Problems

1) A discrete time signal $x(n)$ is defind as

$$x(n) = \begin{cases} 1+n/3, & -3 \le n \le -1 \\ 1 & 0 \le n \le 3 \\ 0 & \text{elsewhere } 0 \end{cases}$$

a) Determaine its value and sketch the signal $x(n)$

$1+\frac{n}{3}$, $-3 \le n \le -1 \Rightarrow$ at $n=-3$ $\quad 0$

$n = -2 \quad 1/3$

$n = -1 \quad 2/3$

from $0 \le n \le 3 \quad 1$

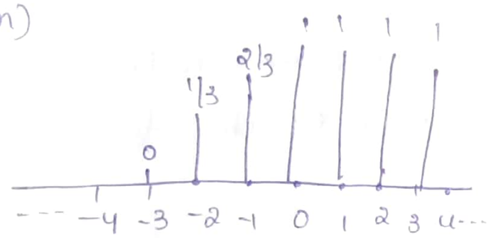
else where 0

so $x(n) = \{ \ldots\ldots 0, 1/2, 2/3, 1, 1, 1, 0 \ldots\ldots \}$

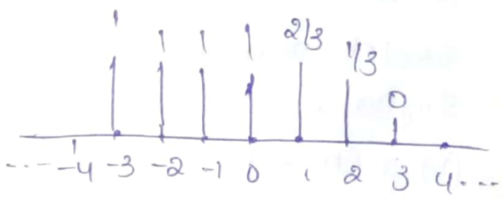
(b) sketch its values and the signals that result if we

(1) first fold $x(n)$ and then delay the resulting by four samples

$x(n)$



folding $x(n) = x(-n)$

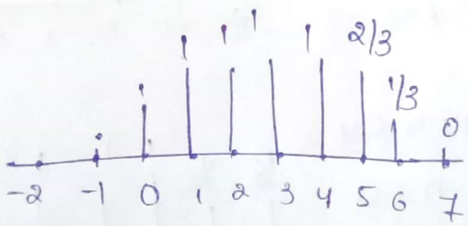


$x(-n+4) \rightarrow$ after folding $(-3 \le -n+4 \le 2$

$-3-4 \le -n \le 2-4$

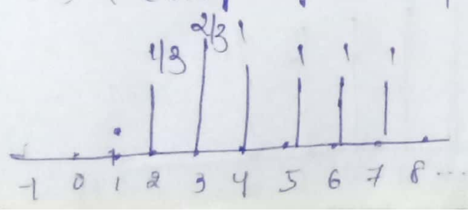
$-7 \le -n \le -2$

$7 \ge n \ge -2$

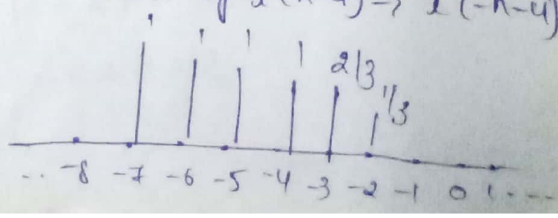


2) first delay $x(+n)$ by four samples and then fold the resulting signal

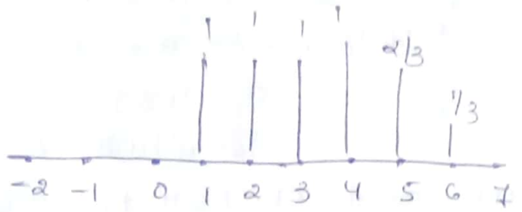
$x(n) \rightarrow$ delay by 4 samples



Folding $x(n-4) \Rightarrow x(-n-4)$



(c) Sketch the signal $x(-n+4)$



(d) Campare the results in parts (b) and (c) and device a rule fore obtaining the signal $x(-n+k)$ from $x(n)$

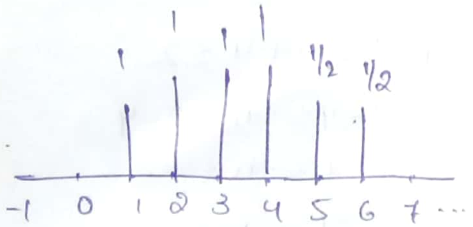
By camparing results in parts (b) and (c) we can say that to get $x(-n+k)$ from $x(n)$ first we need shift by K samples to right of $k>0$ (or) to left if $k<0$ results in $x(-n+k)$

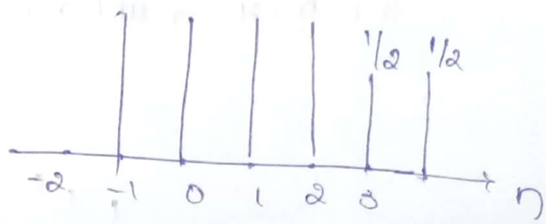
(e) can you express the signal $x(n)$ in terms of signal $\delta(n)$ and ~~(x)~~ $u(n)$

Yes ; $x(n) = \frac{1}{3}\delta(n-2) + \frac{2}{3}\delta(n-1) + u(0) - u(n-4)$

(2) A direct time signal $x(n)$ is shown in the figure. Sketch and label carefully each of the following signals

(a) $x(n-2)$

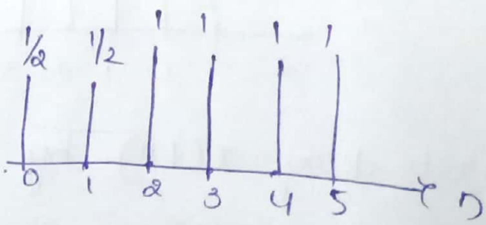




(b) $x(4-n)$ $\cdot -1 \le 4 \le n \le 4$

$-5 \le -n \le 0$

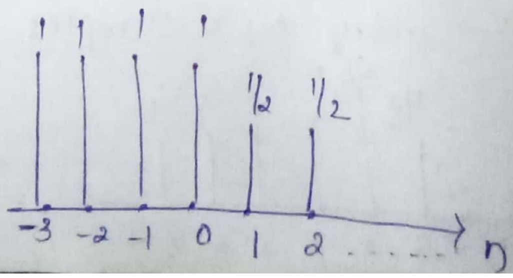
$5 \ge n \ge 0$



(c) $x(n+2)$

$-1 \le n+2 \le 4$

$-3 \le n \le 2$



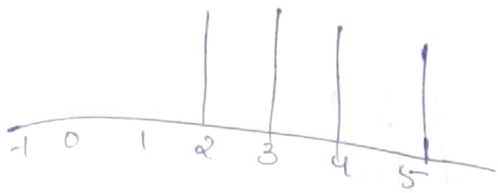
(d) $x(n)u(2-n)$

$x(n)$

$\Rightarrow 2-n \geq 0$

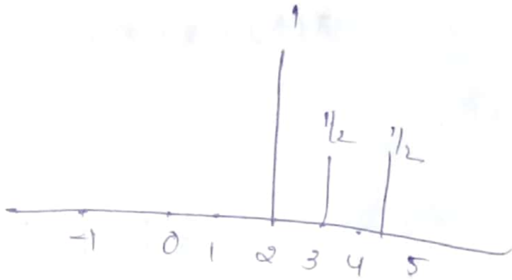
$-n \geq 2$

$n > 2$



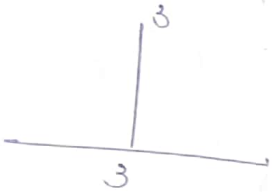
$\Rightarrow$

$x(n)u(2-n)$

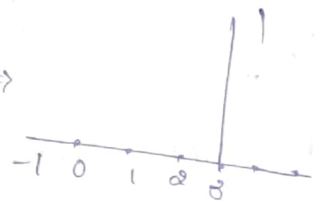


(e) $x(n-1)\delta(n-2)$

$x(n-1)$



$\Rightarrow$



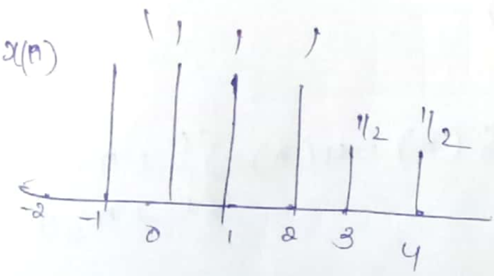
(f) $x(n^2) \Rightarrow x(n) = \{x(-2), x(-1), x(0), x(1), x(2), x(3) \dots\}$

$x(n^2) = \{\dots x(4), x(1), x(0), x(1), x(4), x(9), x(16) \dots\}$

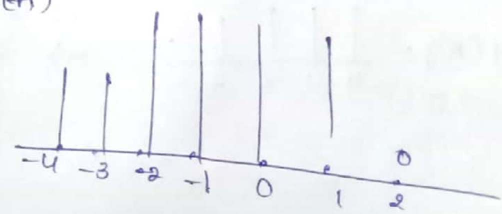
$= \{\dots \tfrac{1}{2}, 1, 1, 1, \tfrac{1}{2}, 0, 0 \dots\}$

(g) even part of $x_e(n) = \dfrac{x(n)+x(-n)}{2}$

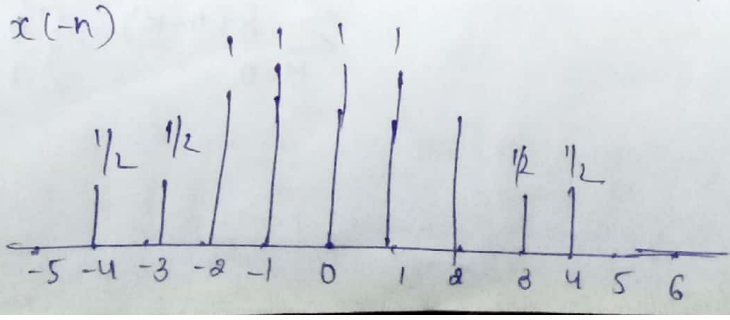
$x(n)$



$x(-n)$

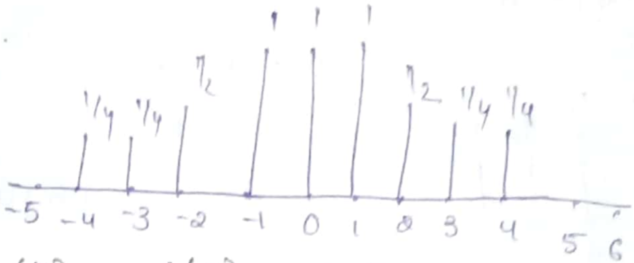


$x(n)+x(-n)$



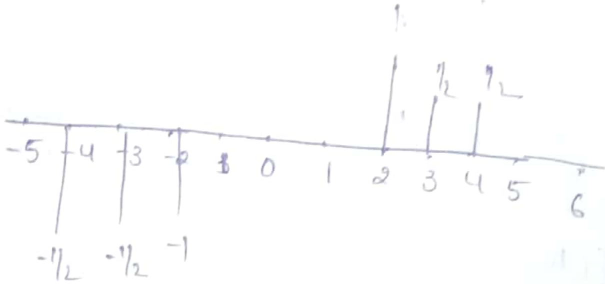
$x(n) + x(-n)$

$\Rightarrow$

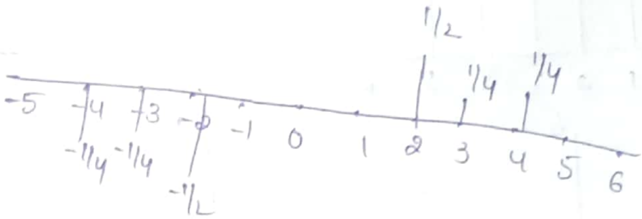


add part $x_o(n) = \dfrac{x(n) - x(-n)}{2}$

$x(n) - x(-n)$

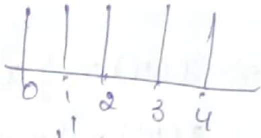


$\dfrac{x(n) - x(-n)}{2}$

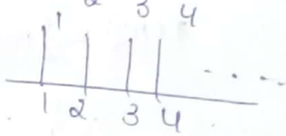


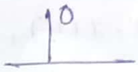
(3) show that $\delta(n) = u(n) - u(n-1)$

we know $\delta(n) = \frac{1}{0}$ $\quad u(n) =$



$u(n) = u(n-1)$ $\quad u(n-1) =$

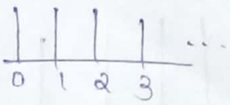




$= \delta(n) = u(n) - u(n-1)$

(b) $u(n) \sum_{k=-a}^{a} \delta(k) = \sum_{k=a}^{a} \delta(n-k)$

$u(n) =$



$\rightarrow \sum_{k=-a}^{n} \delta(k) = u(n) = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$

