28GHz POWDER Measurements

Spatial Consistency Modeling

Bharath Keshavamurthy

September 2023

1 Spatial Autocorrelation Coefficient

The small-scale spatial autocorrelation coefficient is a metric to characterize the coherence between the voltage amplitudes of received signals across variations in distances, alignment accuracies, and relative velocities.

We evaluate the spatial autocorrelation coefficient for separations in distance, Tx-Rx alignment accuracy, and Tx-Rx relative velocity, as detailed next.

2 System Model

With $\mathcal{I}$=$\{1, 2, \ldots, I\}$ as the index set, we define a route in our measurement campaign as a set of 5-tuples, i.e.,

$$\mathcal{R} = \{\mathcal{T}_i = (\mathbf{x}_i, \mathbf{y}_i, \phi_i, v_i, \mathcal{M}_i) \mid i \in \mathcal{I}\}, \tag{1}$$

where $\mathbf{x}_i$ denotes the 3D position vector of the Rx, $\mathbf{y}_i$ denotes the 3D position vector of the Tx, ϕ_i denotes the accuracy of alignment between the Tx and Rx horn antennas (i.e., deviation from perfect alignment), and v_i denotes the relative velocity of the Rx with respect to the Tx.

Let $\mathcal{J}_i$=$\{1, 2, \ldots, J_i\}$ represent the index set for the collection of measurements obtained at the route configuration index $i \in \mathcal{I}$; therefore, the corresponding set of measurements is given by

$$\mathcal{M}_i = \{\mathbf{m}_{i,j} \mid j \in \mathcal{J}_i\}. \tag{2}$$

First, each vector of received samples $\mathbf{m}_{i,j}$ undergoes processing via pre-filtering, time-windowing, and thresholding; subsequently, with propagation delay bins $\boldsymbol{\tau}$=$\{\tau_1, \tau_2, \ldots, \tau_L\}$, we extract the amplitudes of the Multi-Path Components (MPCs) at these delay bins using the SAGE algorithm (reference [1]), i.e.,

$$\tilde{\mathcal{M}}_i = \left\{ \left[A_{i,j}(\tau_1), A_{i,j}(\tau_2), \ldots, A_{i,j}(\tau_L) \right]^\mathsf{T} \,\middle|\, j \in \mathcal{J}_i \right\}. \tag{3}$$

3 Evaluation Conditions

With the Tx fixed ($\mathbf{y}_i$=$\mathbf{y}$, $\forall i \in \mathcal{I}$), we define the evaluation conditions which will be employed in Sec. 4 to compute the spatial autocorrelation coefficient under variations in their corresponding separation variables, i.e., separation in distance, separation in Tx-Rx alignment accuracy, and separation in Tx-Rx relative velocity:

$$\mathcal{I}(\Delta d) = \left\{ (i, i') \in \binom{\mathcal{I}}{2} : \|\mathbf{x}_i - \mathbf{x}_{i'}\| = \Delta d, \phi_i = \phi_{i'}, v_i = v_{i'} \right\}; \tag{4}$$

$$\mathcal{I}(\Delta \phi) = \left\{ (i, i') \in \binom{\mathcal{I}}{2} : \mathbf{x}_i = \mathbf{x}_{i'}, |\phi_i - \phi_{i'}| = \Delta \phi, v_i = v_{i'} \right\}; \tag{5}$$

$$\mathcal{I}(\Delta v) = \left\{ (i, i') \in \binom{\mathcal{I}}{2} : \mathbf{x}_i = \mathbf{x}_{i'}, \phi_i = \phi_{i'}, |v_i - v_{i'}| = \Delta v \right\}. \tag{6}$$

4 Computation

For the MPC corresponding to the delay bin $\tau_l \in \boldsymbol{\tau}$, we define the amplitude sample mean, across the set of measurements $\tilde{\mathcal{M}}_i$ collected at route configuration index $i \in \mathcal{I}$ as

$$\mu_i(\tau_l) = \frac{1}{J_i} \sum_{j=1}^{J_i} A_{i,j}(\tau_l). \tag{7}$$

Using Eq. (7) and the evaluation condition described by Eq. (4), we compute the spatial autocorrelation coefficient vis-à-vis a separation in distance as

$$\rho(\Delta d) = \frac{\frac{1}{|\mathcal{I}(\Delta d)|} \sum_{(i,i') \in \mathcal{I}(\Delta d)} \left[\frac{1}{\min(J_i, J_{i'})} \sum_{j=1}^{\min(J_i, J_{i'})} \left[\sum_{l=1}^{L} \left(\Big(A_{i,j}(\tau_l) - \mu_i(\tau_l) \Big) \Big(A_{i',j}(\tau_l) - \mu_{i'}(\tau_l) \Big) \right) \right] \right]}{\frac{1}{|\mathcal{I}|} \sum_{i \in \mathcal{I}} \left[\frac{1}{J_i} \sum_{j=1}^{J_i} \left[\sum_{l=1}^{L} \Big(A_{i,j}(\tau_l) - \mu_i(\tau_l) \Big)^2 \right] \right]}.$$

Similarly, using Eq. (7) and the evaluation condition described by Eq. (5), we compute the spatial autocorrelation coefficient vis-à-vis a separation in Tx-Rx alignment accuracy as

$$\rho(\Delta \phi) = \frac{\frac{1}{|\mathcal{I}(\Delta \phi)|} \sum_{(i,i') \in \mathcal{I}(\Delta \phi)} \left[\frac{1}{\min(J_i, J_{i'})} \sum_{j=1}^{\min(J_i, J_{i'})} \left[\sum_{l=1}^{L} \left(\Big(A_{i,j}(\tau_l) - \mu_i(\tau_l) \Big) \Big(A_{i',j}(\tau_l) - \mu_{i'}(\tau_l) \Big) \right) \right] \right]}{\frac{1}{|\mathcal{I}|} \sum_{i \in \mathcal{I}} \left[\frac{1}{J_i} \sum_{j=1}^{J_i} \left[\sum_{l=1}^{L} \Big(A_{i,j}(\tau_l) - \mu_i(\tau_l) \Big)^2 \right] \right]}.$$

Lastly, using Eq. (7) and the evaluation condition described by Eq. (6), we compute the spatial autocorrelation coefficient vis-à-vis a separation in Tx-Rx relative velocity as

$$\rho(\Delta v) = \frac{\frac{1}{|\mathcal{I}(\Delta v)|}\sum_{(i,i')\in\mathcal{I}(\Delta v)}\left[\frac{1}{\min(J_i,J_{i'})}\sum_{j=1}^{\min(J_i,J_{i'})}\left[\sum_{l=1}^{L}\left(\Big(A_{i,j}(\tau_l)-\mu_i(\tau_l)\Big)\Big(A_{i',j}(\tau_l)-\mu_{i'}(\tau_l)\Big)\right)\right]\right]}{\frac{1}{|\mathcal{I}|}\sum_{i\in\mathcal{I}}\left[\frac{1}{J_i}\sum_{j=1}^{J_i}\left[\sum_{l=1}^{L}\Big(A_{i,j}(\tau_l)-\mu_i(\tau_l)\Big)^2\right]\right]}.$$

Similar methodologies to evaluate the spatial autocorrelation function are used in references [2], [3], [4], and [5].

5 References

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