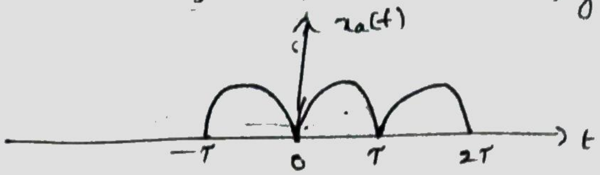
Assignment - 2

1) Consider the full-wave rectified sinusoid in fig



a) Determine its spectrum $X_a(F)$

(b) Compute the PSD (c) Plot the PSD

(d) Check the validity of Parseval's relation for this signal.

Sol

$$x_a(t) = \sum_{k=-\infty}^{\infty} c_k e^{-j2\pi kt/T}$$

where $c_k = \frac{1}{T}\int_0^T A\sin\left(\frac{\pi t}{T}\right) e^{\frac{-j2\pi kt}{T}}\, dt$

$$= \frac{A}{j2T}\int_0^T \left[e^{j\pi t/T} - e^{-j\pi t/T}\right] e^{\frac{-j2\pi kt}{T}}\, dt$$

$$= \frac{A}{j2T}\left[\frac{e^{j\pi(1-2k)t/T}}{\frac{j\pi}{T}(1-2k)} - \frac{e^{-j\pi(1+2k)t/T}}{-j\frac{\pi}{T}(1+2k)}\right]_0^T$$

$$\frac{A}{j2T}\left[\frac{Te^{j\pi(1-2k)}}{j\pi(1-2k)} + \frac{Te^{-j\pi(1+2k)}}{j\pi(1+2k)} - \left[\frac{T}{j\pi(1-2k)} + \frac{T}{j\pi(1+2k)}\right]\right] = \frac{A}{j2T}\left[\frac{-2T}{j\pi(1-2k)} - \frac{2T}{j\pi(1+2k)}\right]$$

$$= \frac{A}{\pi j T}\left[jT\left(\frac{1}{1-2k} + \frac{1}{1+2k}\right)\right] = \frac{A}{\pi}\left[\frac{1}{1-2k} + \frac{1}{2+2k}\right]$$

$$= \frac{2A}{\pi(1-4k^2)}$$

Then $X_a(f) = \int_{-\infty}^{\infty} x_a(t)\, e^{-j2\pi\left(F-\frac{k}{T}\right)t}\, dt$

$$= \sum_{k=-\infty}^{\infty} c_k \int_{-\infty}^{\infty} e^{-j2\pi\left(F-\frac{k}{T}\right)t}\, dt$$

$$= \sum_{k=-\infty}^{\infty} c_k\, \delta\left(F - \frac{k}{T}\right)$$

b) $P_x = \frac{1}{T}\int_0^T x_a^2(t)\,dt = \frac{1}{T}\int_0^T A^2 \sin^2\left(\frac{\pi t}{T}\right) dt = \frac{A^2}{2}$

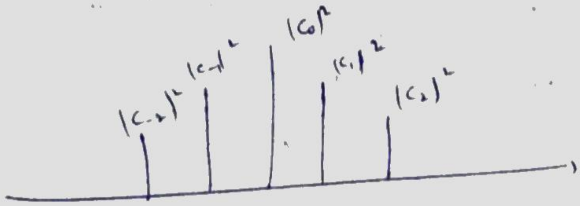
c) The power spectral density spectrum is $|c_k|^2$, $k = 0, \pm 1, \pm 2, \ldots$

d) Parseval's relation -

$$P_x = \frac{1}{T}\int_0^T x_a^2(t)\,dt$$

$$|c_k|^2$$

$$\sum_{k=-\infty}^{\infty} |c_k|^2 = \frac{4A^2}{\pi^2} \sum_{k=-\infty}^{\infty} \frac{1}{(4k^2-1)^2}$$



$$= \frac{4A^2}{\pi^2}\left[1 + \frac{2}{3^2} + \frac{2}{15^2} + \ldots\right]$$

$$\left[1 + \frac{2}{3^2} + \frac{2}{15^2} + \ldots\right] = 1.2337 \simeq \frac{\pi^2}{8}$$

Hence - $\sum_{k=-\infty}^{\infty} |c_k|^2 = \frac{4A^2}{\pi^2}(1.2337)$

$$= \frac{A^2}{4}.$$

2) Compute the Magnitude and phase spectrum for the signals

(a) $x_a(t) = \begin{cases} Ae^{-at}, & t \geq 0 \\ 0, & t < 0 \end{cases}$

Sol)

$x_a(t) = Ae^{-at}\,u(t)$, $a > 0$

$$X_a[f] = \int_0^{\infty} Ae^{-at}\, e^{-j2\pi Ft}\, dt$$

$$= \left[\frac{A}{-a\ 2\pi F_j}\, e^{-(a+j2\pi F)t}\right]_0^{\infty}$$

$$X_a(F) = \frac{A}{a+j2\pi F}$$

$$|X_a(F)| = \frac{A}{\sqrt{a^2+(2\pi F)^2}}$$

$$\angle X_a(F) = -\tan^{-1}\left(\frac{2\pi F}{a}\right)$$

b) $x_a(t) = A e^{-a|t|}$

$$X_a(F) = \int_{-\infty}^{0} A e^{at} e^{-j2\pi F t} dt + \int_{0}^{\infty} A e^{-at} \cdot e^{-j2\pi F t} dt$$

$$= \frac{A}{a-j2\pi F} + \frac{A}{a+j2\pi F} = \frac{2aA}{a^2+(2\pi F)^2}$$

$$|X_a(F)| = \frac{2aA}{a^2+(2\pi F)^2}$$

$$\angle X_a(F) = 0$$

3) $$x(t) = \begin{cases} 1-|t|/T, & |t| \leq T \\ 0, & \text{elsewhere} \end{cases}$$

a) Determine $|X_a(F)|, \angle X_a(F)$

Sol

$$x(t) = \begin{cases} 1-\frac{|t|}{T}, & |t| \leq T \\ 0, & \text{otherwise} \end{cases}$$

$$X_a(F) = \int_{-T}^{0} \left(1+\frac{t}{T}\right) e^{-j2\pi F t} dt + \int_{0}^{T} \left(1-\frac{t}{T}\right) e^{-j2\pi F t} dt$$

Alternatively, we find fourier transform

$$y(t) = x'(t) = \begin{cases} \frac{1}{T} & -T < t \leq 0 \\ -\frac{1}{T} & 0 < t \leq T \end{cases}$$

$$Y(F) = \int_{-T}^{T} y(t)\, e^{-j2\pi F t}\, dt$$

$$= \int_{-T}^{0} \frac{1}{T} e^{-j2\pi F t}\, dt + \int_{0}^{T} -\frac{1}{T} e^{-j2\pi F t}\, dt$$

$$= -\frac{2\sin^2 \pi F T}{j\pi F T}$$

and $$X(F) = \frac{1}{j2\pi F} Y(F)$$

$$= T\left(\frac{\sin \pi F T}{\pi F T}\right)^2$$

$$|X(F)| = T\left(\frac{\sin \pi F T}{\pi F T}\right)^2$$

$$\angle X_a(F) = 0$$

(b)

$$c_k = \frac{1}{T_p}\int_{-T_p/2}^{T_p/2} x(t)\, e^{-j2\pi k t/T_p}\, dt.$$

$$= \frac{1}{T_p}\left[\int_{-T}^{0} \left(1+\frac{t}{T}\right) e^{-j2\pi k t/T_p}\, dt + \int_{0}^{T} \left(1-\frac{t}{T}\right) e^{-j2\pi k t/T_p}\, dt\right]$$

$$c_k = \frac{T}{T_p}\left(\frac{\sin \pi k T/T_p}{\pi k T/T_p}\right)^2$$

$$c_k = \frac{1}{T_p} X_a\left(\frac{k}{T_p}\right)$$

Q) Consider

$$x(n) = \{ \ldots, 1, 0, 1, 2, 3, 2, 1, 0, 1, \ldots \}$$

(a) Sketch $x(n)$, its magnitude & phase spectra.