$\sum_{k=0}^{a} \delta(n-k) = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0. \end{cases}$

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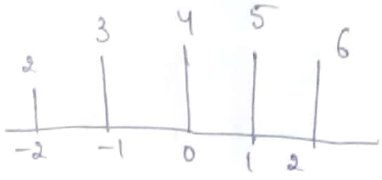
(4) Show that any signal can be decomposed into an even and an odd component. Is the decomposition unique? Illustrate your components using the signal $x(n) = \{2, 3, 4, 5, 6\}$

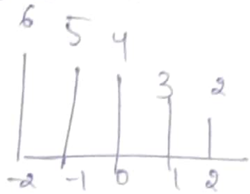
$$x_e(n) = \frac{x(n)+x(-n)}{2}, \quad x_e(n) = x_e(-n)$$

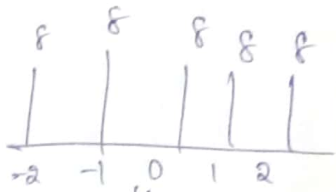
$$x_o(n) = x_o(-n)$$

$$x_o(n) = \frac{x(n)-x(-n)}{2} \Rightarrow x(n) = x_e(n) + x_o(n)$$

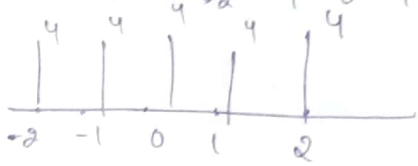
$x(n) = \{2, 3, 4, 5, 6\}$ $\quad x(-n) \quad\quad x(n)+x(-n)$







$$x_e(n) = \frac{x(n)+x(-n)}{2} \Rightarrow$$



(5) Show that the energy (power) of a real-valued energy (power) signal is equal to the sum of the energies (powers) of a even and odd components

first prove that

$$\sum_{n=-\infty}^{\infty} x_e(n) + x_o(n) = 0 \Rightarrow \sum_{n=-\infty}^{\infty} x_e(n)+x_o(n)$$

$$= \sum_{m=-\infty}^{\infty} x_e(-m)x_o(-m)$$

$$= -\sum_{m=\infty} x_e(m)x_o(m)$$

$$= -\sum_{n=\infty}^{\infty} x_e(n)x_o(n)$$

$$= \sum_{n=-\infty}^{\infty} x_e(n)x_o(n)$$

$$= 0$$

Energies (powers)

$$= \sum_{n=-\infty}^{\infty} x^2(n) = \sum_{n=-\infty}^{\infty} [x_e(n) + x_o(n)]^2$$

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$$= \sum_{n=-\infty}^{\infty} xe^2(n) + xo^2(n) + 2xe(n)\, xo(n)$$

$$= \sum_{n=-\infty}^{\infty} xe^2(n) + \sum_{n=-\infty}^{\infty} xo^2(n) + 2\sum_{n=-\infty}^{\infty} xe(n)$$

$$= E_e + E_o + 0$$

$$= E_e + E_e$$

(6) consider the system $y(n) = \mathcal{T}[x(n) = x(n^2)]$

a) Determaine if the system is time invariant.

given $y(n) = \mathcal{T}[x(n)] = x(n^2)$

$$x(n-k) \Rightarrow y_1(n) = x[(n-k)^2]$$

$$= x(n^2 + k^2 - 2nk]$$

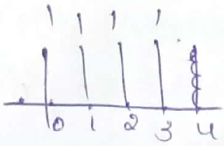
$$x(n-k) \neq y(n-k)$$

so the given system is time variant

(b) To clarify the result in part (a) assume that the signal

$$x(n) = \begin{cases} 1 \dots & 0 \le n \le 3 \\ 0 & \text{else where} \end{cases}$$ is applaied to the system.

(1) sketch the signal $x(n)$ $x(n) = \{\dots 0, 1, 1, 1, 1, 0, 0 \dots\}$



(2) Determaine and sketch the signal a $y(n) = \mathcal{T}[x(n)]$

$$y(n) = \mathcal{T}[x(n)] = x(n^2) = \{x(0), x(1), x(2^2), x(3)^3 \dots\}$$

$$= \{x(0), x(1), x(4), x(9) \dots\}$$

$$y(n) = x(n^2) = \{\underset{\uparrow}{1}, 1, 0, 0, 0 \dots\}$$

0 1 2 3 4

(3) Sketch the signal $y_2(n) = y(n-2)$

$$y(n-2) = \{\dots\dots \underset{\uparrow}{0}, 0, 1, 1, 0, 0, 0 \dots\}$$

(4) sketch the signal $x_2(n) = x(n-2)$

$$x(n-2) = \{\cdots \underset{\substack{\uparrow \\ 0}}{0}, \underset{1}{0}, \underset{2}{1}, \underset{3}{1}, \underset{4}{1}, \underset{5}{1}, \underset{6}{0} \cdots\}$$

(5) Determaine and sketch the signal $y_2(n) = \mathcal{T}[x_2(n)]$

$$y_2(n) = \mathcal{T}[x_2(n-2)] = \{x(0), x(1), x(2), x(3), x(4), x(5)$$

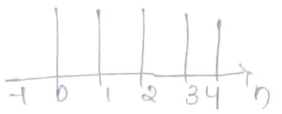
$$= \{\cdots\cdots 0, 1, 0, \underset{\uparrow}{0}, 0, 1, 0 \cdots\}$$

(6) campare the signals $y_2(n)$ and $y(n-2)$ what is your conclusion.

$y_2(n) \neq y(n-2) \Rightarrow$ system is time variant

(c) Repeat part (b) for the system.

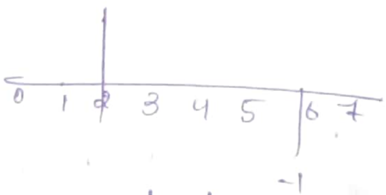
$y(n) = x(n) - x(n-1)$ can you use this result to make any statement about the time invariate of the system ? why ?

(1) $x(n) =$  $\Rightarrow \{1, 1, \underset{\uparrow}{1}, 1, 1\}$

(2) $y(n) = x(n) - x(n-1)$

$y(n) = x(n) - x(n-2)$ 

$= \{0, 1, 0, 0, 0, 0, -1\}$

(3) $y(n-2) \Rightarrow$  $= \{0, 0, \underset{\uparrow}{1}, 0, 0, 0, -1\}$

$x(n-2) \Rightarrow$  $\Rightarrow \{0, 0, 1, 1, 1, 1\}$

$y_2(n) = \{0, 0, 1, 0, 0, 0, -1\}$

(4) $y_2(n) = y(n-2) \Rightarrow$ System is time invariant

(d) Repeat parts (b) and (c) for the system $y(n) = \gamma[x(n)]$

n.d

(1) $y(n) = \gamma n\, x(n)$ $n \to$ integer value from 0

$x(n) = \{\ldots 0, 1, 1, 1, 1, 0 \ldots\}$
$\uparrow$

(2) $y(n) = \{\ldots 0 \cdot 0, 1, 2, 3, 4 \ldots\}$
$\uparrow$

(3) $y(n-2) = \{\ldots 0, 0, 0, 1, 2, 3, 4 \ldots\}$
$\uparrow$

(4) $x(n-2) = \{\ldots 0, 0, 0, 1, 1, 1, 1 \ldots\}$
$\uparrow$

(5) $y_2(n) = \gamma(x(n-2)) = \{\ldots 0, 0, 2, 3, 4, 5\}$

(6) $y_2(n) \neq y(n-2) \Rightarrow$ system is time variant

(7) (1) static or dynamic

(2) linear or non-linear

(3) Time invariant or varying

(4) caused or non caused

(5) stable or unstable

(6) Examine the following systems with respect to the propert above

above

(a) $y(n) = \cos[x(n)]$

(i) Static (only prenent i/p)

(iv) only present i/p → caused

(v) stable

(ii) $y_1(n) = \cos(x(n))$

$y_2(n) = \cos(x_2(n)]$

$y(n) = \cos(x_1(n) + \cos x_2(n)$

$y'(n) = \cos(x_1(n) + x_2(n)]$

(iii) $y(n) = \cos[x(n-n_0)]$

$y'(n) = \cos[x(n-n_0)]$

⇒ Time variant.

(b) $y(n) = \sum_{k=-\infty}^{n+1} x(k)$

Dynamic (depends on future values)

linear, Time variant, non caused (also depends on future value, unstable).

(c) $y(n) = x(n) \cdot \cos(\omega_0(n))$

Static, linear, Time variant, Caused, stable

$y(n) = x(n-n_0)\cos(\omega_0 n - n_0)$

$y'(n) = x(n-n_0)\cos \omega_0 n$

(d) $y(n) = x(-n+2)$

Dynamic

at $n=0 \Rightarrow y(0) = x(2)$

future value

$y_1(n) = x_1(-n+2) + x_2(-n+2)$

$y_2(n) = x + x_2(-n+2) \Rightarrow x(-n+2) + x_2(-n+2)$

linear

non caused, stable, Time variant.

(e) $y(n) = \text{Trun}[x(n)]$

Static, non-linear, time invariant, caused, stable

(f) $y(n) = \text{Round}[x(n)]$

Static, non-linear, time invarient, caused, Stable

(g) $y(n) = |x(n)|$

Static, non-linear, time invariant, Caused, stable

(h) $y(n) = x(n)u(n)$

Static, linear, Time, invariant, caused, stable

(i) $y(n) \pm x(n) + n x(n+1)$

Dynamic, linear, time varient, caused, stable

(J) $y(n) = x(2n)$

Dynamic, linear, time variant, non caused, Stable

(k) $y(n) = x(n)$; if $x(n) \geq 0$

0 ; if $x(n) < 0$

static, linear, time invariant, non-caused stable

(l) $y(n) = x(-n)$

Dynamic, linear, time variant, Caused stable

(m) $y(n) = \text{sign}[x(n)]$

static, non-linear, Time invariant, caused, stable

(n) The ideal Sampling System with input $x_a(t)$ and output

$x(n) = x_a(n) \quad -\alpha < n < \alpha$

$x(n) = x_a(nT)$.

Static, linear, time variant, non caused

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(9) let N be an LTI relaxed and BIBO stable system with input $x(n)$ and output $y(n)$ show that.

(a) if $x(n)$ is periodic with period N [i.e $x(n) = x(n+N)$ for all $n \geq 0$] the output $y(n)$ tends to a periodic signal the same period

$$x(n) = x(n+N) \ \forall n \geq 0$$

$$y(n) = \sum_{K=-\alpha}^{\alpha} h(K)\, x(h-K)$$

$$y(n+N) = \sum_{K=\alpha}^{n+K} h(K)\, x(n+N-K)$$

$$y(n+N) = \sum_{K=n+1}^{n+N} h(K)\, x(n-K) + \sum_{K=-\alpha}^{n} h(K)\, x(n-K)$$

$$y(n+N) = y(n) + \sum_{K=n+1}^{n+N} h(K)\, x(n+K)$$

from BIBO system $\lim |h(n)| = 0$

$$\lim_{n\to\alpha} \sum_{K=n+1}^{n+N} h(K)\, x(n-K) = 0$$

$$\lim_{N\to\alpha} y(n-N) = y(N)$$

$$y(N) = y(n+N)$$

(b) If $x(n)$ is bounded and tendy to a constant, the output will also tend to a constant.

$x(n) = x0(n) + a u(n)$ $\quad x0(n) =$ bounded with $\lim_{n\to\alpha} x0(n) = 0$

$$y(n) = a \sum_{K=0}^{\alpha} n(K)\, u(n-K), \sum_{K=0}^{\alpha} h(K)\, x0(h-K$$

$$a \sum_{K=0}^{n} h(K) + y0(n)$$

(c) $\sum_{n} x0^2(n) < \alpha \Rightarrow \sum_{n} y0^2(n) < a$

hence $\lim_{n\to\alpha} |y0(n)| = 0 \Rightarrow a \sum_{K=0} h(K) = \text{constant}$

(e) if $x(n)$ is a energy signal the output $y(n)$ will also be a energy signal

$$y(n) = \sum_{K} h(K)\, x(n-K)$$

$= \sum_{-\infty}^{K} y_2(n) = \sum_{-\infty}^{\infty} h(K)\,x(n-K)\} = \sum_K \sum_l h(K)\,h(l$

but $\sum x(n-K)\,x(n-l) \leq \sum_n x^2(n)\,|h(x)|$

for BIBO stable system $\sum_K |h(K)| < m$

Hence by $\leq m^2 \epsilon x$ so that $cy < 0$ if $\epsilon x < 0$

(10) The following input - output pairs hare been observed during the operation of a time invariant system :-

$x_1(n) = \{1, \underset{\uparrow}{0}, 2\} \overset{\sim}{\longleftrightarrow} y_1(n) = \{0, \underset{\uparrow}{1}, 2\}$

$x_2(n) = \{\underset{\uparrow}{0}, 0, 3\} \overset{\sim}{\longleftrightarrow} y_2(n) = \{\underset{\uparrow}{0}, 1, 0, 2)$

$x_3(n) = \{\underset{\uparrow}{0}, 0, 0, 1\} \overset{\sim}{\longleftrightarrow} y_3(n) = \{1, 2, \underset{\uparrow}{1}, 1\}$

can you draw any conclusions regarding the linearly of the system what is the impulse respone of the system.

