MATHEMATICAL MODELLING - MINI PROJECT

Team BS330:

- Bharathprasad Sivaprasad (bs330)
- Angel Febiyola (af349)
- Haritha Kalla Veettil (hkv9)

Selected countries: **INDIA, NEW ZEALAND, SOMALIA**

ABSTRACT

The covid-19 or novel corona is the most crucial phenomenon happened in the 21st century which impact almost every aspect of life in earth. This mini project paper on the incidence of the disease in India, New Zealand and Somalia. These countries were selected for the analysis because of the fact that covid-19 impact differently in each of these countries. The mathematical epidemiological models used are logistic growth model considering the exponential and linear growth, Susceptible-Infectious-Recovered (SIR) model and the modified SIR model considering psychological effects of the crowd in reacting to the pandemic at different stages. The modelling and analysis have been done in Microsoft Excel and MATLAB software. The data was extracted from WHO-COVID-19-global-data.csv. [1]

INTRODUCTION

The novel corona virus or widely known as covid-19 is a wide spreading virus which effects mostly the lung on a human being thereby inducing serious damage for the body which can even be fatal. It has been first reported in Wuhan, China in the month of December 2019. The outbreak of the disease spread quicky to other parts of China and afterwards to other parts of the world. By the time this report is created (16-12-2021), over 272 million cases of infected population with over 5.3 million deaths in over 180 countries. [2]

Covid-19 shows similarity to severe acute respiratory syndrome (SARS) virus but with significantly more spreading capability. The spread capability of covid-19 is what made the virus severe in terms of affecting the life of human beings in every sector with high risk on health itself. The effect on covid-19 brought more burden on the medical sector and other economic sectors. The only ways to prevent the spread of the virus were through social distancing, wearing a mask and sanitising the hands. For the mitigation and control of this epidemic, the authorities in power had to close borders to prevent the infected from travelling across. The same factor affected the small and medium business to shut down which indeed affected all the economic sectors of the society negatively.

As the paper considers the three countries – New Zealand, India and Somalia, the explanations will be focused on these countries. The first covid-19 case reported in India was on 30/01/2020, almost one month after the outbreak in Wuhan, China. Currently, India has reported slightly more than 34.7 million cases with a total death of 294,000. The population of India was 1.38 billion when the model was created, and all the calculations has been computed with this population value [3]. As for the case of New Zealand, the first covd-19 case was reported on 28/02/2020, almost one month after the outbreak in India. Currently, New Zealand has reported around 12 thousand cases with a total death of 48 people. The population of New Zealand is 5.08 million [4]. The total number of cases reported in Somalia are 23 thousand with a total death of 1300. The population of Somalia under consideration for this report is 15.8 million. [5]

In this context, mathematical modelling is essential to analyse, observe and evaluate the proliferation of the epidemic in different countries. The mathematical models help to estimate the transmission, recovery, deaths and other parameters of this disease. Moreover, these models can provide wisdom and make predictions about the epidemic, resulting to design efficient strategies and policies to control the situation. For the propose of optimally compare the three countries, the start of the epidemic in each country is considered as the 100^{th} cumulative case. That is, the date the 100^{th} cumulative case reported in the country is taken as day 1. In the case of India, cumulative case 107 is considered as the start of epidemic whereas for New Zealand and Somalia, it is 102 and 116 respectively.

THEORY

The three models used in this project, for the analysis and prediction of covid-19 spread, are general logistic growth model, SIR model and modified SIR model with the psychological factors considered.

Logistic Growth Model:

For modelling the epidemic, continuous logistic model is considered.

The logistic growth model is explained in terms of the reproduction and population size. The model of logistic growth in continuous time follows from the assumption that everyone reproduces at a rate that decreases as a linear function of the population size. The equation for the continuous time model is shown below:

$$\frac{dN}{dt} = r_m * N(1-(N/K))$$

Here, r_m is the maximum rate of growth,
N is the number of individuals,
K is the carrying capacity.

Population sizes that are less than K, the population will increase in size: at population sizes that are greater than K the population size will decline; and at K itself the population neither increases nor decreases. The carrying capacity is therefore a stable equilibrium for the population, and the model exhibits the regulatory properties classically characteristic of intraspecific competition. For the continuous time model, birth and death are continuous. The net rate of such a population will be denoted by dN/dt. This represents the 'speed' at which a population increases in size, N, as time, t, progresses. [6]

SIR model:

The SIR model is a basic statistical tool to analyse infectious disease outbreaks. In this basic model, the total population is divided into three subsets: susceptible (S), infected (I), and removed (R), where the transition of the disease is from S -> I -> R. We consider the whole population as N, which is the sum of susceptible, infected and removed.

ie, $S + I + R = N = constant$.

Further we use dimensionless variables:

$$\{S, I, R\} = \{S/N, I/N, R/N\}$$

$$S + I + R = 1$$

The number of infected people decreases with recoveries and deaths. Recovered individuals can also no longer change to the susceptible state.

Hence, assume that intensity of transitions **S** → **I** is *aI* and the flux **S** → **I** is *aSI*,

where *a* = *constant*.

Assume that the intensity of recovering (or death) is constant (*b*) and, therefore, the flux **I** → **R** is *bI*.

Now we have the following system of ODE ,

$$dS/dt = -aSI,$$

$$dI/dt = aSI - bI,$$

$$dR/dt = bI.$$

Modified SIR model: [7]

In this modified SIR model, we are splitting the factor 'Susceptible' into four parts according to the physiological and psychological aspects of human beings reacting to the pandemic at different stages.

S_ign – "Ignorant people that does not know anything or worry about the epidemic;
S_al – people in the "Alarm phase";
S_res – people in "Resistance" state, who takes necessary steps very rationally to prevent disease from inflicting on them;
S_exh –people in "Exhaustion" state where they are tired of the epidemic, behave really unsafe and do not react on alarm stimuli.

$$S = S_{ign} + S_{al} + S_{res} + S_{exh}$$

At the initial stage, let us consider $S(0) = S_{ign}(0)$. This is due to the fact that people are unknown or ignorant of the intensity of the disease during the initial stage.
Therefore we can consider $S_{al}(0) = S_{res}(0) = S_{exh}(0) = 0$

We are considering the alarm phase partially in S_{ign} and S_{res} , therefore we have a three phase susceptible model which can be written mathematically below as;

$$S = S_{ign} + S_{res} + S_{exh}$$

Now we can modify the SIR model (S → I → R) with the newly introduced psychological factors in susceptible

$$S_{ign} \rightarrow S_{res} \rightarrow S_{exh} \rightarrow S_{ign}$$

With the Mass Action Law formalism, SIR with this additions is;
SIR:

S_ign + I →2I ; I → R

Where *I* is infected population and *R* is removed (recovered and immune or dead) population.

Stress reaction:

S_ign+I → S_res+I;
S_res → S_exh;
S_exh → S_ign.

The people in state S_exh have the same vulnerability as the ignorant people have so we have introduced the reaction S_exh + I → 2I with the same reaction rate constants. The difference between Sign and S_exh is in the absence of the transition S_exh + I → S_res + I – people in "Exhaustion" state that does not move directly to resistant state.

Reactions	Reaction rate	Stoichiometric vector
S_ign+I→2I	r1=aS_ign I	$(-1, 0, 0, 1, 0)^T$
S_ign+I→S_res+I	r2=k2 S_ign I	$(-1, 1, 0, 0, 0)^T$
S_res→S_exh	r3=k3 S_res	$(0, -1, 1, 0, 0)^T$
S_exh+I→2I	r4=aS_exh I	$(0, 0, -1, 1, 0)^T$
I→R	r5=bI	$(0, 0, 0, -1, 1)^T$
S_exh→S_ign	r6=k6 S_exh	$(1, 0, -1, 0, 0)^T$

$$\frac{dc}{dt} = \sum_{\rho=1}^{6} \gamma_\rho r_\rho$$

$$\frac{dc}{dt} = \begin{matrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \end{matrix} r_1 + \begin{matrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{matrix} r_2 + \begin{matrix} 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{matrix} r_3 + \begin{matrix} 0 \\ 0 \\ -1 \\ 1 \\ 0 \end{matrix} r_4 + \begin{matrix} 0 \\ 0 \\ 0 \\ -1 \\ 1 \end{matrix} r_5 + \begin{matrix} 1 \\ 0 \\ -1 \\ 0 \\ 0 \end{matrix} r_6$$

$$
= \begin{matrix}
-aS_{ign}I & -k_2S_{ign}I & & & & k_6S_{exh} \\
 & k_2S_{ign}I & -k_3S_{res} & & & \\
 & & k_3S_{res} & -aS_{exh}I & & -k_6S_{exh} \\
aS_{ign}I & & & aS_{exh}I & -bI & \\
 & & & & bI &
\end{matrix}
$$

$$\frac{dS_{ign}}{dt} = -aS_{ign}I - k_2S_{ign}I + k_6S_{exh}$$

$$\frac{dS_{res}}{dt} = k_2S_{ign}I - k_3S_{res}$$

$$\frac{dI}{dt} = aS_{ign}I + aS_{exh}I - bI$$

$$\frac{dR}{dt} = bI$$

The Stoichiometric conservation laws,

$$\sum_{i=1}^{5} \gamma_{i\rho} b_i = 0 \ \forall \rho.$$

$$-b_1 + b_4 = 0$$

$$-b_1 + b_2 = 0$$

$$-b_2 + b_3 = 0$$

$$-b_3 + b_4 = 0$$

$$-b_4 + b_5 = 0$$

$$-b_3 + b_1 = 0$$

rewriting all equations above as

$$b_1 = b_4 = b_2 = b_3 = b_5 = \beta$$

So, according to conservation law, let us use $\beta = 1/5$:

$$b_1 + b_2 + b_3 + b_4 + b_5 = 1$$

Or

$$S_{ign} + S_{res} + S_{exh} + I + R = 1$$

Therefore,

$$S_{exh} = 1 - S_{ign} - S_{res} - I - R$$

MODEL 1 - ANALYSIS USING LOGISTIC GROWTH MODEL

Here we use Logistic Growth Model, considering linear and exponential growth for the three countries under consideration.

Graph of cumulative cases of India is represented in Fig 1.1.

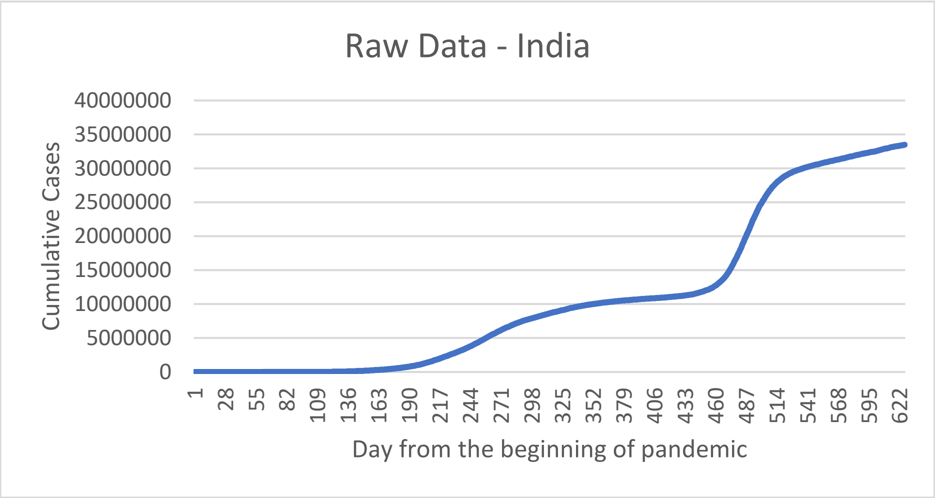


Fig 1.1 Cumulative covid-19 cases - India

The first case in India was reported on 30/01/2020.

Graph of cumulative cases of New Zealand is represented in Fig 1.2

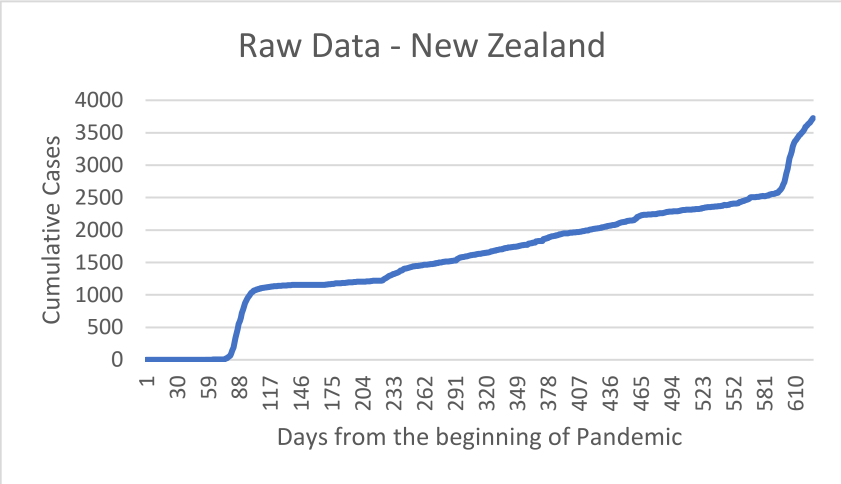


Fig 1.2 Cumulative covid-19 cases – New Zealand

The first case in New Zealand was reported on 28/02/2020.

Graph of cumulative cases of Somalia is represented in Fig1.3.

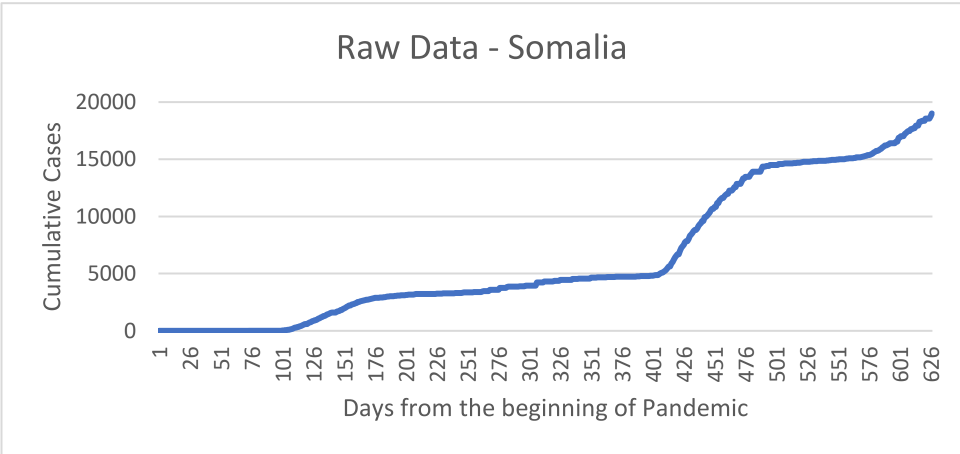


Fig 1.3 Cumulative covid-19 cases - Somalia

The first case in Somalia was reported on 16/03/2020.

Cumulative cases of all three countries are presented as a graph in fig 1.4.

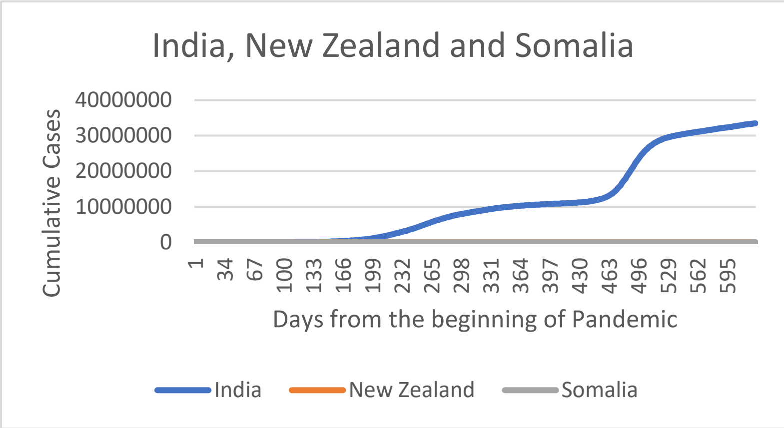


Fig 1.4 Joint graph of cumulative cases in India, New Zealand and Somalia

The number of covid cases in India gradually increased day by day and it was significantly higher when compared with the other two countries as it is nearly invisible in the graph.

As stated above, it was observed that 30/01/2020 as the beginning of epidemic in India , 28/02/2020 for New Zealand and 16/03/2020 for Somalia. Comparison of each country is not possible from figure 1.4 as the cases in India were comparatively high as compared to other two countries.

We have considered the cumulative cases as 107, 102 and 116 as the start of epidemic of India, New Zealand and Somalia respectively.

By considering the horizontal fragments, let us estimate a slope of line as slope of approximation of 11 consequent points. Let us denote 11 points as $i-5, i-4, \dots, i, \dots, i+5$ and values of observed (fraction of) population as P_j. Linear approximation of points is $\hat{P}_j = \alpha + \beta j$. We want to find the best β to approximate $P_j, j = i-5, \dots, i+5$. Let us calculate the quality of approximation as $Err = \sum_{j=i-5}^{i+5} \left(P_j - \hat{P}_j\right)^2 = \sum_{j=i-5}^{i+5} \left(P_j - \alpha + \beta j\right)^2$. This error function is convex and extremum point is point of minimum. This means that we need find β which provide zero values of derivatives

$$\frac{\partial Err}{\partial \alpha} = -2 \sum_{j=i-5}^{i+5} \left(P_j - \alpha - \beta j\right) = 0,$$

$$\frac{\partial Err}{\partial \beta} = -2 \sum_{j=i-5}^{i+5} j\left(P_j - \alpha - \beta j\right) = 0.$$

Now we can rewrite these equations as

$$\sum_{j=i-5}^{i+5} P_j - 11\alpha - 11i\beta = 0,$$

$$\sum_{j=i-5}^{i+5} jP_j - 11ia - (11i^2 + 110)\beta = 0.$$

(I used $i - 5 + i - 4 + \cdots + i + 5 = 11i$, and $(i - k)^2 + (i + k)^2 = i^2 - 2ik + k^2 + i^2 + 2ik + k^2 = 2i^2 + 2k^2$ or $(i - 5)^2 + (i - 4)^2 + \cdots + (i + 5)^2 = 11i^2 + 110$). Now we can multiply the first equation by i and subtract from the second:

$$\sum_{j=i-5}^{i+5} jP_j - 11ia - (11i^2 + 110)\beta - i \sum_{j=i-5}^{i+5} P_j + 11i\alpha + 11i^2\beta$$
$$= \sum_{j=i-5}^{i+5} jP_j - i \sum_{j=i-5}^{i+5} P_j - 110\beta = 0$$

And finally, we have $\beta = (\sum_{j=i-5}^{i+5} jP_j - i \sum_{j=i-5}^{i+5} P_j)/110$. Now we can calculate β for each i and select the closest to zero.

For India, we expect minima in the interval from 250 to 370 and in the interval 460 to 555. By checking the values of β, we found the end of the first wave at day 336 and the end of the wave 2 at the day 556. Figure 2.1 represents the normalized graph of India by taking the cumulative fraction

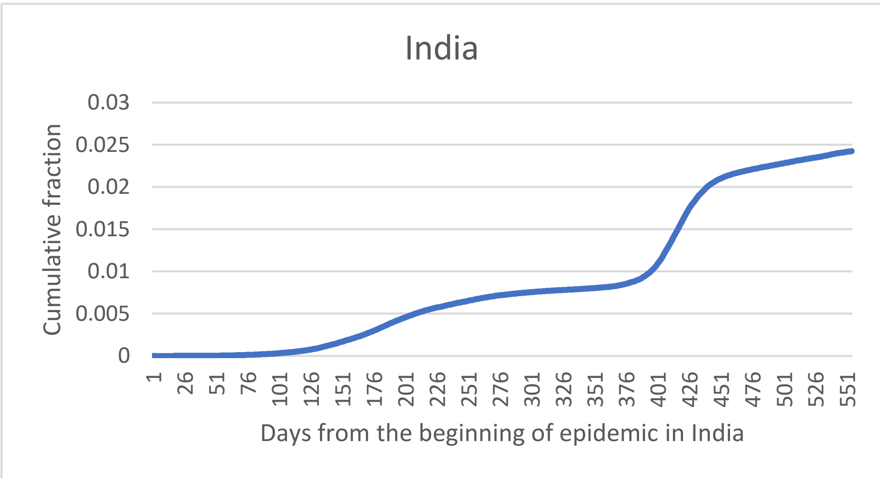


Fig 2.1 Cumulative fraction - India

For New Zealand it is from 32 to 80 and 390 to 471. By checking the values of β, we found end of the first wave at day 68 and end of second wave at the day 431 and the beginning of third wave from the day 432

Fig 2.2 represents the normalized graph of New Zealand by taking the cumulative fraction

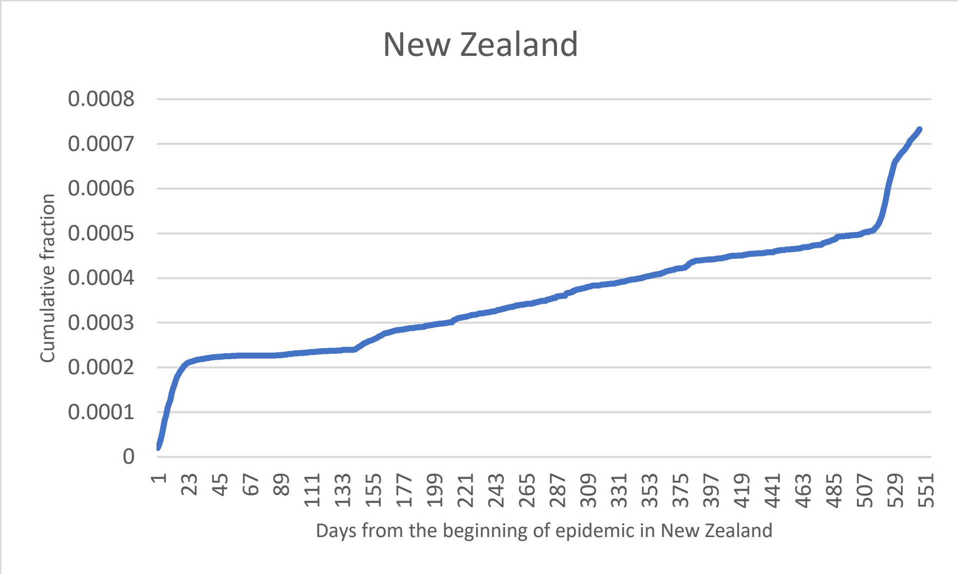


Fig 2.2 Cumulative fraction – New Zealand

By taking the almost horizontal heuristic for Somalia we have found that, the country had four pandemic waves. The horizontal fragments are from 68 to 160, from 230 to 300 and from 390 t0 465. By checking the values of β, we found end of the first wave at day 115, end of second wave at the day 279, end of the third wave at 407 and the starting of wave 4 from the day 408.

Fig 2.3 represents the cumulative fraction graph of Somalia from the beginning of epidemic in the country.

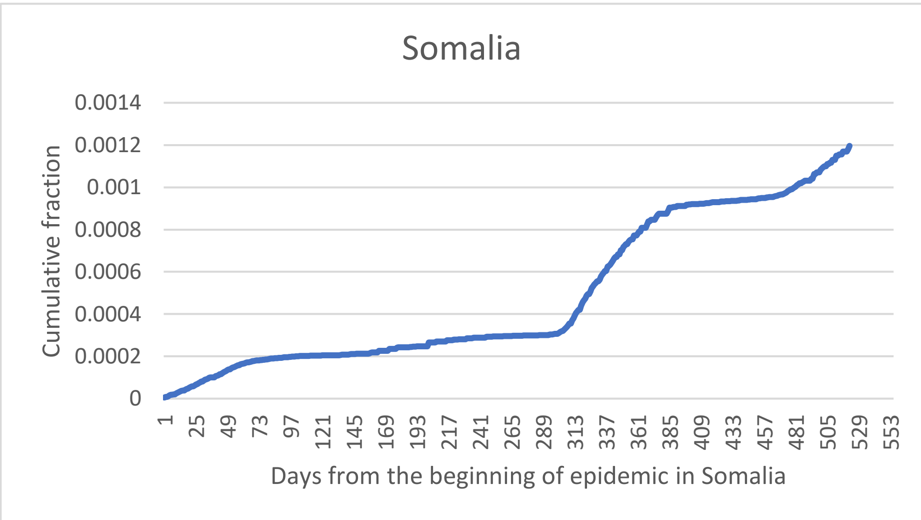


Fig 2.3 Cumulative fraction - Somalia

Fig 2.4 presents the normalized graph of all three countries (India, New Zealand , Somalia) represented together.

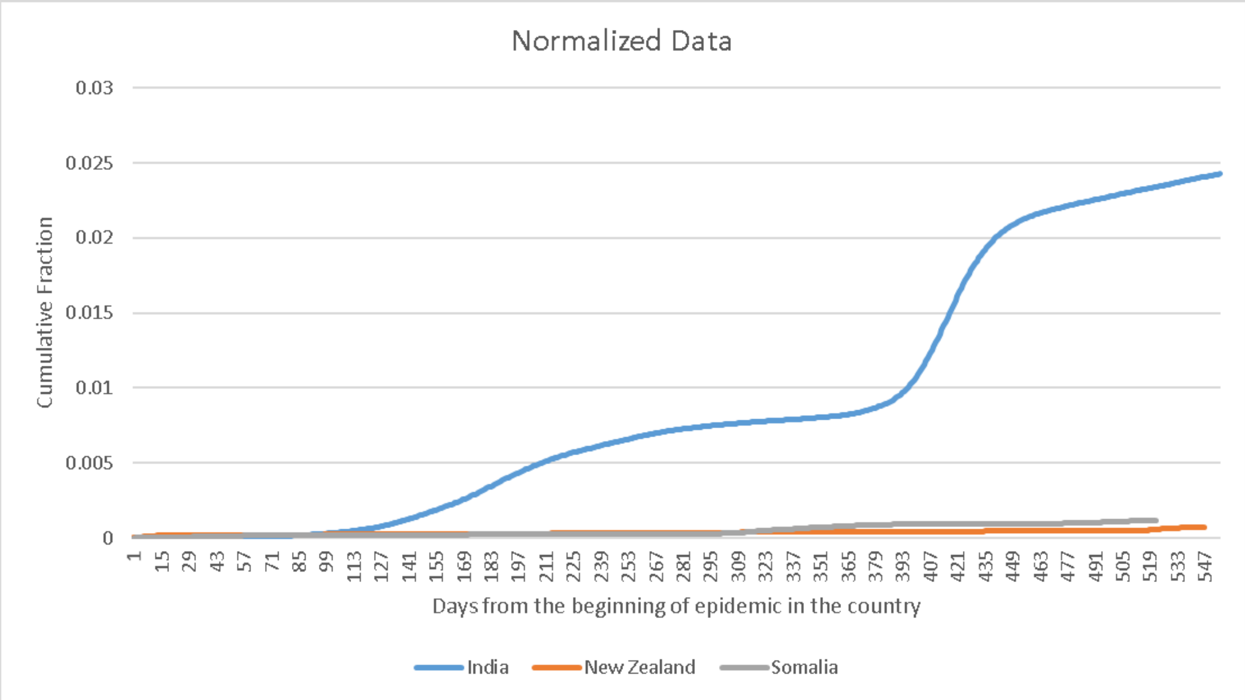


Fig 2.4 Normalized data for India, New Zealand and Somalia

As we can observe, even after normalizing the data, the cumulative fraction of India is very large when compared to the other two countries.

Summary of our study

Table 1: Waves selected by almost horizontal fragments.

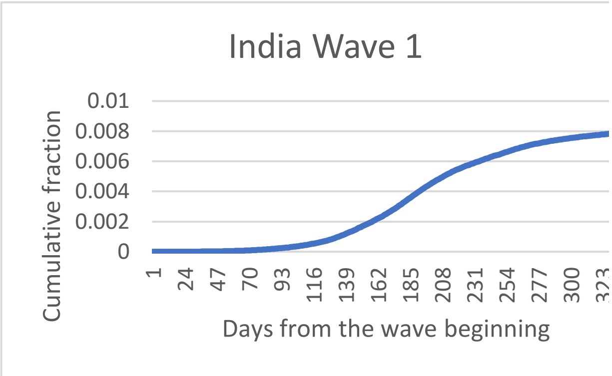
Wave	India		New Zealand		Somalia	
	Almost Horizontal		Almost Horizontal		Almost Horizontal	
	From	To	From	To	From	To
1	1	335	1	68	1	115
2	336	555	69	431	116	279
3			432	547	280	407
4					408	522

Table 1

INDIA:

First wave of India

The first wave for India was estimated by the almost horizontal fragments. The fig 3.1 and 3.2 represents the graph of normalized fractions and logarithm of these values respectively. Approximating from the figure, we can see that the exponential interval is from 30 to 120. By using this we can estimate the parameters of model.



