28GHz POWDER Measurements

Spatial Consistency Modeling

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1 Spatial Autocorrelation Coefficient

The small-scale spatial autocorrelation coefficient is a metric to characterize the coherence between the voltage amplitudes of received signals across variations in distances, alignment accuracies, and relative velocities.

1.1 Separation in Distance

The spatial autocorrelation coefficient ρ vis-à-vis a separation in distance Δd:

$$\rho(\Delta d) = \rho\Big(\mathbf{x}_{\mathrm{Tx}},\ \|\mathbf{x}_{\mathrm{Rx}}(t'){-}\mathbf{x}_{\mathrm{Rx}}(t)\| = \Delta d,\ \phi(t') = \phi(t),\ v(t') = v(t)\Big),\ \text{where}$$

- $\mathbf{x}_{\mathrm{Tx}}(t){=}\mathbf{x}_{\mathrm{Tx}}, \forall t$ denotes the 3-D position vector of the Tx (fixed);
- $\mathbf{x}_{\mathrm{Rx}}(t)$ and $\mathbf{x}_{\mathrm{Rx}}(t')$ denote the Rx 3-D position vectors at times t and t', respectively, with Δd being the 3-D Euclidean distance between them;
- $\phi(t)$ and $\phi(t')$ are the absolute errors in alignment between Tx and Rx antennas (deviation from perfect alignment) at times t and t', respectively;
- $v(t)$ and $v(t')$ denote the relative velocities of the receiver with respect to the transmitter at times t and t', respectively; and
- The conditions $\phi(t'){=}\phi(t)$ and $v(t'){=}v(t)$ ensure that while studying the signal coherence characteristics vis-à-vis a separation in distance Δd, other variables in our measurements (i.e., alignment and velocity) are unchanged.

Mathematically, this spatial autocorrelation coefficient $\rho(\Delta d)$ is given by

$$\frac{\mathbb{E}\left[\Big(\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big){-}\mu\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)\Big)\cdot\Big(\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t'),\mathcal{P}_{\mathrm{Rx}}(t')\Big){-}\mu\Big(\mathcal{P}_{\mathrm{Tx}}(t'),\mathcal{P}_{\mathrm{Rx}}(t')\Big)\Big)\right]}{\sqrt{\mathbb{E}\left[\Big\|\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big){-}\mu\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)\Big\|^2\right]\mathbb{E}\left[\Big\|\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t'),\mathcal{P}_{\mathrm{Rx}}(t')\Big){-}\mu\Big(\mathcal{P}_{\mathrm{Tx}}(t'),\mathcal{P}_{\mathrm{Rx}}(t')\Big)\Big\|^2\right]}}.$$

For a route $\mathcal{R}$, we define the terms employed in this equation as follows.

Tx configuration: $\mathcal{P}_{\mathrm{Tx}}(t) = \mathcal{P}_{\mathrm{Tx}}(t') = \{\mathbf{x}_{\mathrm{Tx}},\ \theta_{\mathrm{Tx}}\}$, $\forall t,\ t' \in \mathcal{R}$;

Rx configuration: $\mathcal{P}_{\mathrm{Rx}}(t) = \{\mathbf{x}_{\mathrm{Rx}}, \theta_{\mathrm{Rx}}, v_{\mathrm{Rx}}\}, \mathcal{P}_{\mathrm{Rx}}(t') = \left\{\mathbf{x}_{\mathrm{Rx}}^{'}, \theta_{\mathrm{Rx}}^{'}, v_{\mathrm{Rx}}^{'}\right\}$, $\forall t, t' \in \mathcal{R}$;

Evaluation conditions E1: $\left\|\mathbf{x}_{\mathrm{Rx}}^{'} - \mathbf{x}_{\mathrm{Rx}}\right\| = \Delta d, \phi(\theta_{\mathrm{Tx}}^{'}, \theta_{\mathrm{Rx}}^{'}) = \phi(\theta_{\mathrm{Tx}}, \theta_{\mathrm{Rx}}), v_{\mathrm{Rx}}^{'} = v_{\mathrm{Rx}}$.

