Date

# Dyad's & Einstein summation convention

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \quad a = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

(a) $c = a_i b_i = a_1 b_1 + a_2 b_2 + a_3 b_3 = 3$

(b) $c = \delta_{ii} = 3$

(c) $c_i = A_{ij} b_j$ • $[c] = [A](b)$ (matrix)

$\cancel{c_1 = A_{1j} b_j}$

(d) $c_i \quad b_j A_{ij} = A_{ij} b_{ij} =$

(e) $c_i = A_{ji} b_j$ $\quad [c] = [A^T](b)$ or $[b^T][A]$

j → dummy var

i → index var

(f) $c_i = A_{ij} B_{ij}$ $\quad c_i = A_{i1} B_{i1} + A_{i2} B_{i2} + A_{i3} B_{i3}$

MATLAB : A .* B

(g) $c = A_{ij} \delta_{ij} = A_{11} + A_{22} + A_{33}$

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Matrix invariance

$\det(A - \lambda I) \rightarrow \lambda^n$ polynomial

$a_0 \lambda^n + a_1 \lambda^{n-1} + \dots a_n \lambda^0$

$a_i \rightarrow$ matrix invariant,

$\lambda_i$ (eig vals) $\rightarrow$ matrix invariant.

$a_0 = 1, \quad a_1 = \text{trace}(A)$

(h) $c_k = B_{jk} b_k$ = Nonsense (Dummy variable on right can't be on LHS)

(index var of RHS has to be in LHS)

(i) $c_i = A_{ij} B_{jk}$ Nonsense, but

$\cancel{C_{ik} = A_{ij} B_{jk} \Rightarrow [C] = [A][B]}$

(j) $c_{ik} = A_{ij} B_{jk} \Rightarrow [C] = [A][B]$ (matrix mult)

(k) $\delta_{ij}\delta_{jk}\delta_{ki}\delta_{ll}$

$= 3\,\delta_{ij}\delta_{jk}\delta_{ki}$

$= 3 \times \delta_{ij}\delta_{ji}$

$= 3 \times \delta_{ii}$

$= 3 \times 3 = 9$

(l) $c_k = \epsilon_{ijk} a_i b_j \Rightarrow$

$\underline{\vec{c} = \vec{a} \times \vec{b}}$