As this is a time-invariant system
$y_2(n)$ should have only 3 elements and
$y_3(n)$ should have 4 eliments
so, it is non-linear

(11) The following input-output have been observed during the operation of a linear system

$x_1(n) = \{-1, \underset{\uparrow}{2}, 1\} \overset{\sim}{\longleftrightarrow} y_1(n) = \{1, \underset{\uparrow}{2}, -1, 0, 1\}$

$x_2(n) = \{1, \underset{\uparrow}{-1}, -1\} \overset{\sim}{\longleftrightarrow} y_2(n) = \{-1, \underset{\uparrow}{1}, 0, 2\}$

$x_3(n) = \{0, 1\} \overset{\sim}{\longleftrightarrow} y_3(n) = \{\underset{\uparrow}{1}, 2, 1\}$ Can you draw any conclusions about the time invariance of the system.

$$x_1(n) + x_2(n) = \delta(n)$$

System is linear, the impulse

$$y_1(n) + y_2(n) = \{0, \underset{\uparrow}{3}, -1, 2, 1\}$$

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if the system were time invariant the response of $x_3(n)$ would be $\{3, 2, 1, 3, 1\}$ with the arrow under the 2

(12) The only available information about the system consists of N input output pairs of signals $y_i(n) = T[x_i(n)]$ $i = 1, 2, 3 \ldots N$

(a) What is the class of input signals for which we can determine the output using the information above if the system is known to be linear.

Any linear combination of signal in the form of $x_i(n)$; $i = 1, 2 \ldots N$

because if we take $i = 1, 3$

$$y_1(n) = x_1(n)$$

$$y_3(n) = x_3(n) \Rightarrow y(n) = y_1(n) + y_3(n) = x_1(n) + x_3(n)$$

$$y(n) = x(n) + x_3(n)$$

linear

(b) Same repeat for the system is invariant

Any $x_i(n+k)$ where k is any integer $i = 1, 2 \ldots N$

1st replace $n = n - n_0 \Rightarrow x_i(n - n_0 - k)$

$x(n)$ by $x(n - n_0) \Rightarrow x_i(n - k - n_0)$

[Time invariant]

(13) Show that the neccesary and sufficient condition for a relaxed LTI system to be BIBO stable is

A system to be BIBO stable only when bounded output should produce bounded input

$$y(n) = \sum_k h(k)\, x(n-k)$$

$$|y(n)| = \sum_k |h(k)|\, |x(n-k)|$$

$$= \sum_k |x(n-k)| < m_n \text{ [some constant]}$$

so $|y(n)| = m_n \sum_k |h(k)|$

$|y(n)| < \infty$ for all n, if and only if $\sum_k |h(k)| < \infty$

so $\sum^{\infty} |y(n)|$

A system to be BIBO stable only when bounded input produce bounded output

$$y(n) = \sum_{K=-\alpha}^{\alpha} h(K)\, x(n-K) \; ; \; n \le n-K$$
$$K \ge 0$$

$$|y(n)| = \sum_{K=-\alpha}^{\alpha} |h(K)|\,|x(n-K)|$$

(a) $\sum_{K=-\alpha}^{\alpha} |x(n-K)| \le mn$ for some constant

$$|y(n)| = mn \sum_{K=-\alpha}^{\alpha} |h(K)| \; ; \; n \le n-K$$

$|y(n)|$ is $< \alpha$ if & only if $\sum_{K=\alpha}^{K\ge 0} |h(K)| < \alpha$

$$\text{so } \sum_{n=-\alpha}^{\alpha} |h(n)| < \alpha$$

(14) show that

a) relaxed line system is caused if & only if for any input $x(n)$ such that $x(n)=0$ for $n<n_0 \Rightarrow y(n)=0$ for $n<n_0$

If a system is caused output depends only on the present and past inputs as $x(n)=0$ for $n<n_0$ then $y(n)$ also becomes zero for $n<n_0$

(b) A relaxed LTI system is caused if and only if $h(n)=0$ for $n<0$

$$y(n) = \sum_{K=-\alpha}^{\alpha} h(K)\, x(n-K)$$

for finite impulse response.

$h(n)=0$, $n<0$ and $n \ge m$

So $y(n)$ reduces to $y(n) = \sum_{K=0}^{n-1} h(K)\, x(n-K)$

If it is infinite impulse response

then $y(n) = \sum_{K=0}^{\alpha} h(K)\, x(n-K)$

15) Show that for any real for complex constant a, and any finite integer number m and N, we have

$$\sum_{n=m}^{N} n = a^n = \begin{cases} \frac{a^m - a^{N+1}}{1-a} & ; \text{if } a \neq 1 \\ N-m+1 & ; \text{if } a = 1 \end{cases}$$

for $a=1$, $\sum_{n=m}^{N} a^n = N-m+1$

for $a \neq 1$, $\sum_{n=m}^{N} a^n = \frac{a^m - a^{N+1}}{1-a}$

$$(1-a)\sum_{n=m}^{N} a^n = a^m + a^{m+1} - a^{m+1} + \cdots - a^N - a^N - a^{N+1}$$

$$= a^m - a^{N+1}$$

(b) for $m=0$ $|a|<1$ and $N \to \infty$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \quad |a|<1$$

(16) : (a). if $y(n) = x(n) * h(n)$ show that $\Sigma_y = \Sigma x \Sigma b$

where $\Sigma_x = \sum_{n=\infty}^{\infty} x(n)$

$$y(n) = \sum_k h(k)\, x(n-k)$$

$$\sum_n y(n) = \sum_n \sum_k h(k)\, x(n-k)$$

$$\sum_n y(n) = \sum_k h(k) \cdot \sum_{n=-\infty}^{\infty} x(n-k)$$

$$\sum_n y(n) = \left(\sum_k h(k)\right)\left(\sum_n x(n)\right)$$

(b) compute the convolution $y(n) * h(n)$ of the following signals and check the connections of the results by using the text in (a)

(i) $x(n) = \{1, 2, 4\}$, $h(n) = \{1, 1, 1, 1, 1\}$

$y(n) = \{1, 3, 7, 7, 7, 6, 4\}$

$\sum_n y(n) = 35$, $\sum_n x(n) = 7$, $\sum_n h(n) = 5$ By Tabular method

h(n) \ x(n)	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4

$$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$$

$35 = 7 \times 5$

$= 35 = 35$

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$x(n) = \{1, 2, -1\}$, $h(n) = x(n)$

$x(n) = \{1, 2, -3\}$, $h(n) = \{1, 2, -1\}$

$y(n) = x(n) * h(n)$

$y(n) = \{1, 4, 2, -4, 1\}$

$h(n) \backslash x(n)$	1	2	-1
1	1	2	-1
2	1	2	-1
-1	1	2	-1

$\sum_n y(n) = 4$; $\sum_n x(n) = 2$, $\sum_n h(n) = 2$

$\sum_n y(n) = \sum_n (x) \sum_n h(n)$

$4 = 4$

$x(n) = \{0, 1, -2, 3, -4\}$, $h(n) = \{1/2, 1/2, 1/2\}$

$y(n) = \{0, 1/2, -1/2, 3/2, -2, 0, -5/2, 2\}$

$\sum_n y(n) = -5$, $\sum_n x(n) = -2$, $\sum_n h(n) = 5/2$

$\sum_n y(n) = \sum_n x(n) h(n)$

$-5 = -5$

$h(n) \backslash x(n)$	0	+1	-2	-3
1/2	0	+1/2	-1	3/2
1/2	0	1/2	-1	3/2
1	0	1	-2	3
1/2	0	1/2	-1	3/2

$x(n) = \{1, 2, 3, 4, 5\}$, $h(n) = \{1\}$

$y(n) = \{1, 2, 3, 4, 5\}$

$\sum_n y(n) = \sum_n x(n) \cdot \sum_n h(n)$

$15 = 15(1)$

$= 15 = 15$

$x(n) = \{1, 2, 3\}$, $h(n) = \{0, 0, 1, 1, 1, 1\}$

$y(n) = \{\underset{\uparrow}{0}, 0, 1, -1, 2, 2, 1, 3\}$

$\sum_n y(n) = 8$ $\sum_n x(n) = 2$; $\sum_n h(n) = 4$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$8 = 8$

$h(n) \backslash x(n)$	1	-1	3
$\rightarrow$ 0	0	0	0
0	0	0	0
1	1	-2	3
1	1	-2	3
1	1	-2	3
1	1	-2	3

$x(n) = \{0, 0, 1, 1, 1, 1\}$, $h(n) = \{1, -2, 3\}$

$y(n) = \{0, 0, 1, -1, 2, 2, 1, 3\}$

$\sum_n y(n) = 8$, $\sum_n x(n) = 4$ $\sum_n h(n) = 2$

$\sum_n y(n) = \sum_n x(n) \cdot \sum h(n)$

$8 = 8$

h(n) \ x(n)	0	0	1	1	1	1
1	0	0	1	1	1	1
-2	0	0	-2	-2	-2	-2
3	0	0	3	3	3	3

$x(n) = \{0, 1, 4, -3\}$, $h(n) = \{1, 4, -1, -1\}$

$y(n) = \{0, 1, 4, -4, -5, -1, 3\}$

$\sum_n y(n) = -2$, $\sum_n x(n) = -2$; $\sum_n h(n) = 1 \to 1$

$\sum_n y(n) = \sum_n x(n) \cdot \sum_n h(n)$

h(n) \ x(n)	0	1	4	-3
1	0	1	4	-3
0	0	0	0	0
-1	0	-1	-4	3
-1	0	-1	-4	3

$x(n) = \{1, 1, 2\}$, $h(n) = u(n)$

$y(n) = \{1, 2, 4, 3, 2\}$

$\sum_n y(n) = 12$, $\sum_n x(n) = 4$, $\sum_n h(n) = 3$

$\sum_n y(n) = \sum_n x(n) \sum h(n)$

$12 = 12$

h(n) \ x(n)	1	1	1
→ 1	1	1	2
1	1	1	2
1	1	1	2

$x(n) = \{1, 1, 0, 1, 1\}$, $h(n) = \{1, -2, -3, 4\}$

$y(n) = \{-1, -5, 2, 3, -5, 1, 4\}$.

