EE201 - Signals and Systems

Indian Institute of Technology Ropar

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Contents

Chapter 1

Analysis of Signals and Systems

1.1 Energy and Power

	Continuous Time Signal, $x(t)$	**Discrete Time Signal**, $x[n]$
Energy	$E_x = \int_{t_1}^{t_2} \lvert x(t)\rvert^2 dt$	$E_x = \sum_{n=N_1}^{N_2} \lvert x[n]\rvert^2$
Power	$E_x = \dfrac{E_x}{t_2 - t_1}$	$P_x = \dfrac{E_x}{N_2 - N_1 + 1}$

In infinite duration signals $(-\infty < t < +\infty, -\infty < n < +\infty)$

	Continuous Time Signal, $x(t)$	**Discrete Time Signal**, $x[n]$
Energy	$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} \lvert x(t)\rvert^2 dt$	$E_\infty = \lim_{N\to\infty} \sum_{n=-N}^{N} \lvert x[n]\rvert^2$
Power	$E_\infty = \lim_{T\to\infty} \dfrac{E_\infty}{2T}$	$P_\infty = \lim_{N\to\infty} \dfrac{E_\infty}{2N+1}$

1.2 Periodic Signals

A periodic signal repeats itself after sometime.

Continuous signals

If $x(t) = x(t+T)\ \forall t,\ T > 0$ then the signal is periodic. The minimum value of T that satisfies the condition is calle the *fundamental period*.

Discrete signals

If $x[n] = x[n+N]\ \forall n,\ N > 0, N \in \mathbb{Z}$ then the signal is periodic. The minimum value of N that satisfies the condition is calle the *fundamental period*.

1.3 Exponential Signals

1.3.1 Continuous Time Exponential Signals

A continuous time exponential signal can be represented as

$$x(t) = Ce^{at} \tag{1.1}$$

where $C, a \in \mathbb{C}$.

[1]Consider the general case where $C = |C|e^{j\theta}$ and $a = \sigma + \mathrm{j}\omega$. The signal becomes

$$x(t) = |C|e^{\sigma t}e^{\mathrm{j}(wt+\theta)}$$

- ω is angular frequency. If $\omega \neq 0$, the signal is sinusoidal, with $f = \frac{\omega}{2\pi}$.
- σ is the attenuation factor. If $\sigma \neq 0$, the signal decays/grows and is bounded by the envelope $|C|e^{\sigma t}$.
- θ is the initial phase.

1.3.2 Discrete Time Exponential Signals

A discrete time exponential signal can be represented as

$$x[n] = Ce^{an} \tag{1.2}$$

where $C, a \in \mathbb{C}$.

Consider the general case where $C = |C|e^{j\theta}$ and $a = \sigma + \mathrm{j}\omega$. The signal becomes

$$x[n] = |C|e^{\sigma n}e^{\mathrm{j}(wn+\theta)}$$

- ω is **related to** angular frequency. If $\omega \neq 2n\pi, n \in \mathbb{Z}$, the signal is periodic.
  Unlike continuous-time signals, the angular frequency cannot take all values. The range is $[0, 2\pi)$ or $[-\pi, \pi)$ because
  $$e^{\mathrm{j}(\omega+2\pi)n} = e^{\mathrm{j}\omega n}e^{\mathrm{j}2n\pi} = e^{\mathrm{j}\omega n}$$
  Rate of oscillation is small for $\omega \sim 2n\pi$, and high for $\omega \sim (2n+1)\pi, n \in \mathbb{Z}$.
- σ is the attenuation factor. If $\sigma \neq 0$, the signal decays/grows and is bounded by the envelope $|C|e^{\sigma n}$.
- θ is the initial phase.

