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Numerical Methods

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Ch-1. Interpolations

Defn: It is a Process of estimating the value of y (dependent Variable or ordinate) corresponding to Intermediate value of x (Independent variable or argument) from the given data

* Finite Differences:

→ The study of F.D. deals with the changes happen in function (or dependent variable) with respect to finite changes in the ~~a~~ independent Variable.

→ Let the function $y = f(x)$ be given for the equally spaced (equidistant) value of x (i.e. $x = x_0, x_0+h, x_0+2h, \ldots x_0+nh$) giving the corresponding of $y = y_0, y_1, y_2 \ldots y_n$

→ To determine the value of function $f(x)$ for any Intermediate value of x, the following 3-types of finite difference are found to be usefull.

1) Forward Differences:

when the differences $y_1 - y_0$, $y_2 - y_1$, $y_3 - y_2, \ldots, y_n - y_{n-1}$ are denoted by $\Delta y_0, \Delta y_1, \Delta y_2 \ldots \Delta y_{n-1}$ respectively then the differences are said to be

→ (First Order) forward differences where, Δ (Delta) is a forward difference operator.

i.e. first order forward differences

$$\Delta y_0 = y_1 - y_0$$
$$\Delta y_1 = y_2 - y_1$$
$$\Delta y_2 = y_3 - y_2$$
$$\vdots \qquad \vdots$$
$$\Delta y_{n-1} = y_n - y_{n-1}$$

Similarly, 2^{nd} order forward difference

$$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$$
$$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$$
$$\vdots \qquad \vdots$$
$$\Delta^2 y_{n-1} = \Delta y_n - \Delta y_{n-1}$$

IIIy, 3^{rd} Order finite difference

$$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$$
$$\Delta^3 y_1 = \Delta^2 y_2 - \Delta^2 y_1$$
$$\vdots \qquad \vdots$$
$$\Delta^3 y_{n-1} = \Delta^2 y_n - \Delta^2 y_{n-1}$$

In general p^{th} order f.d.

$$\Delta^p y_{n-1} = \Delta^{p-1} y_n - \Delta^{p-1} y_{n-1}$$

⇒ The differences are separately given in the following table

x	y	Δ	Δ^2	Δ^3	Δ^4
$x_0 = x_0$	y_0				
		$\Delta y_0 = y_1 - y_0$			
$x_1 = x_0 + h$	y_1		$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$		
		$\Delta y_1 = y_2 - y_1$		$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$	
$x_2 = x_0 + 2h$	y_2		$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$		$\Delta^4 y_0 = \Delta^3 y_1 - \Delta^3 y_0$
		$\Delta y_2 = y_3 - y_2$		$\Delta^3 y_1 = \Delta^2 y_2 - \Delta^2 y_1$	
$x_3 = x_0 + 3h$	y_3		$\Delta^2 y_2 = \Delta y_3 - \Delta y_2$		
		$\Delta y_3 = y_4 - y_3$			
$x_4 = x_0 + 4h$	y_4				

Ex-1 ~~Consulte~~ construct the forward difference table for following data

x	1	2	3	4	5
y	100	212	145	676	786

x	y	Δ	Δ^2	Δ^3	Δ^4
1	$y_0 = 100$				
		$\Delta y_0 = 112$			
2	$y_1 = 212$		$\Delta^2 y_0 = -179$		
		$\Delta y_1 = -67$		$\Delta^3 y_0 = 777$	
3	$y_2 = 145$		$\Delta^2 y_1 = 598$		$\Delta^4 y_0 = -1796$
		$\Delta y_2 = 531$		$\Delta^3 y_1 = -1019$	
4	$y_3 = 676$		$\Delta^2 y_2 = -421$		
		$\Delta y_3 = 110$			
5	$y_4 = 786$				

2) Backward Differences

→ When the difference $y_1 - y_0$, $y_2 - y_1$, ..., $y_n - y_{n-1}$ are denoted by ~~$\Delta y_1, \Delta y_2$~~ ∇y_1, ∇y_2, ∇y_3 ... ~~Δ~~ ∇y_n respectively then the difference are said to be (1st order) Backward difference

where, ∇(nabla), is the backward operator

i.e. first order backward differences

$$\nabla y_1 = y_1 - y_0$$

$$\nabla y_2 = y_2 - y_1$$

$$\nabla y_3 = y_3 - y_2$$

$$\vdots$$

$$\nabla y_n = y_n - y_{n-1}$$

Ily 2nd order backward differences

$$\nabla^2 y_2 = \nabla y_2 - \nabla y_1$$

$$\nabla^2 y_3 = \nabla y_3 - \nabla y_2$$

Ily, $\nabla^n y_n = \nabla y_n - \nabla y_{n-1}$

||y, 3rd order backward difference

$$\nabla^3 y_3 = \nabla^3 y_3 - \nabla^2 y_2$$

in general p^{th} order backward diff.

$$\nabla^p y_n = \nabla^{p-1} y_n - \nabla^{p-1} y_{n-1}$$

These differences are separtly given in following table.

x	y	∇	∇^2	∇^3	∇^4
x_0	y_0				
		$\nabla y_1 = y_1 - y_0$			
x_1	y_1		$\nabla^2 y_2 = \nabla y_2 - \nabla y_1$		
		$\nabla y_2 = y_2 - y_1$		$\nabla^3 y_3 = \nabla^2 y_2 - \nabla^2 y_2$	
x_2	y_2		$\nabla^2 y_3 = \nabla y_3 - \nabla y_2$		$\nabla^4 y_4 = \nabla^3 y_4 - \nabla^3 y_3$
		$\nabla y_3 = y_3 - y_2$		$\nabla^3 y_4 = \nabla^2 y_4 - \nabla^2 y_3$	
x_3	y_3		$\nabla^2 y_4 = \nabla y_4 - \nabla y_3$		
		$\nabla y_4 = y_4 - y_3$			
x_4	y_4				

Ex-1 Construct Backward diff. table for the following table

x	1	2	3	4	5
y	520	321	725	527	777

x	y	∇	∇^2	∇^3	∇^4
1	520				
		$\nabla y_1 = -199$			
2	321		$\nabla^2 y_2 = 603$		
		$\nabla y_2 = 404$		$\nabla^3 y_3 = -1205$	
3	725		$\nabla^2 y_3 = -602$		$\Delta^4 y_4 = 2255$
		$\nabla y_3 = -198$		$\nabla^3 y_4 = 1050$	
4	527		$\nabla^2 y_4 = 448$		
		$\nabla y_4 = 250$			
5	777				

3| Central Differences :

☆ Operators / Relation ship betn operators

<1> Delta (Δ)

$$\Delta f(x) = f(x+h) - f(x)$$

<2> Nabla (∇)

$$\nabla f(x) = f(x) - f(x-h)$$

<3> delta (δ)

$$\delta f(x) = f(x + h/2) - f(x - h/2)$$

<4> Shift operator (E)

Date

It is an operator which increases argument (i.e. x)

i.e. $E(f(x)) = Ef(x) = f(x+h)$

$$E^2(f(x)) = E(Ef(x))$$
$$= E(f(x+h))$$
$$= f(x+h+h)$$
$$= f(x+2h)$$

||ly, $E^3(f(x)) = f(x+3h)$

<5> Inverse shift operator (E^{-1})

It is an operator which decreases the argument (i.e. x) by h

i.e. $E^{-1}f(x) = E^{-1}(f(x))$
$$= f(x-h)$$

||ly, $E^{-2}f(x) = f(x-2h)$

$$\boxed{E^{-n}f(x) = f(x-nh)}$$

<6> Averaging operator (μ)