$\sum_n y(n) = 0$, $\sum_n x(n) = 4$ $\sum_n h(n) = 0$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$0 = 0$

h(n) \ x(n)	1	1	0	1	1
1	1	1	0	1	1
-2	-2	-2	0	-2	-2
-3	-3	-3	0	-3	-3
4	-4	-4	0	-4	4

$x(n) = \{1, 2, 0, 2, 1\}$ $h(n) = x(n)$

$y(n) = \{1, 4, 4, 4, 1, 0, 4, 4, 4\}$

$\sum_n y(n) = 36$ $\sum_n x(n) = 6$ $\sum_n h(n) = 6$

$\sum_n y(n) = \sum_n x(n) \cdot \sum_n h(n)$

$36 = 36$

h(n) \ x(n)	1	2	0	2	1
1	1	2	0	2	1
2	2	4	0	4	2
0	0	0	0	0	0
2	2	4	0	4	2
1	1	2	0	2	1

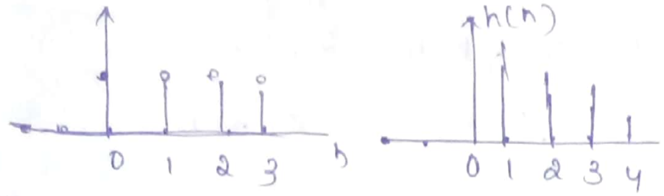
$x(n) = (1/2)^n u(n)$, $h(n) = \left(\frac{1}{4}\right)^n u(n)$

$y(n) = \left[2\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n u(n)\right]$

$\sum_n y(n) = \frac{8}{3}$, $\sum_n h(n) = \frac{4}{3}$, $\sum_n x(n) = 2$

17) compute and plot convolution $x(n) = \frac{1}{n}(n)$ and $h(n) * x(n)$ for the parts of signal shown below

(a)



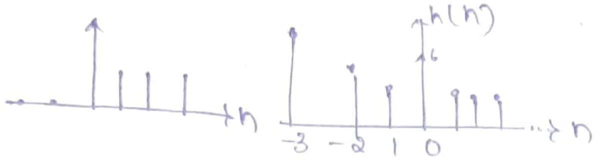
$h(n)$ \ $x(n)$	1	1	1
6	6	6	6
5	5	5	5
4	4	4	4
3	3	3	3
2	2	2	2
1	1	1	1

$x(n) = \{\underset{\uparrow}{1}, 1, 1, 1\}$; $h(n) = \{\underset{\uparrow}{6}, 5, 4, 3, 2, 1, 0\}$

$y(n) = x(n) * h(n)$

$y(n) = \{6, 11, 15, 18, 14, 10, 6, 3, 1\}$

(b)

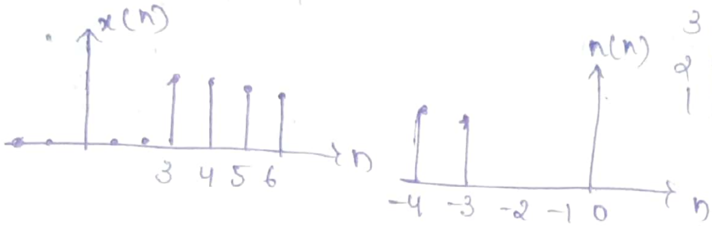


$h(n)$ \ $x(n)$	1	1	1
6	6	6	6
5	5	5	5
4	4	4	4
3	3	3	3
2	2	2	2
1	1	1	1

$x(n) = \{\underset{\uparrow}{1}, 1, 1, 1\}$, $h(n) = \{6, 5, 4, 3, \underset{\uparrow}{2}, 1, 0\}$

$y(n) = \{6, 11, 18, 14, 10, 6, 3, 1\}$

(c)

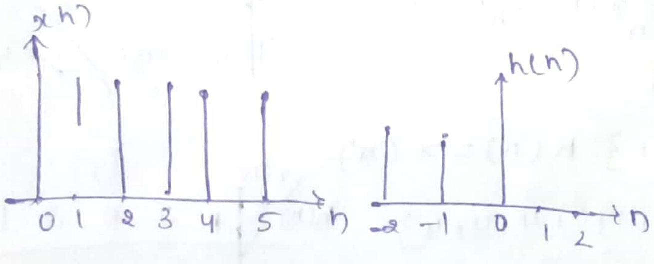


$x(n) = \{0, 0, 0, 1, 1, 1, 1\}$

$h(n) = \{1, 1, 0, 0, 0, \underset{\uparrow}{0}\}$

$y(n) = \{0, 0, 0, 0, 1, 2, 2, 2, 1, 0, 0, 0\}$

$h(n)$ \ $x(n)$	0	0	0	1	1	1	1
1	0	0	0	1	1	1	1
1	0	0	0	1	1	1	1
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

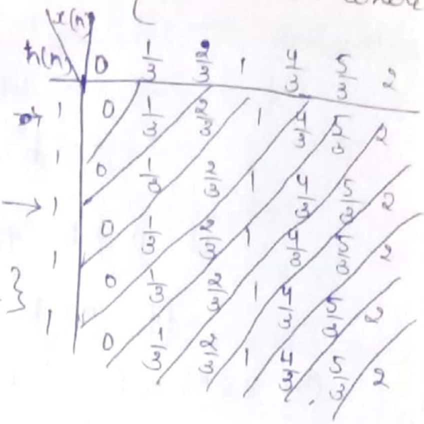


$x(n) = \{0, 0, 1, 1, 1\}$, $h(n) = \{1, 1, \underset{\uparrow}{0}, 0, 0\}$

$y(n) = \{0, 0, \underset{\uparrow}{1}, 2, 2, 2, 1, 0, 0, 0\}$

$h(n)$ \ $x(n)$	0	0	1	1	1	1
1	0	0	1	1	1	1
1	0	0	1	1	1	1
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

18) Determaine and sketch the convolution $y(n)$ of the signals $x(n) = \begin{cases} \frac{1}{3}n & 0 \le n \le 6 \\ 0 ; & \text{elsewhere} \end{cases}$ $\quad h(n) = \begin{cases} 1, & -2 \le n \le 2 \\ 0, & \text{else where} \end{cases}$



(a) graphically

$x(n) = \{ \underset{\uparrow}{0}, \frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}, 2 \}$

$h(n) = \{1, 1, 1, 1, 1\}$

$y(n) = \{0, \frac{1}{3}, \underset{\uparrow}{1}, 2, \frac{10}{3}, 5, \frac{20}{3}, 6, 5, \frac{11}{3}, 2\}$

(b) analytically

$x(n) = \frac{1}{3} n [u(n) - u(n-7)]$

$h(n) = u(n+2) - u(n-3)$

$y(n) = \frac{1}{3} n [u(n) - u(n-7) * u(n+2) - u(n-3)$

$= \frac{1}{3} n u(n) * u(n+2) - \frac{1}{3} n u(n) * u(n-3) - \frac{1}{3} n u(n-u)]$

$\cdots * u(n+2) + \frac{1}{3} n (u(n-7) * u(n-3)$

(20) consider the following three operations.

(a) Multiply the integer numbers 131 and 122

$131 \times 122 = 15982$

(b) compute the convolution

$y(n) = \{15, 9, 8, 2\}$

(c) Multiply the polynomials.

$1 + 3z + z^2$ and $1 + 2z + 2z^2$

$= 2z^4 + z^3 + 9z^2 + 5z + 1$

(d) Repeat part (a) for the numbers 1:3 and 12:2

$1.31 \times 12.2 = 15.982$

(e) comment on your result

These are different ways to perform convolution

21) compute the convolution $y(n) * h(n)$ of the following parts of the signals.

a) $x(n) = a^n u(n), h(n) = b^n u(n)$ when $a \ne b$ and when $a = b$

$y(n) = x(n) * h(n)$

$= a^n u(n) * b^n u(n)$

$= [a^n * b^n] u(n)$

$$y(n) = \sum_{k=0}^{n} a^k u(k) b^{n-k} u(n-k)$$

$$= b^n \sum_{k=0}^{n} a^k u(k) b^{-k}$$

$$= b^n \sum_{k=0}^{n} (ab)^{k}$$

if $a \neq b$ then $y(n) = \dfrac{b^{n+1} - a^{n+1}}{b-a} u(n)$

if $a = b \Rightarrow b^n (n+1) u(n)$

(b) $x(n) = \begin{cases} 1 ; & n = -2, 0, 1 \\ 2 ; & n = -1 \\ 0 ; & \text{else where} \end{cases}$ $\quad h(n) = \delta(n) - \delta(n-1) + \delta(n-4) + \delta(n-3)$

$h(n) = \{1, -1, 0, 0, 1, 1\}$

$x(n) = \{1, 2, 1, 1\}$

$y(n) = \{1, 1, \underset{\uparrow}{-1}, 0, 0, 3, 3, 2, 1\}$

h(n) \ x(n)	1	2	1	1
1	1	2	1	1
-1	-1	-2	-1	-1
0	0	0	0	0
0	0	0	0	0
-1	1	2	1	1
1	1	2	1	1

(c) $x(n) = u(n+1) - u(n-4) - \delta(n-3)$

$h(n) = [u(n+2) - u(n-3)] \{3u(n)\}$

(d) $x(n) = u(n) - u(n-5)$;

$h(n) = u(n-2), u(n+8) + u(n-11) - u(n+17)$

(e) $x(n) = \{1, \underset{\uparrow}{1}, 1, 1, 1, 1, 0, -1\}$

$h(n) = \{1, 2, \underset{\uparrow}{3}, 2, 1\}$

$y(n) = \{1, 3, 6, 8, 9, 8, 5, 1, -2, -2, 1\}$

h(n) \ x(n)	1	1	1	1	1	0	0
1	1	1	1	1	1	0	0
2	2	2	2	2	2	0	0
3	3	3	3	3	3	0	0
2	2	2	2	2	2	0	0
1	1	1	1	1	1	0	0

(d) $x(n) = \{1, 1, 1, 1, 1, 1\}$

$h(x) = \{0, 0, 1, 1, 1, 1, 1\}$

$h(n) = h'(n) + h'(n-9)$

$y(n) = y'(n) + y'(n-9)$ where

$y'(n) = \{0, 0, 1, 2, 3, 4, 5, 5, 4, 3, 2\}$

22) let $x(n)$ be the input signal to a discrete time filter with impulse response $h_i(n)$ and let $y_i(n)$ be the corresponding output.

(a) compute and sketch $x(n)$ and $y_i(n)$ in the following cases using the small scale in all the figures.

$x(n) = \{1, 4, 2, 3, 5, 3, 3, 4, 5, 3, 6, 9\}$

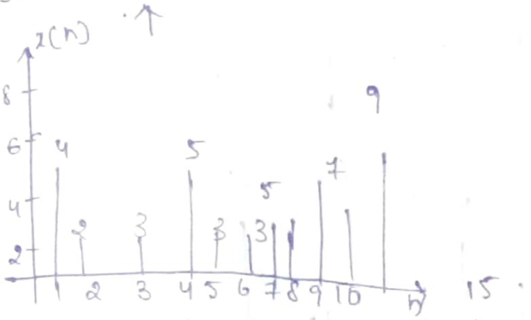
$h_1(n) = \{1, 1\}$, $h_2(n) = \{1, 2, 1\}$, $h_3(n) = \{1/2, 1/2\}$ $h_4(n) = \{1/4, 1/4, 1/4\}$

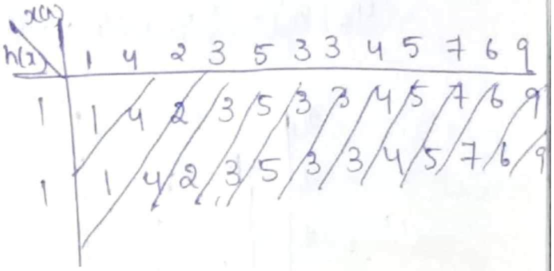
$h_5(n) = \{1/4, -1/2, 1/4\}$

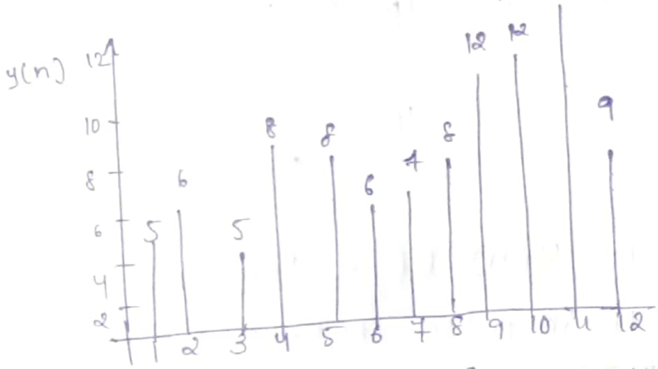
sketch $x(n)$, $y_1(n)$, $y_2(n)$ on the graph and $x(n)$, $y_3(n)$ $y_4(n)$ $y_5(n)$ on other graph.

