AI1110 Assignment2

Bhargava Ram Rajulapati - CS21BTECH11052

ICSE class 12 paper 2018:
19(a): Given the total cost function for x units of a commodity as:

$$C(x) = \frac{1}{3}x^3 + 3x^2 - 16x + 2$$

Find:

(i) Marginal cost function
(ii) Average cost function

Solution: The definitions of above two functions are given in the below table as

Function	Definition
Marginal cost	Rate of change of total cost with respect to the number of units
Average cost	Ratio of total cost to the number of units or Quantity

TABLE 2

(i) Marginal cost is the change in total cost that arises when the quantity produced is changed by one unit.
From Table (2) we can say that Marginal cost function is the derivative of total cost function $C(x)$.

Given,

$$C(x) = \frac{1}{3}x^3 + 3x^2 - 16x + 2 \tag{1}$$

On differentiating,

$$\begin{aligned} M.C(x) &= \frac{d}{dx}[C(x)] && (2)\\ &= \frac{d}{dx}[\frac{1}{3}x^3 + 3x^2 - 16x + 2] && (3)\\ &= \frac{1}{3}\left(3x^2\right) + 3(2x) - 16(1) + 0 && (4)\\ &= x^2 + 6x - 16 && (5) \end{aligned}$$

Therefore the Marginal cost function is,

$$M.C(x) = x^2 + 6x - 16$$

(ii) From the definition in Table (2) the formula for Average cost function is

$$A.C(x) = \frac{C(x)}{Q} \tag{6}$$

Here Q is the quantity produced which is x units.

$$\begin{aligned} A.C(x) &= \frac{C(x)}{x} && (7)\\ &= \frac{\frac{1}{3}x^3 + 3x^2 - 16x + 2}{x} && (8)\\ &= \frac{1}{3}x^2 + 3x - 16 + \frac{2}{x} && (9) \end{aligned}$$

Therefore the Average cost function is,

$$A.C(x) = \frac{1}{3}x^2 + 3x - 16 + \frac{2}{x}$$