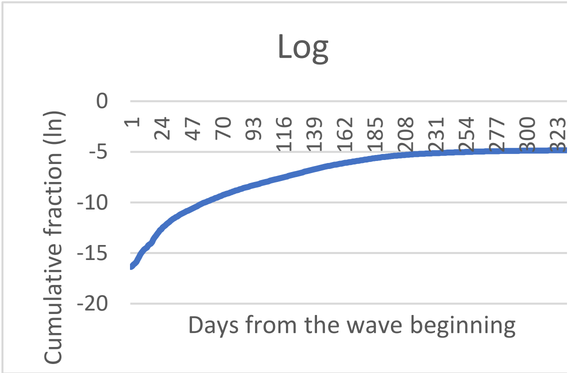
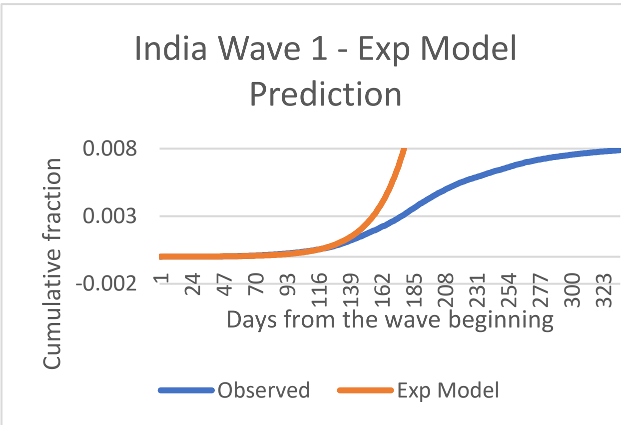


Fig 3.1 Cumulative Fraction

Fig 3.2 Log graph of cumulative fraction

As from the fig 3.2, the exponential model with parameters $a = -12.5968$ and $r = 0.043584$ work only for 120 days.



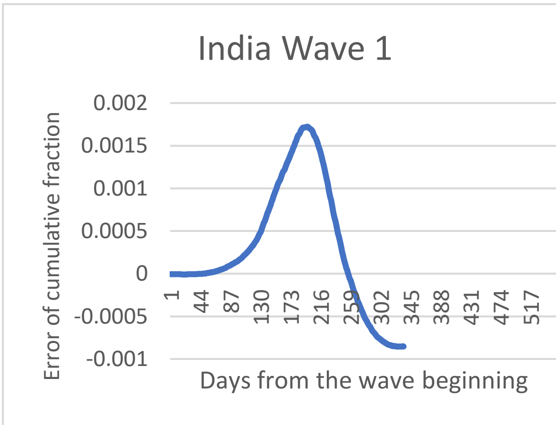
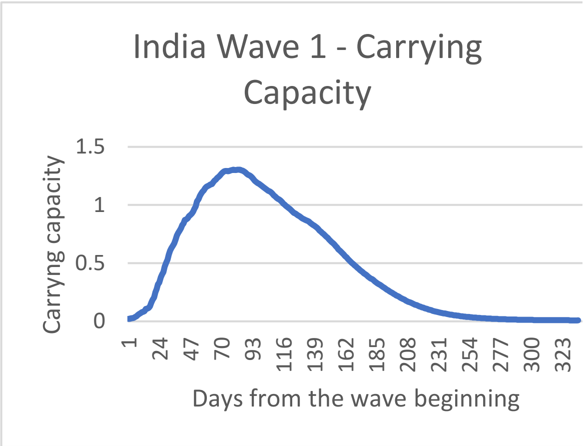


Fig 3.3 Cumulative fraction of exponential model prediction

Fig 3.4 Error of prediction

Now we will calculate the Carrying capacity *K* by using the obtained parameters. As from the fig 3.5, we can see that the value of *K* is small in the beginning. But this value increases to a higher value on day 81 and then decreases hence becoming a constant. We will now consider the value *K* = 0.0089 as the initial estimation and estimate parameters of logistic grows for logistic model.



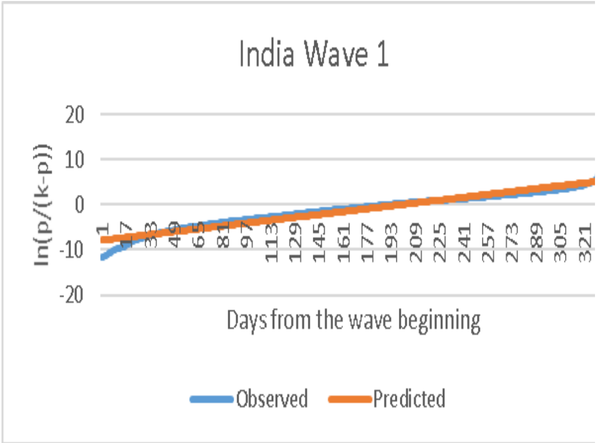
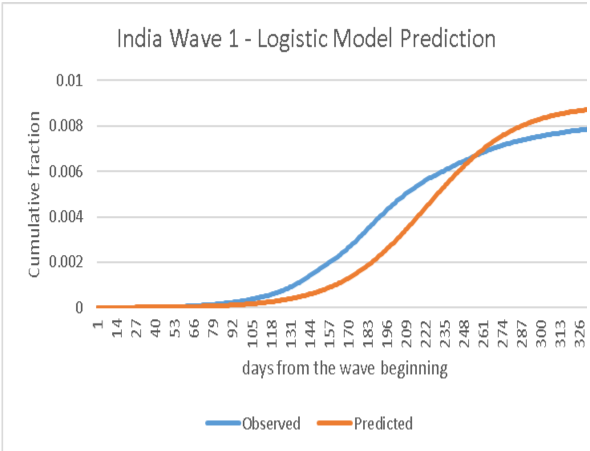


Fig 3.5 Carrying capacity as function of time

Fig 3.6 Estimation of logistic model parameters



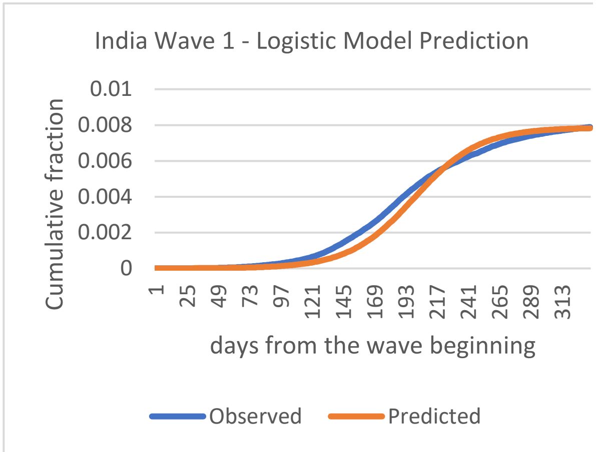


Fig 3.7 Logistic model prediction for initial value of K

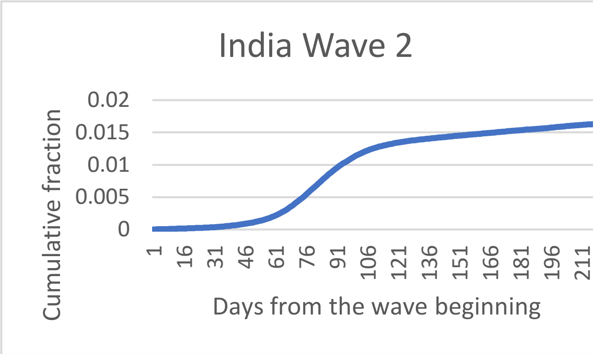
Fig 3.8 Logistic model prediction for optimal value of K

Model	a	r	K	SSE
Exponential	-12.6	0.0436	NA	657.192
Logistic (initial K)	-7.464	0.0334	0.0089	2.187x10^-4
Logistic (optimal K)	-7.938	0.0397	0.00786	4.7191x10^-5

Table 2

Second wave of India

As from the almost horizontal fragments, the second wave in India was started from 336 to 555. The fig 4.1 and 4.2 represents the graph of the normalized fractions and the logarithm of these values respectively. As an approximation, we can see that the exponential interval is from 1 to 95. By using this we can estimate the value of parameters of model.



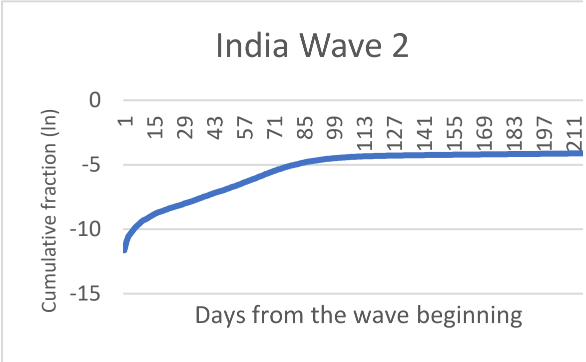
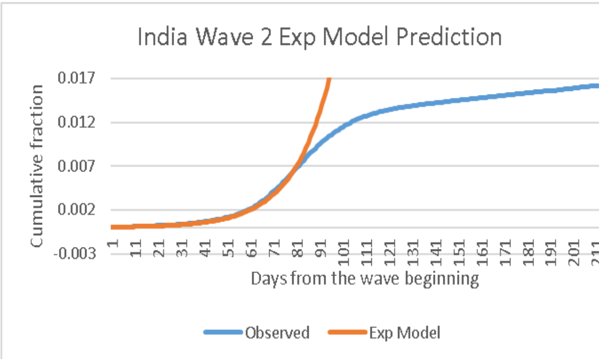


Fig 4.1 Normalized Cumulative fraction

Fig 4.2 Log Graph of cumulative fraction

As from the fig 4.2, the exponential model with parameters *a = -9.99604* and *r = 0.062296* work only for 95 days.

Now we will calculate the Carrying capacity *K* by using the obtained parameters. From fig 4.4, we can see that the value of *K* being smaller in the beginning started to increase steadily and became a constant on reaching day 120. We will now consider the value *K* = 0.01678 as the initial estimation and estimate parameters of logistic growth for the logistic model.



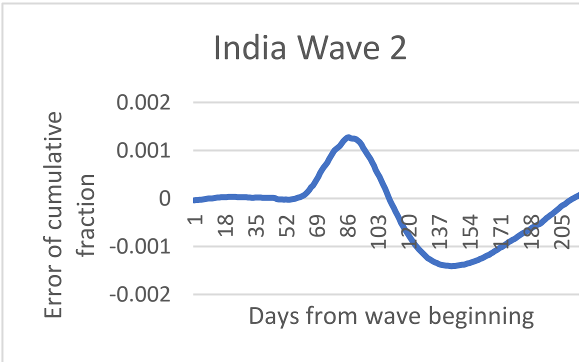
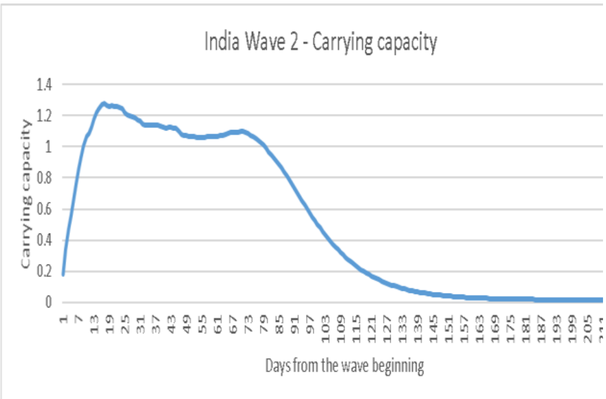


Fig 4.3 Cumulative fraction of Exponential model

Fig 4.4 Error in prediction



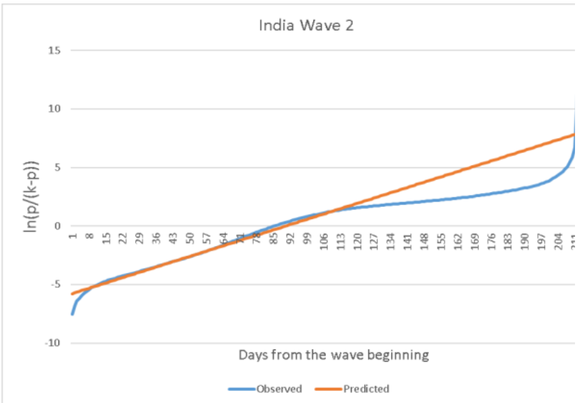
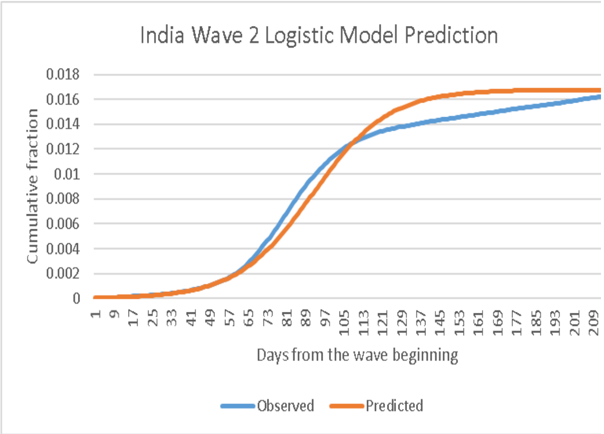


Fig 4.5 Carrying Capacity as a function of time

Fig4.6 Estimation of logistic model parameters



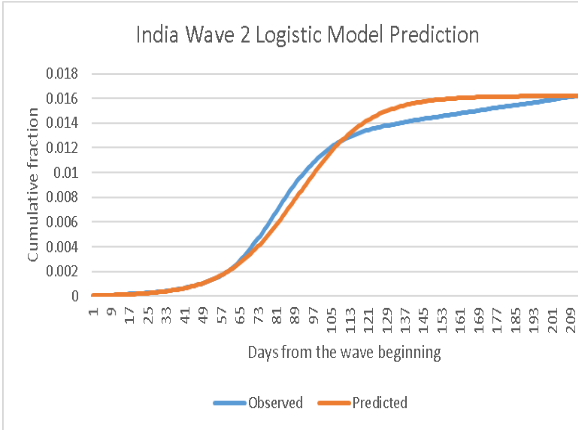


Fig 4.7 Logistic model prediction for initial value of K   Fig 4.8 Logistic model prediction for optimal value of K

Model	a	r	K	SSE
Exponential	-9.966	0.0623		14201.54
Logistic (initial K)	-5.83	0.06345	0.01678	0.00025
Logistic (optimal K)	-5.8376	0.064715	0.0162	0.00013

Table 3

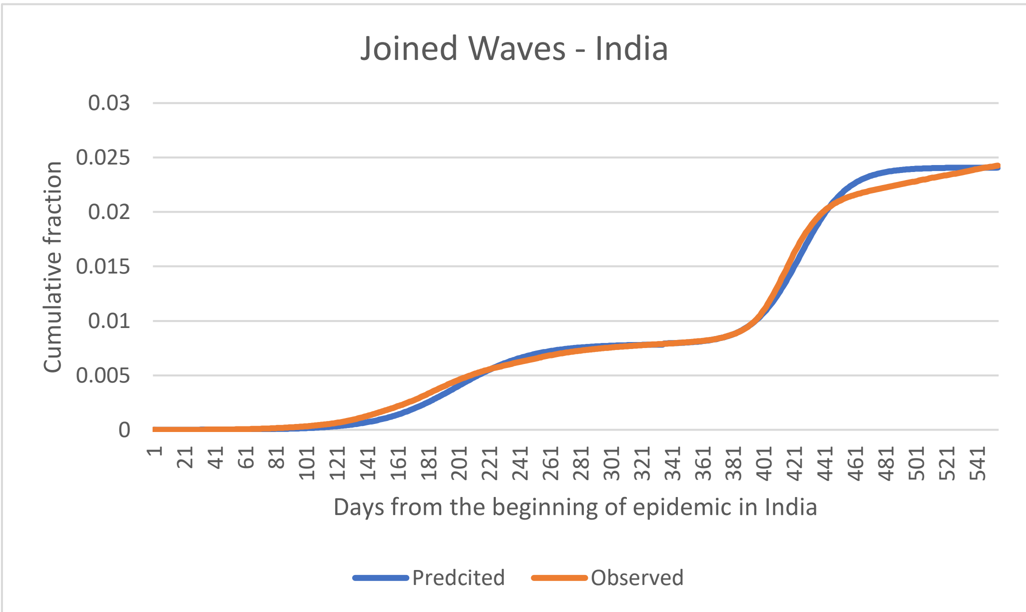


Fig 4.9 Joined Waves – Logistic Model

NEW ZEALAND:

First wave of New Zealand

As from the almost horizontal fragments, the first wave of the epidemic started from day 1 to 79 . The fig 5.1 and 5.2 represents the graph of normalized fractions and their logarithm respectively. Approximating from the graph, the exponential interval is from 1 to 12. Hence we can find the parameters of model by using this.

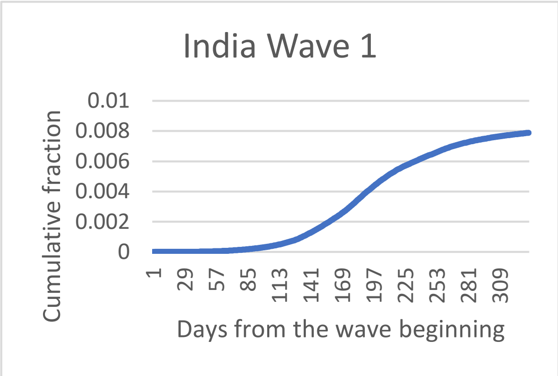


Fig 5.1 Cumulative Fraction

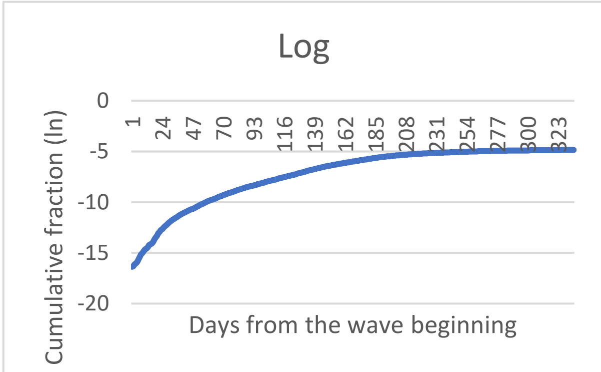


Fig 5.2 Log graph of cumulative fraction

As from the figure 5.2, the exponential model with parameters *a = -10.7614* and *r = 0.1923* work only for the days 1 to 12. By using the values of the parameters, we will calculate the Carrying capacity *K.* The fig 5.4 shows that the value of *K* is increasing till it reaches day 7 and then it decreases and remains constant from the day 35. We will now consider the value *K* = 0.000245 as the initial estimation and estimate parameters of logistic grows for logistic model.

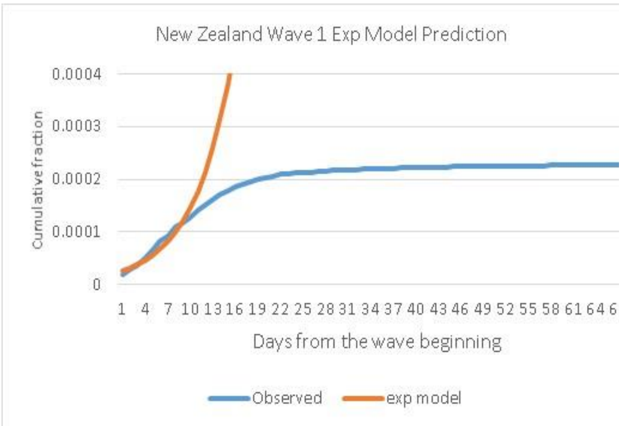


Fig 5.3 Cumulative fraction for exponential model

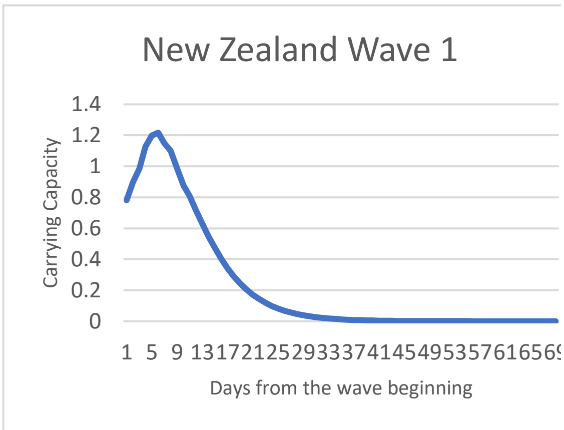
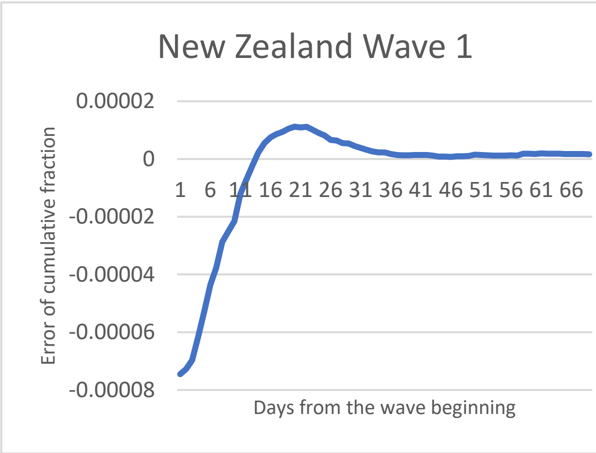


Fig 5.4 Carrying capacity as function of time



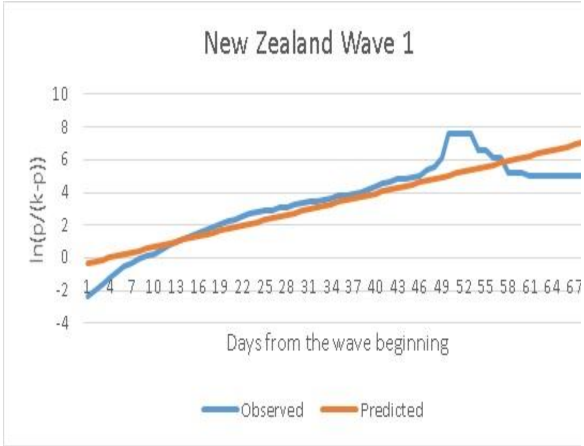
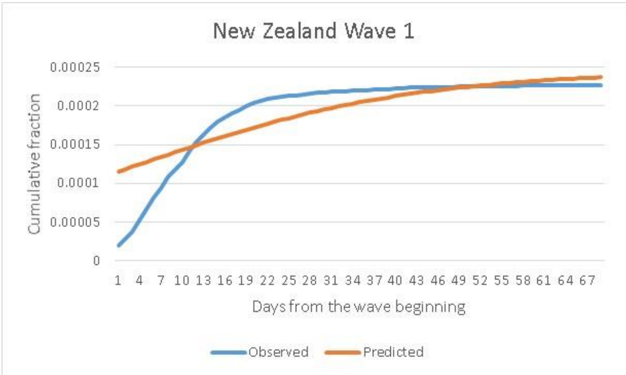


Fig 5.5 Error in prediction

Fig 5.6 Estimation of logistic model parameters



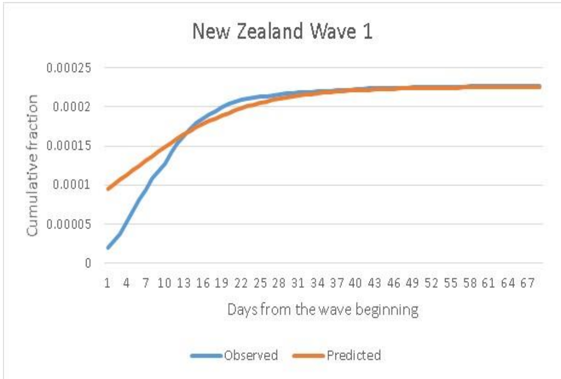


Fig 5.7 Logistic model prediction for initial value of K

Fig5.8 Logistic model prediction for optimal value of K

Model	a	r	K	SSE
Exponential	-10.7614	0.1923		473.7087
Logistic (initial K)	-0.16947	0.051049	0.000245	5.537x10^-8
Logistic (optimal K)	-0.4346	0.10969	0.000226	2.89x10^-8

Table 4

Second wave of New Zealand

The second wave of the pandemic in the New Zealand was from the day 70 till 431. The fig 6.1 and 6.2 represents the graph of normalized fractions and their logarithm respectively. Approximating from the graph, the exponential interval is from 25 to 85. Hence, we can find the parameters of model by using this.

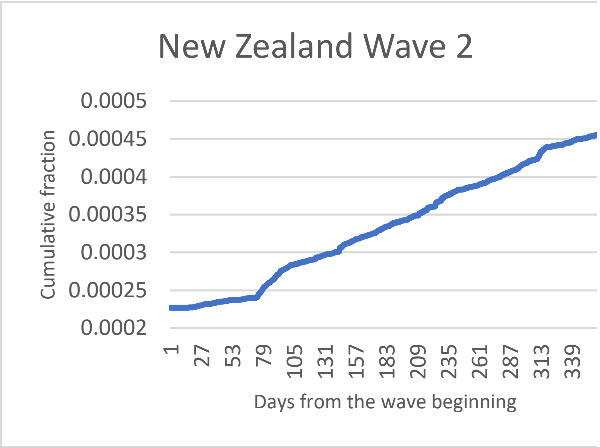


Fig 6.1 Cumulative fraction

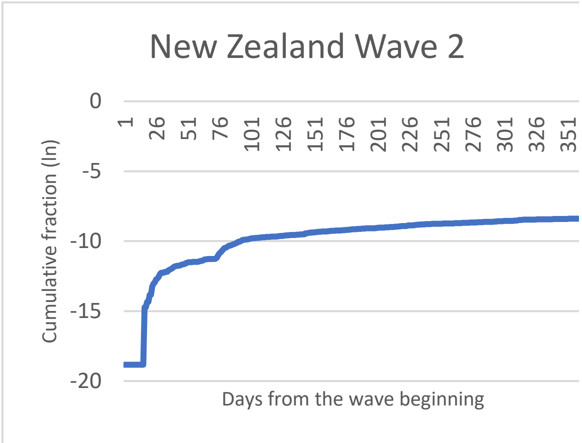


Fig 6.2 Log graph of cumulative fraction

According to the fig 6.3, the exponential model with parameters *a = -13.3674* and *r = 0.03333* work only for the days between 25 and 85.

By using the values of the parameters, we will calculate the Carrying capacity *K.* The fig 6.4 shows that the value of *K* was smaller in the beginning, but after a steady increase it drops down and again increases steadily till the day 100. Then it drops down and became a constant from day 270. We will now consider the value *K* = 0.00025 as the initial estimation and estimate parameters of logistic grows for logistic model.

