Range:

int : $[-2 \times 10^9, 2 \times 10^9)$

long : $[-9 \times 10^{18}, 9 \times 10^{18}]$

⊙ Modular Arithmetic → % → modulo

n % a → Remainder when n is divided by a.

10 % 3 → 1

3 → quotient

3 ⟌ 10 → Dividend

−9

1 → Remainder

divisor

13 % 5 → 3

5 ⟌ 13 (2)

−10

3

10 = 3 × 3 + 1

13 = 5 × 2 + 3

Dividend = divisor × quotient + remainder

remainder = Dividend − divisor × quotient

32 − 5 × 6

= 2

32 % 5

$17 \% 4 = 17 - 4 \times 4$

$= 1$

Remainder = dividend − Greatest multiple of divisor <= dividend

0 <= remainder < divisor

a % b → [0, b-1]

$32 \% 4 \rightarrow 0$

$12 \% 3 \rightarrow 0$

$$
\begin{array}{r|l}
 & 8 \\
\hline
4 & 32 \\
 & -32 \\
\hline
 & 0
\end{array}
$$

if a % b == 0

→ a is a multiple of b

⊙ 150 % 11 ⟶ 150 − greatest multiple of 11 <= 150

150 − 143 = 7 //

⊙ 100 % 7 ⟶ 100 − greatest multiple of 7 <= 100

100 − 98

= 2 //

⊙ 12 % 15 ⟶ 12 //

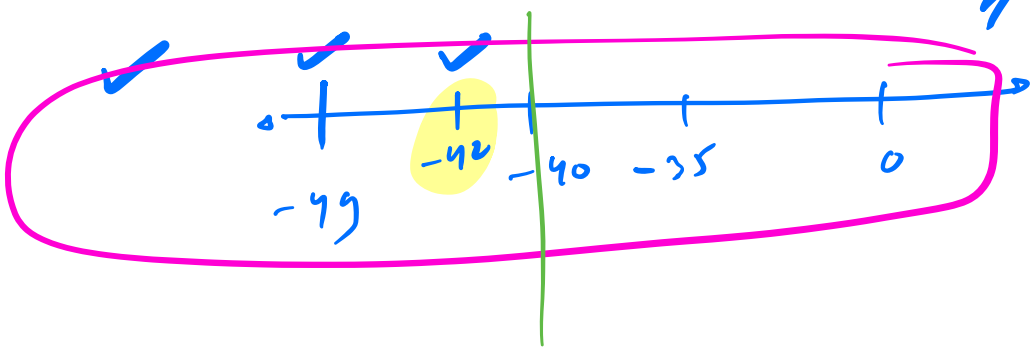
```
    0
15 ) 12
   - 0
    12
```

a % b ⟶ a : a < b

① $-40 \% 7 = -40 -$ greatest multiple of 7 $<= -40$

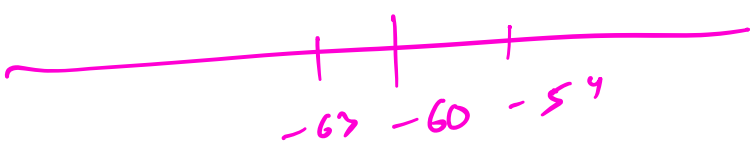
$[0, 6]$

$-40 - (-42) \rightarrow -40 + 42 = 2$



② $-60 \% 9 \longrightarrow -60 -$ greatest multiple of $9 <= -60$

$-60 - (-63) = 3$



C++ / Java

