28GHz POWDER Measurements

Spatial Consistency Modeling

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1 Spatial Autocorrelation Coefficient

The small-scale spatial autocorrelation coefficient is a metric to characterize the coherence between the voltage amplitudes of received signals across variations in distances, alignment accuracies, and relative velocities.

1.1 Separation in Distance

The spatial autocorrelation coefficient ρ vis-à-vis a separation in distance Δd:

$$\rho(\Delta d) = \rho\Big(\mathbf{x}_{\mathrm{Tx}},\ \|\mathbf{x}_{\mathrm{Rx}}(t')-\mathbf{x}_{\mathrm{Rx}}(t)\| = \Delta d,\ \phi(t') = \phi(t),\ v(t') = v(t)\Big),\ \text{where}$$

- $\mathbf{x}_{\mathrm{Tx}}(t)=\mathbf{x}_{\mathrm{Tx}}, \forall t$ denotes the 3-D position vector of the Tx (fixed);
- $\mathbf{x}_{\mathrm{Rx}}(t)$ and $\mathbf{x}_{\mathrm{Rx}}(t')$ denote the Rx 3-D position vectors at times t and t', respectively, with Δd being the 3-D Euclidean distance between them;
- $\phi(t)$ and $\phi(t')$ are the absolute errors in alignment between Tx and Rx antennas (deviation from perfect alignment) at times t and t', respectively;
- $v(t)$ and $v(t')$ denote the relative velocities of the receiver with respect to the transmitter at times t and t', respectively; and
- The conditions $\phi(t')=\phi(t)$ and $v(t')=v(t)$ ensure that while studying the signal coherence characteristics vis-à-vis a separation in distance Δd, other variables in our measurements (i.e., alignment and velocity) are unchanged.

Mathematically, this spatial autocorrelation coefficient $\rho(\Delta d)$ is given by

$$\frac{\mathbb{E}\left[\Big(\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)-\mu\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)\Big)\cdot\Big(\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t'),\mathcal{P}_{\mathrm{Rx}}(t')\Big)-\mu\Big(\mathcal{P}_{\mathrm{Tx}}(t'),\mathcal{P}_{\mathrm{Rx}}(t')\Big)\Big)\right]}{\sqrt{\mathbb{E}\left[\Big\|\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)-\mu\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)\Big\|^2\right]\mathbb{E}\left[\Big\|\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t'),\mathcal{P}_{\mathrm{Rx}}(t')\Big)-\mu\Big(\mathcal{P}_{\mathrm{Tx}}(t'),\mathcal{P}_{\mathrm{Rx}}(t')\Big)\Big\|^2\right]}}.$$

For a route $\mathcal{R}$, we define the terms employed in this equation as follows.

Tx configuration: $\mathcal{P}_{\mathrm{Tx}}(t) = \mathcal{P}_{\mathrm{Tx}}(t') = \{\mathbf{x}_{\mathrm{Tx}},\ \theta_{\mathrm{Tx}}\}\,,\ \forall t,\ t' \in \mathcal{R}$;

Rx configuration: $\mathcal{P}_{\mathrm{Rx}}(t) = \{\mathbf{x}_{\mathrm{Rx}}, \theta_{\mathrm{Rx}}, v_{\mathrm{Rx}}\}\,, \mathcal{P}_{\mathrm{Rx}}(t') = \left\{\mathbf{x}^{'}_{\mathrm{Rx}}, \theta^{'}_{\mathrm{Rx}}, v^{'}_{\mathrm{Rx}}\right\},\ \forall t, t'{\in}\mathcal{R}$;

Evaluation conditions E1: $\left\|\mathbf{x}^{'}_{\mathrm{Rx}}{-}\mathbf{x}_{\mathrm{Rx}}\right\|{=}\Delta d, \phi(\theta^{'}_{\mathrm{Tx}}, \theta^{'}_{\mathrm{Rx}}){=}\phi(\theta_{\mathrm{Tx}}, \theta_{\mathrm{Rx}}), v^{'}_{\mathrm{Rx}}{=}v_{\mathrm{Rx}}$.

