

# 28GHz POWDER Measurements

## Spatial Consistency Modeling

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### 1 Spatial Autocorrelation Coefficient

The small-scale spatial autocorrelation coefficient is a metric to characterize the coherence between the voltage amplitudes of received signals across variations in distances, alignment accuracies, and relative velocities.

#### 1.1 Separation in Distance

The spatial autocorrelation coefficient  $\rho$  vis-à-vis a separation in distance  $\Delta d$ :

$$\rho(\Delta d) = \rho\left(\mathbf{x}_{\text{Tx}}, \|\mathbf{x}_{\text{Rx}}(t') - \mathbf{x}_{\text{Rx}}(t)\| = \Delta d, \phi(t') = \phi(t), v(t') = v(t)\right), \text{ where}$$

- $\mathbf{x}_{\text{Tx}}(t) = \mathbf{x}_{\text{Tx}}, \forall t$  denotes the 3-D position vector of the Tx (fixed);
- $\mathbf{x}_{\text{Rx}}(t)$  and  $\mathbf{x}_{\text{Rx}}(t')$  denote the Rx 3-D position vectors at times  $t$  and  $t'$ , respectively, with  $\Delta d$  being the 3-D Euclidean distance between them;
- $\phi(t)$  and  $\phi(t')$  are the absolute errors in alignment between Tx and Rx antennas (deviation from perfect alignment) at times  $t$  and  $t'$ , respectively;
- $v(t)$  and  $v(t')$  denote the relative velocities of the receiver with respect to the transmitter at times  $t$  and  $t'$ , respectively; and
- The conditions  $\phi(t') = \phi(t)$  and  $v(t') = v(t)$  ensure that while studying the signal coherence characteristics vis-à-vis a separation in distance  $\Delta d$ , other variables in our measurements (i.e., alignment and velocity) are unchanged.

Mathematically, this spatial autocorrelation coefficient  $\rho(\Delta d)$  is given by

$$\frac{\mathbb{E}\left[\left(\mathbf{A}\left(\mathcal{P}_{\text{Tx}}(t), \mathcal{P}_{\text{Rx}}(t)\right) - \boldsymbol{\mu}\left(\mathcal{P}_{\text{Tx}}(t), \mathcal{P}_{\text{Rx}}(t)\right)\right) \cdot \left(\mathbf{A}\left(\mathcal{P}_{\text{Tx}}(t'), \mathcal{P}_{\text{Rx}}(t')\right) - \boldsymbol{\mu}\left(\mathcal{P}_{\text{Tx}}(t'), \mathcal{P}_{\text{Rx}}(t')\right)\right)\right]}{\sqrt{\mathbb{E}\left[\left\|\mathbf{A}\left(\mathcal{P}_{\text{Tx}}(t), \mathcal{P}_{\text{Rx}}(t)\right) - \boldsymbol{\mu}\left(\mathcal{P}_{\text{Tx}}(t), \mathcal{P}_{\text{Rx}}(t)\right)\right\|^2\right] \mathbb{E}\left[\left\|\mathbf{A}\left(\mathcal{P}_{\text{Tx}}(t'), \mathcal{P}_{\text{Rx}}(t')\right) - \boldsymbol{\mu}\left(\mathcal{P}_{\text{Tx}}(t'), \mathcal{P}_{\text{Rx}}(t')\right)\right\|^2\right]}}.$$

For a route  $\mathcal{R}$ , we define the terms employed in this equation as follows.

Tx configuration:  $\mathcal{P}_{\text{Tx}}(t) = \mathcal{P}_{\text{Tx}}(t') = \{\mathbf{x}_{\text{Tx}}, \theta_{\text{Tx}}\}, \forall t, t' \in \mathcal{R};$

Rx configuration:  $\mathcal{P}_{\text{Rx}}(t) = \{\mathbf{x}_{\text{Rx}}, \theta_{\text{Rx}}, v_{\text{Rx}}\}, \mathcal{P}_{\text{Rx}}(t') = \{\mathbf{x}'_{\text{Rx}}, \theta'_{\text{Rx}}, v'_{\text{Rx}}\}, \forall t, t' \in \mathcal{R};$