$-40 \% 7 \longrightarrow -5$ (+7) $\rightarrow 2$

$-60 \% 9 \longrightarrow -6$ (+9) $\rightarrow 3$

python

$-40 \% 7 \longrightarrow 2$

$-60 \% 9 \longrightarrow 3$

C++/- -

36 % 7 ⟶ 36 − greatest multiple of 7 <= 36

$7 \times x <= 36$

$x <= 36/7$

5. —

$36 - (36/7) \times 7$ — int division

$36 - 5 \times 7$

$36 - 35 = 1$ //

~~a % b = a − (a/b) × b~~

$40 \% 9 = 40 - (40/9) \times 9$

$40 - 4 \times 9$

$= 40 - 36 = 4$ //

$-40 \% 7 = -40 - \left(\frac{-40}{7}\right) \times 7$

$-40 - (-5) \times 7$

$-40 + 35 = -5$

Python

$36 \% 7 \longrightarrow 36 - \left(\frac{36}{7}\right) \times 7$

$\left(\frac{36}{7}\right) \rightarrow 5$

$36 - \text{floor}\left(\frac{36}{7}\right) \times 7$

$\text{floor}\left(\frac{36}{7}\right) \rightarrow 5$

$36 - 5 \times 7$

$36 - 35 = 1$

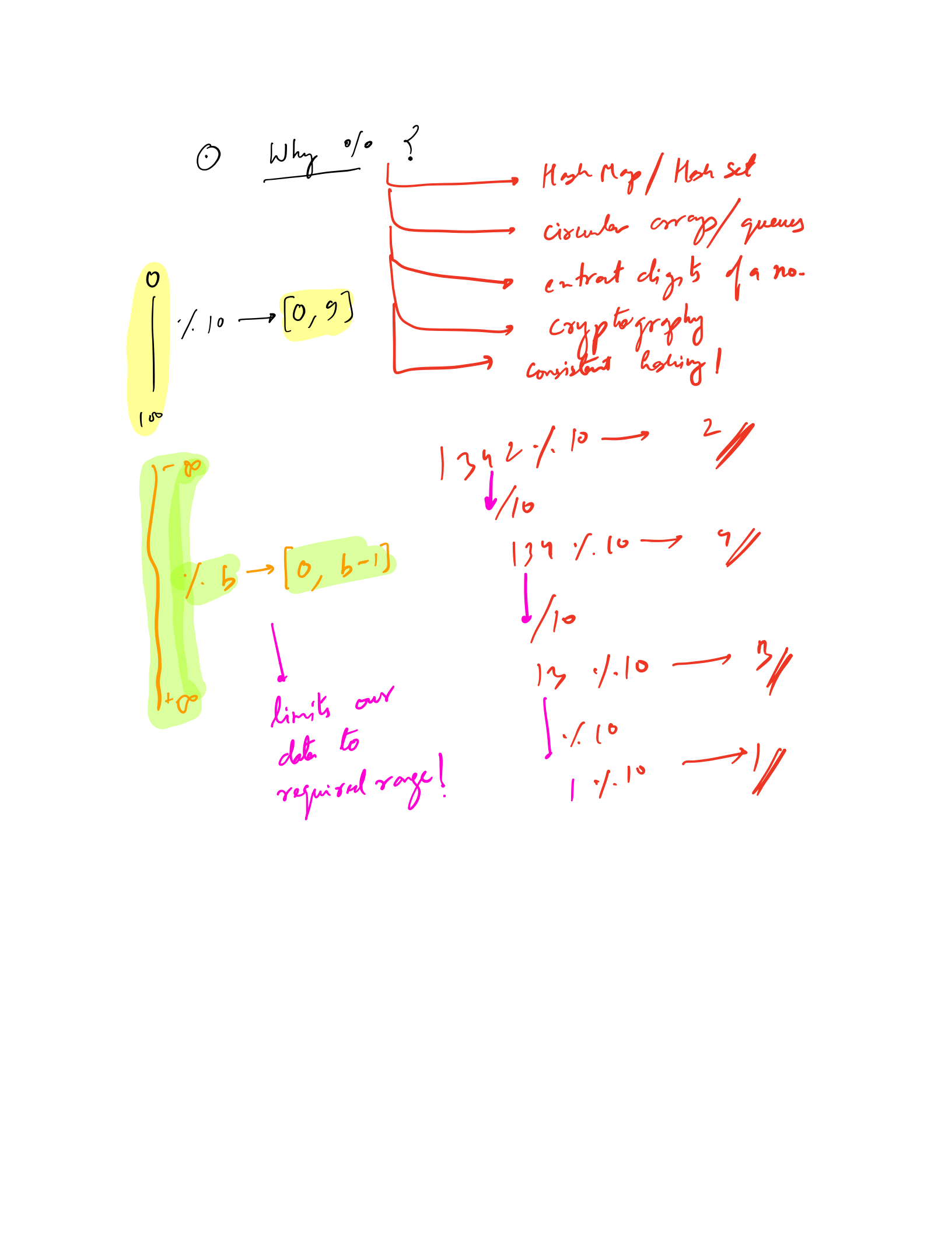
floor (x) → the greatest integer <= x

$-40 \% 7 \longrightarrow -40 - \text{floor}\left(\frac{-40}{7}\right) \times 7$

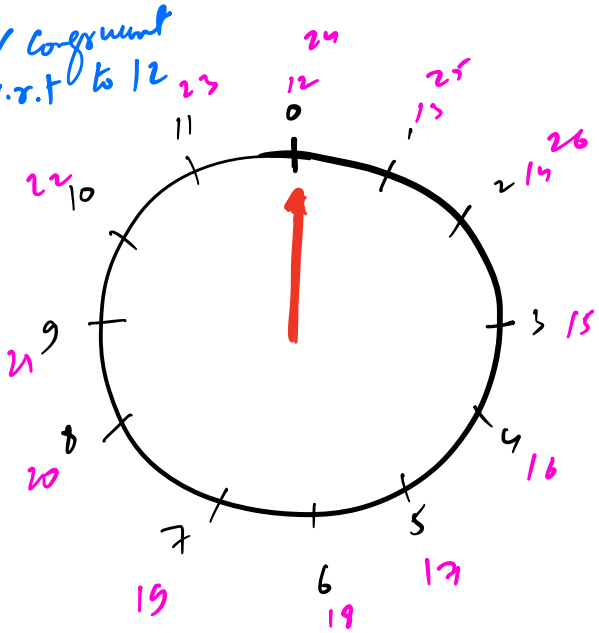
$\text{floor}\left(\frac{-40}{7}\right) \rightarrow -5.\ldots$

$-40 - (-6) \times 7$

2


Why %/ ?
Hash Map / Hash set
circular array / queues
extract digits of a no.
cryptography
consistent hashing!
0
100
%. 10 → [0, 9]
-∞
+∞
%. b → [0, b-1]
limits our data to required range!
