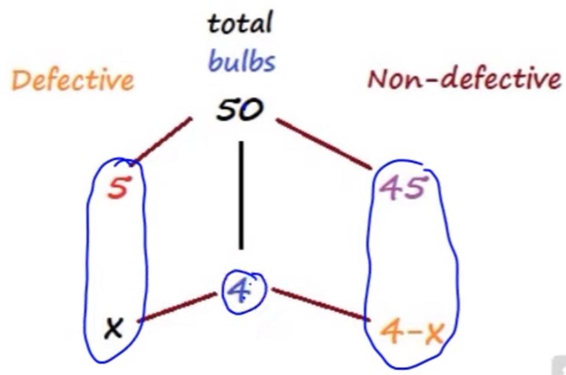
Example: A crate contains 50 light bulbs of which 5 are defective and 45 are not. A Quality Control Inspector randomly samples 4 bulbs without replacement. Let X = the number of defective bulbs selected. Find the probability mass function, f(x), of the discrete random variable X.

Solution: The p.m.f. is

$$P(X=x)=\frac{{}^{5}C_{x}\times {}^{45}C_{4-x}}{{}^{50}C_{4}}$$

$$x=0,1,2,3,4$$

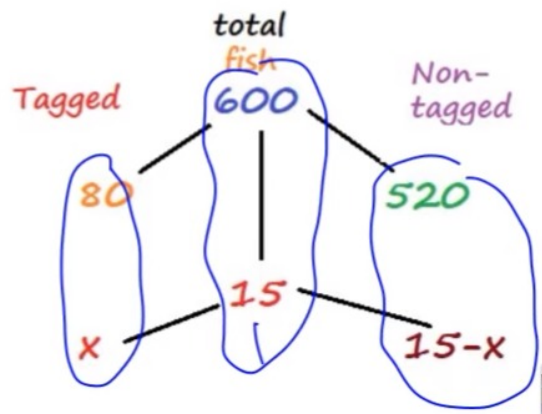


Example: A lake contains 600 fish, eighty (80) of which have been tagged by scientists. A researcher randomly catches 15 fish from the lake. Find a formula for the probability mass function of X, the number of fish in the researcher's sample which are tagged.

Solution: The p.m.f. is

$$P(X=x)=\frac{{}^{80}C_{x}\times {}^{520}C_{15-x}}{{}^{600}C_{15}}$$

$$x=0,1,2,\dots,15$$

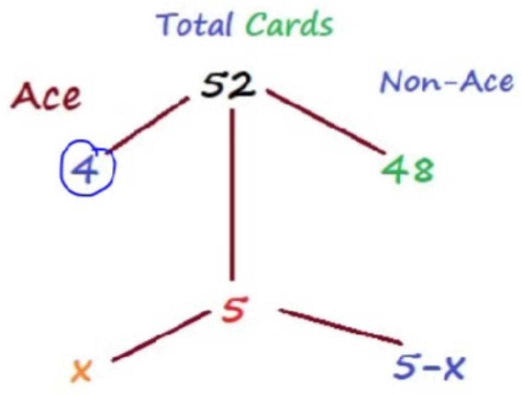


Example: Let the random variable X denote the number of aces in a five-card hand dealt from a standard 52-card deck. Find a formula for the probability mass function of X.

Solution: The p.m.f. is

$$P(X = x) = \frac{{}^{4}C_x \times {}^{48}C_{5-x}}{{}^{52}C_5}$$

$$x = 0,1,2,3,4$$

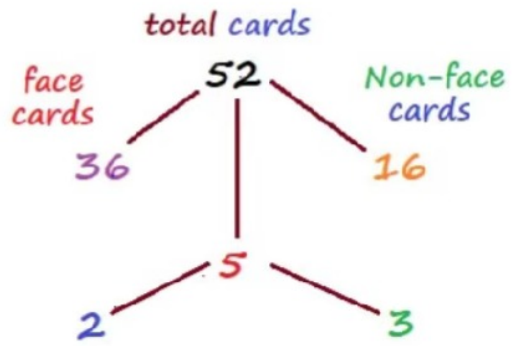


Example: Emma likes to play cards. She draws 5 cards from a pack of 52 cards. What is the probability of that from the 5 cards drawn Emma draws only 2 face cards?

Solution: Let X denotes the number of face cards.