It is defined as

$$\boxed{\mu(f(x)) = \frac{1}{2}\left\{f(x+h/2) + f(x-h/2)\right\}}$$

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* Relation betⁿ Operators

1) $\Delta = E - 1$
2) $\nabla = 1 - E^{-1}$
3) $\delta = E^{1/2} - E^{-1/2}$
4) $\mu = \frac{1}{2}(E^{1/2} + E^{-1/2})$
5) $\Delta = E\nabla = \nabla E = \delta E^{1/2}$
6) $E = e^{hD}$

It useal notations , Prove that

<1> $\Delta f(x) = f(x+h) - f(x)$

$= Ef(x) - f(x)$

$(\because Ef(x) = f(x+h))$

$\Delta f(x) = (E-1)f(x)$

$\Delta = E - 1$

OR

$E = 1 + \Delta$

<2> $\nabla f(x) = f(x) - f(x-h)$

$= f(x) - E^{-1} f(x)$

$(\because E^{-1} f(x) = f(x-h))$

$\nabla f(x) = (1 - E^{-1}) f(x)$

$\nabla = 1 - E^{-1}$

OR

$E^{-1} = 1 - \nabla$

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<3> $\delta = E^{1/2} - E^{-1/2}$

$\delta f(x) = f(x + h/2) - f(x - h/2)$

$= E^{-1/2} f(x) - E^{1/2} f(x)$

$\left(\because E^{1/2} f(x) = f(x + h/2) \quad E^{-1/2} f(x) = f(x - h/2) \right)$

$\delta f(x) = E^{1/2} - E^{-1/2}) f(x)$

$\delta = E^{1/2} - E^{-1/2}$

<4> $\mu = \frac{1}{2}(E^{\frac{1}{2}} + E^{-\frac{1}{2}})$

$\mu f(x) = \frac{1}{2} \{ f(x + h/2) + f(x - h/2) \}$

$= \frac{1}{2} \{ E^{1/2} f(x) + E^{-1/2} f(x) \}$

$(\because E^{1/2} f(x) = f(x + h/2))$

$(\because E^{-1/2} f(x) = f(x - h/2))$

$\mu f(x) = \frac{1}{2}(E^{1/2} + E^{-1/2}) f(x)$

$\mu = \frac{1}{2}(E^{\frac{1}{2}} + E^{-\frac{1}{2}})$

<5> $\Delta = E\nabla = \nabla E = \delta E^{1/2}$

$1 = 2 = 3 = 4$

→ $2 = 1$

$E\nabla f(x) = E(\nabla f(x))$

$= E(f(x) - Ef(x-h))$

$= f(x+h) - f(x - h + h)$

$= f(x+h) - f(x)$

$E\nabla f(x) = \Delta f(x)$

$E\nabla = \Delta$

Date

→ (3) = (1)

$$\nabla E f(x) = \nabla(E f(x))$$
$$= \nabla(f(x+h))$$
$$= f(x+h) - f(x+h-h)$$
$$= f(x+h) - f(x)$$
$$\nabla E f(x) = \Delta f(x)$$
$$\nabla E = \Delta$$

→ (4) = (1)

$$\delta E^{1/2} f(x) = \delta(E^{1/2} f(x))$$
$$= \delta(f(x + h/2))$$
$$= f(x + h/2 + h/2) - f(x + h/2 - h/2)$$
$$= f(x+h) - f(x)$$
$$= \Delta f(x)$$
$$\delta E^{1/2} f(x) \quad \delta E^{1/2} = \Delta$$

<6> $E = e^{hD}$

$$E f(x) = f(x+h)$$

Expand the R.H.S. Taylor's Series

$$= f(x) + h f'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) + \ldots$$

Now,

$$f'(x) = \frac{d}{dx} f(x)$$
$$= D f(x)$$
$$\left(\because D = \frac{d}{dx}\right)$$
$$= f(x) + h D f(x) + \cancel{D^2 f} \frac{h^2}{2!} D^2 f(x) + \frac{h^3}{3!} D^3 f(x) + \ldots$$

$$Ef(x) = \left(I + hD + \frac{h^2D^2}{2!} + \frac{h^3D^3}{3!} + \dots\right) f(x)$$

$$E = e^{hD}$$

$\Rightarrow$ but we know that

$$E = I + \Delta$$

$$I + \Delta = e^{hD}$$

$$\log(I + \Delta) = hD$$

<7>
5

$$hD = \sinh^{-1}(\mu\delta)$$

$$\mu\delta f(x) = \mu(\delta f(x))$$

$$= \mu\left(f(x + h/2) - f(x - h/2)\right)$$

$$= \mu f(x + h/2) - \mu f(x - h/2)$$

$$= \left[\frac{1}{2}\left\{f(x + h/2 + h/2) + f(x + h/2 - h/2)\right\}\right]$$

$$- \left[\frac{1}{2}\left\{f(x - h/2 + h/2) + f(x - h/2 - h/2)\right\}\right]$$

$$= \frac{1}{2}\left[f(x+h) - f(x) - f(x) - f(x-h)\right]$$

$$= \frac{1}{2}\left[f(x+h) - f(x) + f(x) - f(x-h)\right]$$

$$= \frac{1}{2}\left[\Delta f(x) + \nabla f(x)\right]$$

$$= \frac{1}{2}(\Delta + \nabla) f(x)$$

$$= \frac{1}{2}(E - 1 + 1 - E^{-1})$$

$$= \frac{1}{2}(E - E^{-1}) \quad \left\{\because \begin{array}{l} \Delta = E - 1 \\ \nabla = I - E^{-1} \end{array}\right\}$$

$$\mu\delta = \frac{1}{2}\left(e^{hD} - e^{-hD}\right)$$

$$\mu\delta = \sinh(hD)$$

$$\boxed{hD = \sinh^{-1}(\mu\delta)}$$

$$\nabla = 1 - e^{-hD}$$

$$\Delta\nabla = \nabla\Delta = \delta^2$$

> $\nabla = 1 - e^{-hD}$

$$\nabla = 1 - E^{-1}$$

$$\nabla = 1 - e^{-hD}$$

(9) $\Delta\nabla = \nabla\Delta = \delta^2$

(1) = (2) = (3)

⇒ (1) = (3)

$$\Delta\nabla = (-1 + E)(1 - E^{-1})$$

$$= \cancel{1} \cancel{+ E^{-1}} - \cancel{EE^{-1}} \cancel{+ E}$$

$$= E - EE^{-1} - 1 + E^{-1}$$

$$= \cancel{1} - \cancel{E^{-1}} - \cancel{1} \cancel{+ E} = E - 1 - 1 + E^{-1}$$

$$= E + E^{-1} - 2 \quad \text{—— (1)}$$

⇒ $\delta^2 = \left(E^{\frac{1}{2}} - E^{-\frac{1}{2}}\right)^2$

$$= \left(E^{\frac{1}{2}}\right)^2 + \left(E^{-\frac{1}{2}}\right)^2 = E - 2 + E^{-1} \quad \text{—— (2)}$$

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<10> $(I + \Delta)(I - \nabla) = 1$

<11> $\Delta - \nabla = \Delta\nabla$

<12> $\frac{\Delta}{\nabla} - \frac{\nabla}{\Delta} = \Delta + \nabla$

<13> $\mu^2 = 1 + \frac{\delta^2}{4}$

☆ Result :-

Polynomial Differences

→ The n^{th} differences of a Polynomial of the n^{th} ~~negative~~ degree are constant & all higher order differences are zero

for eg ⇒ $\Delta^5 [(1 - ax)(1 - bx^2)(1 - cx^2)]$

$\Rightarrow \Delta^5 [-abc\,x^5 + (\ )x^4 + (\ )x^3 + (\ )x^2 + (\ )x + c$

$\Rightarrow -abc\,(5!)$

☆ Newton Forward Interpolation formula (N.F.I.F.) OR (Gregory-Newton F.I.F.)

$$y_p = y_0 + \frac{p\Delta y_0}{1!} + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$+ \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0 + \ldots$$ is called N-

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0$$
$$+ \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 +$$
$$\frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0 + \dots$$

is called N.F.I.F $\quad p = \frac{x - x_0}{h}$

★ Newton Backward Interpolation Formula (N.B.I.F.)

or

(Gregory - newton B.I.F.)

$$y_p = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n$$

$$+ \frac{p(p+1)(p+2)}{3!} + \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_n +$$

is called N.B.I.F., $p = \frac{x - x_n}{h}$

Note

① Find the tabulated value near the ~~be~~ table. we are N.F.I.F.

