University at Buffalo
The State University of New York

Department of Industrial and Systems Engineering

IE 507: Design and Analysis of Experiments

Mini Project-3

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1. **Using the Design Generators F=ABC, G=ABD, and H=BCDE, show the complete Defining Relation for this design**

Design Summary

Factors:	8	Base Design:	5, 32	Resolution:	IV
Runs:	32	Replicates:	1	Fraction:	1/8
Blocks:	1	Center pts (total):	0		

Design Generators: F = ABC, G = ABD, H = BCDE

Defining Relation: I = ABCF = ABDG = BCDEH = CDFG = ADEFH = ACEGH = BEFGH

Alias Structure (up to order 5)

I + ABCF + ABDG + CDFG + ACEGH + ADEFH + BCDEH + BEFGH
A + BCF + BDG + CEGH + DEFH + ACDFG
B + ACF + ADG + CDEH + EFGH + BCDFG
C + ABF + DFG + AEGH + BDEH + ABCDG
D + ABG + CFG + AEFH + BCEH + ABCDF
E + ACGH + ADFH + BCDH + BFGH + ABCEF + ABDEG + CDEFG
F + ABC + CDG + ADEH + BEGH + ABDFG
G + ABD + CDF + ACEH + BEFH + ABCFG
H + ACEG + ADEF + BCDE + BEFG + ABCFH + ABDGH + CDFGH
AB + CF + DG + ACDEH + AEFGH + BCEGH + BDEFH
AC + BF + EGH + ADFG + BCDG + ABDEH + CDEFH
AD + BG + EFH + ACFG + BCDF + ABCEH + CDEGH
AE + CGH + DFH + BCEF + BDEG + ABCDH + ABFGH
AF + BC + DEH + ACDG + BDFG + ABEGH + CEFGH
AG + BD + CEH + ACDF + BCFG + ABEFH + DEFGH
AH + CEG + DEF + BCFH + BDGH + ABCDE + ABEFG
BE + CDH + FGH + ACEF + ADEG + ABCGH + ABDFH
BH + CDE + EFG + ACFH + ADGH + ABCEG + ABDEF
CD + FG + BEH + ABCG + ABDF + ACEFH + ADEGH
CE + AGH + BDH + ABEF + DEFG + ACDFH + BCFGH
CG + DF + AEH + ABCD + ABFG + BCEFH + BDEGH
CH + AEG + BDE + ABFH + DFGH + ACDEF + BCEFG
DE + AFH + BCH + ABEG + CEFG + ACDGH + BDFGH
DH + AEF + BCE + ABGH + CFGH + ACDEG + BDEFG
EF + ADH + BGH + ABCE + CDEG + ACFGH + BCDFH
EG + ACH + BFH + ABDE + CDEF + ADFGH + BCDGH
EH + ACG + ADF + BCD + BFG
FH + ADE + BEG + ABCH + CDGH + ACEFG + BCDEF
GH + ACE + BEF + ABDH + CDFH + ADEFG + BCDEG
ABE + CEF + DEG + ACDH + AFGH + BCGH + BDFH
ABH + CFH + DGH + ACDE + AEFG + BCEG + BDEF
ACD + AFG + BCG + BDF + ABEH + CEFH + DEGH

2. **Show the linear combination of effects confounded with variable AB in question 1.**

Using the design generators mentioned in question 1, we have defining relation as

Defining Relation: I = ABCF = ABDG = BCDEH = CDFG = ADEFH = ACEGH = BEFGH

Alias structure is obtained by multiplying a factor with the defining relation.
The product of AB with the defining relation gives it's Alias structure (upto order 5)
AB=CF=DG=ACDEH=AEFGH=BCEGH=BDEFH
Therefore, the **linear combination of effects** confounded with variable AB is
l_{AB}=AB+CF+DG+ACDEH+AEFGH+BCEGH+BDEFH

3. **Using the information provided in the *__Problem__* section above, create the most appropriate Treatment design for this experiment using Minitab.**

Answer:

Using the information provided in the ***Problem*** section above, create the most appropriate treatment design for this experiment using Minitab.

The treatment design is a ¼ fractional factorial design with 16 runs and a resolution IV design. The factorial design is represented as a 2^ (6-2) design.

The treatment design for the given problem is shown below:

The factors are:

A-Current

B-Electrode Force

C-Weld Time

D-Cleanliness of Parts

E-Condition of the Electrode

F-Part Alignment

Design Table

Run	Blk	A	B	C	D	E	F
1	1	-	-	-	-	-	-
2	1	+	-	-	-	+	-
3	1	-	+	-	-	+	+
4	1	+	+	-	-	-	+
5	1	-	-	+	-	+	+
6	1	+	-	+	-	-	+
7	1	-	+	+	-	-	-
8	1	+	+	+	-	+	-
9	1	-	-	-	+	-	+
10	1	+	-	-	+	+	+
11	1	-	+	-	+	+	-
12	1	+	+	-	+	-	-
13	1	-	-	+	+	+	-
14	1	+	-	+	+	-	-
15	1	-	+	+	+	-	+
16	1	+	+	+	+	+	+

4. **Display your design in Standard Order. Open the attached Strength.mtw file and merge the Strength data to your design. Analyze this data using the most appropriate model possible with Minitab. Provide a descriptive analysis of all figures and output.**

Answer:

The design is displayed in the standard order along with the response value of strength as shown below:

StdOrder	RunOrder	CenterPt	Blocks	Current	Electrode Force	Weld Time	Cleanliness	Condition	Part Alignment	Strength
1	1	1	1	1.4	125	0.10	Current	New	0.005	4.858
2	2	1	1	1.7	125	0.10	Current	50K	0.005	6.453
3	3	1	1	1.4	175	0.10	Current	50K	0.015	2.648
4	4	1	1	1.7	175	0.10	Current	New	0.015	5.316
5	5	1	1	1.4	125	0.15	Current	50K	0.015	5.465
6	6	1	1	1.7	125	0.15	Current	New	0.015	5.733
7	7	1	1	1.4	175	0.15	Current	New	0.005	5.762
8	8	1	1	1.7	175	0.15	Current	50K	0.005	7.297
9	9	1	1	1.4	125	0.10	More	New	0.015	3.473
10	10	1	1	1.7	125	0.10	More	50K	0.015	5.250
11	11	1	1	1.4	175	0.10	More	50K	0.005	3.663
12	12	1	1	1.7	175	0.10	More	New	0.005	6.545
13	13	1	1	1.4	125	0.15	More	50K	0.005	6.682
14	14	1	1	1.7	125	0.15	More	New	0.005	6.792
15	15	1	1	1.4	175	0.15	More	New	0.015	4.291
16	16	1	1	1.7	175	0.15	More	50K	0.015	5.808

Initial Analysis of the Raw Data:

After importing the given data into Minitab, the experimental design was created containing 6 factors and 16 experimental runs. Then the experiment was analyzed using the 2^k factorial design. The specific design used in this case is the 2^6-2 factorial design. The response variable is the value of Weld Strength. Initially, the test was performed using all the terms in the experiment. This includes both the main factors and the interactions between the factors. After conducting the test once, it was identified that the higher-order terms which included the three, and fourth-order interactions were not significant. These terms were eliminated, and the factorial analysis was performed again. The results are shown below:

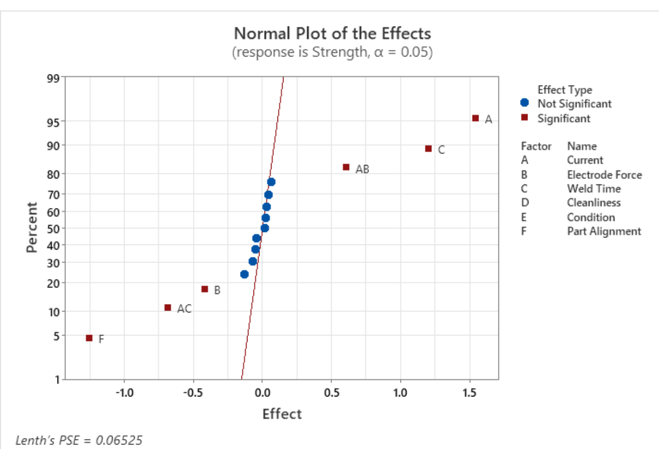
Coded Coefficients

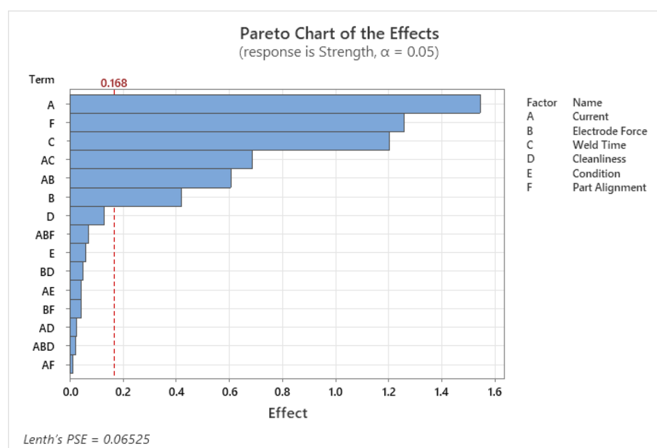
Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		5.377	*	*	*	
Current	1.5440	0.7720	*	*	*	1.00
Electrode Force	-0.4220	-0.2110	*	*	*	1.00
Weld Time	1.2030	0.6015	*	*	*	1.00
Cleanliness	-0.12850	-0.06425	*	*	*	1.00
Condition	0.06200	0.03100	*	*	*	1.00
Part Alignment	-1.2585	-0.6292	*	*	*	1.00
Current*Electrode Force	0.6065	0.3032	*	*	*	1.00
Current*Weld Time	-0.6865	-0.3433	*	*	*	1.00
Current*Cleanliness	0.02750	0.01375	*	*	*	1.00
Current*Condition	0.04350	0.02175	*	*	*	1.00
Current*Part Alignment	0.013500	0.006750	*	*	*	1.00
Electrode Force*Cleanliness	-0.05050	-0.02525	*	*	*	1.00
Electrode Force*Part Alignment	-0.04250	-0.02125	*	*	*	1.00
Current*Electrode Force*Cleanliness	0.02150	0.01075	*	*	*	1.00
Current*Electrode Force*Part Alignment	-0.07150	-0.03575	*	*	*	1.00

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	15	25.8612	1.72408	*	*
Linear	6	22.4536	3.74227	*	*
Current	1	9.5357	9.53574	*	*
Electrode Force	1	0.7123	0.71234	*	*
Weld Time	1	5.7888	5.78884	*	*
Cleanliness	1	0.0660	0.06605	*	*
Condition	1	0.0154	0.01538	*	*
Part Alignment	1	6.3353	6.33529	*	*
2-Way Interactions	7	3.3852	0.48361	*	*
Current*Electrode Force	1	1.4714	1.47137	*	*
Current*Weld Time	1	1.8851	1. 8513	*	*
Current*Cleanliness	1	0.0030	0.00303	*	*
Current*Condition	1	0.0076	0.00757	*	*
Current*Part Alignment	1	0.0007	0.00073	*	*
Electrode Force*Cleanliness	1	0.0102	0.01020	*	*
Electrode Force*Part Alignment	1	0.0072	0.00722	*	*
3-Way Interactions	2	0.0223	0.01115	*	*
Current*Electrode Force Cleanliness	1	0.0018	0.00185	*	*
Current*Electrode Force*Part Alignment	1	0.0204	0.02045	*	*
Error	0	*	*		
Total	15	25.8612			

From both charts, it is evident that we are not able to get the values for all the metrics such as Coefficients of Standard Errors, T Values, and P values. This is because all the degrees of freedom are used up in estimating the effects of the main effects and there are none left to estimate the error term. It is seen that the degrees of freedom for the error term are 0. The Normal and the Pareto chart of effects for this model with a confidence level of 95% are shown in the following figures.





