Ch-3. Numerical Integration

The Process of evaluating a definite from a set of tabulated values of the Integrand f(x) is called 'Numerical Integration'

→ This Process when applied to a function of Single variable is known as 'Quadrature'

☆ Trapezoidal Rule

$$\int_{a=x_0}^{b=x_0+nh} f(x)\,dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+y_3+\dots+y_{n-1})\right]$$

where, $h = \frac{b-a}{n}$

n = no of sub-intervals

☆ observation: The area of each strip is found seperately under the Curve and ordinate x_0 & x_{0+nh} is apparimatly equal to the sum of area of all the n trapezoidal.

☆ Simpson's 1/3rd / Simpson's Rule

$$\int_{a=x_0}^{b=x_0+nh} f(x)\,dx = \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+y_5+\dots)+2(y_2+y_4+y_6+\dots)\right]$$

$h = \frac{b-a}{n}$ $\quad n$ = no of sub interval while

→ ~~By~~ Appying Simpson's Rule the given Intrval must be divided into even numbes of sub Interval Since we find area of two strip at a time.

→ one can Remeber Sim $\frac{1}{3}$rd rule

$$\int_{a=x_0}^{b=x_0+nh} f(x)\,dx = \frac{h}{3}\left[(y_0+y_n) + 4(\text{odd obs}^n) + 2(\text{even}\ldots\right.$$

☆ Simpson's $\frac{3}{8}$th Rule

$$\int_{a=x_0}^{b=x_0+nh} f(x)\,dx = \frac{3h}{8}\left[(y_0+y_n) + 3(y_1+y_2+y_4+y_5+\ldots) + 2(y_3+y_6+y_9+\ldots)\right]$$

where $h = \frac{b-a}{n}$ n = no of Subinterval

→ while Appying above rule one has to take n (i.e. the no of Subintervals in multiple of 3) because we find the area of 3-strip at a time

Ex-1
5

Evaluate $\int_0^6 \frac{1}{1+x^2}\,dx$ using

(i) Tra. (ii) Simp $1/3^{th}$ (iii) Simp $3/8^{th}$

Here $f(x) = \frac{1}{1+x^2}$ $a = 0$ & $b = 6$

$h = \frac{b-a}{n} = \frac{6-0}{6} \Rightarrow \underline{h=1}$ (by taking $n=6$)

x	0	1	2	3	4	5	6
$f(x)$	1	$1/2$	$1/5$	$1/10$	$1/17$	$1/26$	$1/37$

(i) By Trapezoidal Rule

$$\int_{a=0}^{b=6} f(x)\,dx = \frac{h}{2}\left[(y_0+y_6)+2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$\int_0^6 \frac{1}{1+x^2}\,dx = \frac{1}{2}\left[(1+0.02)+2(0.5+0.2+0.1+0.05+0.03)\right]$$

$$\int_0^6 \frac{1}{1+x^2}\,dx \boxed{= 1.39}$$

(ii) $$\int_{a=0}^{b=6} f(x)\cdot dx = \frac{h}{3}\left[(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)\right]$$

$$= \frac{1}{3}\left[(1+0.02)+4(0.5+0.1+0.03)+2(0.2+0.05)\right]$$

$$\boxed{= 1.34}$$

iii) By $3/8^{th}$ Rule

$$\int_a^b f(x)\,dx = \frac{3h}{8}\left[(y_0+y_6)+3(y_1+y_2+y_4+y_5)+2(y_3)\right]$$

$$= \frac{3(1)}{8}\left[(1+0.02)+3(0.5+0.2+0.05+0.03)+2(0.1)\right]$$

$$\boxed{= 1.33}$$

Ex 5 By Simpson's Rule $\int_0^{0.6} e^{-x^2}\,dx$

$f(x) = e^{-x^2}$

$a=0 \quad b=0.6 \quad h = \frac{0.6-0}{6} = 0.1$

x	0	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)$	1	0.99	0.96	0.91	0.85	0.77	0.69