② To find a value over the end of the table. we are N.B.I.F.

Ex-1
5

Estimate $f(22)$, ~~(28)~~ from following data using appropriate Interpolation formula

x	20	25	30	35	40	45
$f(x)$	354	332	291	260	231	204

$\Rightarrow x = 22,\quad h = 5,\quad x_0 = 20$

$$p = \frac{x - x_0}{h} = \frac{22 - 20}{5} = \frac{2}{5} = 0.4$$

x	$f(x)$	Δ	Δ^2	Δ^3	Δ^4
$x_0 = 20$	$y_0 = 354$				
		$\Delta y_0 = -22$			
$x_1 = 25$	$y_1 = 332$		$\Delta^2 y_0 = -19$		
		$\Delta y_1 = -41$		$\Delta^3 y_0 = 29$	
$x_2 = 30$	$y_2 = 291$		$\Delta^2 y_1 = 10$		$\Delta^4 y_0 = -37$
		$\Delta y_2 = -31$		$\Delta^3 y_1 = -8$	
$x_3 = 35$	$y_3 = 260$		$\Delta^2 y_2 = 2$		$\Delta^4 y_1 = 8$
		$\Delta y_3 = -29$		$\Delta^3 y_2 = 0$	
$x_4 = 40$	$y_4 = 231$		$\Delta^2 y_3 = 2$		
		$\Delta y_4 = -27$			
$x_5 = 45$	$y_5 = 204$				

Now N.F.I.F.

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0$$

$$y_p = 354 + (0.4)(-22) + \frac{(0.4)(0.4-1)(-19)}{2!}$$

$$+ \frac{(0.4)(0.4-1)(0.4-2)}{6}(29)$$

$$+ \frac{(0.4)(0.4-1)(0.4-2)(0.4-3)}{24}(-37)$$

$$= 354 - 8.8 + 10 + 1.866 + 1.583$$

$$\boxed{y_p = 358.639}$$

<u>H.W</u>

<10> $(1+\Delta)(1-\nabla) = 1$

$(1+\Delta)(1-\nabla) = (1 + E - 1)(1 - 1 + E^{-1})$

$= (\cancel{1} + E - \cancel{1})(\cancel{1} - \cancel{1} + E^{-1})$

$= 1$

<11> $\Delta - \nabla = \Delta\nabla$

L.H.S. $\Rightarrow E - 1 - 1 + E^{-1}$

$= E + E^{-1} - 2$

R.H.S. $= (E-1)(1 - E^{-1})$

$= (\cancel{E} - \cancel{EE^{-1}} - 1 + \cancel{E^{-1}})$

$= \cancel{E} - \cancel{E^{-1}} - 1 + E^{-1}$

$= E - EE^{-1} - 1 + E^{-1}$

$= E + E^{-1} - 2$

<12> $\frac{\Delta}{\nabla} - \frac{\nabla}{\Delta} = \nabla + \Delta$

L.H.S. $= \frac{\Delta^2 - \nabla^2}{\Delta\nabla}$

$$= \frac{(E-1)^2 - (1-E^{-1})^2}{E+E^{-1}-2}$$

$$= \frac{E-1-1+E^{-1}}{E+E^{-1}-2} = \frac{E+E^{-1}-2}{E+E^{-1}-2} = 1$$

R.H.S. $= \nabla + \Delta$

$$= 1 - E^{-1} + E - 1$$

$$= E^2 - 2E + 1 - 1 - 2E^{-1} + E^{-2}$$

L.H.S. $= \dfrac{\Delta^2 - \nabla^2}{\Delta\nabla} = \dfrac{(\Delta+\nabla)(\Delta-\nabla)}{\Delta\nabla}$

= but we know that $\Delta - \nabla = \Delta\nabla$

$$= \frac{(\Delta+\nabla)(\Delta\nabla)}{\Delta\nabla} = \Delta + \nabla$$

L.H.S.

13) $\mu^2 = 1 + \dfrac{\delta^2}{4}$

$$\mu^2 = 1 + \frac{(E^{1/2} - E^{-1/2})^2}{4}$$

$$= \frac{4 + (E + E^{-1} - 2E^{1/2}E^{-1/2})}{4}$$

$$= \frac{(E + E^{-1} + 2)}{4}$$

$$= \frac{(E^{1/2} + E^{-1/2})^2}{4}$$

$\mu^2 = \mu^2$

Proved

Ex-1 The area of circle of diameter is given for the following values:

d	80	85	90	95	100
A	5026	5674	6362	7088	7854

A--12

Calculate the area, if diameter is 105, using appropriate interpolation formula;

$x = 105$, $x_n = 105$ $h = 5$

$p = \frac{x - x_n}{h} = \frac{105 - 100}{5} = 1$

As $x = 105$ lies near $x_n = 100$ which is at the end of table we use N.B.I.F.

d	A	∇	∇^2	∇^3	∇^4
$x_0 = 80$	$y_0 = 5026$				
		$\nabla y_1 = 648$			
$x_1 = 85$	$y_1 = 5674$		$\nabla^2 y_2 = 40$		
		$\nabla y_2 = 688$		$\nabla^3 y_3 = -2$	
$x_2 = 90$	$y_2 = 6362$		$\nabla^2 y_3 = 38$		$\nabla^4 y_4 = 4$
		$\nabla y_3 = 726$		$\nabla^3 y_4 = 2$	
$x_3 = 95$	$y_3 = 7088$		$\nabla^2 y_4 = 40$		
		$\nabla y_4 = 766$			
$x_4 = 100$	$y_4 = 7854$				

by N.B.I.F.

$y_p = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n +$

$\frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_n$

$$y_p = 7854 + (1)(766) + \frac{(1)(2)(80)}{2} +$$

$$\frac{(1)(2)(3)(2)}{6} + \frac{(1)(2)(3)(4)(4)}{24}$$

$$= 7854 + 766 + 40 + 2 + 4$$

$$\boxed{y_p = 8666}$$

Ex-2.5 The following table is marks of a student in a certain test

Marks	30-40	40-50	50-60	60-70	70-80
No of student	31	42	51	35	31

calculate the number of student who optained marks bet^n 40-45, using appropriate Interporation method

⇒ First we prepare Cumulative frequency table

Marks	40	50	60	70	80
No of student	31	73	124	159	190

Now, we find the no. of student who obtained marks (≤45) (i.e. y_{45})

$x = 45$, $x_0 = 40$

$h = 10$

$$p = \frac{x - x_0}{h} = \frac{45 - 40}{10} = \frac{5}{10} = 0.5$$

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5
$x_0=40$	$y_0=31$					
		$\Delta y_0=42$				
$x_1=50$	$y_1=73$		$\Delta^2 y_0=9$			
		$\Delta y_1=51$		$\Delta^3 y_0=-25$		
$x_2=60$	$y_2=124$		$\Delta^2 y_1=-16$		$\Delta^4 y_0=37$	
		$\Delta y_2=35$		$\Delta^3 y_1=12$		
$x_3=70$	$y_3=159$		$\Delta^2 y_2=-4$			
		$\Delta y_3=31$				
$x_4=80$	$y_4=190$					

by N.F.I.F.