**Evaluation conditions E1:**  $\|\mathbf{x}'_{\text{Rx}} - \mathbf{x}_{\text{Rx}}\| = \Delta d, \phi(\theta'_{\text{Tx}}, \theta'_{\text{Rx}}) = \phi(\theta_{\text{Tx}}, \theta_{\text{Rx}}), v'_{\text{Rx}} = v_{\text{Rx}}.$

Let  $N$  be the number of resolvable Multi-Path Components (MPCs) extracted via the SAGE algorithm at time  $t$  at Rx config  $\mathcal{P}_{\text{Rx}}(t)$ , with Tx config being  $\mathcal{P}_{\text{Tx}}(t)$ ;  $\boldsymbol{\tau}$  represents the discretized delay levels kept consistent throughout our evaluations, i.e.,  $\boldsymbol{\tau} = \{\tau_1, \tau_2, \dots, \tau_N\}$ ; and let  $A(\tau_i)$  be the amplitude of the  $i^{\text{th}}$  MPC, i.e., the MPC at a delay of  $\tau_i$ . Then, the amplitude mean of the MPC at  $\tau_i, \forall i \in \{1, 2, \dots, N\}$ , for Rx config  $\mathcal{P}_{\text{Rx}}(t)$  with Tx config being  $\mathcal{P}_{\text{Tx}}(t)$ :

$$\mu_{t,i} = \mu(\tau_i, \mathcal{P}_{\text{Tx}}(t), \mathcal{P}_{\text{Rx}}(t)) = \frac{1}{M} \sum_{j=1}^M A_j(\tau_i),$$

where  $M$  denotes the set of measurements collected at Rx config  $\mathcal{P}_{\text{Rx}}(t)$  with Tx config being  $\mathcal{P}_{\text{Tx}}(t)$ , and  $A_j(\tau_i)$  denotes the amplitude of the  $i^{\text{th}}$  MPC for the  $j^{\text{th}}$  measurement at this specific Tx-Rx config. Therefore,

$$\boldsymbol{\mu}(\mathcal{P}_{\text{Tx}}(t), \mathcal{P}_{\text{Rx}}(t)) = [\mu_{t,1}, \mu_{t,2}, \dots, \mu_{t,N}]^{\text{T}}.$$

$\mathbf{A}(\mathcal{P}_{\text{Tx}}(t), \mathcal{P}_{\text{Rx}}(t))$  denotes the vector of amplitudes of the MPCs extracted from the measurements collected at the receiver at time  $t$  when the Rx config is  $\mathcal{P}_{\text{Rx}}(t)$  and the Tx config is  $\mathcal{P}_{\text{Tx}}(t)$ . Mathematically,

$$\mathbf{A}(\mathcal{P}_{\text{Tx}}(t), \mathcal{P}_{\text{Rx}}(t)) = [A(t, \tau_1), A(t, \tau_2), \dots, A(t, \tau_N)]^{\text{T}} = [A_{t,1}, A_{t,2}, \dots, A_{t,N}]^{\text{T}}.$$

Note that the dot-product in the numerator denotes the vector inner product of the mean-shifted amplitude vectors at times  $t$  and  $t'$ .

The expectation  $\mathbb{E}[\cdot]$  in the above equation for  $\rho(\Delta d)$  is taken over all Tx and Rx configs along the route  $\mathcal{R}$  that satisfy the evaluation conditions **E1**. Let  $\mathcal{R}_{\text{E1}}$  denote the set of Tx-Rx configs along the route that satisfy these evaluation conditions **E1**. Then, we can elaborate  $\rho(\Delta d)$  as

$$\rho(\Delta d) = \frac{\frac{1}{|\mathcal{R}_{\text{E1}}|} \sum_{(t,t') \in \mathcal{R}_{\text{E1}}} \left[ \sum_{i=1}^N (A_{t,i} - \mu_{t,i})(A_{t',i} - \mu_{t',i}) \right]}{\frac{1}{|\mathcal{R}|} \sum_{t \in \mathcal{R}} \left[ \sum_{i=1}^N (A_{t,i} - \mu_{t,i})^2 \right]}.$$

