Decimal Number System → {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

Base → 10

$(345)_{10}$ ⟶ 345

- 345 ⟶ $3 \times 100 + 4 \times 10 + 5 \times 1$

  $3 \times 10^2 + 4 \times 10^1 + 5 \times 10^0$

- 4 3 2 1 0

  12345 ⟶ $1 \times 10^4 + 2 \times 10^3 + 3 \times 10^2 + 4 \times 10^1 + 5 \times 10^0$

Hexadecimal →

Base → 16

10 11 12 13 14 15

{0, 1, 2 - - , 9, A, B, C, D, E, F}

5 4 3 2 1 0

$(9\,6\,A\,F\,E\,0)_{16}$

↳ $9 \times 16^5 + 6 \times 16^4 + 10 \times 16^3 + 15 \times 16^2 + 14 \times 16^1 + 0 \times 16^0$

⟶ (_____)$_{10}$

Binary Number System → {0, 1} Base → 2

BIN → DEC

$(6)_{10}$

$(110)_2$ → $1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

$4 + 2 + 0 = (6)_{10}$

$(110001)_2$ → $2^5 + 2^4 + 2^0$

$32 + 16 + 1 = (49)_{10}$

DEC → BIN

$(25)_{10}$ → $(\quad)_2$

		rem
2	25	1
2	12	0
2	6	0
2	3	1
2	1	1
	0	

1/2

$(11001)_2$

$16 + 8 + 1$

$2^4 \; 2^3 \; 2^0$

$(25)_{10} = (11001)_2$

2	45	1
2	22	0
2	11	1
2	5	1
2	2	0
2	1	1
	0	

$(101101)_2 \leftarrow (45)_{10}$

ADDITION

```
  2 1 0
0 1 1
  3 6 8
+ 4 5 6
-------
  8 2 4
```

d → s%.10
c → s/10

14 %.10 → 4
14/10 → 1

1+3+4 = 8

= 12 %.10 → 2
12/10 → 1

8 %.10 → 8
8/10 → 0

BINARY

```
  1 1 1 1 1
  1 0 1 1 1 → 23
+ 1 0 1 1   → 11
-------------
1 0 0 0 1 0   34 ✓
```

2 %.2 → 0 2/2 → 1	2 %.2 → 0 2/2 → 1
3 %.2 → 1 3/2 → 1	1 %.2 → 1 1/2 → 0

BITWISE OPERATORS

& AND | OR ^ XOR ~ NEG << Left Shift >> Right Shift

A	B	A & B	A | B	A ^ B	~A
0	0	0	0	0	1
0	1	0	1	1	1
1	0	0	1	1	0
1	1	1	1	0	0

① Bitwise Operations on numbers

5 & 6 → 101 & 110 → 100 → 4

```
int x = 5 & 6;
```
x = 4

111111

= 63
- 2
61

20 | 45 → 10100 | 101101 → 111101 → 61

20 ^ 45 → 10100 ^ 101101 → 111001 → 57

3 2 1 0

| | | | |

$\longrightarrow \quad \underbrace{2^0 + 2^1 + 2^2 + 2^3}_{\hookrightarrow\ 2^4 - 1}$

