Assignment

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CONTENTS

***Abstract*—This manual provides solutions to the Assignment of Random Numbers**

I. UNIFORM RANDOM NUMBERS

Let U be a uniform random variable between 0 and 1.

I.1 Generate 10^6 samples of U using a C program and save into a file called uni.dat .
Solution: Download the following files and execute the C program.

```
https://github.com/bhargav0383/AI1110–Assignments/blob/main/RandomNumbers/codes/1.1.c
https://github.com/bhargav0383/AI1110–Assignments/blob/main/RandomNumbers/codes/functions.h
```

Execute the above C program files using the following commands

```
$ gcc 1.1.c
$ ./a.out
```

I.2 Load the uni.dat file into python and plot the empirical CDF of U using the samples in uni.dat. The CDF is defined as

$$F_U(x) = \Pr(U \leq x) \tag{1}$$

Solution: The python code for the plot in Fig. I.2 is given below,

```
https://github.com/bhargav0383/AI1110–Assignments/blob/main/RandomNumbers/codes/1.2CDF.py
```

Download the above file and execute the command below to produce Fig.I.2

```
$ python3 1.2CDF.py
```

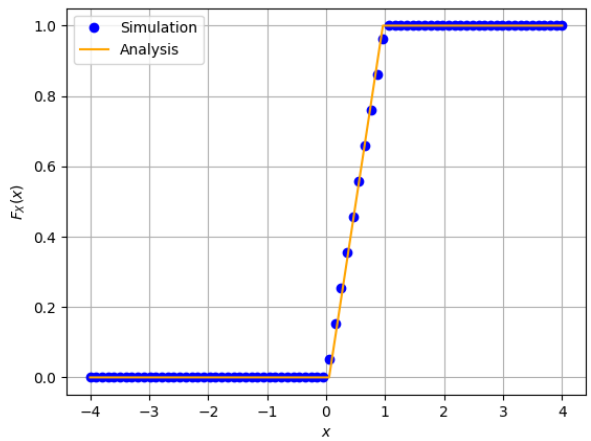

Fig. I.2. The CDF of U

I.3 Find a theoretical expression for $F_U(x)$.
Solution: Given U is a uniform Random Variable between 0 and 1,

$$F_U(x) = \Pr(U \leq x) = \int_{-\infty}^{x} p_U(u)du \tag{2}$$

We have three parts:

i) For $x < 0$; $p_X(x) = 0$, So $F_U(x) = 0$.

ii) For $0 \leq x < 1$;

$$F_U(x) = \int_0^x (1)du = x \tag{3}$$

iii) For $x \geq 1$; CDF is 1 as all the random numbers are between 0 and 1.

Therefore,

$$F_U(x) = \begin{cases} 0 & x < 0 \\ x & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases} \tag{4}$$

I.4 The mean of U is defined as

$$E[U] = \frac{1}{N}\sum_{i=1}^{N} U_i \tag{5}$$

and its variance as

$$\text{var}[U] = E[U - E[U]]^2 \tag{6}$$

Write a C program to find the mean and variance of U.

Solution: Download the following files and execute the C program.

```
https://github.com/bhargav0383/AI1110–
    Assignments/blob/main/RandomNumbers/
    codes/1.4.c
https://github.com/bhargav0383/AI1110–
    Assignments/blob/main/RandomNumbers/
    codes/functions.h
```

Execute the above C program files using the following commands

```
$ gcc 1.4.c
$ ./a.out
```

I.5 Verify your result theoretically given that

$$E\left[U^k\right] = \int_{-\infty}^{\infty} x^k dF_U(x) \tag{7}$$

Solution: W.K.T,

$$\text{var}[U] = E\left[U^2\right] - E[U]^2 \tag{8}$$

Where $E[U]$ is,

$$E[U] = \int_{-\infty}^{\infty} x dF_U(x) \tag{9}$$

$$= \int_{0}^{1} x \tag{10}$$

$$= \frac{1}{2} = 0.5 \tag{11}$$

And $E[U^2]$ is,

$$E\left[U^2\right] = \int_{-\infty}^{\infty} x^2 dF_U(x) \tag{12}$$

$$= \int_{0}^{1} x^2 dF_U(x) \tag{13}$$

$$= \frac{1}{3} \tag{14}$$

(15)

