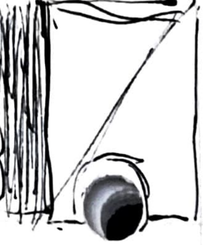

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Numerical Methods HW7

3.1

5a) $f(8.4) = ?$

$f(8.1) = 16.94410$
$f(8.3) = 17.56492$
$f(8.6) = 18.50515$
$f(8.7) = 18.82091$

$x_0 = 8.1 \quad x_1 = 8.3 \quad x_2 = 8.6 \quad x_3 = 8.7$

$$L_0(x) = \frac{(x-8.3)(x-8.6)(x-8.7)}{(8.1-8.3)(8.1-8.6)(8.1-8.7)} = \frac{1}{\frac{-3}{50}}(x-8.3)(x-8.6)(x-8.7)$$

$$L_1(x) = \frac{(x-8.1)(x-8.6)(x-8.7)}{(8.3-8.1)(8.3-8.6)(8.3-8.7)} = \frac{1}{.024}(x-8.1)(x-8.6)(x-8.7)$$

$$L_2(x) = \frac{(x-8.1)(x-8.3)(x-8.7)}{(8.6-8.1)(8.6-8.3)(8.6-8.7)} = \frac{1}{-.015}(x-8.1)(x-8.3)(x-8.7)$$

$$L_3(x) = \frac{(x-8.1)(x-8.3)(x-8.6)}{(8.7-8.1)(8.7-8.3)(8.7-8.6)} = \frac{1}{.024}(x-8.1)(x-8.3)(x-8.6)$$

$P(x) = \cancel{1.01665(x-8.3)(x-8.6)(x-8.7) + 1.42156(x-8.1)(x-8.6)(x-8.7) - .27758(x-8.1)(x-8.3)(x-8.7) + .45170(x-8.1)(x-8.3)(x-8.6)}$

$P(8.4) = 17.87714 \approx f(8.4)$.

7) $f(x) = x \ln x$

$$f(x) - P(x) = \frac{1}{(n+1)!} f^{(n+1)}(\xi_x) \prod_{i=0}^{n} (x - x_i)$$

$f'(x) = 1 + \ln x$

$f''(x) = \frac{1}{x}$

$f^{(3)}(x) = \frac{-1}{x^2}$

$$\frac{1}{3!}(1)\big((x-8.1)(x-8.3)(x-8.6)\big)$$

max around here

$$\frac{1}{6}(1)(.003)$$

$$\approx .0005$$

1) $(0,0)$ $(0.5, y)$ $(1,3)$ $(2,2)$

$$L_0 \quad \frac{(x-.5)(x-1)(x-2)}{(-.5)(-1)(-2)} = -(x-.5)(x-1)(x-2)$$

$$L_1 \quad \frac{x(x-1)(x-2)}{(.5)(-.5)(-1.5)} = \frac{1}{-.375} x(x-1)(x-2)$$

$$L_2 \quad \frac{x(x-0.5)(x-2)}{(1)(.5)(-1)} = \frac{1}{-.5} x(x-0.5)(x-2)$$

$$L_3 \quad \frac{x(x-0.5)(x-1)}{2(1.5)(1)} = \frac{1}{3} x(x-0.5)(x-1)$$

$$P_3(x) = L_0 f(x_0) + L_1 f(x_1) + L_2 f(x_2) + L_3 f(x_3)$$

$$= 0 + \frac{y}{-.375} x(x-1)(x-2) + -6x(x-0.5)(x-2) + \frac{2}{3} x(x-0.5)(x-1)$$

$$= \frac{y}{-.375}\left[x^3 - 3x^2 + 2x\right] - 6\left[x^3 - 2.5x^2 + x\right] + \frac{2}{3}\left[x^3 - 1.5x^2 + .5x\right]$$