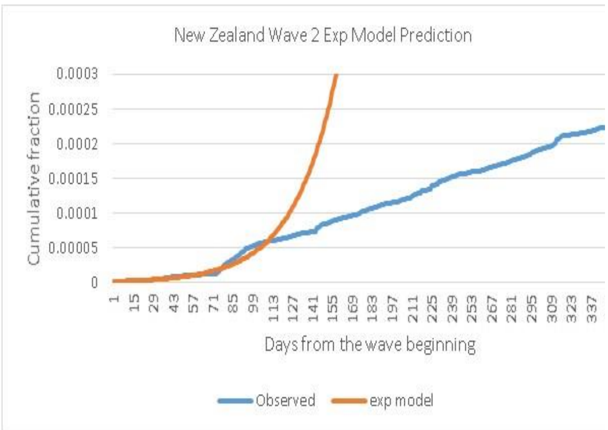


Fig 6.3 Cumulative fraction for Exponential model

Fig 6.4 Carrying capacity as a function of time

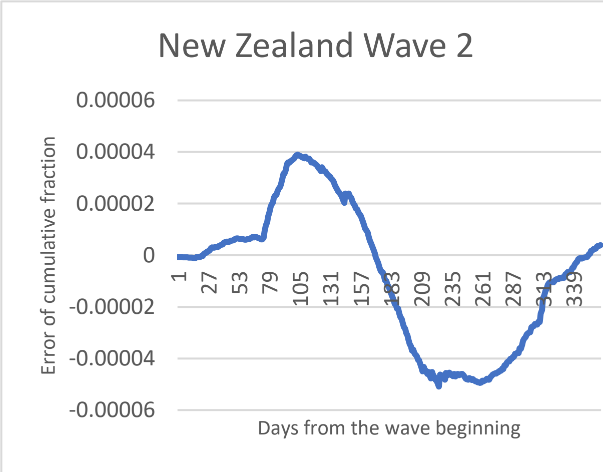


Fig 6.5 Error in prediction

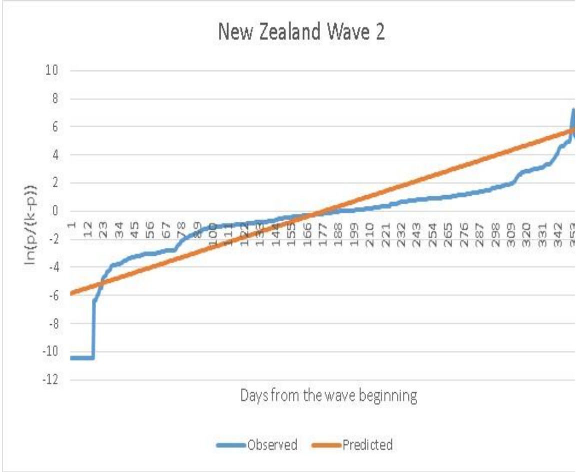


Fig 6.6 Estimation of logistic model parameters

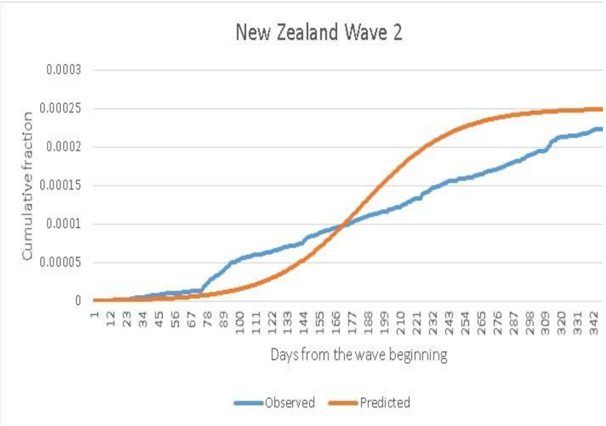


Fig 6.7 Logistic model prediction for initial value of K

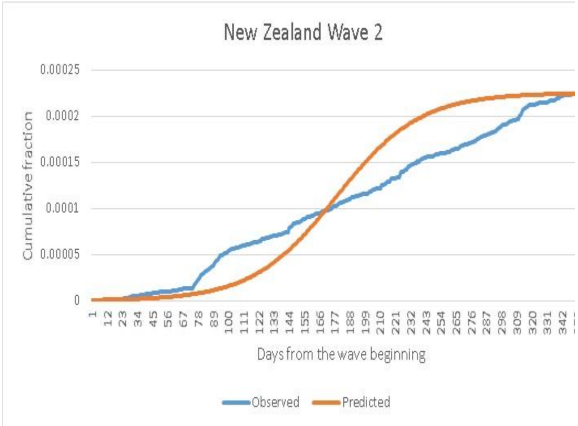


Fig 6.8 Logistic model prediction for optimal value of K

Model	a	r	K	SSE
Exponential	-13.3674	0.03333		1.1441
Logistic (initial K)	-5.9473	0.03229	0.00025	5.405x10^-7
Logistic (optimal K)	-5.8721	0.0329	0.000225	2.968x10^-7

Table 5

Third wave of New Zealand

The third wave of the pandemic in the New Zealand was during 432 and 547. The fig 7.1and 7.2 represents the graph of normalized fractions and their logarithm respectively. Approximating from the graph, the exponential interval is from 15 to 65. Hence, we can find the parameters of model by using this.

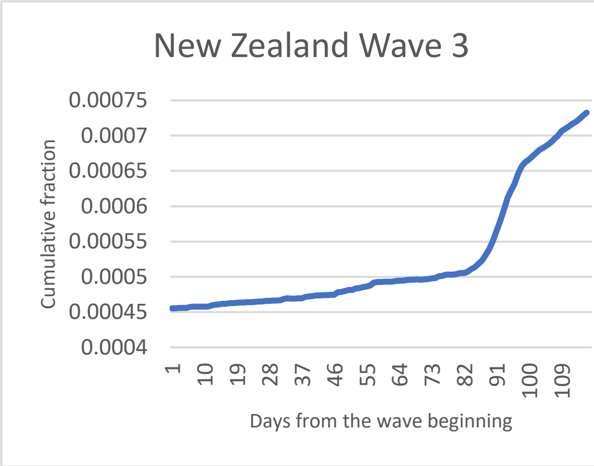


Fig 7.1 Cumulative fraction

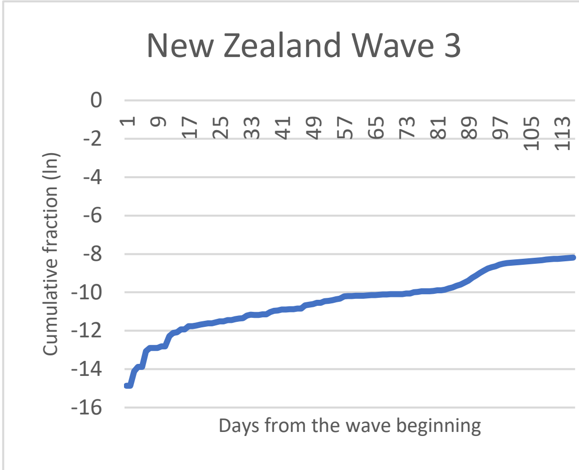


Fig 7.2 Log graph of cumulative fraction

According to the fig 7.3, the exponential model with parameters *a = -12.3116* and *r = 0.032875* work only for days between 15 and 65.

By using the values of the parameters, we will calculate the Carrying capacity *K.* The fig 7.4 shows that the value of *K* was a smaller value in the beginning, later this value started to increase steadily. This is because of the reason that New Zealand was one the best countries to control and restrict the epidemic from spreading. So, the daily new cases were controlled thereby giving an almost linear graph when the cumulative cases are taken into consideration. We will now consider the value *K* = 0.8812 as the initial estimation and estimate parameters of logistic grows for logistic model.

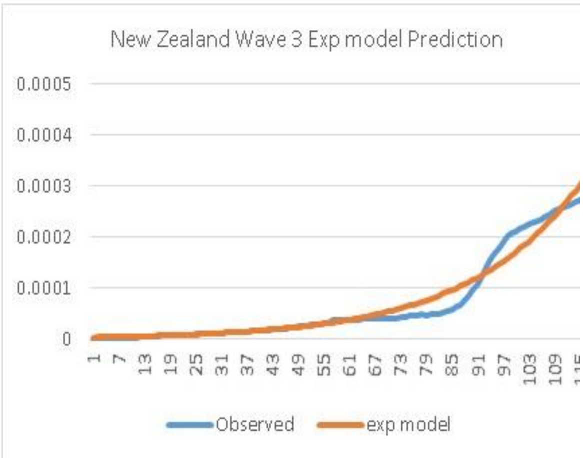


Fig 7.3 Cumulative fraction for exponential model

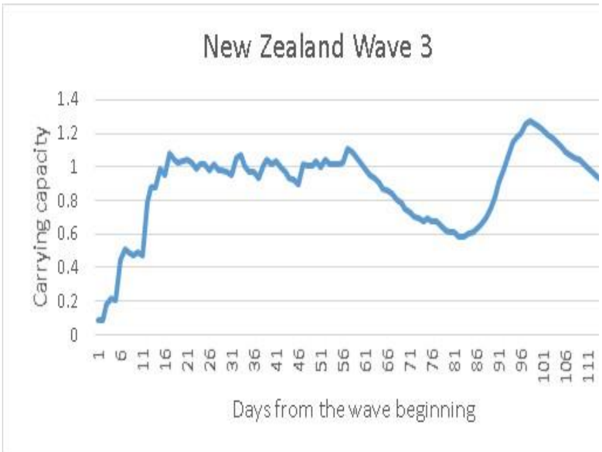
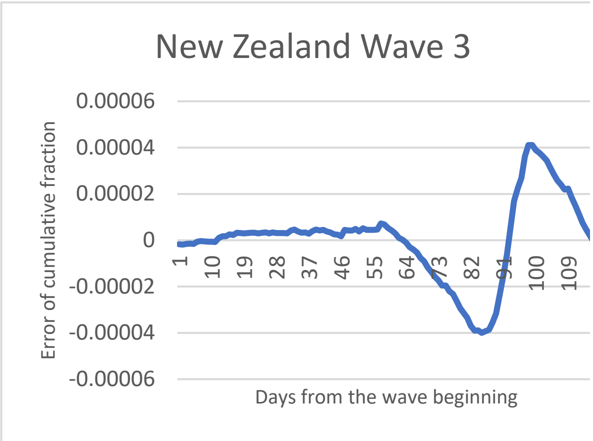


Fig 7.4 Carrying capacity as a function of time



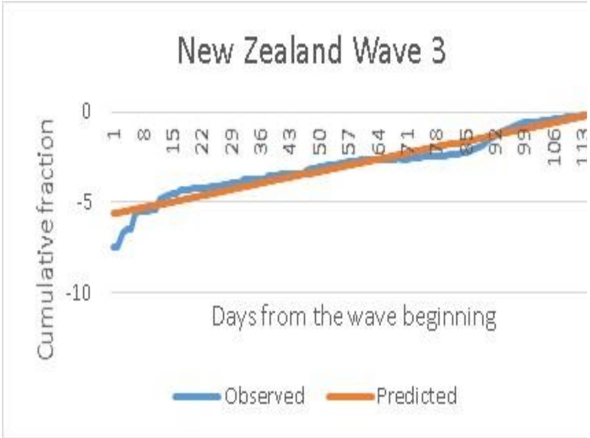
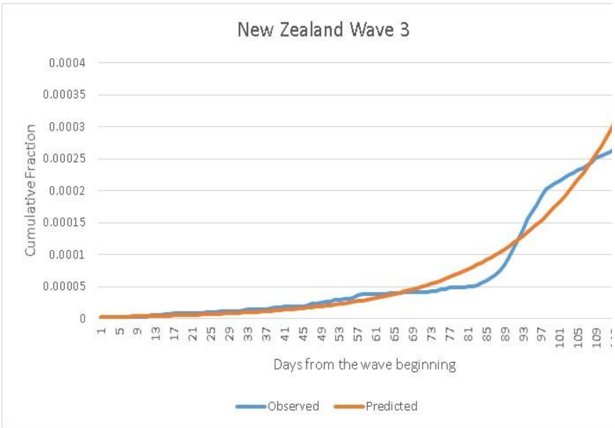


Fig 7.5 Error in prediction

Fig 7.6 Estimation of logistic model parameters



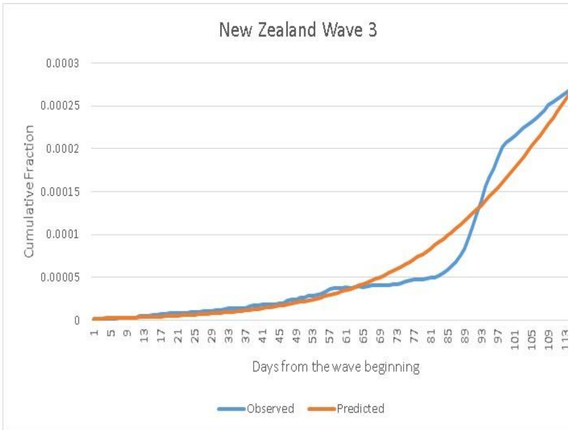


Fig 7.7 Logistic model prediction for initial value of K

Fig 7.8 Logistic model prediction for optimal value of K

Model	a	r	K	SSE
Exponential	-12.4848	0.0381		3.586x10^-8
Logistic (initial K)	-12.8473	0.04323	0.8812	3.9322x10^-8
Logistic (optimal K)	-5.6794	0.0477	0.0006	3.5865x10^-8

Table 6

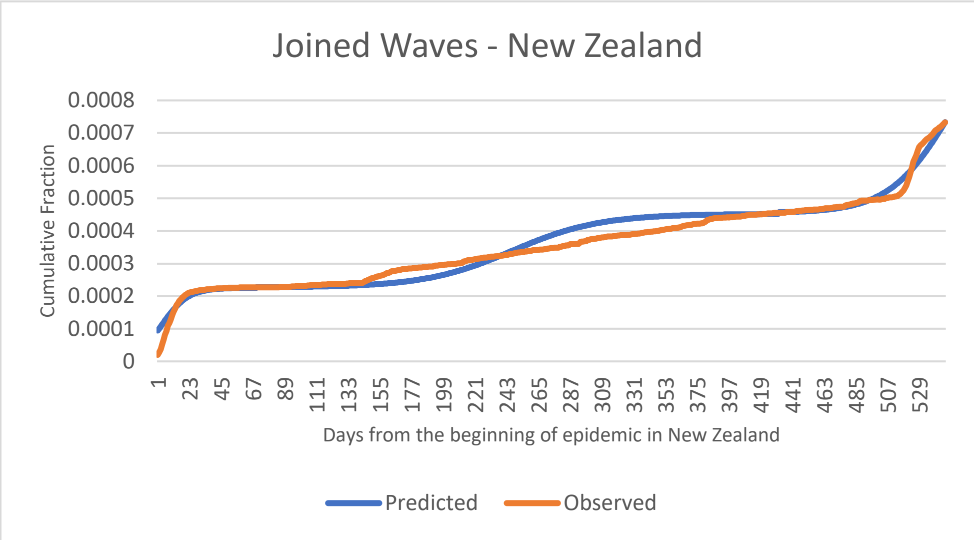


Fig 7.9 Joined Waves – Logistic Model

SOMALIA:

First wave in Somalia

The first wave in Somalia begun from day 1 to 115. The fig 8.1 and 8.2 represents the graph of normalized fractions and their logarithm respectively. Approximating from the graph, the exponential interval is from day 1 to 21. Hence, we can find the parameters of model by using this.

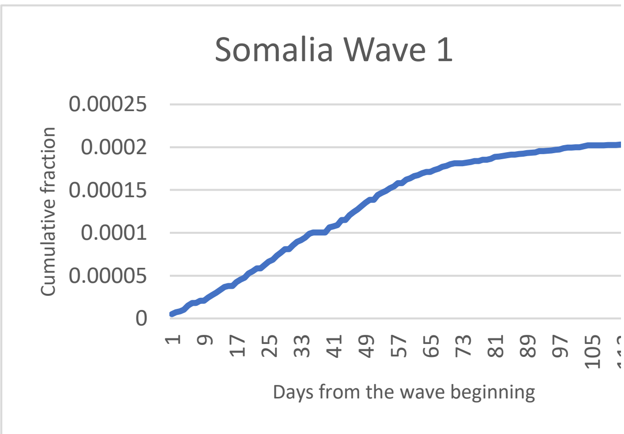


Fig 8.1 Cumulative fraction

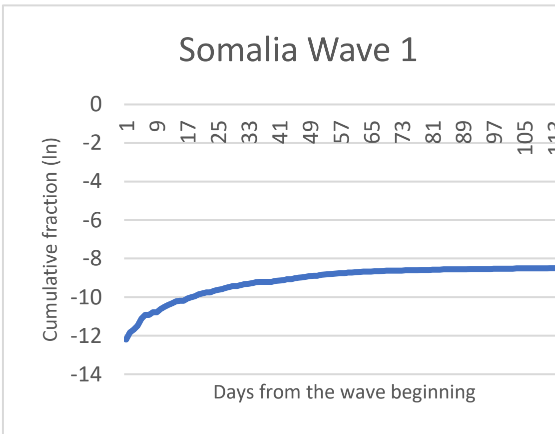


Fig 8.2 Log graph of cumulative fraction

As per the fig 8.3, the exponential model with parameters *a = -11.8491* and *r = 0.10934* work only for days 1 to 21

By using the values of the parameters, we will calculate the Carrying capacity *K.* The fig 8.4 shows that the value of *K* was a smaller value in the beginning, later this value started to increase steadily and then reaching to a constant value from day 69. We will now consider the value *K* = 0.000301 as the initial estimation and estimate parameters of logistic grows for logistic model.

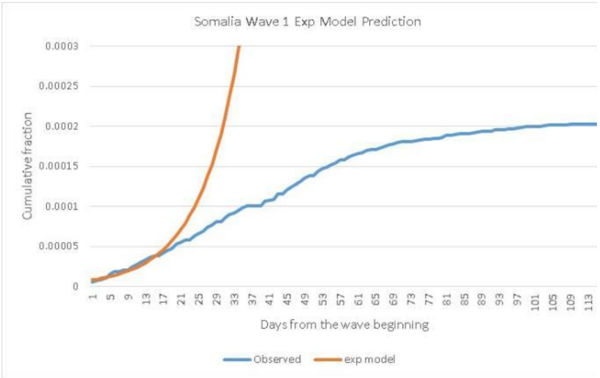


Fig 8.3 Cumulative fraction for exponential model

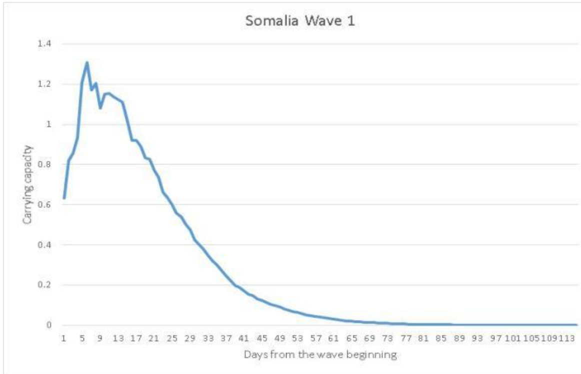


Fig 8.4 Carrying capacity as a function of time

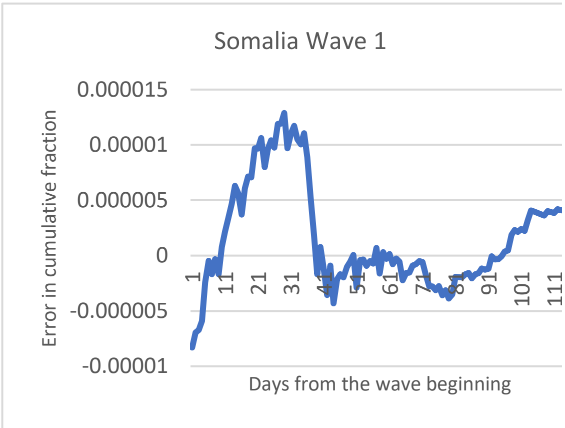


Fig 8.5 Error in prediction

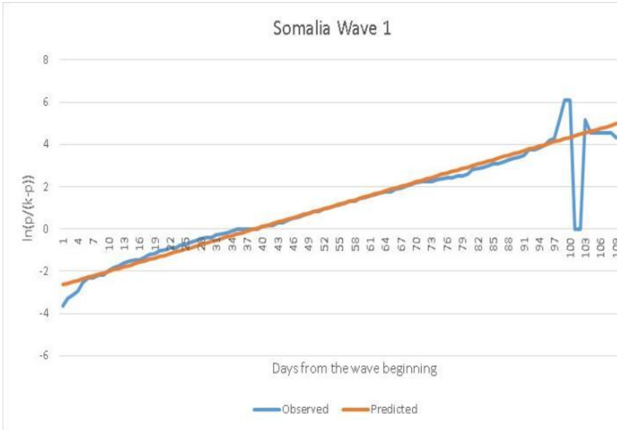


Fig 8.6 Estimation of logistic model parameters

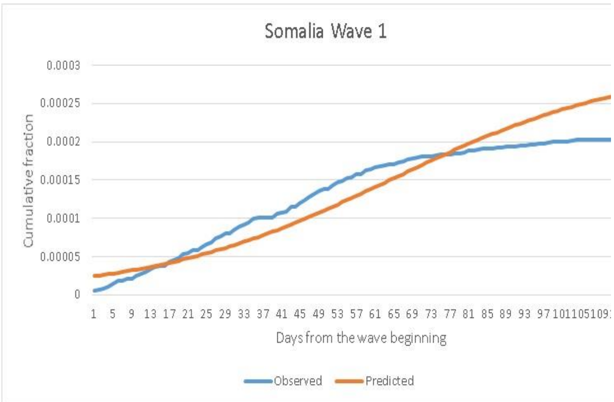


Fig 8.7 Logistic model prediction for initial value of K

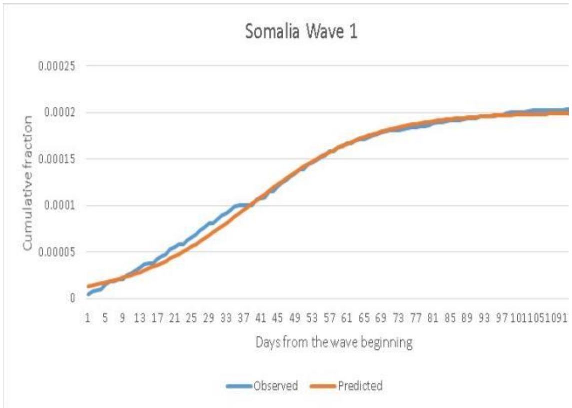


Fig8.8 Logistic model prediction for optimal value of K

Model	a	r	K	SSE
Exponential	-11.8491	0.10934		21.6961
Logistic (initial K)	-2.4782	0.03847	0.000301	8.1997x10^-8
Logistic (optimal K)	-2.70964	0.070482	0.0002	2.7923x10^-9

Table 7

Second wave of Somalia

The second wave in Somalia was started from 116 to 279. The fig 9.1 and 9.2 represents the graph of normalized fractions and their logarithm respectively. Approximating from the graph, the exponential interval is from 4 to 49. Hence, we can find the parameters of model by using this

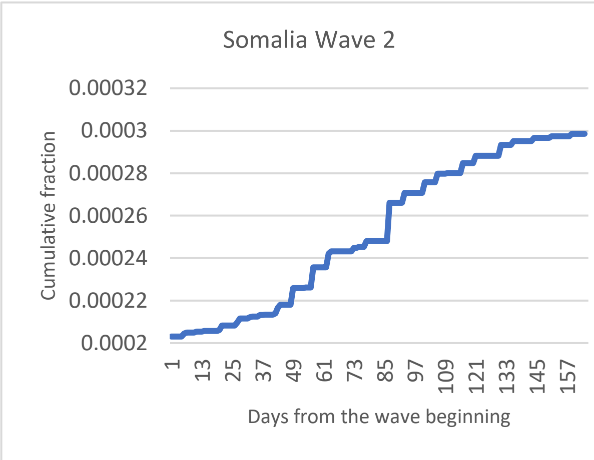


Fig 9.1 Cumulative fraction

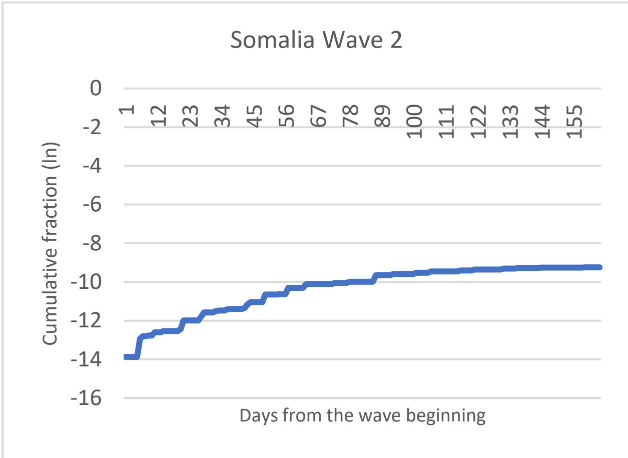


Fig 9.2 Log graph of cumulative fraction

As per the fig 9.3, the exponential model with parameters *a = -13.3657* and *r = 0.053077* work only for the days 4 to 49.

By using the values of the parameters, we will calculate the Carrying capacity *K.* The fig 9.4 shows that the value of *K* became a constant after several continuous fluctuations. We will now consider the value of *K* = 0.00103 as the initial estimation and estimate parameters of logistic grows for logistic model.

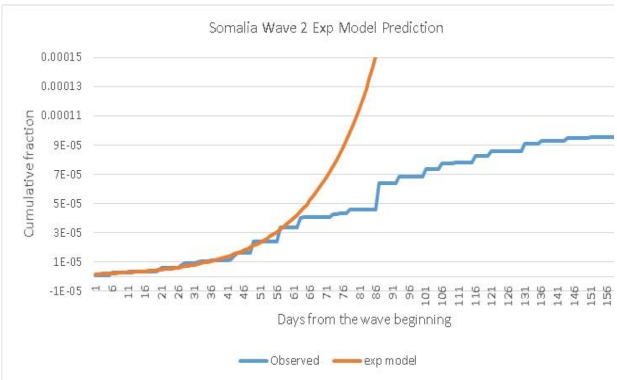


Fig 9.3 Cumulative fraction for Exponential model

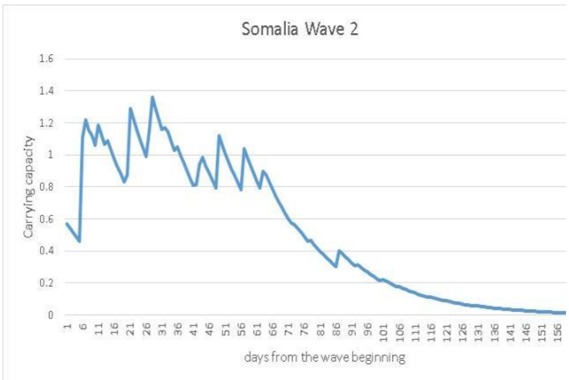
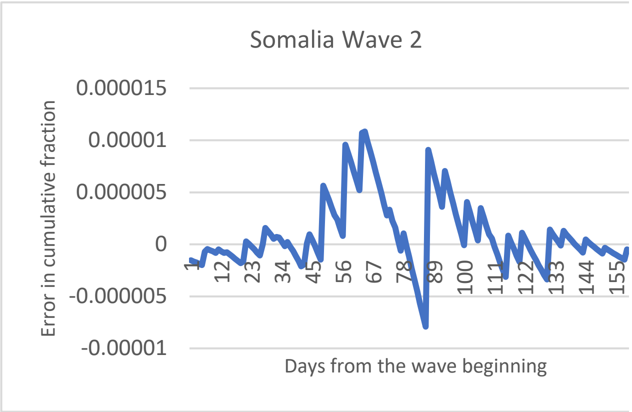


Fig 9.4 Carrying capacity as a function of time



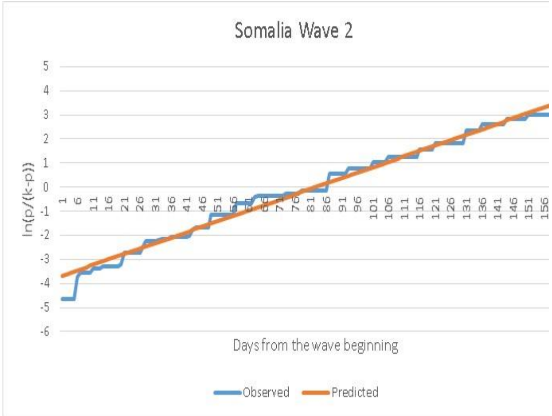
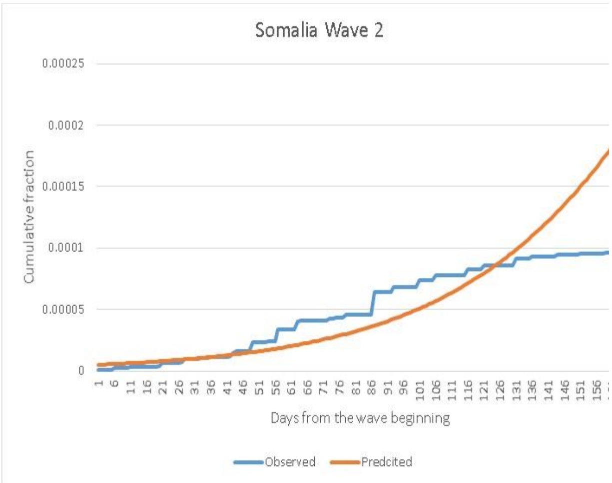


Fig9.5 Error in prediction

Fig 9.6 Estimation of logistic model parameters



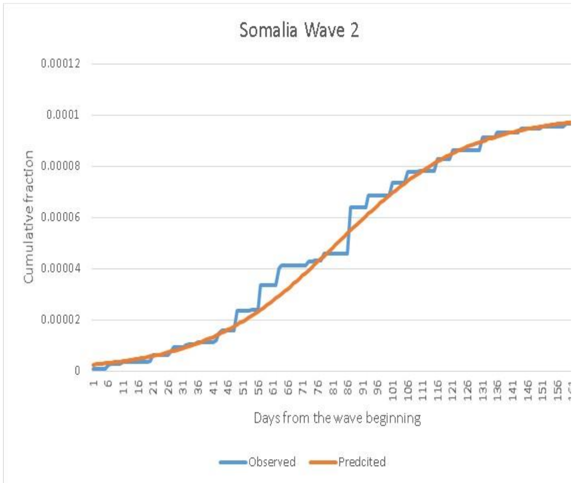


Fig9.7 Logistic model prediction for initial value of K

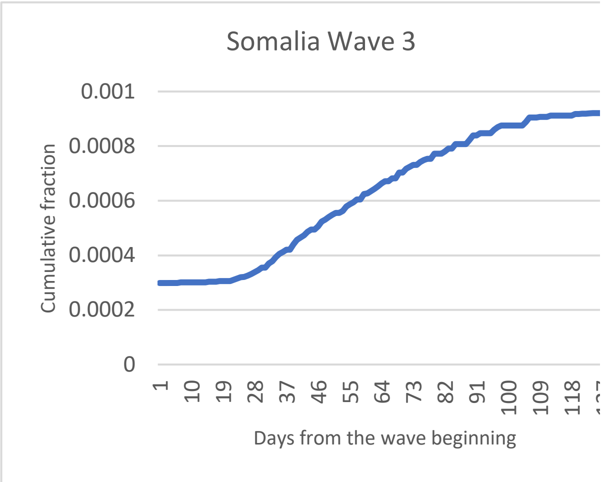
Fig 9.8 Logistic model prediction for optimal value of K

Model	a	r	K	SSE
Exponential	-13.3657	0.053077		0.000855
Logistic (initial K)	-5.35406	0.023728	0.00103	1.2153x10^-7
Logistic (optimal K)	-3.71951	0.045018	0.0001	1.8437x10^-9

Table 8

<u>Third wave of Somalia</u>

It is observed that the third wave in Somalia occurred during days 280 and 407. The fig 10.1 and 10.2 represents the graph of normalized fractions and their logarithm respectively. Approximating from the graph, the exponential interval is from 2 to 30. Hence, we can find the parameters of model by using this.



