New HCSO metric for $\alpha \in [0,1]$:

$$(\mathbf{P.0})\quad \min_{\mathbf{p},\mathbf{v}} \sum_{m=0}^{M-1} \frac{\|\Psi_m\|_2}{v_m}\Big(1-2\alpha+\alpha\frac{P_{\text{mob}}(v_m)}{P_{\max}}\Big) \tag{1}$$

$$\text{s.t. } h_i(\mathbf{p},\mathbf{v}) \triangleq L - \sum_{m=\frac{M}{2}i}^{\frac{M}{2}(i+1)-1} F_m \le 0,\ \ i=0 \text{ and } 1, \tag{$\tilde{\text{C}}$.1}$$

$$\mathbf{x}_0=(r_U,0),\ \|\mathbf{x}_M\|_2=\hat{r}_U, \tag{$\tilde{\text{C}}$.3}$$

where **??** and **??** enforce the data payload and trajectory constraints. To solve $(\mathbf{P.0})$ with CSO, we first convert the constrained problem $(\mathbf{P.0})$ into an unconstrained one, by penalizing constraint violations with a particular solution: if the UAV does not decode (or forward) its data payload by the end of either phase, then it flies along the circumference of a circle (radius $r_{\min}$>0, small) around the current position with its power-minimizing velocity ($v_{P_{\min}}$=22m/s [**?**]) until the transmission/reception is completed. Moreover, we enforce the end radius constraint $\|\mathbf{x}_M\|_2 = \hat{r}_U$ by projecting the penultimate way-point $\mathbf{x}_{M-1}$ to the circle at radius $\hat{r}_U$, i.e. $\mathbf{x}_M=\hat{r}_U\mathbf{x}_{M-1}/\|\mathbf{x}_{M-1}\|_2$.[1] Thus, the penalized objective function is given as

$$\hat{f}_\alpha(\mathbf{p},\mathbf{v}) \triangleq \sum_{m=0}^{M-1}\frac{\|\Psi_m\|_2}{v_m}\Big(1-2\alpha+\alpha\frac{P_{\text{mob}}(v_m)}{P_{\max}}\Big)+(1-2\alpha)(\hat{t}_{P,0}+\hat{t}_{P,1})+\alpha\frac{1}{P_{\max}}(\hat{E}_{P,0}+\hat{E}_{P,1});$$

$$\hat{t}_{P,0}\triangleq\frac{\max\{h_0(\mathbf{p},\mathbf{v}),0\}}{\bar{R}_{GU}(\|\mathbf{x}_{M/2}-\mathbf{x}_G\|_2)};\ \hat{t}_{P,1}\triangleq\frac{\max\{h_1(\mathbf{p},\mathbf{v}),0\}}{\bar{R}_{UB}(\|\mathbf{x}_M\|_2)};\ \hat{E}_{P,i}\triangleq P_{\text{mob}}(v_{P_{\min}})\hat{t}_{P,i},\ \mathbf{x}_M=\hat{r}_U\frac{\mathbf{x}_{M-1}}{\|\mathbf{x}_{M-1}\|_2},$$

$$\frac{1}{\nu P_{\max}}\ell_\nu^*(\mathbf{s};\hat{r}_U,1)=\min_{\Delta,\mathbf{p}_U,t_p} -\ln(eT)\Delta+\frac{1}{P_{\max}}\int_0^\Delta P_{\text{mob}}\Big(\sqrt{r_U'(\eta)^2+r_U^2(\eta)\theta_U'(\eta)^2}\Big)\mathrm{d}\eta \text{ s.t.} constraints \tag{2}$$

where $T=\exp\{-1-(\frac{1}{\nu}-P_{\text{avg}})/P_{\max}\}$. T varies between $T=0$ (when $\nu\to 0$) to $T=1$ (when $\nu\to\infty$ and $P_{\text{avg}}\to P_{\max}$. Let $\Delta_T(\mathbf{s};\hat{r}_U)$ and $E_T(\mathbf{s};\hat{r}_U)$ the optimal delay and energy cost for a certain T (and associated trajectory)/

