Normal approximation to the binomial distribution

Let X be a binomial random variable with parameters n and p. For large n, Z has approximately a normal distribution with $\mu = np$ and $\sigma 2 = np(1-p)$ and

n – total trial

p – prob. of succe in each trial

$n \to \infty$

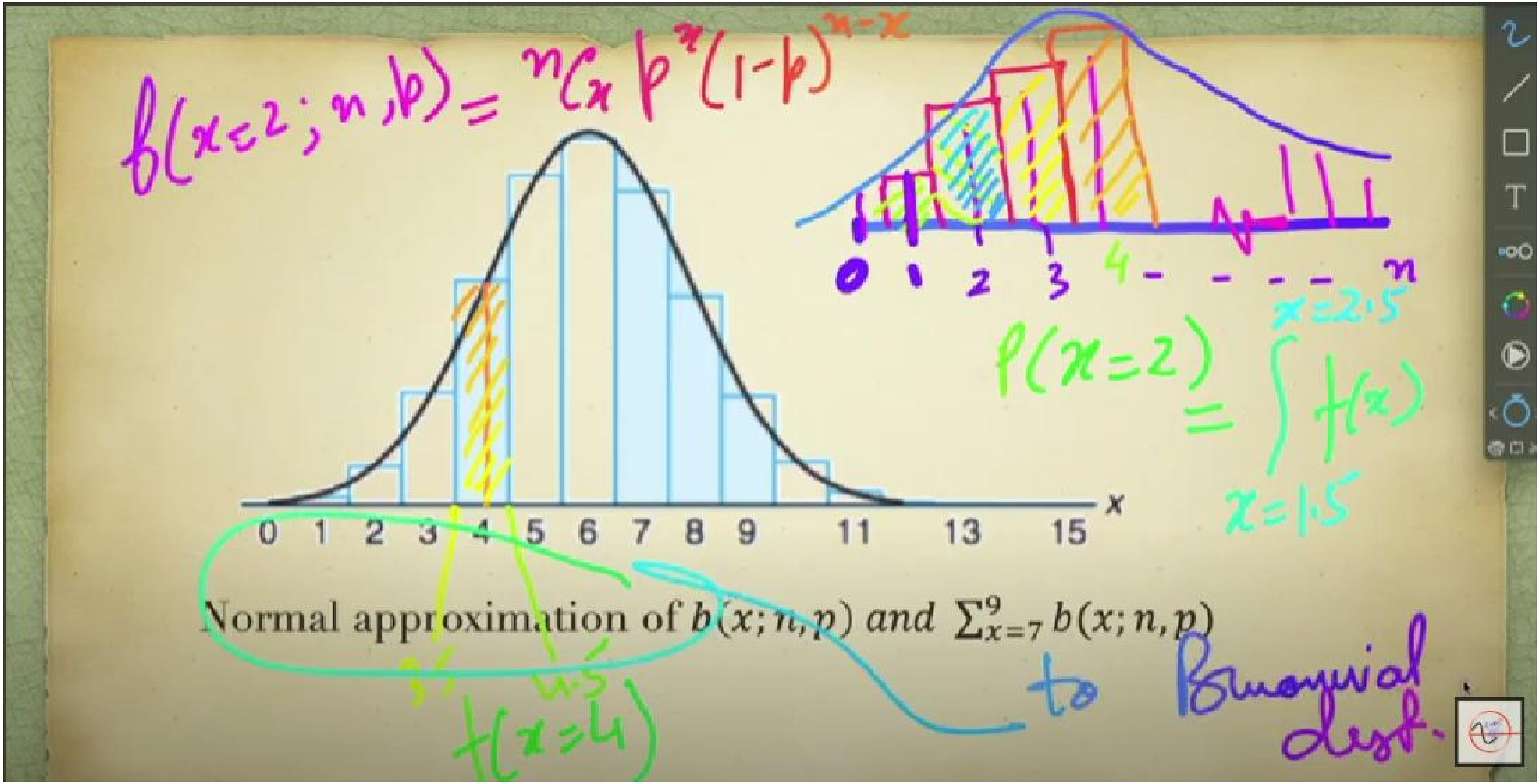
$$P(X \leq x) = \sum_{k=0}^{x} b(k; n, p)$$

$\mu = n$

$$\approx \textit{area under normal curve to the left of } x + 0.5$$

$$= P\left(Z \leq \frac{(x + 0.5) - np}{\sqrt{np(1-p)}}\right).$$

and the approximation will be good if np and n(1-p) are greater than or equal to 5.



