AI1110 Assignment2(ICSE Class 12 2018)

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QUESTION 19(A)

Given the total cost function for x units of a commodity as:

$$c(x) = \frac{1}{3}x^3 + 3x^2 - 16x + 2$$

Find:

(i) Marginal cost function
(ii) Average cost function

SOLUTION

(i) Marginal cost is the change in total cost that arises when the quantity produced is changed by one unit.
Marginal cost function is the derivative of total cost function $C(x)$.
To find the marginal cost(M.C),derive the total cost function to find $C'(x)$ or $\frac{dC}{dx}$.

Given,

$$C(x) = \frac{1}{3}x^3 + 3x^2 - 16x + 2 \tag{1}$$

On differentiating,

$$\begin{aligned} M.C &= \frac{d}{dx}[C(x)] && (2)\\ &= \frac{d}{dx}[\frac{1}{3}x^3 + 3x^2 - 16x + 2] && (3)\\ &= \frac{1}{3}(3x^2) + 3(2x) - 16(1) + 0 && (4)\\ &= x^2 + 6x - 16 && (5) \end{aligned}$$

Therefore the Marginal cost function is,

$$M.C(x) = x^2 + 6x - 16$$

(ii) Average cost function is ratio of total cost of items to the number of items or Quantity,

$$A.C = \frac{C(x)}{Q} \tag{6}$$

Here Q is the quantity produced which is x units.

$$\begin{aligned} A.C &= \frac{C(x)}{x} && (7)\\ &= \frac{\frac{1}{3}x^3 + 3x^2 - 16x + 2}{x} && (8)\\ &= \frac{1}{3}x^2 + 3x - 16 + \frac{2}{x} && (9) \end{aligned}$$

Therefore the Average cost function is,

$$A.C(x) = \frac{1}{3}x^2 + 3x - 16 + \frac{2}{x}$$