SPAVE-28G: Spatial Consistency Modeling

Bharath Keshavamurthy

August 2023

1 Spatial Autocorrelation Coefficient

The small-scale spatial autocorrelation coefficient is a metric to characterize the coherence between the voltage amplitudes of received signals across variations in distances, alignment accuracies, and relative velocities.

Mathematically, the spatial autocorrelation coefficient is described as

$$\rho(\Delta X)=\frac{\mathbb{E}\left[\left(\mathbf{A}(\tau,X_k)-\overline{\mathbf{A}(\tau,X_k)}\right)\cdot\left(\mathbf{A}(\tau,X_k+\Delta X)-\overline{\mathbf{A}(\tau,X_k+\Delta X)}\right)\right]}{\sqrt{\mathbb{E}\left[\left\|\mathbf{A}(\tau,X_k)-\overline{\mathbf{A}(\tau,X_k)}\right\|^2\right]\mathbb{E}\left[\left\|\mathbf{A}(\tau,X_k+\Delta X)-\overline{\mathbf{A}(\tau,X_k+\Delta X)}\right\|^2\right]}},$$

where $\mathbf{A}(\tau, X_k)$ denotes the vector of received signal amplitudes at a specific Rx position X_k along a particular route; ΔX represents change in Rx configuration that is being evaluated (i.e., separation in distance, separation in alignment, or separation in velocity); $\overline{\mathbf{A}(\tau, X_k)}$ denotes the sample mean of the received signal amplitudes across the delay dimension τ; and $\mathbb{E}[\cdot]$ is taken over all Rx positions along the route (henceforth referred to as the *ensemble*).

2 Pseudo-Code

The steps to compute the spatial autocorrelation coefficient $\rho(\Delta X)$ for changes in Rx configuration ΔX (separation in distance, alignment, or velocity) for a particular route (urban, suburban, or foliage) are enumerated below.

1. First, pre-process the received samples via pre-filtering, sample truncation, time-windowing, and noise elimination (by thresholding).

2. A processed Rx sample at a specific Rx position X_k is a vector of received signal amplitudes $\mathbf{A}(\tau, X_k)$ across the delay dimension τ.

3. For a specific Rx configuration change (ΔX), compute $\rho(\Delta X)$ as follows.

   (a) For Rx position X_k along the route, compute the sample mean of the processed sample vector across the delay dimension, i.e., $\overline{\mathbf{A}(\tau, X_k)}$; then compute the mean-shifted vector $\mathbf{A}(\tau, X_k)-\overline{\mathbf{A}(\tau, X_k)}$.

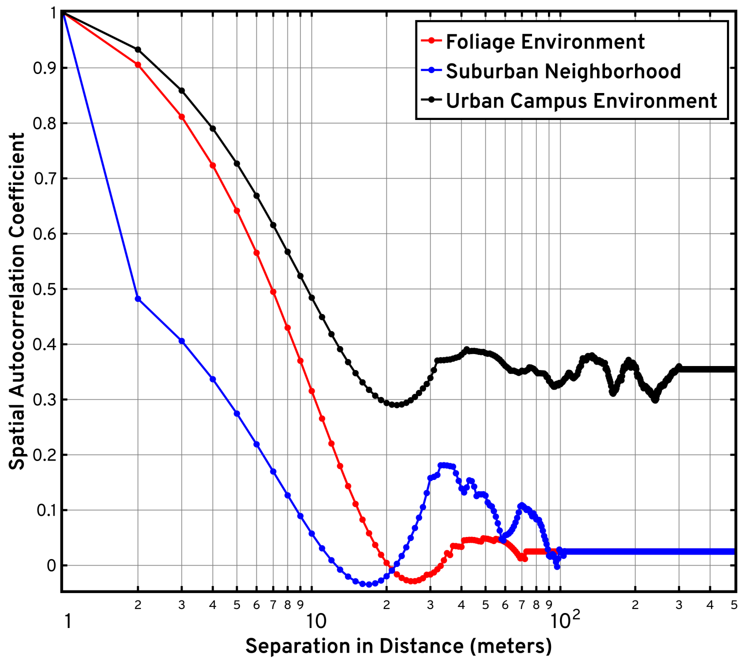

Figure 1: $\rho(\Delta X)$ vs (log-scale) Separation in Distance ΔX (in m).

(b) Repeat step 3(a) for the processed sample vector collected at Rx position a separation of ΔX from X_k, i.e., $X_k{+}\Delta X$.