## 1.2 Separation in Alignment

The spatial autocorrelation coefficient  $\rho$  vis-à-vis a separation in alignment  $\phi$ :

$$\rho(\Delta \phi) = \rho(\mathbf{x}_{\text{Tx}}, \mathbf{x}_{\text{Rx}}(t') = \mathbf{x}_{\text{Rx}}(t), |\phi(t') - \phi(t)| = \Delta \phi, v(t') = v(t)), \text{ where}$$

- $\mathbf{x}_{\text{Tx}}(t) = \mathbf{x}_{\text{Tx}}, \forall t$  denotes the 3-D position vector of the Tx (fixed);
- $\mathbf{x}_{\text{Rx}}(t)$  and  $\mathbf{x}_{\text{Rx}}(t')$  denote the 3-D position vectors of the Rx, at times  $t$  and  $t'$ , respectively;
- $\phi(t)$  and  $\phi(t')$  are the absolute errors in alignment between Tx and Rx antennas (deviation from perfect alignment) at times  $t$  and  $t'$ , respectively, with  $\Delta\phi$  being the difference in alignment accuracies between them;
- $v(t)$  and  $v(t')$  denote the relative velocities of the receiver with respect to the transmitter at times  $t$  and  $t'$ , respectively; and
- The conditions  $\mathbf{x}_{\text{Rx}}(t') = \mathbf{x}_{\text{Rx}}(t)$  and  $v(t') = v(t)$  ensure that while studying the signal coherence characteristics vis-à-vis a separation in alignment  $\Delta\phi$ , other variables (i.e., distance and velocity) remain unchanged.

Mathematically, this spatial autocorrelation coefficient  $\rho(\Delta\phi)$  is given by

$$\frac{\mathbb{E} \left[ \left( \mathbf{A}(\mathcal{P}_{\text{Tx}}(t), \mathcal{P}_{\text{Rx}}(t)) - \boldsymbol{\mu}(\mathcal{P}_{\text{Tx}}(t), \mathcal{P}_{\text{Rx}}(t)) \right) \cdot \left( \mathbf{A}(\mathcal{P}_{\text{Tx}}(t'), \mathcal{P}_{\text{Rx}}(t')) - \boldsymbol{\mu}(\mathcal{P}_{\text{Tx}}(t'), \mathcal{P}_{\text{Rx}}(t')) \right) \right]}{\sqrt{\mathbb{E} \left[ \left\| \mathbf{A}(\mathcal{P}_{\text{Tx}}(t), \mathcal{P}_{\text{Rx}}(t)) - \boldsymbol{\mu}(\mathcal{P}_{\text{Tx}}(t), \mathcal{P}_{\text{Rx}}(t)) \right\|^2 \right] \mathbb{E} \left[ \left\| \mathbf{A}(\mathcal{P}_{\text{Tx}}(t'), \mathcal{P}_{\text{Rx}}(t')) - \boldsymbol{\mu}(\mathcal{P}_{\text{Tx}}(t'), \mathcal{P}_{\text{Rx}}(t')) \right\|^2 \right]}}.$$

For a route  $\mathcal{R}$ , we define the terms employed in this equation as follows.

Tx configuration:  $\mathcal{P}_{\text{Tx}}(t) = \mathcal{P}_{\text{Tx}}(t') = \{\mathbf{x}_{\text{Tx}}, \theta_{\text{Tx}}\}, \forall t, t' \in \mathcal{R}$ ;

Rx configuration:  $\mathcal{P}_{\text{Rx}}(t) = \{\mathbf{x}_{\text{Rx}}, \theta_{\text{Rx}}, v_{\text{Rx}}\}, \mathcal{P}_{\text{Rx}}(t') = \{\mathbf{x}'_{\text{Rx}}, \theta'_{\text{Rx}}, v'_{\text{Rx}}\}, \forall t, t' \in \mathcal{R}$ ;

**Evaluation conditions E2:**  $\mathbf{x}'_{\text{Rx}} = \mathbf{x}_{\text{Rx}}, \left| \phi(\theta'_{\text{Tx}}, \theta'_{\text{Rx}}) - \phi(\theta_{\text{Tx}}, \theta_{\text{Rx}}) \right| = \Delta\phi, v'_{\text{Rx}} = v_{\text{Rx}}$ .