From the normal plot and Pareto chart, we can see that there are 6 significant factors in the model which are A B C F, and the interactions AB and AC (Current, Electrode, Weld Time, Part Alignment). Since there is no error term in the model, we cannot estimate all the statistical measures.

Hence, the above results are inaccurate. This is the reason the residual plots are not plotted in Minitab. The following error is shown in Minitab for Residual Analysis plots:

* NOTE * Could not graph the specified residual type because MSE = 0 or the degrees of freedom for error = 0.

To estimate the error term in the model some of the factors are added to the error term by taking the Coefficients, Effects, Sum of Squares, and Adjusted Mean Squares into consideration that the factors that have very less effect on the entire model are eliminated. These terms are removed one by one instead of all at once to maintain the accuracy of the model. The terms that are included in the final model are as follows:
Current, Electrode Force, Weld Time, Cleanliness of Parts, Condition of the Electrode, Part Alignment, and the interactions Current*Electrode Force and Current*Weld Time.

After the initial pooling of the results, the factorial design is re-analyzed and the results are shown:

The Alias Structure for this design is as follows:

Alias Structure

Factor	Name
A	Current
B	Electrode Force
C	Weld Time
D	Cleanliness
E	Condition
F	Part Alignment

Aliases
I + ABCE + ADEF + BCDF
A + BCE + DEF + ABCDF
B + ACE + CDF + ABDEF
C + ABE + BDF + ACDEF
D + AEF + BCF + ABCDE
E + ABC + ADF + BCDEF
F + ADE + BCD + ABCEF
AB + CE + ACDF + BDEF
AC + BE + ABDF + CDEF

The alias table represents certain factors or variables grouped together in aliases to reduce the number of experimental runs needed to obtain statistical information about their effects on a process or system. Each alias represents a combination of factor settings that are assumed to have similar effects, and conducting experiments on these aliases allows for efficient data collection and analysis.

The interpretation of the alias table is as follows:

- Alias "A" (Current) is aliased with "BCE" (Electrode Force), "DEF" (Weld Time), and "ABCDF" (Part Alignment).
- Alias "B" (Electrode Force) is aliased with "ACE" (Condition), "CDF" (Cleanliness), and "ABDEF" (Part Alignment).
- Alias "C" (Weld Time) is aliased with "ABE" (Current), "BDF" (Cleanliness), and "ACDEF" (Part Alignment).

- Alias "D" (Cleanliness) is aliased with "AEF" (Current), "BCF" (Electrode Force), and "ABCDE" (Part Alignment).
- Alias "E" (Condition) is aliased with "ABC" (Current), "ADF" (Weld Time), and "BCDEF" (Part Alignment).
- Alias "F" (Part Alignment) is aliased with "ADE" (Current), "BCD" (Electrode Force), and "ABCEF" (Cleanliness).
- Alias "AB" (Current and Electrode Force) is aliased with "CE" (Condition), "ACDF" (Cleanliness), and "BDEF" (Part Alignment).
- Alias "AC" (Current and Weld Time) is aliased with "BE" (Electrode Force), "ABDF" (Cleanliness), and "CDEF" (Part Alignment).

Coded Coefficients

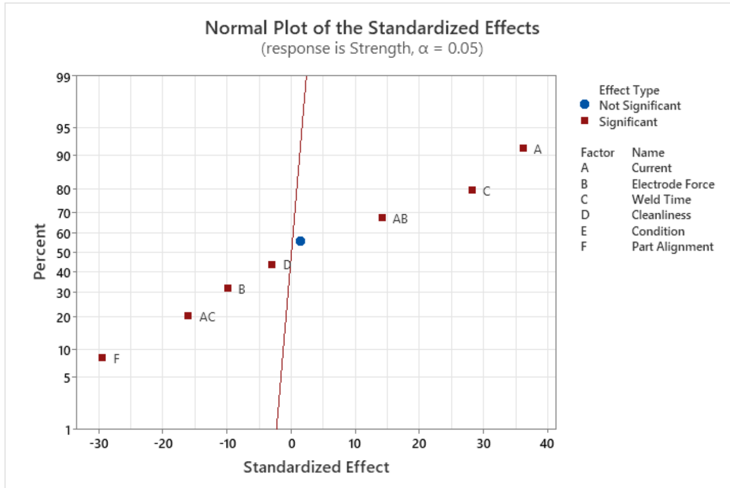
Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		5.3773	0.0213	251.87	0.000	
Current	1.5440	0.7720	0.0213	36.16	0.000	1.00
Electrode Force	-0.4220	-0.2110	0.0213	-9.88	0.000	1.00
Weld Time	1.2030	0.6015	0.0213	28.17	0.000	1.00
Cleanliness	-0.1285	-0.0642	0.0213	-3.01	0.020	1.00
Condition	0.0620	0.0310	0.0213	1.45	0.190	1.00
Part Alignment	-1.2585	-0.6292	0.0213	-29.47	0.000	1.00
Current*Electrode Force	0.6065	0.3032	0.0213	14.20	0.000	1.00
Current*Weld Time	-0.6865	-0.3433	0.0213	-16.08	0.000	1.00

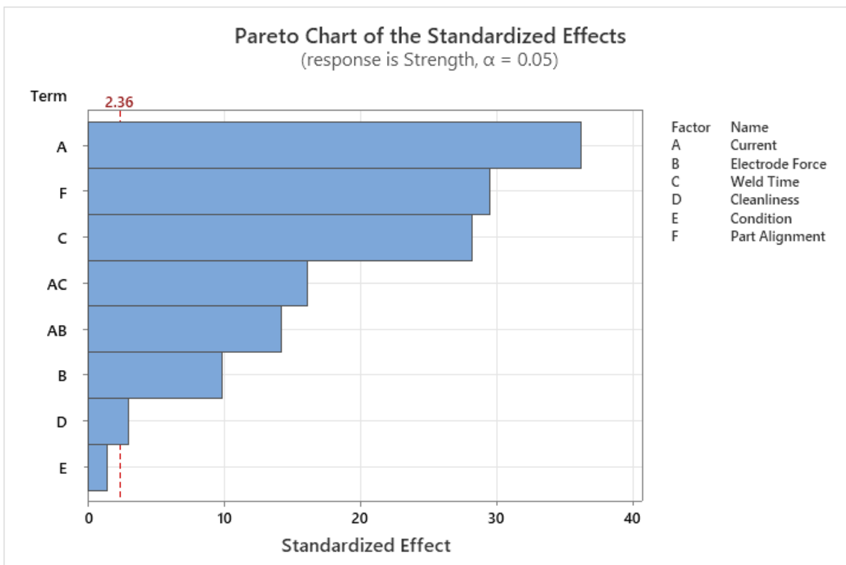
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	8	25.8101	3.22627	442.41	0.000
Linear	6	22.4536	3.74227	513.17	0.000
Current	1	9.5357	9.53574	1307.62	0.000
Electrode Force	1	0.7123	0.71234	97.68	0.000
Weld Time	1	5.7888	5.78884	793.81	0.000
Cleanliness	1	0.0660	0.06605	9.06	0.020
Condition	1	0.0154	0.01538	2.11	0.190
Part Alignment	1	6.3353	6.33529	868.75	0.000
2-Way Interactions	2	3.3565	1.67825	230.14	0.000
Current*Electrode Force	1	1.4714	1.47137	201.77	0.000
Current*Weld Time	1	1.8851	1.88513	258.50	0.000
Error	7	0.0510	0.00729		
Total	15	25.8612			

The Coded Coefficients and the ANOVA Tables show there is now a quantifiable value for the error. Taking the P-Values into account,

Null Hypothesis: The factor does not have a significant effect on the value of the Weld Strength. **Alternative Hypothesis:** The factor has a significant effect on the value of the Weld Strength.
The p-values of the factors – current, electrode force, weld time, cleanliness, condition, and the two-way interactions between current*electrode force and current*weld time are less than the significance value of 0.05 (<0.05). Therefore, for these factors, we must reject the null hypothesis that the factors do not have a significant effect on the response value of strength.
The Normal and the Pareto charts for the standardized effects are shown below. These plots show that the condition of the electrode (Factor) does not have a significant effect on the strength of the weld.





Model Summary:

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.0853957	99.80%	99.58%	98.97%

- This table provides information on the goodness of fit of the regression model from the ANOVA analysis.
- R-sq is 99.80%. This implies that 99.80% of the variation in the response variable is explained by the factors in the model.
- R-sq(adjusted) is 99.58%. This is slightly lower than R-sq as the model includes multiple independent variables.
- R-sq(pred) is 98.97%, which means that the model is expected to explain 98.97% of the variation in the new data. This suggests that the model can be sufficient to use to predict future outcomes.

After analyzing the factorial design, it is clear that the factors A, B, C, F and the two-factor interactions A*B and A*C and have a significant effect on the values of the Strength of the weld, and the values of these factors must be carefully chosen to get the optimal values of CCR. The Residual Analysis is interpreted in the following question.