$y(n) = x(n) * h_1(n)$

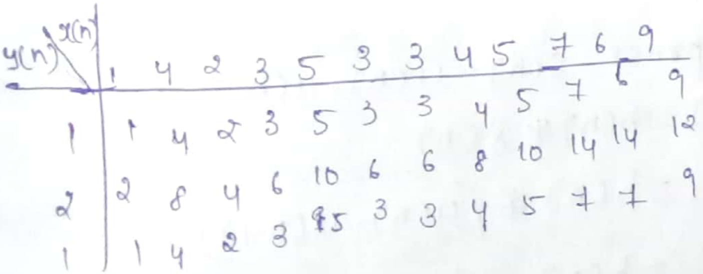
$y_1(n) = \{1, 5, 6, 5, 8, 6, 7, 9, 12, 12, 15, 9\}$
$\uparrow$

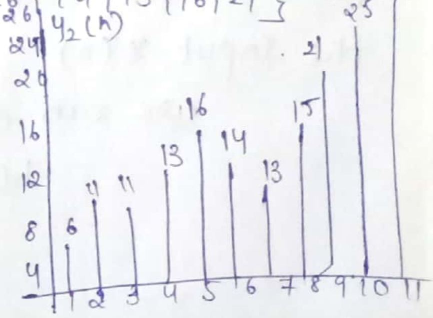




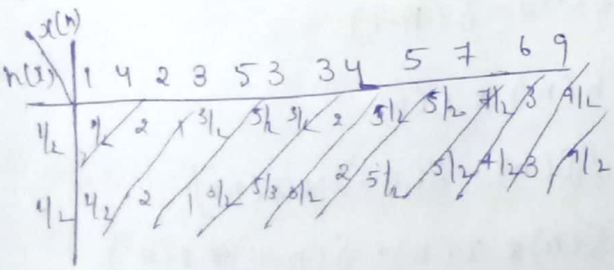


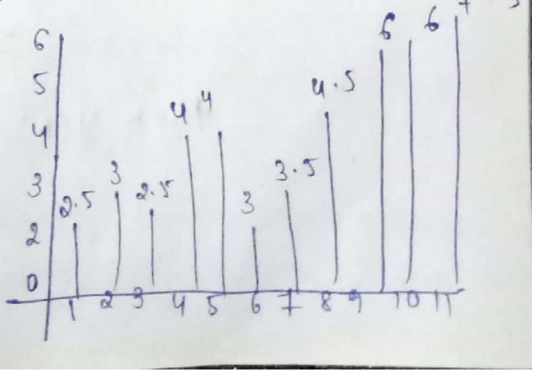
$y_2(n) = x(n) * h_2(n) = \{1, 6, 11, 11, 13, 16, 14, 13, 16, 21\}$





$y_3(n) = x(n) + h_3(n) = \{1/2, 2.5, 3, 2.5, 4, 4, 3, 3, 5, 4, 5, 6, 6, 7.5, 4.5\}$





Comment on the smoothness of $y_2(n)$, $y_4(n)$ which factors effect the smoothness

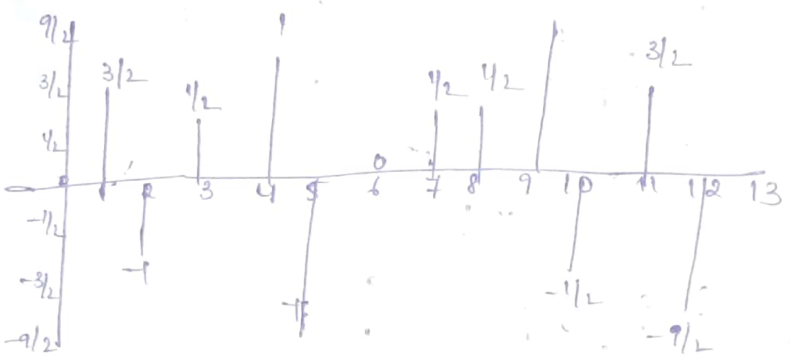
$y_2(n)$ and $y_4(n)$ are smoother then $y_1(n)$ because of Smaller scale factor.

Campare $y_4(n)$ with $y_5(n)$ what is the difference can you explain.

$y_4(n)$ results in smoother output than $y_5(n)$ the negative value of $h_5(0)$ is responisble for the non-smoth characteristics of $y_5(n)$.

let $h_6(n) = \{1/2, -1/2\}$ compute $y_6(n)$ sketch $x(n)$, $y_2(n)$ and $y_6(n)$ on the same figure and comment on the results $y_6(n) = x(n) * h_6(n)$

$y_6(n) = \{1/2, 3/2, 1, -1, 1/2, 1, -1, 0, 1/2, 1/2, 1, -1/2, 3/2, -9/2\}$



23) Express the output $y(n)$ of a linear time-invariant system with impulse response $h(n)$ in form of its step response $s(n) = h(n) * u(n)$ and the input $x(n)$

we can express $\delta(n) = u(n) - u(n-1)$

$h(n) = h(n) * \delta(n)$

$= h(n) * [u(n) - u(n-1)]$

$= h(n) * [u(n) - h(n) * u(n-1)]$

$= S(n) - S(n-1)$

then $y(n) = h(n) * x(n)$

$= [S(n) - S(n-1) * x(n)]$

$= S(n) * x(n) - S(n-1) * x(n)$

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24) The deserate time system $y(n) = ny(n-1) + x(n)$, $n \geq 0$, is atrest [i.e, $y(-1) = 0$] check if the systa is a linear time invariant and BIBO stable

$y(n) = ny(n-1) + x(n)$, $n \geq 0$

$y_1(n) = ny_1(n-1) + x_1(n)$
$y_2(n) = ny_2(n-1) + x_2(n)$

$y(n) = ny_1(n-1) + x_1(n) + ny_2(n-1) + x_2(n)$

$y(n) = ny(n-1) + x(n)$

$x(n) = ax_1(n) + bx_2(n)$

$y(n) = ay_1(n) + by_2(n)$

Hence the system is linear

$\rightarrow y(n-1) = (n-1)y(n-2) + x(n-1)$

delayed $\Rightarrow y(n-1) = ny(n-2) + x(n-1)$

so the system is time variant.

$\rightarrow$ if $x(n) = u(n)$ then $|x(n)| \leq 1$ for this bounded input output is $y(0) = 0$, $y(1) = 2$, $y(2) = 5$ .... un bounded

so system is un stable

25) consider the signal $\delta(n) = a^n u(n)$, $0 < a < 1$

a) show that any sequence $x(n)$ can be decomposed $x(n) = \sum_{n=-\infty}^{\infty} (k \delta(n-k))$ and express c_k in terms of $x(b)$.

$\delta(n) = \delta(n) - a\delta(n-1)$

$\delta(n-1) = \delta(n-k) - a\delta(n-k-1)$

$x(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot \delta(n-k)$

$= \sum_{k=-\infty}^{\infty} x(k) \cdot [\delta(n-k) - a\delta(n-k-1)]$

$x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k) - a\sum_{k=-\infty}^{\infty} x(k)\delta(n-k-1)$

$x(n) = \sum_{k=-\infty}^{\infty} x(k) - ax(x-1)\delta(n-k)$

Thus $c_k = x(k) - ax(k-1)$

(b) use the property of linearity and time invariance to express the output $y(n) = T[x(n)]$ in terms of

of the input $x(n)$ and signal $g(n) = \gamma[\delta(n)]$
where $\gamma[\cdot]$ is an $[\gamma]$ system

$$y(n) = \gamma[x(n)]$$
$$= \gamma\left[\sum_{K=-\infty}^{\infty} c_K \delta(n-K)\right]$$
$$= \sum_{K=-\infty}^{\infty} c_K \gamma[\delta(n-K)]$$
$$= \sum_{K=-\infty}^{\infty} c_K\, g(n-K)$$

(c) Express the impulse response $h(n) = \gamma[\delta(n)]$ in terms of $g(n)$

$$h(n) = \gamma[\delta(n)]$$
$$h(n) = \gamma[\delta(n) - a\delta(n-1)]$$
$$= g(n) - a\,g(n-1).$$

Determain the zero-input responce of the system described by the second order difference equation

$$x(n) - 3y(n-1) - 4y(n-2) = 0$$

with $x(n) = 0$

$$-3y(n-1) - 4y(n-2) = 0 \; [\div(-3)]$$
$$y(n-1) + \frac{4}{3}y(n-2) = 0$$

at $n = 0$

$$y(-1) = -\frac{4}{3}y(-2)$$

at $n = 1$

$$y(0) = -\frac{4}{3}y(-1) = \left(\frac{-4}{3}\right)^2 y(-2)$$
$$y(1) = \left(-\frac{4}{3}\right) y(-2)$$
$$\vdots$$
$$y(K) = \left(-\frac{4}{3}\right)^{K+2} y(-2)$$

→ zero input response

27) Determine the particular solution of the difference equation $y(n) = \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n)$ when the for using function is $x(n) = 2^n u(n)$

$y(n) = \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n)$

Charecteristic equations

$n^2 - \frac{5}{6} + 1/6 - 0$; $n = 1/2, 1/3$

so $y_h(n) = C_1(1/2)^n + C_2(1/3)^n$

$x(n) = 2^n u(n)$

so $K(2^n)u(n) - K(5/6)(2^{n-1})u(n-1) + K(1/6)u(n-2) = 2$

for $n=2$ $\quad 4K - \frac{5K}{3} + \frac{K}{6} = 4$

$$K = \frac{8}{5}$$

Total solution is

$y_p(n) + y_h(n) = y(n)$

$y(n) = \frac{8}{5}(2^n)u(n) + C_1(1/2)^n u(n) + C_2(1/3)^n u(n)$

Assume $y(-2) = y(-1) = 0$, so $y(0) = 1$

then $y(1) = 5/6\ y(0) + 2 = \frac{17}{6}$

so $\frac{8}{5} + C_1 + C_2 = 1$

$C_1 + C_2 = 3/5$ ——— ①

$\frac{16}{5} + \frac{1}{2}C_1 + 1/3\ C_2 = 17/6$

$3C_1 + 2C_2 = -11/5$

By solving 1 & 2

$C_1 = -1$, $C_2 = 2/5$

so the total solution is

$y(n) = \left[\frac{8}{5}(2)^n - (1/2)^n + 2/5(1/3)^n\right]u(n)$

28) In the given equation $y(n) = (-a_1)^{n+1}y(-1) + \frac{(1-(-a_1)^{n+1})}{1+a_1}$ for saparate output sequence $y(n)$ into the transient response steady-state response plot these response for $a_1 = -0.9$.

at $y(-1) = 1$

The given equation $y(n) = (-a)^{n+1} + \frac{(1-(-a)^{n+1})}{1+a}$

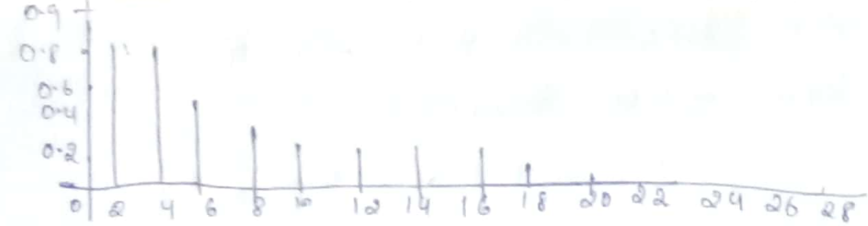
$y(n) = y_1(n) + y_2(n)$

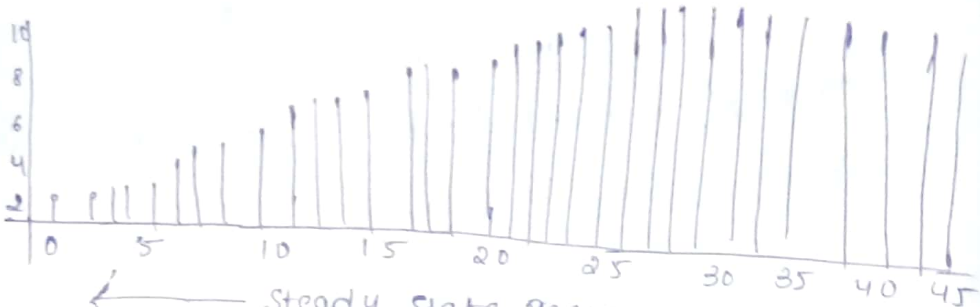
Transient + steady state

$y(0) = 0.9$

$y(1) = 0.87$

$y(2) = 0.71$





← Steady State response →

29) Determine the impulse $y(n)$ $5n \geq 0$ of the system described by the second order difference equation
$y(n) = 3y(n-1) - 4y(n-2) - x(n) + 2x(n-1)$ to the input $x(n) = 4$

$$h(n) = h_1(n) * h_2(n)$$

$$= \sum_{k=-\alpha}^{\alpha} a^k (u(k) u(k-N)] [u(N-k) - u(n-k-m)]$$

$$= \sum_{k=-\alpha}^{\alpha} a^k u(k) u(n-k)] - \sum_{k=-\alpha}^{\alpha} a^k u(k) u(n-k-m)$$

$$= \sum_{k=-\alpha}^{\alpha} a^k u(k-N) u(n-k) + \sum_{k=-\alpha}^{\alpha} a^k u(k-N) u(n-k-m)$$

$$= \left(\sum_{k=0}^{n} a^k - \sum_{k=0}^{n-m} a^k\right) - \left(\sum_{k=n}^{n} a^k - \sum_{k=N}^{n-m} a^k\right)$$

30) Determine the response $y(n)$ $5n \geq 0$ of the describe by the second order difference equation.
$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$ to the input $x(n) = 4^n$