Let N be the number of resolvable Multi-Path Components (MPCs) extracted via the SAGE algorithm at time t at Rx config $\mathcal{P}_{\mathrm{Rx}}(t)$, with Tx config being $\mathcal{P}_{\mathrm{Tx}}(t)$; and let A_i be the amplitude of the i^{th} MPC. Then,

$$\text{Sample Mean of MPC amplitudes: } \mu\Big(\mathcal{P}_{\mathrm{Tx}}(t), \mathcal{P}_{\mathrm{Rx}}(t)\Big) = \frac{1}{N}\sum_{i=1}^{N} A_i.$$

$\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t), \mathcal{P}_{\mathrm{Rx}}(t)\Big)$ denotes the vector of amplitudes of the MPCs extracted from the measurements collected at the receiver at time t when the Rx config is $\mathcal{P}_{\mathrm{Rx}}(t)$ and the Tx config is $\mathcal{P}_{\mathrm{Tx}}(t)$. Mathematically,

$$\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t), \mathcal{P}_{\mathrm{Rx}}(t)\Big) = [A_1, A_2, \ldots, A_N].$$

The expectation $\mathbb{E}[\cdot]$ in the above equation for $\rho(\Delta d)$ is taken over all Tx and Rx configs along the route $\mathcal{R}$ that satisfy the evaluation conditions **E1**.

The product within the $\mathbb{E}[\cdot]$ operator in the numerator of $\rho(\Delta d)$ is a dot-product between the two N-dimensional mean-shifted amplitude vectors.

Note that, to maintain consistency in the vector lengths throughout, we preset the number of MPCs to be considered (N^*). So, at the two time-steps along the route t and t', if SAGE returns a different number of MPCs (N for the Tx-Rx config at time t and N' for the Tx-Rx config at time t', $N \neq N'$) and if $N > N^*$ and/or $N' > N^*$, then we pick the N^*-most amplitude-dominant MPCs.

1.2 Separation in Alignment

The spatial autocorrelation coefficient ρ vis-à-vis a separation in alignment ϕ:

$$\rho(\Delta\phi) = \rho\Big(\mathbf{x}_{\mathrm{Tx}},\ \mathbf{x}_{\mathrm{Rx}}(t') = \mathbf{x}_{\mathrm{Rx}}(t),\ |\phi(t') - \phi(t)| = \Delta\phi,\ v(t') = v(t)\Big), \text{ where}$$

- $\mathbf{x}_{\mathrm{Tx}}(t) = \mathbf{x}_{\mathrm{Tx}}, \forall t$ denotes the 3-D position vector of the Tx (fixed);
- $\mathbf{x}_{\mathrm{Rx}}(t)$ and $\mathbf{x}_{\mathrm{Rx}}(t')$ denote the 3-D position vectors of the Rx, at times t and t', respectively;
- $\phi(t)$ and $\phi(t')$ are the absolute errors in alignment between Tx and Rx antennas (deviation from perfect alignment) at times t and t', respectively, with $\Delta\phi$ being the difference in alignment accuracies between them;

- $v(t)$ and $v(t')$ denote the relative velocities of the receiver with respect to the transmitter at times t and t', respectively; and
- The conditions $\mathbf{x}_{\mathrm{Rx}}(t')=\mathbf{x}_{\mathrm{Rx}}(t)$ and $v(t')=v(t)$ ensure that while studying the signal coherence characteristics vis-à-vis a separation in alignment $\Delta\phi$, other variables (i.e., distance and velocity) remain unchanged.

Mathematically, this spatial autocorrelation coefficient $\rho(\Delta\phi)$ is given by

$$\frac{\mathbb{E}\left[\left(\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)-\mu\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)\right)\cdot\left(\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t'),\mathcal{P}_{\mathrm{Rx}}(t')\Big)-\mu\Big(\mathcal{P}_{\mathrm{Tx}}(t'),\mathcal{P}_{\mathrm{Rx}}(t')\Big)\right)\right]}{\sqrt{\mathbb{E}\left[\left\|\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)-\mu\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)\right\|^2\right]\mathbb{E}\left[\left\|\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t'),\mathcal{P}_{\mathrm{Rx}}(t')\Big)-\mu\Big(\mathcal{P}_{\mathrm{Tx}}(t'),\mathcal{P}_{\mathrm{Rx}}(t')\Big)\right\|^2\right]}}.$$

For a route $\mathcal{R}$, we define the terms employed in this equation as follows.