5. Check model adequacy by analyzing residual plots. Provide a descriptive analysis of all figures and output.

Answer:

Residual Analysis

The following assumptions are made for the residual analysis of the collected data:

Residual Analysis Assumptions:

Normality:

The residuals are assumed to be normally distributed, meaning that the distribution of residuals follows a bell-shaped curve. This assumption is important for making valid inferences about the model parameters and for the accuracy of statistical tests and confidence intervals.

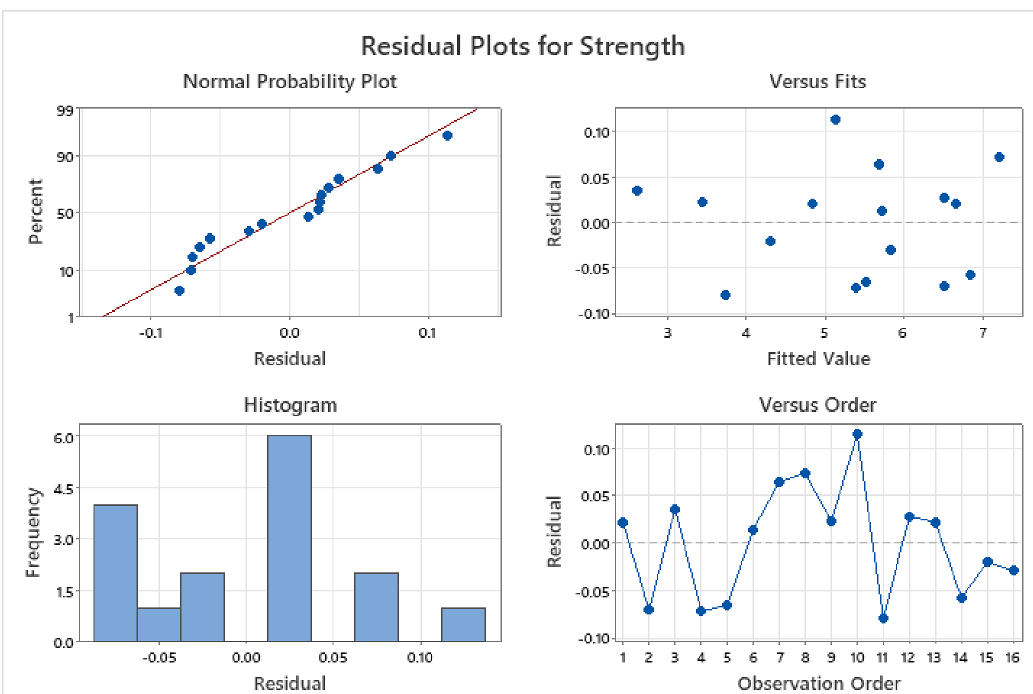
Independence:

The residuals are assumed to be independent of each other, meaning that the error of one observation does not depend on the error of any other observation. This assumption is important for the validity of statistical tests and confidence intervals.

Homoscedasticity:

The variance of the residuals is assumed to be constant across all levels of the independent variables. In other words, the spread of residuals should be constant across the range of predicted values. If the spread of residuals varies systematically with the predicted values, it is called heteroscedasticity.

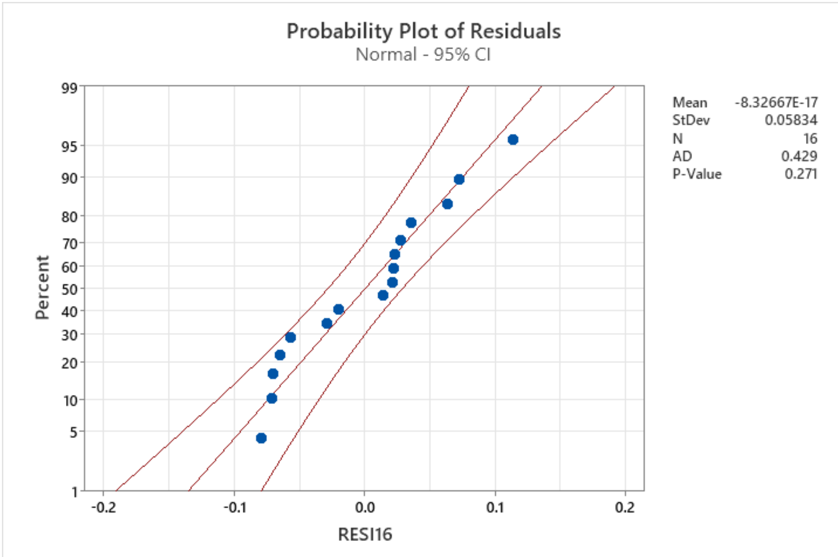
The summary if the residual Analysis is shown in the following figure:



Normality:

To test the normality of the residuals of (Strength of the Weld Values of the significant factors) the probability plot is drawn along with the graphical summary of the given data.

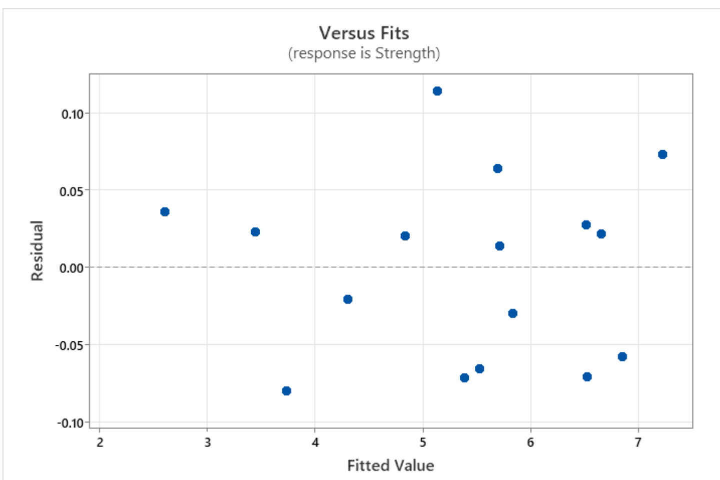
Null Hypothesis: The null hypothesis for the normality test is that the data is normally distributed. **Alternative Hypothesis:** The Alternative Hypothesis is that the data is not normally distributed.



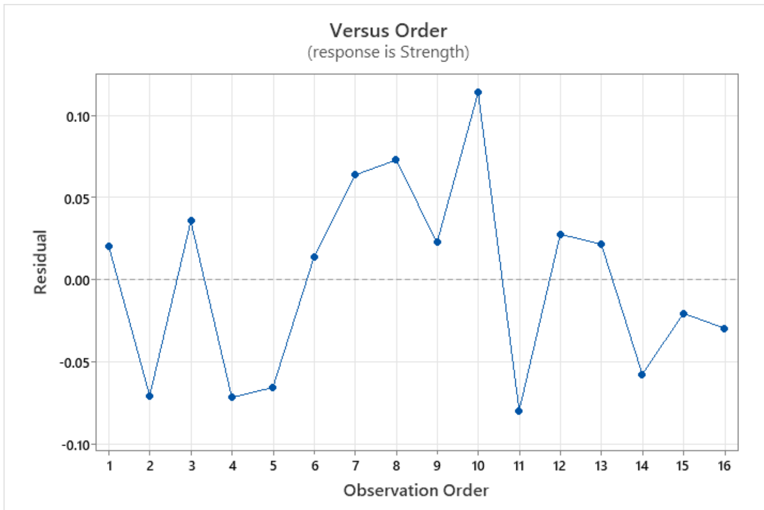
Here the normality plot is shown only for the residuals of the Strength values caused only by the significant factors in the experiment. After the normality test is conducted (Anderson Darling Normality Test), the results show that the p-value for the normality plot is 0.271, which is greater than the significance value of 0.05. This shows that we fail to reject the null hypothesis that the data of the Strength of the weld values form a normal distribution. Hence the assumption that the data is normally distributed is not violated.

Homoscedasticity:

To check the test for equal variances or homoscedasticity, the residuals vs the fits plot is used. It is observed in the plot that the data points of the residuals occur randomly on either side of the 0 lines in the plot. Also, there is no specified pattern such as cyclic or seasonal patterns in the way the points occur. There is no funneling or fanning effects to show non-constant variance for the data.



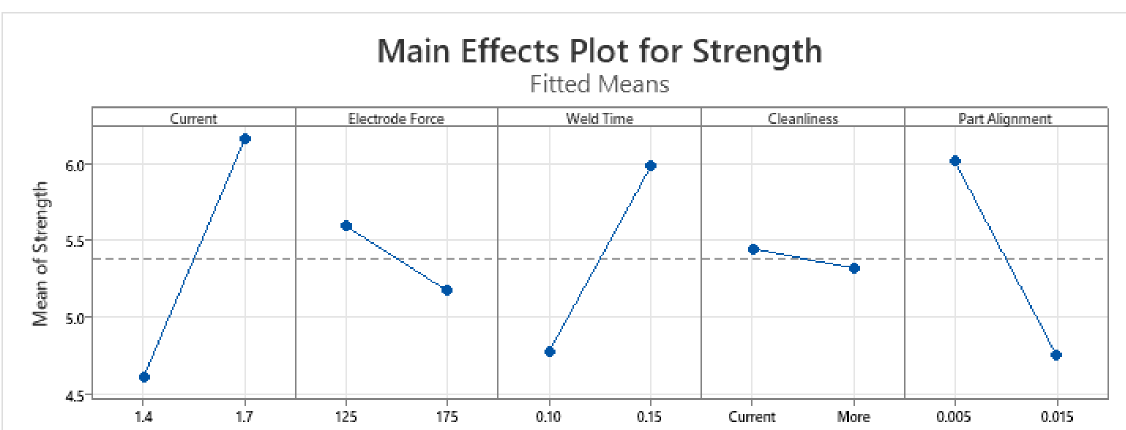
Independence:
To check the independence assumption in Minitab, we used residuals vs order plot. These plots can help to detect any patterns or trends in the data that may violate the independence assumption. In this case, residuals vs order of observations plots is used to verify the independence of the data. The plot is shown here:

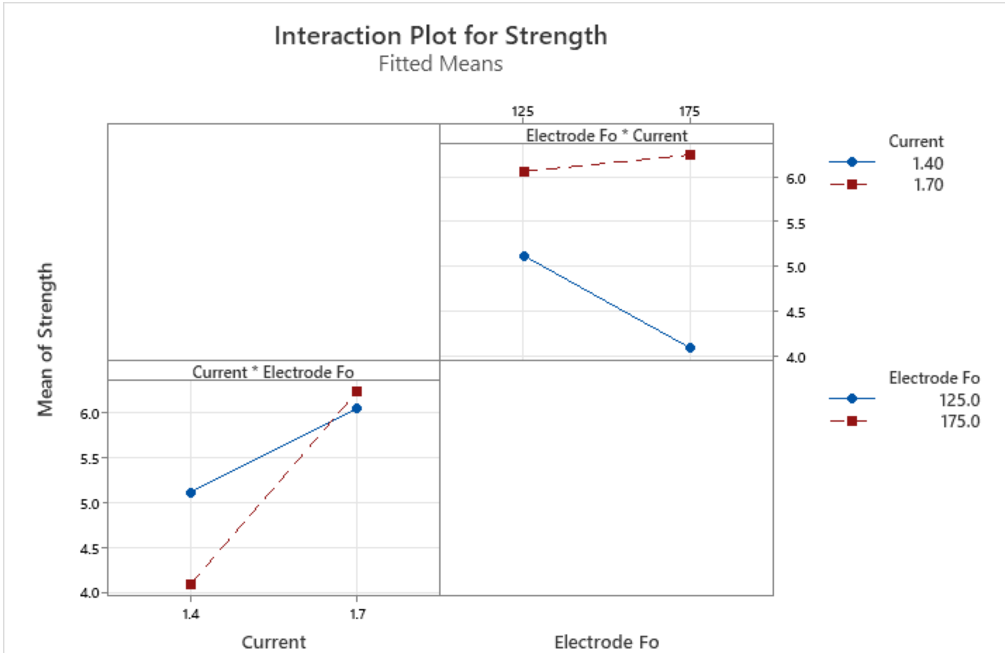


It is clearly seen in the Residual vs the Observation Order Plot that there is no regular pattern in the arrangement of the data points which indicates the criteria for the independence of the residuals is not violated. From the above plots and interpretations, it can be concluded that the assumptions for the residual analysis are not violated.

6. **Create factorial plots for only significant (α = 0.10) factors and interactions. What are your recommendations for improving the Weld Strength?**

Answer:
The factorial plots have been created for the significant factors that affect the weld strength using a confidence level of **α = 0.10.** The results are shown below:





Interpretation for the Main Effects and the Interaction Plots:

Current: The coefficient for Current is positive (1.5440) which indicates that an increase in current leads to an increase in weld strength. The p-value is less than 0.001, which means that the effect of current on the weld strength is statistically significant, indicating that increasing the welding current will result in an increase in weld strength.

Electrode Force: The coefficient for the Electrode Force variable is negative (-0.4220) which means that an increase in the electrode force leads to a decrease in the weld strength. The p-value is less than 0.001, which means that the effect of electrode force on the weld strength is statistically significant. From the plot, a lower value for Electrode Force will ensure a higher value for the weld strength.

Weld Time: The coefficient for the Weld Time variable is positive (1.2030) which indicates that an increase in weld time leads to an increase in weld strength. The p-value is less than 0.001, which means that the effect of weld time on weld strength is statistically significant. This shows that for a higher value of the weld time, we obtain higher weld strength.

Cleanliness: The coefficient for the Cleanliness variable is negative (-0.1285) which means that an increase in the cleanliness leads to a decrease in the weld strength. The p-value is 0.02, which is less than the significance level of 0.1, which means that the effect of cleanliness on the weld strength is statistically significant. Improving cleanliness can result in an increase in weld strength.

Part Alignment: The coefficient for the Part Alignment variable is negative (-1.2585) which means that an increase in the part misalignment leads to a decrease in the weld strength. The p-value is less than 0.001, which means that the effect of part alignment on the weld strength is statistically significant. Improving part alignment can result in an increase in weld strength.

Current * Electrode Force Interaction: The interaction between Current and Electrode Force has a positive coefficient (0.6065) and is statistically significant, the p-value is less than 0.001. The interaction plot indicates that the effect of current on weld strength is stronger when the electrode force is higher.

Recommendation: To obtain a higher value of the weld strength, the current, Weld Time must have higher value, and the electrode force, cleanliness, and part alignment must be set to a lower value.

7. Manually calculate the Main Effect of factors A and B and show all work?

Main Effect of Current (A):
Considering design table

A	B	AB	Strength	Adhesion
-1	-1	1	4.858	1.634
1	-1	-1	6.453	2.598
-1	1	-1	2.648	2.6
1	1	1	5.316	0.684
-1	-1	1	5.465	0.066
1	-1	-1	5.733	0.873
-1	1	-1	5.762	2.545
1	1	1	7.297	0.786
-1	-1	1	3.473	0.615
1	-1	-1	5.25	1.408
-1	1	-1	3.663	3.517
1	1	1	6.545	1.04
-1	-1	1	6.682	0.774
1	-1	-1	6.792	1.5
-1	1	-1	4.291	1.45
1	1	1	5.808	-0.124

Adhesion:
Mean Response when **Current(A)** is at low setting=
(1.634+2.6+0.066+2.545+0.615+3.517+0.774+1.45)/8 = 1.650125
Mean Response when **Current(A)** is at high setting =
(2.598+0.684+0.873+0.786+1.408+1.04+1.5-0.124)/8 = 1.095125
Effect of current on Adhesion is: 1.095125 - 1.650125= -0.555
As factor A moves from its low to high setting, response value (adhesion) decreases by 0.555.

Strength:
Mean Response when **Current(A)** is at low setting=
(4.858+2.648+5.465+5.762+3.473+3.663+6.682+4.291)/8 = 4.60525
Mean Response when **Current(A)** is at high setting=
(5.808+6.792+6.545+5.25+7.297+5.733+5.316+6.453)/8 = 6.14925
Effect of current on Adhesion is: 6.14925-4.60525 =1.544
As factor A moves from its low to high setting, response value (strength) increases by 1.544.

Main Effect of Electrode Force (B):
Adhesion:
Mean Response when **Electrode Force(B)** is at low setting=
(1.634+2.598+0.066+0.873+0.615+1.408+0.774+1.5)/8= 1.1835
Mean Response when **Electrode Force(B)** is at high setting =
(2.6+0.684+0.873+2.545+3.517+1.04+1.45-0.124)/8 = 1.56225
Effect of Electrode force on Adhesion is: 1.56225-1.1835= 0.37875
As factor B moves from its low to high setting, response value (adhesion) increases by 0.37875.
Strength:
Mean Response when **Electrode Force(B)** is at low setting=
(4.858+6.453+5.316+5.465+7.297+3.473+6.545+6.682)/8= **5.58825**
Mean Response when **Electrode Force(B)** is at high setting =
(2.648+5.316+5.762+7.297+3.663+6.454+4.291+5.808)/8= **5.16625**
Effect of Electrode force on Strength is: 5.16625-5.58825= -0.422
As factor B moves from its low to high setting, response value (strength) decreases by 0.422.

8. Manually calculate the AB Interaction Effect and the AB coefficient and show all the work

We define the interaction effect AB as the average difference between the effect of A at the high level of B and the effect of A at the low level of B.

A	B	AB	Strength	Adhesion
-1	-1	1	4.858	1.634
1	-1	-1	6.453	2.598
-1	1	-1	2.648	2.6
1	1	1	5.316	0.684
-1	-1	1	5.465	0.066
1	-1	-1	5.733	0.873
-1	1	-1	5.762	2.545
1	1	1	7.297	0.786
-1	-1	1	3.473	0.615
1	-1	-1	5.25	1.408
-1	1	-1	3.663	3.517
1	1	1	6.545	1.04
-1	-1	1	6.682	0.774
1	-1	-1	6.792	1.5
-1	1	-1	4.291	1.45
1	1	1	5.808	-0.124

AB interaction effect on Adhesion
We find the average response when the interaction is "high" and the average response when the interaction is "low". The difference is the magnitude of the interaction effect.
When the interaction is at '+1'
(1.634+0.684+0.066+0.786+0.615+1.04+0.774-0.124)/8=0.684375
When the interaction is at '-1'
(2.598+2.6+0.873+2.545+1.408+3.517+1.5+1.45)/8=2.061375
Interaction effect of AB on Adhesion is: 0.684375-2.061375= -1.377

Coefficient of AB interaction is calculated as half of the interaction effect value
i.e -1.377/2=-0.6885

AB interaction effect on Strength
When the interaction is at '+1'
(4.858+5.316+5.465+7.297+3.473+6.545+6.682+5.808)/8= 5.6805

When the interaction is at '-1'
(6.453+2.648+5.733+5.762+5.25+3.663+6.792+4.291)/8= 5.074
Interaction effect of AB on Strength is: 5.6805-5.074= 0.6065
Coefficient of AB interaction is calculated as half of the interaction effect value
i.e 0.6065/2=0.3032

9. **With weld parameters set at levels that give high weld strengths, what would you estimate the mean weld strength to be and how much variability would you still expect to see in the data? Produce factorial plots to support your answer and explain how you arrived at your estimate of variability.**

Answer:
Using the predict functionality from factorial tests section in Minitab, we obtained the following settings that would result in high weld strength.

Settings

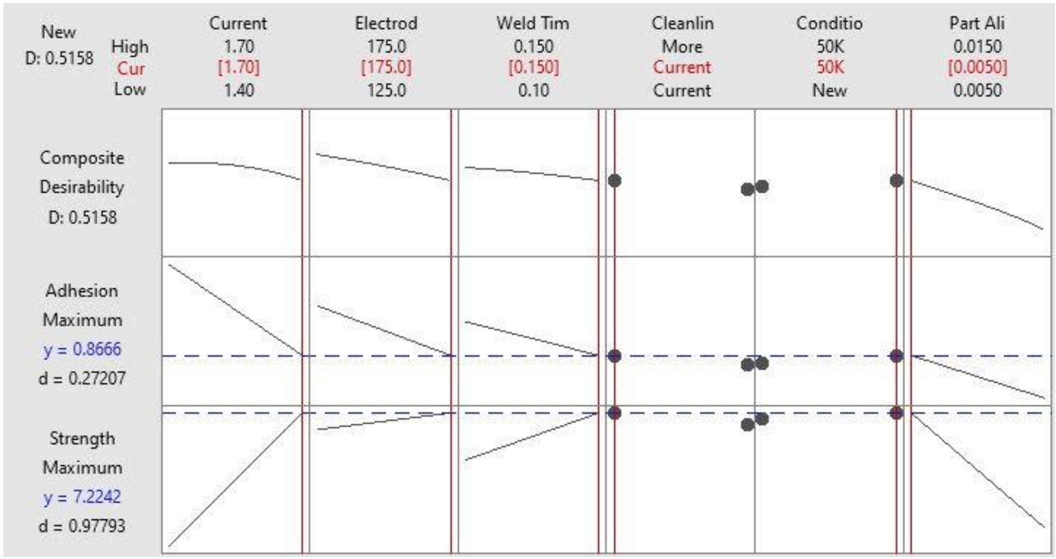
Variable	Setting
Current	1.7
Electrode Force	175
Weld Time	0.15
Cleanliness	Current
Condition	50K
Part Alignment	0.005

Prediction

Fit	SE Fit	95% CI	95% PI
7.22425	0.0640468	(7.07280, 7.37570)	(6.97184, 7.47666)

The estimated mean weld strength with these settings is 7.22425. The variability in the model is explained by the SE Fit value which is 0.0640. Hence, about 6.4 percent of the variability would be expected to be seen in the data. The SE Fit value indicates how much the predicted values for the response variable are likely to vary around the regression line due to the random errors in the data. A smaller SE Fit value indicates less variability and greater precision in the predicted values, whereas a larger SE Fit value indicates more variability and less precision.