The required probability is

$$P(X = 2) = \frac{{}^{36}C_2 \times {}^{16}C_3}{{}^{52}C_5}$$



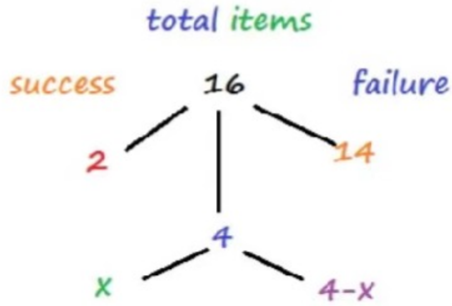
Example: Find the Expectation of a Hypergeometric Distribution such that the probability that a 4-trial hypergeometric experiment results in exactly 2 successes, when the population consists of 16 items.

Solution: Let X denotes the number of successes.

Expected value of X = $\frac{nM}{N}$

$= \frac{2 \times 4}{16}$

= 0.5

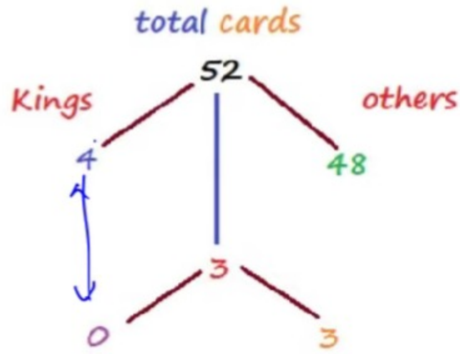


Example: Consider Harish draws 3 cards from a pack of 52 cards. What is the probability of getting no kings?

Solution: Let X represent the number of the king.

The required probability is

$$\boxed{\begin{aligned} P(X=0) &= \frac{{}^{4}C_0 \times {}^{48}C_3}{{}^{52}C_3} \\ &= 0.7826 \end{aligned}}$$



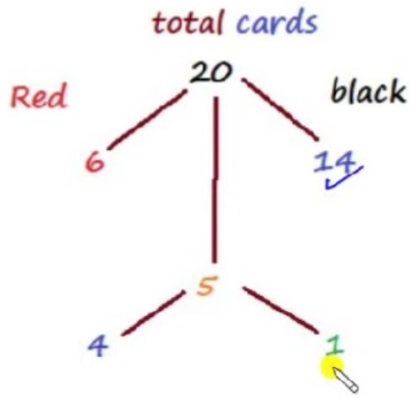
Example: A deck of cards contains 20 cards: 6 red cards and 14 black cards. 5 cards are drawn randomly *without replacement*. What is the probability that exactly 4 red cards are drawn?

Solution: Let X represent the number of red cards.

The required probability is

$$P(X = 4) = \frac{{}^{6}C_4 \times {}^{14}C_1}{{}^{20}C_5}$$

$$= 0.0135$$



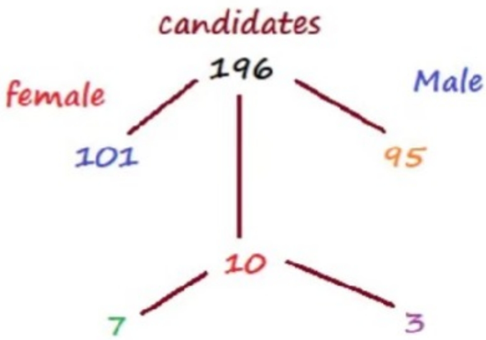
Example: A small voting district has 101 female voters and 95 male voters. A random sample of 10 voters is drawn. What is the probability exactly 7 of the voters will be female?

Solution: Let X represent the number of female candidates.

The required probability is

$$P(X = 7) = \frac{{}^{101}C_7 \times {}^{95}C_3}{{}^{196}C_{10}}$$

$$= 0.130$$



Example: Suppose that a lot contains 100 items, 5 of which do not confirm to requirements. If 10 items are selected at random without replacement, then the probability of finding one or fewer nonconfirming items in the sample.

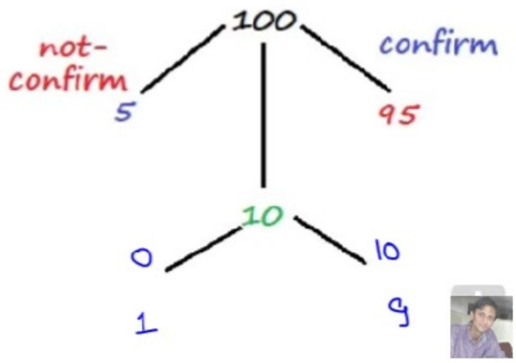
Solution: Let X represents the non-confirming items in the sample.

