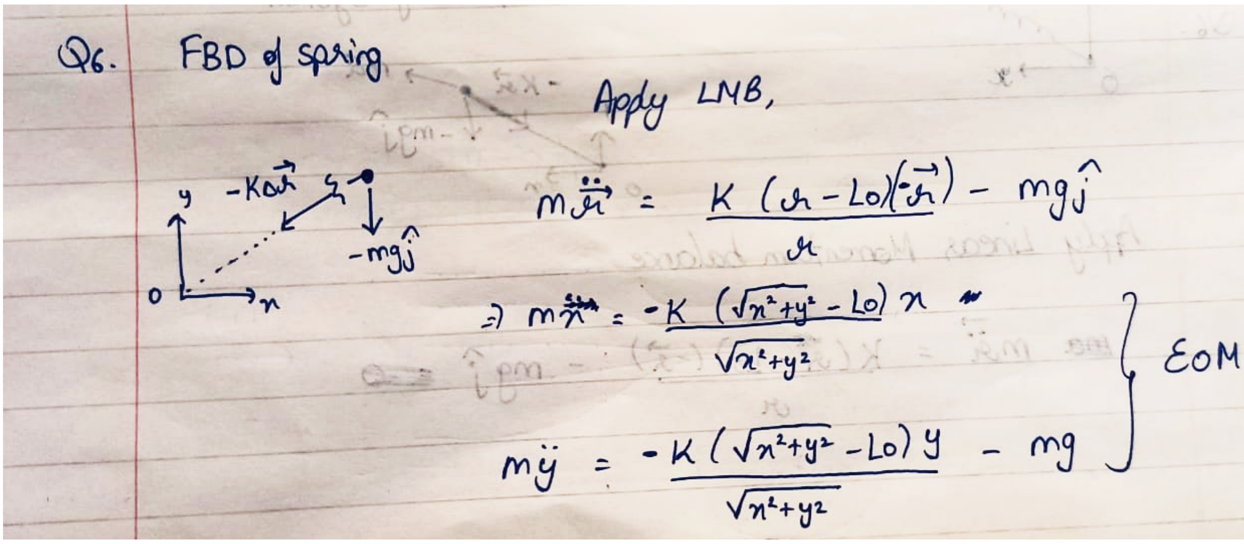
Problem 6

Equations of motion (Spring mass system)



Apply LMB,

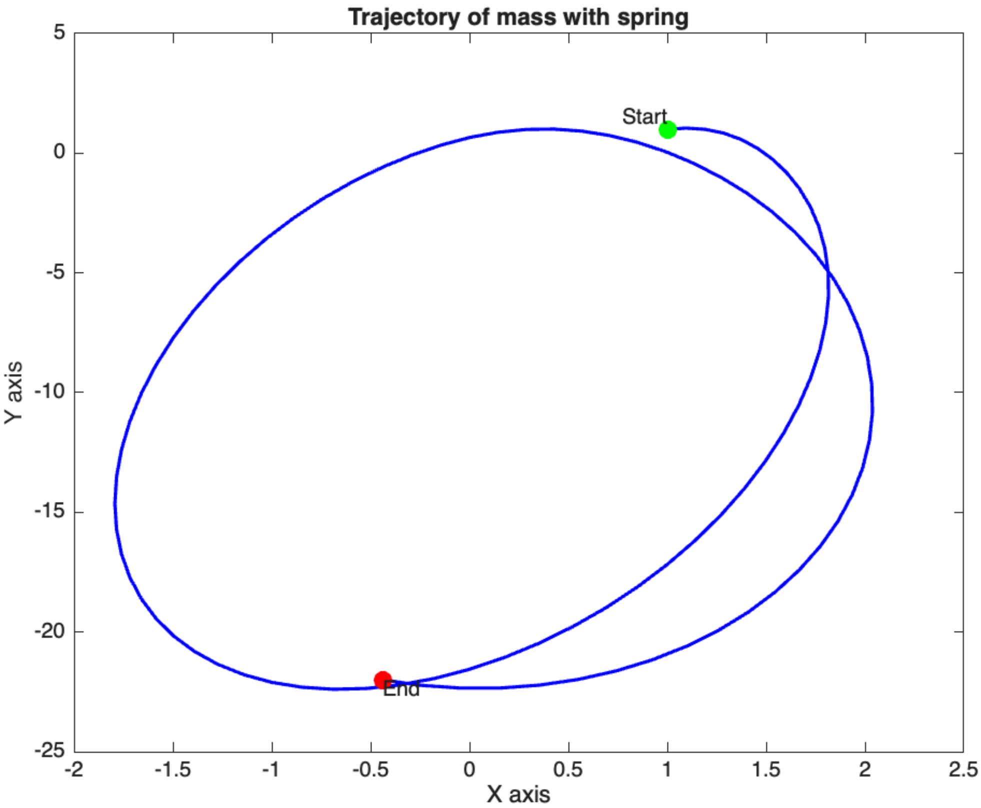
$$m\ddot{\vec{r}} = \frac{K(r - L_0)(-\vec{r})}{r} - mg\hat{j}$$

$$\Rightarrow m\ddot{x} = -\frac{K\left(\sqrt{x^2+y^2} - L_0\right)x}{\sqrt{x^2+y^2}}$$

$$m\ddot{y} = -\frac{K\left(\sqrt{x^2+y^2} - L_0\right)y}{\sqrt{x^2+y^2}} - mg$$

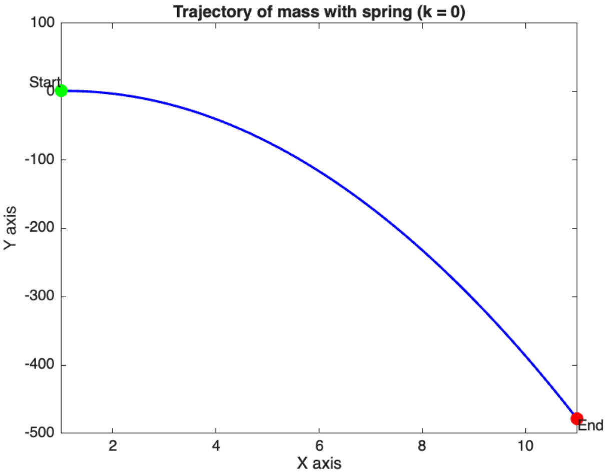
EoM

Trajectory with given initial conditions
(non-zero gravity, initial velocities, positions, rest length)

