Homework 9

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4.4

2b) $\int_{-0.5}^{0.5} x \ln(x+1)\,dx$, $n=6$

$h = \frac{(0.5-(-0.5))}{6} = \frac{1}{6}$

$\approx \frac{h}{2}\left[f(-0.5) + 2\sum_{j=1}^{n-1} f(x_j) + f(0.5)\right]$

$x_j = a + jh$

$\approx \frac{1}{12}\left[-\frac{1}{2}\ln\left(\frac{1}{2}\right) + 2\left[f(-.5+\frac{1}{6}) + f(-.5+\frac{2}{6}) + f(-.5+\frac{3}{6}) + f(-.5+\frac{4}{6}) + f(-.5+\frac{5}{6})\right] + \frac{1}{2}\ln(1.5)\right]$

$\approx .1239$

4) $\int_{-.5}^{.5} x\ln(x+1)\,dx$ $n=6$ use composite simpson's rule

$h = \frac{(0.5-(-.5))}{6} = \frac{1}{6}$

$\approx \frac{h}{3}\left[f(a) + 2\sum_{j=1}^{(n/2-1)} f(x_{2j}) + 4\sum_{j=1}^{n/2} f(x_{2j-1}) + f(b)\right]$

$\approx \frac{1}{18}\left[f(-0.5) + 2\left[f(-0.5+\frac{1}{6}) + f(-0.5+\frac{1}{3})\right] + 4\left[f(-.5) + f(-.5+\frac{1}{6}) + f(-.5+\frac{1}{3})\right] + f(.5)\right]$

$\approx .1627$

6) use composite Midpoint rule $h = \frac{(.5-(-.5))}{8} = 1/8$

$\approx 2h\sum_{j=0}^{n/2} f(x_{2j})$ $x_j = a + (j+1)h$

$\approx 2\left(\frac{1}{8}\right)\left[f(-.5+1/8) + f(-.5+2/8) + f(-.5+3/8) + f(-.5+4/8) + f(-.5+5/8)\right]$

$\approx .0699$

4.5

1b) $\int_0^1 x^2 e^{-x}\,dx$ compute $R_{3,3}$

composite trapezoid

$\frac{h}{2}\left[f(a)+2\sum_{j=1}^{n-1} f(x_j)+f(b)\right]$ $h=\frac{b-a}{n}$ $x_j=a+jh$

$n=1,2,4,8,16$

$n=1$ $R_{1,1}=\frac{1}{2}\left[f(0)+f(1)\right]=.1839$ $h=1$

$n=2$ $R_{2,1}=\frac{1}{4}\left[f(0)+2[f(0.5)]+f(1)\right]=.1678$ $h=1/2$

$n=4$ $R_{3,1}=\frac{1}{8}\left[f(0)+2[f(0.25)+f(0.5)+f(0.75)]+f(1)\right]=.1625$ $h=1/4$

$R_{2,2}=R_{2,1}+\frac{1}{3}(R_{2,1}-R_{1,1})=.1624$

$R_{3,2}=R_{3,1}+\frac{1}{3}(R_{3,1}-R_{2,1})=.2607$

$R_{3,3}=R_{3,2}+\frac{1}{15}(R_{3,2}-R_{2,2})=.1606$

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7) approximate $\int_1^5 f(x)\,dx$

x	1	2	3	4	5
$f(x)$	2.4142	2.6734	2.8974	3.0976	3.2804

$n=1,2,4,8,16,\ldots$

$R_{1,1}=2\left[f(1)+f(5)\right]=11.3892$ $h=4$

$R_{2,1}=1\left[f(1)+2[f(3)]+f(5)\right]=11.4894$ $h=\frac{4}{2}=2$

$R_{3,1}=\frac{1}{2}\left[f(1)+2[f(2)+f(3)+f(4)]+f(5)\right]=11.5157$ $h=\frac{4}{4}=1$

$R_{2,2}=R_{2,1}+\frac{1}{3}(R_{2,1}-R_{1,1})=11.5228$

$R_{3,2}=R_{3,1}+\frac{1}{3}(R_{3,1}-R_{2,1})=11.5245$

$R_{3,3}=R_{3,2}+\frac{1}{15}(R_{3,2}-R_{2,2})=11.5246$ $O(h^6)$

9) $\int_2^3 f(x)\,dx$

$f(2) = 0.51342$, $f(3) = 0.36788$, $R_{31} = 0.43687$, and $R_{33} = 0.43662$

$h = \frac{1}{4}$ find $f(2.5)$

$n = 4$ $R_{31} = 0.43687 = \frac{1}{8}\left[f(2) + 2[f(2.25) + f(2.5) + f(2.75)] + f(3)\right]$

$= \frac{1}{8}\left[.51342 + 2[f(2.25) + f(2.5) + f(2.75)] + 0.36788\right]$

$h = \frac{1}{2}$
$n = 2$ $R_{2,1} = \frac{1}{4}[.51342 + 2f(2.5) + .36788]$

$R_{1,1} = \frac{1}{2}[.51342 + .36788] = .44065$

$R_{2,2} = \frac{4}{3}R_{2,1} - \frac{.44065}{3}$

$R_{32} = .43687 + \frac{1}{3}(.43687) - \frac{1}{3}R_{2,1}$

$R_{3,3} = 0.43662 = .43687 + \frac{1}{3}(.43687) - \frac{1}{3}R_{2,1} + \frac{1}{15}\left(.43687 + \frac{1}{3}(.43687) - \frac{1}{3}R_{2,1}\right) - \frac{1}{15}R_{2,2}$

$0.43662 = .43687 + \frac{1}{3}(.43687) + \frac{1}{15}(.43687) + \frac{1}{45}(.43687) - \frac{1}{3}R_{21} - \frac{1}{45}R_{2,1} - \frac{4}{45}R_{2,1} + \frac{.44065}{45}$

$-.1937 \approx -\frac{20}{45}\left[\frac{1}{4}[.51342 + 2f(2.5) + .36788]\right]$

$\approx -\frac{1}{9}(.51342 + .36788) - \frac{2}{9}f(2.5)$

$\boxed{f(2.5) \approx .43118}$

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2b) $\int_{1}^{1.6} \frac{2x}{x^2-4}\,dx = \int_{-1}^{1} \left[\frac{2(.6)t+2.6}{\left(\frac{2(.6)t+2.6}{2}\right)^2-4}\right]\frac{.6}{2}\,dt = \int_{-1}^{1}\left[\frac{1.2t+2.6}{(.6t+1.3)^2-4}\right].3\,dt$

solve for $n=2$ $\quad 2(2)-1=3$

$\approx c_1 f(x_1) + c_2 f(x_2)$ $\qquad t = \frac{2x-a-b}{b-a}$

$\approx \cancel{1\cdot f(.5773502692) + 1\cdot f(-.5773502692)}$

$\approx -\cancel{.9513}$

actual value: $-.7339$

convert roots from x to t

$.5773502692 \to -2.4088$

$-.5773502692 \to -6.2578$

$f(-2.4088) + f(-6.2578) \approx -.7052$

4) $\approx c_1 f(x_1) + c_2 f(x_2) + c_3 f(x_3)$

$.\bar{5} f(.7745966692) + .\bar{8} f(0) + .\bar{5} f(-.7745966692)$

x		t
$.7745966692$	$\to$	-1.7513
0	$\to$	-4.3333
$-.7745966692$	$\to$	-6.9153

$.\bar{5} f(-1.7513) + .\bar{8} f(-4.\bar{3}) + .\bar{5} f(-6.9153)$

$\approx .0484$