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$
$$+ \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0 + \dots$$

$$= 31 + (0.5)(42) + \frac{(0.5)(0.5-1)(9)}{2}$$
$$+ \frac{(0.5)(0.5-1)(0.5-2)(-25)}{6} + \frac{(0.5)(0.5-1)(0.5-2)(0.5-3)(37)}{24}$$

$$= 31 + 21 - 1.125 - 1.5625 + 2.167$$

$$\boxed{y_p = 47.88 \approx 48}$$

No of student who obtained marks (≤ 45) is 48 & from the table no of students who obtained marks (≤ 40) is 31

$\therefore$ The no of student who obtained marks betn 40-45 = 48-31 = 17

Ex-3 Construct by N.F.I.F Polynomial for the following data :

A-14

x	4	6	8	10
y	1	3	8	16

$x = x$
$x_0 = 4$
$h = 2$

$$p = \frac{x - x_0}{h} = \frac{x-4}{2}$$

$x \quad y \quad \Delta \quad \Delta^2 \quad \Delta^3$

$x_0 = 4 \quad y_0 = 1$

$\Delta y_0 = 2$

$x_1 = 6 \quad y_1 = 3$ $\quad \Delta^2 y_0 = 3$

$\Delta y_1 = 5$ $\quad \Delta^3 y_0 = 0$

$x_2 = 8 \quad y_2 = 8$ $\quad \Delta^2 y_1 = 3$

$\Delta y_2 = 8$

$x_3 = 10 \quad y_3 = 16$

by N.F.I.F

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$= 1 + \left(\frac{x-4}{2}\right)(2) + \frac{\left(\frac{x-4}{2}\right)\left(\frac{x-4}{2} - 1\right)(3)}{2} + 0$$

$$= 1 + (x-4) + \frac{(x-4)(x-6)(3)}{8}$$

$$= 1 + (x-4) + \frac{3(x^2 - 10x - 12x + 72)}{8}$$

$$= \frac{1}{8}(3x^2 - 22x + 48)$$

Apply N.B.I.F. to the data given below & find polynomial of degree 4 in x

x	1	2	3	4	5
y	1	-1	+1	-1	1

Here $x = x$ $\quad p = \frac{x - x_n}{h}$

$x_n = 6$

$h = 1$ $\quad = \frac{x-5}{1} = x - 5$

x	y	∇	∇^2	∇^3	∇^4
$x_0 = 1$	$y_0 = 1$				
		$\nabla y_1 = -2$			
$x_1 = 2$	$y_1 = -1$		$\nabla^2 y_1 = 4$		
		$\nabla y_2 = -2$		$\nabla^3 y_1 = -8$	
$x_2 = 3$	$y_2 = 1$		$\nabla^2 y_2 = -4$		$\nabla^4 y_4 = 16$
		$\nabla y_3 = -2$		$\nabla^3 y_2 = 8$	
$x_3 = 4$	$y_3 = -1$		$\nabla^3 y_3 = 4$		
		$\nabla y_4 = 2$			
$x_4 = 5$	$y_4 = 1$				

by N.F.I.F.

$$y_p = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n +$$

$$\frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_n$$

$$= 1 + (x-5)(-2) + \frac{(x-5)(x-5+1)(4)}{2} +$$

$$\frac{(x-5)(x-5+1)(x-5+2)(-8)}{3!} + \frac{(x-5)(x-5+1)(x-5+2)(x-5 \ldots)}{4}(16)$$

$$= 1 + (x-5)(2) + \frac{(x-5)(x-4)(4)}{2} +$$

$$\frac{(x-5)(x-4)(x-3)(-8)}{6} + \frac{(x-5)(x-4)(x-3)(x-2)(16)}{4}$$

$$y_p = \frac{1}{3}(2x^4 - 24x^3 + 100x^2 - 544x - 387)$$

Ex-4 / 5 H.W

In the following table, the values of y are consecutive terms of a series of which 12.5 is the fifth term. Find the 1^{st} & the 10^{th} term of the series 0.1, 100

x	3	4	5	6	7	8	9
y	2.7	6.4	12.5	21.6	34.3	51.2	72.9

(8) Central Differences : If x takes the value into, $x_0 - 3h$, $x_0 - 2h$, $x_0 - h$, x_0, $x_0 + h$, $x_0 + 2h$, $x_0 + 3h$, etc... then the corresponding value of y are y_{-3}, y_{-2}, y_{-1}, y_0, y_1, y_2, y_3 etc. respectively, & the ~~di~~ ~~back~~ differences are obtain an that of forward & backward differences but the notations are changed.

$x \quad y \quad \Delta \quad \Delta^2 \quad \Delta^3 \quad \Delta^4 \quad \Delta^5$

$x_{-3} = x_0 - 3h \qquad y_{-3}$

$\Delta y_{-3} = y_{-2} - y_{-3}$

$x_{-2} = x_0 - 2h \qquad y_{-2} \qquad \Delta^2 y_{-3} =$

$\Delta y_{-2} = y_{-1} - y_{-2} \qquad \Delta^3 y_{-3}$

$x_{-1} = x_0 - h \qquad y_{-1} \qquad \Delta^2 y_{-2} = \qquad \Delta^4 y_{-3} =$

$\Delta y_{-1} = y_0 - y_{-1} \qquad \Delta^3 y_{-2} \qquad \Delta^5 y_{-3} =$

$x_0 \qquad y_0 \qquad \Delta^2 y_{-1} = \qquad \Delta^4 y_{-2} = \qquad \Delta^6 y_{-3} =$

$\Delta y_0 = y_1 - y_0 \qquad \Delta^3 y_{-1} \qquad \Delta^5 y_{-2} =$

$x_1 = x_0 + h \qquad y_1 \qquad \Delta^2 y_0 = \qquad \Delta^4 y_{-1} =$

$\Delta y_1 = y_2 - y_1 \qquad \Delta^3 y_0$

$x_2 = x_0 + 2h \qquad y_2 \qquad \Delta^2 y_1 =$

$\Delta y_2 = y_3 - y_2$

$x_3 = x_0 + 3h \qquad y_3$

H.W Ex. 4

$x_0 \quad y_0 \quad \Delta \quad \Delta^2 \quad \Delta^3 \quad \Delta^4 \quad \Delta^5 \quad \Delta^6$

3   2.7

$\Delta y_0 = 3.7$

4   6.4 $\Delta^2 y_0 = 2.4$

$\Delta y_1 = 6.1$ $\Delta^3 y_0 = 0.6$

5   12.5 $\Delta^2 y_1 = 3$ $\Delta^4 y_0 = 0$

$\Delta y_2 = 9.1$ $\Delta^3 y_1 = 0.6$ $\Delta^5 y_0 = 0$

6   21.6 $\Delta^2 y_2 = 3.6$ $\Delta^4 y_0 = 0$ $\Delta^6 y_0 = 0$

$\Delta y_3 = 12.7$ $\Delta^3 y_2 = 0.6$ $\Delta^5 y_1 = 0$

7   34.3 $\Delta^2 y_3 = 4.2$ $\Delta^4 y_0 = 0$

$\Delta y_4 = 16.9$ $\Delta^3 y_3 = 0.6$

8   51.2 $\Delta^2 y_4 = 4.8$

$\Delta y_5 = 21.7$

9   72.9

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$x = x \qquad P = \frac{x - x_0}{h}$

