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Q42. (a) given $\vec{r}$, $\hat{e_i}$, $[\vec{r}]_F = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} \vec{r} \cdot \hat{e_1} \\ \vec{r} \cdot \hat{e_2} \\ \vec{r} \cdot \hat{e_3} \end{bmatrix}$

(b) given: R, $\hat{e_i}$

$$R_{ij} = \hat{e_i} \cdot \hat{e_j}'$$
$$= \hat{e_i} \cdot R \cdot \hat{e_j}$$

(c) Given $\hat{e_i}$, $\hat{e_i}'$ find R_{ij}

$$R = \hat{e_i} \cdot \hat{e_i}'$$
$$\Rightarrow R_{ij} = \hat{e_i} \cdot \hat{e_j}'$$

(d) Given R_{ij}, r_i, find r_i^*

$$\vec{r}^* = R \cdot \vec{r}$$
$$\therefore \; r_i^* = R_{ij} r_j$$

(e) Given R_{ij}, r_i, find r_{Bi}^*

$$r_B^* i = r_F i$$
$$\Rightarrow r_B^* i = r_i \quad \text{(assume fixed frame}$$

(f) Given R_{ij}, r_i^* find r_i

$$\vec{r}^* = A \cdot \vec{r}$$
$$\Rightarrow \vec{r} = R^{-1} \vec{r}^*$$
$$\Rightarrow \vec{r} = R^T \vec{r}^* \qquad (R^{-1} = R^T)$$
$$\Rightarrow \vec{r_i} = R_{ji} r_j^*$$

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(g) Given R_{ij}, r_i^* find $r_{\beta i}$

$$r_i = R_{ji} r_j^*$$

$$r_{\beta i} = R^T r_i$$

$$\Rightarrow r_{\beta i} = R_{Ki} r_K$$

$$= \underline{R_{Ki} R_{jK} r_j^*}$$

(h) Given R_{ij}, find $R_{\beta ij}$, $R_{ij} = \hat{e_i} \cdot \hat{e_j}'$

$$R_{\beta ij} = \hat{e_i}' \hat{e_j}'$$

(i) $$\{r^*\}_F = \begin{bmatrix} \vec{r}^* \cdot \hat{e_1} \\ \vec{r}^* \cdot \hat{e_2} \\ \vec{r}^* \cdot \hat{e_3} \end{bmatrix} = R\{r\}_F = \cancel{R_{ij}} = R_{ij} \cdot R \cdot \{r\}_\beta$$

(j) $A = R_{ij} \hat{e_i}' \hat{e_j}'$, $A = A_{ij} \hat{e_i} \hat{e_j}$

$$A_{ij} = \hat{e_i} \cdot A \cdot \hat{e_j}$$

$$= \hat{e_i} \cdot (R_{kl} \hat{e_k}' \hat{e_l}') \cdot \hat{e_j}$$

$$= \hat{e_i} \cdot (\Sigma_{kl} R_{kl} \hat{e_k}' \hat{e_l}') \cdot \hat{e_j}$$

$$= \Sigma_{kl} R_{kl} (\hat{e_i} \cdot \hat{e_k}')(\hat{e_l}' \cdot \hat{e_j})$$

$$= \Sigma_{kl} R_{kl} \delta_{ki} (\hat{e_l}' \cdot \hat{e_j})$$

$$= \Sigma_l R_l (\hat{e_l}' \cdot \hat{e_j})$$

$$= \Sigma_l R_{il} R_{jl}$$

$$= R_{il} R_{jl}$$

$$\Rightarrow A = RR^T = RR^{-1} = I$$

$$\Rightarrow A_{ij} = \delta_{ij}$$

Shipra