Utility Maximization in Cognitive Radio Networks using POMDP Approximate Value Iteration methods

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***Abstract*—This document is a model and instructions for LaTeX. This and the IEEEtran.cls file define the components of your paper [title, text, heads, etc.]. *CRITICAL: Do Not Use Symbols, Special Characters, Footnotes, or Math in Paper Title or Abstract.**

***Index Terms*—Hidden Markov Model, Viterbi Algorithm, Expectation-Maximization, Belief Space, POMDP, Value Iteration, and the PERSEUS Algorithm**

I. Introduction

This document is a model and instructions for LaTeX. Please observe the conference page limits.

II. System Model

A. Observation Model

$$y(n) = \sum_{m=0}^{M-1} h(m)x(n-m) + v(n) \tag{1}$$

Here, $y(n)$ is the wideband signal observed at the SU receiver expressed as a convolution of the PU signal $x(n)$ with the channel impulse response $h(n)$, added with a noise term v(n). Equation (1) can be written in the frequency domain by taking a K-point DFT which decomposes the observed wideband signal into K discrete narrow-band components as shown below,

$$Y_k(i) = H_k X_k(i) + V_k(i) \tag{2}$$

where,
$i \in \{1, 2, 3, ..., T\}$ represents the index of the observation
$k \in \{1, 2, 3, ..., K\}$ represents the index of the channel
$V_k(i) \sim \mathcal{CN}(0, \sigma_V^2)$ represents the circular symmetric additive complex Gaussian noise sample i.i.d across channel indices and across time indices. These noise samples are assumed to be independent of the occupancy state of the channels.
$H_k \sim \mathcal{CN}(0, \sigma_H^2)$ represents the k^{th} DFT coefficient of the impulse response $h(n)$ of the channel in between the PU and the SU receiver - another circular symmetric complex Gaussian random variable i.i.d across channel indices with variance σ_H^2. These impulse response samples are also assumed to be independent of the occupancy state of the channels. We define the set B as the set of channels obtained by discretizing the spectrum of interest, i.e. $B = \{b_1, b_2, b_3, ..., b_K\}$. The PU occupancy behavior in each sub-band $b_k \in B$ is modelled as X_k taking two possible values 0 ($Idle$) and 1 ($Occupied$). Therefore, the PU occupancy behavior in the entire wideband spectrum of interest discretized into narrow-band frequency components can be modelled as a vector of size $|B| = K$ such that,

$$\vec{X} = [X_1, X_2, X_3, ..., X_K]^T \in \{0, 1\}^K \tag{3}$$

B. Correlation Model

The true states encapsulate the actual occupancy behavior of the PU and the measurements at the SU are noisy observations of these true states which are modelled to be the observed states of a Hidden Markov Model. For some sub-band $j \in \{2, 3, 4, ..., K\}$ and time index $i \in \{1, 2, 3, ..., T\}$, the system is assumed to satisfy the Markov property as shown below,

$$\mathbb{P}(X_j(i)|X_{j-1}(i), X_{j-2}(i), .., X_1(i)) = \mathbb{P}(X_j(i)|X_{j-1}(i))$$

And, we will use $\mathbb{P}(X_1(i))$ for $j = 1$. Now, let's expand on the previously discussed observation model. Taking the expectation operator on both sides of equation (2) given X_k has realized as x_k, we have,

$$\mathbb{E}[Y_k(i)|X_k(i) = x_k] = \mathbb{E}[H_k x_k] + \mathbb{E}[V_k(i)]$$
$$\mathbb{E}[Y_k(i)|X_k(i) = x_k] = \mathbb{E}[H_k]\mathbb{E}[x_k] + \mathbb{E}[V_k(i)]$$
$$\mathbb{E}[Y_k(i)|X_k(i) = x_k] = 0 \tag{4}$$

because, $V_k(i) \sim \mathcal{CN}(0, \sigma_V^2)$ and $H_k \sim \mathcal{CN}(0, \sigma_H^2)$.
Furthermore, the variance of $Y_k(i)$ given X_k at observation cycle i has realized as x_k, is calculated to be,

$$Var(Y_k(i)|X_k(i) = x_k) = \mathbb{E}[|H_k X_k(i) + V_k(i)|^2 | X_k(i) = x_k]$$
$$Var(Y_k(i)|X_k(i) = x_k) = \mathbb{E}[|H_k X_k(i)|^2 + |V_k(i)|^2 + 2\Re(H_k X_k(i) V_k^*(i)) | X_k(i) = x_k]$$
$$Var(Y_k(i)|X_k(i) = x_k) = \sigma_H^2 x_k + \sigma_V^2 \tag{5}$$

For the first part of our paper, we assume that the temporal dynamics of the PU Occupancy are slower than the SU's process times. In other words, for the first part of our paper, we assume that the PU is static during our evaluation period.

