# EE201 - Signals and Systems

Indian Institute of Technology Ropar

September 6, 2020

# Contents

# Chapter 1

# Analysis of Signals and Systems

## 1.1 Energy and Power

| | Continuous Time Signal, $x(t)$ | Discrete Time Signal, $x[n]$ |
|---|---|---|
| **Energy** | $E_x = \int_{t_1}^{t_2} |x(t)|^2 dt$ | $E_x = \sum_{n=N_1}^{N_2} |x[n]|^2$ |
| **Power** | $E_x = \dfrac{E_x}{t_2 - t_1}$ | $P_x = \dfrac{E_x}{N_2 - N_1 + 1}$ |

In infinite duration signals $(-\infty < t < +\infty, -\infty < n < +\infty)$

| | Continuous Time Signal, $x(t)$ | Discrete Time Signal, $x[n]$ |
|---|---|---|
| **Energy** | $E_\infty = \lim\limits_{T \to \infty} \int_{-T}^{T} |x(t)|^2 dt$ | $E_\infty = \lim\limits_{N \to \infty} \sum_{n=-N}^{N} |x[n]|^2$ |
| **Power** | $E_\infty = \lim\limits_{T \to \infty} \dfrac{E_\infty}{2T}$ | $P_\infty = \lim\limits_{N \to \infty} \dfrac{E_\infty}{2N + 1}$ |

## 1.2 Periodic Signals

A periodic signal repeats itself after sometime.

**Continuous signals**

If $x(t) = x(t + T)\ \forall t,\ T > 0$ then the signal is periodic. The minimum value of $T$ that satisfies the condition is calle the *fundamental period.*

**Discrete signals**

If $x[n] = x[n + N]\ \forall n,\ N > 0, N \in \mathbb{Z}$ then the signal is periodic. The minimum value of $N$ that satisfies the condition is calle the *fundamental period.*

## 1.3 Exponential Signals

### 1.3.1 Continuous Time Exponential Signals

A continuous time exponential signal can be represented as

$$x(t) = Ce^{at} \tag{1.1}$$

where $C, a \in \mathbb{C}$.

[1]Consider the general case where $C = |C|e^{j\theta}$ and $a = \sigma + j\omega$. The signal becomes

$$x(t) = |C|e^{\sigma t}e^{j(wt+\theta)}$$

- $\omega$ is angular frequency. If $\omega \neq 0$, the signal is sinusoidal, with $f = \frac{\omega}{2\pi}$.

- $\sigma$ is the attenuation factor. If $\sigma \neq 0$, the signal decays/grows and is bounded by the envelope $|C|e^{\sigma t}$.

- $\theta$ is the initial phase.

## 1.3.2   Discrete Time Exponential Signals

A discrete time exponential signal can be represented as

$$x[n] = Ce^{an} \qquad (1.2)$$

where $C, a \in \mathbb{C}$.

Consider the general case where $C = |C|e^{j\theta}$ and $a = \sigma + j\omega$. The signal becomes

$$x[n] = |C|e^{\sigma n}e^{j(wn+\theta)}$$

- $\omega$ is **related to** angular frequency. If $\omega \neq 2n\pi, n \in \mathbb{Z}$, the signal is periodic.
  Unlike continuous-time signals, the angular frequency cannot take all values. The range is $[0, 2\pi)$ or $[-\pi, \pi)$ because
  $$e^{j(\omega+2\pi)n} = e^{j\omega n}e^{j2n\pi} = e^{j\omega n}$$

  Rate of oscillation is small for $\omega \sim 2n\pi$, and high for $\omega \sim (2n+1)\pi, n \in \mathbb{Z}$.

- $\sigma$ is the attenuation factor. If $\sigma \neq 0$, the signal decays/grows and is bounded by the envelope $|C|e^{\sigma n}$.

- $\theta$ is the initial phase.