(b) Verify Parseval's relation by computing power in time & frequency domain

Sol

$$x[n] = \{ \ldots, 1, 0, 1, 2, \underset{\uparrow}{3}, 2, 1, 0, 1, \ldots \}$$

$N = 6$

$$C_k = \frac{1}{6} \sum_{n=0}^{5} x(n)\, e^{-j2\pi kn/6}$$

$$= \left[3 + 2e^{-\frac{j2\pi k}{6}} + e^{-\frac{j2\pi k}{3}} + e^{-\frac{4\pi k}{3}} + 2e^{-j\frac{10\pi k}{6}} \right]$$

$$= \frac{1}{6}\left[3 + 4\cos\frac{\pi k}{3} + 2\cos\frac{2\pi k}{3} \right]$$

Hence: $C_0 = \frac{9}{6}$, $C_1 = 4/6$, $C_2 = 0$, $C_3 = 1/6$, $C_4 = 0$, $C_5 = 4/6$

b)

$$P_t = \frac{1}{6} \sum_{n=0}^{5} |x(n)|^2$$

$$= \frac{1}{6}\left(3^2 + 2^2 + 1^2 + 0^2 + 1^2 + 2^2 \right) = \frac{19}{16}$$

$$P_f = \sum_{n=0}^{5} |c_n|^2$$

$$= \left[\left(\frac{9}{6}\right)^2 + \left(\frac{4}{6}\right)^2 + 0^2 + \left(\frac{1}{6}\right)^2 + 0^2 + \left(\frac{4}{6}\right)^2 \right]$$

$$= \frac{19}{16}.$$

$\therefore P_t = P_f$

5) Consider

$$x(n) = 2 + 2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{2} + \frac{1}{2}\cos\frac{3\pi n}{4}$$

a) Determine and sketch its PSD.

(b) Evaluate Power of signal.

Sol

Given

$$x(n) = 2 + 2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{2} + \frac{1}{2}\cos\frac{3\pi n}{4}$$

here ω's: $\{\pi/4, \pi/2, 3\pi/4\}$

G.C.D $= \frac{3\pi}{4} \Rightarrow 2\pi f = \frac{3\pi}{4} \Rightarrow f = 3/8$

$N = 8$.

(a) $$C_k = \frac{1}{8}\sum_{n=0}^{7} x(n)\, e^{-j\pi k n/4}$$

$$x(n) = \left\{\frac{11}{2},\ 2+\frac{3}{4}\sqrt{2},\ 1,\ 2-\frac{3}{4}\sqrt{2},\ \frac{1}{2},\ 2-\frac{3}{4}\sqrt{2},\ 1,\ 2+\frac{3}{4}\sqrt{2}\right\}$$

$\boxed{C_k = C_{N-k}}$

$$2 + e^{j\pi n/4} + e^{-j\pi n/4} + \frac{e^{\frac{j\pi n}{2}}}{2} + \frac{e^{-\frac{j\pi n}{2}}}{2} + \frac{1}{4}e^{j3\pi n/4} + \frac{1}{4}e^{-j3\pi n/4}$$

Hence - $C_0 = 2$, $C_1 = C_7 = 1$, $C_2 = C_6 = \frac{1}{2}$, $C_3 = C_5 = \frac{1}{4}$, $C_4 = 0$

(b) $$P = \sum_{i=0}^{7} |C(i)|^2$$

$$\Rightarrow 4 + 1 + 1 + \frac{1}{4} + \frac{1}{4} + \frac{1}{16} + \frac{1}{16} = \frac{53}{8}$$

Determine
6) ~~Sketch~~ the magnitude & phase spectra of the following

(a) $x(n) = 4\sin \frac{\pi(n-2)}{3}$

$x(n) = 4\sin \frac{\pi(n-2)}{3} \Rightarrow 4\sin \frac{2\pi(n-2)}{6}$

$C_k = \frac{f}{6}$ $\quad \omega = \pi/3 \Rightarrow 2\pi f = \pi/3$

$N = 6$ $\quad F = 1/6$

$C_k = \frac{1}{6}\sum_{n=0}^{5} x(n)\, e^{-j2\pi kn/6}$

$= \frac{4}{6}\sum_{n=0}^{5} \sin \frac{2\pi(n-2)}{6}\, e^{-j2\pi kn/6}$

$= \cancel{4}\ \cancel{\frac{4}{6}}\ \frac{1}{\sqrt{3}}\left[-e^{-j2\pi k/3} - e^{-j\pi k/3} + e^{-j4\pi k/3} + e^{-j2\pi k/3}\right]$

$= \frac{1}{\sqrt{3}}(-j2)\left[\sin\frac{2\pi k}{6} + \sin\frac{\pi k}{3}\right] e^{-j2\pi k/3}$

Hence $\Rightarrow$ $C_0 = 0,\ C_1 = -j2\, e^{-j2\pi/3},\ C_2 = C_3 = C_4 = 0,\ C_5 = C_1^*$

and $|C_1| = |C_5| = 2,\ |C_0| = |C_2| = |C_3| = |C_4| = 0.$

$\angle C_1 = \pi + \pi/2 - 2\pi/3 = 5\pi/6$

$\angle C_5 = -\frac{5\pi}{6}$, $\angle C_0 = \angle C_2 + = \angle C_3 - C_4 = 0$

$\angle\angle C_5 = -\cancel{5}$

b) $x(n) = \cos \frac{2\pi}{3}n + \sin \frac{2\pi}{5}n \Rightarrow N =$

GCD $\left(2\pi/3,\ 2\pi/5\right) \Rightarrow \frac{2}{15} \Rightarrow N = 15$.