Hence finally,

$$\implies \boxed{\text{var}[U] = \frac{1}{12} = 0.0833} \tag{16}$$

II. Central Limit Theorem

II.1 Generate 10^6 samples of the random variable

$$X = \sum_{i=1}^{12} U_i - 6 \tag{17}$$

using a C program, where $U_i, i = 1, 2, \dots, 12$ are a set of independent uniform random variables between 0 and 1 and save in a file called gau.dat

Solution: Download the following files and execute the C program.

```
https://github.com/bhargav0383/AI1110–
    Assignments/blob/main/RandomNumbers/
    codes/2.1.c
https://github.com/bhargav0383/AI1110–
    Assignments/blob/main/RandomNumbers/
    codes/functions.h
```

Execute the above C program files using the following commands

```
$ gcc 2.1.c
$ ./a.out
```

II.2 Load gau.dat in python and plot the empirical CDF of X using the samples in gau.dat. What properties does a CDF have?

Solution: The CDF of X is plotted in Fig. II.2 using the code below

```
https://github.com/bhargav0383/AI1110–
    Assignments/blob/main/RandomNumbers/
    codes/2.2CDF.py
```

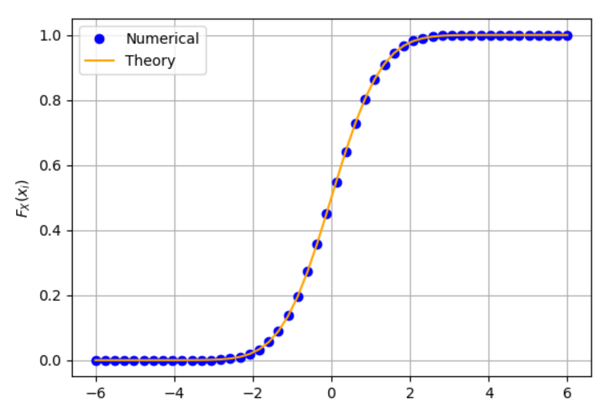

Fig. II.2. The CDF of X

Download the above file and execute the following command to produce Fig.II.2

```
$ python3 2.2CDF.py
```

Some of the properties of CDF

a) $F_X(x)$ is non decreasing function.
b) As $x \to -\infty$, $F_X(x) \to 0$ and when $x \to \infty$, $F_X(x) \to 1$
c) Graph is linear upto some region around $x = 0$.

II.3 Load gau.dat in python and plot the empirical PDF of X using the samples in gau.dat. The PDF of X is defined as

$$p_X(x) = \frac{d}{dx}F_X(x) \tag{18}$$

What properties does the PDF have?

Solution: The PDF of X is plotted in Fig. II.3 using the code below

```
https://github.com/bhargav0383/AI1110–
    Assignments/blob/main/RandomNumbers/
    codes/2.3PDF.py
```

Download the above files and execute the following commands to produce Fig.II.3

```
$ python3 2.3PDF.py
```

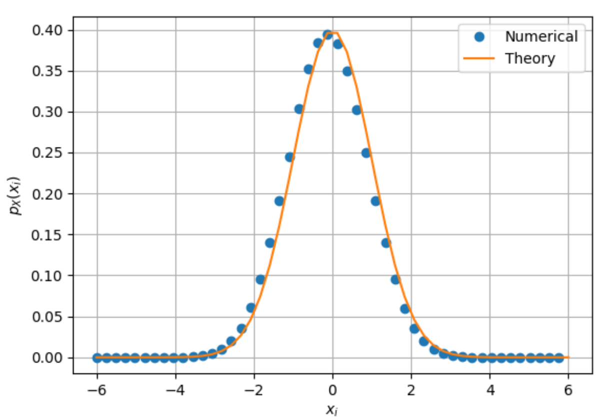

Fig. II.3. The PDF of X

Some of the properties of the PDF:

a) Symmetric about $x = \mu$
b) Area under the PDF graph is unity.
c) Increasing function for $x < \mu$ and decreasing for $x > \mu$ and attains maximum at $x = \mu$.