The required probability is

$$P(X \leq 1) = P(X = 0) + P(X = 1)$$

$$= \frac{{}^{5}C_0 \times {}^{95}C_{10}}{{}^{100}C_{10}} + \frac{{}^{5}C_1 \times {}^{95}C_9}{{}^{100}C_{10}}$$

$=$

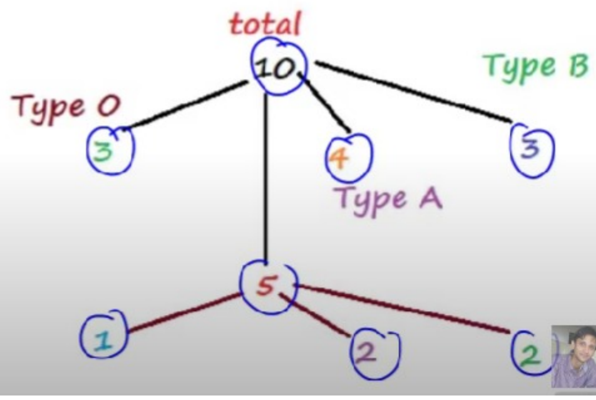


Example: A group of 10 individuals are used for a biological case with the following blood types: Type O - 3 people, Type A - 4 people, Type B - 3 peoples. What is the probability that a random sample of 5 will contain 1 Type O, 2 type A, 2 type B?

Solution: Required probability is

$$= \frac{{}^{3}C_1 \times {}^{4}C_2 \times {}^{3}C_2}{{}^{10}C_5}$$

= 3/14



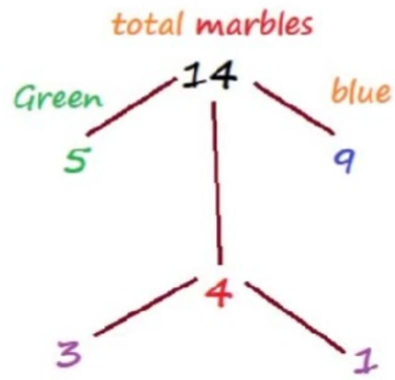
Example: A house contains 5 green marbles and 9 blue marbles. 4 marbles are drawn randomly without replacement. Calculate the probability of getting 3 green marbles.

Solution: Let X represent the variable for the green marbles

The required probability is

$$P(X = 3) = \frac{{}^{5}C_3 \times {}^{9}C_1}{{}^{14}C_4}$$

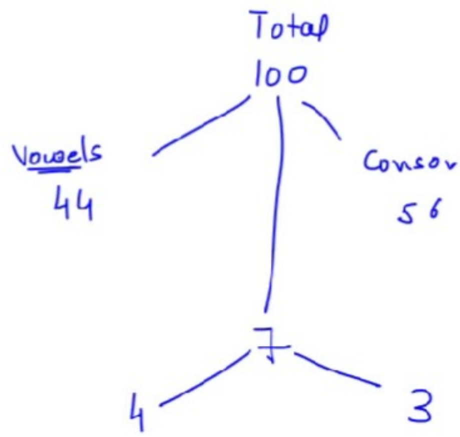
=



Example: A bag contains letter tiles. Forty-four of the tiles are vowels, and 56 are consonants. Seven tiles are picked at random. You want to know the probability that four of the seven tiles are vowels.

Let X — vowels

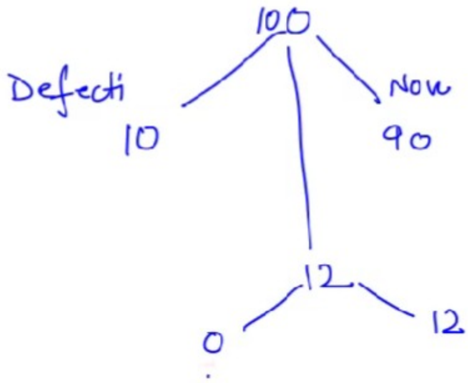
$$P(X=4) = \frac{{}^{44}C_4 \times {}^{56}C_3}{{}^{100}C}$$



Example: Suppose a shipment of 100 DVD players is known to have ten defective players. An inspector randomly chooses 12 for inspection. He is interested in determining the probability that, among the 12 players, at most two are defective.

Let X be def. DVD.

$P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$



Example: A gross of eggs contains 144 eggs. A particular gross is known to have 12 cracked eggs. An inspector randomly chooses 15 for inspection. She wants to know the probability that, among the 15, at most three are cracked.

Let X be crack eggs

$P(X \le 3) = P(X=0) + \cdots + P(X=3)$