Let  $N$  be the number of resolvable Multi-Path Components (MPCs) extracted via the SAGE algorithm at time  $t$  at Rx config  $\mathcal{P}_{\text{Rx}}(t)$ , with Tx config being  $\mathcal{P}_{\text{Tx}}(t)$ ;  $\boldsymbol{\tau}$  represents the discretized delay levels kept consistent throughout our evaluations, i.e.,  $\boldsymbol{\tau} = \{\tau_1, \tau_2, \dots, \tau_N\}$ ; and let  $A(\tau_i)$  be the amplitude of the  $i^{\text{th}}$  MPC, i.e., the MPC at a delay of  $\tau_i$ . Then, the amplitude mean of the MPC at  $\tau_i, \forall i \in \{1, 2, \dots, N\}$ , for Rx config  $\mathcal{P}_{\text{Rx}}(t)$  with Tx config being  $\mathcal{P}_{\text{Tx}}(t)$ :

$$\mu_{t,i} = \mu(\tau_i, \mathcal{P}_{\text{Tx}}(t), \mathcal{P}_{\text{Rx}}(t)) = \frac{1}{M} \sum_{j=1}^M A_j(\tau_i),$$

where  $M$  denotes the set of measurements collected at Rx config  $\mathcal{P}_{\text{Rx}}(t)$  with Tx config being  $\mathcal{P}_{\text{Tx}}(t)$ , and  $A_j(\tau_i)$  denotes the amplitude of the  $i^{\text{th}}$  MPC for the  $j^{\text{th}}$  measurement at this specific Tx-Rx config. Therefore,

$$\boldsymbol{\mu}(\mathcal{P}_{\text{Tx}}(t), \mathcal{P}_{\text{Rx}}(t)) = [\mu_{t,1}, \mu_{t,2}, \dots, \mu_{t,N}]^{\text{T}}.$$

$\mathbf{A}(\mathcal{P}_{\text{Tx}}(t), \mathcal{P}_{\text{Rx}}(t))$  denotes the vector of amplitudes of the MPCs extracted from the measurements collected at the receiver at time  $t$  when the Rx config is  $\mathcal{P}_{\text{Rx}}(t)$  and the Tx config is  $\mathcal{P}_{\text{Tx}}(t)$ . Mathematically,

$$\mathbf{A}(\mathcal{P}_{\text{Tx}}(t), \mathcal{P}_{\text{Rx}}(t)) = [A(t, \tau_1), A(t, \tau_2), \dots, A(t, \tau_N)]^\top = [A_{t,1}, A_{t,2}, \dots, A_{t,N}]^\top.$$

Note that the dot-product in the numerator denotes the vector inner product of the mean-shifted amplitude vectors at times  $t$  and  $t'$ .

The expectation  $\mathbb{E}[\cdot]$  in the above equation for  $\rho(\Delta\phi)$  is taken over all Tx and Rx configs along the route  $\mathcal{R}$  that satisfy the evaluation conditions **E2**. Let  $\mathcal{R}_{\mathbf{E2}}$  denote the set of Tx-Rx configs along the route that satisfy these evaluation conditions **E2**. Then, we can elaborate  $\rho(\Delta\phi)$  as

$$\rho(\Delta\phi) = \frac{\frac{1}{|\mathcal{R}_{\mathbf{E2}}|} \sum_{(t,t') \in \mathcal{R}_{\mathbf{E2}}} \left[ \sum_{i=1}^N (A_{t,i} - \mu_{t,i})(A_{t',i} - \mu_{t',i}) \right]}{\frac{1}{|\mathcal{R}|} \sum_{t \in \mathcal{R}} \left[ \sum_{i=1}^N (A_{t,i} - \mu_{t,i})^2 \right]}.$$

