AI1110
Assignment-11

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Outline

Question

Papoullis 8-17:

Suppose that the IQ scores of children in a certain grade are the samples of an $N(\eta, \sigma)$ random variable x. We test 10 children and obtain the following averages: $\overline{x} = 90$, $s = 5$. Find the 0.95 confidence interval of η and of σ.

Theory

UNKNOWN VARIANCE

If σ is unknown, we form the sample variance to estimate η

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2 \tag{1}$$

This is a unbiased estimate of σ^2 and it tends to σ^2 as $n \to \infty$.

If x is normal, the ratio

$$\frac{\overline{x} - \eta}{s/\sqrt{n}} \tag{2}$$

has a Student t distribution with $n-1$ degrees of freedom.

Denoting by t_u its u percentiles ($u = 1 - \delta$). This yields the interval,

$$\overline{x} - t_{1-\delta/2}\frac{s}{\sqrt{n}} < \eta < \overline{x} + t_{1-\delta/2}\frac{s}{\sqrt{n}} \tag{3}$$

UNKNOWN MEAN

If η is unknown, we use as the point estimate of σ^2 the sample variance s^2. The random variable $(n-1)s^2/\sigma^2$ has a $\chi^2(n-1)$ distribution. This yields the interval,

$$\frac{(n-1)s^2}{{\chi^2}_{1-\delta/2}(n-1)} < \sigma^2 < \frac{(n-1)s^2}{{\chi^2}_{\delta/2}(n-1)} \tag{4}$$

Solution

Given,
Sample size, $n = 10$,
Sample mean, defined as

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{5}$$

from question, $\overline{x} = 90$
Sample standard deviation, $s = 5$.

The 0.95 confidence interval of η

Using equation (3),

$$\overline{x} - t_{0.975}(9)\,\frac{s}{\sqrt{n}} < \eta < \overline{x} + t_{0.975}(9)\,\frac{s}{\sqrt{n}} \tag{6}$$

From Table 8-2 in papoullis book we get the value of $t_{0.975}(9) = 2.26$,

$$t_{0.975}(9)\,\frac{s}{\sqrt{n}} = (2.26)\frac{5}{\sqrt{10}} \tag{7}$$

$$= 3.57 \tag{8}$$

On substituting the above we get,

$$\overline{x} - 3.57 < \eta < \overline{x} + 3.57 \tag{9}$$

$$90 - 3.57 < \eta < 90 + 3.57 \tag{10}$$

$$\therefore 86.43 < \eta < 93.57 \tag{11}$$

The 0.95 confidence interval of σ

Using equation (4),

$$\frac{(n-1)s^2}{\chi^2{}_{1-\delta/2}(n-1)} < \sigma^2 < \frac{(n-1)s^2}{\chi^2{}_{\delta/2}(n-1)} \tag{12}$$

$$\frac{9 \times 5^2}{\chi^2{}_{0.975}(9)} < \sigma^2 < \frac{9 \times 5^2}{\chi^2{}_{0.025}(9)} \tag{13}$$

From Table 8-3 in papoullis book we get the value of $\chi^2{}_{0.975}(9) = 19.02$ and $\chi^2{}_{0.025}(9) = 2.70$,

$$\frac{9 \times 5^2}{19.02} < \sigma^2 < \frac{9 \times 5^2}{2.70} \tag{14}$$

$$11.83 < \sigma^2 < 83.33 \tag{15}$$

$$\therefore 3.44 < \sigma < 9.13 \tag{16}$$