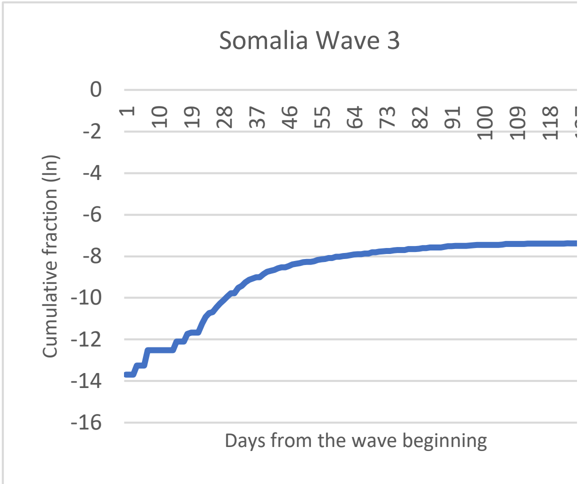
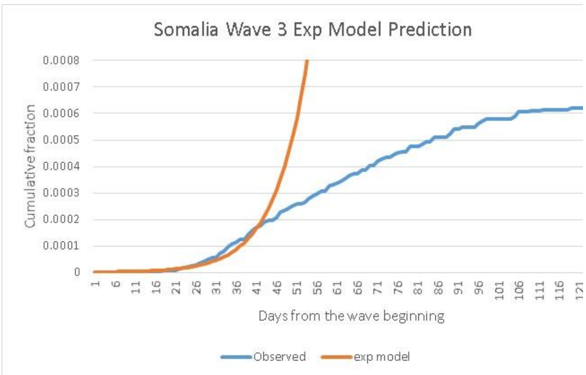


Fig 10.1 Cumulative fraction

Fig 10.2 Log graph of cumulative fraction

From fig 10.3, the exponential model with parameters *a = -13.91538* and *r = 0.12684* work only for days 2 to 30.

By using the values of the parameters, we will calculate the Carrying capacity *K.* From *fig* 10.4 we can see that the value of *K* increases and at day 73 it attains a constant value. We will now consider the value of *K* = 0.000684 as the initial estimation and estimate parameters of logistic grows for logistic model.



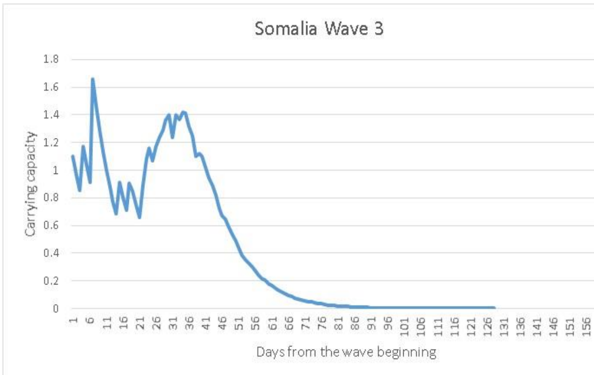
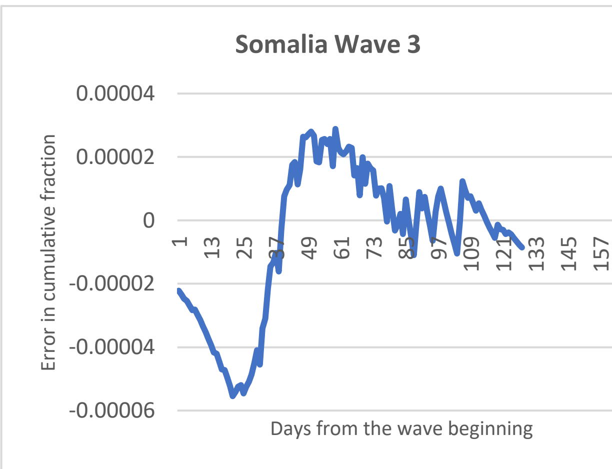


Fig 10.3 Cumulative fraction for exponential model

Fig 10.4 Carrying capacity as a function of time



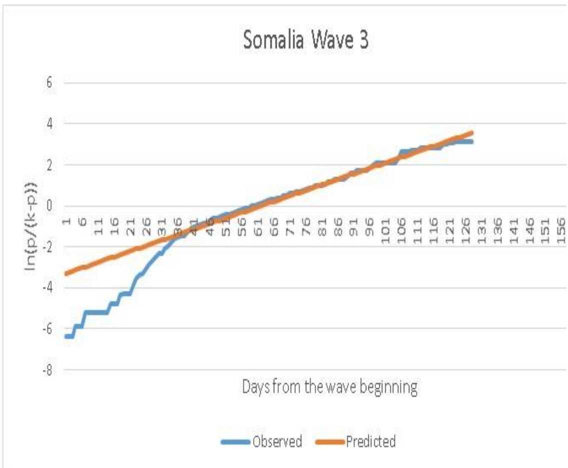
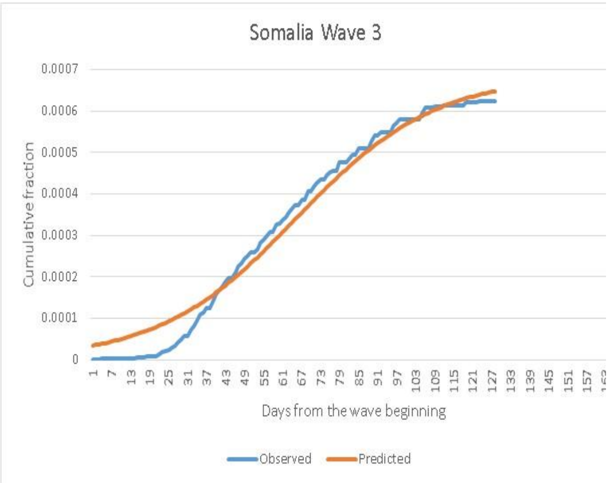


Fig10.5 Error in prediction   Fig10.6 Estimation of logistic model parameters



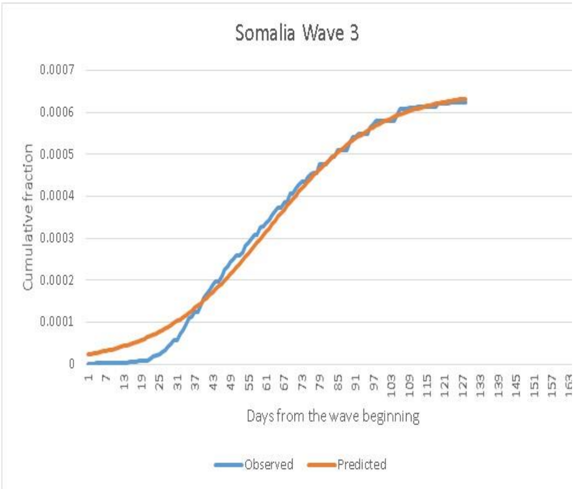


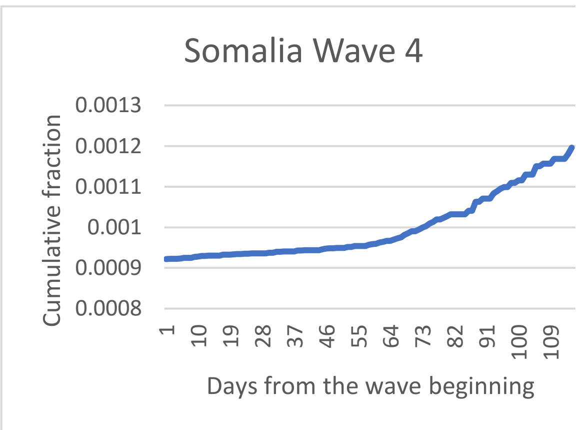
Fig 10.7 Logistic model prediction for initial value of K   Fig 10.8 Logistic model prediction for Optimal value of K

Model	a	r	K	SSE
Exponential	-13.91538	0.12684		462.5905
Logistic (initial K)	-3.00476	0.04586	0.000684	1.4098x10^-7
Logistic (optimal K)	-3.39704	0.05394	0.00065	7.432x10^-8

Table 9

Fourth wave of Somalia

It is observed that the fourth wave in Somalia occurred during the days 409 and 522. The fig11.1 and 11.2 represents the graph of normalized fractions and their logarithm respectively. Approximating from the graph, the exponential interval is from 1 to 12. As a result, we can find the parameters of model by using this.



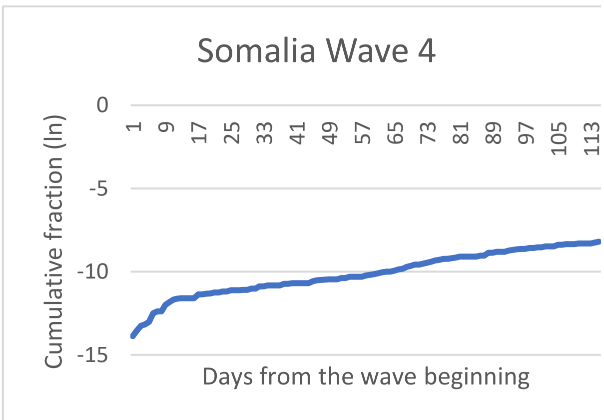


Fig 11.1 Cumulative fraction   Fig 11.2 Log graph of cumulative fraction

From fig 11.3, the exponential model with parameters *a = -13.9936* and *r = 0.216756* work only for days 1 to 12.

By using the values of these parameters, we will calculate the Carrying capacity *K.* From fig 11.4, we find that the value of *K* starts from a high value and increases steadily, then drops down and achieves a constant value from day 38. We will now consider the value of *K* = 0.000275 as the initial estimation and estimate parameters of logistic growth for logistic model.

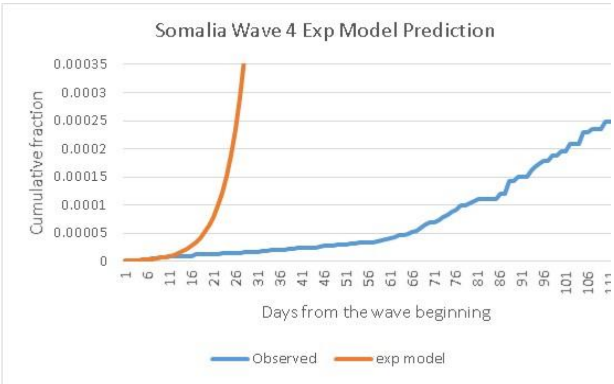


Fig 11.3 Cumulative fraction for exponential model

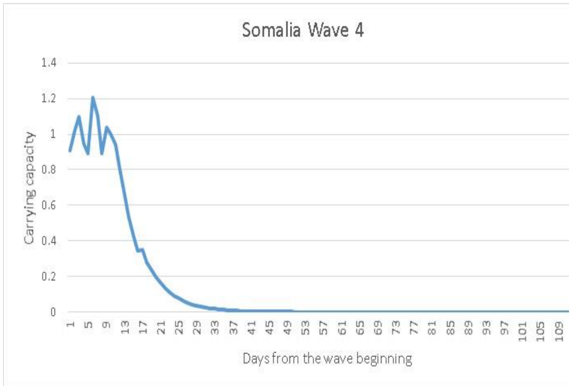


Fig 11.4 Carrying capacity as a function of time

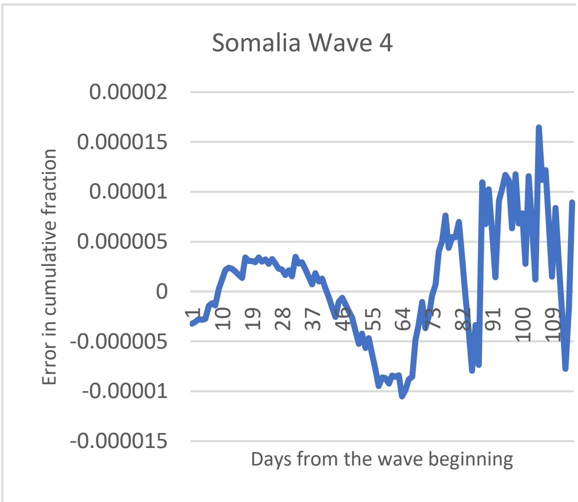


Fig 11.5 Error in Prediction

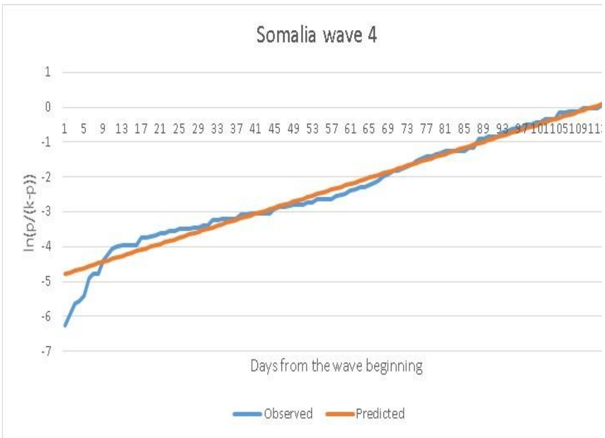
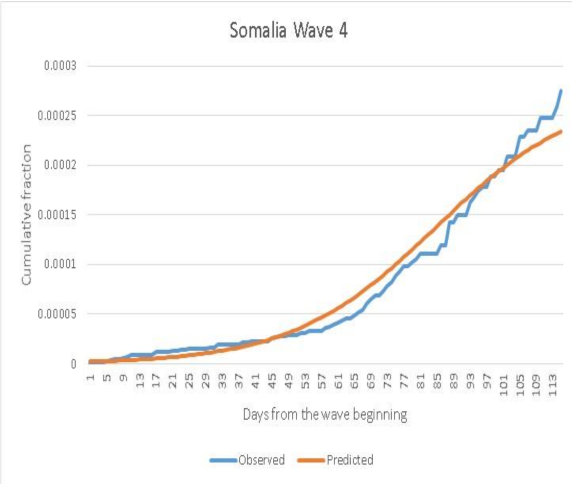


Fig 11.6 Estimation of logistic model parameters



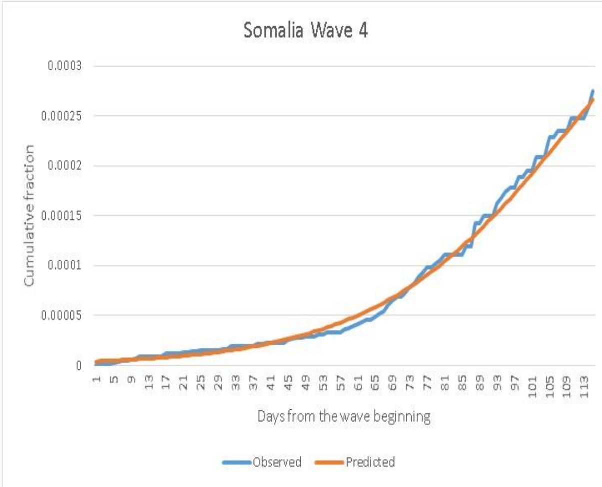


Fig 11.7 Logistic model prediction for initial value of K Fig 11.8 Logistic model prediction for optimal value of K

Model	a	r	K	SSE
Exponential	-13.9936	0.2167		8918x10^6
Logistic (initial K)	-4.87045	0.05746	0.000275	1.6403x10^-8
Logistic (optimal K)	-4.82195	0.04305	0.0005	3.8552x10^-9

Table 10

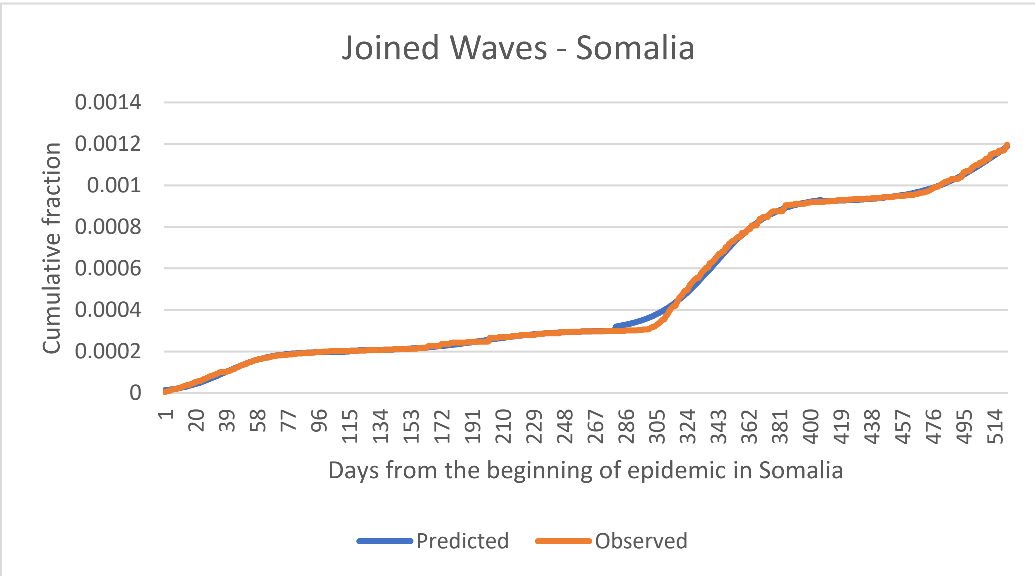


Fig 11.9 Joined Waves – Logistic Model

MODEL 2 - ANALYSING THE SIR MODEL

NEW ZEALAND

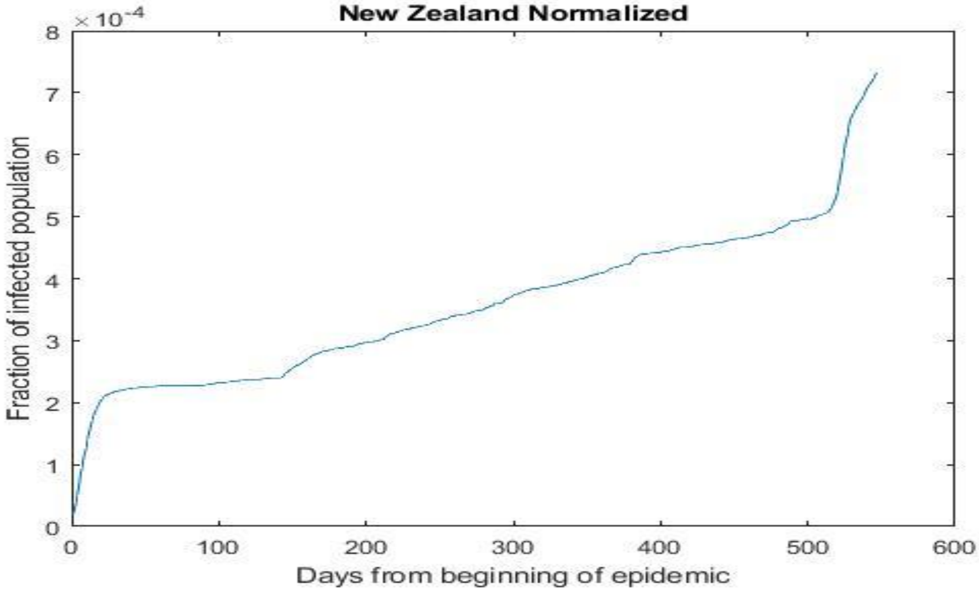


Fig 12.1 Normalized graph of New Zealand

The population of New Zealand is approximately 5.08 million. For the calculation of SIR model, we take the start as 100 cumulative cases.

The fig 12.1 represents normalized graph of New Zealand after the removal of initial fragments before the epidemic start. Here also we have removed the initial fragments of data before the epidemic start for a clear idea, as for each country the start of epidemic was at different times.

From the normalized cumulative fraction plot, we can observe that, after an initial exponential growth in covid cases, New Zealand was successful in preventing the corona from spreading out of control. Therefore, we can observe a linear growth of cumulative cases instead of a logistic growth model until the day 530, there after we can observe another exponential increase in cases.

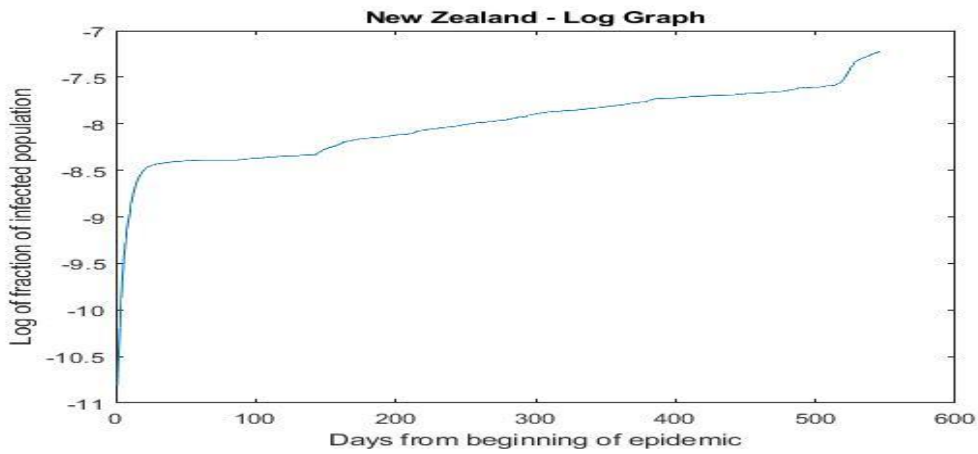


Fig 12.2 Log of cumulative fraction

The Fig 12.2 represents the logarithmic cumulative fraction of the normalized data of New Zealand. From the log graph of the cumulative fraction plotted, we can observe that the initial exponential growth is from days 1 to 20. The first covid-19 wave in New Zealand lasted for 69 days.

In the case of New Zealand, $R_{\circ}$ =12/5.08 million; Because the number of recovered cases 10 days before the considered start cases (ie. 100 cases), is 12 people.

In table.11, we can observe the parameters of the SIR model for each of the intervals as per the plotted graphs. According to the SIR model, ‘r’ is almost equal to the difference between the constants ‘a’ and ‘b’. And the value of the parameter ‘a’ is the sum of ‘r’ and ‘b’.

For all the intervals, we have taken the value of ‘b’ as 0.1 which implies that the number of days taken is 10.

Table 11. Estimation of parameters of SIR model of New Zealand

Interval	r	c	b	a	MSE
1-20	0.1089	-10.3031	0.1	0.2089	2.0205×10^{-4}
Optimal 1w			0.1	0.1196	7.3243×10^{-9}
Optimal all			0.1	0.1	1.4246×10^{-8}

Predicted SIR vs Observed (1 -20)

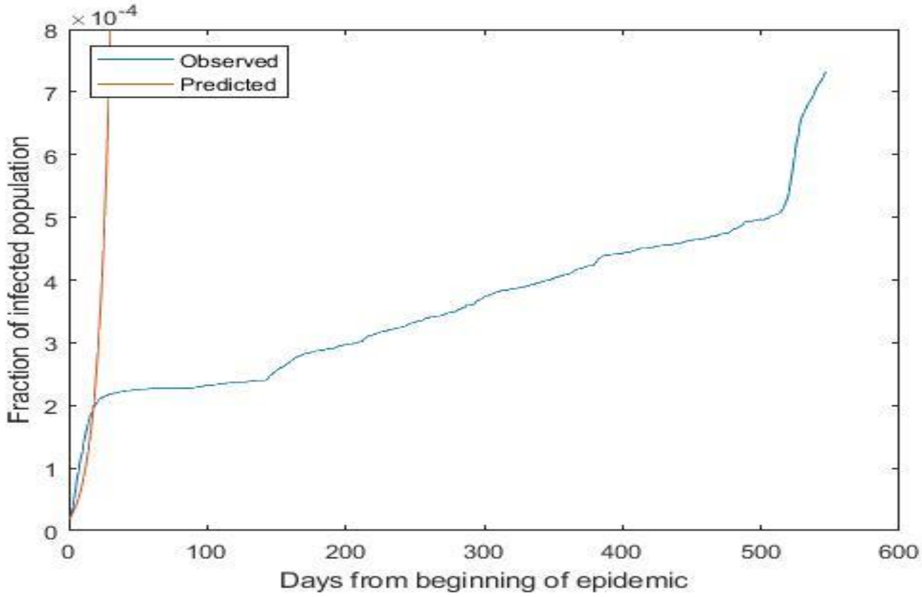


Fig 12.3 For intervals 1 - 20

Fig 12.3 represents the graph plotted between predicted SIR model vs cumulative fraction for the intervals 1 to 20. We can see that the observed graph showing variation from that of predicted one with a mean squared error (average squared difference between the estimated values and the actual value) equals to 2.0205x10^{-4}. Calculating the optimal infectious rate and then we plot the graph, which is given in fig 12.4 and 12.5.

Predicted SIR vs Observed - New Zealand

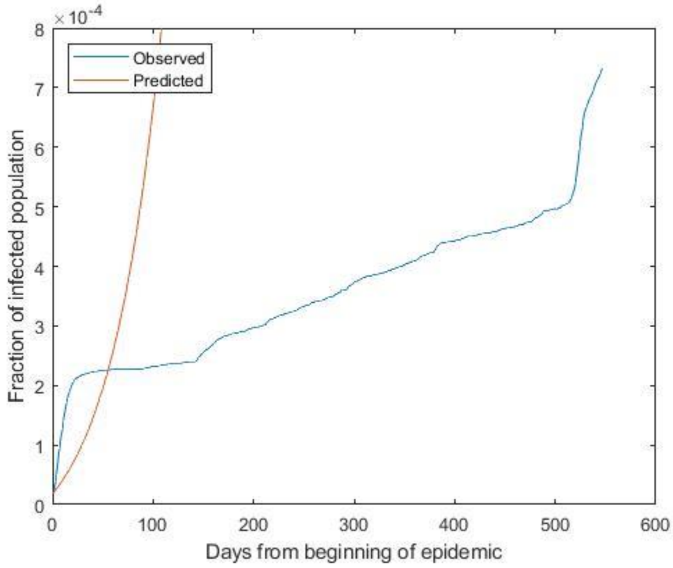


Fig 12.4 For optimal infectious rate (1 to 69)

The above graph represents the predicted SIR model when we take the optimal value of ‘a’ for the whole first wave (1 to 69)

a optimal = 0.1196

The predicted model generated an MSE 0f 7.3243 x 10^-9 which is better than the previous predicted model when only the initial exponential growth was considered.