Let N be the number of resolvable Multi-Path Components (MPCs) extracted via the SAGE algorithm at time t at Rx config $\mathcal{P}_{\mathrm{Rx}}(t)$, with Tx config being $\mathcal{P}_{\mathrm{Tx}}(t)$; and let A_i be the amplitude of the i^{th} MPC. Then,

$$\text{Sample Mean of MPC amplitudes: } \mu\Big(\mathcal{P}_{\mathrm{Tx}}(t), \mathcal{P}_{\mathrm{Rx}}(t)\Big) = \frac{1}{N}\sum_{i=1}^{N} A_i.$$

$\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t), \mathcal{P}_{\mathrm{Rx}}(t)\Big)$ denotes the vector of amplitudes of the MPCs extracted from the measurements collected at the receiver at time t when the Rx config is $\mathcal{P}_{\mathrm{Rx}}(t)$ and the Tx config is $\mathcal{P}_{\mathrm{Tx}}(t)$. Mathematically,

$$\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t), \mathcal{P}_{\mathrm{Rx}}(t)\Big) = [A_1, A_2, \ldots, A_N]\,.$$

The expectation $\mathbb{E}[\cdot]$ in the above equation for $\rho(\Delta d)$ is taken over all Tx and Rx configs along the route $\mathcal{R}$ that satisfy the evaluation conditions **E1**.

1.2 Separation in Alignment

The spatial autocorrelation coefficient ρ vis-à-vis a separation in alignment ϕ:

$$\rho(\Delta\phi) = \rho\Big(\mathbf{x}_{\mathrm{Tx}},\ \mathbf{x}_{\mathrm{Rx}}(t') = \mathbf{x}_{\mathrm{Rx}}(t),\ |\phi(t') - \phi(t)| = \Delta\phi,\ v(t') = v(t)\Big),\ \text{where}$$

- $\mathbf{x}_{\mathrm{Tx}}(t){=}\mathbf{x}_{\mathrm{Tx}}, \forall t$ denotes the 3-D position vector of the Tx (fixed);
- $\mathbf{x}_{\mathrm{Rx}}(t)$ and $\mathbf{x}_{\mathrm{Rx}}(t')$ denote the 3-D position vectors of the Rx, at times t and t', respectively;
- $\phi(t)$ and $\phi(t')$ are the absolute errors in alignment between Tx and Rx antennas (deviation from perfect alignment) at times t and t', respectively, with $\Delta\phi$ being the difference in alignment accuracies between them;
- $v(t)$ and $v(t')$ denote the relative velocities of the receiver with respect to the transmitter at times t and t', respectively; and
- The conditions $\mathbf{x}_{\mathrm{Rx}}(t'){=}\mathbf{x}_{\mathrm{Rx}}(t)$ and $v(t'){=}v(t)$ ensure that while studying the signal coherence characteristics vis-à-vis a separation in alignment $\Delta\phi$, other variables (i.e., distance and velocity) remain unchanged.

Mathematically, this spatial autocorrelation coefficient $\rho(\Delta\phi)$ is given by

$$\frac{\mathbb{E}\left[\left(\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)-\mu\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)\right)\cdot\left(\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t'),\mathcal{P}_{\mathrm{Rx}}(t')\Big)-\mu\Big(\mathcal{P}_{\mathrm{Tx}}(t'),\mathcal{P}_{\mathrm{Rx}}(t')\Big)\right)\right]}{\sqrt{\mathbb{E}\left[\left\|\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)-\mu\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)\right\|^2\right]\mathbb{E}\left[\left\|\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t'),\mathcal{P}_{\mathrm{Rx}}(t')\Big)-\mu\Big(\mathcal{P}_{\mathrm{Tx}}(t'),\mathcal{P}_{\mathrm{Rx}}(t')\Big)\right\|^2\right]}}.$$

For a route $\mathcal{R}$, we define the terms employed in this equation as follows.