By Simpsom's Rule

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)\right]$$

$$= \frac{0.1}{3}\left[(1+0.69)+4(0.99+0.91+0.77)+2(0.966+0.85)\right]$$

$$\boxed{\doteq 1.33}$$

Ex-2 Simpson's Rule $\int_0^{\pi/2} \sin x \, dx$ use 11 ordinates

Here $f(x) = \sin x$

$a = 0 \quad b = \pi/2$

$\therefore h = \frac{b-a}{n} = \frac{\pi/2 - 0}{10} = \frac{\pi}{20}$

x	0	$\pi/20$	$2\pi/20$	$3\pi/20$	$4\pi/20$	$5\pi/20$	$6\pi/20$	$7\pi/20$	$8\pi/20$	$9\pi/20$	$10\pi/20$
$\sin x$	0	0.156	0.309	0.453	0.587	0.707	0.809	0.891	0.915	0.987	1

Simpsons Rule

$= \frac{h}{3}[(0+1) + 4(0.157 + 0.453 + 0.707 + 0.891 + 0.987) + 2(0.308 + 0.587 + 0.809 + 0.915)]$

$= 0.999$

★ Application of Simpson's Rule :

① It is very useful for civil Engg for calculating the amount of earn that must be move to fill the depression of mak a dam.

② Similally if the ordinats (i.e. value of y) denotes the velecities at equal intervals of time, the Simpson's Rule goes the distance travelled.

Ex-1 A rocket is lanched from the ground it's acceleration is registered during the first 80 sec and given in the table as shown below

	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8
t	0	10	20	30	40	50	60	70	80
a	30	31.63	33.34	35.47	37.45	40.33	43.25	46.69	50.59

using simpson's rule find the velocity Of the rocket at $t = 80$ sec.

Simpson 1/3 Rule

$$= \frac{h}{3}\left[(y_0 + y_8) + 4(y_1 + y_3 + y_5 + y_7) + 2(y_2 + y_4 + y_6)\right]$$

$$= \frac{10}{3}\left[(30 + 50.59) + 4(31.63 + 35.47 + 40.33 + 46.69) + 2(33.34 + 37.45 + 43.25)\right]$$

$$= 3083.83 \text{ cm/sec}$$

$$\boxed{V = 30.83 \text{ m/sec}}$$

Ex-2 Evaluate

$$\int_1^{5.2} \log x \, dx \text{ using}$$

Ex. 3

$$\int_0^{+1/2} \sqrt{\cos x} \, dx$$

T. rule

1/3rd Rule

3/8th Rule.

Scanned by CamScanner

Page No.
Date

2) Here $f(x) = \log x$

$a = 1$, $b = 5.2$ ($\because$ by taking $n = 6$)

$$h = \frac{b-a}{n} = \frac{5.2-1}{6} = \frac{4.2}{6} = 0.7$$

x	1	1.7	2.4	3.1	3.8	4.5	5.2
$f(x) = \log x$	0	0.5306	0.8754	1.131	1.335	1.504	1.648
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

★ Simpson 1/3 Rule

$$= \frac{h}{3}\left[(y_0 + y_6) + 4(y_2 + y_4) + 2(y_3 + y_5)\right]$$

$$= \frac{0.7}{3}\left[(0 + 1.648) + 4(0.8754 + 1.335) + 2(1.131 + 1.504)\right]$$

$$\boxed{= 4.37}$$

★ Trapezoidal Rule

$$= \frac{h}{2}\left[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5)\right]$$

$$= \frac{0.7}{2}\left[(0 + 1.648) + 2(0.5306 + 0.8754 + 1.131 + 1.335 + 1.504)\right]$$

$$\boxed{= 3.4034}$$

★ Simpson's 3/8th Rule

$$= \frac{3h}{8}\left[(y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3)\right]$$

$$\boxed{= 4.36}$$

9] $\int_0^{\pi/2} \sqrt{\cos x}\, dx$ $\quad h = \frac{b-a}{n} = \frac{\pi/2 - 0}{6} = \frac{\pi}{12}$

x	0	$\pi/12$	$2\pi/12$	$3\pi/12$	$4\pi/12$	$5\pi/12$	$6\pi/12$
$f(x)$	1	0.982	0.930	0.840	0.707	0.508	0
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

