Numerical Methods
MA346
HW10
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4.6

1) 4.

$$\int_1^{1.6} \frac{2x}{x^2-4}\,dx$$

Inequality 4.38: $\left|S(a,b) - S\left(a, \frac{a+b}{2}\right) - S\left(\frac{a+b}{2}, b\right)\right| < 15\varepsilon$

Inequality 4.39: $\left|\int_a^b f(x)\,dx - S\left(a, \frac{a+b}{2}\right) - S\left(\frac{a+b}{2}, b\right)\right| < \varepsilon$

Simpson's rule: $\frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right]$

$S(1,1.6) = \frac{.3}{3}\left[f(1) + 4f(1.3) + f(1.6)\right] = -0.739105$

$S(1,1.3) = \frac{.15}{3}\left[f(1) + 4f(1.15) + f(1.3)\right] = -0.261412$

$S(1.3,1.6) = \frac{.15}{3}\left[f(1.3) + 4f(1.45) + f(1.6)\right] = -0.473052$

4.38: $\left|-0.739105 + 0.261412 + 0.473052\right| <$

$.00463938 < 15\varepsilon$

$\varepsilon > 3.0929 \times 10^{-4}$

4.39 $\left|\int_1^{1.6} \frac{2x}{x^2-4} + .261412 + 0.473052\right| < \varepsilon$

$\left|-.733969 + .261412 + 0.473052\right| < 3.0929 \times 10^{-4}$

$4.94999 \times 10^{-4} < 3.0929 \times 10^{-4}$ X

~~Inequality does not match up for some reason~~

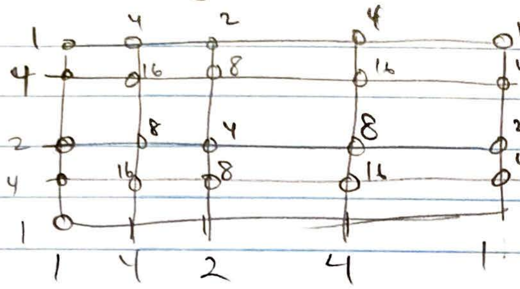
just pick $\varepsilon > 4.9499 \times 10^{-4}$ to match both inequalities.

composite simpson's: $h = \frac{b-a}{n}$ n must be even $x_j = a + jh$

$$\frac{h}{3}\left[f(a) + 2\sum_{j=1}^{n/2-1} f(x_j) + 4\sum_{j=1}^{n/2} f(x_{j-1}) + f(b)\right]$$

1a) $n = m = 4$

$$\int_{2.1}^{2.5}\int_{1.2}^{1.4} xy^2 \, dy \, dx$$



$$\int_{2.1}^{2.5}\int_{1.2}^{1.4} xy^2 \, dy\, dx \approx \frac{(.1)(.05)}{9}\Big[f(2.1,1.2) + 4f(2.2,1.2) + 2f(2.3,1.2) + 4f(2.4,1.2) + f(2.5,1.2)$$

$$+4f(2.1,1.25) + 16f(2.2,1.25) + 8f(2.3,1.25) + 16f(2.4,1.25) + 4f(2.5,1.25)$$

$$+2f(2.1,1.3) + 8f(2.2,1.3) + 4f(2.3,1.3) + 8f(2.4,1.3) + 2f(2.5,1.3)$$

$$+4f(2.1,1.35) + 16f(2.2,1.35) + 8f(2.3,1.35) + 16f(2.4,1.35) + 4f(2.5,1.35)$$

$$+f(2.1,1.4) + 4f(2.2,1.4) + 2f(2.3,1.4) + 4f(2.4,1.4) + f(2.5,1.4)\Big]$$

$h_{outer} = \frac{2.5-2.1}{4} = .1$

$h_{inner} = \frac{1.4-1.2}{4} = .05$

$\approx .3115733$

$$\int_{2.1}^{2.5}\int_{1.2}^{1.4} xy^2 \, dy \, dx$$

$$\frac{1}{3}xy^3\Big]_{1.2}^{1.4} \to \frac{(1.4)^3-(1.2)^3}{3}\int_{2.1}^{2.5} x\,dx = \left(\frac{(1.4)^3-(1.2)^3}{3}\right)\left(\frac{(2.5)^2-(2.1)^2}{2}\right)$$

$= .31157333$

$$\frac{h}{3}\left[f(a)+2\sum_{j=1}^{n/2-1}(x_j)+4\sum_{j=1}^{n/2}(x_{j-1})+f(b)\right]$$

use legendre polynomial coefficients

7) ii) $n=3$ $m=4$ m is along y ; n is along x

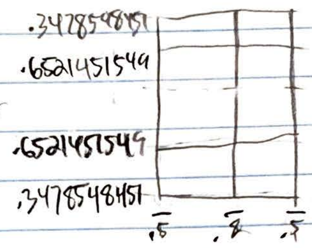
5h) $$\int_{-\pi}^{3\pi/2}\int_{0}^{2\pi}(y\sin(x)+x\cos(y))\,dy\,dx$$

n is odd won't work with simpson's

do we use gaussian quadrature with legendre

$$h_{outer}=\frac{\frac{3\pi}{2}+\pi}{3}=\frac{5\pi}{6}$$

$$h_{inner}=\frac{2\pi-0}{4}=\pi/2$$



have to use gaussian quadrature

$$\int_0^{2\pi} y\sin(x)+x\cos y\,dy \longrightarrow \int_{-1}^{1}\left(\left[(2\pi-1)t+2\pi\right]\sin(x)+x\cos\left[(2\pi-1)t+2\pi\right]\right)(2\pi-1)\,dt$$