Tx configuration: $\mathcal{P}_{\mathrm{Tx}}(t)=\mathcal{P}_{\mathrm{Tx}}(t')=\{\mathbf{x}_{\mathrm{Tx}},\ \theta_{\mathrm{Tx}}\},\ \forall t,\ t'\in\mathcal{R}$;

Rx configuration: $\mathcal{P}_{\mathrm{Rx}}(t)=\{\mathbf{x}_{\mathrm{Rx}},\theta_{\mathrm{Rx}},v_{\mathrm{Rx}}\},\mathcal{P}_{\mathrm{Rx}}(t')=\left\{\mathbf{x}'_{\mathrm{Rx}},\theta'_{\mathrm{Rx}},v'_{\mathrm{Rx}}\right\},\ \forall t,t'\in\mathcal{R}$;

Evaluation conditions E2: $\mathbf{x}'_{\mathrm{Rx}}=\mathbf{x}_{\mathrm{Rx}},\left|\phi(\theta'_{\mathrm{Tx}},\theta'_{\mathrm{Rx}})-\phi(\theta_{\mathrm{Tx}},\theta_{\mathrm{Rx}})\right|=\Delta\phi, v'_{\mathrm{Rx}}=v_{\mathrm{Rx}}$.

Let N be the number of resolvable Multi-Path Components (MPCs) extracted via the SAGE algorithm at time t at Rx config $\mathcal{P}_{\mathrm{Rx}}(t)$, with Tx config being $\mathcal{P}_{\mathrm{Tx}}(t)$; and let A_i be the amplitude of the i^{th} MPC. Then,

$$\text{Sample Mean of MPC amplitudes: } \mu\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)=\frac{1}{N}\sum_{i=1}^{N}A_i.$$

$\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)$ denotes the vector of amplitudes of the MPCs extracted from the measurements collected at the receiver at time t when the Rx config is $\mathcal{P}_{\mathrm{Rx}}(t)$ and the Tx config is $\mathcal{P}_{\mathrm{Tx}}(t)$. Mathematically,

$$\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)=[A_1,A_2,\ldots,A_N].$$

The expectation $\mathbb{E}[\cdot]$ in the above equation for $\rho(\Delta d)$ is taken over all Tx and Rx configs along the route $\mathcal{R}$ that satisfy the evaluation conditions **E2**.

1.3 Separation in Velocity

The spatial autocorrelation coefficient ρ vis-à-vis a separation in velocity v:

$$\rho(\Delta v)=\rho\Big(\mathbf{x}_{\mathrm{Tx}},\ \mathbf{x}_{\mathrm{Rx}}(t')=\mathbf{x}_{\mathrm{Rx}}(t),\ \phi(t')=\phi(t),\ |v(t')-v(t)|=\Delta v\Big),\ \text{where}$$

- $\mathbf{x}_{\mathrm{Tx}}(t)=\mathbf{x}_{\mathrm{Tx}},\forall t$ denotes the 3-D position vector of the Tx (fixed);
- $\mathbf{x}_{\mathrm{Rx}}(t)$ and $\mathbf{x}_{\mathrm{Rx}}(t')$ denote the 3-D position vectors of the Rx, at times t and t', respectively;
- $\phi(t)$ and $\phi(t')$ are the absolute errors in alignment between Tx and Rx antennas (deviation from perfect alignment) at times t and t', respectively;

- $v(t)$ and $v(t')$ denote the relative velocities of the receiver with respect to the transmitter at times t and t', respectively, with Δv being the difference in Rx velocities to get to positions $\mathbf{x}_{\mathrm{Rx}}(t)$ and $\mathbf{x}_{\mathrm{Rx}}(t')$; and
- The conditions $\mathbf{x}_{\mathrm{Rx}}(t')$=$\mathbf{x}_{\mathrm{Rx}}(t)$ and $\phi(t')$=$\phi(t)$ ensure that while the signal coherence characteristics vis-à-vis a separation in velocity Δv are being studied, other variables (i.e., distance and alignment) remain unchanged.