$b(x=2; n,p) = {}^{n}C_{x}\, p^{x} (1-p)^{n-x}$
0 1 2 3 4 - - - - n
$P(x=2) = \int_{x=1.5}^{x=2.5} f(x)$
x
0 1 2 3 4 5 6 7 8 9 11 13 15
Normal approximation of $b(x;n,p)$ and $\sum_{x=7}^{9} b(x;n,p)$
3.5
4.5
$f(x=4)$
to Binomial dist.

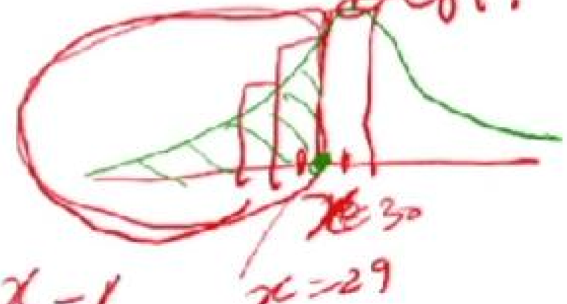
The probability that a patient recovers from a rare blood disease is 0.4. If 100 people are known to have contracted this disease, what is the probability that fewer than 30 survive?

$p = 0.4 \quad n = 100$ } Bernoulli's Process → Binomial dist.

$P(x < 30)$

Normal approximation to Binomial Dist.

$$= \sum_{x=0}^{29} b(x; n, b) = ??$$



$$= \int_{0}^{29.5} f(x) \cdot dx = P(Z < x - 1$$

$$= \int_0 f(x)\cdot dx = P\left(Z < \frac{x-\mu}{\sigma}\right)$$

$$P(x<30) = \int_0^{29.5} f(x)\cdot dx = P\left(Z < \frac{29.5 - np}{\sqrt{np(1-p)}}\right)$$

$$= P(x \le 29)$$

$$= P\left(Z < \frac{29.5 - 100\times 0.4}{\sqrt{100\times 0.4\times 0.6}}\right)$$

$$= P(Z < -2.14)$$

tables of Normal dist (Z-dist)

$$= \int_{0} f(x)\cdot dx = P\left(Z < \frac{x-\mu}{\sigma}\right) \quad x = 2?$$

$$P(x<30) = \int_{0}^{29.5} f(x)\cdot dx = P\left(Z < \frac{29.5 - np}{\sqrt{np(1-p)}}\right)$$

$$= P(x \leq 29)$$

$$= P\left(Z < \frac{29.5 - 100\times 0.4}{\sqrt{100\times 0.4 \times 0.6}}\right)$$

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Normal Approximate to Binom

tables of Normal dist (Z-dist)

$$P(Z<-2.14) = 0.0162 \simeq P(x<30)$$

If $X \sim P_o(\lambda)$ and $\lambda > 20$ then $X \sim N(\lambda, \lambda)$ approximately

In a particular factory the number of accidents occuring in a month follows a Poisson distribution with a mean of 2. Find the probability that that there will be at least 22 accidents in a year.

Let X be the r.v. "Number of accidents occuring in a year" where $X \sim P_o(24)$