### 1.3 Separation in Velocity

The spatial autocorrelation coefficient  $\rho$  vis-à-vis a separation in velocity  $v$ :

$$\rho(\Delta v) = \rho(\mathbf{x}_{\text{Tx}}, \mathbf{x}_{\text{Rx}}(t') = \mathbf{x}_{\text{Rx}}(t), \phi(t') = \phi(t), |v(t') - v(t)| = \Delta v), \text{ where}$$

- $\mathbf{x}_{\text{Tx}}(t) = \mathbf{x}_{\text{Tx}}, \forall t$  denotes the 3-D position vector of the Tx (fixed);
- $\mathbf{x}_{\text{Rx}}(t)$  and  $\mathbf{x}_{\text{Rx}}(t')$  denote the 3-D position vectors of the Rx, at times  $t$  and  $t'$ , respectively;
- $\phi(t)$  and  $\phi(t')$  are the absolute errors in alignment between Tx and Rx antennas (deviation from perfect alignment) at times  $t$  and  $t'$ , respectively;
- $v(t)$  and  $v(t')$  denote the relative velocities of the receiver with respect to the transmitter at times  $t$  and  $t'$ , respectively, with  $\Delta v$  being the difference in Rx velocities to get to positions  $\mathbf{x}_{\text{Rx}}(t)$  and  $\mathbf{x}_{\text{Rx}}(t')$ ; and
- The conditions  $\mathbf{x}_{\text{Rx}}(t') = \mathbf{x}_{\text{Rx}}(t)$  and  $\phi(t') = \phi(t)$  ensure that while the signal coherence characteristics vis-à-vis a separation in velocity  $\Delta v$  are being studied, other variables (i.e., distance and alignment) remain unchanged.

Mathematically, this spatial autocorrelation coefficient  $\rho(\Delta v)$  is given by

$$\frac{\mathbb{E} \left[ \left( \mathbf{A}(\mathcal{P}_{\text{Tx}}(t), \mathcal{P}_{\text{Rx}}(t)) - \boldsymbol{\mu}(\mathcal{P}_{\text{Tx}}(t), \mathcal{P}_{\text{Rx}}(t)) \right) \cdot \left( \mathbf{A}(\mathcal{P}_{\text{Tx}}(t'), \mathcal{P}_{\text{Rx}}(t')) - \boldsymbol{\mu}(\mathcal{P}_{\text{Tx}}(t'), \mathcal{P}_{\text{Rx}}(t')) \right) \right]}{\sqrt{\mathbb{E} \left[ \left\| \mathbf{A}(\mathcal{P}_{\text{Tx}}(t), \mathcal{P}_{\text{Rx}}(t)) - \boldsymbol{\mu}(\mathcal{P}_{\text{Tx}}(t), \mathcal{P}_{\text{Rx}}(t)) \right\|^2 \right] \mathbb{E} \left[ \left\| \mathbf{A}(\mathcal{P}_{\text{Tx}}(t'), \mathcal{P}_{\text{Rx}}(t')) - \boldsymbol{\mu}(\mathcal{P}_{\text{Tx}}(t'), \mathcal{P}_{\text{Rx}}(t')) \right\|^2 \right]}}.$$

For a route  $\mathcal{R}$ , we define the terms employed in this equation as follows.

Tx configuration:  $\mathcal{P}_{\text{Tx}}(t) = \mathcal{P}_{\text{Tx}}(t') = \{\mathbf{x}_{\text{Tx}}, \theta_{\text{Tx}}\}, \forall t, t' \in \mathcal{R}$ ;

Rx configuration:  $\mathcal{P}_{\text{Rx}}(t) = \{\mathbf{x}_{\text{Rx}}, \theta_{\text{Rx}}, v_{\text{Rx}}\}, \mathcal{P}_{\text{Rx}}(t') = \{\mathbf{x}'_{\text{Rx}}, \theta'_{\text{Rx}}, v'_{\text{Rx}}\}, \forall t, t' \in \mathcal{R}$ ;

**Evaluation conditions E3:**  $\mathbf{x}'_{\text{Rx}} = \mathbf{x}_{\text{Rx}}, \phi(\theta'_{\text{Tx}}, \theta'_{\text{Rx}}) = \phi(\theta_{\text{Tx}}, \theta_{\text{Rx}}), |v'_{\text{Rx}} - v_{\text{Rx}}| = \Delta v$ .