$(2\pi, 1)$ $(0,-1)$

$\frac{1}{\pi}$

$t=\frac{1}{\pi}y+b$

$t=\frac{1}{\pi}y-1$

$y=(t+1)\pi$

$$\int_{-1}^{1}\int_{-1}^{1}\left[[\pi t+\pi]\sin\left[\frac{5\pi}{4}w+\frac{\pi}{4}\right]+\left[\frac{5\pi}{4}w+\frac{\pi}{4}\right]\cos(\pi t+\pi)\right]\left(\frac{5\pi}{4}\right)\pi$$

$y=\pi t+\pi$

Rewrite Integral:

$$\cancel{\int_{-1}^{1}\int_{-1}^{1}\left[\left[(2\pi-1)t+2\pi\right]\sin\left[\frac{3\pi/2-1}{1-\pi}w+\frac{\pi/2}{1-\pi}\right]+\left[\frac{3\pi/2-1}{1-\pi}w+\frac{\pi/2}{1-\pi}\right]\cos\left[(2\pi-1)t+2\pi\right]\right](2\pi-1)\left(\frac{3\pi/2}{1-\pi}\right)}$$

$(3\pi/2, 1)$ $(-\pi,-1)$

$w=\frac{4}{5\pi}x+b$

≈ -11.8362

$-1=\frac{4}{5\pi}(-\pi)+b$ $x=\frac{5\pi}{4}w+\pi/4$

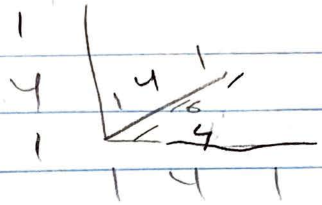
$-1/5=b$

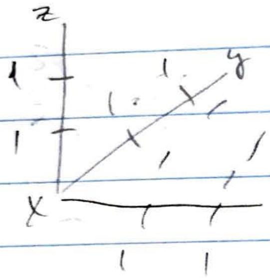
$w=\frac{4}{5\pi}x-1/5$

$(w+1/5)\left(\frac{5\pi}{4}\right)=x$

11) a. $n=m=p=2$

$$\int_0^1 \int_1^2 \int_0^{0.5} e^{x+y+z} \, dz \, dy \, dx$$





$= 1,1,1,1,1,1,1,1$

all the coefficients are 1.

$n=6$ use composite simpson's rule

4.9)

2a) $\int_0^1 \frac{e^{-x}}{\sqrt{1-x}}\,dx \Rightarrow \int_0^1 \frac{e^{u-1}}{\sqrt{u}}\,du$

$\downarrow$

$\frac{e^u e^{-1}}{\sqrt{u}}\,du \to \frac{e^u}{e\sqrt{u}} \to \frac{1}{e}\int_0^1 \frac{e^u}{\sqrt{u}}\,du$

$\frac{1}{e}\int_0^1 \frac{1}{\sqrt{u}} e^u\,du$

use Taylor Polynomial

at $u=0$

$P_4(u) = 1 + u + \frac{u^2}{2} + \frac{u^3}{6} + \frac{u^4}{24}$

$\int_0^1 \left(u^{-1/2} + u^{1/2} + \frac{1}{2}u^{3/2} + \frac{1}{6}u^{5/2} + \frac{1}{24}u^{7/2}\right)du \Rightarrow \int_0^1 u^{-1/2}\,du + \int_0^1 \underbrace{\left(u^{1/2} + \frac{1}{2}u^{3/2} + \frac{1}{6}u^{5/2} + \frac{1}{24}u^{7/2}\right)}_{g}du$

use composite simpson rule for $n=6$

$h = 1/6 \quad x_j = a + jh \qquad \int_a^b f(x)\,dx = \frac{h}{3}\left[f(a) + 2\sum_{j=1}^{n/2-1} f(x_{2j}) + 4\sum_{j=1}^{n/2} f(x_{2j-1}) + f(b)\right]$

$\frac{1}{6}\left[g(0) + 2\left[g(\tfrac{1}{6}) + g(\tfrac{2}{6})\right] + 4\left[g(0) + g(1/6) + g(2/6)\right] + g(1)\right]$

$\simeq 2.92354450 + \int_0^1 G(x)\,dx \simeq 2.92354450 + .0017691$

$\simeq 2.925314$

$G(x) = \begin{cases} \frac{1}{\sqrt{x}}(e^x - P_4(x)) & 0 < x \le 1 \\ 0 & 0 \end{cases}$

$\approx \frac{2.925314}{e}$

$x = 1/t$ $\quad \frac{-1}{t^2} dt = dx$

$dt = -t^2 dx$

2d) $\int_1^\infty x^{-4} \sin x \, dx$

$-\int_1^0 \left(\frac{1}{t}\right)^{-4} \sin\left(\frac{1}{t}\right) t^2 dt$ $\quad \int_0^1 t^4 \sin\left(\frac{1}{t}\right) t^2 dt$

$\int_0^1 t^6 \sin\left(\frac{1}{t}\right) dt$ $\quad n=6$

$$G(t) = \begin{cases} t^6 \sin\left(\frac{1}{t}\right) dt & 0 < t \le 1 \\ 0 & t = 0 \end{cases}$$

$\int_0^1 G(t) \, dt \approx .1290$