Example: An electrical firm manufactures light bulbs that have life, before burn-out, that is normally distributed with mean is 800 hours and a standard deviation of 40 hours. Find the probability that a bulb burns (i) more than 834 hours (ii) between 778 and 834 hours.

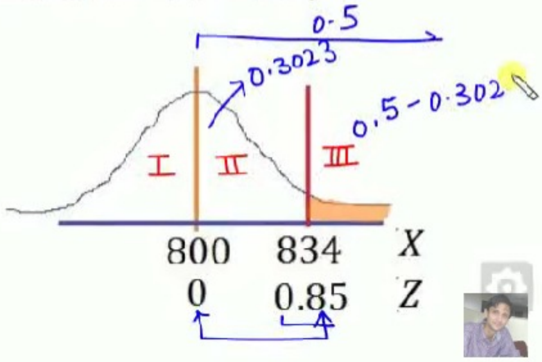
Given that $P(Z < -0.85) = 0.1977$; $P(0 < Z < 0.85) = 0.3023$; $P(-0.55 < Z < 0) = 0.2088$; $P(Z < 0.85) = 0.8023$.

Solution: Let X be the life of the light bulb manufactured by the electrical bulbs. Thus,

$$Z = \frac{X - 800}{40} \qquad \mu = 800; \quad \sigma = 40$$

(i) Required probability $= P(X > 834)$

$$= P(Z > 0.85)$$



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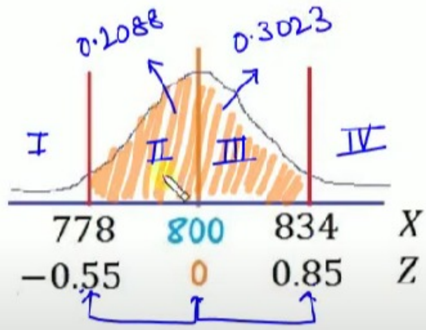
(ii) Required probability $= P(778 < X < 834)$

$$= P(-0.55 < Z < 0.85)$$



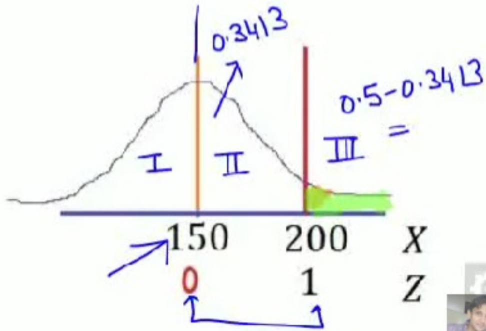
Example: The saving bank account of a customer showed an average balance of \$150 and a standard deviation of \$50. Assuming that the account balances are normally distributed, find what percentage of account is (i) over \$200 (ii) between \$120 and \$170 (iii) less than \$75. Given that $\boxed{P(0 < Z < 1) = 0.3413}$; $P(Z < 0.4) = 0.6554$; $P(Z < -0.6) = 0.2743$; $P(0 < Z < 1.5) = 0.4332$.

Solution: Let X be the balance of savings bank account of a customer. Thus,

$Z = \dfrac{X-150}{50}$ $\qquad \mu = 150; \quad \sigma = 50$

(i) **Required probability** = $P(X > 200)$

$= P(Z > 1)$



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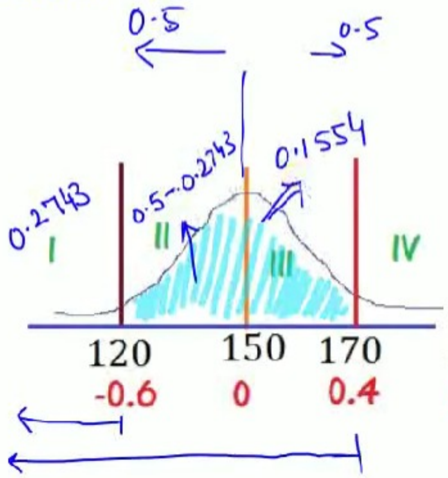
$\underbrace{\boxed{I + II}}_{0.5} + III = 0.6554$

Solution:

(ii) **Required probability** = $P(120 < X < 170)$

$= P(-0.6 < Z < 0.4)$



Example: The saving bank account of a customer showed an average balance of \$150 and a standard deviation of \$50. Assuming that the account balances are normally distributed, find what percentage of account is (i) over \$200 (ii) between \$120 and \$170 (iii) less than \$75. Given that $P(0 < Z < 1) = 0.3413$; $P(Z < 0.4) = 0.6554$; $P(Z < -0.6) = 0.2743$; $P(0 < Z < 1.5) = 0.4332$.