If $X \sim P_o(\lambda)$, $P(X=r) = \dfrac{\lambda^r e^{-\lambda}}{r!}$

$$P(X \geq 22) = 1 - P(X \leq 21)$$

$$= 1 - [P(X=0) + P(X=1) + P(X=2) + \ldots + P(X=21)]$$

$$= 1 - \left[\frac{24^0 e^{-24}}{0!} + \frac{24^1 e^{-24}}{1!} + \frac{24^2 e^{-24}}{2!} + \ldots + \frac{24^{21} e^{-24}}{21!}\right]$$

$$= 0.68607\ldots$$

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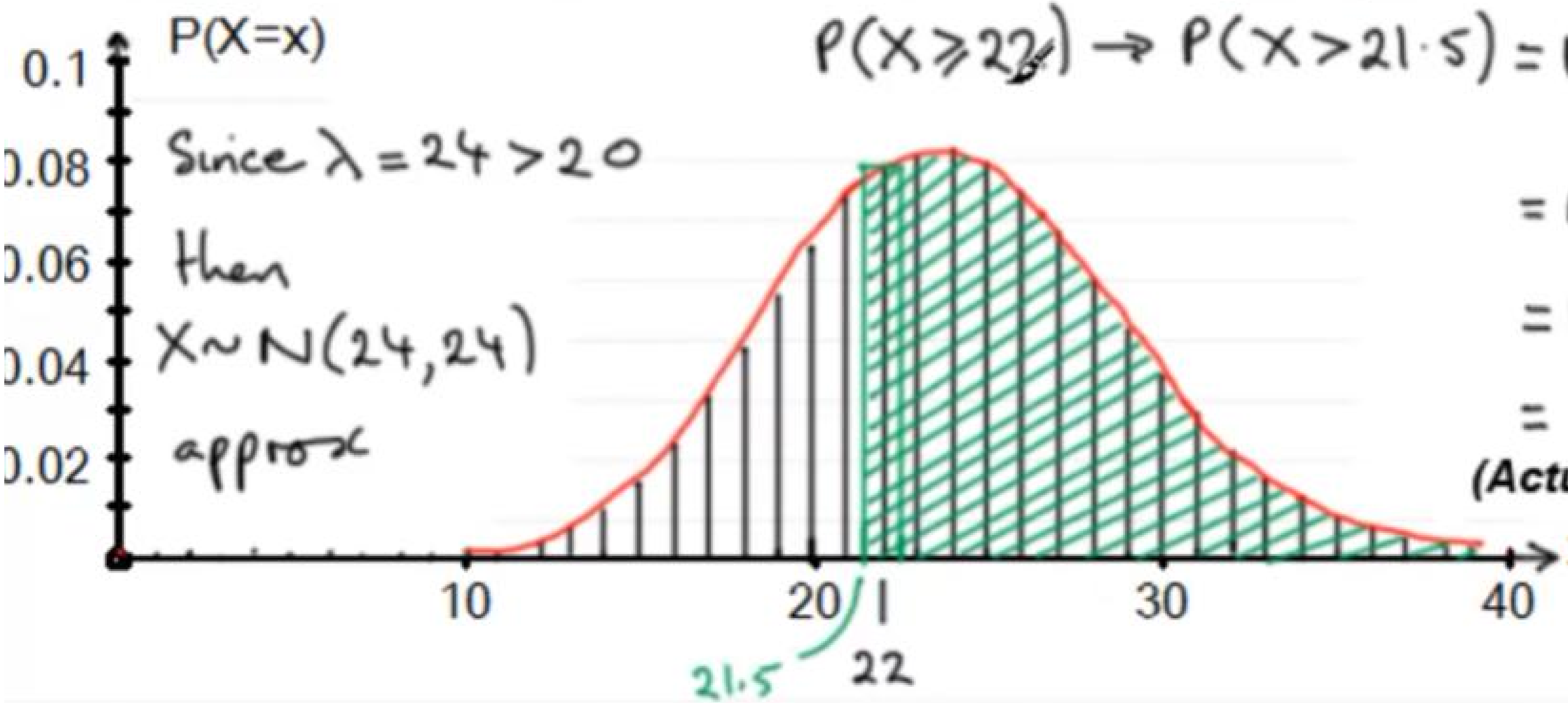

Since $\lambda = 24 > 20$

then

$X \sim N(24, 24)$

approx

$$P(X \geq 22) \rightarrow P(X > 21.5) = P\left(Z > \frac{21.5 - 24}{\sqrt{24}}\right)$$

$$= P(Z > -0.5103\ldots)$$

$$= P(Z < 0.5103\ldots)$$

$$= 0.6950$$

(Actual answer = 0.68607...)