$C_k = C_{k_1} + C_{k_2}$

$$\cos\frac{2\pi n}{3} = \frac{1}{2}\left(e^{j\frac{2\pi n}{3}} + e^{-j\frac{2\pi n}{3}}\right)$$

$$C_{1k} = \begin{cases} \frac{1}{2} & k = 5, 10 \\ 0, & \text{otherwise} \end{cases}$$

Similly

$$\sin\frac{2\pi n}{5} = \frac{1}{2j}\left(e^{+j\frac{2\pi n}{5}} - e^{-j\frac{2\pi n}{5}}\right)$$

$$C_{2k} = \begin{cases} \frac{1}{2j}, & k = 3 \\ -\frac{1}{2j}, & k = 12 \\ 0, & \text{otherwise} \end{cases}$$

$$\therefore C_k = C_{1k} + C_{2k} = \begin{cases} \frac{1}{2j} & k = 3 \\ \frac{1}{2} & k = 5 \\ \frac{1}{2} & k = 10 \\ -\frac{1}{2j} & k = 12 \\ 0 & \text{otherwise} \end{cases}$$

c) $x(n) = \cos\left(\frac{2\pi}{3}\right)n \sin\left(\frac{2\pi}{5}\right)n$

$\therefore x(n) = \frac{1}{2}\sin\frac{16n\pi}{15} - \frac{1}{2}\sin\frac{4\pi n}{15}, \quad N = 15$

$$C_k = \begin{cases} -\frac{1}{4j} & k = 2, 7 \\ \frac{1}{4j} & k = 8, 13 \\ 0, & \text{otherwise} \end{cases}$$

d) $x(n) = \{\ldots, -2, -1, \underset{\uparrow}{0}, 2, -2, -1, 0, 1, 2, \ldots\}$

$N = 5$

$$C_k = \frac{1}{5}\sum_{n=0}^{5} x(n)\, e^{-j\frac{2\pi nk}{5}}$$

$$= \frac{1}{5}\left[e^{-\frac{j2\pi k}{5}} + 2e^{-\frac{j4\pi k}{5}} - 2e^{-\frac{j6\pi k}{5}} - e^{-\frac{j8\pi k}{5}}\right]$$

$c_k = \frac{2j}{5}\left[-\sin\frac{2\pi k}{5} - 2\sin\left(\frac{4\pi k}{5}\right)\right]$

$\therefore\ c_0 = 0,\ c_1 = \frac{2j}{5}\left(-\sin\frac{2\pi}{5} + 2\sin\frac{4\pi}{5}\right)$

$c_2 = \frac{2j}{5}\left[\sin\left(\frac{4\pi}{5}\right) - 2\sin\left(\frac{2\pi}{5}\right)\right],\ c_3 = -c_2,\ c_4 = -c_1$

e) $x(n) = \{\ldots, -1, 2, \underset{\uparrow}{1}, 2, -1, 0, -1, 2, 1, 2, \ldots\}$

$N = 6,\quad c_k = \frac{1}{6}\sum_{n=0}^{5} x(n)\, e^{-j2\pi nk/6}$

$= \frac{1}{6}\left[1 + 2e^{-j\pi k/3} - e^{-j2\pi k/3} - e^{-j4\pi k/3} + 2e^{-j5\pi k/3}\right]$

$= \frac{1}{6}\left[1 + 4\cos\frac{\pi k}{3} - 2\cos\frac{2\pi k}{3}\right]$

$\therefore\ c_0 = 1/2,\ c_1 = 2/3,\ c_2 = 0,\ c_3 = -5/6,\ c_4 = 0,\ c_5 = 2/3$

(f) $x(n) = \{\ldots, 0, 0, \underset{\uparrow}{1}, 1, 0, 0, 0, 1, 1, 0, 0, \ldots\}$

$N = 5,\quad c_k = \frac{1}{5}\sum_{n=0}^{4} x(n)\, e^{-j2\pi nk/5}$

$= \frac{1}{5}\left[1 + e^{-j2\pi nk/5}\right]$

$\Rightarrow\ \frac{2}{5}\cos\left(\frac{\pi k}{5}\right) e^{-j\pi k}$, $\therefore\ c_0 = 2/5,\ c_1 = 2/5\cos(\pi/5)\, e^{-j\pi/5}$

$c_2 = 2/5\cos\left(\frac{2\pi}{5}\right) e^{-j\frac{2}{5}\pi},\ c_3 = 2/5\cos\left(\frac{3\pi}{5}\right) e^{-j\frac{3\pi}{5}}$

$c_4 = 2/5\cos\left(\frac{4\pi}{5}\right) e^{-j4\pi/5}$

(g) $x(n) = 1 \quad , \quad -\infty < n < \infty$.

$N = 1, \qquad C_k : x(0) = 1 \quad (\text{or}) \quad c_0 = 1$

(h) $x(n) = (-1)^n \quad , \quad -\infty < n < \infty$

$N = 2, \qquad C_k = \frac{1}{2} \sum_{n=0}^{1} x(n)\, e^{-jn\pi k}$

$C_k = \frac{1}{2}\left(1 - e^{-j\pi k}\right)$

$C_0 = 0, \quad C_1 = 1$

7) Determine $x(n)$, $N = 8$

(a) $C_k = \cos \frac{k\pi}{4} + \sin \frac{3k\pi}{4}$

Sol $x(n) = \sum_{k=0}^{7} C_k\, e^{\frac{j2\pi nk}{8}}$

$C_k = \frac{1}{2}\left[e^{\frac{j2\pi k}{8}} + e^{\frac{-j2\pi k}{8}} \right] + \frac{1}{2j}\left[e^{\frac{j6\pi k}{8}} - e^{\frac{-j6\pi k}{8}} \right]$

$x(n) = 4\delta(n+1) + 4\delta(n-1) - 4j\,\delta(n+3) + 4j\,\delta(n-3)$

$-3 \leq n \leq 5$

b) $C_k = \begin{cases} \sin \frac{k\pi}{3}, & 0 \leq k \leq 6 \\ 0, & k = 7 \end{cases}$

$C_0 = 0,\ C_1 = \sqrt{3}/2,\ C_2 = \sqrt{3}/2;\ C_3 = 0,\ C_4 = \frac{\sqrt{3}}{2},\ C_5 = -\sqrt{3}/2,$

$C_6, C_7 = 0$

$x(n) = \sum_{k=0}^{7} C_k\, e^{\frac{j2\pi nk}{8}}$

$= \frac{\sqrt{3}}{2}\left[e^{\frac{j\pi n}{4}} + e^{\frac{j2\pi n}{4}} - e^{\frac{j4\pi n}{4}} - e^{\frac{j5\pi n}{4}} \right]$

$= \sqrt{3}\left(\sin\frac{\pi n}{2} + \sin\frac{\pi n}{4} \right) e^{\frac{j\pi(3n-2)}{4}}$

(c) $c_k = \{ \ldots 0, \frac{1}{4}, \frac{1}{2}, 1, \underset{\uparrow}{2}, 1, \frac{1}{2}, \frac{1}{4}, 0, \ldots \}$

$$x(n) = \sum_{k=-3}^{4} c_k e^{j2\pi nk/8}$$

$$= 2 + e^{\frac{j\pi n}{4}} + e^{-\frac{j\pi n}{4}} + \frac{1}{2} e^{\frac{j\pi n}{2}} + \frac{1}{2} e^{-\frac{j\pi n}{2}} + \frac{1}{4} e^{\frac{j3\pi n}{4}} + \frac{1}{4} e^{-j3\pi n/4}$$

$$x(n) = 2 + 2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{2} + \frac{1}{2}\cos\frac{3\pi n}{4}$$

8) Two DT signals $s_k(n)$ & $s_l(n)$ are orthogonal over $[N_1, N_2]$ if

$$\sum_{n=N_1}^{N_2} s_k(n) s_l^*(n) = \begin{cases} A_k, & k=0 \\ 0, & k \neq 0 \end{cases}$$

If $A_k = 1$ the signals are orthogonal.

a) Prove

$$\sum_{n=0}^{N-1} e^{j2\pi kn/N} = \begin{cases} N, & k = 0, \pm N, \pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$$