1342 %. 10 → 2
/10
134 %. 10 → 4
/10
13 %. 10 → 3
%. 10
1 %. 10 → 1

Modular Congruent w.r.t to 12



$0 \xrightarrow{\%12} 0$

$12 \xrightarrow{\%12} 0$

$24 \xrightarrow{\%12} 0$

$1 \xrightarrow{\%12} 1$

$13 \xrightarrow{\%12} 1$

$25 \xrightarrow{\%12} 1$

$10 \xrightarrow{\%12} 10$

$5 \xrightarrow{\%12} 5$

$15 \xrightarrow{\%12} 3$

$27 \xrightarrow{\%12} 3$

$a \,\%\, b == n \,\%\, b$

$\Rightarrow$ a is modular congruent to n w.r.t b

① Modular Arithmetic

$(40 + 37) \% 6$
$77 \% 6$
$= 5$ //

$(40 \% 6 + 37 \% 6) \% 6$
$(4 + 1) \% 6$
$= 5$ //

1) ADDITION

$$(a + b) \% M = ((a \% M) + (b \% M)) \% M$$

$[0, M-1]$ $[0, M-1]$ $[0, M-1]$

$[0 - 2M-2]$
$\% M$
$[0, M-1]$

2) MULTIPLICATION

$$(a \times b) \% M = ((a \% M) \times (b \% M)) \% M$$

a : 10
b : 12
m : 7

$(10 \times 12) \% 7$
$120 \% 7$
$= 1$ //

$((10 \% 7) \times (12 \% 7)) \% 7$
$(3 \times 5) \% 7$
$15 \% 7$
$= 1$ //

$$a \% b = (a \% b) \% b = ((a \% b) \% b) \% b$$

$12 \% 5$

$12 \% 5 = 2$

$(12 \% 5) \% 5$

$2 \% 5 = 2$

$((12 \% 5) \% 5) \% 5$

$(2 \% 5) = 2$

2

Q Given an array. Find the product of all numbers! $\% 10^9$

$M \rightarrow 10^9$

$1 <= N <= 10^5$

$1 <= A[i] <= 10^9$

$$(\underbrace{A_0}_{n} \times A_1 \times A_2 \times \cdots \times A_{N-1}) \% M$$