## 1.3.3   Periodicity

**Continuous-time exponential signals**

For a continuous-time exponential signal to be periodic

$$e^{j\omega(t+T)} = e^{j\omega t}$$
$$\Rightarrow e^{j\omega T} = 1$$
$$\Rightarrow \omega T = 2m\pi, m \in \mathbb{Z}$$

Therefore, continuous-time exponential signal is periodic for all $\omega > 0$ and their *fundamental time period* is

$$T = \frac{2\pi}{\omega}$$

**Discrete-time exponential signals**

For a discrete-time exponential signal to be periodic

$$e^{j\omega(n+N)} = e^{j\omega n}$$
$$\Rightarrow e^{j\omega N} = 1$$
$$\Rightarrow \omega N = 2m\pi, m \in \mathbb{Z}$$

Therefore, discrete-time exponential signal is periodic only if $\frac{\omega}{2\pi} = \frac{m}{N}$ i.e. $\frac{w}{2\pi}$ is a rational number. The *fundamental period* is

$$N = m\frac{2\pi}{\omega}$$

where $m$ is smallest positive integer such that $N$ evaluates to an integer.

### 1.3.4 Harmonics

**Continuous-time exponential signals**

$$\phi_k(t) = e^{jk[\frac{2\pi}{T}]t} = e^{jk\omega t}$$

**Discrete-time exponential signals**

$$\phi_k[n] = e^{jk[\frac{2\pi}{N}]n} = e^{jk\frac{\omega}{m}n}$$

## 1.4 Unit Impulse and Step Functions

### 1.4.1 Discrete-Time Case

**Unit Impulse**

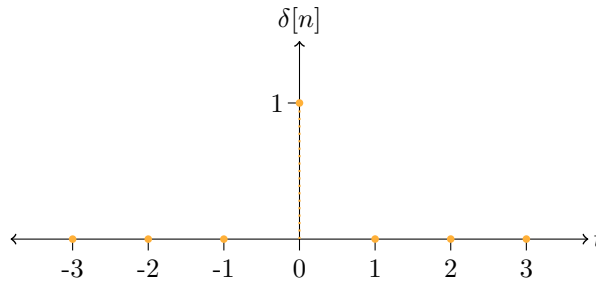$$\delta[n] = \begin{cases} 0 & n \neq 0 \\ 1 & n = 0 \end{cases}$$



Figure 1.1: Unit impulse function in discrete-time

**Unit Step**

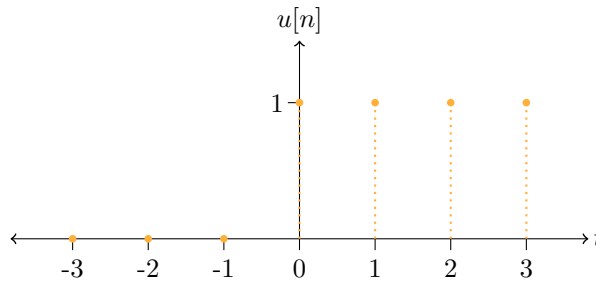$$u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$$



Figure 1.2: Unit step function in discrete-time

**Properties**

1. $\delta[n] = u[n] - u[n-1]$

2. $u[n] = \sum\limits_{m=-\infty}^{n} \delta[n] = \sum\limits_{k=0}^{\infty} \delta[n-k]$

3. **Sampling Property of unit impulse function**

$$x[n] \cdot \delta[n] = x[0] \cdot \delta[n]$$
$$x[n] \cdot \delta[n-n_0] = x[n_0] \cdot \delta[n-n_0]$$

## 1.4.2   Continuous-Time Case

**Unit Step**

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$

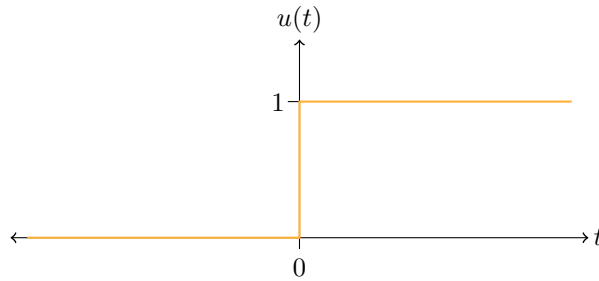This function is discontinuous at $t = 0$. It is modified a little bit to make it continuous.