- Radii levels: K_R+1 levels $r_j=aj/K_R$ for $j=0,\dots,K_R$ (encoding UAV positions)
- Radii levels: G_R+1 levels $r_\ell=a\ell/G_R$ for $\ell=0,\dots,G_R$ (encoding GN radii positions)
- Angular levels (for GN request generation): for radius level ℓ, $G_{A,\ell}+1$ angular levels $\phi_{\ell,z}=$

[1] We assume that $\frac{\mathbf{x}}{\|\mathbf{x}\|_2}$=$(1,0)$ for a point in the origin, $\mathbf{x}$=$(0,0)$.

$z/G_{A,\ell}\pi$, $z = 0, \dots, G_{A,\ell}$ (note that value function is symmetric (with symmetric trajectries), wrt ϕ hence only $\phi \in [0, \pi]$ is considered).

- Radial velocity level: $2K_V + 1$ levels $v_u = u/K_V V_{\max}$ where $u = -K_V, \dots, K_V$.

Trajectory optimziation during comm:

Step 1: For every $(j, \ell, z, \hat{j})$ (encoding the current UAV radius r_j, GN position $(r_\ell, \phi_{\ell,z})$, next UAV radius position $r_{\hat{j}}$) find a set of $Q+1$ trajectories by solving the problem

$$\min_{\Delta, \mathbf{p}_U, t_p} -\ln(eT_q)\Delta + \frac{1}{P_{\max}} \int_0^{\Delta} P_{\text{mob}}\Big(\sqrt{r_U'(\eta)^2 + r_U^2(\eta)\theta_U'(\eta)^2}\Big)\mathrm{d}\eta \text{ s.t.} constraints \tag{3}$$

via HCSO. Here $T_q = q/Q$ for $q = 0, \dots Q$. $T_q = 0$ ($\nu = 0$) is a special case:

$$\min_{\Delta, \mathbf{p}_U, t_p} \Delta \text{ s.t.} constraints \tag{4}$$

Note that this involves $Q+1$ HCSO calls, for every state-outer action $(j, \ell, z, \hat{j})$. Since there are $(K_R+1)^2 \sum_{j=0}^{G_R} G_{A,j}$ state - outer actions, total number of HCSO calls is

$$(Q+1)(K_R+1)^2 \sum_{j=0}^{G_R} G_{A,j}$$

For each trajectory indexed by $q = 0 \dots Q$, you should have its delay and energy costs:

$$\Delta_q(j, \ell, z, \hat{j}), \;\; E_q(j, \ell, z, \hat{j})$$

Let $V_C(j)$ be the value function when a request arrives with the UAV in radial position $r_U = aj/K_R$, averaged out with respect to the request position. This is computed as

$$\begin{aligned} V_C(j) &= \int_0^{2\pi} \frac{1}{2\pi} \int_0^a \frac{2r'}{a^2} V(r_U, r', \psi')\mathrm{d}r'\mathrm{d}\psi' \\ &= \int_0^{\pi} \frac{1}{\pi} \int_0^a \frac{2r'}{a^2} V(r_U, r', \psi')\mathrm{d}r'\mathrm{d}\psi' \end{aligned}$$

(angular symmetry) and

$$\int_0^a \frac{2r'}{a^2} V(r_U, r', \psi')\mathrm{d}r' = \sum_{\ell=0}^{G_R-1} \int_{a\ell/G_R}^{a(\ell+1)/G_R} \frac{2r'}{a^2} V(r_U, r', \psi')\mathrm{d}r'$$

and approximating $V(r_U, r', \psi')$ for $r' \in [a\ell/G_R, a(\ell+1)/G_R]$ via linear interpolation as

$$V(r_U, r', \psi') \approx [(\ell+1) - r'G_R]V(r_U, a\ell/G_R, \psi') + [r'G_R - \ell]V(r_U, a(\ell+1)/G_R, \psi')$$