$$= \frac{-y}{.375}x^3 - 6x^3 + \frac{2}{3}x^3$$

Coefficient of $x^3 = 6$

$$\frac{-y}{.375} - 6 + \frac{2}{3} = 6$$

$$\frac{-y}{.375} = 11\frac{1}{3}$$

$$\boxed{y = -4.25}$$

13b) $f(x) = \sin(\ln x)$ $\quad x_0 = 2.0$

$x_1 = 2.4$

$x_2 = 2.6$

$L_0 \quad \dfrac{(x-2.4)(x-2.6)}{(2-2.4)(2-2.6)} =$

$L_1 \quad \dfrac{(x-2.0)(x-2.6)}{(2.4-2)(2.4-2.6)} =$

$L_2 \quad \dfrac{(x-2)(x-2.4)}{(2.6-2)(2.6-2.4)} =$

$|f(x) - P(x)| \le \frac{1}{3!} \left| f^{(3)}(\xi_x) \right| \left| (x-2)(x-2.4)(x-2.6) \right|$

$\frac{1}{6} * .336 * .017 =$

$.000952$

9.52×10^{-4}

$f'(x) = \frac{1}{x}\cos(\ln x)$

$f''(x) = \frac{-1}{x^2}\sin(\ln x) + (\cos(\ln x))\left(-\frac{1}{x^2}\right)$

$= -\frac{1}{x^2}\left[\sin(\ln x) + \cos(\ln x)\right]$

$f^{(3)}(x) = -\frac{1}{x^2}\left[\frac{1}{x}\cos(\ln x) - \frac{1}{x}\sin(\ln x)\right] + \frac{2}{x^3}\left[\sin(\ln x) + \cos(\ln x)\right]$

got the max $= .336$ at $x = 2$

$(x-2)(x-2.4)(x-2.6) = (x^2 - 4.4x + 4.8)(x-2.6)$

got max $= .017$

$x^3 - 4.4x^2 + 4.8x - 2.6x^2 + 11.44x - 12.48$

$g(x) \quad x^3 - 7x^2 + 16.24x - 12.48$

$g'(x) = 3x^2 - 14x + 16.24$

$g'(x) = 0$

$\dfrac{14 \pm \sqrt{196 - 4(16.24)(12)}}{6} =$

section 3.2

$f(.4)$

5) $P_{0123} = \dfrac{(x-.75)P_{012} - (x-0)\,2.96}{0-.75} = 3.016$

$P_{012} = 3.08 = \dfrac{(x-.5)\,2.6 - (x-0)P_{12}}{0-.5}$

$P_{12} = 3.2 = \dfrac{(x-.25)\,2 - (x-.5)P_2}{.5-.25}$

$P_2 = 8$

$j = 0,1,2,3 \qquad x_j = j$

7) $P_{0,1}(x) = 2x+1 \quad P_{0,2}(x) = x+1 \quad P_{1,2,3}(2.5) = 3$

find $P_{0,1,2,3}(2.5)$ need P_{123} (given) and P_{012} (to be found)

Section 3.3 7, 12

2)

x	$f(x)$	DD-1	DD-2	DD-3
-0.1	5.30000	-33.00000	129.83333	~~-452.49999~~
0.0	2.00000	5.95000	~~-[illegible]~~	-556.66667
0.2	3.19000	-21.90000	-92.83333	
0.3	1.00000			

a.

b.

x	$f(x)$	DD-1	DD-2	DD-3	DD-4
-0.1	5.30000	-33.00000	129.83333	~~-[illegible].80000~~	~~[illegible]~~
0.0	2.00000	5.95000	~~-[illegible]~~	-556.66667	2733.29101
0.2	3.19000	-21.90000	-92.83333	~~[illegible]~~	
0.3	1.00000	-.47600	142.82667	673.31428	
0.35	.97260				

12)

x	0	1	2	3
$P(x)$	4	9	15	18

Determine coefficient of x^3 if all 4th order forward differences are 1.