$x_0 = 3$

$h = 1 \qquad = \frac{x-3}{1} \qquad \underline{P = x - 3}$

$$y_p = \frac{y_0}{0!} + \frac{P \Delta y_0}{1!} + \frac{P(P-1)}{2!}\Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!}\Delta^3 y_0$$

$$y_p = 2.7 + (x-3)(3.7) + \frac{(x-3)(x-4)(2.4)}{2}$$

$$+ \frac{(x-3)(x-4)(x-5)(0.6)}{6}$$

$$= 2.7 + 3.7x - 11.1 + (x^2 - 4x - 3x + 12)(1.2) + (x-5)(x^2 - 4x - 3x + 12)(0.1)$$

$$= 2.7 + 3.7x - 11.1 + 1.2x^2 - 8.4x + 14.4 + (x-5)(0.1x^2 - 0.7x + 1.2)$$

$$= 2.7 + 3.7x - 11.1 + 1.2x^2 + 8.4x + 14.4 + \underline{0.1x^3} - 0.7x^2 + 1.2x - 0.5x^2 + 3.5x - 6$$

$$\boxed{y_x = 0.1x^3} +$$

take $x = 1$

$$\boxed{y_x = 0.1}$$

take $x = 10$

$$\boxed{y_x = 100}$$

x	y		Δ	Δ^2	Δ^3	Δ^4	
x_{-2}	y_2						
			Δy_{-2}				
x_{-1}	y_{-1}			$\Delta^2 y_{-2}$			
		G.B.I.F.	Δy_{-1}		$\Delta^3 y_{-2}$		
x_0	y_0			$\Delta^2 y_{-1}$		$\Delta^4 y_{-2}$	Central line
		G.F.I.F.	Δy_0		$\Delta^3 y_{-1}$		
x_1	y_1			$\Delta^2 y_0$			
			Δy_1				
x_2	y_2						

☆ G.F.I.F. (Gauss Forward Interpolation formula)

$$y_x = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_{-1} + \frac{p(p-1)(p+1)}{3!}\Delta^3 y_{-1}$$
$$+ \frac{p(p-1)(p+1)(p-2)}{4!}\Delta^4 y_{-2}$$

where $p = \frac{x - x_0}{h}$

☆ (G.B.I.F.) (Gauss backward Interpolation formula)

$$y_x = y_0 + p\Delta y_0 + \frac{p(p+1)}{2!}\Delta^2 y_{-1} + \frac{p(p+1)(p-1)}{3!}\Delta^3 y_{-1}$$
$$+ \frac{p(p+1)(p-1)(p+2)}{4!}\Delta^4 y_{-2}$$

where $p = \frac{x - x_0}{h}$

<u>Note</u> (1) GFIF is applicable whenever p lies bet^n 0 to 1 i.e. $0 < p < 1$

(2) GBIF is applicable whenever p lies bet^n -1 to 0 i.e. $-1 < p < 0$

☆ Stirling's Formula

By taking the average of GFIF & GBIF we get, a formula called S.F.

$$y_x = [y_0] + \left[p\left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right)\right] + \left[\frac{p^2}{2!}\Delta^2 y_{-1}\right] + \left[\frac{p(p^2-1)}{3!} \times \left(\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right)\right] + \left[\frac{p^2(p^2-1)}{4!}\right]$$

$p = \frac{x - x_0}{h}$

Date

Note Stirling's Formula is applicable whenever P lies betn $-\frac{1}{4}$ to $\frac{1}{4}$

i.e. $-\frac{1}{4} < P < \frac{1}{4}$

Ex-1 using S.F. evaluate $f(3.75)$

A = 17

x	2.5	3	3.5	4	4.5	5
$f(x)$	24.145	22.043	20.225	18.644	17.267	16.047

Here $x = 3.75$ & we need to find $f(3.75)$

$x_0 = 3.5$ $\therefore P = \frac{x - x_0}{h} = \frac{3.75 - 3.5}{0.5} = 0.5$

$h = 0.5$

x	$f(x)$	Δ	Δ^2	Δ^3	Δ^4	Δ^5
$x_{-2} = 2.5$	$y_{-2} = 24.145$					
		$\Delta y_{-2} = -2.103$				
$x_{-1} = 3$	$y_{-1} = 22.043$		$\Delta^2 y_{-2} = 0.285$			
		$\Delta y_{-1} = -1.818$		$\Delta^3 y_{-2} = -0.048$		
$x_0 = 3.5$	$y_0 = 20.225$		$\Delta^2 y_{-1} = 0.237$		$\Delta^4 y_{-2} = 0.009$	
		$\Delta y_0 = -1.58$		$\Delta^3 y_{-1} = -0.237$		$\Delta^5 y_{-2} = 0.395$
$x_1 = 4$	$y_1 = 18.644$		$\Delta^2 y_0 = 0.198$		$\Delta^4 y_{-1} = 0.206$	
		$\Delta y_1 = -1.382$		$\Delta^3 y_0 = -0.031$		
$x_2 = 4.5$	$y_2 = 17.267$		$\Delta^2 y_1 = 0.167$			
		$\Delta y_2 = -1.215$				
$x_3 = 5$	$y_3 = 16.047$					

$$y_x = 20.225 + (0.5)(-1.58) + \frac{(0.5)(0.5-1)(0.237)}{2}$$

$$+ \frac{(0.5)(0.5-1)(0.5+1)(-0.034)}{3 \times 2 \times 1} + \frac{(0.5)(0.5-1)(0.5+1)(0.5-2)(0.009)}{4 \times 3 \times 2 \times 1}$$

$$\boxed{y_x = 19.390}$$

Date

Ex. 2 Apply G.B.F to find $\sin 45°$ from the following is data :

θ	20°	30°	40°	50°	60°	70°	80°
$\sin\theta$	0.34202	0.500	0.64279	0.7660	0.86603	0.93969	0.9848

A=18

let $x = 45°$; $h = 10°$; $P = \frac{x - x_0}{h} = \frac{45 - 50}{10} = -0.5$

$x_0 = 50°$

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5
$x_{-3} = 20$	$y_{-3} = 0.34202$					
		$\Delta y_{-3} = 0.157$				
$x_{-2} = 30$	$y_{-2} = 0.5$		$\Delta^2 y_{-3} = -0.015$			
		$\Delta y_{-2} = 0.142$		$\Delta^3 y_{-3} = -4\times10^{-3}$	$\Delta^4 y_{-3} = -1\times10^{-3}$	
$x_{-1} = 40$	$y_{-1} = 0.64279$		$\Delta^2 y_{-2} = 0.019$			
		$\Delta y_{-1} = 0.123$		$\Delta^3 y_{-2} = -5\times10^{-3}$		$\Delta^5 y_{-3} = 4\times10$
$x_0 = 50$	$y_0 = 0.76604$		$\Delta^2 y_{-1} = -0.024$		$\Delta^4 y_{-2} = 3\times10^{-3}$	
		$\Delta y_0 = 0.099$		$\Delta^3 y_{-1} = -2\times10^{-3}$		$\Delta^5 y_{-2} = -0.0279$
$x_1 = 60$	$y_1 = 0.86603$		$\Delta^2 y_0 = -0.026$		$\Delta^4 y_{-1} = -0.0299$	
		$\Delta y_1 = 0.073$		$\Delta^3 y_0 = -0.0319$		
$x_2 = 70$	$y_2 = 0.93969$		$\Delta^2 y_1 = -0.0579$			
		$\Delta y_2 = 0.0151$				
$x_3 = 80$	$y_3 = 0.9848$					