Sol

If $k = 0, \pm N, \pm 2N, \ldots$

$$\sum_{n=0}^{N-1} e^{j2\pi kn/N} = \sum_{n=0}^{N-1} 1 = N$$

If $k \neq 0, \pm N, \pm 2N$

$$\sum_{n=0}^{N-1} e^{j2\pi kn/N} = \frac{1 - e^{j2\pi k}}{1 - e^{j2\pi k/N}} = \frac{1-1}{1 - e^{j2\pi k/N}}$$

$$= 0$$

9) Compute the F.T of following signals.

a) $x(n) = u(n) - u(n-6)$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n} = \sum_{n=0}^{5} e^{-j\omega n}$$

$$= \frac{1 - e^{-j6\omega}}{1 - e^{-j\omega}}$$

b) $x(n) = 2^n u(-n)$

$$X(\omega) = \sum_{n=-\infty}^{0} 2^n\, e^{j\omega n}$$

$$= \sum_{m=0}^{\infty} \left(\frac{e^{j\omega}}{2}\right)^n = \frac{2}{2 - e^{j\omega}}$$

c) $x(n) = \left(\frac{1}{4}\right)^n u(n+4)$

$$X(\omega) = \sum_{n=-4}^{\infty} \left(\frac{1}{4}\right)^n e^{-j\omega n}$$

$$= \left(\sum_{m=0}^{\infty} \left(\frac{1}{4}\right)^m e^{-j\omega n}\right) 4^4\, e^{j\omega(4)}$$

$$= \frac{4^4\, e^{j\omega 4}}{1 - \frac{1}{4} e^{-j\omega}}$$

(d) $x(n) = \alpha^n (\sin \omega_0 n)\, u(n)\ ;\ |\alpha| < 1$

$$X(\omega) = \sum_{n=0}^{\infty} \alpha^n \left[\frac{e^{j\omega_0 n} - e^{-j\omega_0 n}}{2j}\right] e^{-j\omega n}$$

$$= \frac{1}{2j} \sum_{n=0}^{\infty} \left[\alpha\, e^{-j(\omega-\omega_0)}\right]^n - \frac{1}{2j} \sum_{n=0}^{\infty} \left[\alpha\, e^{-j(\omega+\omega_0)}\right]^n$$

$$\Rightarrow \frac{1}{2j}\left[\frac{1}{1-\alpha e^{-j(\omega-\omega_0)}} - \frac{1}{1-\alpha e^{-j(\omega+\omega_0)}}\right]$$

c) $$\frac{\alpha \sin \omega_0 \, e^{-j\omega}}{1 - 2\alpha \cos\omega_0 \, e^{-j\omega} + \alpha^2 e^{-j2\omega}}$$

e) $x(n) = |\alpha|^n \sin \omega_0 n$, $|\alpha| < 1$

Note that $\sum_{n=-\infty}^{\infty} |x(n)| = \sum_{n=-\infty}^{\infty} |\alpha|^n |\sin\omega_0 n|$

If $\omega_0 = \pi/2$, $|\sin \omega_0 n| = 1$

$$\sum_{n=\infty}^{\infty} |\alpha|^n = \sum_{n=-\infty} |x(n)| = \infty$$

$\therefore$ F.T does not exist

(f) $x(n) = \begin{cases} 2-(\frac{1}{2})^n, & |n| \le 4 \\ 0, & \text{elsewhere} \end{cases}$

$$X(\omega) = \sum_{n=-4}^{4} x(n) e^{-j\omega n}$$

$$= \sum_{n=-4}^{4} \left(2-\left(\tfrac{1}{2}\right)^n\right) e^{-j\omega n}$$

$$= \frac{2e^{j4\omega}}{1-e^{-j\omega}} - \frac{1}{2}\left[-4e^{j\omega n} + 4e^{-j\omega n} - 3e^{j3\omega} + e^{-j3\omega} - 2e^{j2\omega} + 2e^{-j2\omega} - e^{j\omega} + e^{-j\omega}\right]$$

$$\Rightarrow \frac{2e^{j4\omega}}{1-e^{-j\omega}} + j\left[4\sin 4\omega + 3\sin 3\omega + 2\sin 2\omega + \sin\omega\right]$$

g) $x(n) = [-2, -1, \underset{\uparrow}{0}, 1, 2]$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} \Rightarrow -2e^{j2\omega} - e^{j\omega} + e^{j\omega} + 2e^{-j2\omega}$$

$$\Rightarrow -2j\left[2\sin 2\omega + \sin\omega\right]$$

h) $$x(n) = \begin{cases} A(2M+1+|n|) & ; |n| < M \\ 0 & ; |n| > M \end{cases}$$

$$X(\omega) = \sum_{n=-M}^{M} x(n) e^{-j\omega n}$$

$$= A \sum_{n=-M}^{M} (2M+1-|n|) e^{-j\omega n}$$

$$= (2M+1)A + A\sum_{k=1}^{M} (2M+1-k)(e^{-j\omega k} + e^{j\omega k})$$

$$= (2M+1)A + 2A\sum_{K=1}^{M} (2M+1-k)\cos\omega k$$

10) Determine the signals

(a) $$X(\omega) = \begin{cases} 0, & 0 \le |\omega| \le \omega_0 \\ 1, & \omega_0 < |\omega| \le \pi \end{cases}$$

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega) e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\omega_0} e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{\omega_0}^{\pi} e^{j\omega n}\, d\omega$$

$$x(0) = \frac{1}{2\pi}(\pi - \omega_0) + \frac{1}{2\pi}(\pi - \omega_0) = \frac{\pi - \omega_0}{\pi}$$

For $n \neq 0$, $$\int_{-\pi}^{-\omega_0} e^{j\omega n}\, d\omega = \left[\frac{1}{jn} e^{j\omega n}\right]_{-\pi}^{-\omega_0} = \frac{1}{jn}\left[e^{-j\omega_0 n} - e^{-j\pi n}\right]$$

$$\int_{\omega_0}^{\pi} e^{j\omega n}\, d\omega = \left[\frac{1}{jn} e^{j\omega n}\right]_{\omega_0}^{\pi} = \frac{1}{jn}\left[e^{jn\pi} - e^{+j\omega_0 n}\right)$$

Hence $x(n) = -\dfrac{\sin(n\omega_0)}{n\pi}$, $n \neq 0$

(b) $X(\omega) = \cos^2 \omega$

$$X(\omega) = \left(\tfrac{1}{2} e^{j\omega} + \tfrac{1}{2} e^{j\omega}\right)^2 = \tfrac{1}{4}\left(e^{j2\omega} + 2 + e^{-j2\omega}\right)$$

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega) e^{j\omega n}\, d\omega$$

$$= \frac{1}{8\pi}\left[2\pi\,\delta(n+2) + 4\pi\,\delta(n) + 2\pi\,\delta(n-2)\right]$$

$$\Rightarrow \tfrac{1}{4}\left(\delta(n+2) + \delta(n) + \delta(n-2)\right)$$

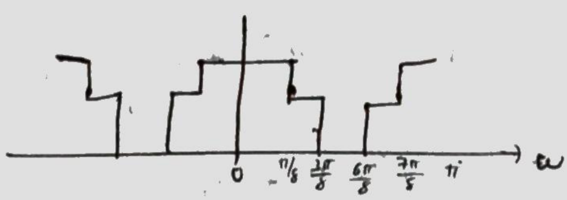
c)
$$X(\omega) = \begin{cases} 1, & \omega_0 - \frac{\delta\omega}{2} \le |\omega| \le \omega_0 + \delta\omega/2 . \\ 0, & \text{elsewhere.} \end{cases}$$