1.3.3 Periodicity

Continuous-time exponential signals

For a continuous-time exponential signal to be periodic

$$\begin{aligned} e^{\mathrm{j}\omega(t+T)} &= e^{\mathrm{j}\omega t} \\ \Rightarrow e^{\mathrm{j}\omega T} &= 1 \\ \Rightarrow \omega T &= 2m\pi, m \in \mathbb{Z} \end{aligned}$$

Therefore, continuous-time exponential signal is periodic for all $\omega > 0$ and their *fundamental time period* is

$$T = \frac{2\pi}{\omega}$$

Discrete-time exponential signals

For a discrete-time exponential signal to be periodic

$$\begin{aligned} e^{\mathrm{j}\omega(n+N)} &= e^{\mathrm{j}\omega n} \\ \Rightarrow e^{\mathrm{j}\omega N} &= 1 \\ \Rightarrow \omega N &= 2m\pi, m \in \mathbb{Z} \end{aligned}$$

Therefore, discrete-time exponential signal is periodic only if $\frac{\omega}{2\pi} = \frac{m}{N}$ i.e. $\frac{w}{2\pi}$ is a rational number. The *fundamental period* is

$$N = m\frac{2\pi}{\omega}$$

where m is smallest positive integer such that N evaluates to an integer.

1.3.4 Harmonics

Continuous-time exponential signals

$$\phi_k(t) = e^{\mathrm{j}k[\frac{2\pi}{T}]t} = e^{\mathrm{j}k\omega t}$$

Discrete-time exponential signals

$$\phi_k[n] = e^{\mathrm{j}k[\frac{2\pi}{N}]n} = e^{\mathrm{j}k\frac{\omega}{m}n}$$

1.4 Unit Impulse and Step Functions

1.4.1 Discrete-Time Case

Unit Impulse

$$\delta[n] = \begin{cases} 0 & n \neq 0 \\ 1 & n = 0 \end{cases}$$

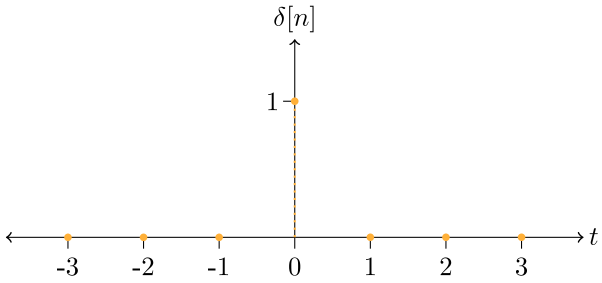

Figure 1.1: Unit impulse function in discrete-time

Unit Step

$$U[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$$

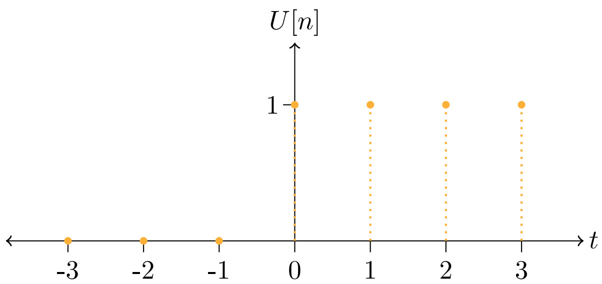

Figure 1.2: Unit step function in discrete-time

Properties

1. $\delta[n] = U[n] - U[n-1]$
2. $U[n] = \sum_{m=-\infty}^{n} \delta[n] = \sum_{k=0}^{\infty} \delta[n-k]$
3. **Sampling Property of unit impulse function**

$$x[n] \cdot \delta[n] = x[0] \cdot \delta[n]$$
$$x[n] \cdot \delta[n-n_0] = x[n_0] \cdot \delta[n-n_0]$$

1.4.2 Continuous-Time Case

Unit Step

$$U(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$

This function is discontinuous at $t = 0$. It is modified a little bit to make it continuous.