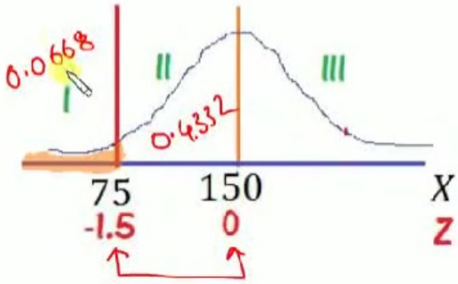
Solution:

(iii) Required probability $= P(X < 75)$

$$= P(Z < -1.5)$$

$$P(0 < Z < 1.5) = 0.4332$$

$$\Rightarrow P(-1.5 < Z < 0) = 0.4332$$



Example: The marks obtained by a number of students in a certain subject are approximately normally distributed with mean 65 and standard deviation 5. If 3 students are selected at random from this group, what is the probability that at least one of them would have scored above 75? Given that $P(Z < -2) = 0.0228$

Solution: Let X be marks obtained by a student in a subject. Thus,

$$\mu = 65, \qquad \sigma = 5$$

$-\infty \leftrightarrow -2$
$2 \leftrightarrow \infty$

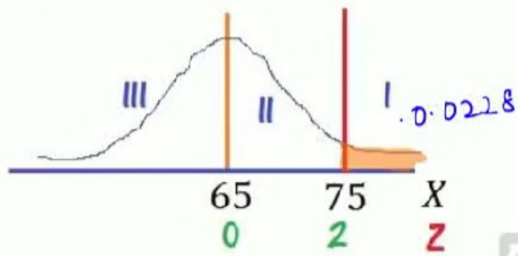
The probability that a student would have scored above 75 is

$P(Z < -2) = 0.0228$
$P(Z > 2) = 0.0228$

$$= P(X > 75)$$

$$= P(Z > 2)$$

$=$



Since 3 students are selected at random from given group, thus the required probability that at least one of them would have scored above 75 is

$$= P(X \geq 1)$$

$$= 1 - P(X < 1)$$

$$= 1 - P(X = 0)$$

$$= 1 - {}^{3}C_0 p^0 q^3$$

$$= 1 - (0.9772)^3$$

$$= 0.0669$$

Normal — Marks

\+ Binomal — Student

Binomial Distribution

$n = 3;$

$p = 0.0228$

Example: The weekly wages of 1000 workers are normally distributed around a mean of \$500 with a standard deviation of \$50. Estimate the number of workers, whose weekly wages will be (i) between \$400 and \$600 (ii) less than \$400 (iii) more than \$600. Given that $P(0 < Z < 2) = 0.4772$; $P(Z < 0.4) = 0.6554$; $P(Z < -0.6) = 0.2743$

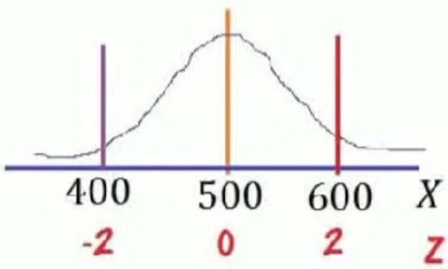
Solution: Let X be the weekly wage of worker. Thus, $\mu = 500$; $\sigma = 50$