(c) Make sure that when $\rho(\Delta X)$ is being computed for a particular Rx separation (e.g., distance), the other separation parameters (e.g., alignment accuracy) remain consistent for X_k and $X_k{+}\Delta X$.

(d) To evaluate the numerator term in $\rho(\Delta X)$, compute the dot-product between the mean-shifted vectors computed in steps 3(a) and 3(b); with $\mathbb{E}[\cdot]$ taken over the *ensemble*.

(e) To evaluate the norm-squared terms in the denominator, compute the self dot-product of the vectors computed in steps 3(a) and 3(b); with $\mathbb{E}[\cdot]$ taken over the *ensemble*.

(f) Evaluate the product of the expectations of the two norm-squared terms in step 3(e), then compute the square-root of this product.

(g) Using the numerator in step 3(d) and the denominator in step 3(f), compute the spatial autocorrelation coefficient $\rho(\Delta X)$.

The plots obtained from these evaluations are shown in Figs. 1, 2, and 3.

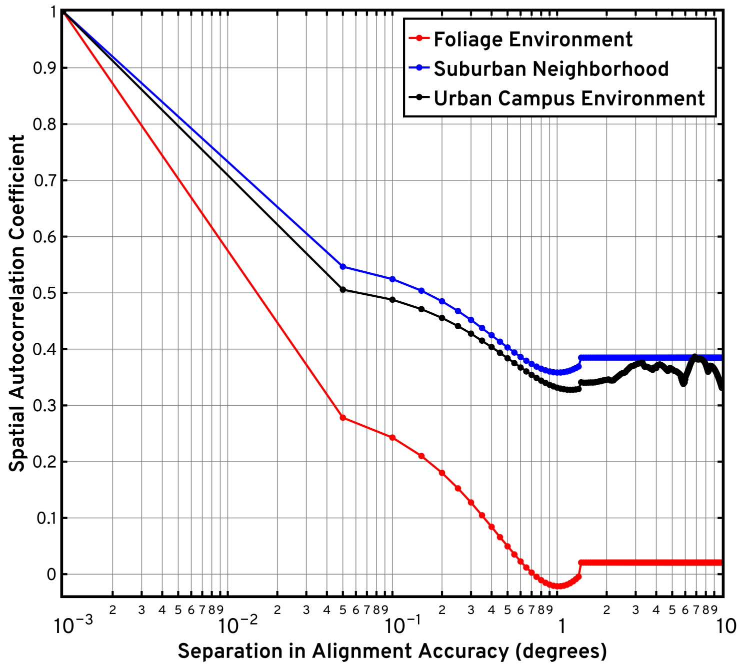

Figure 2: $\rho(\Delta X)$ vs (log-scale) Separation in Alignment Accuracy ΔX (in deg).

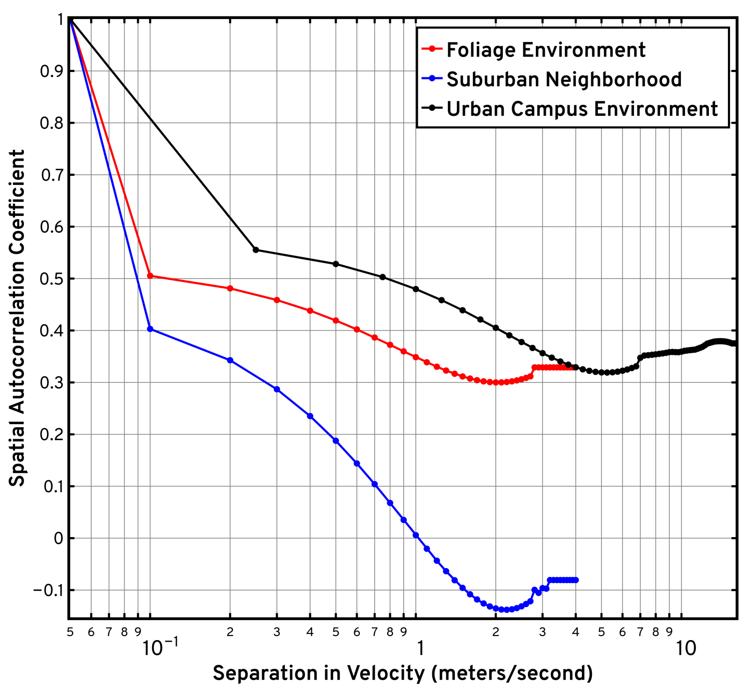

Figure 3: $\rho(\Delta X)$ vs (log-scale) Separation in Velocity ΔX (in m/s).