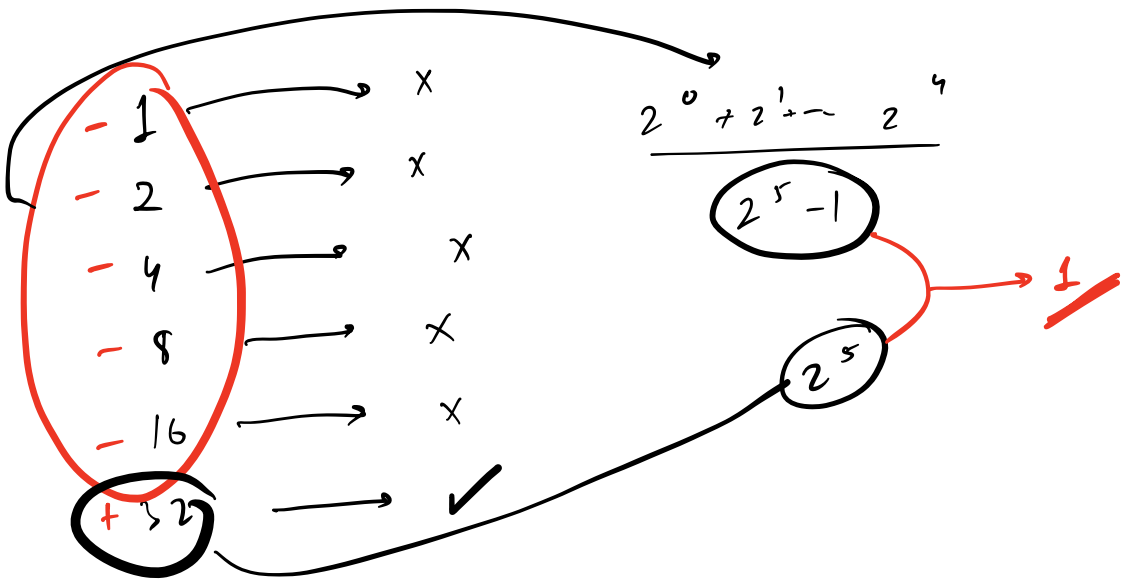
$a\left(\frac{r^n - 1}{r - 1}\right)$

5 4 3 2 1 0

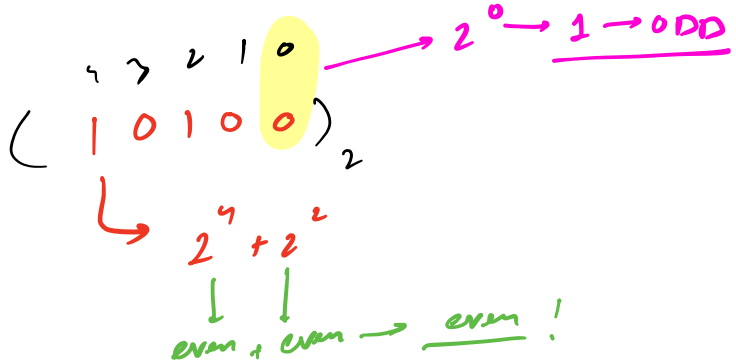
| | | | | | |

$\longrightarrow \quad \underbrace{2^0 + 2^1 + \cdots + 2^5} = 2^6 - 1 < 2^6$

$$2^0 + 2^1 + 2^2 + \cdots + 2^k < 2^{k+1}$$

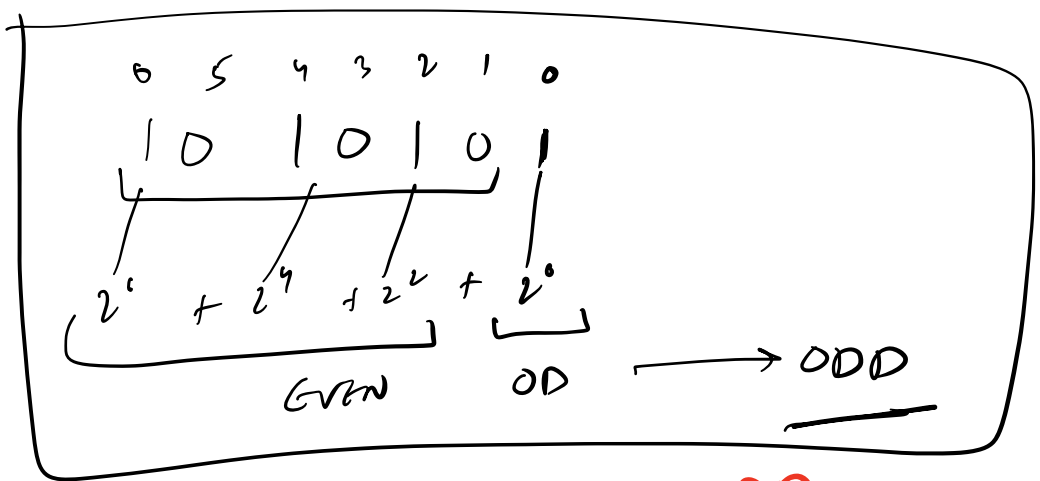


Properties →



→ If a NUM is ODD → it's 0th bit is 1 [SET]

