Example: The number of personal computers (PC's) sold daily at Alfa Computer is uniformly distributed with a minimum of 2000 PC's and a maximum of 5000 PC's.

(i) Find the probability that the daily sales will fall between 2500 and 3000 PC's.

(ii) What is the probability that Alfa Computer will sell at least 4000 PC's. $P(X \geq 4000)$

(iii) What is the probability that Alfa computer will exactly sell 2500 PC's. $P(X = 2500)$

Solution: Let X be the number of PC's sold daily at Alfa Computer, then X follow uniform distribution over the (2000, 5000). Thus, its pdf is

$$f(x) = \begin{cases} \dfrac{1}{3000}; & 2000 < x < 5000 \\ 0 \;; & otherwise \end{cases}$$

CDF is

$$F(x) = \begin{cases} 0 \;; & if\ x \leq 2000 \\ \dfrac{x-2000}{3000} \;; & if\ 2000 < x < 5000 \\ 1 & if\ x \geq 5000 \end{cases}$$

(i) Required probability $= P(2500 < X < 3000) = \int_{2500}^{3000} f(x)dx$

By PDF

$$= \int_{2500}^{3000} \frac{1}{3000} dx$$

$$= \frac{500}{3000}$$

$$= \frac{1}{6}$$

PDF is

$$f(x) = \begin{cases} \dfrac{1}{3000}; & 2000 < x < 5000 \\ 0 \;; & otherwise \end{cases}$$

(i) Required probability = $P(2500 < X < 3000)$

By PDF

$$= \int_{2500}^{3000} \frac{1}{3000} dx$$

$$= \frac{500}{3000}$$

$$= \frac{1}{6}$$

BY CDF

$$P(2500 < X < 3000)$$

$$= F(3000) - F(2500)$$

$$= \frac{1000}{3000} - \frac{500}{3000}$$

CDF is

$$F(x) = \begin{cases} 0 & ; \; if \; x \le 2000 \\ \dfrac{x - 2000}{3000} & ; \; if \; 2000 < x < 5000 \\ 1 & \; if \; x \ge 5000 \end{cases}$$

(ii) Required probability = $P(X > 4000)$

By PDF

$$= \int_{4000}^{\infty} f(x) dx = \int_{4000}^{5000} \frac{1}{3000} + \int_{5000}^{\infty} 0$$

$$= \int_{4000}^{5000} \frac{1}{3000} dx$$

$$= \frac{1000}{3000}$$

$$= \frac{1}{3}$$

PDF is

$$f(x) = \begin{cases} \dfrac{1}{3000}; & 2000 < x < 5000 \\ 0 \; ; & otherwise \end{cases}$$

(ii) Required probability = $P(X > 4000)$

By PDF

$$= \int_{4000}^{\infty} f(x)dx$$

$$= \int_{4000}^{5000} \frac{1}{3000} dx$$

$$= \frac{1000}{3000}$$

$$= \frac{1}{3}$$

By CDF

$$P(X > 4000)$$

$$= 1 - P(X \leq 4000)$$

$$= 1 - F(4000)$$

$$= 1 - \frac{2000}{3000}$$

$$= \frac{1}{3}$$

(iii) Required probability = $P(X = 2500)$ OR $P(2500 \leq X \leq 2500)$

$$= 0$$

$$= \int_{2500}^{2500} f(x)dx = 0$$

Because X is a continuous probability distribution and in it, probability at a single point is **ZERO Always**.

Example: The cumulative distribution function of a random variable X is given by

$$F(x) = \begin{cases} 0 & ; if \ x \le -\pi \\ \dfrac{x+\pi}{2\pi} & ; \ if -\pi < x < \pi \\ 1 & ; \ if \ x > \pi \end{cases}$$

Find

(i) The probability density function of X. (ii) $P\left(-\frac{\pi}{2} < X < \frac{\pi}{2}\right)$

Solution:

The pdf is computed by taking derivative of cdf F(x), i.e.,

Pdf ·

$$f(x) = \frac{d}{dx} F(x) \Rightarrow \boxed{f(x) = \begin{cases} \dfrac{1}{2\pi}; & -\pi < x < \pi \\ 0 \ ; & otherwise \end{cases}}$$

Required probability = $P\left(-\frac{\pi}{2} < X < \frac{\pi}{2}\right)$

By PDF

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)dx$$

$$= \int_{-\pi/2}^{\pi/2} \frac{1}{2\pi} dx$$

$$= \frac{1}{2\pi}\left(\frac{\pi}{2} + \frac{\pi}{2}\right)$$

$$= \frac{1}{2}$$

By CDF

$$P\left(-\frac{\pi}{2} < X < \frac{\pi}{2}\right)$$

$$= F\left(\frac{\pi}{2}\right) - F\left(-\frac{\pi}{2}\right)$$

$$= \frac{\frac{\pi}{2} + \pi}{2\pi} - \frac{-\frac{\pi}{2} + \pi}{2\pi}$$

$$= \frac{3}{4} - \frac{1}{4}$$

$$= \frac{1}{2} \checkmark$$

Example: If X is uniform random variable over the interval (a,b). Find the mean and variance of X.

Solution: X is uniform random variable over the interval (a,b). So its pdf is

$$f(x) = \begin{cases} \frac{1}{b-a} ; & a < x < b \\ 0 ; & otherwise \end{cases}$$

Solution: X is uniform random variable over the interval (a,b). So its pdf is

Mean of X = $E(X)$

$$= \int_{-\infty}^{\infty} x\, f(x) dx$$

$$= \int_a^b \frac{x}{b-a} dx$$

$$= \frac{1}{b-a}\left(\frac{x^2}{2}\right)_a^b$$

$$= \frac{1}{b-a}\left(\frac{b^2-a^2}{2}\right)$$

$$= \frac{b+a}{2}$$

Variance of X = $E(X^2) - [E(X)]^2$

$$= \int_{-\infty}^{\infty} x^2 f(x) dx - \left(\frac{b+a}{2}\right)^2$$

$$= \int_a^b \frac{x^2}{b-a} dx - \frac{b^2+a^2+2ab}{4}$$

$$= \frac{b^3-a^3}{3(b-a)} - \frac{b^2+a^2+2ab}{4}$$

$$= \frac{b^2+ab+a^2}{3} - \frac{b^2+a^2+2ab}{4}$$

$$= \frac{b^2-2ab+a^2}{12}$$

$$= \frac{(b-a)^2}{12}$$

Example: If X is uniform random variable in [-1, 1]. Find the probability density function, mean and variance of X.

Solution: X is uniform random variable in [-1, 1], so the probability density function is given as

$$f(x) = \begin{cases} \frac{1}{2} & ; -1 < x < 1 \\ 0 & ; otherwise \end{cases} \quad (i)$$

Mean $= \frac{b+a}{2} = \frac{1-1}{2} = 0$

Variance $= \frac{(b-a)^2}{12} = \frac{(1+1)^2}{12} = \frac{1}{3}$