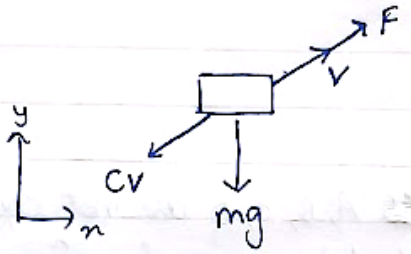
Dynamics and Simulation

Homework Q1

1. The system is a jetpack in motion through air.
It experiences air drag (linear) and acceleration due to gravity.

Free body diagram :



Force F is the thrust provided by the jetpack in direction of its velocity.

Apply Linear Momentum balance,

$$m\vec{a} = -mg\hat{j} - c\vec{v} + F\frac{\vec{v}}{|\vec{v}|}$$

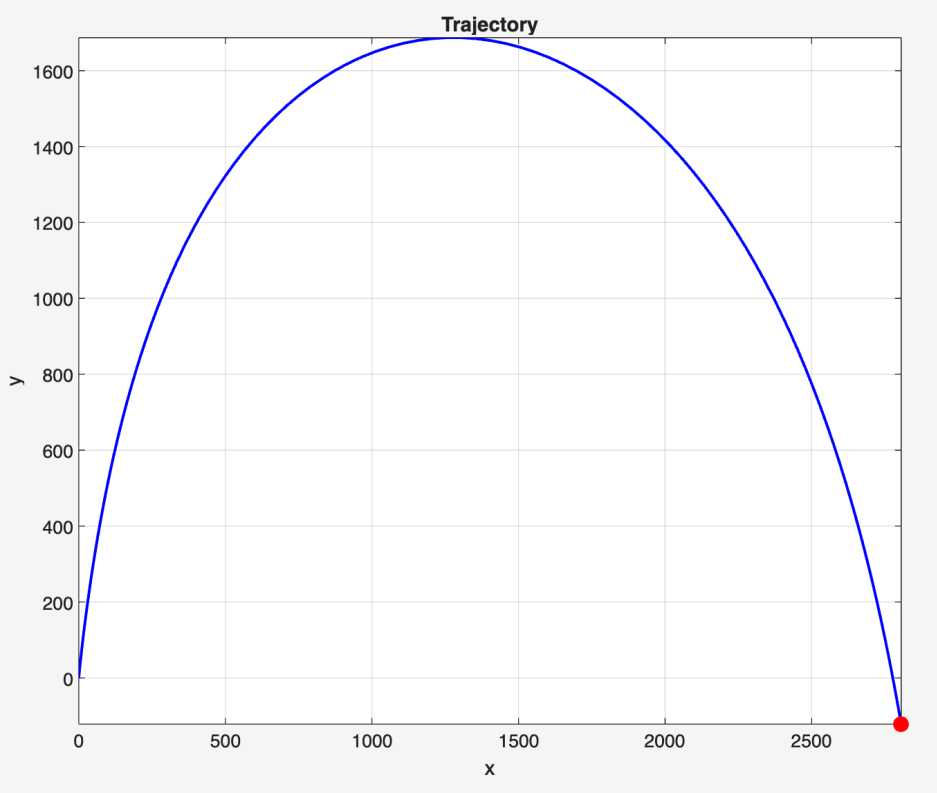
$$\Rightarrow m\ddot{\vec{r}} = -mg\hat{j} - c\dot{\vec{r}} + \frac{F\dot{\vec{r}}}{|\dot{\vec{r}}|}$$

Representing the EoMs in state space.

