

## Subarrays

→ Continuous portion of the array!

→ Single element ✓

→ Whole array ✓

→ empty part X

A : 

|   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 2 | 4 | 6 | 2 | 9 | 3 | 2 | 5 | 7 |

indices : [ 0, 1, 2, 3 ] ✓

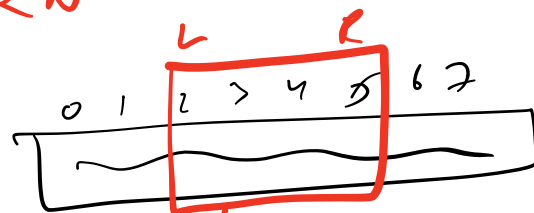
— : [ 1, 2, 4, 7 ] X

— : [ 4 ] ✓

④ Uniquely identify a S.A using [st, end] or [L, R]

$$0 \leq st \leq end < N$$

$$\text{or } 0 \leq L \leq R < N$$

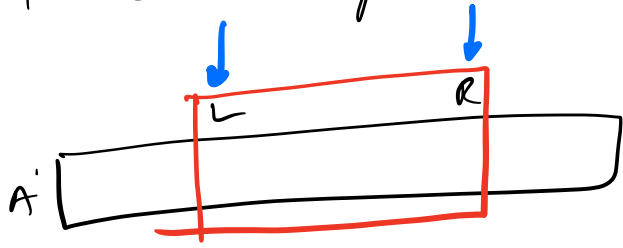


# elements in SA →  $R - L + 1$

[2, 5]

Q Given an array. Given  $[L, R]$ , print the  $SA[L, R]$ .

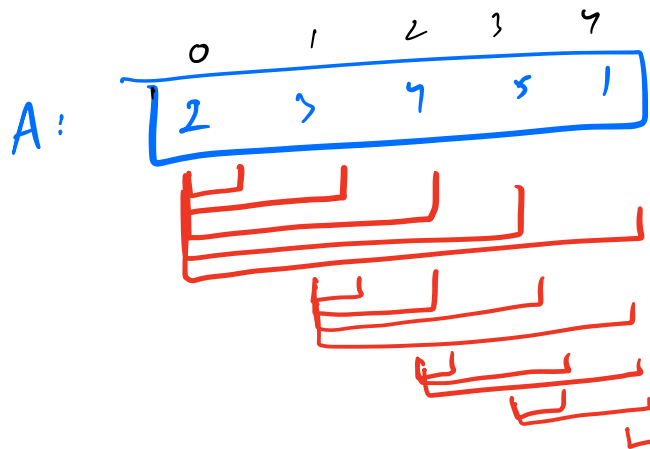
```
f(i = L; i <= R; i++) {  
    print(A[i]);  
}
```



TC:  $O(R-L)$

~~TC:  $O(N)$~~

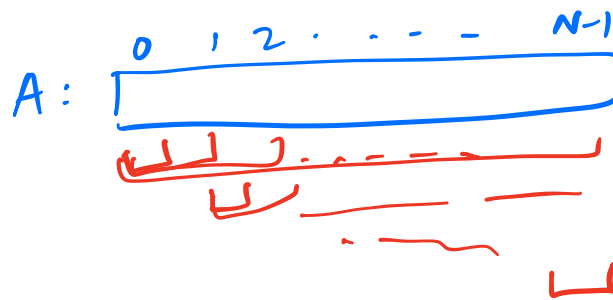
Q Given an array of size  $N$ .  
Find # SA.



$N = 5$

$$5 + 4 + 3 + 2 + 1 = 15$$

Generalize :



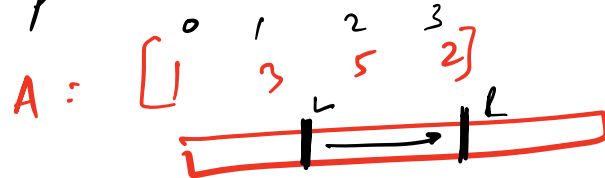
$$N + N-1 + \dots + 1$$

$$\# \text{ Sub Array} = \frac{N(N+1)}{2} \rightarrow O(N^2)$$

Q Given an arr, print all the Sub-array!