_____ is EVEN → _____ 0 [UNSET]



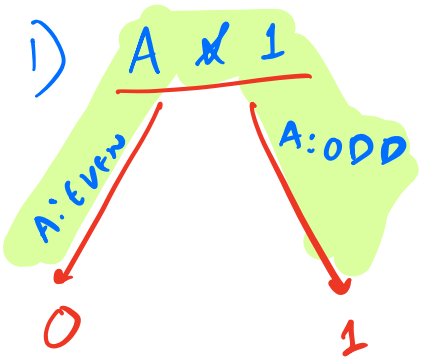
EVEN

x = 20 → 10100
16 → 1000
24 → 11000
14 → 1110
2 → 10

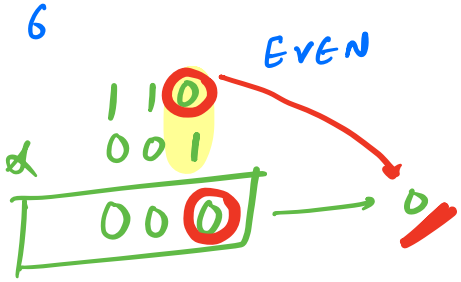
ODD

1 → 1
3 → 11
5 → 101
7 → 111
9 → 1001
11 → 1011

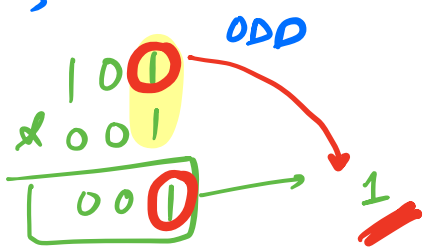
1) A & 1



A = 6



A → 5



if(A % 2 == 0)
↳ EVEN

else
↳ ODD

NOTE: PRE of BITWISE OP. is LESS

---

if ((A & 1) == 0)
↳ EVEN

else
↳ ODD

2) A & 0 → 0
3) A & A → A
4) A | 0 → A
5) A ^ 0 → A
6) A ^ A → 0

A: 1 0 1
A: 1 0 1
0 0 0 → 0

0 ^ 0 → 0
1 ^ 1 → 0

③ Commutative property →

$a \barwedge b = b \barwedge a$

$a \mid b = b \mid a$

$a \wedge b = b \wedge a$

$a \barwedge b \barwedge c = c \barwedge a \barwedge b$

$a \barwedge b \barwedge c \rightarrow$
- $a \barwedge c \barwedge b$ — XOR
- $b \barwedge a \barwedge c$ — OR
- $b \barwedge c \barwedge a$
- $c \barwedge a \barwedge b$
- $c \barwedge b \barwedge a$