We can incorporate the above assumption into our correlation model as shown below by eliminating the time dependence.

$$\mathbb{P}(X_j|X_{j-1}, X_{j-2}, ..., X_1) = \mathbb{P}(X_j|X_{j-1}),\ for\ j > 1,$$

And, we will continue to use $\mathbb{P}(X_1)$ for $j = 1$. Now, we know that,

$$\vec{X} = [X_1, X_2, X_3, ..., X_K]^T$$

which realizes as $\vec{x} = [x_1, x_2, x_3, ..., x_K]^T$, so,

$$\mathbb{P}(\vec{X} = \vec{x}) = \mathbb{P}(X_1 = x_1) \prod_{k=2}^{K} \mathbb{P}(X_k = x_k|X_{k-1} = x_{k-1}) \tag{6}$$

Since $x_k \in \{0, 1\}$, for $k \in \{1, 2, 3, ..., K\}$, let,

$$\mathbb{P}(X_k = 1) \triangleq \Pi, \forall k$$

Furthermore, let,

$$\mathbb{P}(X_k = 1|X_{k-1} = 0) \triangleq p, \forall k$$

And,

$$\mathbb{P}(X_k = 0|X_{k-1} = 1) \triangleq q, \forall k$$

From the above definitions, we have,

$$\mathbb{P}(X_k = 1) = \Pi = \frac{p}{p + q}, \forall k$$

Moreover, we also assume that the Markov Property is satisfied when we traverse the spectrum in the descending order of the channel indices, i.e, the reverse direction. Mathematically,

$$\mathbb{P}(\vec{X} = \vec{x}) = \mathbb{P}(X_K = x_K) \prod_{k=1}^{K-1} \mathbb{P}(X_k = x_k|X_{k+1} = x_{k+1})$$

We now proceed with our discussions assuming that there is only one Primary User (PU), i.e. licensed incumbent in the wideband spectrum of interest and that there is only one Secondary User (SU) learning to intelligently access the *spectrum holes* or *white spaces* both spatially and temporally. In the next section of our paper, assuming a static PU, model knowledge, and complete observations, we discuss an algorithm to estimate the PU occupancy behavior. Later, we methodically relax these assumptions and detail occupancy behavior estimation results and optimal POMDP policy search methods for cases where we have incomplete observations, dynamic PU occupancy behavior, and no model information.

III. Prepare Your Paper Before Styling

Before you begin to format your paper, first write and save the content as a separate text file. Complete all content and organizational editing before formatting. Please note sections III-A–III-E below for more information on proofreading, spelling and grammar.

Keep your text and graphic files separate until after the text has been formatted and styled. Do not number text heads—LaTeX will do that for you.

A. Abbreviations and Acronyms

Define abbreviations and acronyms the first time they are used in the text, even after they have been defined in the abstract. Abbreviations such as IEEE, SI, MKS, CGS, ac, dc, and rms do not have to be defined. Do not use abbreviations in the title or heads unless they are unavoidable.

B. Units

- Use either SI (MKS) or CGS as primary units. (SI units are encouraged.) English units may be used as secondary units (in parentheses). An exception would be the use of English units as identifiers in trade, such as "3.5-inch disk drive".
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C. Equations

Number equations consecutively. To make your equations more compact, you may use the solidus (/), the exp function, or appropriate exponents. Italicize Roman symbols for quantities and variables, but not Greek symbols. Use a long dash rather than a hyphen for a minus sign. Punctuate equations with commas or periods when they are part of a sentence, as in:

$$a + b = \gamma \tag{7}$$

Be sure that the symbols in your equation have been defined before or immediately following the equation. Use "(7)", not "Eq. (7)" or "equation (7)", except at the beginning of a sentence: "Equation (7) is . . ."

D. LaTeX-Specific Advice

Please use "soft" (e.g., `\eqref{Eq}`) cross references instead of "hard" references (e.g., `(1)`). That will make it possible to combine sections, add equations, or change the order of figures or citations without having to go through the file line by line.

Please don't use the `{eqnarray}` equation environment. Use `{align}` or `{IEEEeqnarray}` instead. The `{eqnarray}` environment leaves unsightly spaces around relation symbols.