With these settings, we have produced the factorial plots in Minitab using response optimizer. In the following matrix, 3rd row provides the maximum strength that would be expected with the weld parameters set at levels that give high weld strengths.



10. **At this point in your analysis, what steps could you take to verify that your model for Weld Strength is performing as expected?**

Answer:

To verify that the model for Weld Strength is performing as expected, we can take several steps, including:

Residual Analysis: We analyze the residuals of your regression model. Residuals are the differences between the observed values of the dependent variable which is Weld Strength and the predicted values from the regression model. Plotting the residuals against the predicted values or the independent variables will help to identify any patterns or trends that may indicate issues with model performance. Ideally, as shown in the analysis conducted, the residuals are randomly distributed around zero with constant variance, indicating that the model is capturing the underlying patterns in the data.

Model Fit Measures: We assess the goodness of fit of the model using appropriate model fit measures. Common measures include R-squared, adjusted R-squared, and root mean squared error (RMSE). These measures provide information about how well the model explains the variation in the data and how accurately it predicts the dependent variable. Higher R-squared values and lower RMSE values generally indicate better model performance, but it's important to interpret them in the context of the specific study objectives and data characteristics which are the factors affecting the weld strength and powdered coated adhesion values.

11. **After welding, the joint is power-coated to create a protective finish. Again display your design in Standard Order. Open the attached Strength.mtw file and merge the Adhesion data to your design. Analyze this data using the most appropriate model possible with Minitab. Provide a descriptive analysis of all figures and output.**

Answer:

The design is displayed in the standard order along with the response value of strength as shown below:

StdOrder	RunOrder	CenterPt	Blocks	Current	Electrode Force	Weld Time	Cleanliness	Condition	Part Alignment	Adhesion
1	1	1	1	1.4	125	0.10	Current	New	0.005	1.634
2	2	1	1	1.7	125	0.10	Current	50K	0.005	2.598
3	3	1	1	1.4	175	0.10	Current	50K	0.015	2.600
4	4	1	1	1.7	175	0.10	Current	New	0.015	0.684
5	5	1	1	1.4	125	0.15	Current	50K	0.015	0.066
6	6	1	1	1.7	125	0.15	Current	New	0.015	0.873
7	7	1	1	1.4	175	0.15	Current	New	0.005	2.545
8	8	1	1	1.7	175	0.15	Current	50K	0.005	0.786
9	9	1	1	1.4	125	0.10	More	New	0.015	0.615
10	10	1	1	1.7	125	0.10	More	50K	0.015	1.408
11	11	1	1	1.4	175	0.10	More	50K	0.005	3.517
12	12	1	1	1.7	175	0.10	More	New	0.005	1.040
13	13	1	1	1.4	125	0.15	More	50K	0.005	0.774
14	14	1	1	1.7	125	0.15	More	New	0.005	1.500
15	15	1	1	1.4	175	0.15	More	New	0.015	1.450
16	16	1	1	1.7	175	0.15	More	50K	0.015	-0.124

Initial Analysis of the Raw Data:

After importing the given data into Minitab, the experimental design was created containing 6 factors and 16 experimental runs. Then the experiment was analyzed using the 2^k factorial design. The specific design used in this case is the 2^6-2 factorial design. The response variable is the value of Adhesion. Initially, the test was performed using all the terms in the experiment. This includes both the main factors and the interactions between the factors. After conducting the test once, it was identified that the higher-order terms which included the three, and fourth-order interactions were not significant. These terms were eliminated, and the factorial analysis was performed again. The results are shown below:

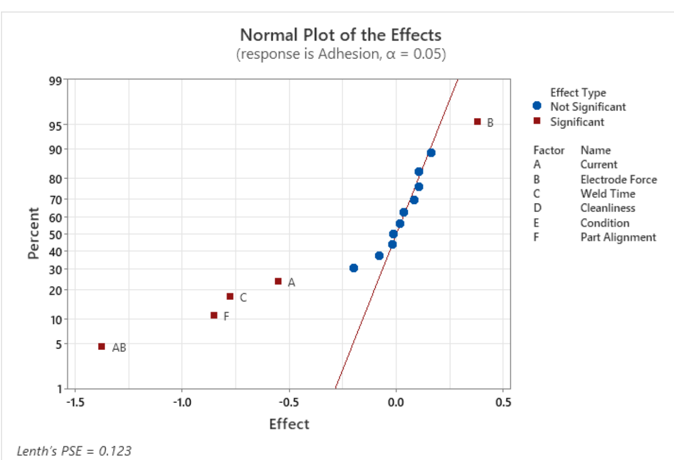
Coded Coefficients

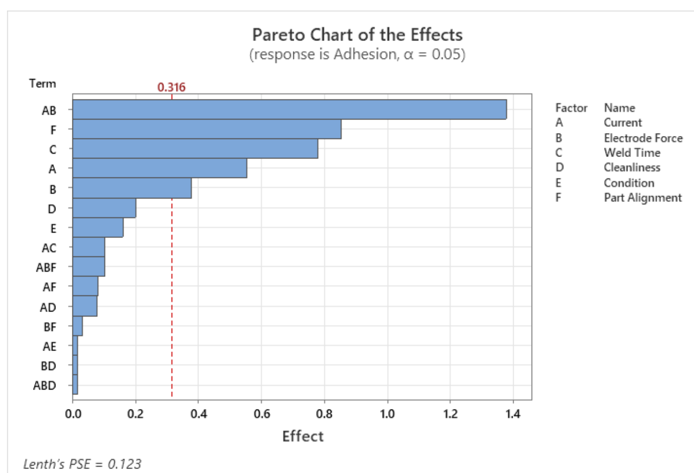
Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		1.373	*	*	*	
Current	-0.5545	-0.2772	*	*	*	1.00
Electrode Force	0.3788	0.1894	*	*	*	1.00
Weld Time	-0.7783	-0.3891	*	*	*	1.00
Cleanliness	-0.2008	-0.1004	*	*	*	1.00
Condition	0.16050	0.08025	*	*	*	1.00
Part Alignment	-0.8527	-0.4264	*	*	*	1.00
Current*Electrode Force	-1.3770	-0.6885	*	*	*	1.00
Current*Weld Time	0.10450	0.05225	*	*	*	1.00
Current*Cleanliness	-0.07850	-0.03925	*	*	*	1.00
Current*Condition	-0.017750	-0.008875	*	*	*	1.00
Current*Part Alignment	0.08200	0.04100	*	*	*	1.00
Electrode Force*Cleanliness	0.017750	0.008875	*	*	*	1.00
Electrode Force*Part Alignment	0.03325	0.01662	*	*	*	1.00
Current*Electrode Force*Cleanliness	-0.015500	-0.007750	*	*	*	1.00
Current*Electrode Force*Part Alignment	0.10450	0.05225	*	*	*	1.00

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	15	15.1307	1.00871	*	*
Linear	6	7.3994	1.23323	*	*
Current	1	1.2299	1.22988	*	*
Electrode Force	1	0.5738	0.57381	*	*
Weld Time	1	2.4227	2.42269	*	*
Cleanliness	1	0.1612	0.16120	*	*
Condition	1	0.1030	0.10304	*	*
Part Alignment	1	2.9087	2.90873	*	*
2-Way Interactions	7	7.6867	1.09810	*	*
Current*Electrode Force	1	7.5845	7.58452	*	*
Current*Weld Time	1	0.0437	0.04368	*	*
Current*Cleanliness	1	0.0246	0.02465	*	*
Current*Condition	1	0.0013	0.00126	*	*
Current*Part Alignment	1	0.0269	0.02690	*	*
Electrode Force*Cleanliness	1	0.0013	0.00126	*	*
Electrode Force*Part Alignment	1	0.0044	0.00442	*	*
3-Way Interactions	2	0.0446	0.02232	*	*
Current*Electrode Force*Cleanliness	1	0.0010	0.00096	*	*
Current*Electrode Force*Part Alignment	1	0.0437	0.04368	*	*
Error	0	*	*		
Total	15	15.1307			

From both charts, it is evident that we are not able to get the values for all the metrics such as Coefficients of Standard Errors, T Values, and P values. This is because all the degrees of freedom are used up in estimating the effects of the main effects and there are none left to estimate the error term. It is seen that the degrees of freedom for the error term are 0. The Normal and the Pareto chart of effects for this model with a confidence level of 95% are shown in the following figures.





From the normal plot and Pareto chart, we can see that there are 6 significant factors in the model which are A B C F, and the interaction AB (Current, Electrode, Weld Time, Part Alignment). Since there is no error term in the model, we cannot estimate all the statistical measures. Hence, the above results are inaccurate. This is the reason the residual plots are not plotted in Minitab. The following error is shown in Minitab for Residual Analysis plots:

* NOTE * Could not graph the specified residual type because MSE = 0 or the degrees of freedom for error = 0.

To estimate the error term in the model some of the factors are added to the error term by taking the Coefficients, Effects, Sum of Squares, and Adjusted Mean Squares into consideration that the factors that have very less effect on the entire model are eliminated. These terms are removed one by one instead of all at once to maintain the accuracy of the model. The terms that are included in the final model are as follows:
Current, Electrode Force, Weld Time, Cleanliness of Parts, Condition of the Electrode, Part Alignment, and the interactions Current*Electrode Force.