II.4 Find the mean and variance of X by writing a C program.

Solution: Download the following files and execute the C program.

```
https://github.com/bhargav0383/AI1110–
    Assignments/blob/main/RandomNumbers/
    codes/2.4.c
https://github.com/bhargav0383/AI1110–
    Assignments/blob/main/RandomNumbers/
    codes/functions.h
```

Execute the above C program files using the following commands

```
$ gcc 2.4.c
$ ./a.out
```

II.5 Given that

$$p_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right), -\infty < x < \infty, \tag{19}$$

repeat the above exercise theoretically.

Solution: CDF is defined as

$$F_X(x) = \int_{-\infty}^{\infty} p_X(x)dx \tag{20}$$

$$\text{W.K.T,} \quad \boxed{F_X(x) = 1} \tag{21}$$

Mean is given by

$$E(x) = \int_{-\infty}^{\infty} x p_X(x)dx \tag{22}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x \exp\left(-\frac{x^2}{2}\right) dx \tag{23}$$

It is a odd function function, So its value is 0.

$$\boxed{E(x) = 0} \tag{24}$$

Variance is given by

$$\text{var}[U] = E(U^2) - (E(U))^2 \tag{25}$$

$$\therefore \boxed{\text{var}[U] = \sqrt{2}} \tag{26}$$

III. From Uniform to Other

III.1 Generate samples of

$$V = -2\ln(1 - U) \tag{27}$$

and plot its CDF.

Solution: Download the following files and execute the C program.

https://github.com/bhargav0383/AI1110–Assignments/blob/main/RandomNumbers/codes/3.1.c
https://github.com/bhargav0383/AI1110–Assignments/blob/main/RandomNumbers/codes/functions.h

Execute the above C program files using the following commands

```
$ gcc 3.1.c -lm
$ ./a.out
```

The CDF of V is plotted in Fig. III.1 using the code below

https://github.com/bhargav0383/AI1110–Assignments/blob/main/RandomNumbers/codes/3.1CDF.py

Download the above files and execute the following commands to produce plot Fig.III.1

```
$ python3 3.1CDF.py
```

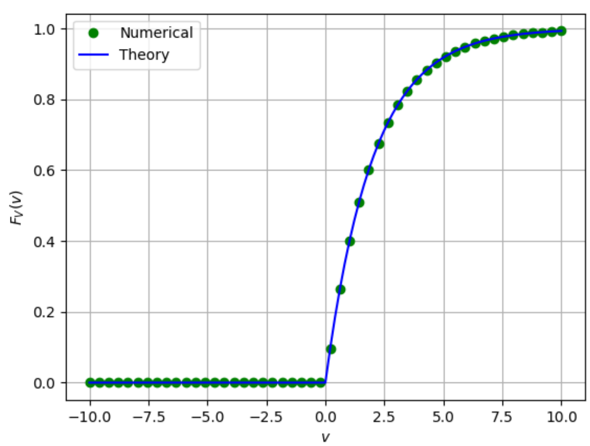

Fig. III.1. The PDF of X

III.2 Find a theoretical expression for $F_V(x)$.
Solution: If Y = g(X),
W.K.T,

$$F_Y(y) = F_X(g^{-1}(y)) \tag{28}$$

$$V = -2\ln(1-U) \tag{29}$$

$$1-U = e^{\frac{-V}{2}} \tag{30}$$

$$U = 1-e^{\frac{-V}{2}} \tag{31}$$

$$F_V(x) = F_U(1-e^{\frac{-x}{2}}) \tag{32}$$

$$\implies F_V(x) = \begin{cases} 0 & x<0 \\ 1-e^{\frac{-x}{2}} & x \geq 0 \end{cases} \tag{33}$$