Please note that the `{subequations}` environment in LaTeX will increment the main equation counter even when there are no equation numbers displayed. If you forget that, you might write an article in which the equation numbers skip from (17) to (20), causing the copy editors to wonder if you've discovered a new method of counting.

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Do not use `\nonumber` inside the `{array}` environment. It will not stop equation numbers inside `{array}` (there won't be any anyway) and it might stop a wanted equation number in the surrounding equation.

E. Some Common Mistakes

- The word "data" is plural, not singular.
- The subscript for the permeability of vacuum μ_0, and other common scientific constants, is zero with subscript formatting, not a lowercase letter "o".
- In American English, commas, semicolons, periods, question and exclamation marks are located within quotation marks only when a complete thought or name is cited, such as a title or full quotation. When quotation marks are used, instead of a bold or italic typeface, to highlight a word or phrase, punctuation should appear outside of the quotation marks. A parenthetical phrase or statement at the end of a sentence is punctuated outside of the closing parenthesis (like this). (A parenthetical sentence is punctuated within the parentheses.)
- A graph within a graph is an "inset", not an "insert". The word alternatively is preferred to the word "alternately" (unless you really mean something that alternates).
- Do not use the word "essentially" to mean "approximately" or "effectively".
- In your paper title, if the words "that uses" can accurately replace the word "using", capitalize the "u"; if not, keep using lower-cased.
- Be aware of the different meanings of the homophones "affect" and "effect", "complement" and "compliment", "discreet" and "discrete", "principal" and "principle".
- Do not confuse "imply" and "infer".
- The prefix "non" is not a word; it should be joined to the word it modifies, usually without a hyphen.
- There is no period after the "et" in the Latin abbreviation "et al.".
- The abbreviation "i.e." means "that is", and the abbreviation "e.g." means "for example".

An excellent style manual for science writers is [7].

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The class file is designed for, but not limited to, six authors. A minimum of one author is required for all conference articles. Author names should be listed starting from left to right and then moving down to the next line. This is the author sequence that will be used in future citations and by indexing services. Names should not be listed in columns nor group by affiliation. Please keep your affiliations as succinct as possible (for example, do not differentiate among departments of the same organization).

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Headings, or heads, are organizational devices that guide the reader through your paper. There are two types: component heads and text heads.

Component heads identify the different components of your paper and are not topically subordinate to each other. Examples include Acknowledgments and References and, for these, the correct style to use is "Heading 5". Use "figure caption" for your Figure captions, and "table head" for your table title. Run-in heads, such as "Abstract", will require you to apply a style (in this case, italic) in addition to the style provided by the drop down menu to differentiate the head from the text.

Text heads organize the topics on a relational, hierarchical basis. For example, the paper title is the primary text head because all subsequent material relates and elaborates on this one topic. If there are two or more sub-topics, the next level head (uppercase Roman numerals) should be used and, conversely, if there are not at least two sub-topics, then no subheads should be introduced.

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a) Positioning Figures and Tables: Place figures and tables at the top and bottom of columns. Avoid placing them in the middle of columns. Large figures and tables may span across both columns. Figure captions should be below the figures; table heads should appear above the tables.

TABLE I
TABLE TYPE STYLES

Table Head	**Table Column Head**		
	Table column subhead	***Subhead***	***Subhead***
copy	More table copy[a]		

[a]Sample of a Table footnote.

Figure Labels: Use 8 point Times New Roman for Figure labels. Use words rather than symbols or abbreviations when writing Figure axis labels to avoid confusing the reader. As an example, write the quantity "Magnetization", or "Magnetization, M", not just "M". If including units in the label, present them within parentheses. Do not label axes only with units. In the example, write "Magnetization (A/m)" or "Magnetization {A[m(1)]}", not just "A/m". Do not label axes with a ratio of quantities and units. For example, write "Temperature (K)", not "Temperature/K".

ACKNOWLEDGMENT

The preferred spelling of the word "acknowledgment" in America is without an "e" after the "g". Avoid the stilted

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References

Please number citations consecutively within brackets [1]. The sentence punctuation follows the bracket [2]. Refer simply to the reference number, as in [3]—do not use “Ref. [3]” or “reference [3]” except at the beginning of a sentence: “Reference [3] was the first …”

Number footnotes separately in superscripts. Place the actual footnote at the bottom of the column in which it was cited. Do not put footnotes in the abstract or reference list. Use letters for table footnotes.

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