MA346 Numerical Methods

HW8

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Section 4.1

4b) Using exercise 2b for the data.

$f(x) = x^2 \ln x + 1$

2b)

x	$f(x)$	$f'(x)$
1.0	1.0000	1.0125
1.2	1.2625	1.6850
1.4	1.6595	2.2850

use forward/backward difference formulas

$$\boxed{\text{formula: } f'(x_0) = \frac{f(x_0+h) - f(x_0)}{h} - \frac{h}{2} f''(\xi)}$$

$$f'(x) = x^2\left(\frac{1}{x}\right) + 2x(\ln x) = x + 2x\ln x = x(1 + 2\ln x)$$

$$f''(x) = x\left(\frac{2}{x}\right) + (1 + 2\ln x) = 3 + 2\ln x$$

~~the max would be at x = 1.4 because log is an increasing function, but I'll pick x = 1.0 for less rounding error (?)~~

$$f''(1.0) = 3 + 2\ln(1.0) = 3$$

Approximations

$$f'(1.0) = \frac{f(1.0 + .2) - f(1.0)}{.2} - \frac{.2}{2}\left(f''(1.2)\right) = .9760$$

$$f'(1.2) = \frac{f(1.2 + .2) - f(1.2)}{.2} - \frac{.2}{2}\left(f''(1.4)\right) = 1.6177$$

$$f'(1.4) = \frac{f(1.4 - .2) - f(1.4)}{-.2} + \frac{.2}{2}\left(f''(1.4)\right) = 2.3523$$

$$f'(1.0) = 1.0(1 + 2\ln(1.0)) = 1$$

$$f'(1.2) = (1.2)(1 + 2\ln(1.2)) = 1.6376$$

$$f'(1.4) = 1.4(1 + 2\ln(1.4)) = 2.3421$$

Errors

.024

.0199

.0102

use 3 pt. Midpoint Formula and endpt formula

	x	$f(x)$	$f'(x)$
6b)	7.4	-68.3193	-16.6929
	7.6	-71.6982	-17.0959
	7.8	-75.1576	-17.4981
	8.0	-78.6974	-17.8997

Endpoint: $f'(x_0) = \frac{1}{2h}\left[-3f(x_0) + 4f(x_0+h) - f(x_0+2h)\right] + \frac{h^2}{3} f^{(3)}(\xi_0)$

Midpoint: $f'(x_0) = \frac{1}{2h}\left[f(x_0+h) - f(x_0-h)\right] - \frac{h^2}{6} f^{(3)}(\xi_1)$

from 8b we know actual function is $f(x) = \ln(x+2) - (x+1)^2$

$$f'(x) = \frac{1}{x+2} - 2(x+1)$$

$$f''(x) = \frac{-1}{(x+2)^2} - 2$$

$$f^{(3)}(x) = \frac{2}{(x+2)^3}$$

$$f'(7.4) = \frac{1}{2(.2)}\left[-3(-68.3193) + 4(-71.6982) + 75.1576\right] + \frac{(.2)^2}{3}\left(\frac{2}{(7.4+2)^2}\right)$$

$$= -16.6929$$

$$f'(7.6) = \frac{1}{2(.2)}\left[-75.1576 + 68.3193\right] - \frac{(.2)^2}{6}\left(\frac{2}{(7.4+2)^2}\right) = -17.0959$$

$$f'(7.8) = \frac{1}{2(.2)}\left[-78.6974 + 71.6982\right] - \frac{(.2)^2}{6}\left(\frac{2}{(7.6+2)^2}\right) = -17.4981$$

$$f'(8.0) = \frac{1}{-2(.2)}\left[-3(-78.6974) + 4(-75.1576) + 71.6982\right] + \frac{.04}{3}\left(\frac{2}{(7.6+2)^2}\right)$$