(1) By Trapezoidal Rule

$$= \frac{\pi}{12\times 2}\left[(1+0 + 2(0.982+0.930+0.840+0.707+0.508)\right]$$

$$= 1.1688$$

(2) By Simpson's 1/3 Rule

$$= \frac{\pi}{12\times 3}\left[(1+0) + 4(0.982+0.840+0.508) + 2(0.930+0.707)\right]$$

$$= 1.1856$$

(3) By Simpson's 3/8 Rule

$$= \frac{3\pi}{12\times 8}\left[(1+0) + 3(0.982+0.930+0.707+0.508) + 2(0.840)\right]$$

$$= 1.1834$$

Scanned by CamScanner

(Quadrature for.)

Gaussian Integration

So far the formulae derived for evaluation of $\int_a^b f(x)\,dx$, required the value of the function at equidistant points of interval.

Gauss derived a formula which uses the same number of functional values but with different spacing and provided better accuracy.

Gauss formula is Expressed as

$$\int_{-1}^{1} f(x)\,dx = \omega_1 f(x_1) + \omega_2 f(x_2) + \ldots \omega_n f(x_n)$$

$$= \sum_{i=1}^{x} \omega_i f(x_i)$$

where, ω_i & x_i are the weights & arguement respectively the arguement & weight for different values (points) of n is given below.

n	Formula	ω_i	x_i
$n=2$	$\int_{-1}^{1} f(x)\,dx = \omega_1 f(x_1) + \omega_2 f(x_2)$	$\omega_1 = 1$ $\omega_2 = 1$	$x_1 = -0.57735$ $x_2 = 0.57735$

Date

$n=3$ $\int_{-1}^{1} f(x)\,dx = \omega_1 f(x_1) + \omega_2 f(x_2) + \omega_3 f(x_3)$

$\omega_1 = 0.55555$ $x_1 = -0.77460$
$\omega_2 = 0.88889$ $x_2 = 0$
$\omega_3 = 0.55555$ $x_3 = 0.77460$

$n=4$ $\int_{-1}^{1} f(x)\,dx = \omega_1 f(x_1) + \omega_2 f(x_2) + \omega_3 f(x_3) + \omega_4 f(x_4)$

$\omega_1 = 0.34785$ $x_1 = -0.86114$
$\omega_2 = 0.65214$ $x_2 = -0.33998$
$\omega_3 = 0.65214$ $x_3 = 0.33998$
$\omega_4 = 0.34784$ $x_4 = 0.86114$

$n=5$ $\int_{-1}^{1} f(x)\,dx = \omega_1 f(x_1) + \omega_2 f(x_2) + \omega_3 f(x_3) + \omega_4 f(x_4) + \omega_5 f(x_5)$

$\omega_1 = 0.23693$ $x_1 = -0.9618$
$\omega_2 = 0.47863$ $x_2 = -0.53847$
$\omega_3 = 0.56889$ $x_3 = 0$
$\omega_4 = 0.47863$ $x_4 = 0.53847$
7985+ $\omega_5 = 0.23693$ $x_5 = 0.90618$

* Gauss formula imposes a restriction on the limit of integration to be of the form $\int_{-1}^{1} f(x)\,.dx$
10,143