(i) Required probability $= P(400 < X < 600)$

$$= P(-2 < Z < 2)$$

$$= 0.4772 + 0.4772$$

$$= 0.9544$$



Thus, expected number of workers

$$= 1000 \times 0.9544 \; \boxed{= 954.4 \simeq 954}$$

Example: The weekly wages of 1000 workers are normally distributed around a mean of \$500 with a standard deviation of \$50. Estimate the number of workers, whose weekly wages will be (i) between \$400 and \$600 (ii) less than \$400 (iii) more than \$600. Given that $P(0 < Z < 2) = 0.4772$; $P(Z < 0.4) = 0.6554$; $P(Z < -0.6) = 0.2743$

Solution: Let X be the weekly wage of worker. Thus, $\mu = 500;\ \sigma = 50$

(ii) Required probability $= P(X < 400)$

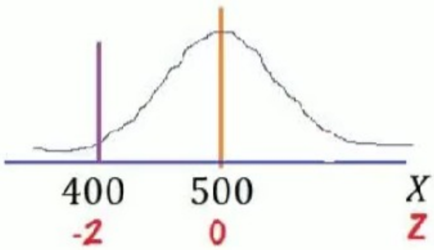
$$= P(Z < -2)$$

$$= 0.0228$$

Thus, expected number of workers

$$= 1000 \times 0.0228$$

$$= 22.8 \simeq 23$$



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Solution: Let X be the weekly wage of worker. Thus, $\mu = 500;\ \sigma = 50$

(iii) Required probability $= P(X > 600)$

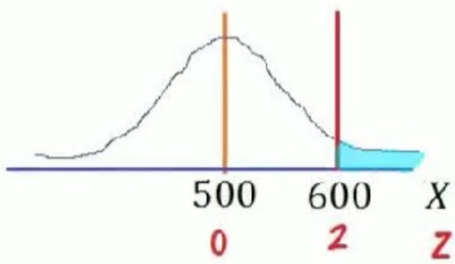
$$= P(Z > 2)$$

$$= 0.0228$$

Thus, expected number of workers

$$= 1000 \times 0.0228$$

$$= 22.8 \simeq 23$$



Example: In an intelligence test administered on 1000 children, the average was 60 and the standard deviation was 20. Assuming that the marks obtained by the children follow a normal distributed, find the number of children who have scored (i) over 90 marks, (ii) below 40 marks and (iii) between 50 and 80 marks. Given that

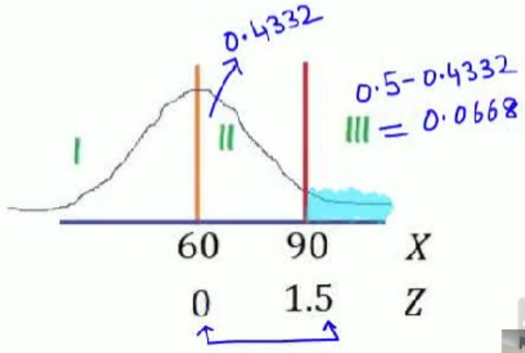
$$P(0 < Z < 1.5) = 0.4332; \; P(Z < \cancel{1}) = 0.8413; \; P(Z < \cancel{0.5}) = 0.6915$$

Solution: Let X be the marks obtained by the children in the intelligence test. Thus,

$$\mu = 60; \qquad \sigma = 20$$

(i) Required probability $= P(X > 90)$

$= P(Z > 1.5)$



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$$P(0 < Z < 1.5) = 0.4332; \; P(Z < 1) = 0.8413; \; P(Z < 0.5) = 0.6915$$

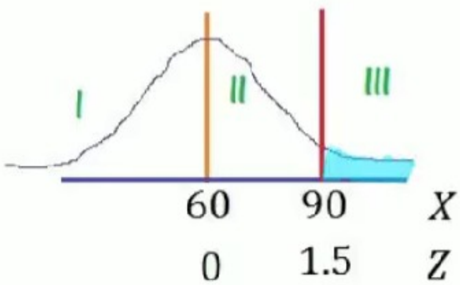
Solution: Let X be the marks obtained by the children in the intelligence test. Thus,

$$\mu = 60; \qquad \sigma = 20$$

(i) Required probability $= P(X > 90)$

$= P(Z > 1.5)$

$= 0.0668$



Thus, expected number of children

$= 1000 \times 0.0668$

$= 66.8 \simeq 69$

Example: In an intelligence test administered on 1000 children, the average was 60 and the standard deviation was 20. Assuming that the marks obtained by the children follow a normal distributed, find the number of children who have scored (i) over 90 marks, (ii) below 40 marks and (iii) between 50 and 80 marks. Given that