Mathematically, this spatial autocorrelation coefficient $\rho(\Delta v)$ is given by

$$\frac{\mathbb{E}\left[\left(\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)-\mu\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)\right)\cdot\left(\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t'),\mathcal{P}_{\mathrm{Rx}}(t')\Big)-\mu\Big(\mathcal{P}_{\mathrm{Tx}}(t'),\mathcal{P}_{\mathrm{Rx}}(t')\Big)\right)\right]}{\sqrt{\mathbb{E}\left[\left\|\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)-\mu\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)\right\|^2\right]\mathbb{E}\left[\left\|\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t'),\mathcal{P}_{\mathrm{Rx}}(t')\Big)-\mu\Big(\mathcal{P}_{\mathrm{Tx}}(t'),\mathcal{P}_{\mathrm{Rx}}(t')\Big)\right\|^2\right]}}.$$

For a route $\mathcal{R}$, we define the terms employed in this equation as follows.

Tx configuration: $\mathcal{P}_{\mathrm{Tx}}(t)=\mathcal{P}_{\mathrm{Tx}}(t')=\{\mathbf{x}_{\mathrm{Tx}},\ \theta_{\mathrm{Tx}}\},\ \forall t,\ t'\in\mathcal{R};$

Rx configuration: $\mathcal{P}_{\mathrm{Rx}}(t)=\{\mathbf{x}_{\mathrm{Rx}},\theta_{\mathrm{Rx}},v_{\mathrm{Rx}}\},\mathcal{P}_{\mathrm{Rx}}(t')=\Big\{\mathbf{x}_{\mathrm{Rx}}^{'},\theta_{\mathrm{Rx}}^{'},v_{\mathrm{Rx}}^{'}\Big\},\ \forall t,t'{\in}\mathcal{R};$

Evaluation conditions E3: $\mathbf{x}_{\mathrm{Rx}}^{'}{=}\mathbf{x}_{\mathrm{Rx}},\phi(\theta_{\mathrm{Tx}}^{'},\theta_{\mathrm{Rx}}^{'}){=}\phi(\theta_{\mathrm{Tx}},\theta_{\mathrm{Rx}}),\Big|v_{\mathrm{Rx}}^{'}{-}v_{\mathrm{Rx}}\Big|{=}\Delta v.$

Let N be the number of resolvable Multi-Path Components (MPCs) extracted via the SAGE algorithm at time t at Rx config $\mathcal{P}_{\mathrm{Rx}}(t)$, with Tx config being $\mathcal{P}_{\mathrm{Tx}}(t)$; and let A_i be the amplitude of the i^{th} MPC. Then,

$$\text{Sample Mean of MPC amplitudes: } \mu\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)=\frac{1}{N}\sum_{i=1}^{N}A_i.$$

$\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)$ denotes the vector of amplitudes of the MPCs extracted from the measurements collected at the receiver at time t when the Rx config is $\mathcal{P}_{\mathrm{Rx}}(t)$ and the Tx config is $\mathcal{P}_{\mathrm{Tx}}(t)$. Mathematically,

$$\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)=[A_1,A_2,\ldots,A_N].$$

The expectation $\mathbb{E}[\cdot]$ in the above equation for $\rho(\Delta d)$ is taken over all Tx and Rx configs along the route $\mathcal{R}$ that satisfy the evaluation conditions **E3**.

The plots obtained from these evaluations are shown in Figs. 1, 2, and 3.

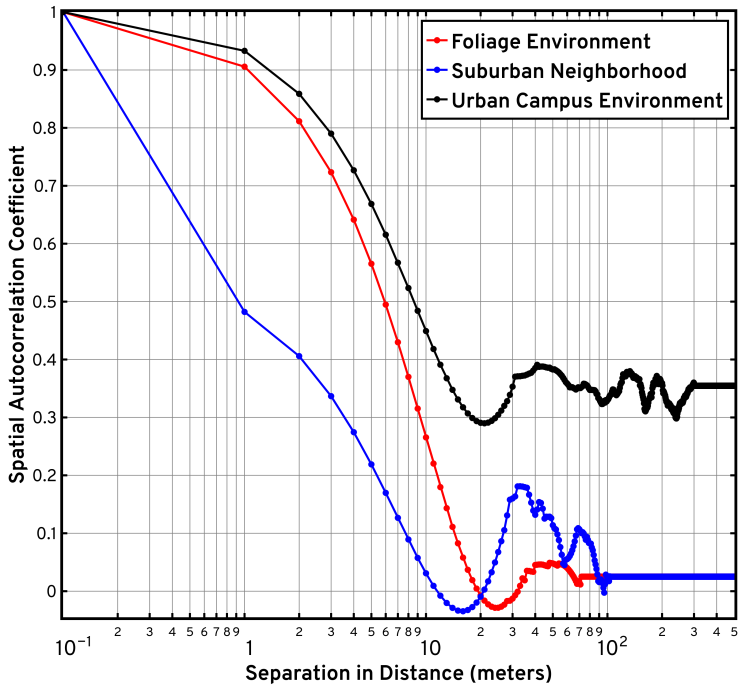

Figure 1: $\rho(\Delta d)$ vs (log-scale) Separation in Distance Δd (in m).