Figure 1.3: Unit step function in continuous-time

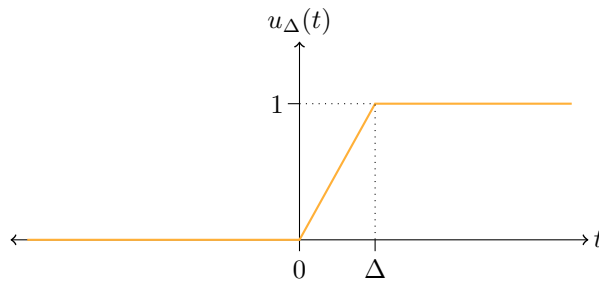$$U_\Delta(t) = \begin{cases} 0 & t < 0 \\ \frac{t}{\Delta} & 0 \le t \le \Delta \\ 1 & t > \Delta \end{cases}$$



Figure 1.4: Continuous unit step function in continuous-time

**Unit Impulse**

$$u(t) = \int\limits_{-\infty}^{t} \delta(\tau)d\tau$$

$$\delta(t) = \frac{du(t)}{dt}$$

Since, $u(t)$ is not differentiable.

$$\delta(t) = \lim_{\Delta \to 0} \frac{dU_\Delta(t)}{dt}$$

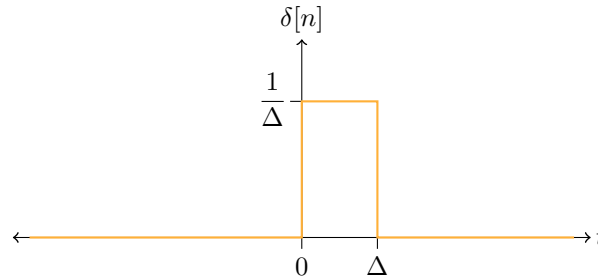$$\delta(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{\Delta} & 0 \le t \le \Delta \\ 0 & t > \Delta \end{cases}$$



Figure 1.5: Unit impulse function in continuous-time

**Properties**

1. $\int_{-\infty}^{+\infty} \delta(\tau)d\tau = 1$

2. **Sampling Property of unit impulse function**

$$x(t) \cdot \delta(t) = x(0) \cdot \delta(t)$$
$$x(t) \cdot \delta(t - t_0) = x(t_0) \cdot \delta(t - t_0)$$

## 1.5 Systems

System is the interconnection of components/devices/subsystems. A *continuous-time system* is represented by *differential equation* and a *discrete-time system* is represented by *difference equation*.

### 1.5.1 Interconnection of Systems
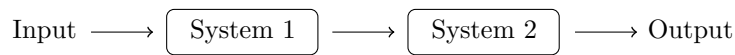
1. **Series (Cascade) Connection**



Figure 1.6: Example of series interconnection
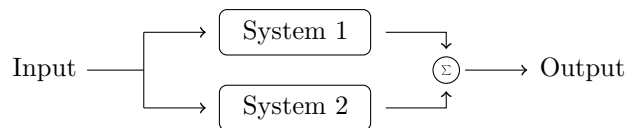
2. **Parallel Connection**



Figure 1.7: Example of parallel interconnection

3. **Series-Parallel Connection**
   It is combination of series and parallel connections.

4. **Feedback Interconnection**
   In this kind of interconnection, some part of output is again fed as input to the system.
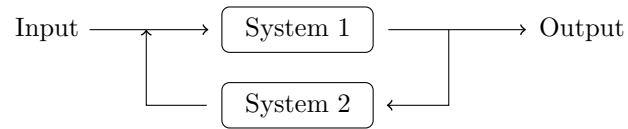


Figure 1.8: Example of feedback interconnection

## 1.5.2  Basic Properties

Some basic properties of a system are

1. Memory

2. Invertibility

3. Causality

4. Stability

5. Time-invariance

6. Linearity

# Bibliography

[1] Continuous Time Complex Exponential Signal - College. http://pilot.cnxproject.org/content/collection/col10064/latest/module/m10060/latest. Accessed: 2020-09-03.