Tx configuration: $\mathcal{P}_{\mathrm{Tx}}(t)=\mathcal{P}_{\mathrm{Tx}}(t')=\{\mathbf{x}_{\mathrm{Tx}},\ \theta_{\mathrm{Tx}}\},\ \forall t,\ t'\in\mathcal{R};$

Rx configuration: $\mathcal{P}_{\mathrm{Rx}}(t)=\{\mathbf{x}_{\mathrm{Rx}},\theta_{\mathrm{Rx}},v_{\mathrm{Rx}}\},\mathcal{P}_{\mathrm{Rx}}(t')=\left\{\mathbf{x}_{\mathrm{Rx}}^{'},\theta_{\mathrm{Rx}}^{'},v_{\mathrm{Rx}}^{'}\right\},\ \forall t,t'\in\mathcal{R};$

Evaluation conditions E2: $\mathbf{x}_{\mathrm{Rx}}^{'}=\mathbf{x}_{\mathrm{Rx}},\left|\phi(\theta_{\mathrm{Tx}}^{'},\theta_{\mathrm{Rx}}^{'})-\phi(\theta_{\mathrm{Tx}},\theta_{\mathrm{Rx}})\right|=\Delta\phi, v_{\mathrm{Rx}}^{'}=v_{\mathrm{Rx}}.$

Let N be the number of resolvable Multi-Path Components (MPCs) extracted via the SAGE algorithm at time t at Rx config $\mathcal{P}_{\mathrm{Rx}}(t)$, with Tx config being $\mathcal{P}_{\mathrm{Tx}}(t)$; and let A_i be the amplitude of the i^{th} MPC. Then,

$$\text{Sample Mean of MPC amplitudes: } \mu\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)=\frac{1}{N}\sum_{i=1}^{N}A_i.$$

$\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)$ denotes the vector of amplitudes of the MPCs extracted from the measurements collected at the receiver at time t when the Rx config is $\mathcal{P}_{\mathrm{Rx}}(t)$ and the Tx config is $\mathcal{P}_{\mathrm{Tx}}(t)$. Mathematically,

$$\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)=[A_1,A_2,\ldots,A_N].$$

The expectation $\mathbb{E}[\cdot]$ in the above equation for $\rho(\Delta\phi)$ is taken over all Tx and Rx configs along the route $\mathcal{R}$ that satisfy the evaluation conditions **E2**.

The product within the $\mathbb{E}[\cdot]$ operator in the numerator of $\rho(\Delta\phi)$ is a dot-product between the two N-dimensional mean-shifted amplitude vectors.

Note that, to maintain consistency in the vector lengths throughout, we preset the number of MPCs to be considered (N^*). So, at the two time-steps along the route t and t', if SAGE returns a different number of MPCs (N for the Tx-Rx config at time t and N' for the Tx-Rx config at time t', $N\neq N'$) and if $N>N^*$ and/or $N'>N^*$, then we pick the N^*-most amplitude-dominant MPCs.

1.3 Separation in Velocity

The spatial autocorrelation coefficient ρ vis-à-vis a separation in velocity v:

$\rho(\Delta v) = \rho\Big(\mathbf{x}_{\mathrm{Tx}},\ \mathbf{x}_{\mathrm{Rx}}(t') = \mathbf{x}_{\mathrm{Rx}}(t),\ \phi(t') = \phi(t),\ |v(t') - v(t)| = \Delta v\Big),$ where

- $\mathbf{x}_{\mathrm{Tx}}(t)=\mathbf{x}_{\mathrm{Tx}}, \forall t$ denotes the 3-D position vector of the Tx (fixed);
- $\mathbf{x}_{\mathrm{Rx}}(t)$ and $\mathbf{x}_{\mathrm{Rx}}(t')$ denote the 3-D position vectors of the Rx, at times t and t', respectively;
- $\phi(t)$ and $\phi(t')$ are the absolute errors in alignment between Tx and Rx antennas (deviation from perfect alignment) at times t and t', respectively;
- $v(t)$ and $v(t')$ denote the relative velocities of the receiver with respect to the transmitter at times t and t', respectively, with Δv being the difference in Rx velocities to get to positions $\mathbf{x}_{\mathrm{Rx}}(t)$ and $\mathbf{x}_{\mathrm{Rx}}(t')$; and
- The conditions $\mathbf{x}_{\mathrm{Rx}}(t')=\mathbf{x}_{\mathrm{Rx}}(t)$ and $\phi(t')=\phi(t)$ ensure that while the signal coherence characteristics vis-à-vis a separation in velocity Δv are being studied, other variables (i.e., distance and alignment) remain unchanged.