[L, R] → S.A

- [0, 0] → 1
- [0, 1] → 1, 3
- [0, 2] → 1, 3, 5
- [0, 3] → 1, 3, 5, 2
- [1, 1] → 3
- [1, 2] → 3, 5
- [1, 3] → 3, 5, 2
- [2, 2] → 5
- [2, 3] → 5, 2
- [3, 3] → 2



```

f ( L = 0; L < N; L++ ) {
    f ( R = L; R < N; R++ ) {
        // [L, R]
        f ( i = L; i <= R; i++ ) {
            print ( A[i] );
        }
        print ( New Line );
    }
}

```

$$\frac{N(N+1)}{2} \times \frac{N}{2}$$

$$\# \text{ TC: } O(N^3)$$

Q Given an array - Print all the Subarray Sums.

$[L, R] \rightarrow$  S.A Sum

$[0, 0] \rightarrow 1 \rightarrow 1$   
 $[0, 1] \rightarrow 1, 3 \rightarrow 4$   
 $[0, 2] \rightarrow 1, 3, 5 \rightarrow 9$   
 $[0, 3] \rightarrow 1, 3, 5, 2 \rightarrow 11$   
 $[1, 1] \rightarrow 3 \rightarrow 3$   
 $[1, 2] \rightarrow 3, 5 \rightarrow 8$   
 $[1, 3] \rightarrow 3, 5, 2 \rightarrow 10$   
 $[2, 2] \rightarrow 5 \rightarrow 5$   
 $[2, 3] \rightarrow 5, 2 \rightarrow 7$   
 $[3, 3] \rightarrow 2 \rightarrow 2$

$A = [1^0, 3^1, 5^2, 2^3]$

$(L = 0; L < N; L++) \{$

$\{ R = L; R < N; R++ \} \{$

//  $[L, R]$   $sum = 0$

$\{ i = L; i \leq R; i++ \}$

$sum += A[i];$

$\}$

$print(sum);$   
 $print(New Line);$

$\}$

$\}$

$TC = O(N^3)$

II) Use PS →

1. Build the PS Array! →

| TC     | SC     |
|--------|--------|
| $O(N)$ | $O(N)$ |

2. 
$$\begin{aligned} & \text{for } (L = 0; L < N; L++) \{ \\ & \quad \text{for } (R = L; R < N; R++) \{ \end{aligned}$$

→  $\frac{TC}{O(N^2)}$

$$\begin{aligned} & \quad \quad // [L, R] \\ & \quad \quad \text{if } (L == 0) \{ \\ & \quad \quad \quad \text{print}(PS[R]); \\ & \quad \quad \} \\ & \quad \quad \text{else } \{ \\ & \quad \quad \quad \text{print}(PS[R] - PS[L-1]); \\ & \quad \quad \} \\ & \quad \text{print}(\text{new line}); \end{aligned}$$

$\frac{SC}{O(1)}$

}

}

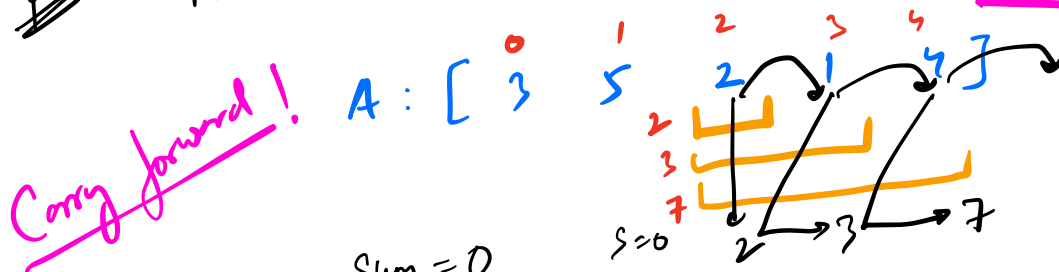
$$TC = O(N + N^2)$$

$$\boxed{TC = O(N^2)}$$

$$\boxed{SC = O(N)}$$

PS[]

Print all the S.A. sums start at index 2!



```
f(R = 2; R < N; R++) {  
    Sum += A[R];  
    print(Sum);  
}
```

⊛ Solving the prev problem

```
f(L = 0; L < N; L++) {  
    Sum = 0  
    f(R = L; R < N; R++) {  
        Sum += A[R];  
        print(Sum);  
    }  
}
```

TC =  $O(N^2)$

SC =  $O(1)$

~~Q~~ Given an array.  
Find the sum of all the sub-array sums!