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$

Charecteristic equation is

$$\lambda^2 - 3\lambda - 4 = 0$$

$$\lambda = 4, -1$$

so $y_\lambda(n) = C_1 4^n + C_2 (-1)^n$

$$x(n) = 4^n u(n)$$

$$y_p(n) = kn\, 4^n u(n)$$

$$kn 4^n u(n) - 3k(n-1) 4^{n-1} u(n-1) - 4k(n-2) 4^{n-2} u(n-2) =$$

$4^n u(n-1) - 2(4)^{n-1} u(n-1)$

for $n=2$ $k(32-2) = 4^2 + 8 = 24 \longrightarrow k = \frac{6}{5}$

The total solution is

$y(n) = y_p(n) + y_h(n)$

$= \left[\frac{6}{5} n 4^n + c_1 4^n + c_2 (-1)^n\right] u(n)$

to find c_1 and c_2 let $y(-2)$ & $y(-1) = 0$ then $y(0) = 1$

$y(1) = 3y(0) + 4 + 2 = 9$

$c_1 + c_2 = 1$ —— (1)

$\frac{24}{5} + 4c_1 - c_2 = 9 \Rightarrow 4c_1 - c_2 = \frac{21}{5}$ —— (2)

from (1) & (2)

$c_1 = \frac{26}{25}$ & $c_2 = 1/25$

so $y(n) = \left[\frac{6}{5} n 4^n + \frac{26}{25} 4^n - \frac{1}{25}(-1)^n\right] u(n)$

31) Determine the impulse response of the following caused systems $y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

Charecteristic equation $\lambda^2 - 3\lambda - 4 = 0$

$\lambda = -4, 1$

$y_h(n) = c_1 4^n + c_2 (-1)^n$

$x(n) = \delta(n)$

$y(0) = 1$ and $y(1) - 3y(0) = 2$

$y(1) = 3$

So $c_1 + c_2 = 1$ —— (1)

$4c_1 - c_2 = 5$

from (1) & (2) $c_1 = \frac{6}{5}$, & $c_2 = -1/5$

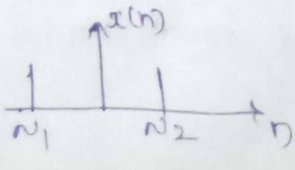
$h(n) = \left[\frac{6}{5} 4^n - 1/5 (-1)^n\right] u(n)$

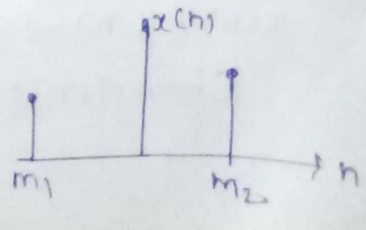
32) let $x(n)$, $N_1 \le n \le N_2$ and $h(n)$, $m_1 \le n \le m_2$ be two finite signals.

a) Determine the range $L_1 \le n \le L_2$ of their convertors terms of N_1, N_2, m_1 and m_2

$L_1 = N_1 + m_1$

$L_2 = N_2 + m_2$

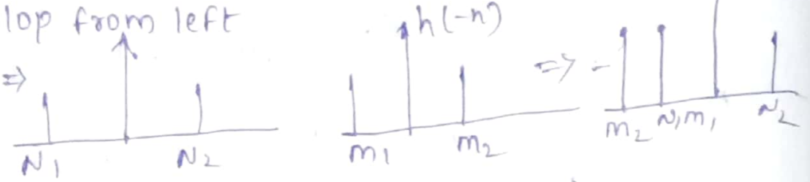




b) Determine the limits of the cases of partial overlop from the left, full overlop and partial overlop from the right convince assume that $h(n)$ has shortest duration on than $x(n)$.

partial overlop from left

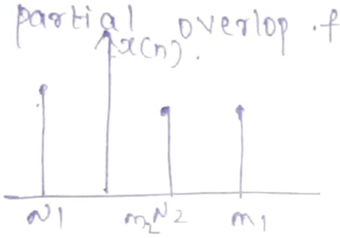
$\to x(n) * h(n) \Rightarrow$



low $N_1 + m_1$ & high $m_2 + N_1 - 1$

If fully overlop then $N_1 + m_2$ (low) & high $N_2 + m_1$

partial overlop from right



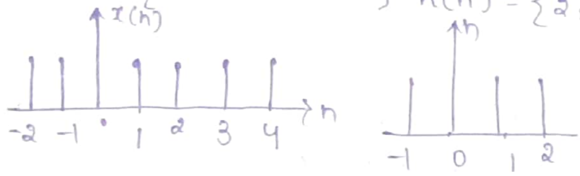
low $\to N_2 + m_1 + 1$

high $\to N_2 + m_2$

If fully overlopped high $N_2 + m_1$; low $= N_1 + m_2$

(c) illutrate the validity of your results, by computing the convolution of the signal $x(n) = \begin{cases} 1, & -2 \le n \le 4 \\ 0, & \text{elsewhere} \end{cases}$

$x(n) = \{1, 1, 1, 1, 1, 1, 1\}$ $h(n) = \{2, 2, \underset{\uparrow}{2}, 2\}$

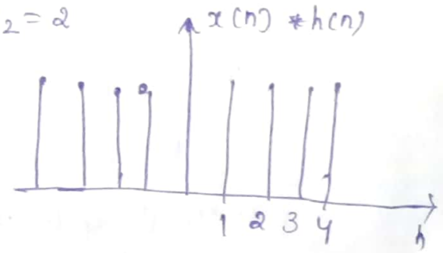


$N_1 = 2,\ N_2 = 4$ $m_1 = -1,\ m_2 = 2$

partial overlop from that

low $N_1 + m_1 = -3$

high $m_2 + N_1 - 1 = -2 - 2 - 1 = 1$



full overlap $m = 0,\ n = 3$

partial right $n = 4, n = 6, h_2 = 6$

33) Determine the impulse response and described by the difference equation.

(a) $y(n) = 0.6\,y(n-1) - 0.08\,y(n-2) + x(n)$

$x(n) = y(n) - 0.6\,y(n-1) - 0.08\,y(n-2)$

Charecteristics equation

$\lambda^2 - 0.6\lambda + 0.08 = 0$

$\lambda = 1/2, 2/3$

$y_h(n) = c_1(1/5)^n + c_2(2/5)^n$

Impulse response $x(n) = \delta(n)$ with $y(0) = 1$

$y(1) - 0.6\,y(0) = 0 \Rightarrow y(1) = 0.6$

so $c_1 + c_2 = 1$ —— ①

$1/5\, c_1 + 2/5\, c_2 = 0.6$ —— ②

from ① & ② $\cdot$ $c_1 = -1$, $c_2 = 3$

$h(n) = \left[-(1/5)^n + 2(2/5)^n\right] u(n)$

Step response $x(n) = u(n)$

$s(n) = \sum_{k=0}^{n} h(n-k), \; n \geq 0$

$\sum_{k=0}^{n} \left(2\left(\frac{2}{5}\right)^{n-k} - (1/5)^{n-k}\right)$

$= 2\left(\frac{2}{5}\right)^n \sum_{k=0}^{n} \left(\frac{5}{2}\right)^k - \left(\frac{1}{5}\right)^n$

$= 2\left(\frac{2}{5}\right)^n \left(5/2\right)^{n+1} - 1$

b) $y(n) = 0.7\,y(n-1) - 0.1\,y(n-2) + 2x(n) - x(n)$

$2x(n) - x(n-2) = y(n) . 0.7\,y(n-1) + 0.1\,y$

Charecteristics equations

$\lambda^2 - 0.7\lambda + 0.1 = 0$

$\lambda = 1/2, 1/5$

$y_h(n) = c_1(1/2)^n + c_2(1/5)^n$

Impulse response $x(n) = \delta(n)$, $y(0) = 2$

$y(1) - 0.7\,y(0) = 0 \Rightarrow y(1)$

$c_1 + c_2 = 2$

$1/2\, c_1 + 1/5\, c_2 = \frac{7}{5}$ —— ①

$c_1 + \frac{2}{5} c_2 = \frac{14}{5}$ —— ②

Solving ① & ②

$c_1 = \frac{10}{3}$, $c_2 = \frac{-4}{3}$

so $h(n) = \left[\frac{10}{3}(1/2)^n - \frac{4}{3}(1/5)^n\right] u(n)$

step response $s(n) = \sum_{k=0}^{n} h(n-k)$

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$$= \frac{10}{3}\sum_{k=0}^{n}\left(\frac{1}{2}\right)^{n-k} - \frac{4}{3}\sum$$

$$= \frac{10}{3}\left[\frac{1}{2}^{n}\left(2^{n+1}-1\right) - u(n)\right]u(n)$$

34) consider a system with impulse response

$$h(n) = \begin{cases} (1/2)^n & ; 0 \le n \le 4 \\ 0 & ; \text{elsewhere} \end{cases}$$

Determine the input $x(n)$ for $0 \le n \le 2$ the output sequence $y(n) = \{1, 2, 2, 5, 3, 3, 3, 2, 1, 0\}$

$h(n) = \{1, 1/2, 1/4, 1/8, 1/16 \ldots\ldots\}$

$y(n) = \{\underset{\uparrow}{1}, 2, 2, 5, 3, 3, 8, 2, 1, 0 \ldots\}$

$y(0) = x(0), h(0)$

$y(0) = x(0)\cdot 1 \Rightarrow x(0) = 1$

$y(1) = x(1) + h(1)\,x(0)$

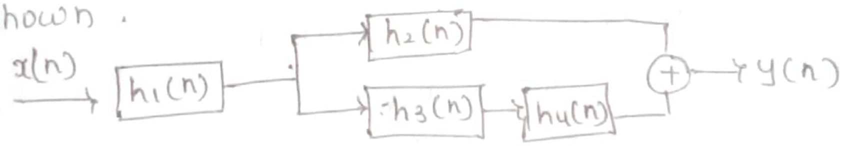
$2 = x(1) + 1/2\,(0) = x(1) = 3/2$

$y(2) = x(2) + h(2)\,x(1) + h_1 x(0)$

$2.5 = x(2) + 1/4\,(3/2) + 1/2\,(1)$

so $x(n) = \{1, 3/2, 3/2, 7/4, 3/2 \ldots\}$

35) consider the interconnection of LTI system as shown.



Express the overall impulse response in terms of $h_1(n)$, $h_3(n)$ and $h_4(n)$.

$$h(n) = h_1(n) * \left[h_2(n) - \{h_3(n) * h_4(n)\}\right]$$

b) Determine $h(n)$ when $h_1(n) = \{1/2, 1/4, 1/2\}$

$$h_2(n) = h_3(n) = (n+1)u(n)$$

$$h_4(n) = \delta(n-2)$$

$$h_3(n) * h_4(n) = (n+1)u(n) * \delta(n-2)$$

$$= (n+1)u(n+2) = (n+1)u(n-2)$$

$$h_2(n) = \left[h_3(n) * h_4(n)\right] = (n+1)u(n) - (n+1)u(n)$$

$$= 2u(n) - \delta(n)$$

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$h(n) = [\frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)] * 2(u(n)$

$= \frac{1}{2}\delta(n) + \frac{5}{4}\delta(n-1) + 2\delta(n-2) + \frac{5}{4}u(n-3)$

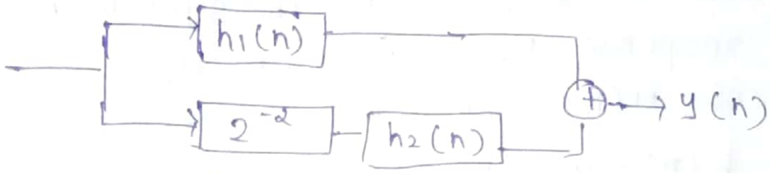
36) Determine the response of the system in part(b

$x(n) = \delta(n+2) + 3\delta(n) + 4\delta(n-3)$

$x(n) = \{1, 0, 0, 3, 0, -4\}$

consider the system fig with $h(n) = a^n u(n)$, $-1 < a < 1$ response $y(n)$ of the system to the existation.