x	$f(x)$	$\Delta f(x_n)$ ~~DD-1~~	$\Delta^2 f(x_n)$ ~~DD-2~~	$\Delta^3 f(x_n)$
0	4	5	1/2	~~14~~ -2/3
1	9	6	-3/2 ~~[illegible]~~	
2	15	3		
3	18			

$$P(x) = 4 + 5(x-0) + \frac{1}{2}(x-0)(x-1) + \frac{-2}{3}(x-0)\underbrace{(x-1)(x-2)}_{x^2-3x+2} + \frac{1}{4!\,1^4}\underbrace{(x-0)(x-1)(x-2)(x-3)}_{(x^2-3x+2)(x^2-3x)}$$

$$= 4 + 5x + \frac{1}{2}x^2 - \frac{1}{2}x - \frac{2}{3}x^3 + 2x^2 - \frac{4}{3}x + \frac{1}{24}x^4 - \frac{1}{4}x^3 + \frac{11}{24}x^2 - \frac{1}{4}x$$

$$-\frac{2}{3}x^3 - \frac{1}{4}x^3$$

coefficient of $x^3 = -11/12$

3.4

1a)

x	$f(x)$	$f'(x)$
8.3	17.56492	3.116256
8.6	18.50515	3.151762

		DD-1	DD-2	DD-3
8.3	17.56492	3.116256	.059480	-.002022
8.3	17.56492	3.134100	.058873	
8.6	18.50515	3.151762		
8.6	18.50515			

$P(x) = 17.56492 + 3.116256(x-8.3) + .059480(x-8.3)(x-8.3) - .00202(x-8.3)^2(x-$

3a) $P(8.4) = 17.877144$

$f(8.4) = 17.877146$

error: 0.000002

or

2.0×10^{-6}

3.5/1) $(0,0)\ (1,1)\ (2,2)$

$S_0(x) = a_0 + b_0(x-0) + c_0(x-0)^2 + d_0(x-0)^3$

$S_1(x) = a_p + b_p(x-1) + c_p(x-1)^2 + d_p(x-1)^3$

$a_0 = 0$

$b_0 =$

$c_0 =$

$d_0 =$

$0 = f(0) = a_0$ $\quad 1 = f(1) = b_0 + c_0 + d_0$

$1 = f(1) = a_1$ $\quad 2 = f(2) = b_1 + c_1 + d_1 + 1$

$S(x) = \{$

$S_0'(1) = S_1'(1)$ and $S_0''(1) = S_1''(1)$

$b_0 + 2c_0(1) + 3d_0(1)^2 = b_1 + \cancel{2c_1+3d_1}$ $\quad 2c_0 + 6d_0 = 2c_1$

$S_0''(0) = 0$ $\quad S_1''(2) = 0$

$2c_0 = 0$ $\quad 2c_1 + 6d_1 = 0$

$0 = a_0 \quad c_0 = 0$

$1 = a_1$

$1 = b_0 + c_0 + d_0 \longrightarrow 1 = b_0 + d_0$

$1 = b_1 + c_1 + d_1 \rightarrow 1 =$

$b_1 = b_0 + 2c_0 + 3d_0 \rightarrow b_1 = b_0 + 3d_0$

$0 = 2c_0$

$2c_1 = 2c_0 + 6d_0 \rightarrow c_1 = 3d_0$

$0 = 2c_1 + 6d_1 \quad c_1 = -3d_1$

used algorithm provided

$$s(x) = \begin{cases} 0 + 1(x-0) + 0(x-0)^2 + 0(x-0)^3 \\ 1 + 1(x-1) + 0(x-1)^2 + 0(x-1)^3 \end{cases}$$

$$s(x) = \begin{cases} x & \text{if } [0,1] \\ x & [1,2] \end{cases}$$

??

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2)

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1) $b_0 = 2 \quad c_0 = 0 \quad d_0 = -1$

$s_0'(1) = s_1'(1) = 0$

$s_0'(x) = 2 - 3x^2$

$s_1'(x) = b + 2c(x-1) + 3d(x-1)^2$

$b = -1 \quad c = -3 \quad d = 1$