Q15 Potential Energy.

(a) $\vec{F} = -mg\hat{j}$

$E_p = -\int -mg\hat{j} \cdot dy\hat{j}$

$= \underline{mgy}$

(b) $\vec{F} = -Kr\,\hat{e}_r$

$dr = dr\hat{r} + r\,d\theta_\theta + r\sin\theta\, d\phi\,\hat{\phi}$

$E_p = -\int F \cdot dr$

$= -\int -Kr\, dr$

$= \underline{\frac{Kr^2}{2}}$

(c) $\vec{F} = -K(l - l_0)\,\hat{e}_r$

$= -K(r - l_0)\,\hat{e}_r$

$E_p = -\int \vec{F} \cdot d\vec{r}$

$= \int K(r - l_0)\, dr$

$= \underline{K\left(\frac{r^2}{2} - l_0 r\right)}$

(d) $F = -\frac{C\vec{r}}{r^3} = -\frac{C}{r^2}\,\hat{e}_r$

$E_p = -\int \vec{F} \cdot dr$

$= \int \frac{C}{r^2}\, dr$

$= -\frac{C}{r}$

(e) $\vec{F} = -f(r)\,\hat{e}_r$

$E_p = -\int \vec{F} \cdot d\vec{r}$

$= \int_{r_0}^{r} f(r)\, dr$

(f) $F = -K(r_2 - r_1 - l_0)\,\hat{r}$

$E_p = \int K(r_2 - r_1 - l_0)\, dr$

$= \int K(r - l_0)\, dr$

$= \frac{Kr^2}{2} - l_0 r$

$= \underline{\frac{K(l_{12})^2}{2} - l_0(l_{12})}$

(g) $\vec{F} = \frac{-C}{r^2}\hat{\lambda}$

$E_p = -\int \vec{F} \cdot d\vec{r}$

$= \int_{l_1}^{l_2} \frac{C}{r^2}\, dr = \frac{-C}{l_2 - l_1} = \underline{\frac{-C}{l_{12}}}$

(h) $\vec{F} = -C\hat{\lambda}$

$E_p = -\int \vec{F} \cdot d\vec{r}$

$= \int_0^r C\hat{\lambda} \cdot d\vec{r}$

$= \underline{C\hat{\lambda} \cdot \vec{r}}$

(e) $\vec{F} = -f(x)\,\hat{i}$

$E_p = -\int \vec{F} \cdot d\vec{x}$

$= \underline{\int_{x_0}^{x} f(x)\, dx}$