④ Associative Property →

$(a \barwedge b) \barwedge c = a \barwedge (b \barwedge c)$

$(a \mid b) \mid c = a \mid (b \mid c)$

$(a \wedge b) \wedge c = a \wedge (b \wedge c)$

Q $a \wedge b \wedge a \wedge d \wedge b$

$\rightarrow (a \wedge a) \wedge (b \wedge b) \wedge d$

$(0 \wedge 0) \wedge d$

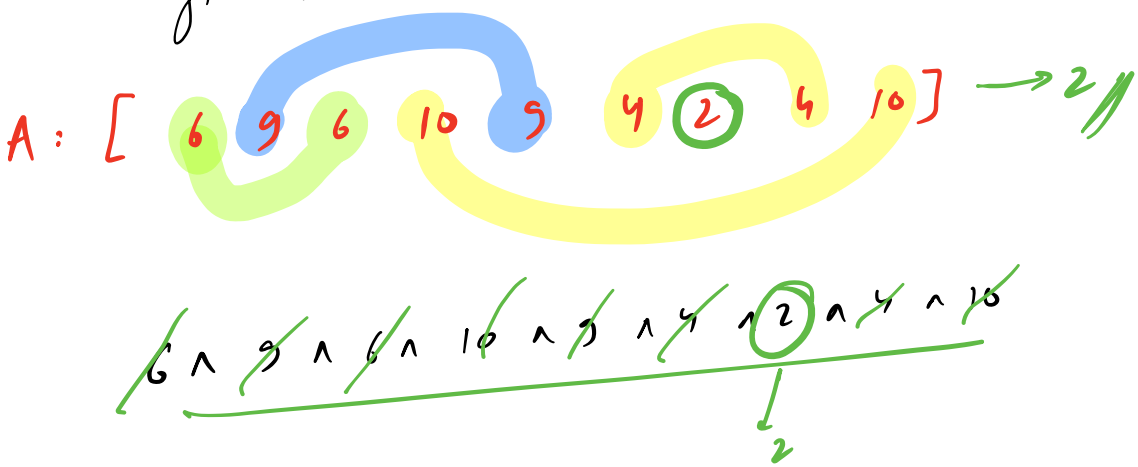
$0 \wedge d$

d //

Q Given an array,

→ every element repeats twice except one element

→ find that element!




```
x = 0;
for (i=0; i<N; i++) {
    x = x ^ A[i];
}
ret x;
```

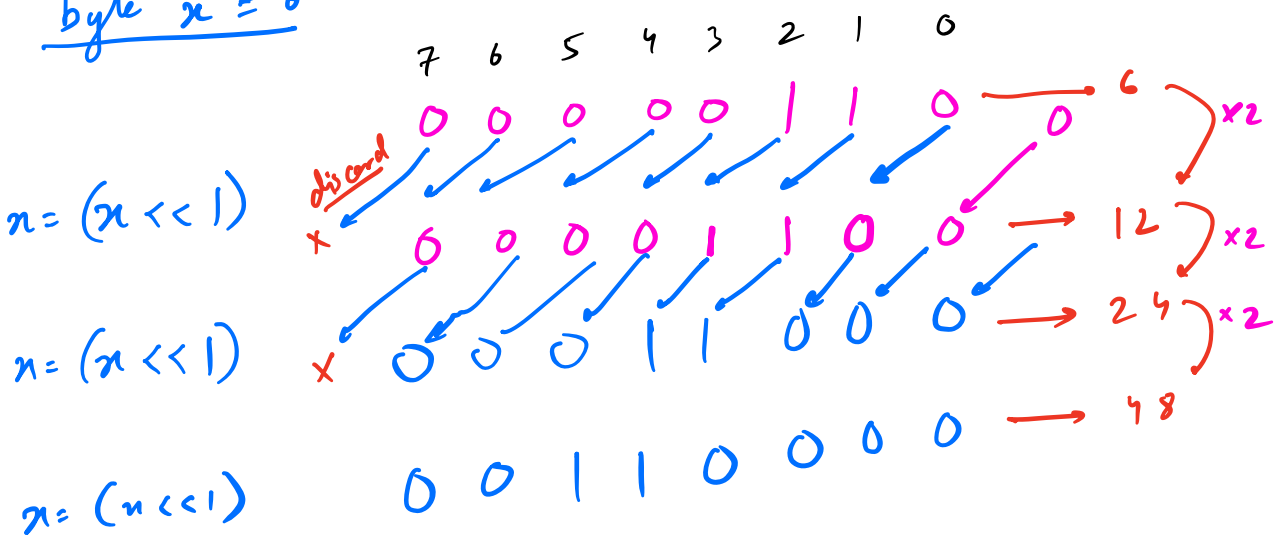
TC : O(N)