In general, the limit of integral $\int_a^b f(x)\,.dx$ are changed to -1 to 1 by means of a transformation

$$x = \frac{1}{2}(b-a)u + \frac{1}{2}(b+a)$$

Ev-1 ? Evaluate (3.0) $\int_0^1 \frac{1}{1+x} \cdot dx$

$\Rightarrow$ first, we change the limit of Integral from (0, 1) to (-1, 1) using the Substitution as

$$x = \tfrac{1}{2}(b-a)u + \tfrac{1}{2}(b+a)$$

$$= \frac{1}{2}(1-0)u + \frac{1}{2} = \frac{u+1}{2}$$

$$\frac{dx}{du} = \frac{1}{2}$$

$$dx = \tfrac{1}{2}du$$

$$\int_0^1 \frac{1}{1+x} dx = \int_{-1}^{1} \frac{1}{u+\left(\frac{u+1}{2}\right)} \frac{1}{2} \cdot du$$

$$= \int_{-1}^{1} \frac{1}{\frac{2+u+1}{2}} \cdot \frac{1}{2} \cdot du$$

$$= \int_{-1}^{1} \frac{1}{u+3} \cdot du$$

$$= w_1 f(u_1) + w_2 f(u_2) + w_3 f(u_3)$$

$$(0.55555)\left(\frac{1}{-0.7747+3}\right) + (0.8889)\left(\frac{1}{0+3}\right) + (0.55555)\left(\frac{1}{0.7747+3}\right)$$

$$\boxed{= 0.693}$$

3 - Point G.q.f.

Ex-2 $\int_{1}^{5} \frac{1}{x} \, dx$ Ex-3 $\int_{0.2}^{1.5} e^{-x^2} dx$ Ex-4 $\int_{2}^{4} (1+x^4) \, dx$

2] $\int_{1}^{5} \frac{1}{x} \, dx$

$x = \frac{1}{2}(b-a)u + \frac{1}{2}(b+a)$

$= \frac{1}{2}(5-1)u + \frac{1}{2}(5+1)$

$x = 2u+3$

$dx = 2du$

$\int_{1}^{5} \frac{1}{x} \, dx = \int_{-1}^{1} \frac{1}{2u+3} \cdot 2 \, du = \int_{-1}^{1} \frac{1}{u+3/2} du$

$= 2 [\omega_1 f(u_1) + \omega_2 f(u_2) + \omega_3 f(u_3)]$

$= 2 [0.55555 \left(\frac{1}{(-0.7747 + 3/2)} \right) +$

$0.88889 \left(\frac{1}{0 + 3/2} \right) + 0.555 \left(\frac{1}{0.7747 + 3/2} \right)$

$= 0.7659 + 0.592 + 0.2442$

$= 1.6021$

3] $\int_{0.2}^{1.5} e^{-x^2} \, dx$

$= (1.5, 0.2)$ to $(-1, 1)$

$x = \frac{1}{2}(b-a)u + \frac{1}{2}(b+a)u$

$$= \tfrac{1}{2}(1.5 - 0.2)u + \tfrac{1}{2}(1.5 + 0.2)$$
$$= 0.65u + 0.85$$

$$\frac{dx}{du} = 0.65$$

$$\int_{0.2}^{1.5} e^{-x^2}\, dx = \int_{-1}^{1} \frac{1}{0.65u + 0.85}\, du$$

$$= \frac{1}{0.65} \int_{-1}^{1} \frac{1}{u + 1.307}$$

$$= \frac{1}{0.65} \left\{ 0.5555 \left(\frac{1}{-0.7747 + 1.307} \right) + 0.8889 \left(\frac{1}{0.1.307} \right) + 0.5555 \left(\frac{1}{0.7747 + 1.307} \right) \right.$$

$$= 3.072$$

4) $$\int_{2}^{4} (1 + x^4)\, dx$$

$(4, 2)$ to $(-1, 1)$

$$x = \tfrac{1}{2}(4 - 2)u + \tfrac{1}{2}(4 + 2)$$

$$x = 2u + 3$$

$$\frac{dx}{du} = 2$$

$$dx = 2du$$

$$\int_{0.2}^{1.5} e^{-x^2}\,dx = \int_{-1}^{1} e^{-(0.65u+0.85)^2} \cdot 0.65\, du$$