Mathematically, this spatial autocorrelation coefficient $\rho(\Delta v)$ is given by

$$\frac{\mathbb{E}\left[\left(\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)-\mu\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)\right)\cdot\left(\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t'),\mathcal{P}_{\mathrm{Rx}}(t')\Big)-\mu\Big(\mathcal{P}_{\mathrm{Tx}}(t'),\mathcal{P}_{\mathrm{Rx}}(t')\Big)\right)\right]}{\sqrt{\mathbb{E}\left[\left\|\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)-\mu\Big(\mathcal{P}_{\mathrm{Tx}}(t),\mathcal{P}_{\mathrm{Rx}}(t)\Big)\right\|^2\right]\mathbb{E}\left[\left\|\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t'),\mathcal{P}_{\mathrm{Rx}}(t')\Big)-\mu\Big(\mathcal{P}_{\mathrm{Tx}}(t'),\mathcal{P}_{\mathrm{Rx}}(t')\Big)\right\|^2\right]}}.$$

For a route $\mathcal{R}$, we define the terms employed in this equation as follows.

Tx configuration: $\mathcal{P}_{\mathrm{Tx}}(t) = \mathcal{P}_{\mathrm{Tx}}(t') = \{\mathbf{x}_{\mathrm{Tx}},\ \theta_{\mathrm{Tx}}\},\ \forall t,\ t' \in \mathcal{R};$

Rx configuration: $\mathcal{P}_{\mathrm{Rx}}(t) = \{\mathbf{x}_{\mathrm{Rx}}, \theta_{\mathrm{Rx}}, v_{\mathrm{Rx}}\}, \mathcal{P}_{\mathrm{Rx}}(t') = \Big\{\mathbf{x}^{'}_{\mathrm{Rx}}, \theta^{'}_{\mathrm{Rx}}, v^{'}_{\mathrm{Rx}}\Big\},\ \forall t, t'\in\mathcal{R};$

Evaluation conditions E3: $\mathbf{x}^{'}_{\mathrm{Rx}}=\mathbf{x}_{\mathrm{Rx}}, \phi(\theta^{'}_{\mathrm{Tx}}, \theta^{'}_{\mathrm{Rx}})=\phi(\theta_{\mathrm{Tx}}, \theta_{\mathrm{Rx}}), \Big|v^{'}_{\mathrm{Rx}}-v_{\mathrm{Rx}}\Big|=\Delta v.$

Let N be the number of resolvable Multi-Path Components (MPCs) extracted via the SAGE algorithm at time t at Rx config $\mathcal{P}_{\mathrm{Rx}}(t)$, with Tx config being $\mathcal{P}_{\mathrm{Tx}}(t)$; and let A_i be the amplitude of the i^{th} MPC. Then,

$$\text{Sample Mean of MPC amplitudes: } \mu\Big(\mathcal{P}_{\mathrm{Tx}}(t), \mathcal{P}_{\mathrm{Rx}}(t)\Big) = \frac{1}{N}\sum_{i=1}^{N} A_i.$$

$\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t), \mathcal{P}_{\mathrm{Rx}}(t)\Big)$ denotes the vector of amplitudes of the MPCs extracted from the measurements collected at the receiver at time t when the Rx config is $\mathcal{P}_{\mathrm{Rx}}(t)$ and the Tx config is $\mathcal{P}_{\mathrm{Tx}}(t)$. Mathematically,

$$\mathbf{A}\Big(\mathcal{P}_{\mathrm{Tx}}(t), \mathcal{P}_{\mathrm{Rx}}(t)\Big) = [A_1, A_2, \ldots, A_N].$$

