E9 241: DIGITAL IMAGE PROCESSING

ASSIGNMENT 2; DUE AUGUST 29/30, 2019

TOPIC: 2-D FOURIER TRANSFORMS

HANDWRITTEN TASKS (DUE: AUGUST 29, 5.15 PM)

(1) Considering the standard definition of the inner-product in $\ell^2(\mathbb{Z}^2)$, state and prove the Cauchy-Schwartz inequality.

(2) Show that if a sequence is absolutely summable, then it is also square summable.

(3) (*Differentiation property*) Consider two 2-D square-integrable functions $f(x)$ and $\widehat{f}(\omega)$ that form a Fourier transform pair. Assume that partial derivatives of all orders of f exist and that they are square integrable. Determine the Fourier transform of $\dfrac{\partial^{n+m} f}{\partial x^n \partial y^m}$, where $n, m \in \mathbb{N}$.

(4) Consider a 2-D Fourier pair: $f(x) \xleftrightarrow{\mathscr{F}} \widehat{f}(\omega)$. Determine the Fourier transform of $f(Ax)$, where A is a 2×2 nonsingular matrix. How would the Fourier transform be affected if A happens to be singular?

(5) Consider a linear, shift-invariant, 2-D system with impulse response $h(x) = \begin{pmatrix} 0 & -1 & 0 \\ -1 & \boxed{+4} & -1 \\ 0 & -1 & 0 \end{pmatrix}$, where $\square$ indicates the coefficient at the origin. If the input is $0.25\cos(0.2\pi\, m + 0.3\pi\, n) + 0.5\cos(0.7\pi\, m - 0.2\pi\, n), m, n \in \mathbb{Z}$, determine the output.

PROGRAMMING TASKS (DUE AUGUST 30)

(1) This exercise would help you understand **frequency-domain discretization** of the discrete space Fourier transform. Optimal discretization would be addressed in a subsequent assignment.
Write a Matlab/Python script to accept a 256×256 or a 512×512 image as input and compute the discrete-space Fourier transform on a uniform grid of size $m \times m$. Display the magnitude and phase responses for the following cases: (i)

$m = 64$; (ii) $m = 128$; (iii) $m = 256$; (iv) $m = 512$; and (v) $m = 1024$. The frequency axes must cover the range $[-\pi, +\pi] \times [-\pi, +\pi]$. Introduce a radio button feature in your program to enable a user to select the value of m from the given options. For each figure, display titles (MAGNITUDE RESPONSE or PHASE RESPONSE, as the case may be, in UPPERCASE). Show your results on the *Cameraman* and *Zelda* images. What do you think should be the relationship between the dimension of the image and m?

(2) This exercise would help you understand the **rotation property of the Fourier transform.**
Write a Matlab/Python script to compute the discrete-space Fourier transform of the 2-D sequence $f[k, m] = \cos(\omega_0(k \cos\theta + m \sin\theta)), 1 \leq k, m \leq 256$. The variable ω_0 takes values between 0 and π, and θ takes values between 0 and 2π. Incorporate a linear slider to select the value of ω_0 and a round slider to select the value of θ. Each slider must accommodate at least 120 values in the given range. Display your results on three panels side-by-side: the first panel for showing the 2-D sequence f, the second one for the Fourier magnitude, and the third one for the Fourier phase. For each figure, display the titles as well (A PLANE WAVE, FOURIER MAGNITUDE, FOURIER PHASE, as the case may be, in UPPERCASE). Also display the values of ω and θ in the legend of the first panel. All the figure panels must get dynamically updated depending on the choice of ω and θ.