m) \right|^2}_{\text{minimizing this is like finding the movement in 3D space that orthogonalizes the channel further}}.$$

Iterative Gradient Descent (IGD)

$$\nabla f(q_m) = \sum_{(l,k)\in K} \frac{4\pi}{\lambda} \left(\mathrm{Re}\{ \underline{h}^*_{-m,l}, \underline{h}_{-m,k} \} \sin a - \mathrm{Im}\{ \underline{h}^*_{-m,l}, \underline{h}_{-m,k} \} \cos a \right) \times \left(u(p_l - q_m) - u(p_k - q_m) \right)$$

$\underline{h}_{-m,l}$ is column l of $H_{N_R \times N_T}$ except element m,

$$a = \frac{2\pi}{\lambda} \left(\|p_l - q_m\| - \|p_k - q_m\| \right)$$

p_l: position of l^{th} Tx antenna

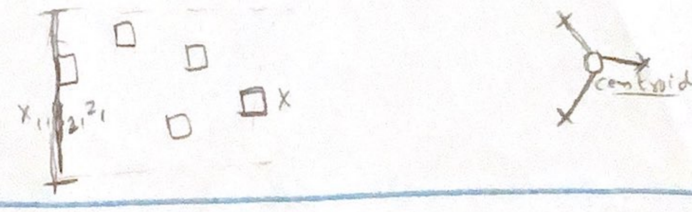
p_k: position of k^{th} Tx antenna

$b_{GD}^{(0)} = 0.05$

$$\underline{r}_m^{(i)} = -b_{GD}^{(i)} \nabla f(q_m^{(i)})$$

step size : $b_{GD}^{(i+1)} = \text{decay factor} \; b_{GD}^{(i)}$

decay factor < 1 (0.999)



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(IGD)

for i = 1 to # Iterations @ the {Master} do
    estimatePositionAll() {Master, UAVs}
    for all d in UAVs {Master} do
        calcGradientAndMoveUAV(d) {UAV d}
        estimateBroadcastCSIUAV(d) {UAV d}
    end for
    c = evalChannelOrthogonality() {Master}
    if meetsCriterion(c) {Master} then
        exitStartCommStage() {Master}
    end if
end for.

initial random voxels in the cluster domain

her brother

クロさき
CLINIC

でいおしましょ

(IBF)

$$\underline{r}_m^{(i)} = b_{BF}^{(i)}\left(\underset{\underline{z}\in\{\underline{0},\,\pm\underline{e}_1,\,\pm\underline{e}_2,\,\pm\underline{e}_3\}}{\operatorname{argmin}} f\left(\underline{q}_m + \cancel{\underline{x}_m^{(i)}}\,\underline{z}\right)\right)$$

?

$$\begin{bmatrix}0\\0\\0\end{bmatrix} \pm \begin{bmatrix}1\\0\\0\end{bmatrix} \pm \begin{bmatrix}0\\1\\0\end{bmatrix} \pm \begin{bmatrix}0\\0\\1\end{bmatrix}$$

$\underline{e}_i \in \mathbb{R}^3,\ i = \{1, 2, 3\}$

$$\underline{e}_1 = \begin{bmatrix}1\\0\\0\end{bmatrix},\quad \underline{e}_2 = \begin{bmatrix}0\\1\\0\end{bmatrix},\quad \underline{e}_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}$$

single-unit movement / single-voxel movement

for i = 1 to # Iterations @ the {Master} do
    for all d in UAVs {Master} do
        for all $\underline{z} \in Z$ {UAV d} do
            moveUAV(d, $\underline{z}$) {UAV d}
            estimateCSIEvalObjectiveUAV(d) {UAV d}
            moveUAV(d, $-\underline{z}$) {UAV d}
        end for
        $\hat{\underline{z}}$ = findMinObj ; moveUAV(d, $\hat{\underline{z}}$) {UAV d}
        broadcastCSIUAV(d) {UAV d}
    end for
If evalChannelOrthogonality() meets crit : exitStartCommStage {Master} endif

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