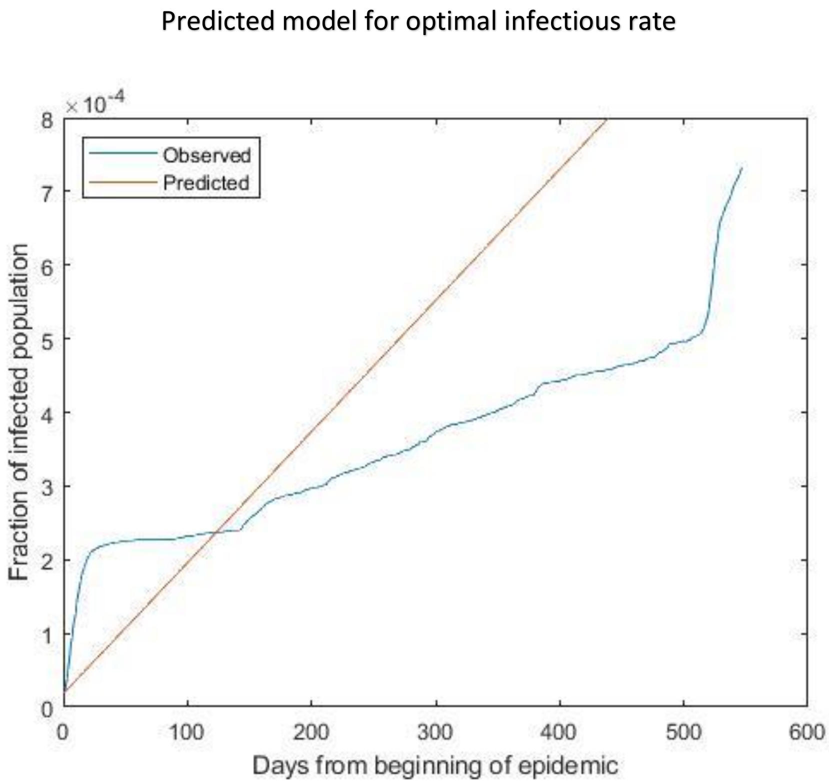


Fig 12.5 For optimal infectious rate – all data

When we consider the length of the interval from the start of pandemic to the current observed day, the predicted model can be observed in figure 12.5.

The optimal ‘a’ generated was 0.1 with an MSE of 1.4245x10^-8.

Therefore, we can say that the predicted model for optimal infectious rate for first wave is better than the one we considered all data when the mean square error is considered. But when we observe the graph, we can say that the model for optimal infectious rate for all data creates a linear model which is approximately identical to the real scenario in New Zealand.

Different values of I(t) - New Zealand

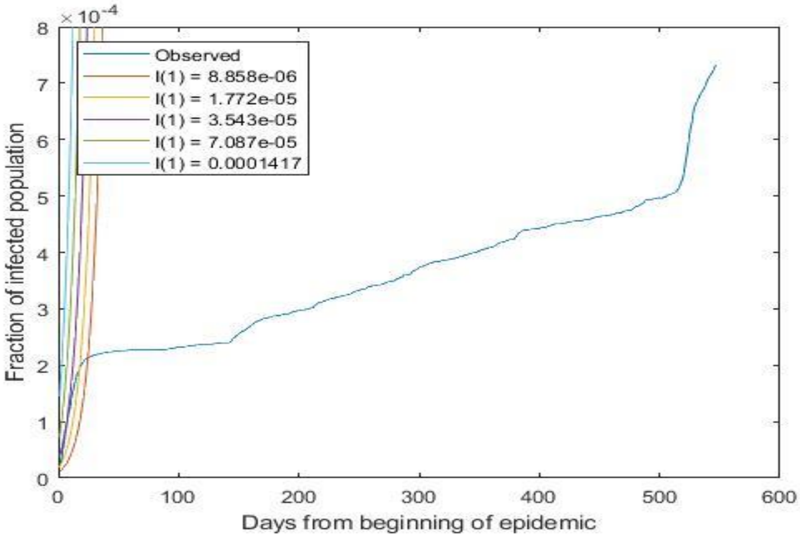


Fig 12.6 Different values of I(t) for interval 1 - 20

Now let us change the initial value of infected fraction to predict a better SIR model.

Fig 12.6 is plotted for the predicted model for 5 different values of I0 for the initial exponential segment of 1-20.

As we can observe, even for different values of initial infectious fraction, there is no significant observation.

So, in order to correct the graph, we tried to plot it by taking the optimized value of a with different I0 values. which is given below in Fig 12.7.

Different I(t) for optimal a value -New Zealand

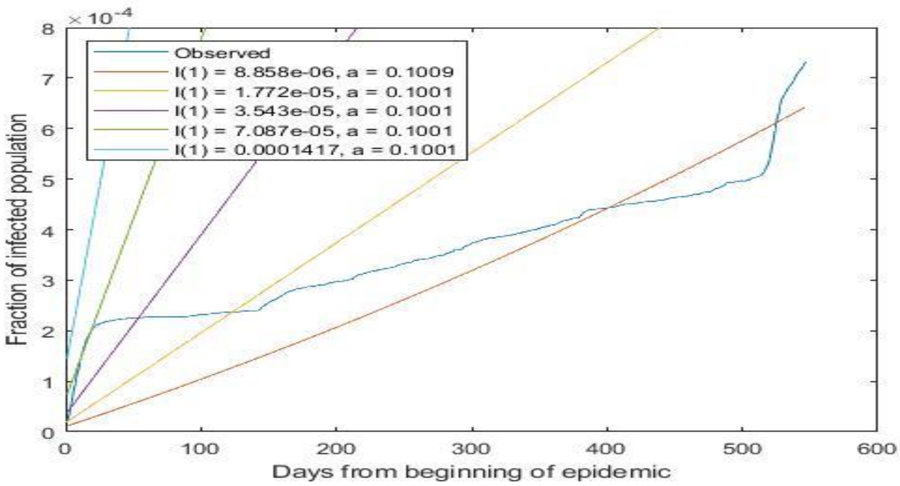


Fig 12.7 Different I(t) for optimal a value

Fig 12.7 gives a group of curves of different I value which almost goes with the observed and predicted values.

Further prediction for 2000 days

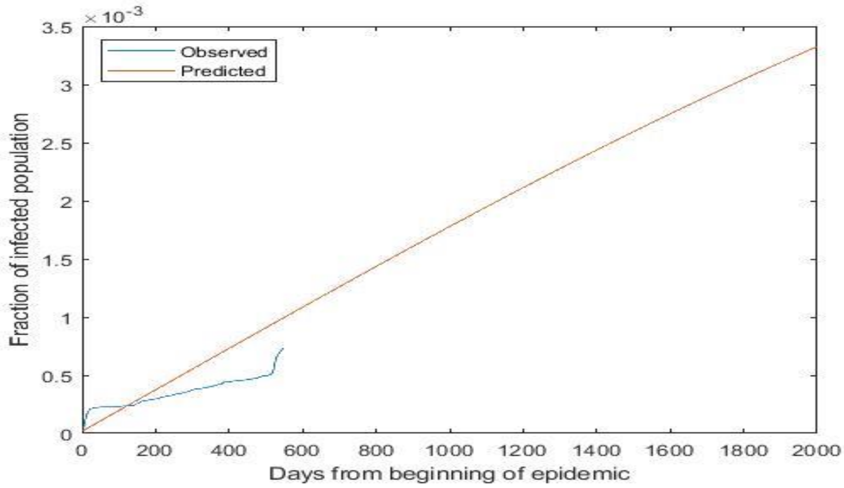


Fig 12.8 Further prediction with b = 0.1

Fig 12.8 represents the prediction with b = 0.1 as we have considered. As we can observe from the plot, it yields really a good, approximated prediction with optimized a value is 0.1000 and mean squared error (MSE) IS 1.4246X10^{-8} .

Further prediction for 2000 days

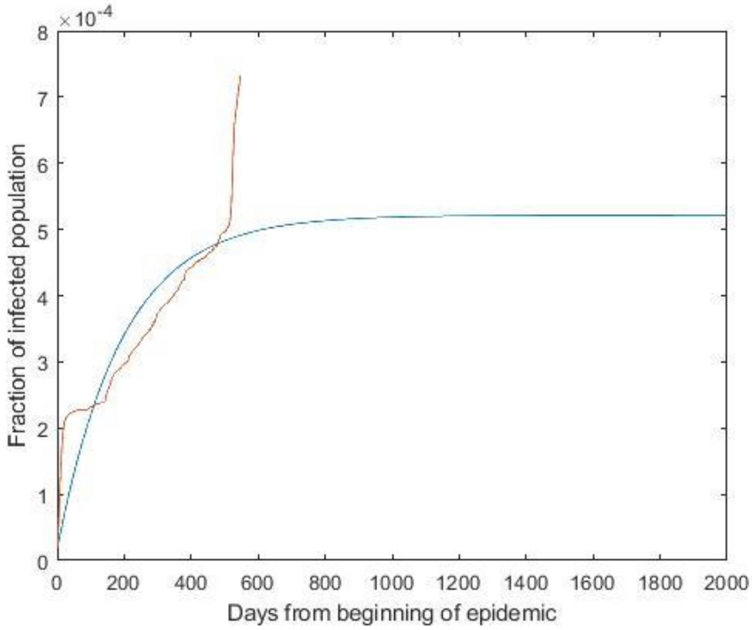


Fig 12.9 Further prediction with b = 0.15

Fig 12.9 represents the plot for further prediction of SIR model till day 2000 with b = 0.15. This value of b is manually taken to show that b = 0.1 is better than b = 0.15. As we have mentioned before, New Zealand was one of the best countries to prevent Covid 19 spread and the recovery day closely follows the 10-day mark. Therefore b = 0.1 plots a better graph.

<u>INDIA</u>:

For the calculation of SIR model, we take the start as 100 cumulative cases.

Graph of Normalized cumulative fraction

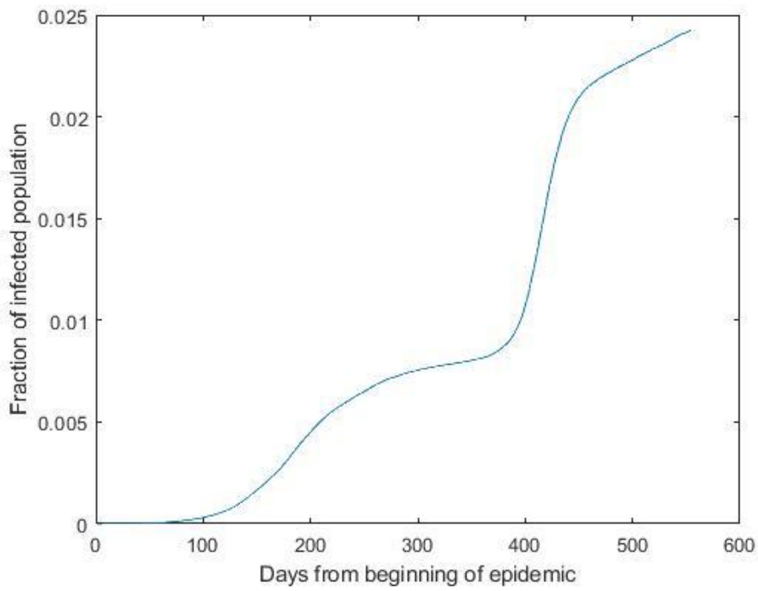


Fig 13.1 Normalized cumulative fraction

The above figure shows the normalized cumulative fraction of the cumulative cases of covid-19 in India.

Log graph of cumulative fraction

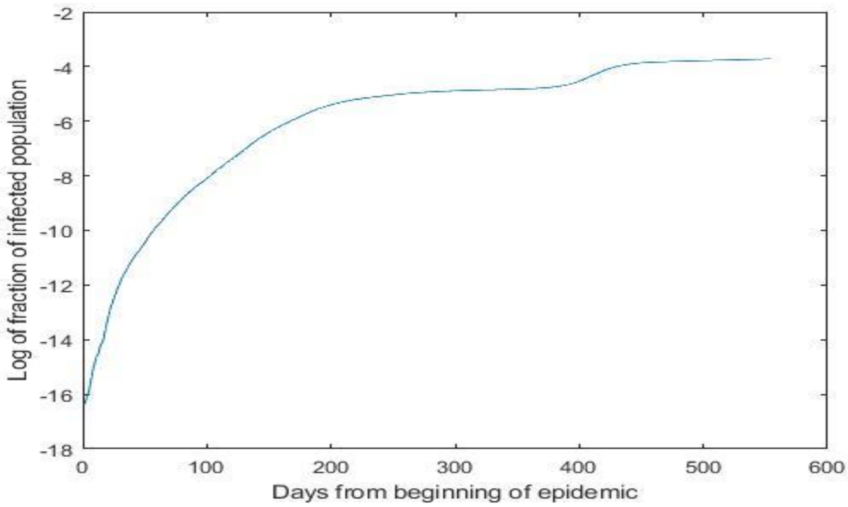


Fig 13.2 Log of cumulative fraction

From the log graph of the cumulative fraction plotted, we can observe that the initial exponential growth is from days 1 to 30 and from 31 to 149. The first covid-19 wave in India lasted for 336 days.

We consider the recovery time, for a person, from covid to be 10 days. Therefore, we analyse the model with b = 0.1.

So, 3 different predicted SIR model vs observed cumulative fraction can be plotted when we consider the intervals 1 to 30, 31 to 149 and 1 to 149.

Graph of predicted SIR model vs cumulative fraction

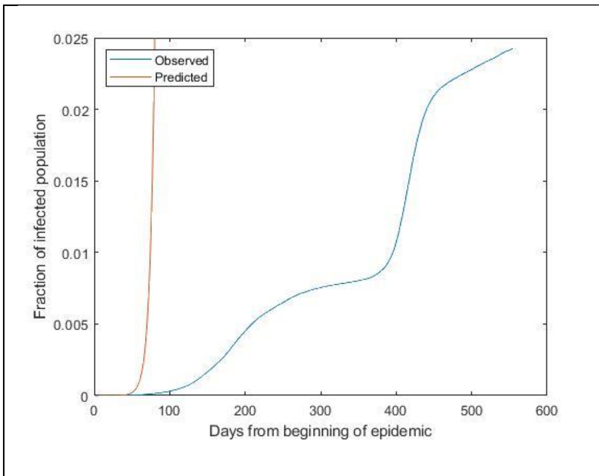


Fig 13.3 For intervals 1 – 30

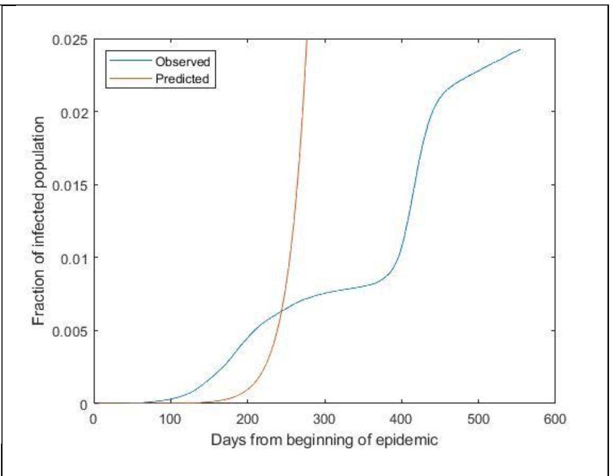


Fig 13.4 For intervals 31 – 149

Fig 13.3 and 13.4 represents the graph plotted between predicted SIR model vs cumulative fraction for the intervals 1 to 30 and 31 to 149 respectively.

As we can observe, the predicted value for the intervals 1 to 30 has a larger margin of error.

ie, MSE = 0.5392.

So, the predicted model for this particular interval is not a good approximation.

But the predicted model for the interval 31 to 149 is slightly better than the previous one with the mean square error of approximately 0.0017.

Graph of predicted SIR model (1 - 149) - India

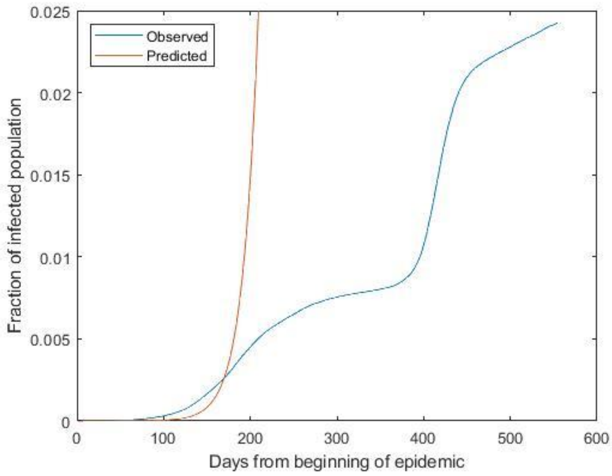


Fig 13.5 For intervals 1 to 149

Now we can consider the whole initial exponential segment (1 to 149) and plot the predicted model for the same. The mean square error observed for this interval is 0.0580, which is better than the predicted model for the interval 1 to 30 but inferior to interval 31 to 149.

Graph of optimal infectious rate (1 - 336)

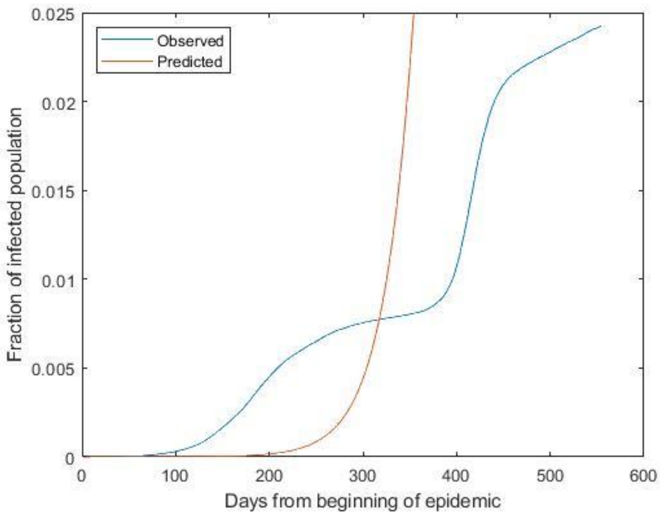


Fig 13.6 For optimal infectious rate (1 to 336)

The above graph represents the predicted SIR model when we take the optimal value of 'a' for the whole first wave (1 to 336)

a optimal = 0.1331

The predicted model generated an MSE 0f 1.0141 x 10^-5 which is better than the previous predicted models when only the initial exponential growth was considered.

For optimal infectious rate – all data

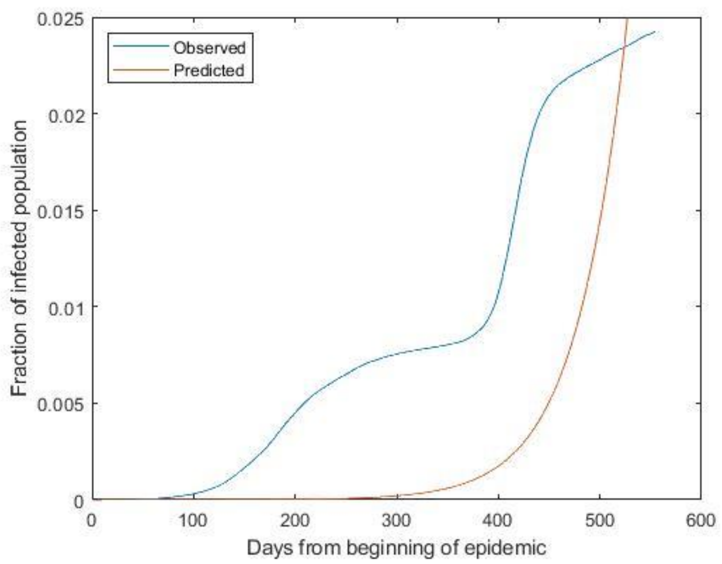


Fig 13.7 For optimal infectious rate – all data

When we consider the length of the interval from the start of pandemic to the current observed day (1 to 555), the predicted model can be observed in fig 13.7.

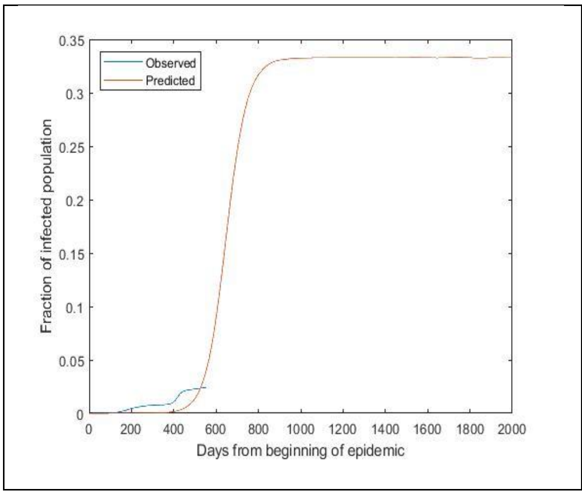
The optimal a generated was 0.1216 with an MSE of 1.9511x10^-5.

Therefore, we can say that the predicted model for optimal infectious rate for first wave is better than the one we considered all data.

Interval	**r**	**c**	**b**	**a**	**MSE**
1 - 30	0.1599	-16.5356	0.1	0.2599	0.5392
31 - 149	0.0430	-12.5341	0.1	0.1430	0.0017
1 - 149	0.0581	-14.1100	0.1	0.1581	0.0580
Optimal Wave			0.1	0.1331	1.0141x10^-5
Optimal all			0.1	0.1216	1.9511x10^-5

Table 12 – Estimation of parameters of SIR model

Graph of further prediction



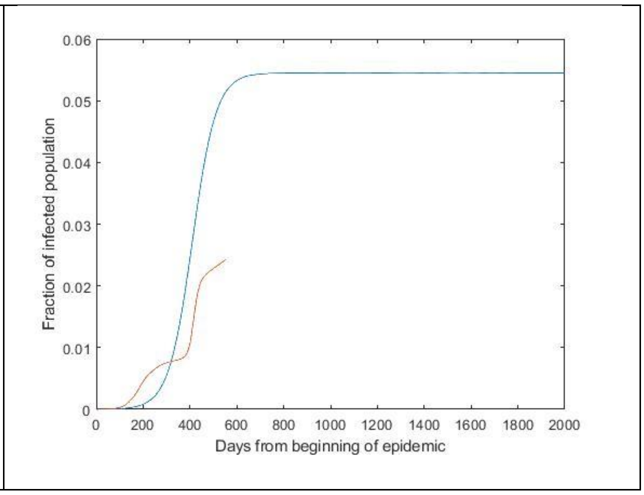


Fig 13.8 Further prediction with b = 0.1 Fig 13.9 Further prediction with b = 0.7 and a = 0.7198

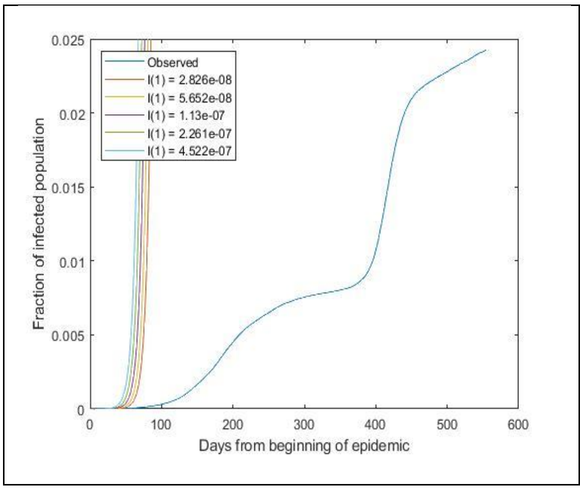
The above two figures (2.8 and 2.9) represents the further prediction of SIR model for 2000 days from the beginning of epidemic considered.

Fig 13.8 represents the prediction with b = 0.1 as we have considered. As we can observe from the plot, it doesn't yield a good approximation. But when we manually change the value of b to 0.7, we can observe a better prediction of the pandemic for 2000 days. I t is represented in fig 13.9.

b = 1/t ; where t is the number of days to recover from covid. So, when b = 0.7, t ≈ 1.5

But it is impossible to recover from covid-19 in 1.5 days.

Graph of different initial value



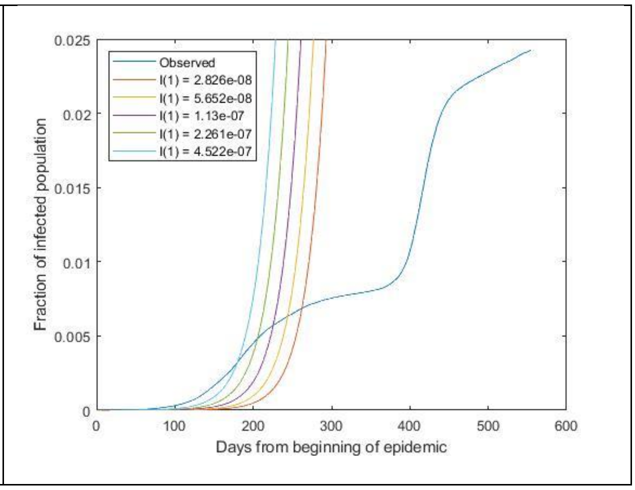


Fig 13.10 different I(t) for interval 1-30 Fig 13.11 Different I(t) for interval 31-149

Now let us change the initial value of infected fraction to predict a better SIR model.

Fig 13.10 is plotted for the predicted model for 5 different values of I0 for the initial exponential segment of 1-30.

Fig 13.11 is plotted for the predicted model for 5 different values of I0 for the initial exponential segment of 31-149.

As we can observe, even for different values of initial infectious fraction, there is no significant observation.

Graph of different initial values - India

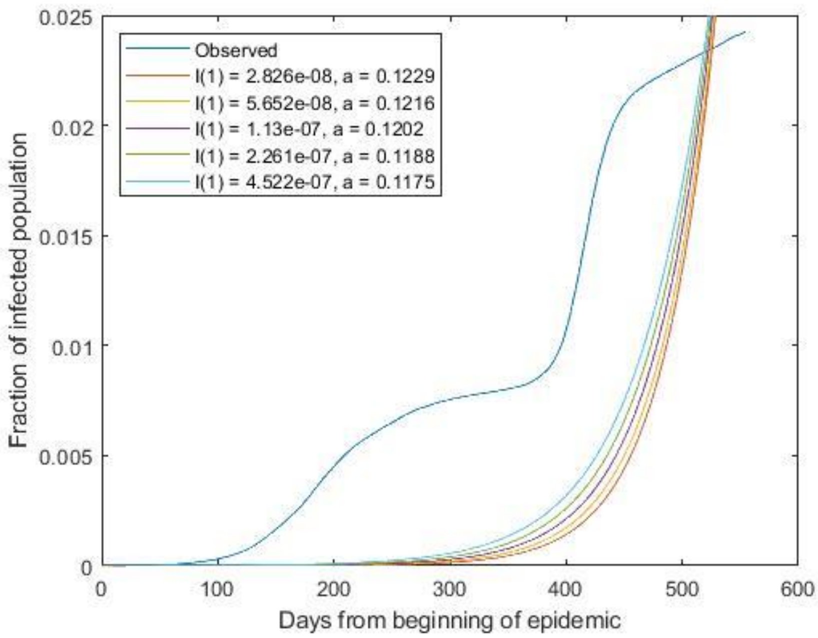


Fig 13.12 Different I(t) for optimal a value.

When the optimal value of ‘a’ is taken, a better predicted model can be created as shown in fig 13.12.

SOMALIA

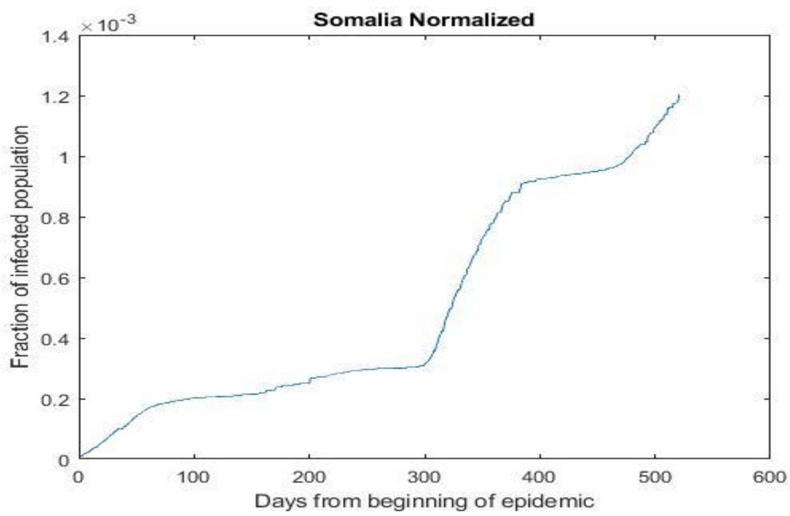


Fig. 14.1 Normalized graph of Somalia

The figure fig.14.1 represents the normalized graph of Somalia after the removal of initial fragments before the epidemic start.

