Example: The daily consumption of milk in a city in excess of 20000 galllons is approximately distributed as a Gamma distribution with parameters  $\beta=\frac{1}{10000}$  and  $\alpha=2$ . The city has a daily stock of 30000 gallons. What is the probability that the stock is insufficient on a particular day.

Solution: Let X denotes the daily consumption of milk (in litres) in a city

Then the random variable Y = X - 20000 has a Gamma distribution with pdf

$$f(y) \le \frac{1}{(10000)^2 \Gamma(2)} y^{2-1} e^{-\frac{y}{10000}}; \quad y > 0$$

Since the city has a daily stock of 30000 gallons, the probability that the city is insufficient for a (single) day is

$$P(X > 30000) = P(Y > 10000) = \iint f(y) dy$$

$$P(X > 30000) = P(Y > 10000)$$

$$= \int_{10000}^{\infty} \frac{ye^{-\frac{y}{10000}}}{(10000)^2} dy$$

$$= \int_{1}^{\infty} ze^{-z} dz$$

$$= (-ze^{-z})_{1}^{\infty} - (e^{-z})_{1}^{\infty}$$

$$= \frac{2}{e}$$

$$Put z = \frac{y}{10000}$$

$$\Rightarrow dz = \frac{dy}{10000}$$



**Note:** Here  $\alpha = 2$  so the integration is easily done.

However, for general values of  $\alpha$ ,  $\beta$  the integral is evaluated by using table of incomplete Gamma integral of the form  $\int_0^\infty \frac{x^{n-1}e^{-x}}{\Gamma(n)} dx$ , which has been tabulated for different values of lpha,eta .



Example: Let X be a random variable with the probability density function (P-d-{-)

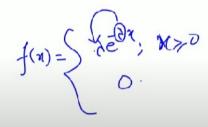
$$f(x) = \begin{cases} a e^{-\frac{x}{3}}; & x > 0\\ 0 & ; otherwise \end{cases}$$

(ii) 
$$P(X > 3)$$

Find (i) Value of 
$$a$$
 (ii)  $P(X > 3)$  (iii)  $P(1 < X < 4)$ 

Solution:

$$\Rightarrow$$
  $\alpha = \frac{1}{3}$ 





$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\Rightarrow \int_{0}^{\infty} a e^{-x/3} dx = 1$$

$$\Rightarrow a \left(\frac{e^{-\frac{x}{3}}}{\frac{-1}{3}}\right)_{0}^{\infty} = 1$$

$$\Rightarrow -3a(0-1) = 1$$

$$\Rightarrow a = \frac{1}{3}$$

$$f(x) = \begin{cases} a e^{-\frac{x}{3}}; & x > 0\\ 0 & ; otherwise \end{cases}$$



(ii) 
$$P(X > 3)$$

1st method by P.D.F.

$$= \int_{3}^{\infty} \frac{1}{3} e^{-x/3} dx$$

$$= \frac{1}{3} \left( \frac{e^{-\frac{x}{3}}}{-\frac{1}{3}} \right)_{3}^{\infty}$$

$$= -1(0 - e^{-1})$$

$$= \frac{1}{e}$$

2nd method by using CDF

$$P(X > 3)$$
  
= 1 -  $P(X \le 3)$   
= 1 -  $F(3)$   
= 1 -  $[1 - e^{-\lambda(3)}]$   
=  $e^{-1}$ 



(iii) 
$$P(1 < X < 4)$$

1st method by using P.D.F.

$$= \int_{1}^{4} \frac{1}{3} e^{-x/3} dx$$

$$= \frac{1}{3} \left( \frac{e^{-\frac{x}{3}}}{-\frac{1}{3}} \right)_{1}^{4}$$

$$= -1 \left( e^{-4/3} - e^{-1/3} \right)$$

$$= e^{-1/3} - e^{-4/3}$$

## 2<sup>nd</sup> method by using CDF

$$P(1 < X < 4)$$

$$= F(4) - F(1)$$

$$= [1 - e^{-\lambda(4)}] - [1 - e^{-\lambda(1)}]$$

$$= e^{-1/3} - e^{-4/3} \text{ Ags}$$



Example: The time (in hours) required to repair a machine is exponential distributed with parameter 1/3. What is the probability that the repair time exceeds 3 hours?

Solution: Required probability = P(X > 3)

$$= \int_{3}^{\infty} \frac{1}{3} e^{-x/3} dx$$

$$= \frac{1}{3} \left( \frac{e^{-\frac{x}{3}}}{-\frac{1}{3}} \right)_{3}^{\infty}$$

$$= -1(0 - e^{-1})$$

$$= \frac{1}{e}$$

2nd method by using CDF

$$P(X > 3)$$
= 1 - P(X \le 3)
= 1 - F(3)
= 1 - [1 - e^{\lambda(3)}]
= e^{-1}

