MA346 HW 11

5.1

2d) $\frac{dy}{dt} = \frac{ty+y}{ty+t} = \frac{y(t+1)}{t(y+1)}$

$\int \frac{(y+1)}{y}\,dy = \int \frac{(t+1)}{t}\,dt$

$\ln|y| + y = \ln|t| + t + c$

$y \ne 4$

$dy(2) = 4$

$4 = \frac{3y}{2y+2}$

$8y+8 = 3y$

$y = -8/5$

then $y = -8/5$

which doesn't work

$\ln|4| + 4 = \ln|2| + 2 + c$

$\ln\left|\frac{4}{2}\right| + \cancel{3}2 = c$

$\boxed{\ln|y| + y = \ln|t| + t + \ln|2| + 2}$

well posed?

- Continuous on interval ✓
- convex ✓

$\left|\frac{ty+y}{ty+t}\right| \le 1 = L$ ✓

Lipschitz in y so this problem is well posed.

5.2 $y(t) = \frac{4 - \cos(2) - \cos(2t)}{2t^2}$ $y(2)=4$ $2 \le t \le 4$

2d) $w_0 = y(1) = 2$ $h = 0.25$ $1 \le t \le 2$ $y' = t^{-2}(\sin(2t) - 2ty)$

$w_1 = 4 + 0.25 f(1, 2) = 1.2273$

$\longrightarrow$

4e)

approximations

(1,2)

(1.25, 1.2273)

(1.5, 0.83715)

(1.75, 0.57045)

(2, 0.37883)

(1,2)

(1.25, 1.40319)

(1.5, 1.0164101)

(1.75, 0.7380089)

(2, 0.5296870)

Section 5.3

Taylor Method of order n to approximate $y(t_i)$ where $t_i = a + ih$ by w_i

9a)

$$w_0 = \alpha \quad w_{i+1} = w_i + hT^{(n)}(t_i, w_i) \text{ for each } i = 0,1,2,\ldots, N-1$$

where $T^{(n)}(t_i, w_i) = f(t_i, w_i) + \frac{h}{2} f'(t_i, w_i) + \cdots + \frac{h^{n-1}}{n!} f^{(n-1)}(t_i, w_i)$.

$h = 0.1$ $\quad T^{(2)}(t_i, w_i) = f(t_i) + hf'(t_i) + \frac{h^2}{2} f''(t_i)$

$y(1) = 0$

$a = 1$

$w_0 = 0$

$y(t) = t^2(e^t - e)$

$y' = \frac{2}{t} y + t^2 e^t$

$f(t, y(t)) = \frac{2}{t} y + t^2 e^t$

$f'(t, y(t)) = \frac{2}{t} y' + 2y \ln|t| + (t^2 + 2t)e^t = \frac{2}{t} f(t, y(t)) + 2y \ln(t) + (t^2 + 2t)e^t$

$1 + .1 = 1.1$

$W_1 = 0 + 0.1\left[f(1.1, 0) + \frac{0.1}{2} f'(1.1, 0)\right] = .381679$

$w_2 = w_1 + 0.1\left[f(1.2, w_1) + \frac{0.1}{2} f'(1.2, w_1)\right] = .9504744$

$w_3 = 1.7551298$

$w_4 = 2.8529594$

$w_5 = 4.3111727679$

$w_6 = 6.2083893$

$w_7 = 8.63637732$

$w_8 = 11.7020447$

actual values

.3459198

.8666425

1.6072507

2.62035955

3.967666294

5.7209615256

7.9638735

10.7936246605

14.3230815

$w_9 = 15.5297169$

at 1.5 $w_5 = 4.311173$

9b) ii) $y(1.55)$ at 1.6 $w_6 = 6.2083893$

$$\approx \left(\frac{1.55-1.5}{1.6-1.5}\right)6.2083893 + \left(\frac{1.55-1.6}{1.5-1.6}\right)4.311173 \approx 5.25978$$

actual value: 4.788635

Section 5.4

4a, 8a, 12a, 16a

4a) $y' = \frac{2-2ty}{t^2+1}$ $0 \le t \le 1$, $y(0)=1$ $h=0.1$

actual: $y(t) = \frac{2t+1}{t^2+1}$

$w(0) = 1$

$w(0+0.1) = 1 + \frac{1}{2}\left[0.1f(0,1) + 0.1f(0+0.1, 1+0.1f(0,1))\right] = 1.1871$

approximation	actual
(0, 1)	(0, 1)
(0.1, 1.1871)	(0.1, 1.1881)
(0.2, 1.3444)	(0.2, 1.3462)
(0.3, 1.4655)	(0.3, 1.4679)
(0.4, 1.5491)	(0.4, 1.5517)
(0.5, 1.5973)	(0.5, 1.6)
(0.6, 1.615)	(0.6, 1.6176)
(0.7, 1.6083)	(0.7, 1.6107)
(0.8, 1.5832)	(0.8, 1.5854)
(0.9, 1.5451)	(0.9, 1.547)
(1, 1.4984)	(1, 1.5)

8a) $y' = \frac{2-2ty}{t^2+1}$ $0 \le t \le 1$, $y(0)=1$ $h=0.1$ actual $y(t) = \frac{2t+1}{t^2+1}$

$w_0 = 1$

$w_1 = 1 + 0.1\left[f\left(0+0.1/2,\ 1+\frac{0.1}{2}f(0,1)\right)\right] = 1.1885$

approximations

(0,1)
(0.1, 1.1885)
(0.2, 1.3467)
(0.3, 1.4683)
(0.4, 1.5517)
(0.5, 1.5994)
(0.6, 1.6165)
(0.7, 1.6092)
(0.8, 1.5835)
(0.9, 1.545)
(1, 1.4975)

actual

(0,1)
(0.1, 1.1881)

just check previous problem

12a $w_0 = 1$

$w_1 = 1 + \frac{0.1}{4}\left[f(0,1) + 3f\left(0+\frac{2(0.1)}{3},\ 1+\frac{2(0.1)}{3}f\left(0+\frac{0.1}{3},\ 1+\frac{0.1}{3}f(0,1)\right)\right)\right]$

$= 1.1881$

approximations

(0,1)
(0.1, 1.1881)
(0.2, 1.3461)
(0.3, 1.4678)
(0.4, 1.5517)
(0.5, 1.5999)
(0.6, 1.6176)
(0.7, 1.6107)
(0.8, 1.5854)
(0.9, 1.547)
(1, 1.5)

might be most accurate?

$k_1 = 0.1\, f(0,1)$

$k_2 = 0.1\, f(0 + \frac{.1}{2}, 1 + \frac{k_1}{2})$

$k_3 = 0.1 f(0 + \frac{.1}{2}, 1 + k_2/2)$

$k_4 = 0.1 f(0 + .1, 1 + k_3)$

16g) $w_0 = 1$

$w_1 = 1 + \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right] = 1.1881$

approximations

(0,1)
(0.1, 1.1881)
(0.2, 1.3462)
(0.3, 1.4679)
(0.4, 1.5517)
(0.5, 1.6)
(0.6, 1.6176)
(0.7, 1.6107)
(0.8, 1.5854)
(0.9, 1.547)
(1, 1.5)