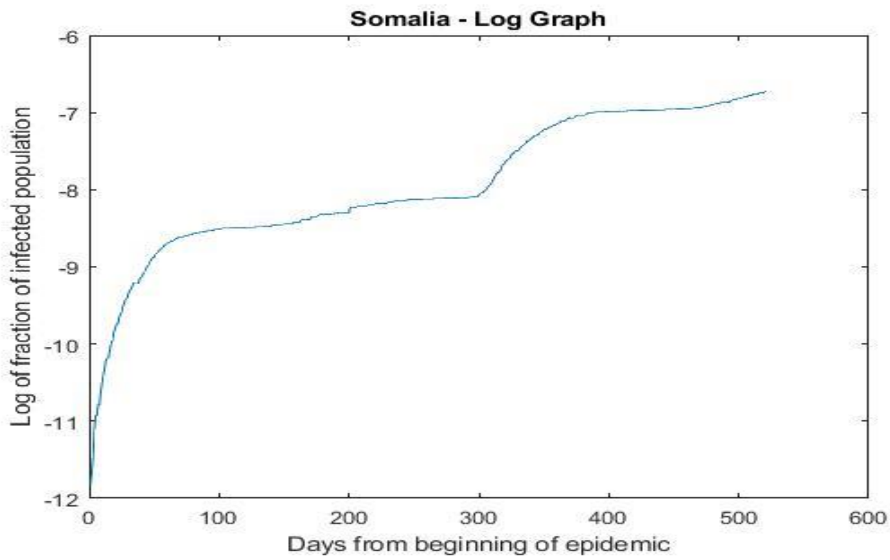


Fig.14.2 logarithmic graph of Somalia

The figure, fig.14.2 represents the logarithmic cumulative fraction of the normalized data of India. From the log graph of the cumulative fraction plotted, we can observe that the initial exponential growth is from days 1 to 15 and from 17 to 47. The first covid-19 wave in Somalia lasted for 115 days.

Table.13, observes the parameters of the SIR model for each of the intervals as per the plotted graphs. According to the SIR model, ‘r’ is almost equal to the difference between the constants ‘a’ and ‘b’. And the value of the parameter ‘a’ is the sum of ‘r’ and ‘b’.

For all the intervals, we have taken the value of ‘b’ as 0.1 which implies that the number of days to recover from covid is approximated as 10 days.

Table 13. Estimation of parameters of SIR model

Interval	r	c	b	a	MSE
1-15	0.1164	-11.7214	0.1	0.2164	0.0615
17-47	0.0329	-10.4213	0.1	0.1329	8.0982×10^{-8}
1-47	0.0533	-11.1166	0.1	0.1533	5.1398×10^{-6}
Optimal 1w			0.1	0.1175	2.6688×10^{-9}
Optimal all			0.1	0.1044	8.2568×10^{-9}

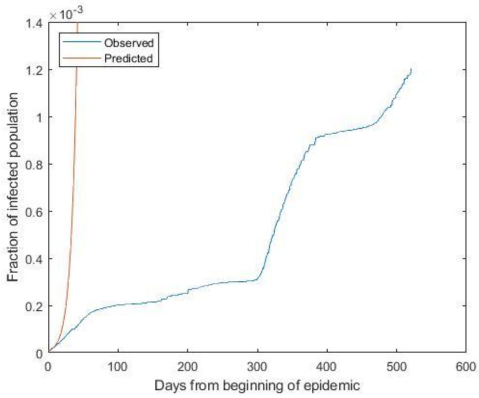


Fig.14.3 for interval 1 - 15

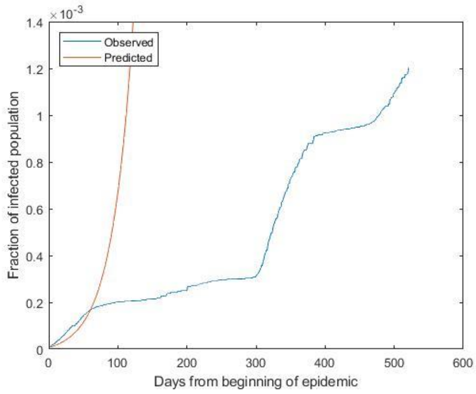


Fig.14.4 for interval 17 - 47

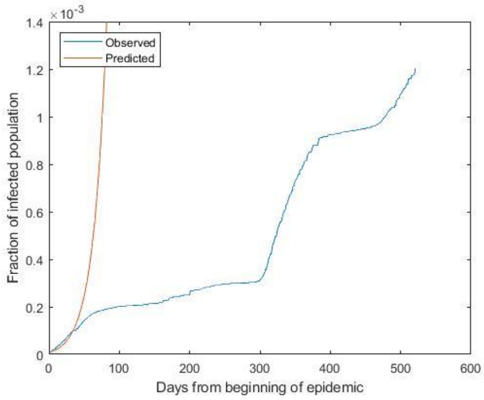


Fig.14.5 for interval 1 - 47

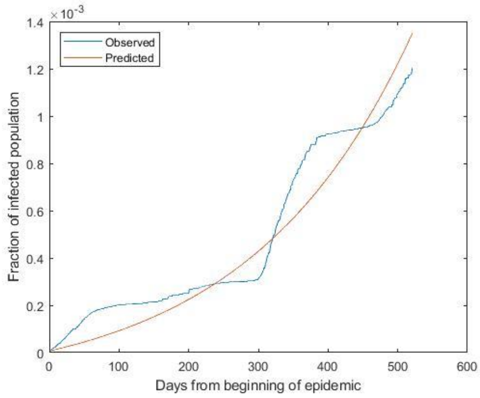


Fig.14.6 Optimal infectious rate in Somalia

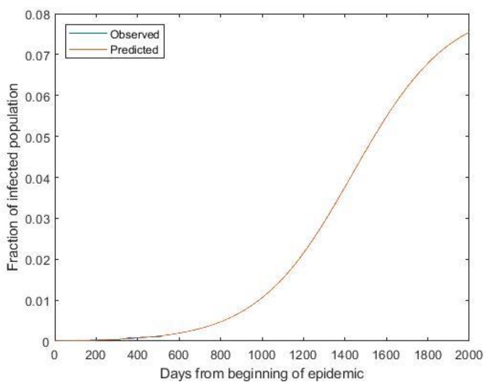


Fig.14.7 Further prediction of cases in Somalia

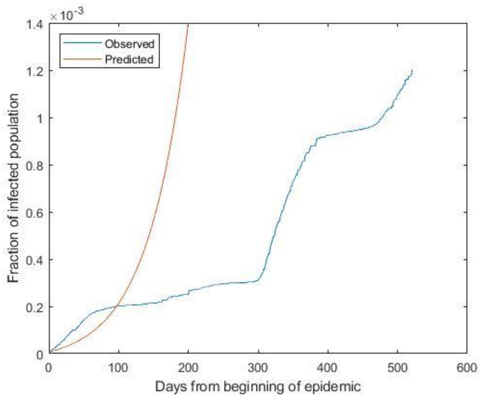


Fig.14.8For optimal infectious rate

(1 - 115)

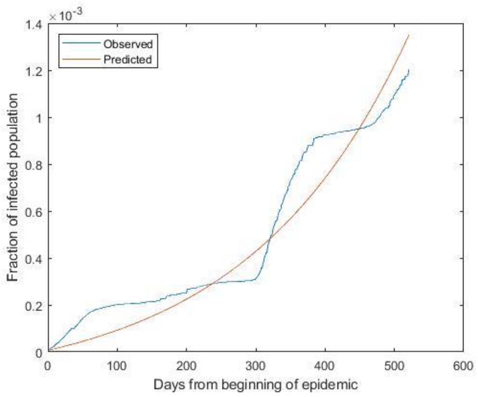


Fig.14.9For optimal infectious rate (all data)

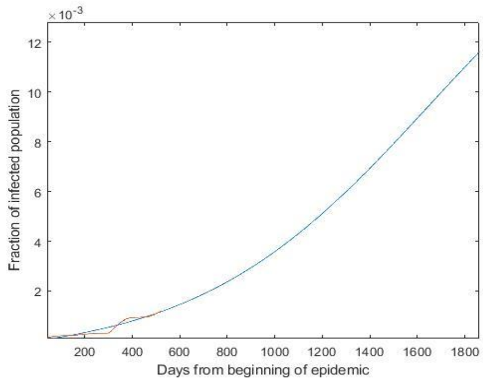


Fig.14.10Further prediction for $\mathrm{b} = 0.2$

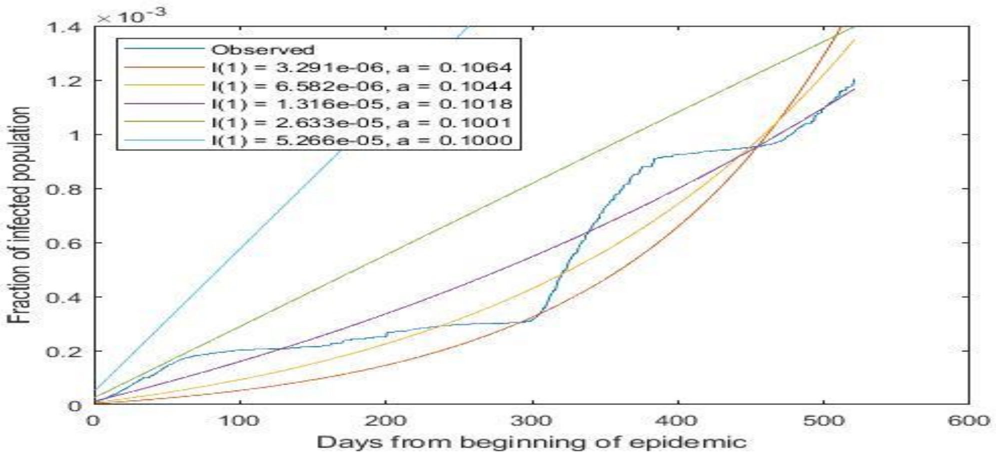


Fig.14.11 for different value of I(t) for optimal a

During both the intervals, (1 - 15) and (17 - 47), we find that the graph obtained shows a huge variation between the observed and the predicted values of the cases in Somalia. Even if we consider the figure fig.14.5, we get the same result. Hence, in order to get a correct analysis, we calculate the optimal infectious rate and then we plot the graph.

The figure fig.14.6 represents the graph obtained using the optimal infectious rate for the interval (1 - 47). Here we find that there is a correspondence between the predicted and observed values.

From the figure fig.14.7, we are considering the prediction from the beginning of the epidemic to 2000 days, here we can see that the fraction of the infected population lies between 0.7 and 0.8.

Consider the figure fig.14.10 which represents the graph of further prediction in Somalia with b = 0.2. We can derive a smooth graph which is marked in purple colour.

From figure fig.14.11we get a smooth graph when the value of I(1) is 1.316e - 0.5 and a = 0.1018

MODEL 3 - ANALYSING THE MODIFIED SIR MODEL

NEW ZEALAND

The Fig 15.1 represents the normalized cumulative fraction graph of the country New Zealand.

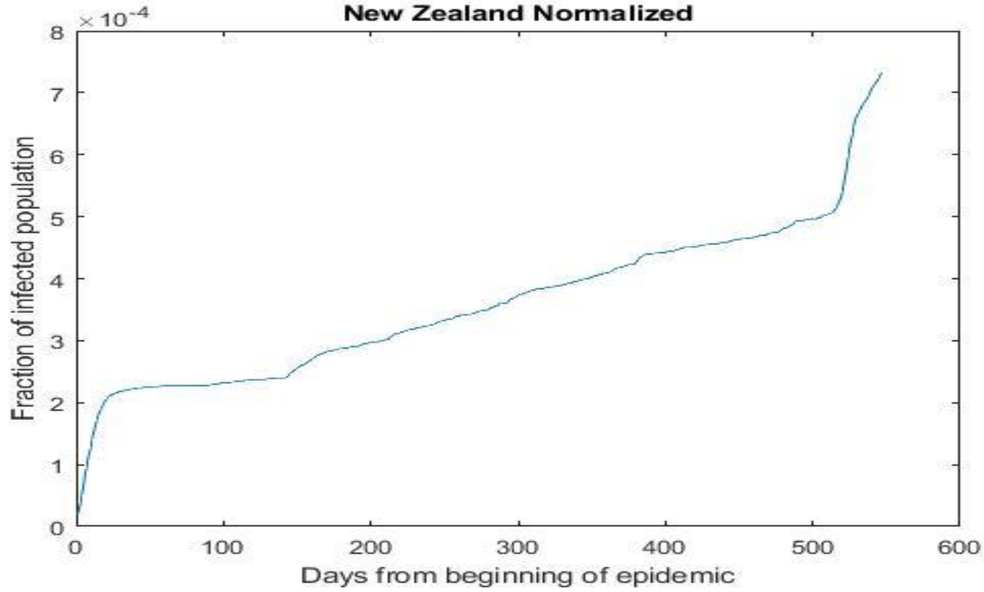


Fig 15.1 Normalized graph of New Zealand

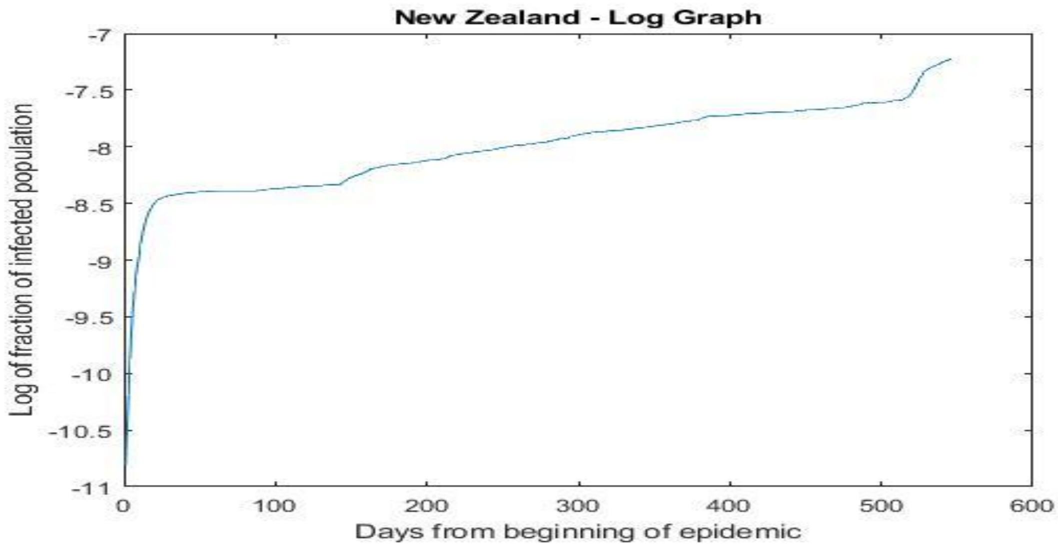


Fig 15.2 Log of cumulative fraction

The Fig 15.2 represents the logarithmic cumulative fraction of the normalized data of New Zealand. From the log graph of the cumulative fraction plotted, we can observe that the initial exponential growth is from days 1 to 20. The first covid-19 wave in New Zealand lasted for 69 days. In the case of New Zealand, $R_{\circ}$ =12/5.08 million; Because the number of recovered cases 10 days before the considered start cases (ie, 100 cases), is 12 people.

Graphs of SIR and modified SIR models for the interval 1 – 20:

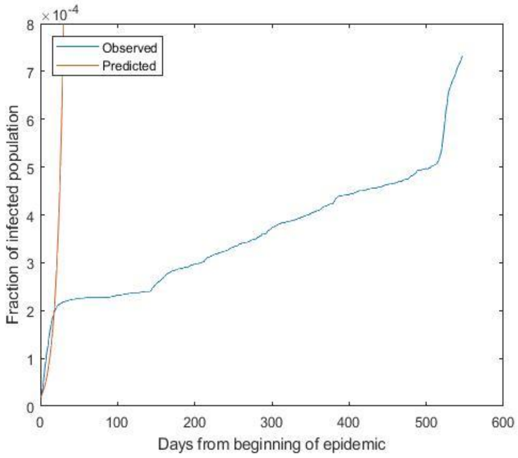


Fig 15.3 SIR model

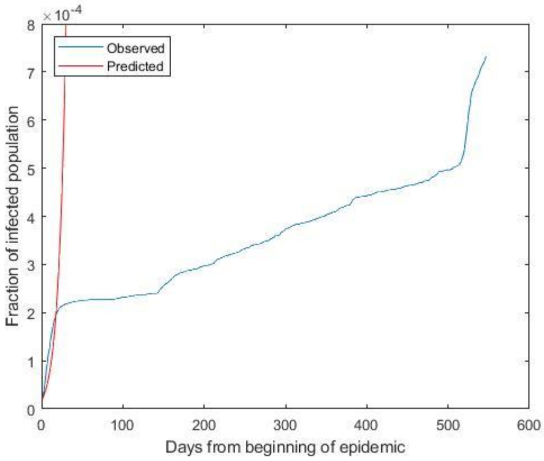


Fig 15.4 Modified SIR model

As we can observe, the SIR model created in chapter 2 and the modified SIR model with different susceptible parameters doesn't show much difference when the exponential growth from day 1 to 20 is considered. We can assume that this is because of the fact that the exponential growth of Covid-19 cases in New Zealand lasted just 20 days, there after New Zealand was successful in preventing the Covid-19 spread by lockdown and proper awareness.

Graphs of SIR and modified SIR models for optimal value of *a* in all data:

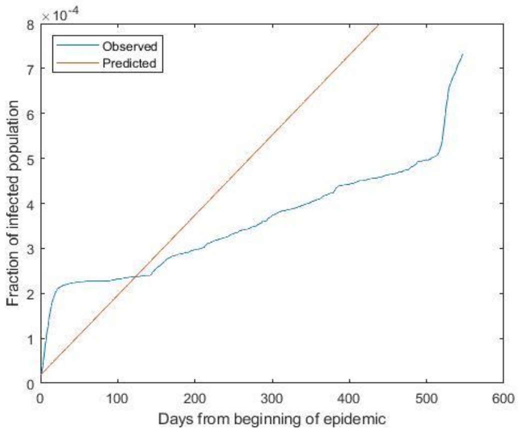


Fig 15.5 SIR model – a optimal

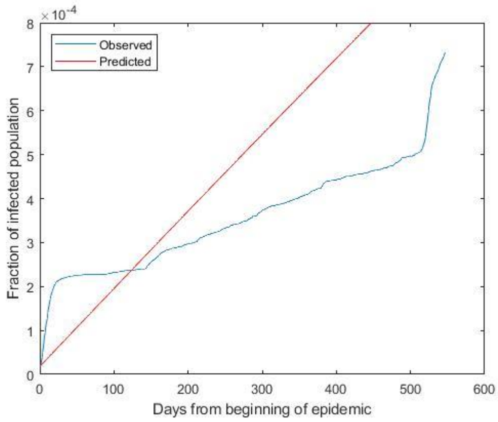


Fig 15.6 Modified SIR – a optimal

When we consider the length of the interval from the start of pandemic to the current observed day, the predicted model for SIR and modified SIR can be observed in fig 15.5 and 15.6 respectively.

The optimal 'a' generated was 0.1 with an MSE of 1.4245x10^-8.

When we observe the graph, we can say that the model for optimal infectious rate for all data creates a linear model which is approximately identical to the real scenario in New Zealand. When b = 0.1, we could observe that the SIR and modified SIR doesn't show much difference.

Graph with S ignorant, S Resistant along with observed and predicted lines:

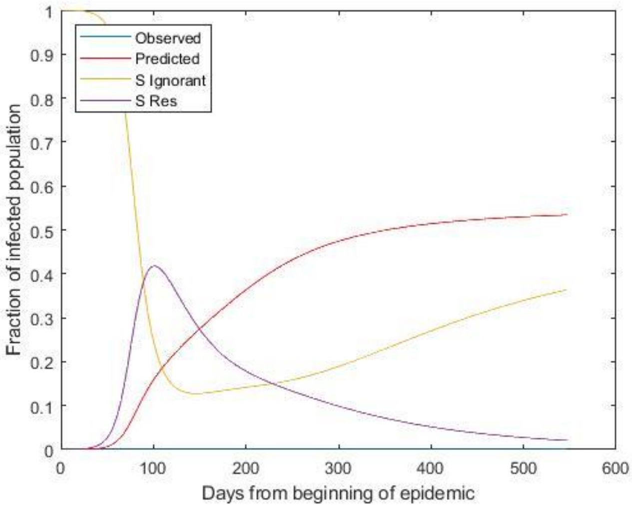


Fig 15.7 modified SIR

In Fig 15.7, we have plotted the change in S ignorant and S resistant along with observed and predicted lines. we were not able to observe the observed line with eye due to the fact that the exponential interval taken was only for first 20 days, which predicted a model that doesn't resembles the actual observed one. Therefore, we were unable to analyse clearly and extract observations from this graph. But by altering the value of b along with the optimized value of a, we were able to observe a clear difference between the two models. The modified SIR model created a better prediction which can be analysed better than the traditional SIR model. Let us now alter the value of b to create a better predictive model. The comparison between SIR and modified SIR models are plotted and described below.

Graph for SIR and modified SIR model with altered b value and predicted to 2000 days

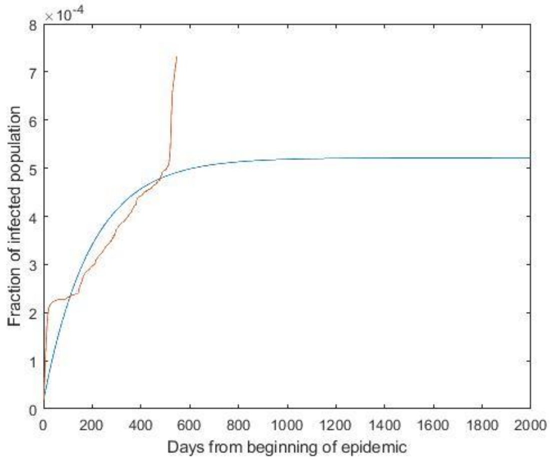


Fig 15.8 SIR model – b =0.15

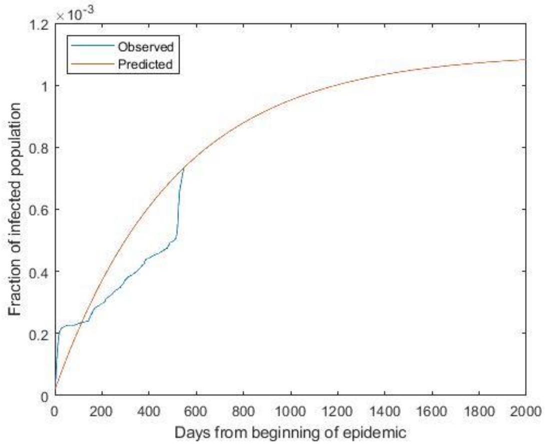


Fig 15.9 modified SIR – b = 0.12155

The Fig 15.8 represents the predicted SIR model for 2000 days with b = 0.15.
The Fig 15.9 represents the predicted modified SIR model for 2000 days with b = 0.12155.

The b values are taken manually to generate a better graph and for further prediction.
New Zealand was one of the best countries to prevent Covid 19 spread and the recovery day closely follows the 10-day mark. Therefore b = 0.12155 plots a better graph with recovery days t = 8.

As for New Zealand, there was an initial exponential growth of covid cases for first 20 days. Then we can observe that covid cases growth were kept linear for almost next 550 days. There after there was again an exponential growth. The predicted model resembles an approximate linear graph which turns almost horizontal after 1000 days.

Graph of modified SIR model for all data – New Zealand

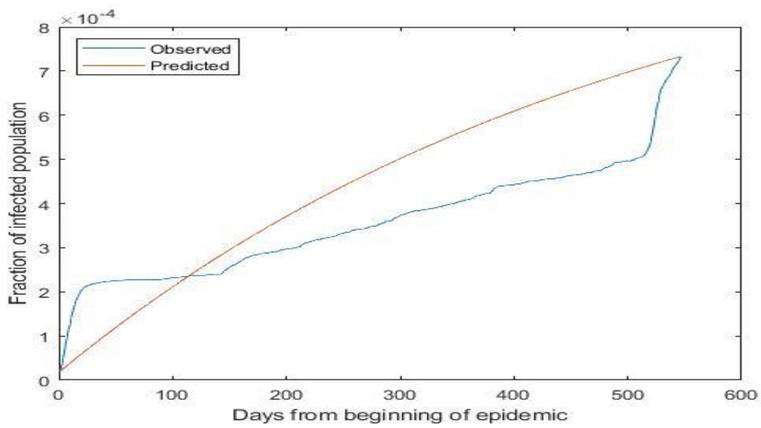
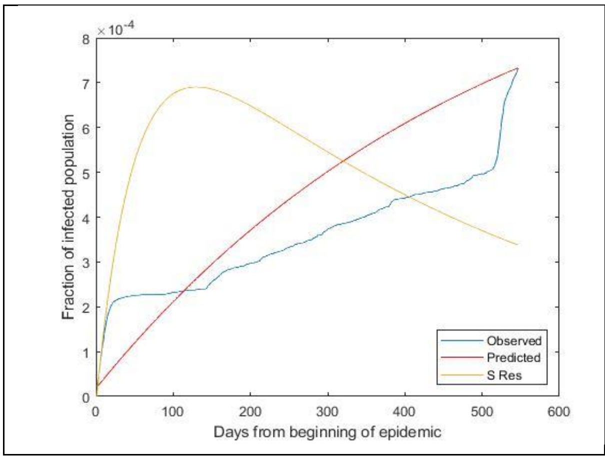


Fig 15.10 modified SIR model – for all data b = 0.12155

The Fig 15.10 represents the graph with b = 0.12155 for the length of data available. We can observe a better prediction line vs the observed data.



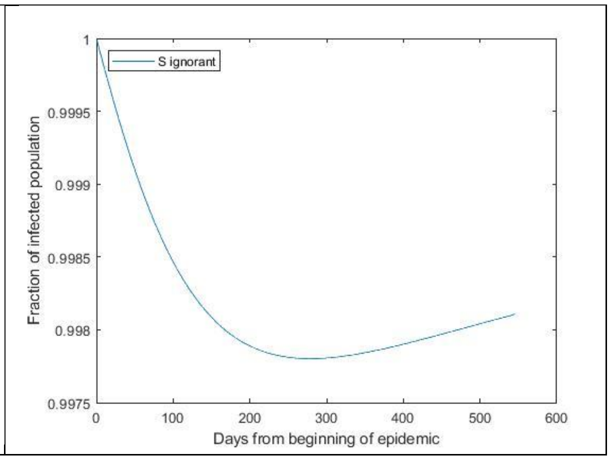


Fig 15.11 modified SIR, b = 0.12155

Fig15.12 S ignorant plotted separately

The Fig 15.11 represents the graph of predicted modified SIR model against the observed data along with S resistant. The Fig 15.12 is plotted for S ignorant separately due to the reason that if the S ignorant is plotted along with the other graph, it distorts the whole plot.

As we can observe from both graphs, S ignorant starts at approximately 1, there after it starts decreasing because more people started becoming aware of Covid-19 crisis and started moving into resistant stage. We can observe a steady increase in S_res throughout the first wave in New Zealand.

After approximately 200 days, more people start to move from resistance stage to exhausted stage. We can observe an increase in S ignorant line after 250 days and interpret that people starts to move from resistance to exhausted, then from exhausted to ignorant at these days.

INDIA

The Fig 16.1 represents the normalized cumulative fraction graph of the country India.

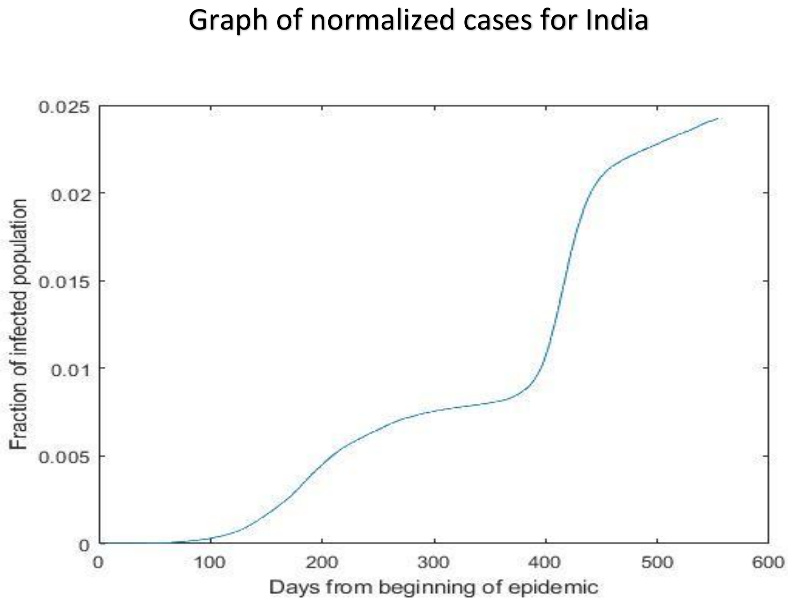


Fig 16.1Normalized graph of India

Log graph - India

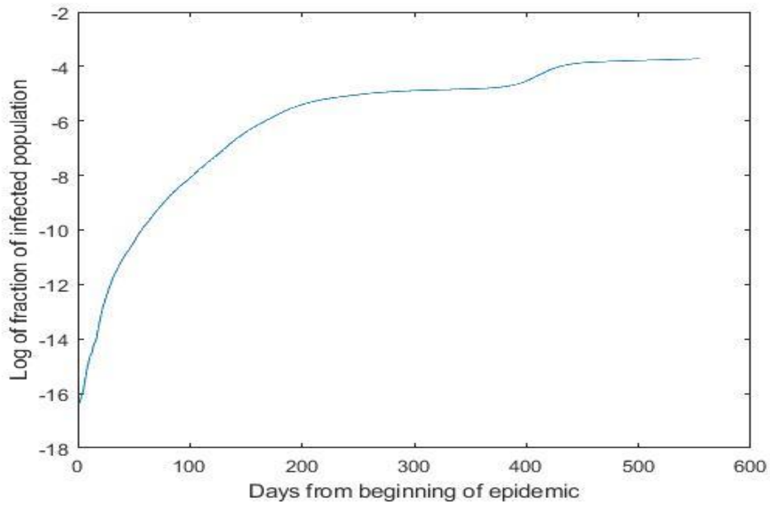


Fig 16.2 Log of cumulative fraction of India

As seen in the case of New Zealand, Fig 16.2 represents the logarithmic cumulative fraction of the normalized data of India. From the log graph of the cumulative fraction plotted, we can observe that the initial exponential growth is from days 1 to 149. The first Covid-19 wave in India lasted for 336 days.

