AI1110
Assignment-7

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Outline

Abstract

- This document contains the solution to a Question
- In NCERT Class 12 Textbook
- In Chapter 13 (Probability)

Question

Exercise 13.2.7

Given that the events A and B are such that $\Pr(A) = \frac{1}{2}$, $\Pr(A \cup B) = \frac{3}{5}$ and $\Pr(B) = p$. Find p if they are

1. mutually exclusive
2. independent.

Theory

Inclusion-Exclusion Principle

If A and B are two events, the individual probabilities and probability of occurrence of both events at same time are known,then probability of occurrence of either event A or B is given as

$$\Pr(A+B) = \Pr(A) + \Pr(B) - \Pr(AB) \tag{1}$$

Mutually Exclusive Events

Two events A and B are said to be mutually exclusive if they cannot occur at the same time or simultaneously. Mutually exclusive events are also called Disjoint events. So,

$$\Pr(AB) = 0 \tag{2}$$

Independent Events

Two events A and B are said to be independent events if the probability of occurrence of one of them is not affected by occurrence of the other.

$$\Pr(AB) = \Pr(A) \times \Pr(B) \tag{3}$$

Solution

We have,

$$\Pr(A) = \frac{1}{2} \tag{4}$$

$$\Pr(A+B) = \frac{3}{5} \tag{5}$$

$$\Pr(B) = p \tag{6}$$

let, $\Pr(AB) = x$

Using Inclusion-Exclusion Principle and substituting,

$$\Pr(A+B) = \Pr(A) + \Pr(B) - \Pr(AB) \tag{7}$$

$$\frac{3}{5} = \frac{1}{2} + p - x \tag{8}$$

$$\implies p - x = \frac{3}{5} - \frac{1}{2} \tag{9}$$

$$\implies p - x = \frac{1}{10} \tag{10}$$

1. When events A and B are mutually exclusive,
   From definition,

$$\Pr(AB) = 0 \tag{11}$$

$$x = 0 \tag{12}$$

On substituting (12) in equation (10),

$$p - 0 = \frac{1}{10} \tag{13}$$

$$p = \frac{1}{10} \tag{14}$$

2. When events A and B are independent,
From definition,

$$\Pr(AB) = \Pr(A) \times \Pr(B) \tag{15}$$

$$x = \frac{1}{2} \times p \tag{16}$$

$$x = \frac{p}{2} \tag{17}$$

On substituting (17) in equation (10),

$$p - \frac{p}{2} = \frac{1}{10} \tag{18}$$

$$\frac{p}{2} = \frac{1}{10} \tag{19}$$

$$p = \frac{1}{5} \tag{20}$$