$x(n) = u(n+5) - u(n-1)$



$$= \sum_{k=0}^{\infty} h(n-k)$$

$$= \sum_{k=0}^{\infty} a^{n-k}$$

$$= \frac{a^{n+1} - 1}{a-1}, \quad n \geq 0$$

for $x(n) = u(n+5), -u(n-10)$ then

$\delta(n+5) - \delta(n-10) = \frac{a^{n+6} - 1}{a-1} u(n+5) = \frac{a^{3-0}}{a-}$

from given figure $y(n) = x(n) * h(n) - x(n) * h(n)$

$$y(n) = \frac{a^{n+6} - 1}{a-1} u(n+9) - \frac{a^{n-9} - 1}{a-1} u(n-10)$$

$$\frac{a^{n+4} - 1}{a-1} u(n+3) + \frac{a^{n+1} - 1}{a-1} u(n-2)$$

37) compute and sketch step response of the system

$$y(n) = \frac{1}{m} \sum_{k=0}^{m-1} x(n-k)$$

$$h(n) = \left[\frac{u(n) - u(n-m)}{m}\right]$$

$$S(n) = \sum_{k=\infty}^{\infty} u(k)\, h(n-k)$$

$$= \sum_{k=0}^{n} h(n-k) = \begin{cases} \frac{n+1}{m}, & n < m \\ 1, & n \geq m \end{cases}$$

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38) Determine the range of values of the parameter the linear time-invariant system with impulse response $h(n) = \begin{cases} a^n; & n \geq 0 \text{ even} \\ 0; & \text{elsewhere} \end{cases}$ is stable

$$\sum_{K=-\alpha}^{\alpha} |h(n)| = \sum_{\substack{K=\alpha \\ n=\text{even}}}^{\alpha} |a|^n$$

$$= \sum_{n=0}^{\alpha} |a|^{2n} = \frac{1}{1-|a|^2}$$

Stable if $|a|<1$

39) Determine the response of the system with impulse response $h(n) = a^3 u(n)$ to i/p signal $x(n) = u(n) = u(n-10)$

$$h(n) = a^3 u(n)$$

$$y_1(n) = \sum_{K=0}^{\alpha} u(K) h(n-K)$$

$$= \sum_{K=0}^{n} a^{n-K}$$

$$= a^m \sum_{K=0}^{n} a^{-K}$$

$$= \frac{1-a^{n+1}}{1-a} u(n)$$

$$y(n) = y_1(n) - y_1(n-10)$$

$$= \frac{1}{1-a}\left[(1+a^{3+1}) u(n) - (1-a^{n-9}) u(n-10)^3\right]$$

40) Determine the response of the relaxed system charecteristics the impulse response : $h(n) = (1/2)^n u(n)$ to the input signal.

$$x(n) = \begin{cases} 1, & 0 \leq n \leq 10 \\ 0, & \text{otherwise} \end{cases}$$

from 36 problem with $a = 1/2$

$$y(n) = 2\left[\left(1 - 1/2\right)^{n+1}\right] u(n) - 2\left[1 - (1/2)^{n-9}\right] u(n-10)$$

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41) Determine the response of system charecterized by response $h(n) = (1/2)^n u(n)$ to the input signal

$x(n) = 2^n u(n)$

$$y(n) = \sum_{k=-\alpha}^{\alpha} h(k) x(n-k)$$

$$= \sum_{k=0}^{n} (1/2)^k 2^{n-k}$$

$$= 2^n \left(1-(1/4)^{n+1}\right) \cdot \left[4/2\right]$$

$$= \frac{2}{3} \left[2^{n+1} - (1/2)^{n+1}\right] u(n)$$

(b) $x(n) = u(-n)$

$$y(n) = \sum_{k=-\alpha}^{\alpha} h(k) x(n-k)$$

$$= \sum_{k=0}^{\alpha} h(k) = \sum_{k=0}^{\alpha} (1/2)^k = 2, n<0$$

$$y(n) = \sum_{k=n}^{\alpha} h(k) = \sum_{k=n}^{\alpha} (1/2)^k$$

$$= \sum_{k=0}^{\alpha} (1/2)^k - \sum_{k=0}^{n+1} (1/2)^k$$

$$= 2 - \frac{(1-(1/2)^n)}{1/2}$$

$$= 2(1/2)^n, n \geq 0$$

42) Three systems with impulse response $h(n) = \delta(n), \delta(n-1)$, $h_2(n) = h(n)$ and $h_3(n) = u(n)$ are connected in cascade

a) What is the impulse response $h_c(n)$ of the overall system,

$$h_c(n) = h_1(n) * h_2(n) * h_3(n)$$

$$= \delta(n) - \delta(n-1) * u(n) * h(n)$$

$$= [u(n) - u(n-1)] * h(n)$$

$$= \delta(n) * h(n) = h(n)$$

b) Does the order of inter connection offect the overlap

3 43) a) prove and explain graphical the differences relations $x(n)\,\delta(n-n_0) = x(n_0)\,\delta(n-n_0)$ and

$x(n) * \delta(n-n_0) = x(n-n_0)$

$x(n)\,\delta(n-n_0) = x(x_0)$ Thuse only the value of invariant

$x(n) * \delta(n-n_0) = x(n-n_0)$ Thus we obtain shiftting

$x(n)$ sequency

44) Show that a discrete time system with is described convolution summation, is LTI and related

$$y(n) = \sum_{k=-\alpha}^{\alpha} h(k)\,x(n-k)$$

$$= h(n) * x(n)$$

linearily: $x_1(n) \to y_1(n) = h(n) * x_1(n)$

$x_2(n) \to y_2(n) = h(n) * x_2(n)$

$\alpha h(n) + x_1(n-1)\ \beta h(n) * x_e(n)$

$= \alpha y_1(n) + \beta y_2(n)$

Time invariance

$x(n) \to y_1(n) = h(n) * x(n)$

$x(n-n_0) \to y_1(n) = h(n) * x(n-n_0)$

$$= \sum_k h(k)\,x(n-n_0-k)$$

$$= y(n-n_0)$$

45) What is the impulse response of the system described $n(n) = \delta(n-n_0)$

$y(n) = x(n-n_0)$

46) compute the zero state response of the system consider by difference equation $y(n) + 1/2\,y(n-1)$ $= x(n) - 2x(n-2)$ input $x(n) = \{1,2,3,4,2,1\}$ by

Solving the differential equation

$y(n) = -\frac{1}{2}\,y(n-1) + x(n_1) + 2x(n-2)$

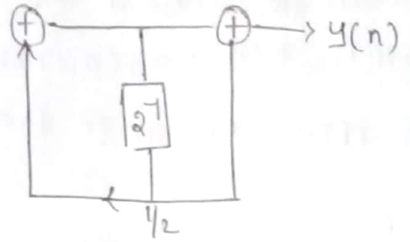
at $y(-2) = -1/2\,y(-3) + x(-2) + 2x(-4) = 1$

$y(-1) = -1/2\,y(-2) + x(-1) + 2x(-3) = 3/2$

$y(0) = -1/2\,y(-1) + 2x(-2) + x(0) = \frac{17}{4}$

$y(1) = -1/2\,y(0) + x(1) + 2x(-1) = \frac{47}{8}$

47) consider the discrete time system in fig



cumpute the 10 first samples of its impulse response

b) find the input -output relation.

apply the input $x(n)=\{1,1,1\ldots\}$ and compute the first samples of the output

d) compute the first 10 samples of the output for the input given in part c) by using convolusion

e) is the system caused ? it is stable ?

$x(n)=\{\underset{\uparrow}{1},0,0\ldots\}$

$y(n)=1/2\,y(n-1)+x(n)+x(n-1)$

$y(0)=x(0)=1$

$y(1)=1/2\,y(1)+x(2)+x(1)=3/4$ thus we obtain

$y(n)=\{1,3/2,3/4,3/8,3/16,3/32\ldots\}$

b) $y(n)=1/2\,y(n-1)+x(n)+x(n-1)$

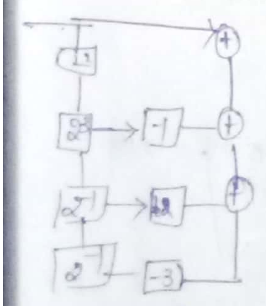
c) as in part a we obtain

$y(n)=\{1,5/2,13/4,29/8,\frac{61}{16}\ldots\}$

d) $y(n)=u(n)*h(n)$

$=\sum_{K}u(K)\,h(n-15)$

$=\sum_{K=0}^{n}h(n-K)$



$y(0)=h(0)=1$

$y(1)=h(0)+h(1)=5/2$

$y(2)=h(0)+h(1)+h(2)=13/4$ etc

from porat (a), $h(n)=0$ for $n<0\Rightarrow$ the system caused

$\sum_{n=0}^{\infty}h(1)=1+3/2(1+1/2+1/4+\ldots)=9=4$. system is stable

48) Consider the system described by the differential equation $y(n) = ay(n-1) + bx(n)$

a) determine b in terms of a so that

$$\sum_{n=-\alpha}^{\alpha} h(n) = 1$$

b) compute the zero-state step response $s(n)$ of the system and choose b so that $s(\alpha) = 1$

c) campare the values of b obtained in parts (a) and (b) what did you notice

$$y(n) = ay(n-1) + bx(n)$$

$$h(n) = ba^n u(n)$$

$$\sum_{n=0}^{\alpha} h(n) = \frac{b}{1-a} = 1$$

$$b = 1-a$$

b) $$s(n) = \sum_{k=0}^{n} h(n-k)$$

$$= b\left[\frac{1-a^{n+1}}{1-a}\right]u(n)$$

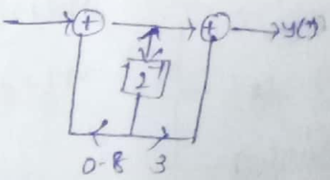
$$b = 1-a$$

c) $b = 1-a$ in both the cases

49) A discrete time system is relized by the structure shown in fig 2.49

a) Determine the impulse response

b) Determine a realization for its inverse system that is the system which produces $x(n)$ as an output when $y(n)$ is used as input



$$y(n) = 0.8\,y(n-1) + 2x(n) + 3x(n-1)$$

$$y(n) - 0.8\,y(n-1) = 2x(n) + 3x(n-1)$$

The charecteristic equation is

$$\lambda - 0.8 = 0$$

$$\lambda = 0.8$$

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$y_h(n) = c(0.8)^n$

let by first consider the response of the system
$y(n) - 0.8y(n-1) = x(n)$
to $x(n) = \delta(n)$ Since $y(0) = 1$, it follows that $c = 1$
Then the impulse response of the original
System is $h(n) = 2(0.8)^n u(n) + 3(0.8)^{n-1} u(n-1)$
$= 2\delta(n) + 4.6\,(0.8)^{n-1} u(n-1)$

b) The inverse system is characterized by the difference equation

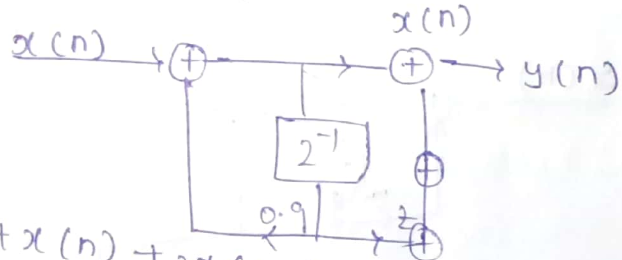
$$x(n) = 0 - 1.5x(n-1) + 1/2\, y(n) - 0.4y(n-1)$$

50) consider the discrete time system shown below

a) compute the first six values of the $x(n)$ impulse response of the System.

b) compute the first six values of the zero state step response of the system

c) Determine an analytical expression for the impulse response of the system

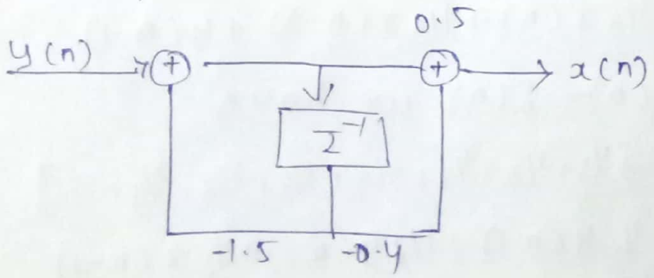


$y(n) = 0.9y(n-1) + x(n) + 2x(n-1) + 3x(n-2)$

$y(n) = 0.9y(n-1) = x(n) + 2x(n-1) + 3x(n-2)$

a) for $x(n) = \delta(n)$ we have

$y(0) = 1$, $y(1) = 2.9$, $y(2) = 5.61$, $y(3) = 5.049$
$y(4) = 4.544$,



b) $s(0) = y(0) = 1$
$s(1) = y(0) + y(1) = 3.9$
$s(2) = y(0) + y(1) + y(2) = 9.51$