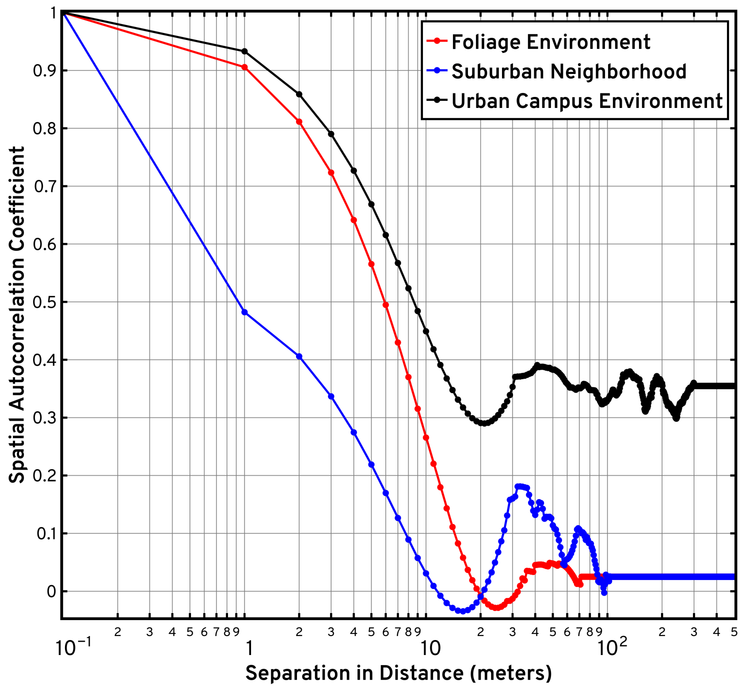

Figure 1: $\rho(\Delta d)$ vs (log-scale) Separation in Distance Δd (in m).

The expectation $\mathbb{E}[\cdot]$ in the above equation for $\rho(\Delta v)$ is taken over all Tx and Rx configs along the route $\mathcal{R}$ that satisfy the evaluation conditions **E3**.

The product within the $\mathbb{E}[\cdot]$ operator in the numerator of $\rho(\Delta v)$ is a dot-product between the two N-dimensional mean-shifted amplitude vectors.

Note that, to maintain consistency in the vector lengths throughout, we preset the number of MPCs to be considered (N^*). So, at the two time-steps along the route t and t', if SAGE returns a different number of MPCs (N for the Tx-Rx config at time t and N' for the Tx-Rx config at time t', $N \neq N'$) and if $N > N^*$ and/or $N' > N^*$, then we pick the N^*-most amplitude-dominant MPCs.

The plots obtained from these evaluations are shown in Figs. 1, 2, and 3.

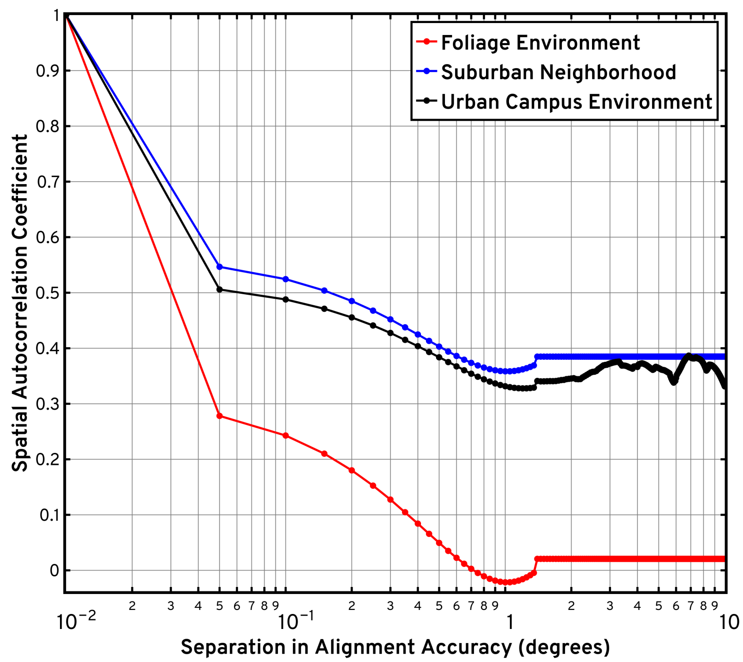

Figure 2: $\rho(\Delta\phi)$ vs (log-scale) Separation in Alignment Accuracy $\Delta\phi$ (in deg).