Δ^6

$\Delta^6 y_{-3} = -0.0319$

$$y_x = y_0 + P\Delta y_{-1} + \frac{P(P+1)\Delta^2 y_{-1}}{2!}$$

$$+ \Delta^3 y_{-2} \frac{P(P+1)(P-1)}{3!} + \Delta^4 y_{-2} \frac{P(P+1)(P-1)(P+2)}{4!}$$

$$= 0.7660 + (-0.5)(0.123) + (-0.5)\frac{(-0.5+1)(-0.014)}{2!}$$

$$+ (-4\times10^{-3})\frac{(-0.5)(-0.5+1)(-0.5-1)}{3!}$$

$$+ -1\times10^{-3}\frac{(-0.5)(-0.5+1)(-0.5+1)(-0.5+2)}{4!}$$

$$\boxed{y_x = 7.0702}$$

Ex-3/5 Use Stirling's formula to evaluate $f(1.22)$ from the following data

Assignment Q-27

x	1	1.1	1.2	1.3	1.4
$f(x)$	0.841	0.891	0.930	0.960	0.485

Hence $x = 1.22$, $h = 0.1$

$x_0 = 1.2$

$$P = \frac{x - x_0}{h} = \frac{1.22 - 1.20}{0.1}$$

$$= \frac{0.02}{0.1} \quad \underline{P = 0.20}$$

x	$f(x)$	Δ	Δ^2	Δ^3	Δ^4
$x_{-2} = 1$	$y_{-2} = 0.841$				
		$\Delta y_{-2} = 0.05$			
$x_{-1} = 1.1$	$y_{-1} = 0.891$		$\Delta^2 y_{-2} = -0.011$		
		$\Delta y_{-1} = 0.039$		$\Delta^3 y_{-2} = 0.005$	
$x_0 = 1.2$	$y_0 = 0.930$		$\Delta^2 y_{-1} = 0.006$		$\Delta^4 y_{-2} = -0.01$
		$\Delta y_0 = 0.033$		$\Delta^3 y_{-1} = 0.005$	
$x_1 = 1.3$	$y_1 = 0.963$		$\Delta^2 y_0 = -0.011$		
		$\Delta y_1 = 0.022$			
$x_2 = 1.4$	$y_2 = 0.985$				

Now, by Stirling's Formula

$$y_x = y_0 + \left| p\frac{(\Delta y_0 + \Delta y_{-1})}{2} \right| + \left| \frac{p^2 \Delta^2 y_{-1}}{2!} \right| + \left| p(p^2-1)\frac{(\Delta^3 y_{-2} + \Delta^3 y_{-1})}{2} \right|$$

$$+ \left| p^2(p^2-1)\Delta^4 y_{-2} \right| + \ldots$$

$$= 0.930 + (0.2)\left(\frac{0.033 + 0.066}{2}\right) + (0.2)^2\frac{(0.066)}{2}$$

$$+ \frac{(0.2)((0.2)^2-1)}{2!}\left(\frac{0.05 + 0.005}{2}\right) + (0.2)((0.2)^2-1)(-0.01)$$

$$y_x = 0.930 + 0.0105 + (0.2)^2\frac{(0.006)^2}{2\times 1} + 0$$

$$+ (0.2)^2((0.2)^2-1)(-0.01)$$

$$= \cancel{0.930}\ 0.9405 + 7.2\times 10^{-7} + 3.84\times 10^{-4}$$

$$\boxed{y_x = 0.940}$$

Ex-4 The Pressure P of wind corresponding to velocity V is given in following data : Estimate P when [V = 21] using Stirling formula

V	10	20	30	40
P	1.1	2	4.4	7.9

Ans :- 2.1792

$p = \frac{x - x_0}{h} = \frac{21-20}{10} = \frac{1}{10} = 0.1$

Ex.5 given that

θ	$0°$	$5°$	$10°$	$15°$	$20°$	$25°$	$30°$
$\tan\theta$	0	0.0875	0.1763	0.2679	0.3600	0.4663	0.5774

using Stirling's formula find $\tan 16°$

Ex 4

V	P	Δ	Δ^2	Δ^3
$x_{-1} = 10$	$y_{-1} = 1.1$			
		$\Delta y_{-1} = 0.9$		
$x_0 = 20$	$y_0 = 2$		$\Delta^2 y_{-1} = 1.5$	
		$\Delta y_0 = 2.4$		$\Delta^3 y_{-1} = -0.4$
$x_1 = 30$	$y_1 = 4.4$		$\Delta^2 y_0 = 1.1$	
		$\Delta y_1 = 3.5$		
$x_2 = 40$	$y_2 = 7.9$			

$$p = \frac{x - x_0}{h} = \frac{21 - 20}{10} = 0.1$$

Now Stirling formula

$$y_p = y_0 + p\left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right) + \frac{p^2 \Delta^2 y_{-1}}{2!}$$

$$+ p(p^2-1)\left(\frac{\Delta^3 y_{-2} + \Delta^3 y_{-1}}{2}\right)$$

$$y_p = 20 + (0.1)\left(\frac{2.4 + 0.9}{2}\right) + \frac{(0.1)^2}{2}(1.5)$$

$$+ (0.1)((0.1)^2 - 1)$$

$$\boxed{y_p = 20.1725}$$

Ex-5

θ	$\tan\theta$	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
$x_{-3} = 0$	$y_{-3} = 0$						
		$\Delta y_{-3} = 0.0875$					
$x_{-2} = 5$	$y_{-2} = 0.0875$		$\Delta^2 y_{-3} = 1.3\times10^{-3}$				
		$\Delta y_{-2} = 0.0888$		$\Delta^3 y_{-3} = 1.5\times10^{-3}$			
$x_{-1} = 10$	$y_{-1} = 0.1763$		$\Delta^2 y_{-2} = 2.8\times10^{-3}$		$\Delta^4 y_{-3} = 0.7\times10^{-3}$		
		$\Delta y_{-1} = 0.0916$		$\Delta^3 y_{-2} = 2.2\times10^{-3}$		$\Delta^5 y_{-3} = 0.0108$	
$x_0 = 15$	$y_0 = 0.2679$		$\Delta^2 y_{-1} = 5\times10^{-4}$		$\Delta^4 y_{-2} = 0.0115$		$\Delta^6 y_{-3} = -0.04$
		$\Delta y_0 = 0.0921$		$\Delta^3 y_{-1} = 0.0137$		$\Delta^5 y_{-2} = -0.0296$	
$x_1 = 20$	$y_1 = 0.3600$		$\Delta^2 y_0 = 0.0142$		$\Delta^4 y_{-1} = -0.0181$		
		$\Delta y_1 = 0.1063$		$\Delta^3 y_0 = -9.4\times10^{-3}$			
$x_2 = 25$	$y_2 = 0.4663$		$\Delta^2 y_1 = 4.8\times10^{-3}$				
		$\Delta y_2 = 0.1111$					
$x_3 = 30$	$y_3 = 0.5774$						

$\underline{P = 0.2}$

$$y_p = 0.2679 + (0.2)\left\{\frac{0.0921 + 0.0916}{2}\right\} + \frac{(0.2)^2}{2!}(5\times10^{-4})$$

$$+ (0.2)((0.2)^2 - 1)\frac{(2.2\times10^{-3} + 0.0137)}{2} + \ldots$$

$$= 0.2679 + 0.01837 + 1\times10^{-5} + 1.5264\times10^{-3}$$

$$\boxed{y_p = 0.2847}$$

Ex-1/5 Evaluate $\sin 56^\circ$, using Stirling's formula

x	45°	50°	55°	60°	65°
$\sin x$	0.7071	0.7660	0.8191	0.8660	0.9063

x	$\sin x$	Δ	Δ^2	Δ^3	Δ^4
$x_{-2} = 45^\circ$	$y_{-2} = 0.7071$				
		$\Delta y_{-2} = 0.0584$			
$x_{-1} = 50^\circ$	$y_{-1} = 0.7660$		$\Delta^2 y_{-2} = -5.8\times10^{-3}$		
		$\Delta y_{-1} = 0.0531$		$\Delta^3 y_{-2} = 0.4\times10^{-3}$	
$x_0 = 55^\circ$	$y_0 = 0.8141$		$\Delta^2 y_{-1} = -6.2\times10^{-3}$		$\Delta^4 y_{-2} = 0$
		$\Delta y_0 = 0.0469$		$\Delta^3 y_{-1} = -0.4\times10^{-3}$	
$x_1 = 60^\circ$	$y_1 = 0.8660$		$\Delta^2 y_0 = -6.6\times10^{-3}$		
		$\Delta y_1 = 0.0403$			
$x_2 = 65^\circ$	$y_2 = 0.9063$				