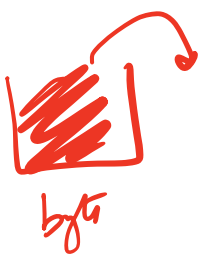
SC : O(1)

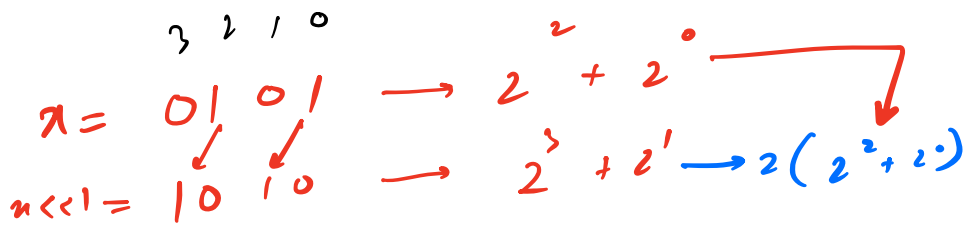
⑨ Left Shift (<<)

1 byte → 8 bits

byte x = 6







$x << 1 \longrightarrow x \times 2$

$x << 2 \longrightarrow x \times 2^2$

$x << 3 \longrightarrow x \times 2^3$

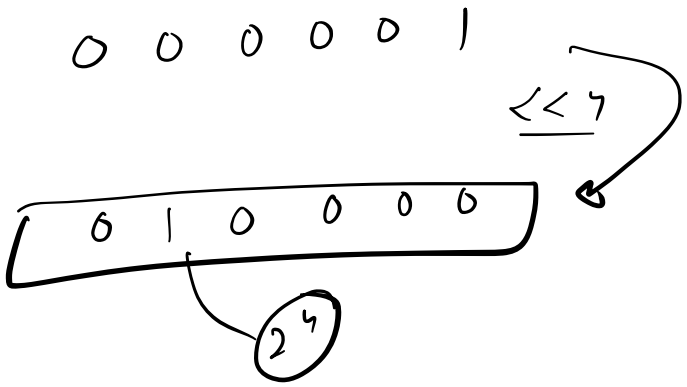
$x << K \longrightarrow x \times 2^K$

$1 << K \longrightarrow 2^K$

```
x = (1<<20);
```

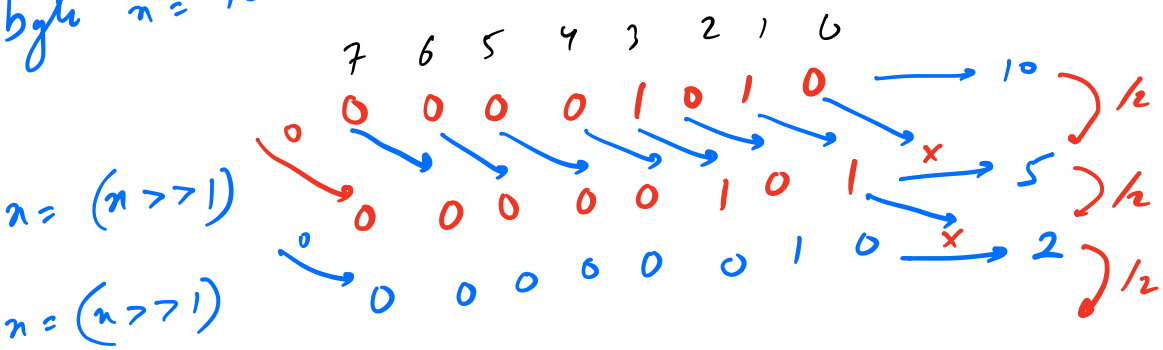
2^{20}

int n = (1 << 4); ⟶ 2^4



Right Shift (>>)

byte n = 10



$(n >> 1) \longrightarrow \lfloor n/2 \rfloor$

$(n >> 2) \longrightarrow \lfloor n/2^2 \rfloor$

$(n >> 3) \longrightarrow \lfloor n/2^3 \rfloor$

$n >> k \longrightarrow \lfloor n/2^k \rfloor$