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega) e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{\omega_0 - \frac{\delta\omega}{2}}^{\omega_0 + \frac{\delta\omega}{2}} e^{j\omega n}\, d\omega$$

$$\Rightarrow \frac{2}{\pi}\,\delta\omega \left(\frac{\sin(\pi\,\delta\omega/2)}{\pi\,\delta\omega/2}\right) e^{jn\omega_0}$$

(d) The signal



$$x(n) = \frac{1}{2\pi}\,\mathrm{Re}\left[\int_0^{\pi/8} 2e^{j\omega n}\, d\omega + \int_{\pi/8}^{3\pi/8} e^{j\omega n}\, d\omega + \int_{6\pi/8}^{7\pi/8} e^{j\omega n}\, d\omega + \int_{7\pi/8}^{\pi} e^{j\omega n}\, d\omega\right]$$

$$x(n) = \frac{1}{\pi}\left[\int_0^{\pi/8} 2\cos wn\, dw + \int_{\pi/8}^{3\pi/8} \cos wn\, dw + \int_{6\pi/8}^{7\pi/8} \cos wn\, dw + \int_{7\pi/8}^{\pi} 2\cos wn\, dw\right]$$

$$= \frac{1}{\pi n}\left[\sin\frac{7\pi n}{8} + \sin\frac{6\pi n}{8} - \sin\frac{3\pi n}{8} \quad \sin\frac{\pi n}{8}\right]$$

11) $x(n) = (1, 0, -1, \underset{\uparrow}{2}, 3)$ with $X(\omega) = X_R(\omega) + j(X_I(\omega))$

determine $y(n)$ if $Y(\omega) = X_I(\omega) + X_R(\omega)e^{j2\omega}$

Sol)

$$x_e(n) = \frac{x(n) + x(-n)}{2} = [\tfrac{1}{2}, 0, 1, \underset{\uparrow}{2}, 1, 0, \tfrac{1}{2}]$$

$$x_o(n) = \frac{x(n) - x(n)}{2} \quad [\tfrac{1}{2}, 0, -2, \underset{\uparrow}{0}, 2, 0, \tfrac{1}{2}]$$

Then $X_R(\omega) = \sum_{n=-3}^{3} x_e(n)e^{-j\omega n}$

$$jX_I(\omega) \quad \sum_{n=-3}^{3} x_o(n)e^{-j\omega n}$$

Now $Y(\omega) = X_I(\omega) + X_R(\omega)e^{j2\omega}$,

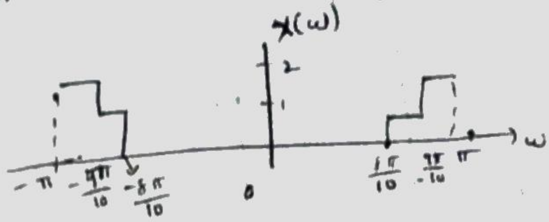
$$\therefore y(n) = F^{-1}(X_I(\omega)) + F^{-1}(X_R(\omega)e^{j2\omega})$$

$$- j x_o(n) + x_e(n+2)$$

$$= \{\tfrac{1}{2}, 0, 1 - \tfrac{j}{2}, 2, 1 + \tfrac{j}{2}, \underset{\uparrow}{0}, \tfrac{1}{2} - j2, 0, \tfrac{1}{2}\}$$

12) Determine $x(n)$

a)



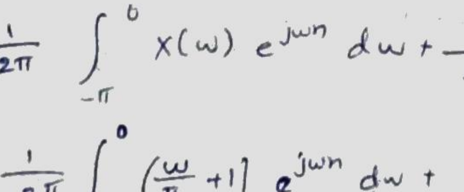
$$x(n) = \frac{1}{2\pi}\left[\int_{8\pi/10}^{9\pi/10} e^{j\omega n}\,d\omega + \int_{-9\pi/10}^{-8\pi/10} e^{j\omega n}\,d\omega + 2\int_{9\pi/10}^{\pi} e^{j\omega n}\,d\omega + 2\int_{-\pi}^{-9\pi/10} e^{j\omega n}\,d\omega\right]$$

$$\Rightarrow \frac{1}{2\pi}\left[\frac{1}{jn}\left[e^{j9\pi n/10} - e^{-j9\pi n/10} - e^{j8\pi n/10} + e^{-j8\pi n/10}\right] + \frac{2}{jn}\left(-e^{j9\pi n/10} + e^{-9\pi n/10} + e^{jn\pi} - e^{-j\pi n}\right)\right]$$

$$\Rightarrow \frac{1}{n\pi}\left(\underbrace{\sin \pi n}_{(0)} + - \sin 8\pi n/10 - \sin 9\pi/10\right)$$

$$\Rightarrow \frac{-1}{n\pi}\left[\sin\frac{4\pi n}{5} + \sin\frac{9\pi n}{10}\right]$$

(b)



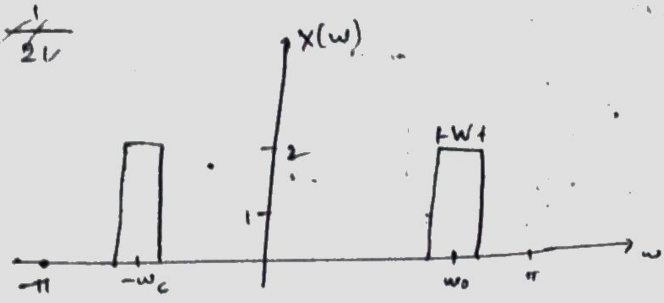
$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{0} X(\omega)\, e^{j\omega n}\,d\omega + \frac{1}{2\pi}\int_{0}^{\pi} X(\omega)\, e^{j\omega n}\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{0}\left(\frac{\omega}{\pi}+1\right) e^{j\omega n}\,d\omega + \frac{1}{2\pi}\int_{0}^{\pi}\frac{\omega}{n} e^{j\omega n}\,d\omega.$$

$$= \frac{1}{2\pi}\left[\frac{\omega}{jn\pi} e^{j\omega n}\right]_{-\pi}^{\pi} + \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{-\pi}^{0}$$

$$x(n) = \frac{1}{\pi n}\sin\left(\frac{\pi n}{2}\right), e^{-j\pi n/2}.$$

(c) $x(n) = \frac{1}{2\pi}$



$$x(n) = \frac{1}{2\pi}\int_{w_c - \frac{w}{2}}^{w_c - \frac{w}{2}} 2e^{jwn}\,dw + \frac{1}{2\pi}\int_{-w_c - \frac{w}{2}}^{-w_c + w/2} 2e^{jwn}\,dw$$

$$= \frac{1}{\pi}\left[\frac{1}{jn\pi}\left[e^{jwn}\right]_{w_c - w/2}^{w_c - w/2} + \left(\frac{e^{jwn}}{jn}\right)_{-w_c + w/2}^{-w_c + w/2} \right]$$