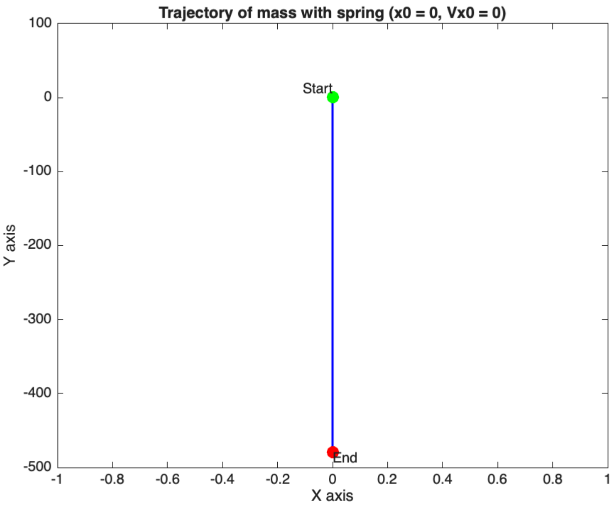


Trajectory with special initial conditions (checking solution)

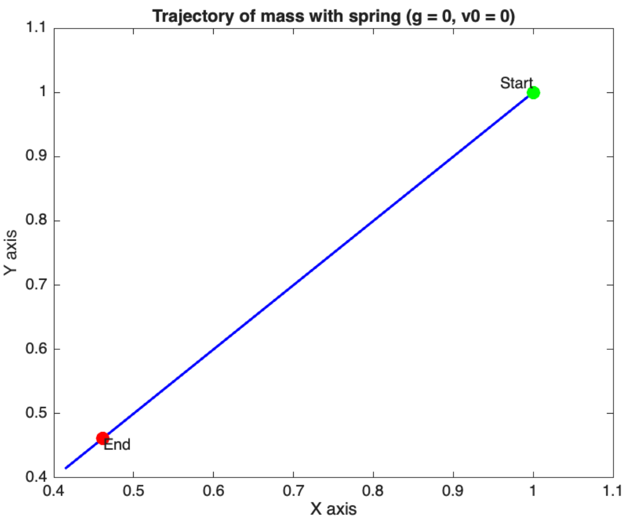
K = 0 gives us parabolic flight under the influence of gravity as shown below



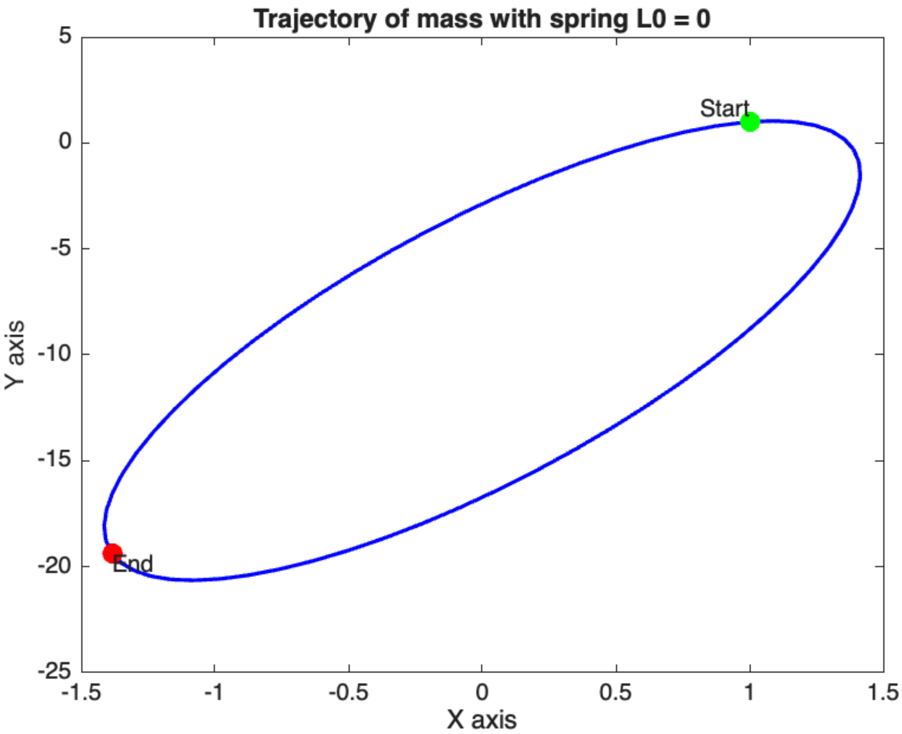
For $x_0 = 0$ and $v_{x0} = 0$, the motion stays on the y axis as shown below



For g = 0 and $v_0 = 0$, motion is radial as the spring force acts in the radial direction.



For $L_0 = 0$, the mass follows an elliptical orbit with periodic motion due to the spring force and gravity



For $L_0 = 0$, $g = 0$, and v_0 in the direction of r_0, the spring undergoes oscillatory motion about the origin along the line of r_0.