$S(3) = y(0) + y(1) + y(2) + y(3)$

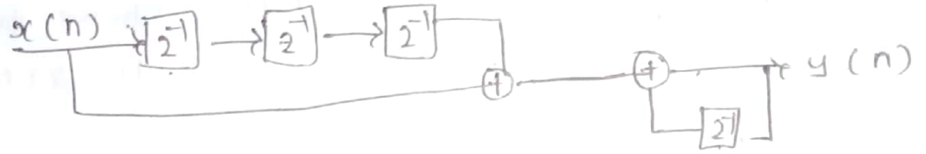
$S(4) = \sum_{0}^{4} y(n) = 19.10$

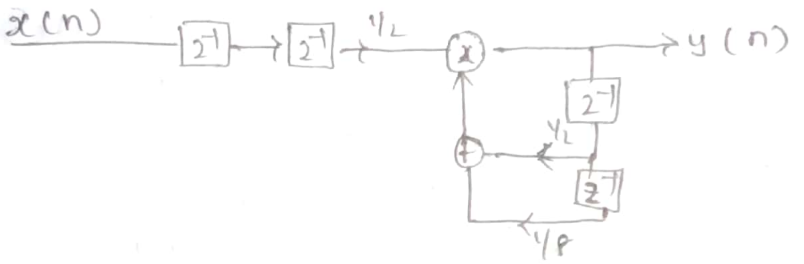
$S(5) = \sum_{0}^{5} y(n) = 23.19$

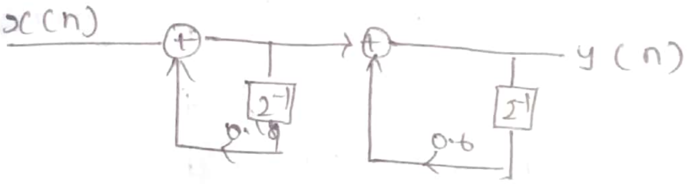
c) $h(n) = (0.9)^n u(n) + 2(0.9)^{n-1} u(n-1) + 3(0.9)^{n-2} u(n-2)$

$= \delta(n) + 2.9\,\delta(n-1) + 5.61(0.9)^{n-2} u(n-2)$

51) Determine and sketch the impulse response of the following systems for $n = 0, 1, \ldots, 9$







a) Fig P.2.5 (a) b) Fig P2.5 b) c) Fig P.251 (c)

b) Classify the systems above as FIR or IIR

c) Find an explain expression for the impulse response of the systems in part (c)

a) $y(n) = 1/3\, x(n) + 1/3\, x(n-3) + y(n-1)$

for $x(n) = \delta(n)$ we have

$h(n) = \{1/3, 1/3, 1/3, 2/3, 2/3, 2/3, 2/3 \ldots\}$

b) $y(n) = 1/2\, y(n-1) + 1/8\, y(n-2) + 1/2\, x(n-2)$

with $x(n) = \delta(n)$ and

$y(-1) = y(-2) = 0$ we obtain

53) All three systems are IIR

$y(n) = 1.4\, y(n-1) - 0.48\, y(n-2) + x(n)$

The charecteristics equations

$\lambda^2 - 1.4\lambda + 0.48 = 0$ Hence

$\lambda = 0.48, 0.6$ and

$y_h(n) = C_1(0.8)^n + C_2(0.6)^n$ for $x(n) = \delta(n)$ we have

$C_1 + C_2 = 1$ and

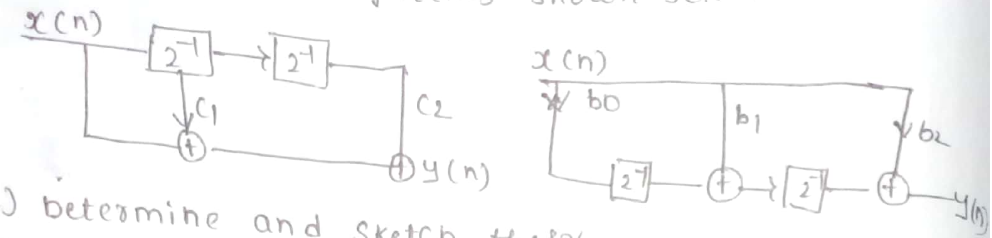
$0.8 4 + 0.6 C_2 = 1.4$

$C_1 = 4$

$C_2 = -3$

$h(n) = [4(0.8)^n - 3(0.6)^n] u(n)$

54) consider the systems shown below



a) Determine and sketch their impulse response $h_1(n)$, $h_2(n)$ and $h_3(n)$

b) Is it possible to choose the cofficient of these systems in such a way that

$h_1(n) = h_2(n) = h_3(n)$

a) $h_1(n) = c_0\,\delta(n) + c_1\,\delta(n-1) + c_2\,\delta(n-2)$

$h_2(n) = b_2\,\delta(n) + b_1\,\delta(n-1) + b_0\,\delta(n-2)$

$h_3(n) = a_0\,\delta(n) + (a_1 + a_0 a_2)\,\delta(n-1) + a_1 a_2\,\delta(n-2)$

$h_3(n) = h_2(n) = h_1(n)$

let $a_0 = c_0$

$a_1 + a_2 c_0 = c_1 \Rightarrow a_1 + a_2 c_0 = 0$

$a_2 a_1 = c_2 \Rightarrow \frac{a_2}{a_2} = 9$

$\frac{c_2}{a_2} + a_2 c_0 - c_1 = 0$

$\Rightarrow c_0 a_2^2 - c_1 a_2 + c_2 = 0$

for $c_0 \neq 0$ the quadratic has a real solution if and if $c_1^2 - 4 c_0 c_2 \geq 0$

55) The zero static response of a caused LTI systems to the input $x(n) = \{1, 3, 3, 1\}$ is $y(n) = \{1, 4, 6, 4, 1\}$ Determine the impulse response $x(n) * y(n) = h(n)$

log lengths of $h(n) = 2$, $h(n) = \{h_0, h_1\}$

$$h(n) = \{h_0, h_1\}$$

$$h_0 = 1$$

$$3h_0 + h_1 = 4$$

$$\Rightarrow h_0 = 1, h_1 = 1$$

56) prove by direct substitution the equivalence of equation (2,5,9) and (2,5,6) which describes the direct from II structure to the relation (2,5,6) which describes the dereat from I structure

$$(2,5,6)\quad y(n) = \sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{m} b_k x(n-k)$$

$$(2,5,9)\quad \omega(n) = -\sum_{k=1}^{n} a_k \omega(n-k) + x(n)$$

$$(2,5,10)\quad y(n) = \sum_{k=0}^{m} b_k \omega(n-k)$$

from 2,5,9 we obtain $x(n) = \omega(n) + \sum_{k=1}^{N} a_k \omega(n-k)$

by substituting (2,5,10) for $y(n)$ and (A) into (2,5,6) we obtain L.H.S = R.H.S

57) Determine the response $y(n)$, $n \geq 0$ of the system described by the second-order difference equation $y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$ when the input is $x(n) = (-1)^n u(n)$ and intial conditions are $y(-1) = y(2) < 0$

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-2)$$

The charecteristic equation

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda = 2, 2 \quad \text{Hence}$$

$$y_h(n) = c_1 2^n + C_2 n_2^n$$

The particular solution is

$$y_p(n) = k(-1)^n u(n)$$

Substituting this solution into the difference equation we obtain

$$k(-1)^n u(n) - 4k(-1)^{n-1} u(n-1) + 4k(-1)^{n-2} u(n-2)$$

$$= (-1)^n u(n) - (1)^{n-1} u(n-1)$$

for $n=2$, $k(1+4+4)=2 \Rightarrow k=\frac{2}{9}$ total solution is

$$y(n) = \left[c_1 2^n + c_2 n_2^n + \frac{2}{9}(-1)^n\right] u(n)$$

from the initial conditions we obtain

$$y(0)=1,\ y(1)=2,$$

$$c_1 + \frac{2}{9} = 1$$

$$\Rightarrow c = \frac{7}{9}$$

$$2c_1 + 2c_2 - \frac{2}{9} = 2$$

$$c_2 = 1/3$$

58) Determine the impulse response $h(n)$ for the system the second-order difference equation

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

$$h(n) = \left[c_1 2^n + c_1 n_2^n\right] u(n)$$

with $y(0)=1$, $y(1)=3$; we have

$$c_1 = 1,\ 2c_1 + 2c_2 = 3$$

$$c_2 = 1/2$$

Thuse $h(n) = \left[2^n + 1/2\, n 2^n\right] u(n)$

59) show that only discrete time signal $x(n)$ can

$$x(n) = \sum_{k=-\alpha}^{\alpha} \left[x(k) - x(k-1)\right] u(n-k)$$

where $u(n-k)$ is a unit skip delayed by k we have that is

$$u(n-k) = \begin{cases} 1, & n \geq k \\ 0, & \text{otherwise} \end{cases}$$

$$x(n) = x(n) * \delta(n)$$
$$= x(n) * [u(n) - u(n-1)]$$
$$= [x(n) - x(n-1) * u(n)]$$
$$x(n) = \sum_{k=-\alpha}^{\alpha} [x(k) - x(k-1)] u(n-k)$$

60) show that the output of an LTI system can be expressed in terms of units' step response $s(n)$ as follow.

$$y(n) = \sum_{k=-\alpha}^{\alpha} [s(k) - s(k-1)] x(n-k)$$

$$\sum_{k=-\alpha}^{\alpha} [x(k) - x(k-1)] s(n-k)$$

let $h(n)$ be the impulse response of the system.

$$s(k) = \sum_{m=-\alpha}^{\alpha} h(m)$$

$$h(k) = s(k) - s(k-1)$$

$$y(n) = \sum_{k=-\alpha}^{\alpha} h(k) x(n-k)$$

$$= \sum_{k=-\alpha}^{\alpha} [s(k) - s(k-1)] x(n-k)$$

61) $x(n) = \begin{cases} 1, & -N \leq n \leq N \\ 0, & \text{otherwise} \end{cases}$

$$r_{xx}(l) = \sum_{n=-\alpha}^{\alpha} x(n) x(n-l)$$

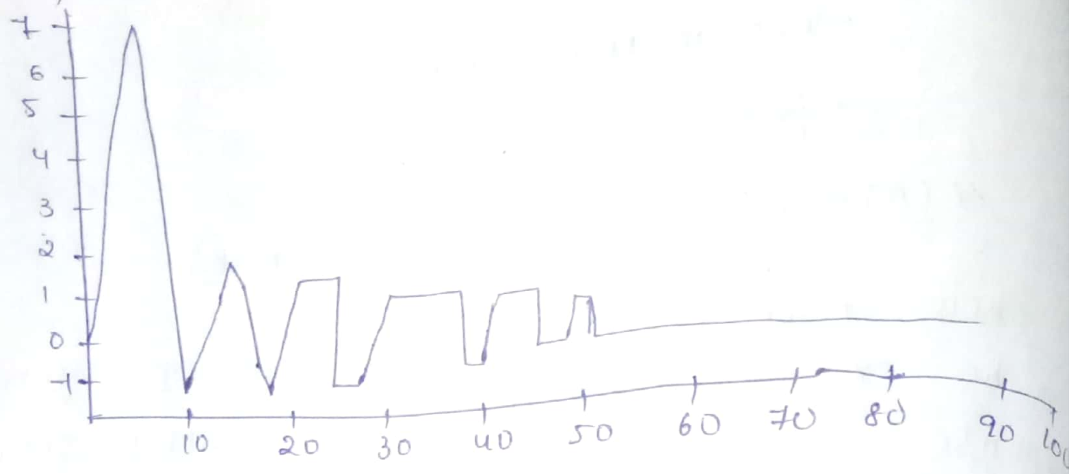
$$= \begin{cases} 2N+1, (l) & , -2N \leq l \leq 2N \\ 0 & ; \text{else} \end{cases}$$

$$r_{xx}(0) = 2N+1$$

The normalised auto connections

$$\rho_{xx}(l) = \begin{cases} \frac{1}{2N+1}(2N+1\ (l)) & ; -2N \leq l \leq 4 \\ 0 & , \text{else} \end{cases}$$

62)



63) $x(n) = \sum_{k=\alpha}^{\alpha} [x(k) - 2(k-1)] u(n-k)$ where

$u(n-6)$ delogs by k unit

$$u(n-1) = \begin{cases} 1, & n \geq k \\ 0, & \text{otherwise} \end{cases}$$

A) $x(n) = x(n) * \delta(n)$

$= x(n) * [u(n) - u(n-1)]$

$= [x(n) - x(n-1)] * u(n)$

$$x(n) = \sum_{k=\alpha}^{\alpha} [x(k) - x(k-1)] u(n-k)$$

64) $x(n) = \{\underset{\uparrow}{1}, 3, 3, 1\}$; $y(m) = \{\underset{\uparrow}{1}, 4, 6, 4, 1\}$

$x(n) * y(n)$, $h(n)$

lengthed $h(n) = 2$ $h(n) = \{h_0, h_1\}$

$h_0 = 1$

$3h_0 + h_1 = 4$

$\Rightarrow h_0 = 1$, $h_1 = 1$