$$\Rightarrow \frac{2}{\pi n}\left[\frac{e^{j(w_c + w/2)n} - e^{j(w_c - w/2)n} + e^{-j(w_c - w/2)n} - e^{-j(w_c + w/2)n}}{2j} \right]$$

$$\Rightarrow \frac{2}{\pi n}\left[\sin(w_c + w/2)n - \sin(w_c - w/2)n \right]$$

13) We have $x(n) = \begin{cases} 1, & -M \le n \le M \\ 0, & \text{otherwise} \end{cases}$

and its FT is $X(\omega) = 1 + 2\sum_{n=1}^{M} \cos wn$ then show that

$x_1(n) = \begin{cases} 1, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$ and $x_2(n) = \begin{cases} 1, & -M \le n \le -1 \\ 0, & \text{otherwise} \end{cases}$

and show that

$$X_1(\omega) = \frac{1 - e^{j\omega(M+1)}}{1 - e^{jw}}$$

$$X_2(\omega) = \frac{e^{jw} - e^{jw(M+1)}}{1 - e^{jw}}$$

then $X(\omega) = X_1(\omega) + X_2(\omega)$

$$= \frac{\sin(M+1/2)\omega}{\sin(\omega/2)}$$ and

$$\therefore \quad 1 + 2\sum_{n=1}^{M} \cos \omega n = \frac{\sin(M+1/2)\omega}{\sin(\omega/2)}$$

Sol

$$x_1(n) = \begin{cases} 1, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$

$$X_1(\omega) = \sum_{n=0}^{M} e^{-j\omega n} = \frac{1 - e^{-j\omega(M+1)}}{1 - e^{-j\omega}}$$

$$x_2(n) = \begin{cases} 1, & -M \le n \le -1 \\ 0, & \text{otherwise} \end{cases}$$

$$X_2(\omega) = \sum_{n=-M}^{-1} e^{-j\omega n} \Rightarrow \sum_{n=1}^{M} e^{j\omega n} \Rightarrow \left[\frac{1 - e^{j\omega M}}{1 - e^{j\omega}}\right] e^{j\omega}$$

$$X(\omega) = X_1(\omega) + X_2(\omega)$$

$$= \frac{1 + e^{j\omega} - e^{j\omega} - 1 - e^{-j\omega(M+1)} - e^{j\omega(M+1)} + e^{j\omega M} + e^{-j\omega M}}{2 - e^{-j\omega} - e^{+j\omega}}$$

$$= \frac{2\cos\omega M - 2\cos\omega(M+1)}{2 - 2\cos\omega}$$

$$= \frac{2\sin(\omega M + \omega/2)\cos\omega/2}{2\sin^2\omega/2}$$

$$X(\omega) = \frac{\sin(M+1/2)\omega}{\sin(\omega/2)}$$

14) $x(n) = \{-1, 2, -3, \cancel{2, -1}\}$ find

a) $X(0)$ b) $\angle X(\omega)$ c) $\int_{-\pi}^{\pi} X(\omega)\,d\omega$ d) $X(\pi)$

e) $\int_{-\pi}^{\pi} |X(\omega)|^2\,d\omega$

Sol

a) $X(0) = \sum_n x(n) = -1$

(b) $\angle X(\omega) = \pi$ for all ω

(c) $x(0) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\,d\omega \Rightarrow 2\pi\,x(0) = 2\pi(-3) = -6\pi$

(d) $X(\pi) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\pi n} \Rightarrow \sum_n (-1)^n x(n) = -3-4-2 = -9$

(e) $\int_{-\pi}^{\pi} |X(\omega)|^2\,d\omega = 2\pi \sum_n |x(n)|^2 \Rightarrow 2\pi(19) = 38\pi$

15) Centre of gravity of a signal $x(n)$ is defined as

$$c = \frac{\sum_{n=-\infty}^{\infty} n\,x(n)}{\sum_{n=-\infty}^{\infty} x(n)}$$

and provides a measure of "time delay" of signal.

(a) Express 'c' in terms of $X(\omega)$

$$X(\omega) = \sum_n x(n) e^{-j\omega n}$$

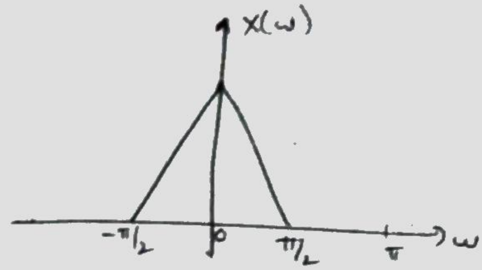
$$X(0) = \sum_n x(n)$$

$$\left.\frac{dX(\omega)}{d\omega}\right|_{\omega=0} = -j\sum_n n\,x(n)\,e^{-j\omega n}\Big|_{\omega=0}$$

$$= -j\sum_n n\,x(n)$$

$$\therefore \quad c = \frac{j\left.\frac{dX(\omega)}{d\omega}\right|_{\omega=0}}{X(0)}$$

(b) compute 'c' for signal $x(n)$



$X(0) = 1$ $\therefore$ $c = 0/1 = 0$

(6) Consider the FT pair

$$a^n u(n) \xleftrightarrow{F} \frac{1}{1-ae^{-jw}} \quad |a| < 1$$

use the differentiation in frequency theorem and induction to show that

$$x(n) = \frac{(n+l-1)!}{n!\,(l-1)!} a^n u(n) \xleftrightarrow{F} X(w) = \frac{1}{(1-ae^{-jw})^l}$$

Sol

$$x_1(n) \equiv a^n u(n)$$

$$\xleftrightarrow{F} \frac{1}{1-ae^{-jw}}$$

Now, suppose that

$$x_k(n) = \frac{(n+k-1)!}{n!\,(k-1)!} a^n u(n)$$

$$\xleftrightarrow{F} \frac{1}{(1-ae^{-jw})^k}$$

holds, then

$$x_{k+1}(n) = \frac{(n+k)!}{n!\,k!} x_k(n)$$

$$x_{k+1}(n) = \frac{n+k}{k} x_k(n)$$

$$X_{k+1}(\omega) = \frac{1}{k}\sum_n n\, x_k(n) e^{-j\omega n} + \sum_n x_k(n) e^{-j\omega n}$$

$$= \frac{1}{k} j \frac{dX_k(\omega)}{d\omega} + X_k(\omega)$$

$$X_{k+1}(\omega) = \frac{a e^{-j\omega}}{(1 - a e^{-j\omega})^{k+1}} + \frac{1}{(1 - a e^{-j\omega})^{k}}$$

17) Let $x(n)$ is an arbitary signal, not necessary real-valued, with FT $X(\omega)$. Express FT of the following signals in terms of $X(\omega)$

(a) $x^*(n)$

$$\sum_n x^*(n) e^{-j\omega n} = \sum_n \left(x(n) e^{-j(-\omega)n}\right)^* = X^*(-\omega)$$

(b) $x^*(-n)$

$$\sum_n x^*(-n) e^{-j\omega n} = \sum_{n=-\infty}^{\infty} x^*(n) e^{j\omega n} = X^*(\omega)$$

