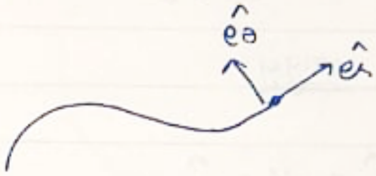
Problem 8

Equations of motion

Q8. A particle on a plane can be represented by $\hat{e}_r$ and $\hat{e}_\theta$



$$\vec{r} = r\hat{e}_r$$

$$\vec{V} = \frac{d}{dt}\vec{r}$$

$$= \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\vec{a} = \frac{d}{dt}\vec{V} = \ddot{r}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta - r\dot{\theta}^2\hat{e}_r$$

$$= (\ddot{r} - r(\dot{\theta})^2)\hat{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{e}_\theta$$

As $\vec{a} = 0$,

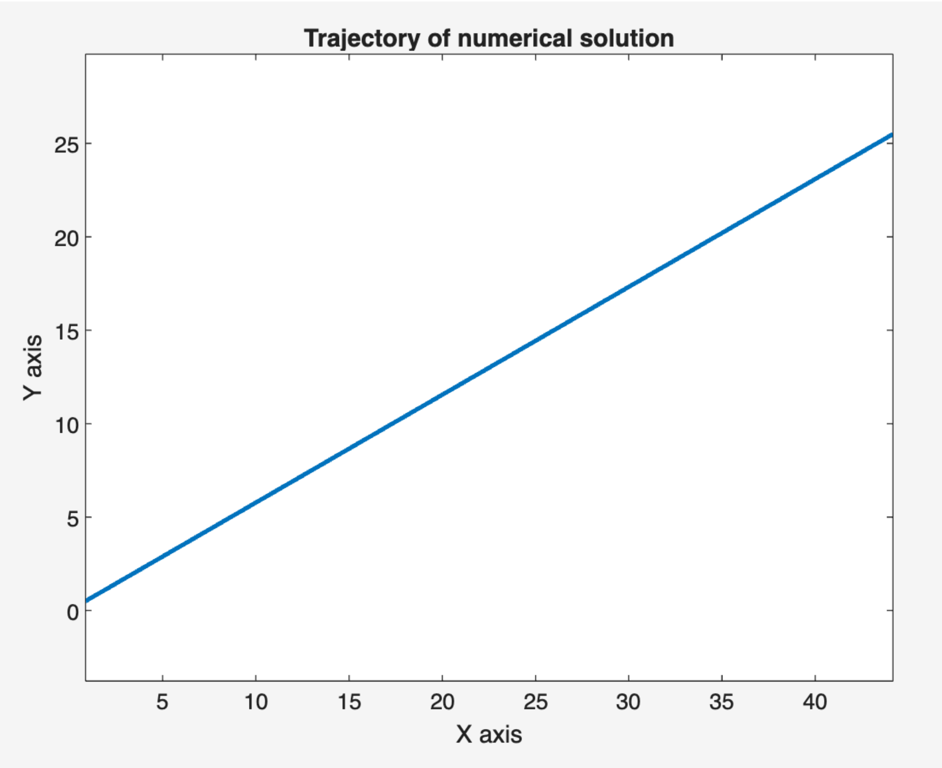
① $\ddot{r} = r(\dot{\theta})^2$

② $\ddot{\theta} = -\dfrac{2\dot{r}\dot{\theta}}{r}$

Writing EoMs in state space,

$$Z = \begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} \qquad \dot{Z} = \begin{bmatrix} \dot{r} \\ \dot{\theta} \\ r(\dot{\theta})^2 \\ -\dfrac{2\dot{r}\dot{\theta}}{r} \end{bmatrix}$$

Straight line motion of solution



Constant velocity plot