$$= 0.65\left[w_1 f(u_1) + w_2 f(u_2) + w_3 f(u_3)\right]$$

$$= 0.65\left[(0.55555\left(e^{-(0.65(-0.7745)+0.85)^2}\right)\right.$$

$$+ (0.88884)\left(e^{-(0.65(0)+0.85)^2}\right)$$

$$\left. + 0.55555\left(e^{-(0.65(0.7746)+0.85)^2}\right)\right]$$

$$= \cancel{0.85}\quad 0.65860$$

Assignment - 6

Q-46 Evaluate $\int_0^1 e^{-x^2} dx$ by using trapezoidal rule and simpson's 1/3 Rule with $n = 10$

$f(x) = e^{-x^2}$ $\quad h = \frac{b-a}{n} = \frac{1}{10} = 0.1$

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$f(x)$	1	0.99	0.96	0.91	0.85	0.77	0.69	0.61	0.52	0.44	0.36
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}

By trapezoidal rule

$$f(x) = \frac{h}{2}\left[(y_0 + y_{10}) + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9)\right]$$

$$= 0.778$$

By Simpson's 1/3 Rule

$$f(x) = \frac{h}{3}\left[(y_0 + y_{10}) + 4(y_1 + y_3 + y_5 + y_7 + y_9) + 2(y_2 + y_4 + y_6 + y_8)\right]$$

$$= \frac{0.1}{3}\left[(1 + 0.36) + 4(0.99 + 0.91 + 0.77 + 0.61 + 0.44) + 2(0.96 + 0.85 + 0.69 + 0.52)\right]$$

$$= 0.742$$

Scanned by CamScanner

47) $\int_{-2}^{6} (1+x^2)^{3/2}$ Simpson $\frac{1}{3}$ Rule $n=6$

four digit after decimal

$h = \frac{6+2}{6} = \cancel{1.3333} = 4/3$

x	-2	$-2/3$	$2/3$	2	$10/3$	$14/3$	6
$f(x)$	11.1803	1.7360	1.7360	11.1803	42.1479	108.7094	225.0622
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$$f(x) = \frac{h}{3}\left[(y_0+y_6) + 2(y_2+y_4) + 4(y_1+y_3+y_5)\right]$$

$$= \frac{4}{9}\left\{\left[(11.1803 + 225.0622) + 2(1.7360 + 42.1479) + 4(1.7360 + 11.1803 + 108.7094)\right]\right\}$$

$$\boxed{= 360.2280}$$

48) $\int_0^1 \frac{1}{1+x^2}$ $n=10$ Simpson's $\frac{3}{8}$ Rule

$h = \frac{1}{10}$

~~x 0 $\frac{1}{10}$ $\frac{1}{20}$ $\frac{1}{30}$ $\frac{1}{40}$ $\frac{1}{50}$ $\frac{1}{60}$ $\frac{1}{70}$ $\frac{1}{80}$ $\frac{1}{90}$ $\frac{1}{10}$~~

~~$f(x)$~~

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$f(x)$	1	0.99	0.961	0.917	0.862	0.8	0.735	0.671	0.609	0.55	0.5
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}

$$f(x) = \frac{3h}{8}\left[(y_0 + y_{10}) + 2(y_3 + y_6 + y_9)\right.$$

$$\left. + 3(y_2 + y_4 + y_5 + y_7 + y_8)\right]$$

$$= \frac{3(0.1)}{8}\left[(1+0.5)+2(0.917+0.735+0.55) + 3(0.961+0.862+0.8+0.671+0.609)\right]$$