(c) $y(n) = x(n) - x(n-1)$

$$\sum_n y(n) e^{-j\omega n} = \sum_n x(n) e^{-j\omega n} - \sum_n x(n-1) e^{-j\omega n}$$

$$Y(\omega) = X(\omega) + X(\omega) e^{-j\omega}$$

$$Y(\omega) = (1 - e^{-j\omega}) X(\omega)$$

d) $y(n) = \sum_{k=-\infty}^{n} x(k)$

$\cancel{Y(\omega) = X}$

$$y(n) = y(n) - y(n-1)$$
$$= x(n)$$

$$X(\omega) = \left(1-e^{-j\omega}\right)Y(\omega)$$

$$Y(\omega) = \frac{X(\omega)}{1-e^{-j\omega}}$$

e) $y(n) = x(2n)$

$$Y(\omega) = \sum_n x(2n)\, e^{-j\omega n} = \sum_n x(n)\, e^{-j\omega/2\, n}$$

$$Y(\omega) = X(\omega/2)$$

f) $y(n) = \begin{cases} x(n/2) & , \quad n \text{ even} \\ 0 & , \quad n \text{ odd} \end{cases}$

$$Y(\omega) = \sum_n x(n/2)\, e^{-j\omega n}$$

$$= \sum_n x(n)\, e^{-j2\omega n}$$

$$Y(\omega) = X(2\omega)$$

18) Find F.T

(a) $x_1(n) = \{1, 1, \underset{\uparrow 0}{1}, 1, 1\}$

$$X_1(\omega) = \sum_n x(n)\, e^{-j\omega n} \Rightarrow e^{j2\omega} + e^{j\omega} + 1 + e^{-j\omega} + e^{-j2\omega}$$

$$X_1(\omega) = 1 + 2\cos\omega + 2\cos 2\omega$$

b) $x_2(n) = \{1, 0, 1, 0, \underset{\uparrow}{1}, 0, 1, 0, 1\}$

$$\cancel{X_2}(n) = \sum_n \cancel{x_2}(n)\, e^{-j\omega n} \Rightarrow e^{j4\omega} + e^{j2\omega} + 1 + e^{-j2\omega} + e^{-j4\omega}$$

$$X_2(\omega) \Rightarrow 1 + 2\cos 2\omega + 2\cos 4\omega$$

c) $x_3(n) = \{1,0,0,1,0,0,\underset{\uparrow}{1},0,0,1,0,0,1\}$

$$X_3(\omega) = \sum_n x_3(n) e^{-j\omega n}$$

$$\Rightarrow e^{j6\omega} + e^{j\omega 3} + e^{-j3\omega} + 1 + e^{-j6\omega}$$

$$X_3(\omega) \Rightarrow 1 + 2\cos 3\omega + 2\cos 6\omega$$

d) Is there any relation b/w $X_1(\omega)$, $X_2(\omega)$, $X_3(\omega)$

$$X_2(\omega) = X_1(2\omega) \quad \& \quad X_3(\omega) = X_1(3\omega)$$

e) Show that

$$x_k(n) = \begin{cases} x(n/k), & \text{if } n/k \text{ integer} \\ 0, & \text{otherwise} \end{cases}$$

then $X_k(\omega) = X(k\omega)$

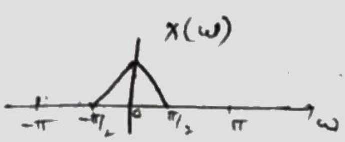
e)

$$x_k(n) = \sum_{n/k \text{ an integer}} x_k(n) e^{-j\omega n}$$

$$= \sum_n x(n) e^{-jk\omega n}$$

$$x_k(n) = X(k\omega)$$

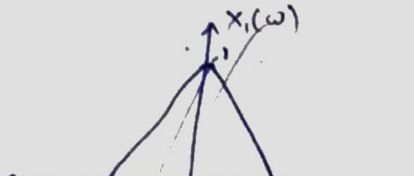
19) Find the F.T Given

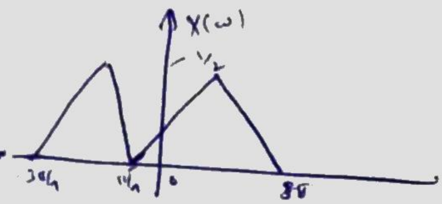


a) $x_1(n) = x(n)\cos(\pi n/4)$

$$x_1(n) = \frac{1}{2}\left(e^{j\pi/4} + e^{-j\pi/4}\right) x(n)$$

$$X_1(\omega) = \frac{1}{2}\left(X(\omega - \pi/4) + X(\omega + \pi/4) \right]$$





b) $x_2(n) = x(n)\sin(\pi n/2)$

$$x_2(n) = \frac{1}{2j}\left(e^{j\pi n/2} + e^{-j n\pi/2}\right) x(n)$$

$$X_2(\omega) = \frac{1}{2j}\left[X(\omega-\pi/2) + X(\omega+\pi/2)\right]$$

c) $x_3(n) = x(n)(\cos n\pi/2)$

$$x_3(n) = \frac{1}{2}\left(e^{jn\pi/2} + e^{-jn\pi/2}\right) x(n)$$

$$X_3(\omega) = \tfrac{1}{2}\left(X(\omega-\pi/2) + X(\omega+\pi/2)\right]$$

d) $x_4(n) = x(n)\cos\pi n$

$$x_4(n) = \tfrac{1}{2}\left(e^{j\pi n} + e^{-jn\pi}\right) x(n)$$

$$X_4(\omega) = \tfrac{1}{2}\left(X(\omega-\pi) + X(\omega+\pi)\right)$$

$$= \tfrac{1}{2}\, X(\omega-\pi)$$

20) Consider an aperiodic signal $x(n)$ with fourier transform $X(\omega)$. Show that fourier series co-efficients C_k^y of periodic signal

$$y(n) = \sum_{l=-\infty}^{\infty} x(n-lN)$$ are given by

$$C_k^y = \frac{1}{N} X\left(\frac{2\pi}{N}k\right), \quad k = 0, 1, \ldots, N-1$$

sol

$$C_k = \frac{1}{N}\sum_{n=0}^{N-1} y(n)\, e^{-j\frac{2\pi k}{N}\cdot n}$$

$$= \frac{1}{N}\sum_{n=0}^{N-1}\left[\sum_{l=-\infty}^{\infty} x(n-lN)\right] e^{-j\frac{2\pi k}{N}\cdot n}$$

$$= \frac{1}{N}\sum_{l=-\infty}^{\infty}\ \sum_{m=-lN}^{N-1-lN} x(m)\, e^{-j2\pi k(m+lN)/N}$$

but $\displaystyle\sum_{l=-\infty}^{\infty} \sum_{m=-lN}^{N-1-lN} x(m)\, e^{-jw(m+lN)} = X(w)$

$$C_k^{y} = \frac{1}{N} X\left(\frac{2\pi k}{N}\right).$$

21) Prove that

$$X_N(w) = \sum_{n=-N}^{N} \frac{\sin w_c n}{\pi n} e^{-jwn}.$$

may be expressed as

$$X_N(w) = \frac{1}{2\pi} \int_{w_c}^{-w_c} \frac{\sin\left[(2N+1)(w-\theta/2)\right]}{\sin\left[(w-\theta)/2\right]} d\theta.$$

sol

Let

$$x_N(n) = \frac{\sin w_c n}{\pi n} \quad ; \quad -N \le n \le N$$

$$= x(n)\, w(n)$$

where $x(n) = \dfrac{\sin w_c n}{\pi n} \quad ; \quad -\infty \le n < +\infty$

$$w(n) = 1 \quad ; \quad -N \le n \le N$$
$$= 0 \quad ; \quad \text{otherwise}$$

Then $\dfrac{\sin w_c n}{\pi n} \overset{F}{\longleftrightarrow} X(w)$

$$= 1, \quad |w| \le w_c$$
$$= 0, \quad \text{otherwise}$$

$$X_N(w) = X(w) * W(w)$$

$$= \int_{-\pi}^{\pi} X(\theta)\, W(w-\theta)\, d\theta$$

$$= \int_{-w_c}^{w_c} \frac{\sin(2N+1)(w-\theta)/2}{\sin(w-\theta)/2}\, d\theta.$$