Let  $N$  be the number of resolvable Multi-Path Components (MPCs) extracted via the SAGE algorithm at time  $t$  at Rx config  $\mathcal{P}_{\text{Rx}}(t)$ , with Tx config being  $\mathcal{P}_{\text{Tx}}(t)$ ;  $\tau$  represents the discretized delay levels kept consistent throughout our evaluations, i.e.,  $\tau = \{\tau_1, \tau_2, \dots, \tau_N\}$ ; and let  $A(\tau_i)$  be the amplitude of the  $i^{\text{th}}$  MPC, i.e., the MPC at a delay of  $\tau_i$ . Then, the amplitude mean of the MPC at  $\tau_i, \forall i \in \{1, 2, \dots, N\}$ , for Rx config  $\mathcal{P}_{\text{Rx}}(t)$  with Tx config being  $\mathcal{P}_{\text{Tx}}(t)$ :

$$\mu_{t,i} = \mu(\tau_i, \mathcal{P}_{\text{Tx}}(t), \mathcal{P}_{\text{Rx}}(t)) = \frac{1}{M} \sum_{j=1}^M A_j(\tau_i),$$

where  $M$  denotes the set of measurements collected at Rx config  $\mathcal{P}_{\text{Rx}}(t)$  with Tx config being  $\mathcal{P}_{\text{Tx}}(t)$ , and  $A_j(\tau_i)$  denotes the amplitude of the  $i^{\text{th}}$  MPC for the  $j^{\text{th}}$  measurement at this specific Tx-Rx config. Therefore,

$$\boldsymbol{\mu}(\mathcal{P}_{\text{Tx}}(t), \mathcal{P}_{\text{Rx}}(t)) = [\mu_{t,1}, \mu_{t,2}, \dots, \mu_{t,N}]^{\text{T}}.$$

$\mathbf{A}(\mathcal{P}_{\text{Tx}}(t), \mathcal{P}_{\text{Rx}}(t))$  denotes the vector of amplitudes of the MPCs extracted from the measurements collected at the receiver at time  $t$  when the Rx config is  $\mathcal{P}_{\text{Rx}}(t)$  and the Tx config is  $\mathcal{P}_{\text{Tx}}(t)$ . Mathematically,

$$\mathbf{A}(\mathcal{P}_{\text{Tx}}(t), \mathcal{P}_{\text{Rx}}(t)) = [A(t, \tau_1), A(t, \tau_2), \dots, A(t, \tau_N)]^{\text{T}} = [A_{t,1}, A_{t,2}, \dots, A_{t,N}]^{\text{T}}.$$

Note that the dot-product in the numerator denotes the vector inner product of the mean-shifted amplitude vectors at times  $t$  and  $t'$ .

The expectation  $\mathbb{E}[\cdot]$  in the above equation for  $\rho(\Delta v)$  is taken over all Tx and Rx configs along the route  $\mathcal{R}$  that satisfy the evaluation conditions **E3**. Let  $\mathcal{R}_{\text{E3}}$  denote the set of Tx-Rx configs along the route that satisfy these evaluation conditions **E3**. Then, we can elaborate  $\rho(\Delta v)$  as

$$\rho(\Delta v) = \frac{\frac{1}{|\mathcal{R}_{\text{E3}}|} \sum_{(t,t') \in \mathcal{R}_{\text{E3}}} \left[ \sum_{i=1}^N (A_{t,i} - \mu_{t,i})(A_{t',i} - \mu_{t',i}) \right]}{\frac{1}{|\mathcal{R}|} \sum_{t \in \mathcal{R}} \left[ \sum_{i=1}^N (A_{t,i} - \mu_{t,i})^2 \right]}.$$

## 2 References

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