$$= 0.6604$$

49) $\int_0^{1.2} \ln(1+x^2)\,dx$ using Simpson's Rule

take $n=6$ $\therefore h = \frac{1.2-0}{6} = 0.2$

x	0	0.2	0.4	0.6	0.8	0.1	1.2
$f(x)$	~~0~~	~~0.693~~	~~1.609~~	~~2.302~~	~~2.833~~	~~3.258~~	~~0.788~~
	0	0.039	0.148	0.307	0.494	9.9×10^{-3}	0.891
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Simpson's Rule

$$= h/3\left[(y_0+y_6)+4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$

$$= \frac{0.2}{2}\left[(0+0.891)+4(0.039+0.307+9.9\times10^{-3}) + 2(0.148+0.494)\right]$$

$$= 0.35986$$

50) calculate the value of Integral $\int_4^{5.2} \ln x \cdot dx$
$n = K$

$h = \frac{5.2-4}{6}$ $\qquad h = 0.2$

x	4	4.2	4.4	4.6	4.8	5	5.2
$f(x)$	1.386	1.435	1.481	1.526	1.568	1.609	1.648
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

using Simpson's Rule

$$f(x) = h/3 \left[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_4 + y_2) \right]$$

$$= \frac{0.2}{3} \left[(1.386 + 1.648) + 4(1.435 + 1.526 + 1.609) + 2(1.4[illegible] + 1.568) \right]$$

$= 1.827$

51] Apply Trapezoidal & Simpson Rule $\int_0^1 \sqrt{1-x^2}\, dx$

$$h = \frac{1-0}{6} = \frac{1}{6}$$

~~x~~	~~0~~	~~1/1~~	~~1/2~~	~~1/3~~	~~1/4~~	~~1/5~~	~~1/6~~
~~f(x)~~	~~1~~	~~0~~	~~0.866~~	~~0.942~~	~~0.968~~		

x	0	$1/6$	$1/3$	$1/2$	$2/3$	$5/6$	1
$f(x)$	1	0.986	0.942	0.866	0.745	0.552	0
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Trapezoidal Rule

$$= h/2 \left[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5) \right]$$

$= 0.7651$

0.0216

Simpson 1/3 Rule

$$= h/3 \left[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_4 + y_2) \right]$$

$= 0.7772$

Scanned by CamScanner

52) Evaluate $\int_0^1 e^{-x^2} dx$ by using gaussian Integration formula for $n=3$

$x = \frac{1}{2}(b-a)u + \frac{1}{2}(b+a)$

$= \frac{1}{2}(1-0)u + \frac{1}{2}(1+0)$

$x = \frac{u+1}{2}$

$dx = \frac{1}{2} du$

$\int_0^1 e^{-x^2} \cdot dx = \int_{-1}^{1} \frac{1}{2} e^{-\left(\frac{u+1}{2}\right)^2} \cdot du$

$= w_1 f(u_1) + w_2 f(u_2) + w_3 f(u_3)$

$= 0.55555\left(e^{-\left(\frac{-0.77460+1}{2}\right)^2}\right) + 0.88889 \left(e^{-\left(\frac{0+1}{2}\right)^2}\right) + 0.55555\left(e^{-\left(\frac{0.77460+1}{2}\right)^2}\right)$

$= (0.\cancel{1094}5151 + 0.692 + 0.1094)\frac{1}{2}$

$= \cancel{1.3165}$ $\quad 0.6582$

53) $\int_0^{\pi/2} \sin x \, dx \quad n=3 \quad$ g.q

$x = \frac{1}{2}(\pi/2 - 0)u + \frac{1}{2}(\pi/2 + 0)$

$= \pi/4 \, u + \pi/4$

$= \frac{\pi u + \pi}{4}$

$dx = \pi/4 \cdot du$

$\int_{-1}^{1} \sin \frac{\pi}{4}(u+1) \cdot \pi/4 \cdot du$

Scanned by CamScanner

⑥

$= \frac{\pi}{4} \{ 0.55555(0.1770) + 0.88889(0.70...)$
$0.55555(0.984) \}$

$= 0.9996$

54) $\int_{-1}^{1} \frac{1}{1+x^2} dx$ $n=2$, $n=3$

$n=2$

$= \omega_1 f(x_1) + \omega_2 f(x_2)$

$= 1(0.75) + 1(0.75) = 1.5$

$= \omega_1 f(x_1) + \omega_2 f(x_2) + \omega_3 f(x)$

$= 0.55555(0.6249) + 0.8889(1) + 0.55555(0.6249)$

$= 1.5832$

55) $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta$ Simpson's Rule $h = \pi/12$