$h = -.2$

$$= -17.8997$$

4.2

b) $M = \int_0^{3\pi/2} \cos x \, dx$

$N_1(h) = 2.356194, \quad N_1\left(\frac{h}{2}\right) = -0.4879837$

$N_1\left(\frac{h}{4}\right) = -0.8815732 \quad N_1\left(\frac{h}{8}\right) = -0.9709157$

$M - N_1(h) = K_1 h + K_2 h^2 + K_3 h^3 + K_4 h^4 + O(h^{10})$

$M = \sin x \Big]_0^{3\pi/2} = -1$

$M = 2.356194 + K_1 h + K_2 h^2 + K_3 h^3 + K_4 h^4 + \ldots$

$M = -0.4879837 + K_1 h/2 + K_2 h^2/4 + K_3 h^3/8 + K_4 h^4/16 + \ldots$

$2M = 2(-0.4879837) + K_1 h + K_2 h^2/2 + K_3 h^3/4 + K_4 h^4/8$

$-M = 2.356194 + K_1 h + K_2 h^2 + K_3 h^3 + K_4 h^4$

$M = -3.3321614 + K_2\left(\frac{h^2}{2} - h^2\right) + K_3\left(\frac{h^3}{4} - h^3\right) + K_4\left(\frac{h^4}{8} - h^4\right) + \ldots$

$N_2(h) = -3.3321614$

9) ① $M = N(h) + K_1 h + K_2 h^2 + K_3 h^3 + \dots$

② $M = N(h/3) + K_1 h/3 + K_2 \frac{h^2}{9} + K_3 \frac{h^3}{27} + \dots$

$3②-①$

$$2M = \left[3N(h/3) - N(h)\right] + K_2\left(\frac{h^2}{3} - h^2\right) + K_3\left(\frac{h^3}{9} - h^3\right)$$

$$M = \left[\frac{3N(h/3) - N(h)}{2}\right] - K_2 \frac{h^2}{3} - \frac{4h^3}{9} K_3$$

$N_2(h) = \downarrow$

$$M = N_2(h) - K_2 \frac{h^2}{3} - K_3 \frac{4h^3}{9}$$

$$- 9\left(M = N_2(h/3) - K_2 \frac{h^2}{27} - K_3 \frac{4h^3}{9 \cdot 27}\right)$$

$$-8M = \left(N_2(h) - 9N_2(h/3)\right) - K_3\left(\frac{4h^3}{9} - \frac{4h^3}{27}\right) + \dots$$

$$M = \frac{-\left(N_2(h) - 9N_2(h/3)\right)}{8} + \frac{1}{8} K_3 \left(\frac{4h^3}{9} - \frac{4h^3}{27}\right)$$

$O(h^3)$ approximation would be $N_3(h) = \frac{-\left(N_2(h) - 9N_2(h/3)\right)}{8}$

or $N_3(h) = -\frac{1}{8}\underbrace{\left[\frac{3N_1(h/3) - N_1(h)}{2} - 9\left(\frac{3N_1(h/9) - N_1(h/3)}{2}\right)\right]}_{\text{written in terms of the first order approximations.}}$

$n=1$

Trapezoidal rule: $\frac{h}{2}[f(x_0)+f(x_1)] - \frac{h^3}{12}f''(\xi)$

$n=2$

Simpson's rule: $\frac{h}{3}[f(x_0)+4f(x_1)+f(x_2)] - \frac{h^5}{90}f^{(4)}(\xi)$

8) 2b: $\int_{-0.5}^{0} x\ln(x+1)\,dx$

make $h = .25$ for even spacing

$\frac{.25}{3}[f(-.5)+4f(-.25)+f(0)] - \frac{.25^5}{90}(8)$

$.346 + .288 + 0$

$\boxed{.0528}$

$f'(x) = x\frac{1}{x+1} + \ln(x+1)$

$f''(x) = x\frac{-1}{(x+1)^2} + \frac{2}{x+1}$

$f^{(3)}(x) = x\frac{2}{(x+1)^3} + \frac{-1}{(x+1)^2} + \frac{-2}{(x+1)^2} = x\frac{2}{(x+1)^3} - \frac{3}{(x+1)^2}$