y

$$((A_0 \% m) \times \cdots \% m) \% M$$

$$\times (A_1 \% m) \times \cdots \% m \quad \% m$$

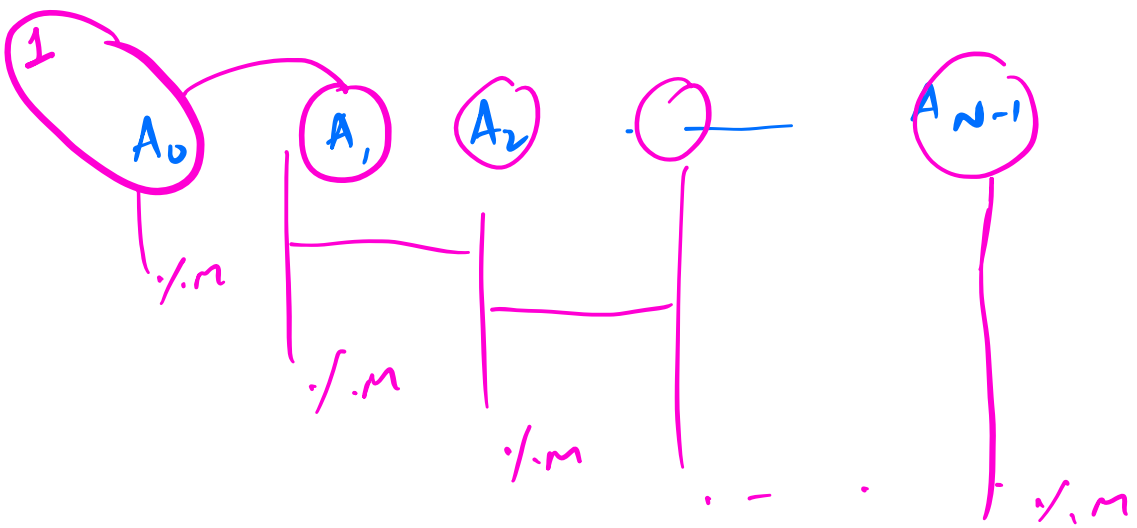
$$(((A_0 \% M) \times (A_1 \% M) \times (A_2 \% M) - -) \% M$$

10^9 10^9

$\times$

10^{19}

$$(((((A_0 \% M) \times (A_1 \% M)) \% M \times (A_2 \% M)) \% M - -$$




```
long p = 1;
for (i = 0; i < N; i++) {
    p = ( p  x A[i] ) % M;
}
ret p;
```

Q Calc $a^b \% M$?