$$Z = \begin{bmatrix} \vec{r} \\ \dot{\vec{r}} \end{bmatrix}$$

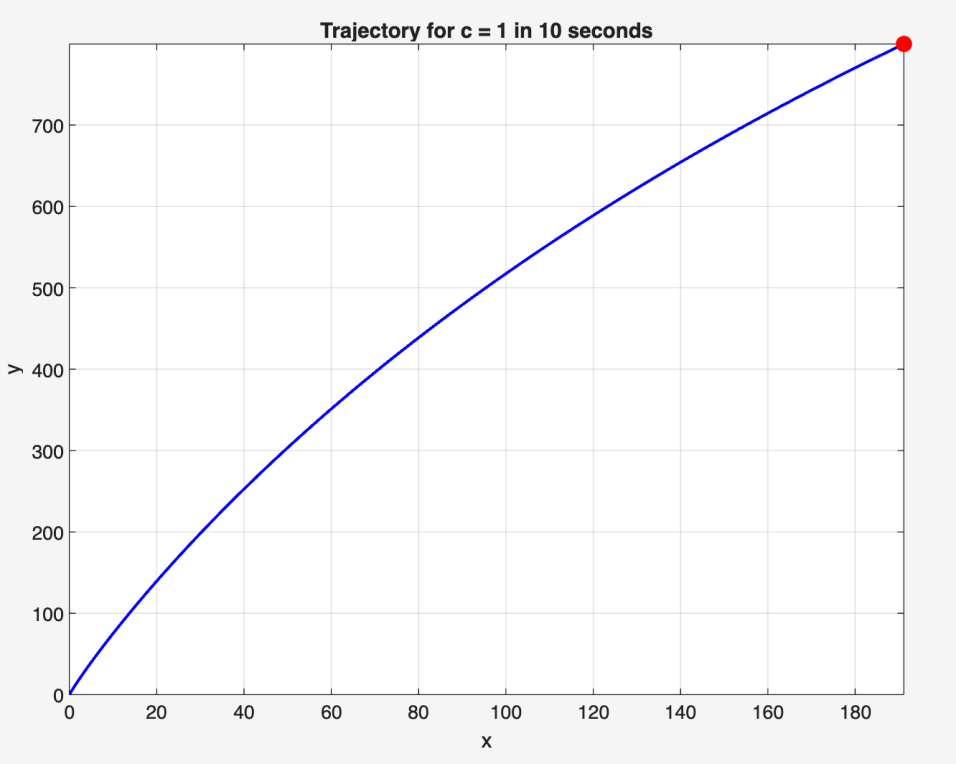
$$\dot{Z} = \begin{bmatrix} \dot{\vec{r}} \\ \ddot{\vec{r}} \end{bmatrix} = \begin{bmatrix} \dot{\vec{r}} \\ \frac{1}{m}\left[-mg\hat{j} - c\dot{\vec{r}} + \frac{F\dot{\vec{r}}}{|\dot{\vec{r}}|}\right] \end{bmatrix}$$

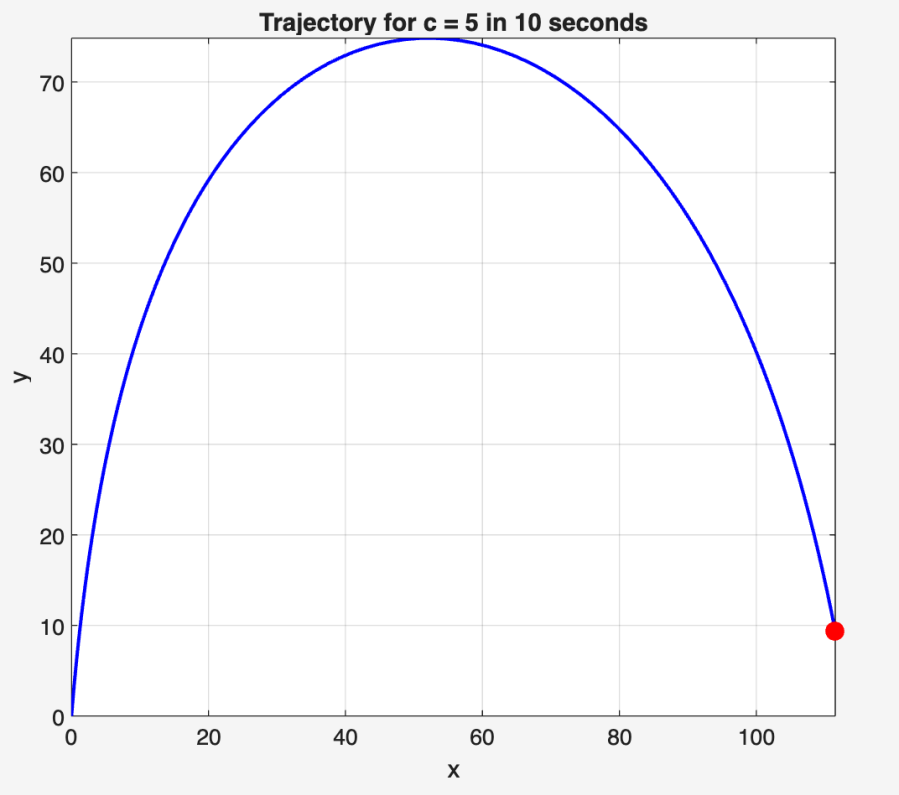
The numerical solution of this EoM is obtained on MATLAB.



The Trajectory of the jetpack found through the numerical solution of the Equations of motion is parabolic. The jetpack moves upwards, then downwards.

Initially, the initial conditions dominate and the trajectory is in the direction of initial velocity. Acceleration due to gravity dominates, changing the trajectory as it moves downward.





The coefficient of air friction affects the time taken to fall back to the ground.
In the plots above, for c = 1, the jetpack is still travelling upwards after 10 seconds, while it almost reaches the ground for c = 5. Higher air friction seems to makes the jetpack fall faster.