Graphs of SIR and modified SIR models for the interval 1 – 149

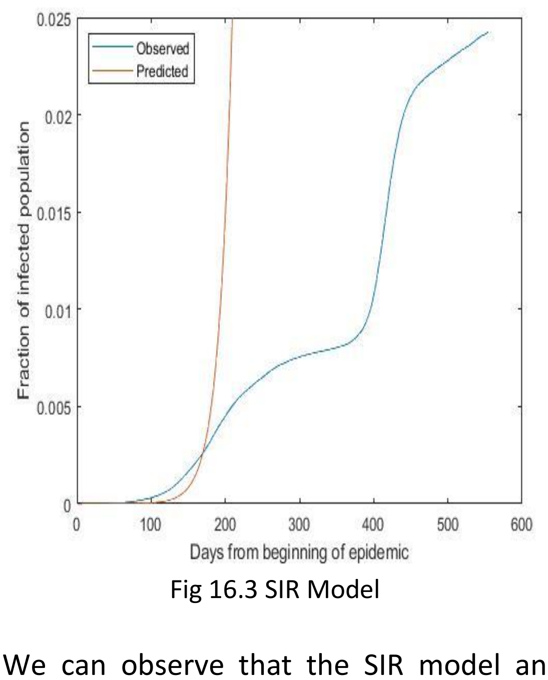


Fig 16.3 SIR Model

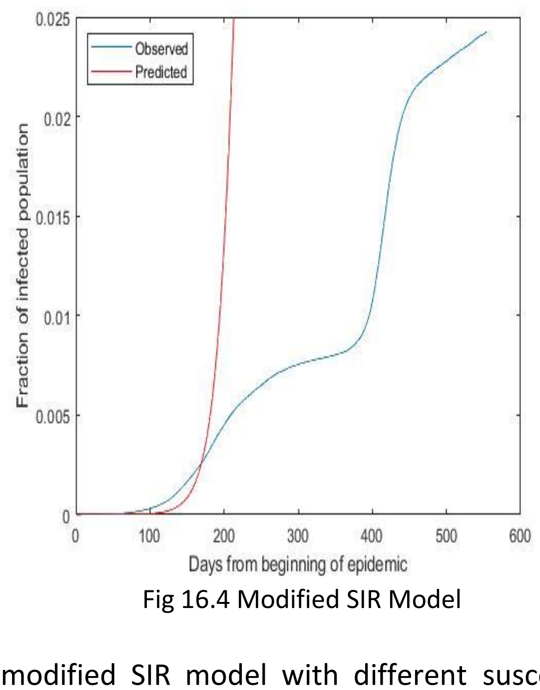


Fig 16.4 Modified SIR Model

We can observe that the SIR model and modified SIR model with different susceptible parameters doesn't show much difference when the exponential growth from day 1-149 is considered, which is similar to the case of New Zealand's graphs. But according to India, the exponential growth lasted for approximately 149 days as compared to New Zealand which was just 20 days. As India could not control the exponential growth so fast as New Zealand did. At a glance there are no such notable differences between the graphs obtained by both the hypothesis.

Graphs of SIR and modified SIR models for optimal value of *a* in all data

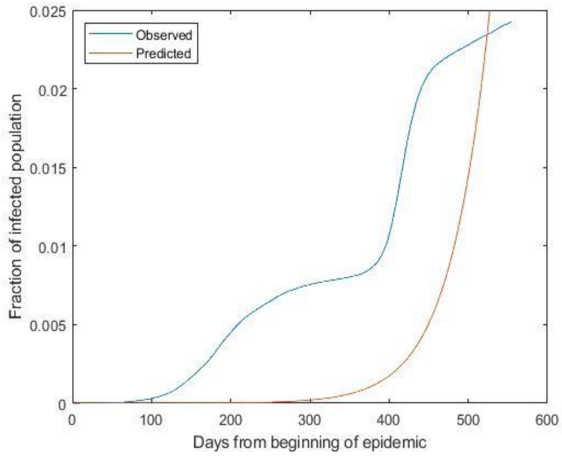


Fig 16.5 SIR model – a optimal

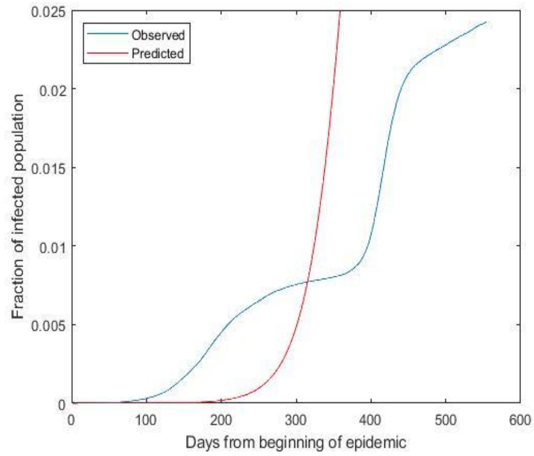


Fig 16.6 Modified SIR – a optimal

Considering the length of the interval from the start of pandemic to the current observed day, the predicted model for SIR and modified SIR can be observed in fig 16.5 and 16.6 respectively. The optimal 'a' generated was 0.1336 with an MSE of 9.7643×10^{-6}.
Here, in the case of India both the graphs are not similar as we obtained in the case of New Zealand. That is, when we compare the same between New Zealand and India, India shows a huge variation in Fig 16.5 and Fig 16.6 of SIR and modified SIR models for optimal value of *a* in all data. In our point of view, here the modified SIR model stands more close to the predicted line and SIR model is to be improved from this case.

Graph with S ignorant, S Resistant along with observed and predicted lines:

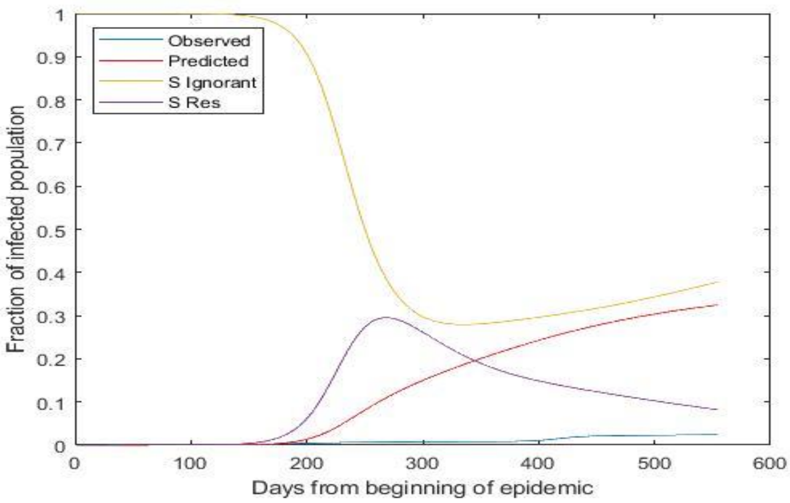


Fig 16.7 Modified SIR

From Fig 16.7, we have the change in S ignorant and S resistant along with observed and predicted lines. The observed line is very close to the X- axis. In the case of India, exponential interval was taken for first 149 days. The predicted value and observed lines does not closely follows because the graph was plotted with initial values of parameters without any optimization. So, analysing this graph and arriving at conclusions is difficult in this case too. But as mentioned in the case of New Zealand, by altering the value of b along with the optimized value of a, we were able to observe a clear difference between the two models, which is represented in the next task below.

Graph for SIR and modified SIR model with altered b value and predicted to 2000 days:

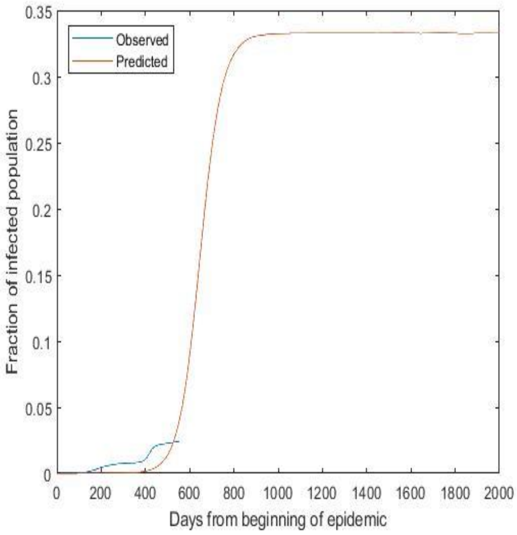


Fig 16.8 SIR model

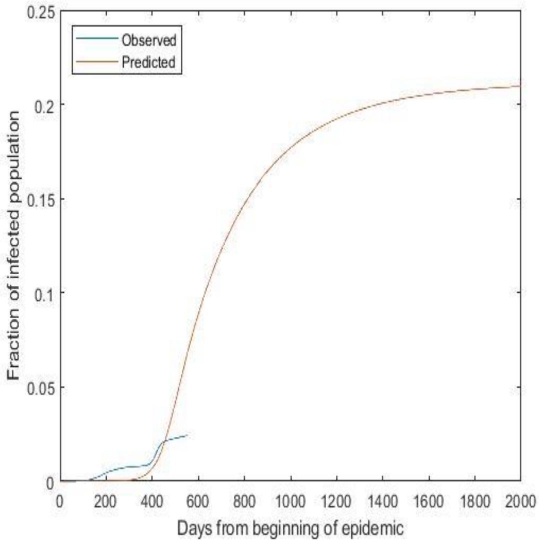


Fig 16.9 Modified SIR model

The Fig 16.8 represents the predicted SIR model for 2000 days with b = 0.1.
The Fig 16.9 represents the predicted modified SIR model for 2000 days with b = 0.10801.

Comparing Fig 16.8 and Fig 16.9, the difference can be clearly observed. In the case of SIR model, predicted value lies between 0.3 and 0.35 from 650 days to the end which is almost a constant horizontal value. When we analyze the modified SIR model, predicted value is between 0.2 and 0.25 with the predicted line closely following the observed line, with minimum mean squared error, from the beginning of pandemic to the current stage.

Plot with optimized parameters

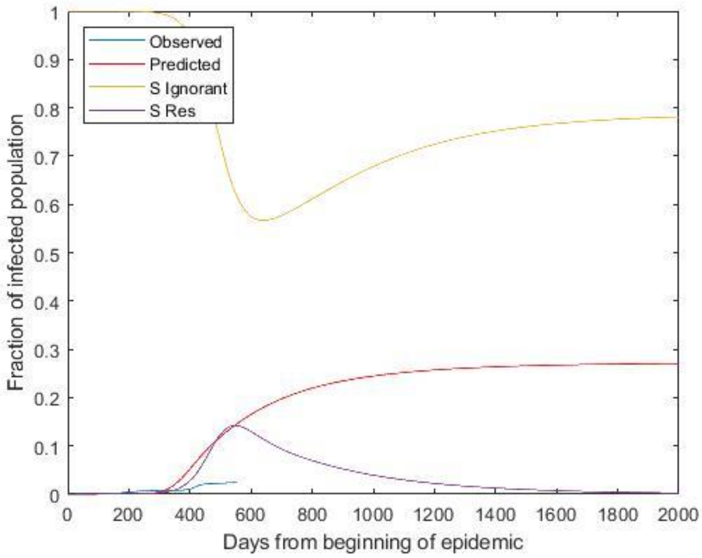


Fig 16.10 Graph with optimized parameters

The Figure 16.10 represents the graph of b optimized for fraction infected population against days from beginning of epidemic. Considering the days from 1 to 300, fraction of infected population showing ignorant behavior is at the maximum value in the graph. Whereas for the same interval, fraction of infected population for S resistant is at the minimum value. The S_ign and S_res shows almost opposite nature in their curves. From the day 400, the value of S ignorant declined to 0.55 and from day 600 it showed a slight exponential growth hence reaching nearly a value of 0.8 at the end of the period. Conversely, the fraction of infected population for S_res increased after 300 days to a value of 1.15 and reduces to zero gradually from day 500 to 1600.

SOMALIA

Fig 17.1 represents the normalized cumulative fraction graph of the country Somalia.

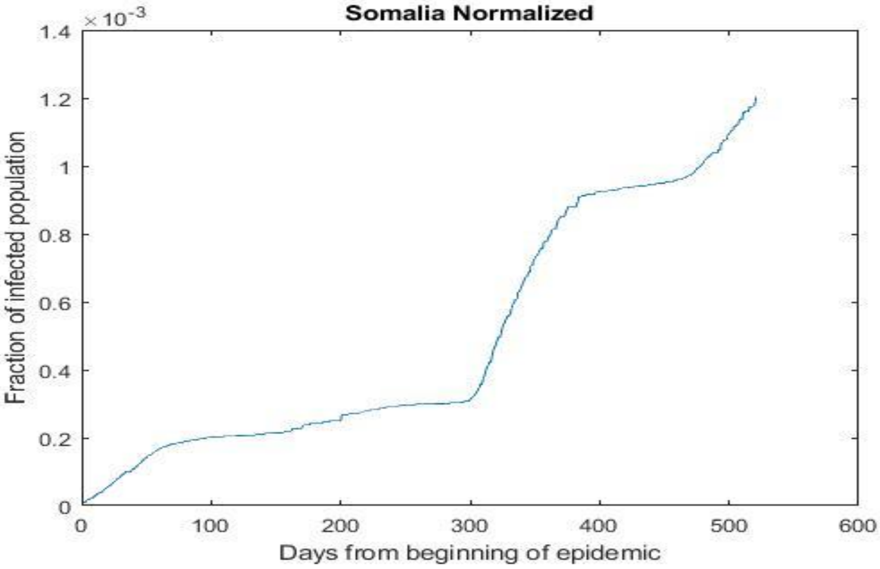


Fig 17.1 Normalized graph of Somalia

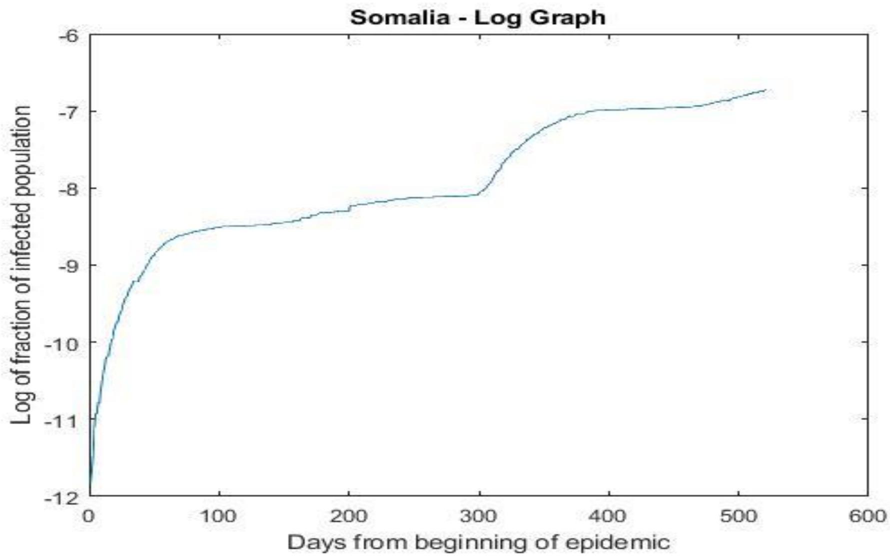


Fig 17.2 Log of cumulative fraction of Somalia

The Fig.17.2 represents the logarithmic cumulative fraction of the normalized data of Somalia. From the log graph of the cumulative fraction plotted, we can observe that the initial exponential growth is from days 1 to 15 and from 17 to 47. Where the population of Somalia is approximately 15.8 million. The first covid-19 wave in Somalia lasted for 115 days.

Graphs of SIR and modified SIR models for the interval 1 – 47:

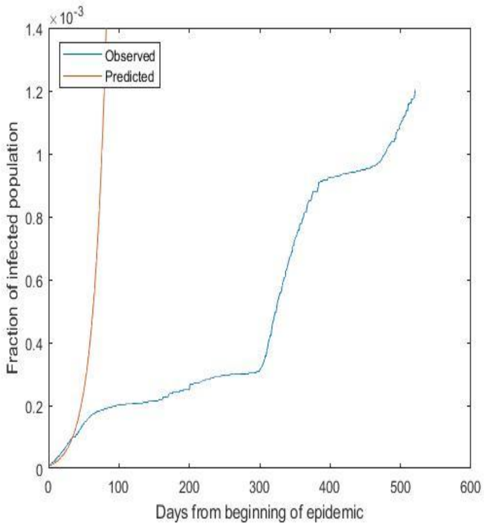


Fig 17.3 SIR model

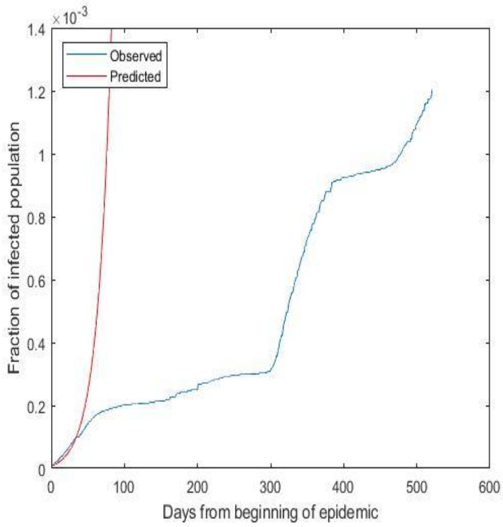
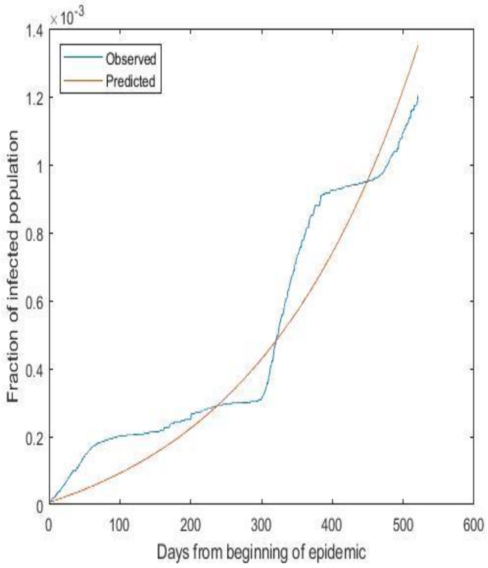


Fig 17.4 Modified SIR Model

In the case of the country Somalia, we get a similar pattern as that of the other two countries**.** Here also, the SIR model and the modified SIR model constructed with distinct susceptible parameters looks similar.

Graphs of SIR and modified SIR models for optimal value of *a* in all data:



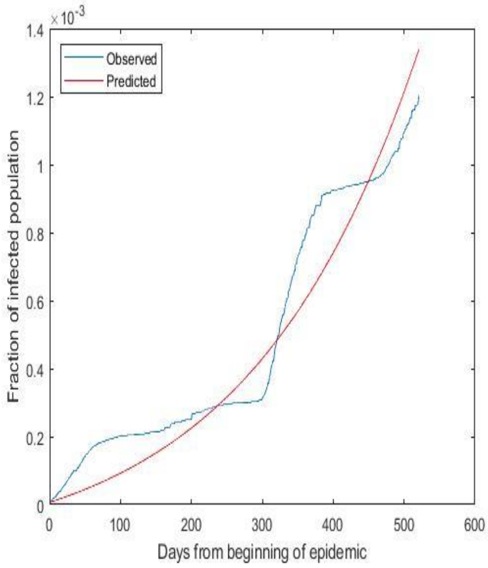


Fig 17.5 SIR model – a optimal

Fig 17.6 Modified SIR – a optimal

Similar to the case of New Zealand and India, when we consider the length of the interval from the start of pandemic to the current observed day, the predicted model for SIR and modified SIR can be observed from Fig 17.5 and Fig 17.6 respectively.

The optimal 'a' generated was 0.1044with an MSE of 8.2568×10^{-9}.

Graph with S ignorant, S Resistant along with observed and predicted lines:

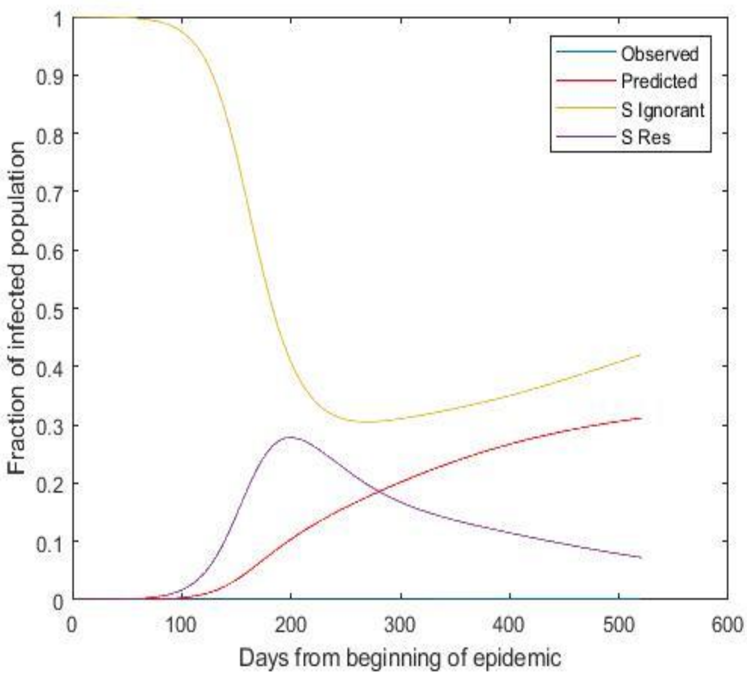


Fig 17.7 modified SIR

The Fig 17.7 depicts the changes in S ignorant and S resistant along with observed and predicted lines. Here, in the case of Somalia too, the observed line is not in correlation with the predicted line. This is because of the fact that the graph was plotted without optimizing the parameters. The observed line is marked in blue on the x axis whereas the predicted line is seen in red colour. So let us consider altering the value of b to create a better predictive model which is done in task 3 below.

Graph for SIR and modified SIR model with altered b value and predicted to 2000 days:

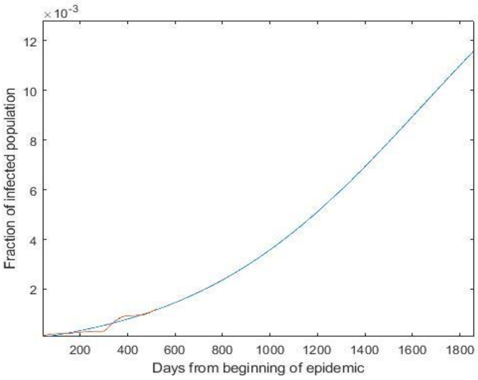


Fig 17.8 SIR model

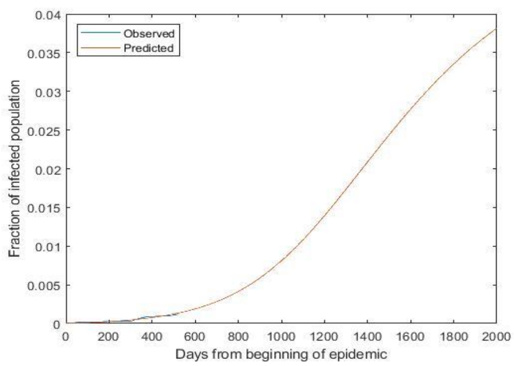


Fig 17.9 Modified SIR model

Fig 17.8 represents the predicted SIR model for 2000 days with b = 0.2.

Fig 17.9 represents the predicted modified SIR model for 2000 days with b = 0.1135.

The b values are taken manually to generate a better graph and for further prediction.

Considering SIR and modified SIR model of Somalia for altered b value predicted to 2000 days, we find that both the graphs show a similar pattern but with different maximum cumulative fraction values at day 2000.

Fig 17.8 and Fig 17.9 we can see that the graph of the predicted values shows an increasing trend from the same duration till the predicted epidemic period (ie, day 2000). If we consider the graph of observed cases in Somalia, here also we find that they share a similar trend from day 1 to day 520.

<u>MODIFIED SIR MODEL WITH CROWD EFFECT:</u>

Now let us consider the crowd effect to the modified SIR model. Crowd effect is the assumption that the alarm increases linearly, and the proper reaction form is Sign +2I → Sres +2I. The reaction rate is qSignI2, where q is a new constant.
We evaluate q assuming that for some selected proportion of infected I the reaction rate is the same as for the linear reaction. Say, let for I = Ip = 0:02 (2% of population) qSignI2
p = kSignIp. Then q = k=Ip The number Ip characterises the "visibility" of epidemic and depends on activity of mass-media.

INDIA

Predicted vs Observed - India

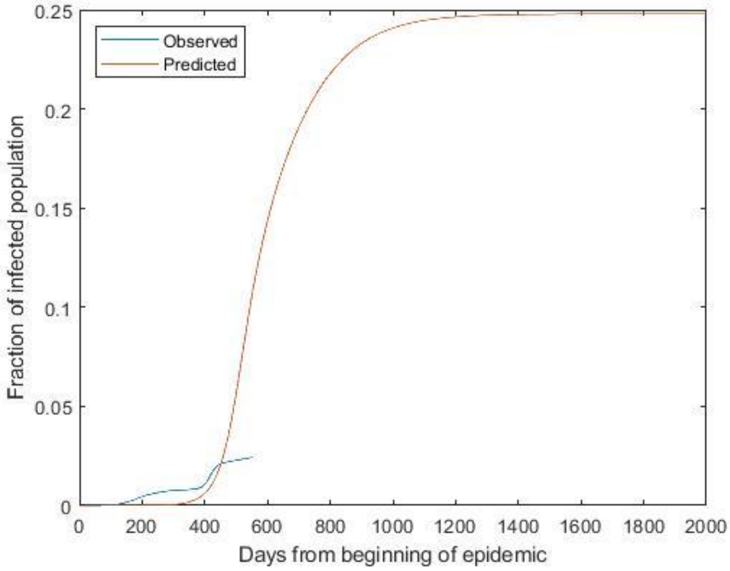


Fig 18.1 – Modified SIR with Ip = 0.02

When we consider Ip = 0.02 as the initial condition in altering the q value, we observe a graph as shown in figure 18.1. the predicted line approximately resembles the observed line. The saturation of the prediction line reaches a value of approximately 0.25 of the cumulative fraction after 1200 days.

Predicted vs Observed - India

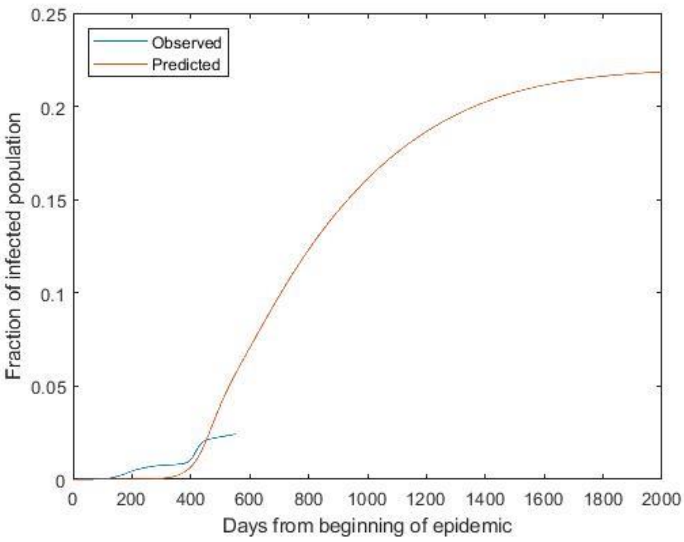


Fig 18.2 – Modified SIR with Ip = 0.002

Now, let us alter the value of Ip to 0.002. We can observe a change in the graph when compared to Ip = 0.02 as shown in the figure 18.2. The graph follows a logistic model with a saturation of prediction line attained after 1400 days with a cumulative fraction value between 0.2 and 0.25.