I

ANS = 0

{ (L = 0; L < N; L++) }

Sum = 0

{ (R = L; R < N; R++) }

Sum += A[R];

ANS += Sum

}

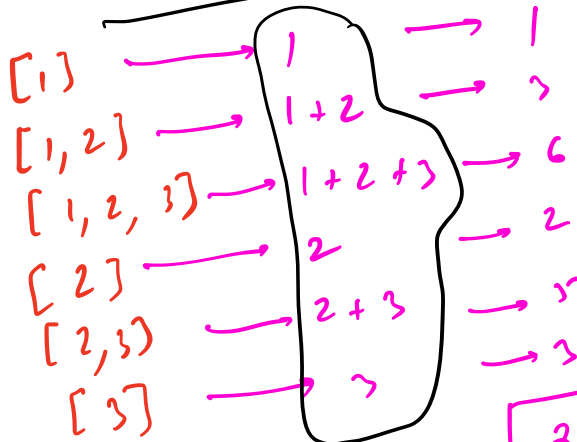
}

// print ANS

~~TC = O(N<sup>2</sup>)~~

~~SC = O(1)~~

A: [ <sup>0</sup>1, <sup>1</sup>2, <sup>2</sup>3 ]

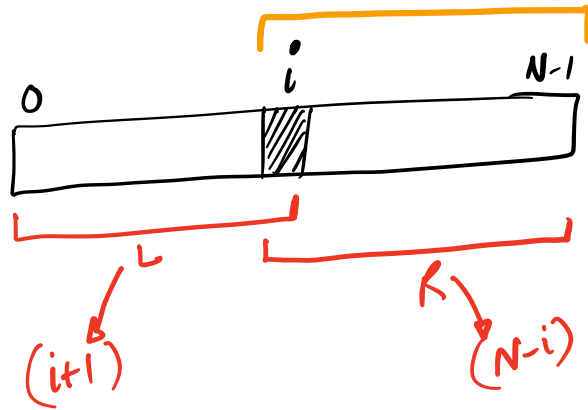


A[i] Contribution

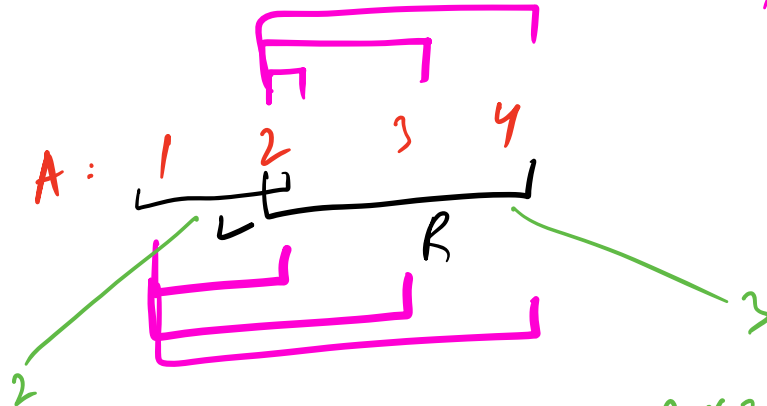
1 x 3  
+ 2 x 4  
+ 3 x 3

20

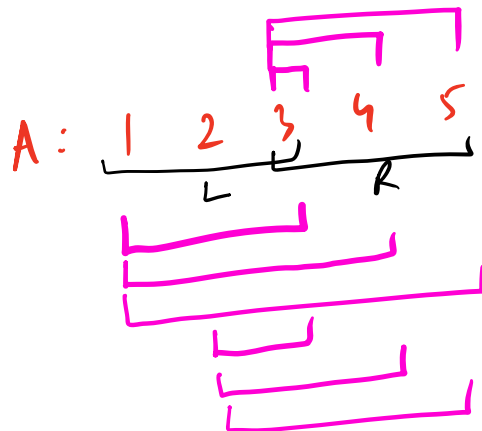
Idea: find contribution of every element!  
 → find the # SAs  $i^{\text{th}}$  element is a part of!



# SAs containing  $i^{\text{th}}$  element =  $(i+1)(N-i)$



$$2 \times 3 = 6!$$



$$3 \times 3 = 9!$$



$$ANS = \sum_{i=0}^{N-1} A[i] \times (i+1) (N-i)$$

ANS = 0  
{ ( i = 0; i < N; i++ ) {  
    ANS += A[i] \* (i+1) \* (N-i);  
}  
print(ANS)

TC = O(N)

SC = O(1)