$$(\underbrace{a \times a \times a \cdots a}_{b \text{ times}}) \% M$$

$1 <= a <= 10^9$

$1 <= b <= 10^5$

$M \rightarrow 10^9 + 7$

Q Given a no. in an array!
Calc. the no. % M! $\rightarrow 10^9 + 7$

$1 <= N <= 10^5$

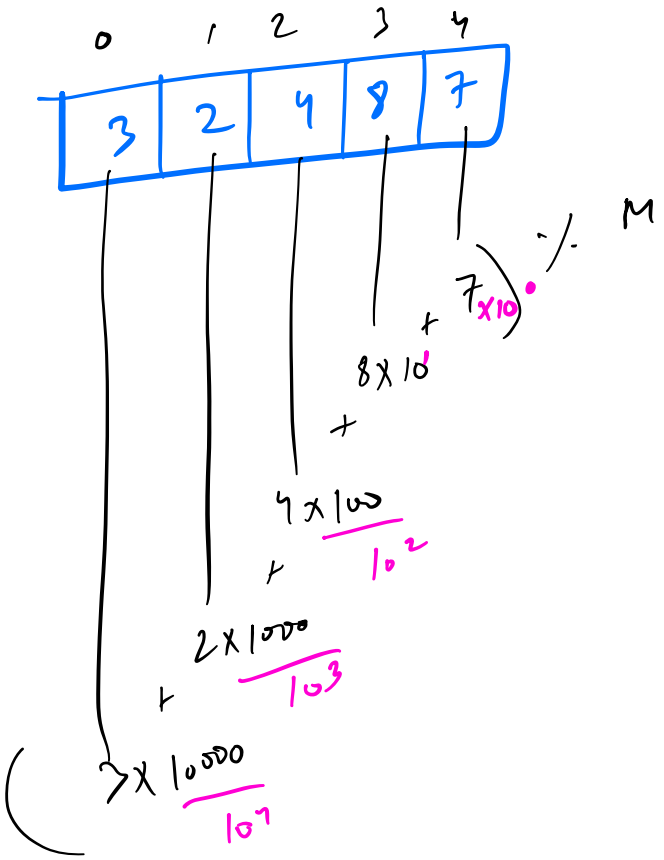
4	5	2	6	3	8	3	5	4	7	4	3	5	2

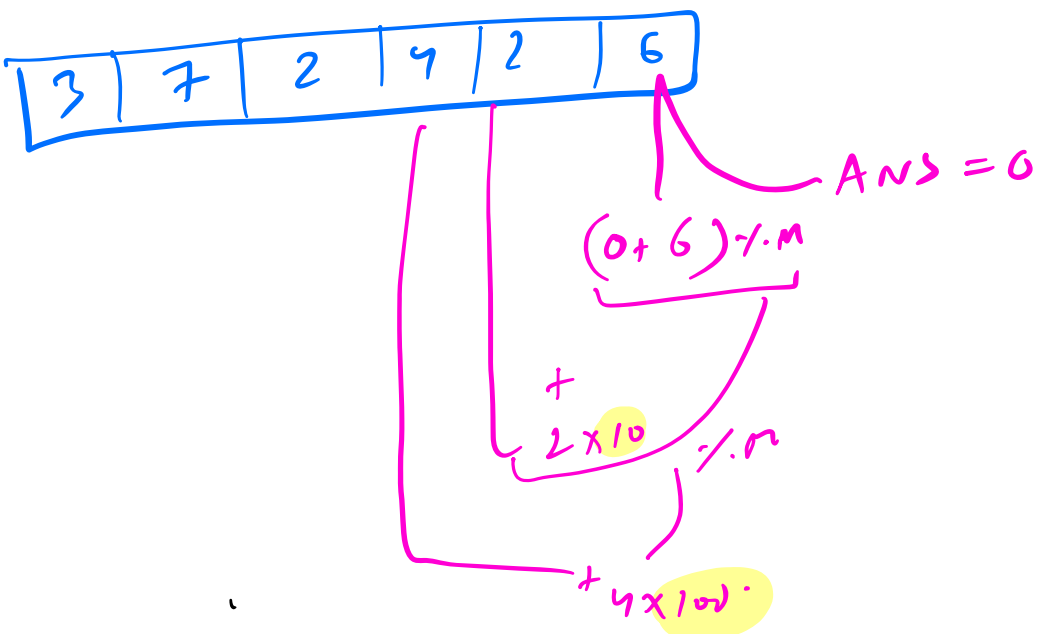
$(4526838547435 2) \% M$

$M = 10$

A: | 3 | 4 | 6 |

$346 \% 10 \rightarrow 6$


0 1 2 3 4
3 2 4 8 7
7x10^0
8x10^1
4x10^2
2x10^3
3x10^4
%M

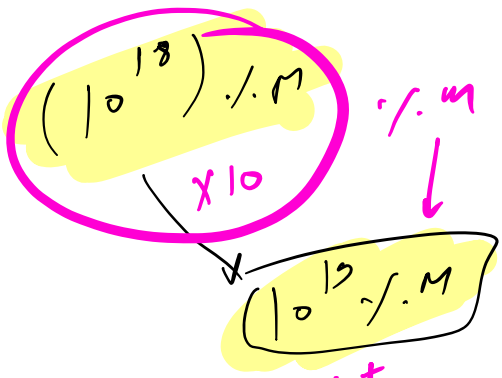

3 7 2 4 2 6
ANS = 0
(0+6)%M
+ 2x10
%M
+ 4x10^2

$(10^n) \% M$

$(10^{n+1}) \% M$

$(10^n \times 10) \% M$

$((10^n) \% M \times 10) \% M$



long → int

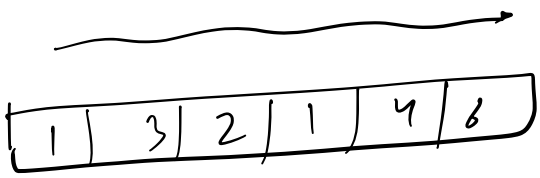
```
pT = 1;  Sum = 0;
for( i = N-1; i >= 0; i--) {
    Sum = (Sum + A[i] * pT) % M;
    pT = (pT * 10) % M;
}
```

10^9 × 10 → 10^9

3457

345 × 10 + 7

× 10 + 8 → 34578




```
s=0
for(i=0; i<n; i++){
    s= s*10 + A[i];
}
```

? %

⊙ Divisibility Rules →

HW/→ 5

⑧ Div. rule 3 ?

Sum of digits is div by 3

123 % 3 = 0

↓

6 % 3 → 0

$(1234)\%3 \rightarrow 0$

$(1\times10^3 + 2\times10^2 + 3\times10^1 + 4\times10^0)\%3$

$((1\times10^3)\%3 + (2\times10^2)\%3 + (3\times10^1)\%3 + (4\times10^0)\%3)\%3$

$(1\times 10^3\%3 + 2\times 10^2\%3 + 3\times 10^1\%3 + 4\times 10^0\%3)\%3$

$(1\times1 + 2\times1 + 3\times1 + 4\times1)\%3$

$(1+2+3+4)\%3$

① Div rule for 4 ?

↳ No formed by last 2 digits should be div by 4.

$(12345)\%4$

$(1\times10^4 + 2\times10^3 + 3\times10^2 + 45)\%4$

$(1\times10^4\%4 + 2\times10^3\%4 + 3\times10^2\%4 + 45)\%4$

$(0+45)\%4$

$= 45\%4$