we obtain

$$\int_0^a \frac{2r'}{a^2} V(r_U, r', \psi')\mathrm{d}r' = \sum_{\ell=0}^{G_R} p_{R,\ell} V(r_U, a\ell/G_R, \psi')$$

where $p_{R,\ell}$ is defined as

$$p_{R,0} = \frac{1/3}{G_R^2},\ p_{R,\ell} = \frac{2\ell}{G_R^2}\ \forall \ell = 1 \dots G_R - 1,\ p_{R,G_R} = \frac{G_R - 1/3}{G_R^2}$$

(note that they sum to one) Therefore

$$V_C(j) = \int_0^{2\pi} \frac{1}{2\pi} \int_0^a \frac{2r'}{a^2} V(r_U, r', \psi')\mathrm{d}r'\mathrm{d}\psi'$$

$$= \sum_{\ell=0}^{G_R} p_{R,\ell} \int_0^{\pi} \frac{1}{\pi} V(r_U, a\ell/G_R, \psi')\mathrm{d}\psi'$$

$$= \sum_{\ell=0}^{G_R} p_{R,\ell} \sum_{z=0}^{G_{A,\ell}-1} \int_{z\pi/G_{A,\ell}}^{(z+1)\pi/G_{A,\ell}} \frac{1}{\pi} V(r_U, a\ell/G_R, \psi')\mathrm{d}\psi'$$

and approximating $V(r_U, a\ell/G_R, \psi')$ for $\psi' \in [z\pi/G_{A,\ell}, (z+1)\pi/G_{A,\ell}]$ via linear interpolation

$$V(r_U, a\ell/G_R, \psi') \approx [(z+1) - G_{A,\ell}\psi'/\pi] V(r_U, a\ell/G_R, z\pi/G_{A,\ell}) + (G_{A,\ell}\psi'/\pi - z) V(r_U, a\ell/G_R, (z+1)\pi/G_{A,\ell})$$

we obtain

$$\int_{z\pi/G_{A,\ell}}^{(z+1)\pi/G_{A,\ell}} \frac{1}{\pi} V_{C,i}(r_U, a\ell/G_R, \psi')\mathrm{d}\psi' = \frac{V(r_U, a\ell/G_R, z\pi/G_{A,\ell}) + V(r_U, a\ell/G_R, (z+1)\pi/G_{A,\ell})}{2G_{A,\ell}}$$

and

$$\sum_{z=0}^{G_{A,\ell}-1} \int_{z\pi/G_{A,\ell}}^{(z+1)\pi/G_{A,\ell}} \frac{1}{\pi} V_{C,i}(r_U, a\ell/G_R, \psi')\mathrm{d}\psi' = \sum_{z=0}^{G_{A,\ell}} p_{A,\ell,z} V(r_U, a\ell/G_R, z\pi/G_{A,\ell})$$

where $p_{A,\ell,z}$ is defined as

$$p_{A,\ell,0} = \frac{1}{2G_{A,\ell}},\ p_{A,\ell,z} = \frac{1}{G_{A,\ell}}\ \forall z = 1 \dots G_{A,\ell} - 1,\ p_{A,\ell,G_{A,\ell}} = \frac{1}{2G_{A,\ell}}$$

(note that they sum to one for each ℓ) yielding

$$V_C(j) = \sum_{\ell=0}^{G_R} p_{R,\ell} \sum_{z=0}^{G_{A,\ell}} p_{A,\ell,z} V(r_U, a\ell/G_R, z\pi/G_{A,\ell})$$

(note that $\sum_{\ell=0}^{G_R} p_{R,\ell} \sum_{z=0}^{G_{A,\ell}} p_{A,\ell,z} = 1$, as it should be)

Algorithm 1 $(O^*, U^*, g(\nu), \bar{\mathcal{E}}, V_{W,0}^{next}, V_{C,0}^{next}, \mathcal{E}_{W,0}^{next}, \mathcal{E}_{C,0}^{next}) = \mathrm{VITER}(\nu, V_{W,0}, V_{C,0}, \mathcal{E}_{W,0}, \mathcal{E}_{C,0})$

Initialization: i=0; stop criterion δ.