After the initial pooling of the results, the factorial design is re-analyzed and the results are shown:

Alias Structure

Factor	Name
A	Current
B	Electrode Force
C	Weld Time
D	Cleanliness
E	Condition
F	Part Alignment

Aliases
I + ABCE + ADEF + BCDF
A + BCE + DEF + ABCDF
B + ACE + CDF + ABDEF
C + ABE + BDF + ACDEF
D + AEF + BCF + ABCDE
E + ABC + ADF + BCDEF
F + ADE + BCD + ABCEF
AB + CE + ACDF + BDEF

The interpretation of the alias table is as follows:

- Alias "A" (Current) is aliased with "BCE" (Electrode Force), "DEF" (Weld Time), and "ABCDF" (Part Alignment).
- Alias "B" (Electrode Force) is aliased with "ACE" (Condition), "CDF" (Cleanliness), and "ABDEF" (Part Alignment).
- Alias "C" (Weld Time) is aliased with "ABE" (Current), "BDF" (Cleanliness), and "ACDEF" (Part Alignment).
- Alias "D" (Cleanliness) is aliased with "AEF" (Current), "BCF" (Electrode Force), and "ABCDE" (Part Alignment).
- Alias "E" (Condition) is aliased with "ABC" (Current), "ADF" (Weld Time), and "BCDEF" (Part Alignment).
- Alias "F" (Part Alignment) is aliased with "ADE" (Current), "BCD" (Electrode Force), and "ABCEF" (Cleanliness).
- Alias "AB" (Current and Electrode Force) is aliased with "CE" (Condition), "ACDF" (Cleanliness), and "BDEF" (Part Alignment).

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		1.3729	0.0339	40.54	0.000	
Current	-0.5545	-0.2772	0.0339	-8.19	0.000	1.00
Electrode Force	0.3788	0.1894	0.0339	5.59	0.001	1.00
Weld Time	-0.7783	-0.3891	0.0339	-11.49	0.000	1.00
Cleanliness	-0.2008	-0.1004	0.0339	-2.96	0.018	1.00
Condition	0.1605	0.0802	0.0339	2.37	0.045	1.00
Part Alignment	-0.8527	-0.4264	0.0339	-12.59	0.000	1.00
Current*Electrode Force	-1.3770	-0.6885	0.0339	-20.33	0.000	1.00

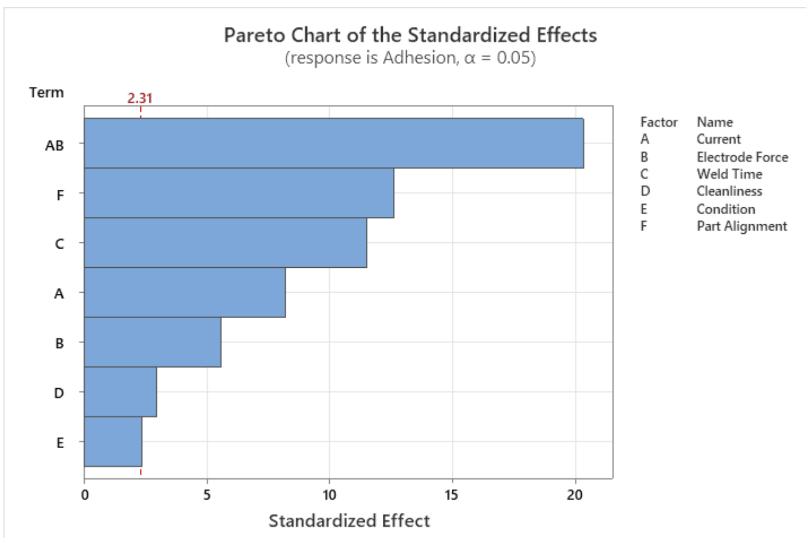
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	7	14.9839	2.14055	116.64	0.000
Linear	6	7.3994	1.23323	67.20	0.000
Current	1	1.2299	1.22988	67.02	0.000
Electrode Force	1	0.5738	0.57381	31.27	0.001
Weld Time	1	2.4227	2.42269	132.02	0.000
Cleanliness	1	0.1612	0.16120	8.78	0.018
Condition	1	0.1030	0.10304	5.61	0.045
Part Alignment	1	2.9087	2.90873	158.50	0.000
2-Way Interactions	1	7.5845	7.58452	413.29	0.000
Current*Electrode Force	1	7.5845	7.58452	413.29	0.000
Error	8	0.1468	0.01835		
Total	15	15.1307			

The Coded Coefficients and the ANOVA Tables show there is now a quantifiable value for the error. Taking the P-Values into account,

Null Hypothesis: The factor does not have a significant effect on the value of the Adhesion.
Alternative Hypothesis: The factor has a significant effect on the value of the Adhesion.

The p-values of the factors – current, electrode force, weld time, cleanliness, condition, part alignment and the two-way interaction between current*electrode force time are less than the significance value of 0.05 (<0.05). Therefore, for these factors, we must reject the null hypothesis that the factors do not have a significant effect on the response value of Adhesion.
The Normal and the Pareto charts for the standardized effects are shown below. These plots show that the condition of the electrode (Factor) does not have a significant effect on the Adhesion of the weld.



The Normal and the Pareto Charts for the Standardized Effects of the Adhesion values show that the significant factors affecting the Adhesion values are Current, Electrode Force, Weld Time, Cleanliness, Condition, Part Alignment, and the 2-way interaction of Current and Electrode Force.

Model Summary:

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.135467	99.03%	98.18%	96.12%

- This table provides information on the goodness of fit of the regression model from the ANOVA analysis.
- R-sq is 99.03%. This implies that 99.03% of the variation in the response variable is explained by the factors in the model.

- R-sq(adjusted) is 98.18%. This is slightly lower than R-sq as the model includes multiple independent variables.
- R-sq(pred) is 96.12%, which means that the model is expected to explain 96.12% of the variation in the new data. This suggests that the model can be sufficient to use to predict future outcomes.

12. Check model adequacy by analyzing residual plots. Provide a descriptive analysis of all figures and output.

Answer:

Residual Analysis

The following assumptions are made for the residual analysis of the collected data:

Residual Analysis Assumptions:

Normality:

The residuals are assumed to be normally distributed, meaning that the distribution of residuals follows a bell-shaped curve. This assumption is important for making valid inferences about the model parameters and for the accuracy of statistical tests and confidence intervals.

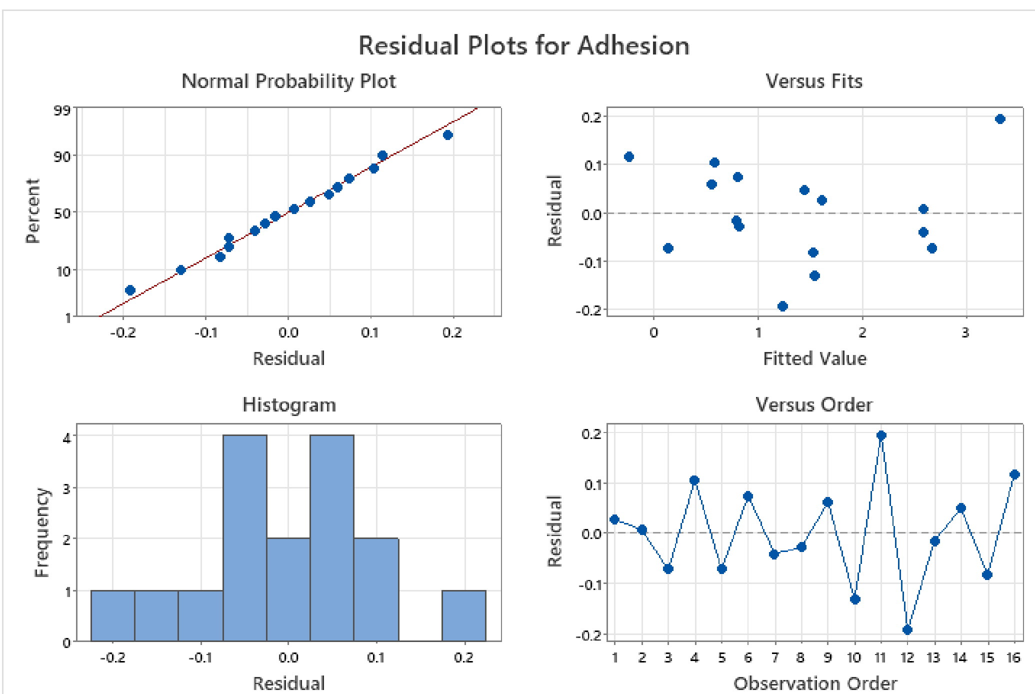
Independence:

The residuals are assumed to be independent of each other, meaning that the error of one observation does not depend on the error of any other observation. This assumption is important for the validity of statistical tests and confidence intervals.

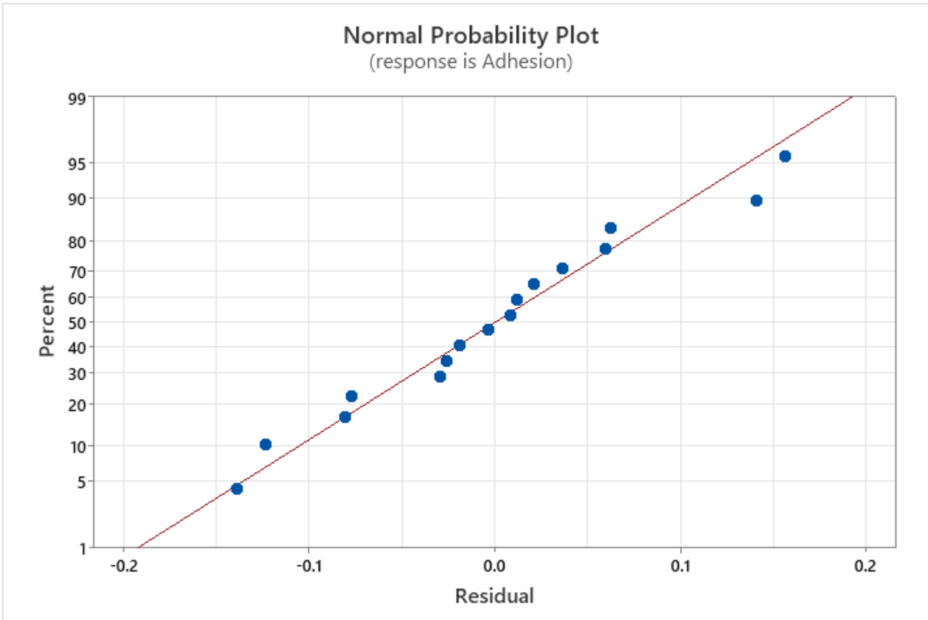
Homoscedasticity:

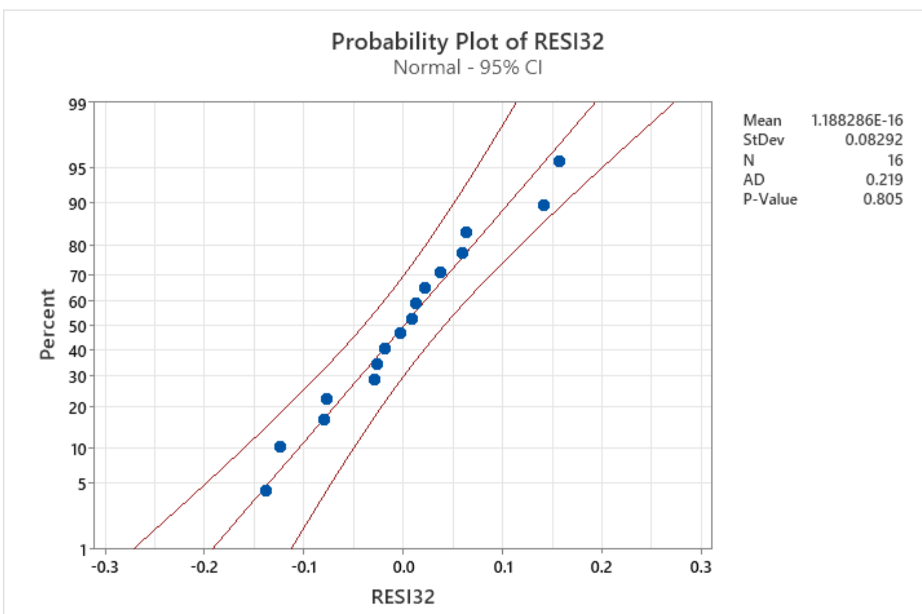
The variance of the residuals is assumed to be constant across all levels of the independent variables. In other words, the spread of residuals should be constant across the range of predicted values. If the spread of residuals varies systematically with the predicted values, it is called heteroscedasticity.

The summary if the residual Analysis is shown in the following figure:



Normality:
To test the normality of the residuals (Adhesion of the Weld Values of the significant factors) the probability plot is drawn along with the graphical summary of the given data.
Null Hypothesis: The null hypothesis is that the residuals are normally distributed.
Alternative Hypothesis: The Alternative Hypothesis is that the residuals are not normally distributed.



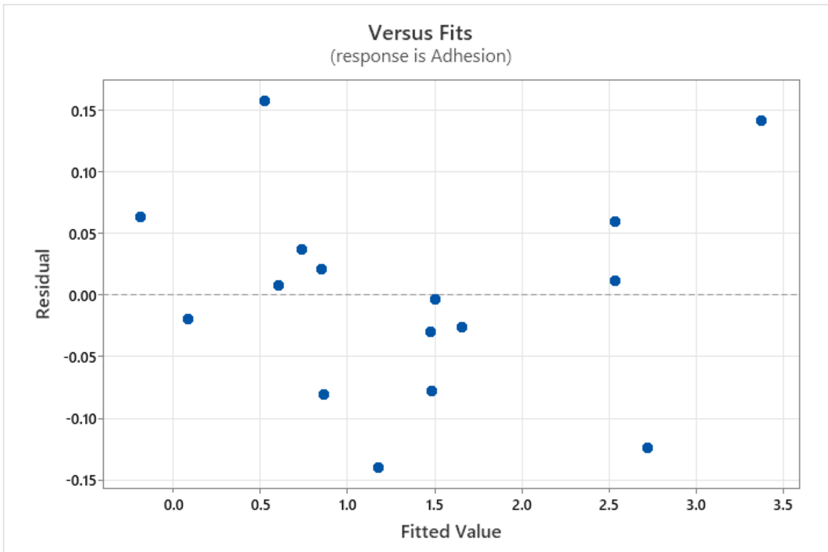


Here the normality plot is shown only for the Residuals of the Adhesion values caused only by the significant factors in the experiment. After the normality test is conducted (Anderson Darling Normality Test), the results show that the p-value for the normality plot is 0.805, which is greater than the significance value of 0.05. This shows that we fail to reject the null hypothesis that the Residuals of the

Strength of the weld values form a normal distribution. Hence the assumption that the data is normally distributed is not violated.

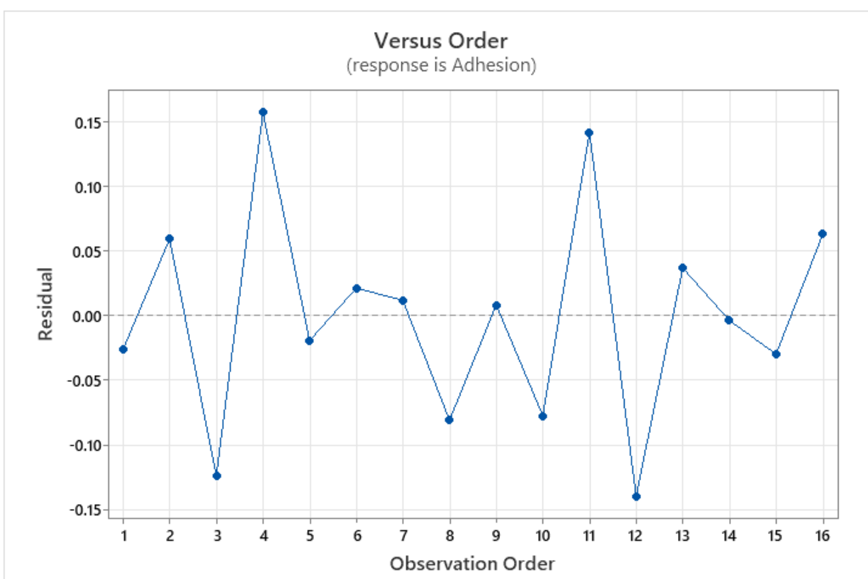
Homoscedasticity:

To check the test for equal variances or homoscedasticity, the residuals vs the fits plot is used. It is observed in the plot that the data points of the residuals occur randomly on either side of the 0 lines in the plot. Also, there is no specified pattern such as cyclic or seasonal patterns in the way the points occur. There is no funneling or fanning effects to show non-constant variance for the data.



Independence:

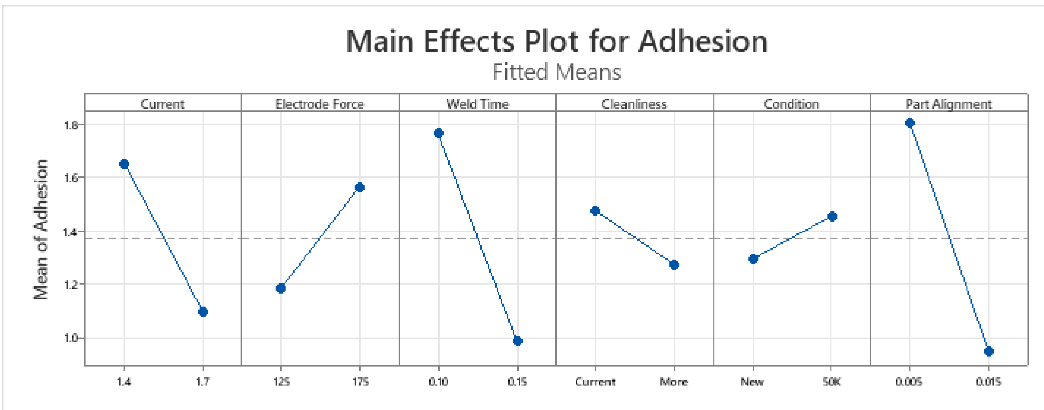
To check the independence assumption in Minitab, we used residuals vs order plot. These plots can help to detect any patterns or trends in the data that may violate the independence assumption. In this case, residuals vs order of observations plots is used to verify the independence of the data. The plot is shown here:

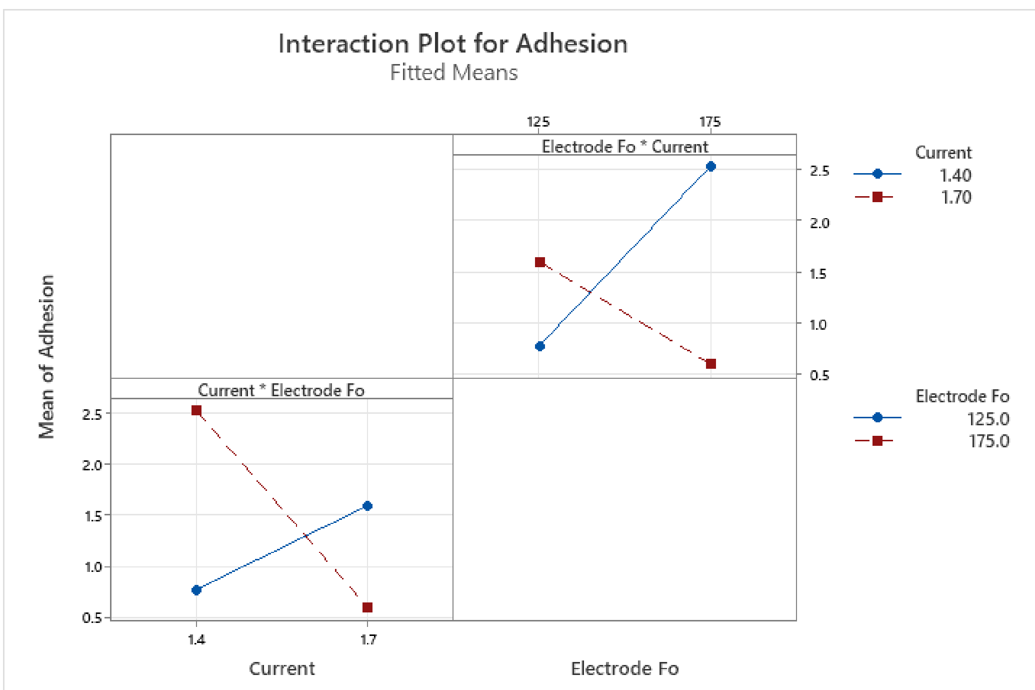


It is clearly seen in the Residual vs the Observation Order Plot that there is no regular pattern in the arrangement of the data points which indicates the criteria for the independence of the residuals is not violated. From the above plots and interpretations, it can be concluded that the assumptions for the residual analysis are not violated.

13. Create factorial plots for only significant (α = 0.10) factors and interactions. What are your recommendations for improving the Powder-Coat Adhesion?