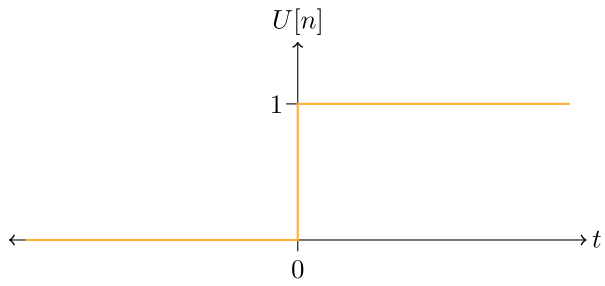

Figure 1.3: Unit step function in continuous-time

$$U_\Delta(t) = \begin{cases} 0 & t < 0 \\ \frac{t}{\Delta} & 0 \le t \le \Delta \\ 1 & t > \Delta \end{cases}$$

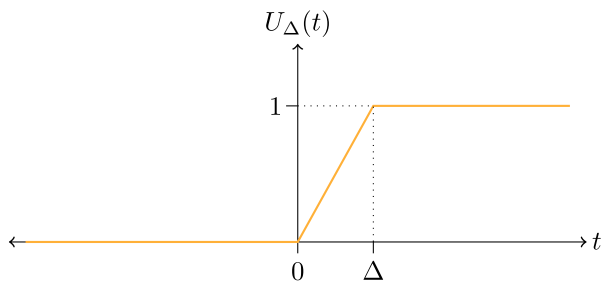

Figure 1.4: Continuous unit step function in continuous-time

Unit Impulse

$$U(t) = \int_{-\infty}^{t} \delta(\tau) d\tau$$
$$\delta(t) = \frac{dU(t)}{dt}$$

Since, $U(t)$ is not differentiable.

$$\delta(t) = \lim_{\Delta \to 0} \frac{dU_\Delta(t)}{dt}$$
$$\delta(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{\Delta} & 0 \leq t \leq \Delta \\ 0 & t > \Delta \end{cases}$$

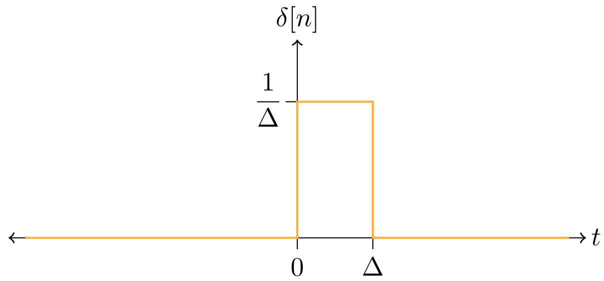

Figure 1.5: Unit impulse function in continuous-time

Properties

1. $\int\limits_{-\infty}^{+\infty} \delta(\tau) d\tau = 1$

2. **Sampling Property of unit impulse function**

$$x(t) \cdot \delta(t) = x(0) \cdot \delta(t)$$
$$x(t) \cdot \delta(t - t_0) = x(t_0) \cdot \delta(t - t_0)$$

1.5 Systems

System is the interconnection of components/devices/subsystems. A *continuous-time system* is represented by *differential equation* and a *discrete-time system* is represented by *difference equation.*

1.5.1 Interconnection of Systems

1. **Series (Cascade) Connection**

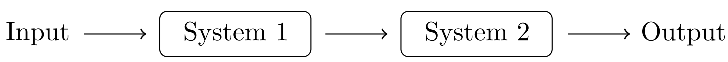

Figure 1.6: Example of series interconnection

2. **Parallel Connection**

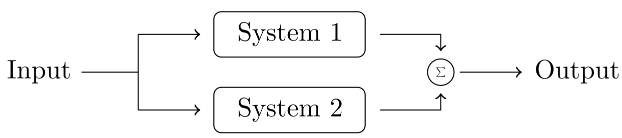

Figure 1.7: Example of parallel interconnection

3. **Series-Parallel Connection**
   It is combination of series and parallel connections.

4. **Feedback Interconnection**
   In this kind of interconnection, some part of output is again fed as input to the system.

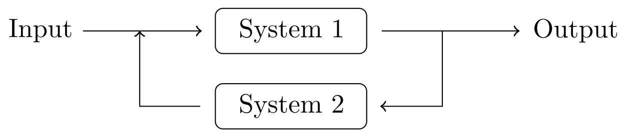

Figure 1.8: Example of feedback interconnection

Bibliography

[1] Continuous Time Complex Exponential Signal - College. http://pilot.cnxproject.org/content/collection/col10064/latest/module/m10060/latest. Accessed: 2020-09-03.