22) Given

$$X(\omega) = \frac{1}{1-ae^{-j\omega}} \quad \text{for} \quad x(n)$$

Determine the F.T

(a) $x(2n+1)$

$$X_1(\omega) = \sum_n x(2n+1)\, e^{-j\omega n}$$

$k = 2n+1$

$n = \frac{k-1}{2}$

$$= \sum_k x(k)\, e^{-j\omega k/2}\, e^{j\omega/2}$$

$$= X(\omega/2)\, e^{j\omega/2}$$

$$= \frac{e^{j\omega/2}}{1 - a e^{-j\omega/2}}$$

(b) $e^{\pi n/2}\, x(n+2)$

$$X_2(\omega) = \sum_n x(n+2)\, e^{\pi n/2}\, e^{-j\omega n}$$

$n+2 = k$

$n = k-2$

$$= -\sum_k x(k)\, e^{-jk(\omega + j\pi/2)}\, e^{j2\omega}$$

$$= -X(\omega + j\pi/2)\, e^{j2\omega}$$

(c) $x(-2n)$

$$X_3(\omega) = \sum_n x(-2n)\, e^{-j\omega n}$$

$k = -2n$

$n = -k/2$

$$\Rightarrow \; -\sum_k x(k)\, e^{-j\omega k/2}$$

$$\Rightarrow \; X(-\omega/2)$$

d) $x(n)\cos(0.3\pi n)$

$$X_4(\omega) = \sum_n \frac{1}{2}\left(e^{j0.3\pi n} + e^{-j0.3\pi n}\right) x(n)\, e^{-j\omega n}$$

$k =$

$$= \frac{1}{2} \sum_n x(n) \left[e^{-j(\omega - 0.3\pi)n} + e^{-j(\omega + 0.3\pi)n}\right]$$

$$\Rightarrow \frac{1}{2}\left[X(\omega - 0.3\pi) + X(\omega + 0.3\pi)\right]$$

e) $x(n) * x(n-1)$

$$X_5(\omega) = X(\omega)\left[X(\omega)\, e^{-j\omega}\right] \Rightarrow X^2(\omega)\, e^{-j\omega}$$

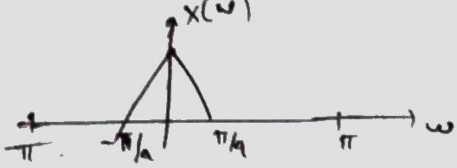
f) $x(n) * x(-n)$

$$X_6(\omega) = X(\omega)\, X(-\omega)$$

$$= \frac{1}{(1 - ae^{-j\omega})(1 - ae^{j\omega})}$$

$$X_6(\omega) = \frac{1}{(1 - 2a\cos\omega + a^2)}$$

23) From a discrete signal $x(n)$ with F.T $X(\omega)$. shown in fig



a) $y_1(n) = \begin{cases} x(n) & n \text{ even} \\ 0 & n \text{ odd.} \end{cases}$

Note than $y(n) = x(n)\, s(n)$ where

$$s(n) = \{\ldots\ 0, 1, 0, \underset{\uparrow}{1}, 0, 1, 0, 1$$

Sol. $Y_1(\omega) = \sum_n y_1(n) e^{-j\omega n} \Rightarrow \sum_n$

b) $y_2(n) = x(2n)$

$$Y_2(\omega) = \sum_n y_2(n) e^{-j\omega n} = \sum_n x(2n) e^{-j\omega n}$$

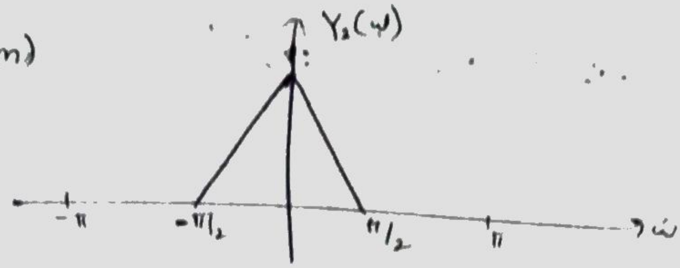
$m = 2n$

$n = m/2$

$$= \sum_m x(m) e^{-j\omega m/2}$$

$$= X(\omega/2)$$

a) $y_3(n)$



c) $y_3(n) = \begin{cases} x(n/2), & n\text{ even} \\ 0, & n = \text{odd} \end{cases}$

$y_3(n) =$

$$Y_3(\omega) = \sum_n y_3(n) e^{-j\omega n}$$

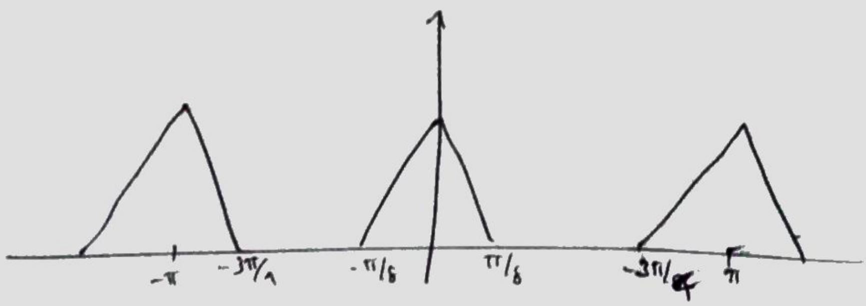
$m = n/2$

$n = 2m$

$$= \sum_{n\text{-even}} x(n/2)\, e^{-j\omega n}$$

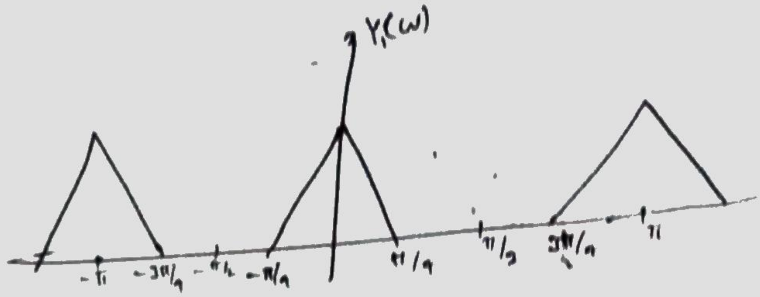
$$= \sum_m x(m) e^{-j2\omega m}$$

$$Y_3(\omega) = X(2\omega)$$



We now return to part (a) Note that $y_1(n)$ may be expressed as $y_1(n) = \begin{cases} y_0(n/2) & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$

Hence: $Y(\omega) = Y_0(2\omega)$



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