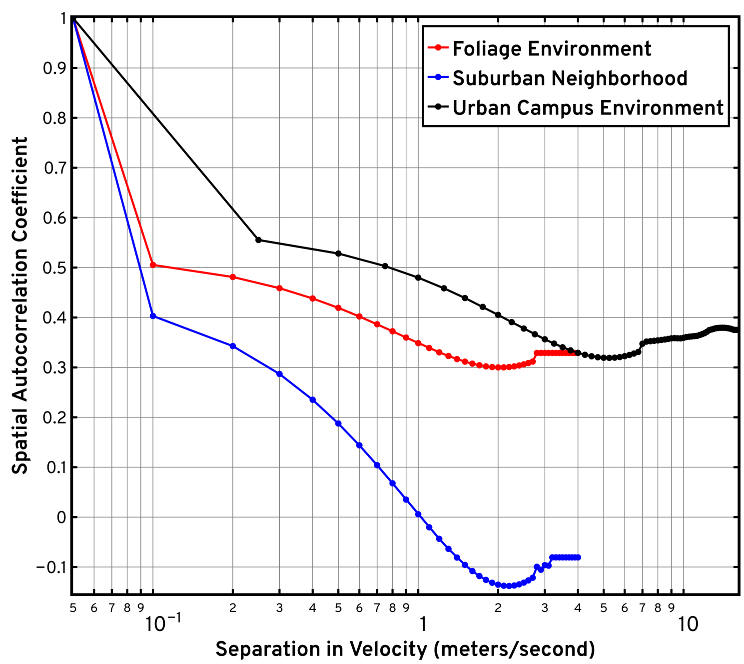

Figure 3: $\rho(\Delta v)$ vs (log-scale) Separation in Velocity Δv (in m/s).

1.4 Physical Interpretation

- The physical interpretation of this spatial autocorrelation coefficient is that ($-1 \leq \rho \leq 1$) is similar to the correlation coefficient metric typically used in probability theory, i.e., are the signals received at two Tx-Rx configs separated in distance/alignment/velocity correlated (if yes, how correlated) or are they independent from each other (i.e., de-correlated).
- So, studying this spatial correlation coefficient in different propagation environments, the idea behind these evaluations is two-fold: analyze the gradient at which 28GHz signals become de-correlated as the separation between two points/configs increases in distance, alignment accuracy, and velocity; and determine the distances or errors in alignment or relative velocity changes at which the 28GHz signals received at two counterpart configurations become completely de-correlated (i.e., ρ=0).
- However, since our propagation modeling campaign involved beam-steered measurements, the LoS component remains significant, thereby ensuring that even at large separations, the signals remain relatively correlated with each other.
- Since the MPC delay parameter is a continuous random variable, with no discretization possible because of the variations from one Tx-Rx config along the route to another, only the amplitude parameter of the MPCs is considered.

Similar definitions of ρ are used in the following works.

2 References

1. S. Sun, H. Yan, G. R. MacCartney and T. S. Rappaport, "Millimeter wave small-scale spatial statistics in an urban microcell scenario," 2017 IEEE International Conference on Communications (ICC), Paris, France, 2017.
2. M. K. Samimi, G. R. MacCartney, S. Sun and T. S. Rappaport, "28 GHz Millimeter-Wave Ultrawideband Small-Scale Fading Models in Wireless Channels," 2016 IEEE 83rd Vehicular Technology Conference (VTC Spring), Nanjing, China, 2016, pp. 1-6.
3. M. K. Samimi, S. Sun and T. S. Rappaport, "MIMO channel modeling and capacity analysis for 5G millimeter-wave wireless systems," 2016 10th European Conference on Antennas and Propagation (EuCAP), Davos, Switzerland, 2016, pp. 1-5, doi: 10.1109/EuCAP.2016.7481507.
4. T. S. Rappaport, S. Y. Seidel and K. Takamizawa, "Statistical channel impulse response models for factory and open plan building radio communicate system design," in IEEE Transactions on Communications, vol. 39, no. 5, pp. 794-807, May 1991, doi: 10.1109/26.87142.