$f^{(4)}(x) = x\frac{-6}{(x+1)^4} + \frac{2}{(x+1)^3} + \frac{6}{(x+1)^3} = \frac{-6x}{(x+1)^4} + \frac{8}{(x+1)^3}$

max would be based on desmos and knowing it's decreasing

$x = 0$ so $f^{(4)}(0) = 8$

$n=1$

Midpoint rule

14) $\int_0^2 f(x)\,dx \approx$ 5 (trapezoidal rule)

4 (midpoint rule)

trapezoidal $\frac{h}{2}\left[f(x_0)+f(x_1)\right]-\frac{h^3}{12}f''(\xi_2)$

$h=2$ $\quad f(x_0)+f(x_1)-\frac{3}{4}f''(\xi_2)=5$

$n=1$ midpoint rule $\frac{3h}{2}\left[f(x_0)+f(x_1)\right]+\frac{3h^3}{4}f''(\xi_2)=4$

$h=2$

$3f(x_0)+3f(x_1)+6f''(\xi)=4$

Simpsons rule : $\frac{2}{3}\left[f(x_0)+4f(x_1)+f(x_2)\right]-\frac{32}{90}f^{(4)}(\xi)=?$

17)

x	1.8	2.0	2.2	2.4	2.6
$f(x)$	3.12014	4.42569	6.04241	8.03014	10.46675

$\int_{1.8}^{2.6} f(x)dx$

using trapezoidal rule

$h=.8 \qquad \frac{.8}{2}\left[3.12014+10.46675\right] \approx 5.434756$

simpson's rule

$h=.4 \qquad \frac{.4}{3}\left[3.12014+4(6.04241)+10.46675\right] \approx 5.034204$

20) $h=\frac{b-a}{3} \qquad x_0=a \qquad x_1=a+h \qquad x_2=b$

$\int_a^b f(x)\,dx \approx \frac{9}{4}hf(x_1)+\frac{3}{4}hf(x_2)$

$\int_a^b 1\,dx = b-a \approx \frac{9h}{4}(1)+\frac{3h}{4}(1)=3h=3\frac{(b-a)}{3}=(b-a)$ ✓

$\frac{1}{2}x^2\Big]_a^b \qquad \int_a^b x\,dx = \frac{b^2-a^2}{2} \approx \frac{9h}{4}(a+h)+\frac{3h}{4}(b) \approx \frac{9ha}{4}+\frac{9h^2}{4}+\frac{3hb}{4}$

$\approx \frac{3(b-a)a}{4}+\frac{(b-a)^2}{4}+\frac{(b-a)b}{4}=\frac{3ab-3a^2}{4}+\frac{b^2-2ab+a^2}{4}+\frac{b^2-ab}{4}$

$\frac{-2a^2+2b^2}{4}=\frac{b^2-a^2}{2}$ ✓

$\frac{1}{3}x^3\Big]_a^b \qquad \int_a^b x^2\,dx = \frac{b^3-a^3}{3} \approx \frac{9h}{4}(a+h)^2+\frac{3h}{4}(b)^2 \approx \frac{9h(a^2+2ah+h^2)}{4}+\frac{3hb^2}{4}$

≈ 9

22) $\int_0^2 f(x)\,dx = c_0 f(0) + c_1 f(1) + c_2 f(2)$

Must be exact for $f(x) = 1, f(x) = x, x^2$

$$\int_0^2 1\,dx = 2 = c_0 + c_1 + c_2$$

$$\int_0^2 x\,dx = 2 = c_1 + 2c_2$$

$$\int_0^2 x^2\,dx = \frac{8}{3} = c_1 + 4c_2$$

$c_2 = 5/6 \quad c_1 = -4/6 \quad c_0 = 11/6$