Answer:
The factorial plots have been created for the significant factors that affect the Powder Coat Adhesion using a confidence level of **α = 0.10.** The results are shown below:





Interpretation for the Main Effects and the Interaction Plots:

Reduce the Current: Current has a negative coefficient of -0.2772, indicating that as the current used during the welding process increases, the adhesion of the powder coat decreases. Therefore, a lower value of the current used during the welding process may help improve powder-coat adhesion.

Increase the Electrode Force: Electrode Force has a positive coefficient of 0.1894, indicating that as the electrode force increases, the adhesion of the powder coat also increases. Therefore, increasing the electrode force used during the welding process may help improve powder-coat adhesion.

Reduce Weld Time: Weld Time has a negative coefficient of -0.3891, indicating that as the weld time increases, the adhesion of the powder coat decreases. Therefore, reducing the weld time used during the welding process may help improve powder-coat adhesion.

Improve Part Alignment: Part Alignment has a negative coefficient of -0.4264, indicating that as the part alignment deviates from the correct position, the adhesion of the powder coat decreases. Therefore, ensuring that the parts are properly aligned before applying the powder coat may help improve powder-coat adhesion.

Condition: The condition has a positive coefficient of 0.0803, indicating that as the condition increases, the adhesion of the powder coat also increases. Therefore, the higher value of condition which is after 50000 cycles for the electrodes, there will be a higher value of the powder-coated adhesion.

Improve Cleanliness: Cleanliness has a negative coefficient of -0.1004, indicating that as the cleanliness of the parts decreases, the adhesion of the powder coat also decreases. Therefore, ensuring that the parts are clean before applying the powder coat may help improve powder-coat adhesion.

Current*Electrode Force: The interaction between Current and Electrode Force also has a significant effect on Powder-Coat Adhesion. The regression equation shows that the interaction term has a negative coefficient of -0.6885, indicating that the combined effect of high current and low electrode force is particularly detrimental to powder-coat adhesion. Therefore, optimizing the settings for both Current and Electrode Force may also help improve powder-coat adhesion.

Finally, the recommendations for improving Powder-Coat Adhesion include reducing the current used during welding, increasing electrode force, reducing weld time, ensuring the cleanliness of parts, optimizing environmental conditions, properly aligning parts, and optimizing the settings for both Current and Electrode Force.

14. **Using the Response Optimizer, find the best process settings for simultaneously maximizing Weld Strength and Adhesion. Provide your best estimates for Weld Strength and Powder-Coat Adhesion with 95% confidence intervals for each response.**

Answer:
The Optimization plot is used to determine the optimal settings for the predictors of the optimal values of the Weld Strength and Adhesion given the parameters for the significant factors. The optimization plot displays the fitted values for the predicted settings. The results of the response optimizer and the optimization plot for the Weld Strength and the Adhesion responses are shown below:

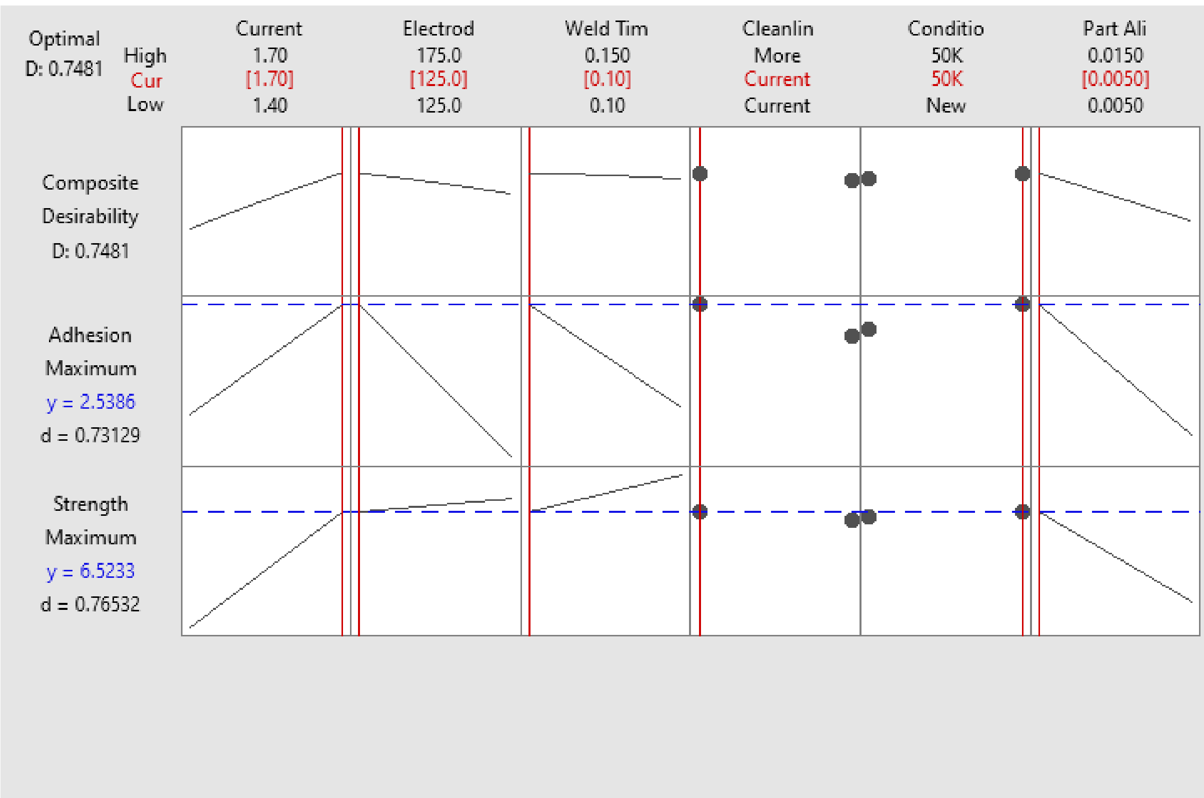
Solution

Solution	Current	Electrode Force	Weld Time	Cleanliness	Condition	Part Alignment	Adhesion Fit
1	1.7	125	0.1	Current	50K	0.005	2.53862

Solution	Strength Fit	Composite Desirability
1	6.52325	0.748110

Parameters

Response	Goal	Lower	Target	Upper	Weight	Importance
Adhesion	Maximum	-0.124	3.517		1	1
Strength	Maximum	4.000	7.297		1	1



For the given data, the composite desirability is 0.7481. Since this number is close to 1, all the response values are within acceptable limits. The goal is to maximize both the Weld Strength and the Adhesion values. The predicted value of strength is 6.5233 and its desirability is 0.76532. The predicted value of Adhesion is 2.5386 and its desirability is 0.73129.

For the Strength and Adhesion data, the composite desirability is 0.7750 The columns shown in the graph and their corresponding optimal values are described below:

Column 1 – Current – The Optimal Value for Current is 1.70
Column 2 – Electrode Force – The Optimal Value for Electrode Force is 125.
Column 3 – Weld Time – The Optimal Value for Weld Time is 0.10.
Column 4 – Cleanliness – The Optimal Value for Cleanliness is Current.
Column 5 – Condition – The Optimal Value for the Condition is 50K.
Column 6 – Part Alignment – The Optimal Value for Part Alignment is 0.0050.

Current: Current is a continuous variable and increasing the current value increases both the weld strength and Adhesion values.

Electrode Force: Electrode Force is also a continuous variable. The plot indicates that increasing the electrode force decreases the adhesion considerably. The plot also shows that the effect of increasing the electrode force value is very less on the strength of the weld. Since our objective is to increase both the strength and adhesion, the optimal value for the electrode force is the lower value which is 125.

Weld Time: Weld time is a continuous variable. The optimization plot shows that the effect of the weld time is less on strength when compared with that of adhesion. Hence, the optimal value for Weld Time is taken as 0.10 as a compromise to achieve a common goal of attaining high strength and adhesion.

Cleanliness: The two points for each cell in this column represent the two levels of the categorical variable cleanliness. It appears that the amount of cleanliness levels that is current or more clean is the same on the values of strength and for adhesion, it is shown that by increasing the cleanliness, the adhesion decreases.

Condition of the electrode: Condition is a categorical variable. The optimization plot shows that for adhesion, the value increases as the number of cycles for the electrodes reached 50000 whereas for strength, there is no significant difference in the values of weld strength when the electrodes are new or after 50000 cycles. Hence, the optimal value for the Condition of the electrode is after 50000 cycles.

Part Alignment: Part Alignment is a continuous variable. It is shown in the plot that as the clearance between the parts increases the strength and adhesion of the weld decrease. Since the objective of the experiment is to maximize the weld strength and adhesion, it is determined that the optimal value for Part Alignment will be 0.0050".

Multiple Response Prediction

Variable	Setting
Current	1.7
Electrode Force	125
Weld Time	0.1
Cleanliness	Current
Condition	50K
Part Alignment	0.005

Response	Fit	SE Fit	95% CI	95% PI
Adhesion	2.5386	0.0910	(2.3234, 2.7539)	(2.1799, 2.8974)
Strength	6.5233	0.0640	(6.3718, 6.6747)	(6.2708, 6.7757)

For a 95% Confidence Interval,

- The values for maximum strength will lie between 6.3718 and 6.6747 and the mean value is 6.5233.
- The values for maximum adhesion will lie between 2.3234 and 2.7539. and the mean value is 2.5386.

15. Compare and contrast your recommended settings from questions 6, 13, and 14. If there are (or are not) differences in the recommended settings, explain why.

Answer:

While question 6 provides recommended settings to obtain a higher value of the Strength, question 13 provides recommended settings to obtain a higher value of adhesion. Response optimizer in question 14th is used to simultaneously maximize both the responses.

These are shown in the following table:

Each row represents the recommended setting from questions 6,13 and 14 respectively.

Factor	Current	Electrode Force	Weld Time	Cleanliness	Condition	Part Alignment
Strength	1.7	125	0.15	Current	________	0.005
Adhesion	1.4	175	0.1	Current	50K	0.005
Response Optimizer	1.7	125	0.1	Current	50K	0.005

After comparing, we observed the following similarities and contrast from three questions:

- From the above table it is evident that the categorical variables – Cleanliness and Condition are at same level. However, condition factor is not significant for the question 6. It is because of the different significant values considered.
- To maximize the response values, the clearance between the parts must be low. The lower setting (0.005) of part alignment in all three interpretations is the same.
- Current factor should be low if only Adhesion is prioritized. To optimize both responses, current should be set at higher setting.
- Electrode force should be at high setting only if Adhesion is prioritized. To optimize both responses electrode force should be set at low setting.
- Weld time should be at low setting only if current is prioritized. To optimize both responses weld time should be at high setting.
- The difference in these settings can be attributed to the difference in significance level.