$$P = \frac{x - x_0}{h} = \frac{1}{5}$$

$$\underline{P = 0.2}$$

$$y_x = y_0 + P\left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right) + \frac{P^2 \Delta^2 y_{-1}}{2!} + P^2(P-1)\left(\frac{\Delta^3 y_{-2} + \Delta^3 y_{-1}}{2}\right)$$

$$= 0.8191 + (0.2)\left(\frac{0.0469 + 0.0531}{2}\right) + \frac{(0.2)^2(-6.2\times10^{-3})}{2!} + (0.2)^2(0.2-1)\left(\frac{-0.4\times10^{-3} - 0.4\times10^{-3}}{2}\right)$$

$$= 0.8191 + 0.01 - 1.2\times10^{-4} + 1.28\times10^{-5}$$

$$\boxed{y_x = 0.8289}$$

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★ Intropolation with un-equal Intervals

→ The various Interpolation formulae seen because before having disadvantage of being applicable only to equally space (equidistant) values of x

→ So it is desirable to divlop Interpolation formula for not equidistant value of x

Now, we learn the following formula

1) Lagrange Interpolation formula (L.I.F.)

2) Newton divided difference formula (N.D.D.F.)

★ L.I.F.

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots(x_0-x_n)} y_0 + \frac{(x-x_0)(x-x_2)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_n)} y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)\ldots(x-x_n)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)\ldots(x_2-x_n)} y_2$$

★ N. D. D. F.

$$\frac{(x-x_0)(x-x_1)(x-x_2)\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)(x_n-x_2)\ldots(x_n-x_{n-1})}y$$

★ Inverse Lagrange's Interpolation formula

$$f(y) = \frac{x = y}{}$$

Ex-1
5
Use L.I.F. to find the value of y when $x=10$, If the following value of x & y are given

	x_0	x_1	x_2	x_3
x	5	6	9	11
y	12	13	14	16
	y_0	y_1	y_2	y_3

Here, we want to find y for $x=10$

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 +$$

$$\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1 +$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

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$$= \frac{(10-6)(10-9)(10-11)}{(5-6)(5-9)(5-11)} (12) +$$

$$\frac{(10-5)(10-9)(10-11)}{(5-5)(5-9)(5-11)} (12) +$$

$$\frac{(10-5)(10-9)(10-11)}{(6-5)(6-9)(6-11)} (13) +$$

$$\frac{(10-5)(10-6)(10-11)}{(9-5)(9-6)(9-11)} (14) +$$

$$\frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)} (16)$$

$$\boxed{f(x) = 14.66}$$

Ex-2 given $y_0 = -12$, $y_1 = 0$ $y_3 = 6$, $y_4 = 12$
find $y_4 = 12$ find y_2 using L.I.F.

	x_0	x_1	x_2	x_3
x	0	1	3	4
y	-12	0	6	12
	y_0	y_1	y_2	y_3

find y when $\underline{x=2}$

$$f(x) = \frac{(2-1)(2-3)(2-4)}{(0-1)(0-3)(0-4)} (-12)$$

$$+ \frac{(2-0)(2-3)(2-4)}{(1-0)(1-3)(1-4)} (0) +$$

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$$+ \frac{(2-0)(2-1)(2-3)(2-4)(6)}{(3-0)(3-1)(3-4)} +$$

$$+ \frac{(2-0)(2-1)(2-3)(12)}{(4-0)(4-1)(4-3)}$$

$$\boxed{f(x) = 4}$$

Ex-9/5 Using L.I.F. find the from of f(x)

	x_0	x_1	x_2	x_3
x	0	2	3	6
$f(x)$	648	704	729	792
	y_0	y_1	y_2	y_3

⇒ Here $x = x$ by L.I.F.

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 +$$

$$\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

-18

$$f(x) = \frac{(x-2)(x-3)(x-6)}{(0-2)(0+3)(0+6)}(648) +$$

88

$$\frac{(x-0)(x-3)(x-6)}{(2-0)(2-3)(2-6)}(704) +$$

-81

$$\frac{(x-0)(x-2)(x-6)}{(3-0)(3-2)(3-6)}(729) +$$

11

$$\frac{(x-0)(x-2)(x-3)}{(6-0)(6-2)(6-3)}(729)$$

(4) (3)

$$f(x) = (x-3)(x-6)\,[-18(x-2) + 88x] + x(x-2)\,[-81(x-6) + 11(x-3)]$$

$$= x^2 - 9x + 18\,[-18x + 36 + 88x] + (x^2 - 2x)\,[-81x + 486 + 11x - 33]$$

$$= (x^2 - 9x + 18)\,[36 + 70x] + (x^2 - 2x)\,(453 - 70x]$$

$$= 36x^2 - 324x + 648 + 70x^3 - 630x^2 + 1260x + 453x^2 - 906x - 70x^3 + 140x^2$$

$$f(x) = -x^3 + 30x + 648$$

given $f(0)=-18$, $f(1)=0$, $f(3)=0$
$f(5)=-248$, $f(6)=0$, $f(9)=13104$,
find $f(x)$ using L.I.F.

A
Q-3

	x_0	x_1	x_2	x_3	x_4	x_5
x	0	1	3	5	6	9
$f(x)$	+18	0	0	-248	0	13104
	y_0	y_1	y_2	y_3	y_4	y_5

$$f(x)=\frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_5)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)(x_0-x_5)}y_0$$

$+$

$$\frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)(x-x_5)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)(x_1-x_5)}y_1$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)(x-x_5)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)(x_2-x_5)}y_2$$

$+$

$$\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)(x-x_5)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)(x-x_5)}y_3$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_5)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)(x-x_5)}y_4$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_5-x_0)(x_5-x_1)(x_5-x_2)(x_5-x_3)(x_5-x_4)}y_5$$

$$=\frac{\cancel{(x-0)}(x-1)(x-3)(x-5)(x-6)(x-9)}{(0+1)(0+3)(0+5)(0+6)(0-9)}(18)$$

$$+\frac{(x-0)(x-1)(x-3)(x-6)(x-9)}{\cancel{(-248-18)(-248)}\;(5-0)(5-1)(5-3)(5-6)(5\ldots}(-248)$$

$$\frac{(x-0)(x-1)(x-3)(x-5)(x-6)(13104)}{(9-0)(9-1)(9-3)(9-5)(9-6)}$$

$$f(x) = \{(x-1)(x-3)(x-5)(x-6)(x-9)\}\frac{1}{45} - \frac{18}{810}$$

$$+ (x)(x-1)(x-3)(x-6)(x-9) \div \frac{248}{160} +$$

$$(x)(x-1)(x-3)(x-5)(x-6) \quad \frac{13104}{5184}$$

$$= (x-1)(x-3)(x-6)\Big[(x-5)(x-9)\left(\frac{-1}{45}\right)$$

$$+ x(x-9)\left(\frac{-31}{30}\right) + x(x-5)\left(\frac{91}{36}\right)\Big]$$

$$= (x-1)(x-3)(x-6)\Big[\frac{-4(x^2-14x+45-31\times 9(x^2-9x}{180}$$

$$+ 5(x^5 - 5x)\Big]$$

$$\cancel{= \frac{(x-1)(x-3)(x-6)}{180}\left[-4x^2 + 56x - 180\right.}$$

$$\cancel{+ 2511x - 279x^2 + 5x^2 - 25x}$$

$$= (x-1)(x-3)(x-6)\Big[\frac{-4(x^2-14x+45)}{180}$$

$$-31\times 9(x^2-9x) + 91\times 5(x^2-5x)\Big]$$

$= (x^2 - 4x + 3)(x - 6)\left[\frac{-4x^2 + 56x - 180 - 279x^2 + 2571x + 455x^2 - 2275x}{180}\right]$

$= (x^3 - 4x^2 + 3x - 6x^2 + 24x - 18)\left[\frac{172x^2 + 292x - 180}{180}\right]$

$= \frac{1}{180}\left\{172x^5 - 1168x^2\right.$

$f(x) = +0.96x^5 - 7.95x^4 + 8.59x^3 + 86.59x^2 - 56.19x + 18$

Ex-5 using L.I.F. express the function $\frac{x^2+6x-1}{(x^2-1)(x-4)(x-6)}$ as a sum of Partial fraction.