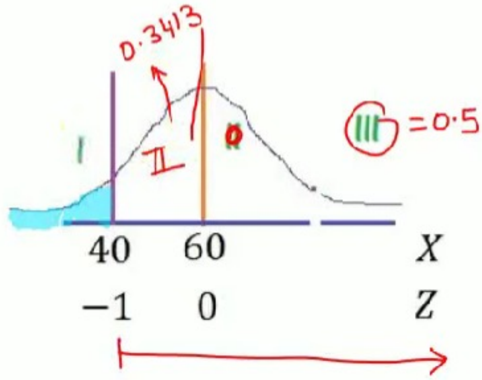
$$P(0 < Z < 1.5) = 0.4332; \ \boxed{P(Z < 1) = 0.8413}; \ P(Z < 0.5) = 0.6915$$

Solution: Let X be the marks obtained by the children in the intelligence test. Thus,

$$\mu = 60; \quad \sigma = 20$$

(ii) Required probability $= P(X < 40)$

$= P(Z < -1)$



$-\infty \quad 1$

$P(Z < 1) = 0.8413$

$\Rightarrow P(Z > -1) = 0.8413$

$II + III = 0.8413$

Example: In an intelligence test administered on 1000 children, the average was 60 and the standard deviation was 20. Assuming that the marks obtained by the children follow a normal distributed, find the number of children who have scored (i) over 90 marks, (ii) below 40 marks and (iii) between 50 and 80 marks. Given that

$$P(0 < Z < 1.5) = 0.4332; \ P(Z < 1) = 0.8413; \ P(Z < 0.5) = 0.6915$$

Solution: Let X be the marks obtained by the children in the intelligence test. Thus,

$$\mu = 60; \quad \sigma = 20$$

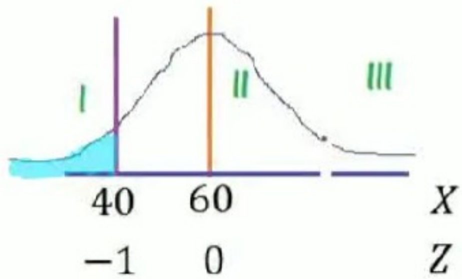
(ii) Required probability $= P(X < 40)$

$= P(Z < -1)$

$= 0.1587$

Thus, expected number of children

$= 1000 \times 0.1587 = 158.7 \simeq 159$



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$$P(0 < Z < \cancel{1.5}) = 0.4332;\ P(Z \cancel{<} 1) = 0.8413;\ P(Z < 0.5) = 0.6915$$

Solution:

(iii) Required probability $= P(50 < X < 80)$

$= P(-0.5 < Z < 1)$

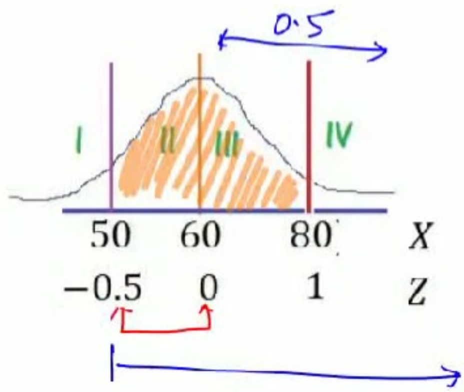
$-\infty$ 0.5 ∞

-0.5

$P(Z < 0.5)$

$P(Z > -0.5) = 0.6915$

$\mathrm{I} + \boxed{\mathrm{II} + \mathrm{IV}} = 0.6915$



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$$P(0 < Z < 1.5) = 0.4332;\ P(Z < 1) = 0.8413;\ P(Z < 0.5) = 0.6915$$

Solution:

(iii) Required probability $= P(50 < X < 80)$

$= P(-0.5 < Z < 1)$

$= 0.1915 + 0.3413$

$= 0.5328$