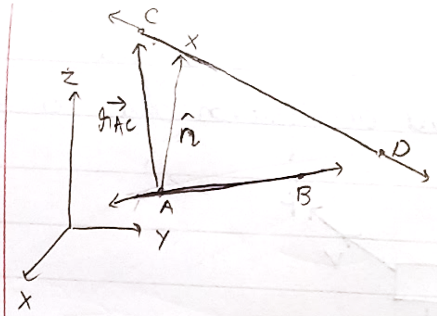
Dynamics and Simulation

Homework Q2

2.



a) Assuming A, B, C, D are not coplanar,
The shortest distance is along $\hat{n}$ perpendicular to $\vec{r}_{AB}$ and $\vec{r}_{CD}$

$$\Rightarrow \hat{n} \text{ (unit vector)} = \frac{\vec{r}_{AB} \times \vec{r}_{CD}}{|\vec{r}_{AB} \times \vec{r}_{CD}|}$$

The distance between the lines is the projection of a vector joining any points on AB to any point on CD on the vector $\hat{n}$

$$\Rightarrow \text{distance } (d) = \vec{r}_{AC} \cdot \hat{n}$$

$$= \vec{r}_{AC} \cdot \frac{(\vec{r}_{AB} \times \vec{r}_{CD})}{|\vec{r}_{AB} \times \vec{r}_{CD}|}$$

b) There can be multiple shortest lines segments between the two lines
Let us consider a line segment from A to point X on line CD

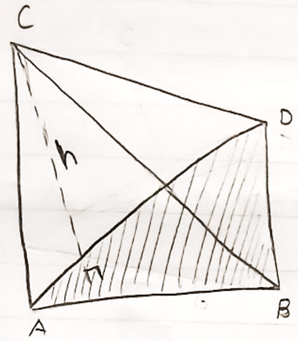
$\vec{r}_{AX}$ is a vector along $\hat{n}$ with magnitude = d (shortest distance)

$$\vec{r}_{AX} = \left[\frac{\vec{r}_{AC} \cdot (\vec{r}_{AB} \times \vec{r}_{CD})}{|\vec{r}_{AB} \times \vec{r}_{CD}|}\right]\left[\frac{\vec{r}_{AB} \times \vec{r}_{CD}}{|\vec{r}_{AB} \times \vec{r}_{CD}|}\right]$$

$$\Rightarrow \vec{r}_{X} = \vec{r}_{A} + \vec{r}_{An} = \vec{r}_{A} + \frac{(\vec{r}_{AC} \cdot (\vec{r}_{AB} \times \vec{r}_{CD}))(\vec{r}_{AB} \times \vec{r}_{CD})}{|\vec{r}_{AB} \times \vec{r}_{CD}|^2}$$

The endpoints of the line segment are given by vectors $\vec{r}_{A}$ and $\vec{r}_{X}$

(c)



Let ABD be the base of the tetrahedron

The volume of a ‖ piped formed by AB, AD and AC is given by

$$V = \vec{AC} \cdot (\vec{AB} \times \vec{AD}) \quad \text{—} ①$$

~~The volume of a tetrahedron~~

A tetrahedron is 1/6th of a parallelopiped in terms of volume

∴ Volume of tetrahedron ABCD

$$V = \frac{1}{6} \vec{AC} \cdot (\vec{AB} \times \vec{AD}) \quad \text{—} ②$$

The base area of the tetrahedron $= \frac{1}{2} \| \vec{AB} \times \vec{AD} \|$

The height (h) is the projection of AC on a vector ⊥ to the base

$$h = \frac{AC \cdot (AB \times AD)}{|AB \times AD|}$$

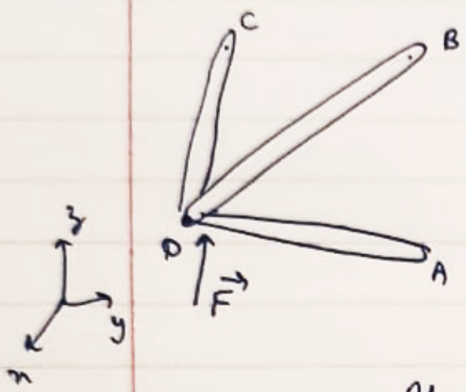
$$\frac{1}{3} \times \text{base area} \times h = \frac{1}{3} \times \frac{1}{2} \cancel{|\vec{AB} \times \vec{AD}|} \times \frac{\vec{AC} \cdot (\vec{AB} \times \vec{AD})}{\cancel{|\vec{AB} \times \vec{AD}|}}$$

$$= \frac{1}{6} \vec{AC} \cdot (\vec{AB} \times \vec{AD})$$

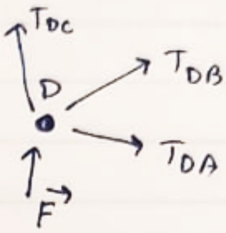
Which is the same as the Volume in eqⁿ ②

Here, the volume of a parallelopiped can be ~~written~~ expressed as

$\frac{1}{3} \times$ base area $\times$ height



FBD of D



At equilibrium, $\Sigma \vec{F} = \vec{0}$

$\Rightarrow \vec{F} + \vec{T_{DA}} + \vec{T_{DB}} + \vec{T_{DC}} = \vec{0}$

$\Rightarrow \vec{F} + \frac{T_{DA}\vec{r_{DA}}}{|\vec{r_{DA}}|} + \frac{T_{DB}\vec{r_{DB}}}{|\vec{r_{DB}}|} + \frac{T_{DC}\vec{r_{DC}}}{|\vec{r_{DC}}|} = \vec{0}$

Dot product with $\vec{r_{DB}} \times \vec{r_{DC}}$

$\Rightarrow \vec{F}\cdot(\vec{r_{DB}} \times \vec{r_{DC}}) + \frac{T_{DA}}{|\vec{r_{DA}}|}\vec{r_{DA}}\cdot(\vec{r_{DB}} \times \vec{r_{DC}}) = \vec{0}$

$$\Rightarrow T_{DA} = \frac{-[\vec{F}\cdot(\vec{r_{DB}} \times \vec{r_{DC}})]\,|\vec{r_{DA}}|}{\vec{r_{DA}}\cdot(\vec{r_{DB}} \times \vec{r_{DC}})}$$