Inner optimization in waiting states: $\forall j = 0, \dots, K_R, \forall u = -K_V, \dots, K_V$, calculate

$$\ell_\nu^*(j;u) = \nu\left(P_{\mathrm{mob}}(\max\{|u/K_V|V_{\max}, V_{\min}\}) - P_{\mathrm{avg}}\right)\Delta_0;$$

compute excess energy cost

$$\epsilon^*(j;u) = P_{\mathrm{mob}}(\max\{|u/K_V|V_{\max}, V_{\min}\})\Delta_0 - P_{avg}\Delta_0.$$

Inner optimization in communication states: $\forall(j,\ell,a,\hat{j})$, with $j = 0\dots K_R$, $\hat{j} = 0 \dots K_R$, $\ell = 0\dots, G_R$, $a = 0 \dots G_{A,\ell}$: calculate

$$\ell_\nu^*(j,\ell,a,\hat{j},1) = \min_{q=0,\dots,Q}(1-\nu P_{avg})\Delta_q(j,\ell,a,\hat{j}) + \nu E_q(j,\ell,a,\hat{j});$$

with the minimizer q^*, compute excess energy cost

$$\epsilon^*(j,\ell,a,\hat{j}) = E_{q^*}(j,\ell,a,\hat{j}) - P_{avg}\Delta_{q^*}(j,\ell,a,\hat{j}).$$

repeat

for each $j = 0\dots, K_R$ **do** ▷ Outer optimization in waiting states

$$V_{W,i+1}(j) \leftarrow \min_{u=-K_V,\dots,K_V}\left[\ell_\nu^*(j;u) + e^{-\Lambda'\Delta_0}[\alpha V_{W,i}(\tilde{j}) + (1-\alpha)V_{W,i}(\tilde{j}+1)] + (1-e^{-\Lambda'\Delta_0})[\alpha V_{C,i}(\tilde{j}) + (1-\alpha)V_{C,i}(\tilde{j}+1)]\right],$$

where for each u, $\alpha \in [0,1]$ and (nearest index) $\tilde{j}$ are such that $|aj/K_R + u/K_V V_{\max}\Delta_0| = \alpha a\tilde{j}/K_R + (1-\alpha)a(\tilde{j}+1)/K_R$ (linear interpolation).

$O_{i+1}(j) \leftarrow u^*$, where u^* is the $\arg\min$, with α and $\tilde{j}$ (below) corresponding linear interpolation.

$\mathcal{E}_{W,i+1}(j) \leftarrow \epsilon^*(j;u) + e^{-\Lambda'\Delta_0}[\alpha\mathcal{E}_{W,i}(\tilde{j}) + (1-\alpha)\mathcal{E}_{W,i}(\tilde{j}+1)] + (1-e^{-\Lambda'\Delta_0})[\alpha\mathcal{E}_{C,i}(\tilde{j}) + (1-\alpha)\mathcal{E}_{C,i}(\tilde{j}+1)]$.

end for

for each $j = 0, \dots, K_R$ **do** ▷ Outer optimization in communication states

for each $\ell = 0, \dots, G_R$, $a = 0, \dots, G_{A,\ell}$ **do**

$TEMP_V(j,\ell,a) \leftarrow \min\{\frac{L}{R_{GB}(\ell/G_R)} + V_{W,i+1}(j), \min_{\hat{j}=0\dots,K_R}\left[\ell_\nu^*(j,\ell,a,\hat{j},1) + V_{W,i+1}(\hat{j})\right]$

$U_{i+1}(j,\ell,a) \leftarrow \hat{j}^*$, where $\hat{j}^*$ is the $\arg\min$ and scheduling decision $\xi_{i+1}(j,\ell,a) \leftarrow \xi^*$.

