E9 241: DIGITAL IMAGE PROCESSING

ASSIGNMENT 4; DUE SEPTEMBER 19/24, 2019

TOPIC: SAMPLING AND QUANTIZATION

HANDWRITTEN TASKS (DUE: SEPTEMBER 19, 5.15 PM)

(1) Show that the set of functions $\{\text{sinc}(x-m)\text{sinc}(y-n), x, y \in \mathbb{R}, m, n \in \mathbb{Z}\}$ constitutes an orthogonal family of functions. Before you proceed, observe that the functions in the set are all separable. Obviously, you would be required to compute inner products, but use suitable Fourier transform properties to simplify the computation.

(2) Let f be a 2π-periodic, zero-mean, square-integrable, real-valued function. Let f' denotes its derivative and let f' also be square-integrable. Show that

$$\int_0^{2\pi} f'^2(x)\,\mathrm{d}x \geq \int_0^{2\pi} f^2(x)\,\mathrm{d}x.$$

Under what condition does the equality hold? This is known as *Wirtinger's inequality*.

(3) Consider $f \in \mathrm{L}^2(\mathbb{R})$ and its representation using the set of orthogonal functions $\{\text{sinc}(t-k), t \in \mathbb{R}, k \in \mathbb{Z}\}$ as follows:

$$f(t) = \sum_{k\in\mathbb{Z}} c_k \text{sinc}(t-k) + \epsilon(t),$$

where $\epsilon(t)$ is the representation/approximation error. Since $f \in \mathrm{L}^2(\mathbb{R})$ may not necessarily be bandlimited, whereas $\{\text{sinc}(t-k)\}$ contains only bandlimited functions, there would be an error in the representation, which is precisely why there is $\epsilon(t)$ in the above equation. Determine the optimal coefficients $\{c_k\}$ such that $\|\epsilon\|^2$ is minimized. This exercise would help you understand the optimality of the anti-aliasing filter.

(4) Go through the following *Black & White* videos on YouTube (hyperlinked):
- Charlie Chaplin – The Lion's Cage
- The Salt Satyagraha, 1930

- Adolf Hitler's speech
- The Great Dictator

Explain why the videos look a bit choppy and unnatural. For instance, why do people appear to be walking faster than usual?

(5) Design the optimal two-level Lloyd-Max quantizer for the amplitude distribution described as follows:

$$f(x) = \begin{cases} 1 - |x|, |x| \leq 1, \\ 0, \text{elsewhere.} \end{cases}$$

You could analytically derive the equations first and then use a Python/Matlab script to recursively solve for the *decision* and *reconstruction* levels based on the equations. Sketch the input-output characteristics (also known as *transfer characteristics* of the resulting quantizer. The solution would become simpler if you take advantage of the symmetry of $f(x)$.

In practice, a uniform quantizer is what one prefers to use in many applications, but this exercise is aimed at training you to determine optimal thresholding schemes.

PROGRAMMING TASKS (DUE SEPTEMBER 24)

(1) This exercise would help you understand the **effect of the DFT size** in performing the forward and inverse transformations.

Write a Matlab/Python script to do the following in order. **Do not use the 2-D FFT algorithm from Assignment-3 to implement the DFT.** Instead, use the matrix-vector product approach to answer this question, even if it means that the implementation is not going to be computationally efficient.

$M \times M$ image $\rightarrow$ $N \times N$ DFT $\rightarrow$ $P \times P$ IDFT. M and N are powers of 2. Use the *Lenna* image to report your results. Report the results for the following cases: (i) $P = N = M$, (ii) $P = N = 2M$, (iii) $2P = N = M$, (iv) $P = N = M/2$. These four cases must appear as the options for a radio button. Display the input image (title: LENNA IMAGE) and the output image (title: RECONSTRUCTED LENNA IMAGE). Based on the selected option, the corresponding results must be computed dynamically and displayed. Explain why the output appears the way it does in each case.

Titbit: The *Lenna* image was cropped from the centrefold of a men's lifestyle and entertainment magazine dating back to 1972. It was first used in a conference paper and became popular in the image processing community ever since. Beware what images you use for reporting your research results for they could potentially have a lasting impact! Lenna was also the *guest of honor* at the banquet of IEEE International Conference on Image Processing (ICIP) 2015.

(2) Write a Matlab/Python script to implement the 1-bit dithering scheme. Use the *Samantha* image to report your results. The image has been oversampled already. Use a slider to adjust the variance of the additive noise from a minimum of 1 to a maximum of 21 in steps of 2 and dynamically update your result.
Titbit: Samantha is a famous south Indian actress who won numerous awards for her versatile acting skills. The lesser known fact is that she also runs an NGO to provide medical aid for women and children.

(3) Write a Matlab/Python script to implement an r-bit uniform quantizer. The number of bits r could take any value between 1 and 8, both values included. Your program must have a radio button option to select the number of bits and display the corresponding quantized result. Also display the peak signal-to-(quantization) noise ratio (PSQNR) computed as follows:

$$\text{PSQNR} = 20\log_{10}\left(\frac{255}{\frac{1}{N}\|f - \hat{f}_r\|}\right) \text{ dB},$$

where f is the given image, $\hat{f}_r$ is the r-bit quantized image, and N is the total number of pixels in the image. Report your results on the *Mr. Bean* image. Generate a rate-distortion trade-off plot showing the PSQNR (in dB) on the y-axis versus the number of bits r on the x-axis.
Titbit: Rowan Atkinson is not only the title character, but also a creator of the British sitcom *Mr. Bean*. He holds an M.Sc. degree in Electrical Engineering.