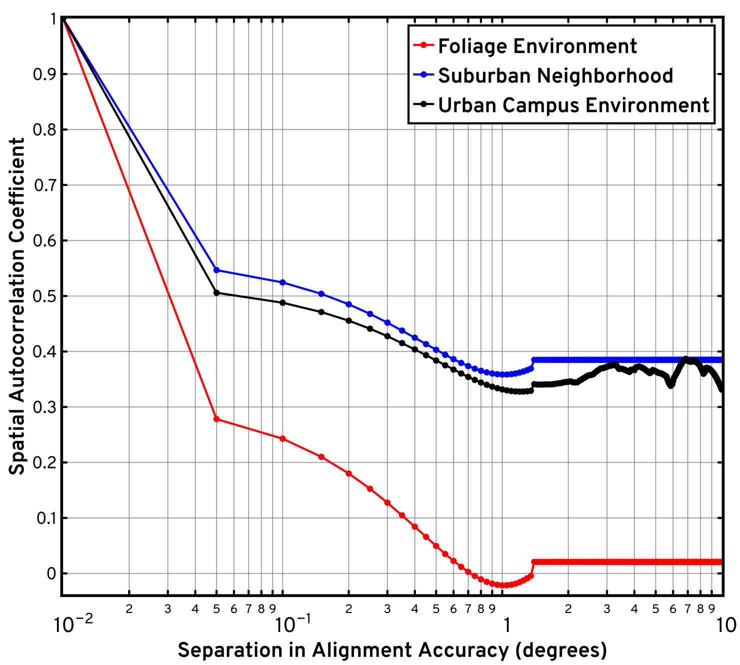

Figure 2: $\rho(\Delta\phi)$ vs (log-scale) Separation in Alignment Accuracy $\Delta\phi$ (in deg).

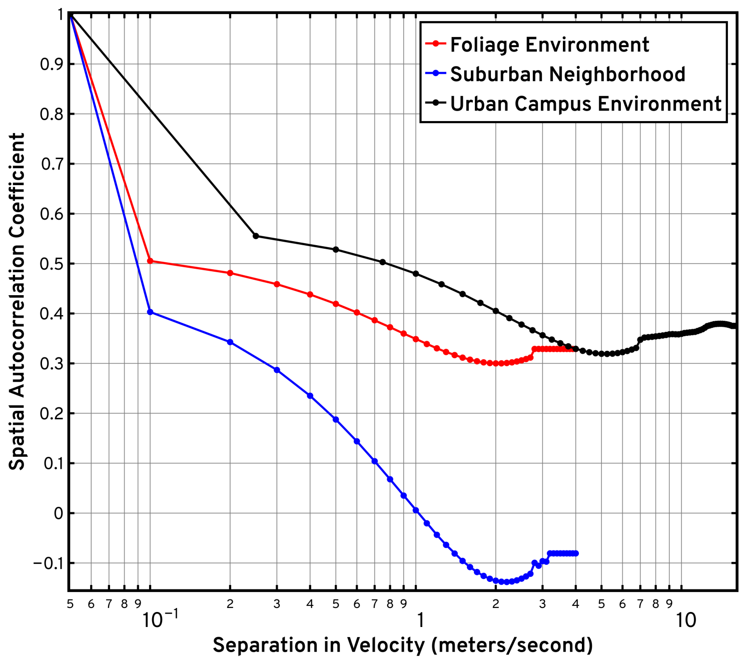

Figure 3: $\rho(\Delta v)$ vs (log-scale) Separation in Velocity Δv (in m/s).