E9 241: DIGITAL IMAGE PROCESSING

ASSIGNMENT 1; DUE AUGUST 22, 2019

HANDWRITTEN TASKS.

(1) Evaluate the sum: $S = \sum_{n=0}^{+\infty} n\, x^{n-1}, |x| < 1, x \in \mathbb{R}$.

(2) Evaluate the sum: $S = \sum_{n=-\infty}^{+\infty} a^{|n|}\, z^{-n}, |a| < 1, z \in \mathbb{C}$. Indicate the region of convergence of the sum.

(3) Consider a cartesian coordinate system with orthogonal x and y axes. The axes are rotated counterclockwise by an angle of θ to obtain a transformed coordinate system. Derive a mapping between the points in the original coordinate system and those in the transformed coordinate system. Is the mapping linear? Is it invertible? Justify your answer.

(4) Consider a unit cube in $\mathbb{R}^3$, subjected to a 3×3 transformation A. Compute the volume of the transformed cube in each of the following cases:

$$(a)\quad A = \begin{pmatrix} 1 & 2 & 3 \\ 7 & 9 & 9 \\ 4 & 7 & 6 \end{pmatrix} \qquad (b)\quad A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{pmatrix}$$

$$(c)\quad A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 5 & 7 & 9 \end{pmatrix} \qquad (d)\quad A = \begin{pmatrix} 2 & 2 & -1 \\ 1 & 3 & 0 \\ -1 & 1 & 4 \end{pmatrix}$$

(5) Given n orthonormal column vectors $\boldsymbol{a}_i \in \mathbb{R}^n, i = 1, 2, \ldots, n$, evaluate

$$S = \sum_{i=1}^{n} \boldsymbol{a}_i\, \boldsymbol{a}_i^{\mathrm{T}},$$

where T denotes the transpose operation.

PROGRAMMING TASKS.

(1) Write a Matlab/Python script to implement the following input-output operation on an input image (f denotes an 8-bit greyscale input, g denotes the output, $0 < \tau_1 \leq \tau_2 < 255$):

$$g(x,y) = \begin{cases} f(x,y), & \text{if} \quad 0 < \tau_1 \leq f(x,y) \leq \tau_2 < 255, \\ \tau_1, & \text{if} \quad f(x,y) < \tau_1, \\ \tau_2, & \text{if} \quad f(x,y) > \tau_2. \end{cases}$$

Sketch the input-output characteristics. Also display the input image as well as the output images on greyscale. The program should provide a slider control to select τ_1 and τ_2 respecting $0 < \tau_1 \leq \tau_2 < 255$, and dynamically update the output.

(2) Write a Matlab/Python script that accepts two greyscale images f_1 and f_2 as the inputs and gives the output $g = \alpha f_1 + (1-\alpha) f_2$, where $0 \leq \alpha \leq 1$. Display the input images as well as the output. The program should provide a slider control to select α on a scale of 0 to 1 and dynamically update the output.

(3) Write a Matlab/Python script to accept a greyscale image as input and generate a 3-D mesh plot with the pixel coordinates on the x and y axes and the greyscale intensity on the z axis.

(4) Write a Matlab/Python script to accept two greyscale images f_1 and f_2 as the input and generate the output g as follows:

$$g(x,y) = \begin{cases} 0, & \text{if} \quad f_1(x,y) < f_2(x,y), \\ 255, & \text{if} \quad f_1(x,y) \geq f_2(x,y). \end{cases}$$

(5) Write a Matlab/Python script to generate and display a 2-D sinusoid $f(m,n) = \cos(2\pi\, 0.2\, m + 2\pi\, 0.3\, n + \phi); m, n = 1, 2, \ldots, 256$. The program should provide a slider control to select $\phi \in [0, 2\pi)$ and dynamically update the display.