Predicted vs Observed - India

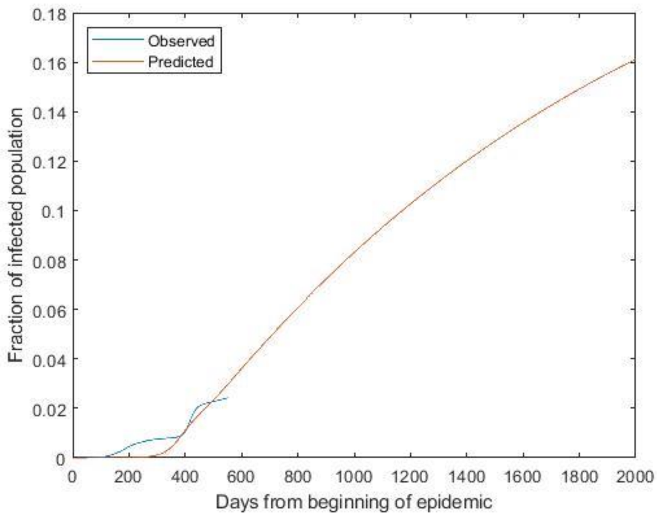


Fig 18.3 – Modified SIR with Ip = 0.0002

When the Ip value is altered to 0.0002, we can observe a graph as shown in figure 18.3.

The logistic model doesn't reach saturation even at 2000 days when Ip = 0.0002. But the predicted line closely follows the observed cumulative fraction of covid cases.

SOMALIA

Predicted vs Observed - Somalia

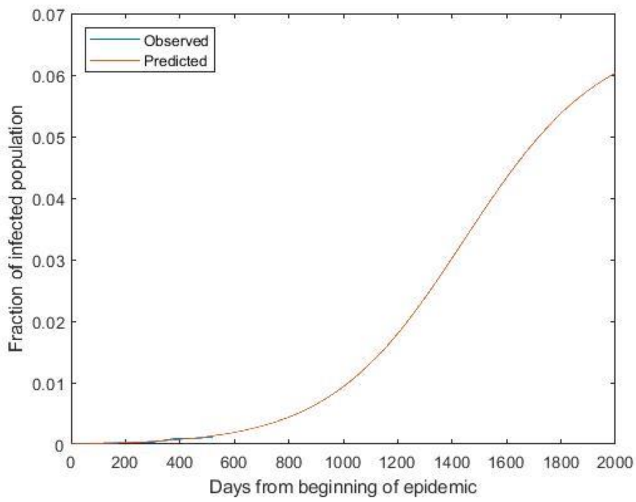


Fig 19.1 – Modified SIR with Ip = 0.02

Figure fig 19.1represents the predicted vs observed graph of cumulative fraction using modified SIR model for 2000 days of prediction for the country Somalia with initial Ip=0.02.

The predicted model closely follows the observed one as we can observe the same in fig 19.1. When Ip = 0.02, the predicted model reaches a cumulative fraction value of 0.06 at 2000 days. We can analyse that by taking Ip = 0.02, the country doesn't reach saturation for the logistic model even at 2000 days but on the path of attaining saturation.

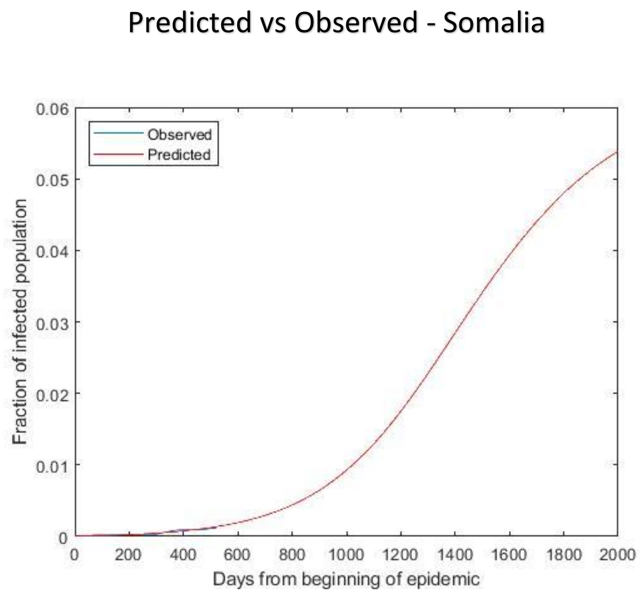


Fig 19.2 – Modified SIR with Ip = 0.002

Figure 19.2 represents the predicted modified SIR model when Ip = 0.002.

At 2000 days from the beginning of epidemic, the cumulative fraction reaches a value between 0.05 and 0.055. But this predicted model also follows the observed line closely.

Predicted vs Observed - Somalia

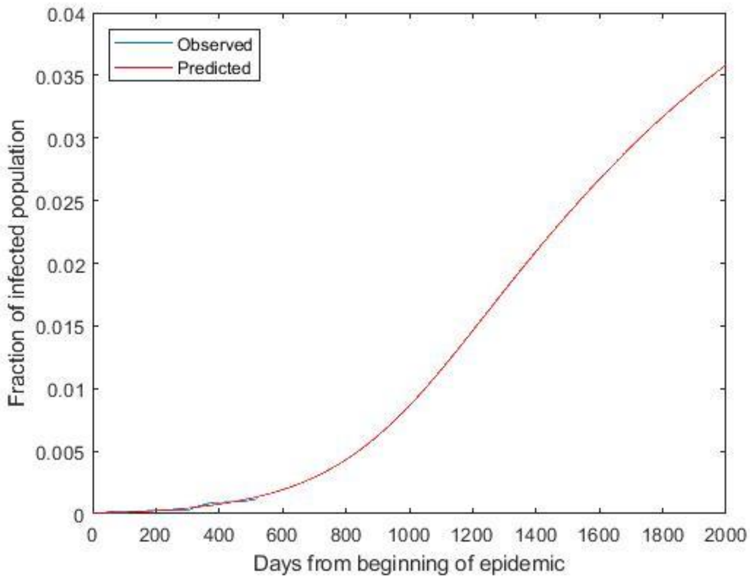


Fig 19.3 Modified SIR with Ip = 0.0002

The predicted modified SIR model when Ip = 0.0002 is shown in figure 19.3. Even at 2000 days, the logistic growth is nowhere near to reaching the saturation. It reaches a value of approximately 0.035 at 2000 days from the beginning of pandemic in the respective country. But this model also closely follows the observed cumulative fraction.

NEW ZEALAND

Predicted vs Observed – New Zealand

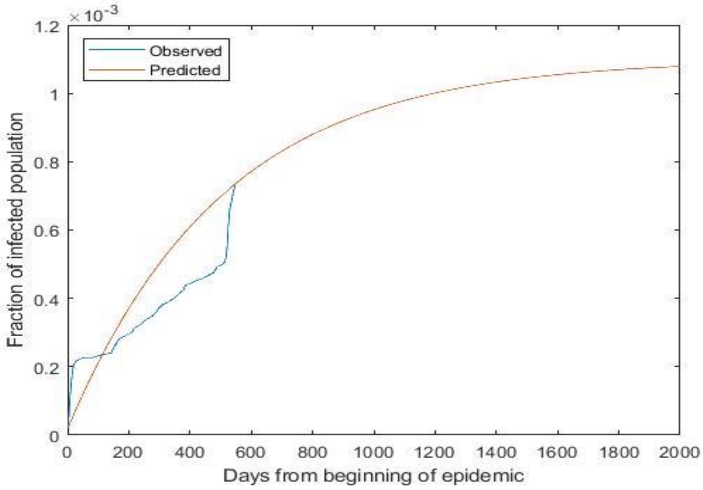


Fig 20.1 Modified SIR with Ip = 0.02

Predicted vs Observed – New Zealand

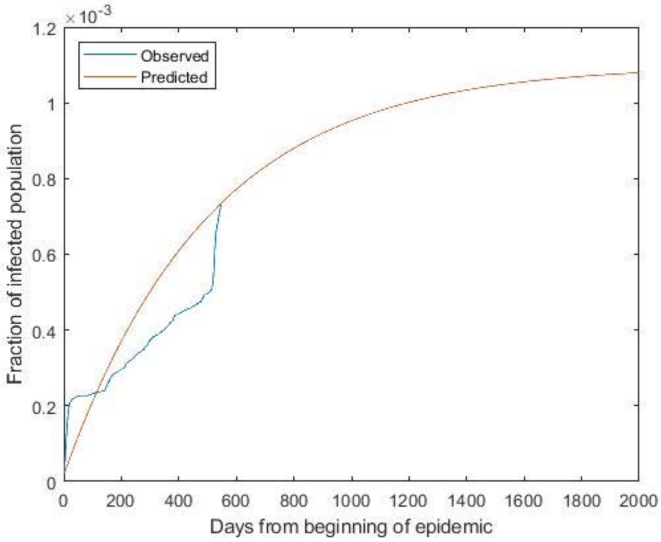


Fig 20.2 Modified SIR with Ip = 0.002

Predicted vs Observed – New Zealand

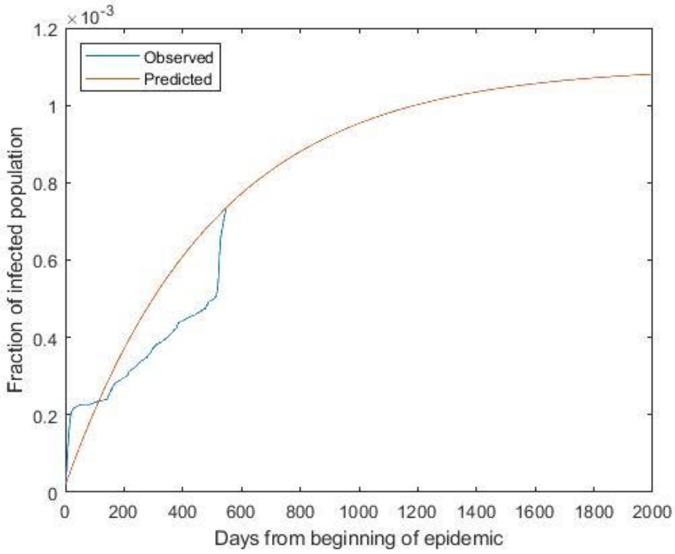


Fig 20.3 Modified SIR with Ip = 0.0002

Figure 20.1, 20.2 and 20.3 represents the predicted modified SIR model of New Zealand when crowd effect is considered with Ip values as 0.02, 0.002 and 0.0002 respectively.

We could not find any difference in the predictive models when different Ip values are considered. So we assume that uniquely for New Zealand, the predictive model is affected mostly by constants such as 'b' and 'a' more than the crowd effect.

CONCLUSION

Analysis of three models combined for India

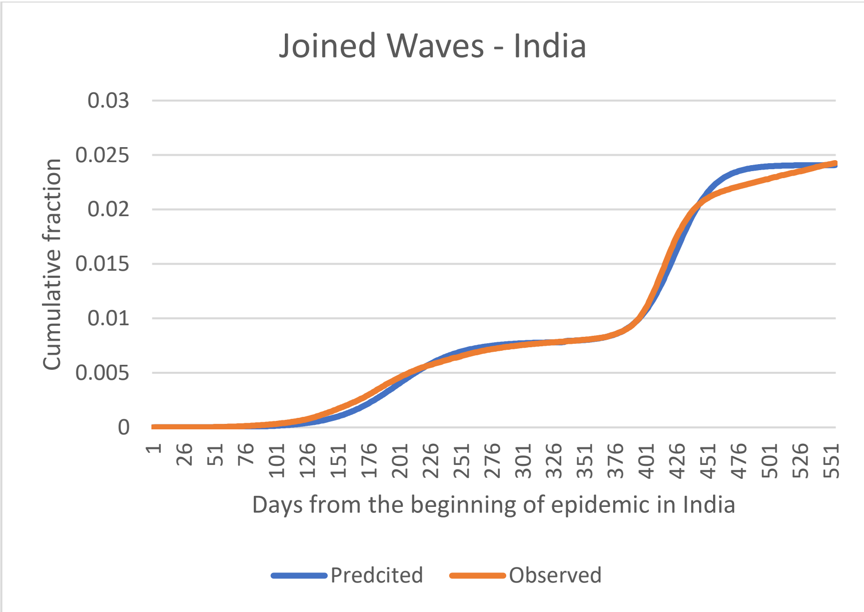


Fig 21.1 Logistic Growth model

Graph of SIR model - India

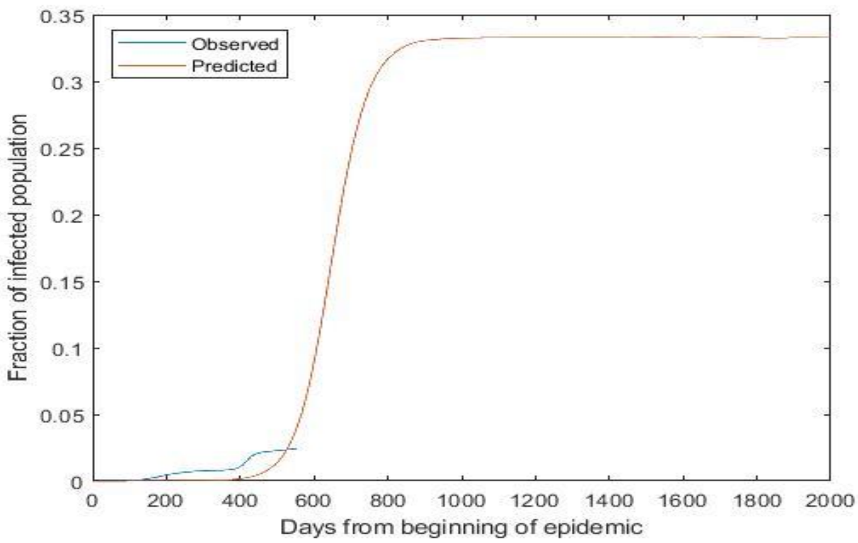


Fig 21.2 SIR model

Graph of modified SIR model - India

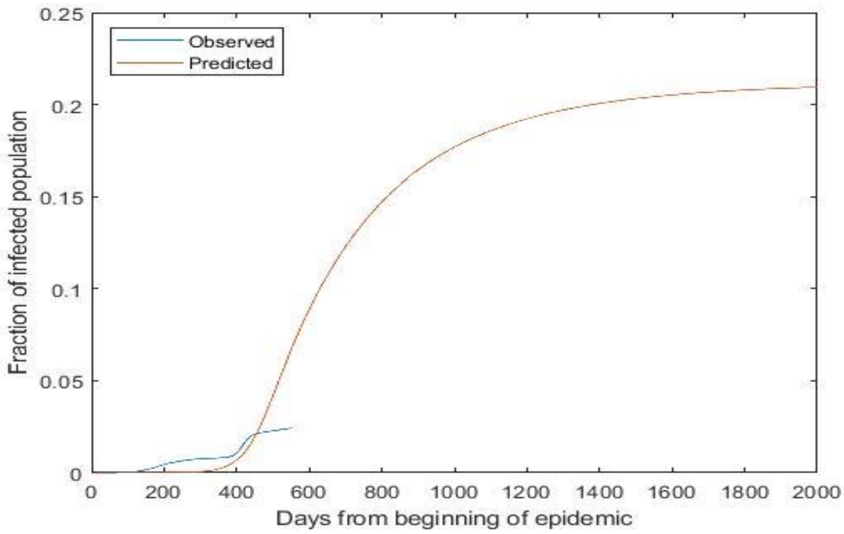


Fig 22.3 Modified SIR model

When we analyse the 3 models for India, we can observe that the logistic model gives more accurate analysis compared to traditional SIR and modified SIR models. Because, in fig 22.1 we can clearly observe that the predicted curve and the observed curve are almost similar, whereas for the other two models, predicted curve shows a significant difference with the observed curve.

Analysis of three models combined for Somalia

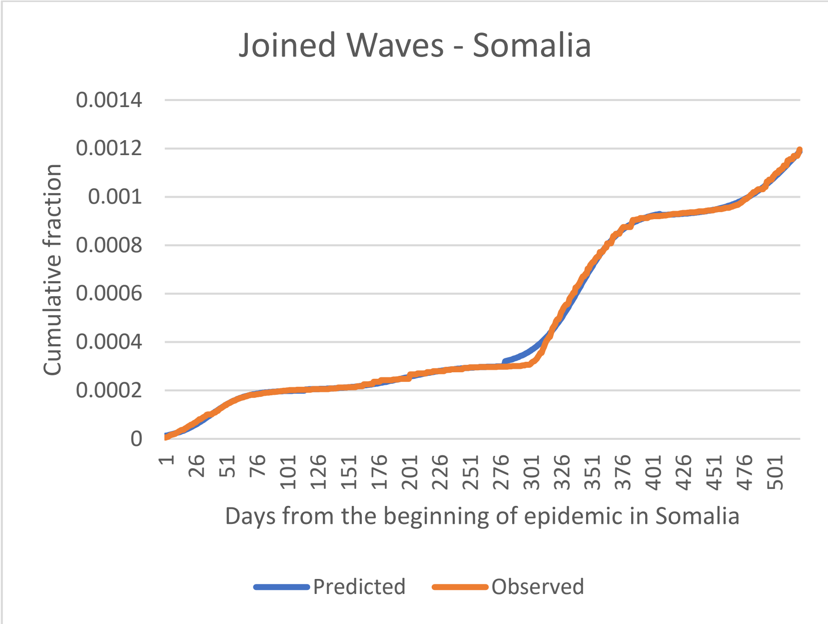


Fig 23.1 Logistic Growth Model

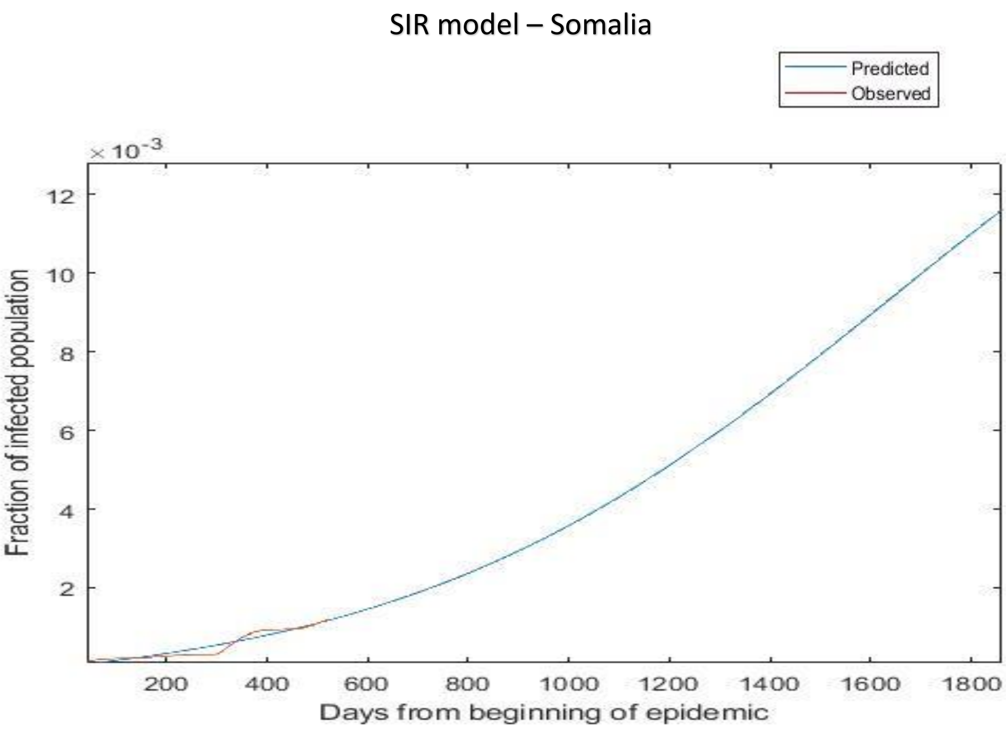


Fig 23.2 SIR model

Modified SIR model – Somalia

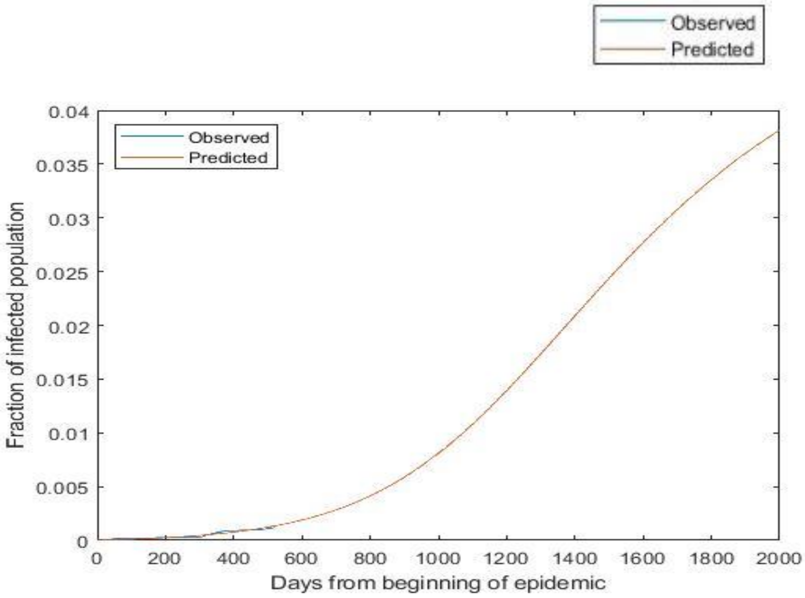


Fig 23.3 Modified SIR model

Similarly, here in the case of Somalia too, we observe that the logistic model is more accurate than the other two models. Even though both the SIR and modified SIR models have similar pattern, modified SIR model with optimized parameters have yield better result. The SIR and modified SIR models cannot be seen as perfect models to analyse the given data but can be used to approximately predict the pandemic for future. Compared to this, logistic model is more useful here in analysing the data but not for future predictions of the pandemic.

Analysis of three models combined for New Zealand

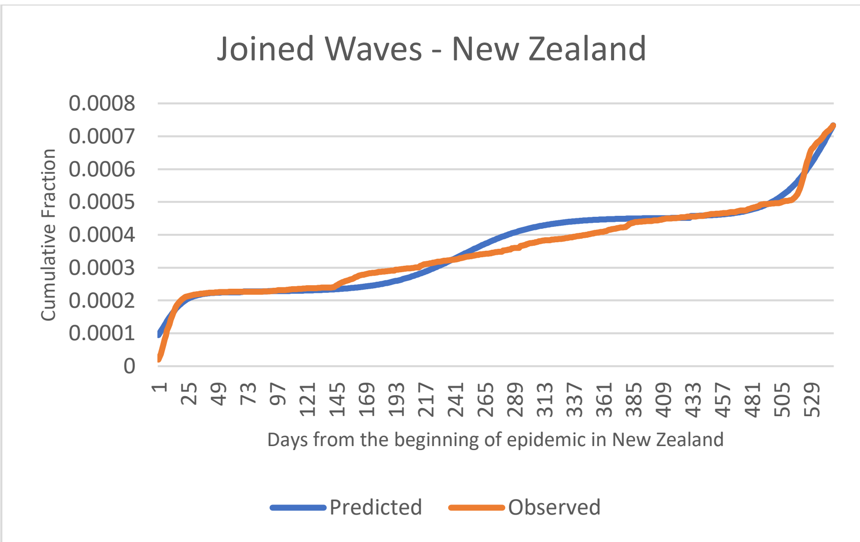


Fig 24.1 Logistic growth model

SIR Model – New Zealand

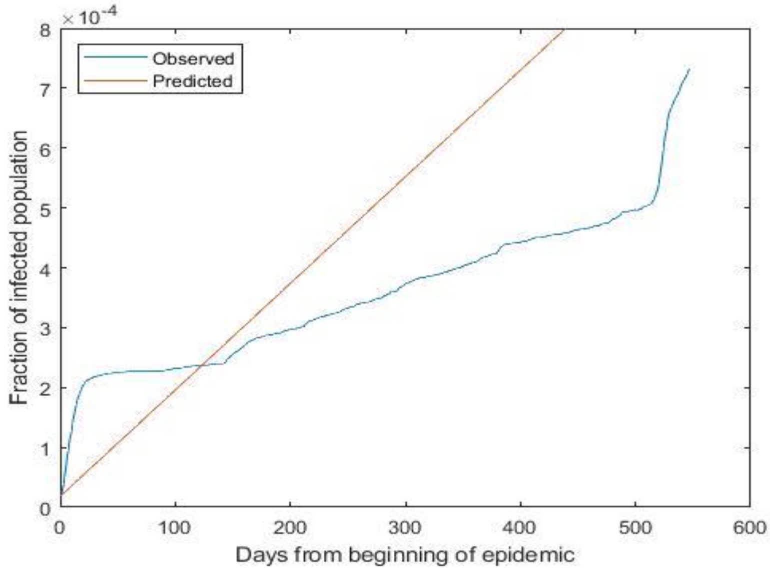


Fig 24.2 SIR model

Modified SIR Model – New Zealand

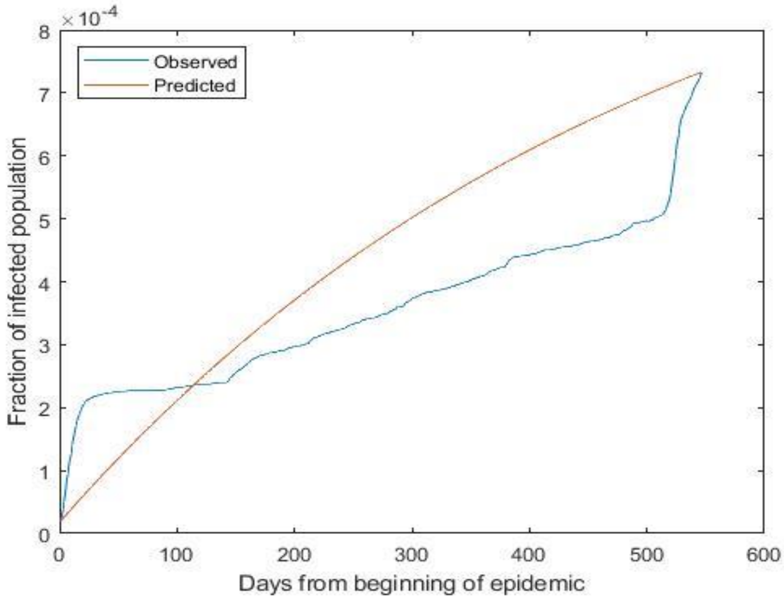


Fig 24.3 Modified SIR model

When we examine, logistic model, SIR model and modified SIR model represented by the figures 24.1,24.2 and 24.3 respectively, we find that the graph of the modified SIR model gives a clear picture of the observed and predicted data. For the cases of India and Somalia, we spotted that the logistic model is closer to the observed and predicted value compared to the

SIR and modified SIR models. But here in the case of New Zealand, modified SIR model is better than the logistic model as New Zealand had a linear growth in case of covid 19 after the initial exponential growth of covid cases. So, we could plot better graph with modified SIR model.

SIR model can be used to predict the outcome of a pandemic with very limited resources or parameters. If we know the population of the country and the time it takes a person to recover from the disease, we can easily analyse and predict the impact of pandemic for specific countries. But the limitation of SIR model is its simplicity itself. It works only in ideal situations where the population of the country remains constant or the recovery rate is absolutely known which in real world covid 19 crises, is entirely a different scenario. SIR model can be used to predict or analyse how well the country managed to prevent the spread of disease or vice versa. In our analysis of countries New Zealand, India and Somalia, we could observe that New Zealand was so much better than India and Somalia because we could plot the predicted model with the observed cumulative cases with the 'b' value which we have taken. Whereas for India and Somalia, we needed to take non ideal 'b' values to predict an approximate SIR model. SIR model doesn't take into consideration the cultural element of countries, which proved to be a major factor in determining the spread of disease in those respective countries.

From our observation SEIR, SEIS and MSEIR models are better than SIR model. These models can be a better choice to predict and analyse the covid-19 pandemic as covid-19 virus strain shows unusual behavioural characteristics. The recovery rate depends on different factors with 80% of the infected never shows any symptoms. The symptomatic patients are said to be the 'spreaders'. The non-attainment of immunity or in other words repeated infection of covid-19 for the same person makes it difficult to use SIR model to predict the impact of disease in real world scenario.

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