$\frac{\pi}{12} = \frac{\pi/2}{n} \Rightarrow n = 6$

x	0	$\pi/12$	$\pi/6$	$\pi/4$	$\pi/3$	$5\pi/12$	$\pi/2$
$f(x)$	0	0.5087	0.7071	0.8409	0.9306	0.9828	1.0
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$= \frac{\pi}{36} [(0+1) + 4(0.5087 + 0.8409 + 0.9828) + 2(0.7071 + 0.9306)]$

$= 1.1873$

56) $\int_0^3 \sin x^2 \, dx$, Simpson's 3/8, Simpson's 1/3

$n = 6$

$h = \frac{3-0}{6} = \frac{1}{2}$

x	0	0.5	1	1.5	2	2.5	3
$f(x)$	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	0	4.36×10^{-3}	0.017	0.039	0.069	0.108	0.156
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Simpson's 3/8 Rule =

$$= \frac{3h}{8}\left[(y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3)\right]$$

$= 0.1824$

Simpson's 1/3 Rule

$$= h/3\left[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$$

$= 0.1555$

57) $\int_0^1 \frac{dt}{1+t}$ G.Q. three Point

$t = \frac{1}{2}(1-0)x + \frac{1}{2}(1+0)$

$= \frac{x+1}{2}$

$dt = \frac{1}{2} \cdot dx$

$= w_1 f(x_1) + w_2 f(x_2) + w_3 f(x_3)$

$$\frac{1}{2}\int_{-1}^{1} \frac{dx}{1+\left(\frac{x+1}{2}\right)}$$

$$= \frac{1}{2}\{w_1 f(x_1) + w_2 f(x_2) + w_3 f(x_3)\}$$

$$= \frac{1}{2}\{0.55555(0.8987) + (0.88889)(0.6666) + 0.55555(0.5298)\}$$

$$= \underline{0.6930}$$

58) $\int_{1}^{3} \sin x \, dx$ ... $n = 5$

$$x = \frac{1}{2}(3-1)y + \frac{(3+1)}{2}$$

$$= \underline{y+2}$$

$$dx = \underline{dy}$$

$$f(x) = \int_{-1}^{1} \sin(y+2)\, dy$$

$$= w_1 f(y_1) + w_2 f(y_2) + w_3 (f(y_3)) + w_4 f(y_4) + w_5 f(y_5)$$

$$= 0.23693(0.0181) + (0.47863)(0.0255) + 0.56889(0.0348) + 0.47863(0.0442) + 0.23693(0.0507)$$

$$= \underline{0.0694}$$

Scanned by CamScanner

Date

59) A River is 80 meters wide, the depth 'd' in meters at distance x meters from one bank is given by the following table. Calculate the area of cross section of the river using Sim. 1/3 Rule

x	0	10	20	30	40	50	60	70	80
y	0	4	7	9	12	15	14	8	7

$\Rightarrow$ $a = 0$ $\quad b = 80$ $\quad n = 8$

$h = \frac{80 - 0}{8} = 10$

Simpson's 1/3 Rule

$$= \frac{10}{3}\left[(y_0 + y_8) + 4(y_1 + y_3 + y_5 + y_7) + 2(y_2 + y_4 + y_6) \right]$$

$$= 10/3 \left[(0+7) + 4(4+9+15+8) + 2(7+12+14) \right]$$

$$= 723.33\, m^2$$