$TEMP_{energy}(j,\ell,a) \leftarrow \xi^* \cdot \epsilon^*(j,\ell,a,\hat{j}^*) + \mathcal{E}_{W,i+1}(\hat{j}^*)$

end for

$V_{C,i+1}(j) \leftarrow AVG[TEMP_V(j,\cdot,\cdot)]$.

$\mathcal{E}_{C,i+1}(j) \leftarrow AVG[TEMP_{energy}(j,\cdot,\cdot)]$.

Where $AVG[TEMP_X(j,\cdot,\cdot)] = \sum_{\ell=0}^{G_R} p_{R,\ell}\sum_{z=0}^{G_{A,\ell}} p_{A,\ell,z} TEMP_X(j,\ell,z)$

end for

$\forall j = 0, \dots, K_R$ and $X \in \{C, W\}$ calculate $\delta_X^V(j) = V_{X,i+1}(j) - V_{X,i}(j)$, $\delta_X^{\mathcal{E}}(j) = \mathcal{E}_{X,i+1}(j) - \mathcal{E}_{X,i}(j)$; $i \leftarrow i+1$.

until $\max_{j,X}\delta_X^V(j) - \min_{j,X}\delta_X^V(j) < \delta$ AND $\max_{j,X}\delta_X^{\mathcal{E}}(j) - \min_{j,X}\delta_X^{\mathcal{E}}(j) < \delta$. ▷ Value Iteration termination condition

Approximate $g(\nu) = \delta_W^V(0)/\pi_{\mathrm{comm}}$ and $\bar{\mathcal{E}} = \delta_W^{\mathcal{E}}(0)$. Compute $V_{X,0}^{next}(j) = V_{X,i}(j) - V_{W,i}(0)$, $\mathcal{E}_{X,0}^{next}(j) = \mathcal{E}_{X,i}(j) - \mathcal{E}_{W,i}(0), \forall j, \forall X$ (relative values; **note: you need to subtract the SAME quantity** $V_{W,i}(0)$ **and** $\mathcal{E}_{W,i}(0)$ **from ALL states**)

return $O^*(j) = O_i(j), \forall j = 0\dots, K_R$, $U^*(j,\ell,a) = U_i(j,\ell,a), \forall j,\ell,a$, $g(\nu)$, $\bar{\mathcal{E}}$, $V_{C,0}^{next}, \mathcal{E}_{W,0}^{next}, \mathcal{E}_{C,0}^{next}$. ▷ Optimal waiting and communication policies

Algorithm 2 Projected Sub-gradient Ascent: PSGA()

Initialization: dual variable $\nu \geq 0$; step-size $\{\rho_j = \frac{\rho_0}{(j+1)}, j \geq 0\}$; $V_{W,0}(j) = V_{C,0}(j) = \mathcal{E}_{W,0}(j) = \mathcal{E}_{C,0}(j) = 0, \forall j$

for k=0, 1, . . . **do**

$(O^*, U^*, g, \bar{\mathcal{E}}, V_{W,0}, V_{C,0}, \mathcal{E}_{W,0}, \mathcal{E}_{C,0}) \leftarrow \mathrm{VITER}(\nu, V_{W,0}, V_{C,0}, \mathcal{E}_{W,0}, \mathcal{E}_{C,0})$ via Alg. 1.

if $\bar{\mathcal{E}} < \epsilon_{PF}$; $\nu|\bar{\mathcal{E}}| < \epsilon_{CS}$ **then** ▷ Check KKT optimality conditions [NM: NOTE: you had $|g_k - g_{k-1}| < \epsilon_{DI}$ but I dont think is needed! VI is already guaranteeing the minimization of the Lagrangian which is one of the KKT conditions]

return: optimal outer policy (O^*, U^*);

else

Update $\nu \leftarrow \max\{\nu + \rho_k\bar{\mathcal{E}}, 0\}$; $k \leftarrow k+1$. ▷ Dual variable value update

end if

end for