$f(x) = \frac{x^2 + 6x - 1}{(x^2-1)(x-4)(x-6)} = \frac{f(x)}{g(x)}$

If $g(x) = 0$.

$\Rightarrow (x^2-1)(x-4)(x-6) = 0$

$\Rightarrow (x+1)(x-1)(x-4)(x-6) = 0$

$\Rightarrow x = 1, -1, 4, 6$

	x_0	x_1	x_2	x_3
x	1	-1	4	6
$f(x)$	6	-6	39	71
	y_0	y_1	y_2	y_3

$$\Rightarrow f(x) = \frac{(x-(-1))(x-4)(x-6)}{(1-(-1))(1-4)(1-6)}(6) +$$

$$\frac{(x-1)(x-4)(x-6)}{(-1-1)(-1-4)(-1-6)}(-6) +$$

$$\frac{(x-1)(x-(-1))(x-6)}{(4-1)(4-(-1))(4-6)}(39) +$$

$$\frac{(x-1)(x-(-1))(x-4)}{(6-1)(6-(-1))(6-4)}(71)$$

$$= \frac{(x+1)(x-4)(x-6)}{(2)(+3)(+5)}(6) + \frac{(x-1)(x-4)(x-6)}{}$$

$$+ \frac{(x-1)(x+1)(x-6)}{(3)(5)(-2)}(\overset{13}{\cancel{39}}) +$$

$$\frac{(x-1)(x+1)(x-4)}{(5)(7)(2)}(71)$$

$$F(x) = \frac{f(x)}{g(x)}$$

$$= \frac{(x\cancel{+1})(x\cancel{-4})(x-\cancel{6})}{(x+1)(x+1)(x\cancel{-4})(x\cancel{-6})}\left(\frac{1}{5}\right) +$$

$$\frac{(x\cancel{-1})(x\cancel{-4})(x-\cancel{6})}{(x\cancel{-1})(x+1)(x\cancel{-4})(x\cancel{-6})}\left(\frac{3}{35}\right) +$$

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$$\frac{(x-1)(x+1)(x-6)}{(x-1)(x+1)(x-4)(x-6)}\left(\frac{19}{-10}\right)+$$

$$\frac{(x-1)(x+1)(x-4)}{(x+1)(x-1)(x-4)(x-6)}\left(\frac{71}{70}\right)$$

$$F(x)=\frac{1}{5}\left(\frac{1}{x-1}\right)+\frac{3}{35}\left(\frac{1}{x+1}\right)$$

$$-\frac{19}{10}\cdot\frac{1}{x-4}+\frac{71}{70}\cdot\frac{1}{x-6}$$

11. H.W. $\frac{x^2-5x+10}{(x^2-1)(x-3)(x-2)}$

A Divided differences :-

When the differences are

$\frac{y_1-y_0}{x_1-x_0}$, $\frac{y_2-y_1}{x_2-x_1}$ ...... etc

are denoted by $[x_0, x_1]$, $[x_1, x_2]$ ...... etc

resply, then the differences are called as divided differences

→ The divided differences are given in the following table serately.

x $\quad$ y $\quad$ 1^{st} $\quad$ 2^{nd} $\quad$ 3^{rd}

x_0 $\quad$ y_0

$$[x_0, x_1] = \frac{y_1 - y_2}{x_1 - x_0}$$

x_1 $\quad$ y_1 $\quad$ $$[x_0, x_1, x_2] = \frac{[x_1, x_2] - [x_0 - x_1]}{x_2 - x_0}$$

$$[x_1, x_2] = \frac{y_2 - y_1}{x_2 - x_1}$$

x_2 $\quad$ y_2 $\quad$ $$[x_1, x_2, x_3] = \frac{[x_2, x_3] - [x_1, x_2]}{x_3 - x_1}$$

$$[x_2, x_3] = \frac{y_3 - y_2}{x_3 - x_2}$$

x_3 $\quad$ y_3 $\quad$ $$[x_2, x_3, x_4] = \frac{[x_3, x_4] - [x_2, x_3]}{x_4 - x_2}$$

$$[x_3, x_4] = \frac{y_4 - y_3}{x_4 - x_3}$$

x_4 $\quad$ y_4

3rd

$$[x_0, x_1, x_2, x_3] = \frac{[x_1, x_2, x_3] - [x_0, x_1, x_2]}{x_3 - x_0}$$

$$[x_1, x_2, x_3, x_4] = \frac{[x_2, x_3, x_4] - [x_1, x_2, x_3]}{x_4 - x_1}$$

4rd

$$[x_0, x_1, x_2, x_3, x_4] = \frac{[x_1, x_2, x_3, x_4] - [x_0, x_1, x_2, x_3]}{x_4 - x_0}$$

H.W $f(x) = \dfrac{x^2-5x+10}{(x^2-1)(x-3)(x+2)} = \dfrac{f(x)}{g(x)}$

If $g(x)=0$

$\Rightarrow (x+1)(x-1)(x-3)(x-2)=0$

$\Rightarrow x=-1,1,3,2$

	x_0	x_1	x_2	x_3
x	+1	-1	3	2
$f(x)$	15	6	4	4
	y_0	y_1	y_2	y_3

$f(x) = \dfrac{(x-x_1)(x-x_2)(x-x_3)\,y_0}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} +$

$\dfrac{(x-x_0)(x-x_2)(x-x_3)\,y_1}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} +$

$\dfrac{(x-x_0)(x-x_1)(x-x_3)\,y_2}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} +$

$\dfrac{(x-x_0)(x-x_1)(x-x_2)\,y_3}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$

$= \dfrac{(x+1)(x-3)(x-2)(15)}{(1+1)(1-3)(1-2)} +$

$\dfrac{(x-1)(x+3)(x-2)(6)}{(-1-1)(-1-3)(-1-2)} +$

$\dfrac{(x-1)(x+1)(x-2)(4)}{(3-1)(3+1)(3-2)} +$

$$\frac{(x-1)(x+1)(x-3)(4)}{(4-1)(4+1)(4-3)}$$

$$F(x) = \frac{(x+1)(x-3)(x-2)(15)}{(2)(+2)(+2)} +$$

$$\frac{(x-1)(x+3)(x-2)(8)}{(+2)(-4)(+3)} +$$

$$\frac{(x-1)(x+1)(x-2)(9)}{(2)(4)(1)} +$$

$$\frac{(x-1)(x+1)(x-3)(4)}{(3)(5)(1)}$$

$$F(x) = \frac{f(x)}{g(x)}$$

$$= \frac{(x+1)(x-3)(x-2)(15)}{(x+1)(x-1)(x-2)(x-3)(8)} +$$

$$\frac{(x-1)(x-2)(x+3)}{(x+1)(x-1)(x-2)(x-3)} \frac{1}{(-4)} +$$

$$\frac{(x-1)(x+1)(x-2)}{(x+1)(x-1)(x-2)(x-3)} \frac{1}{(2)} +$$

$$\frac{(x-1)(x+1)(x-3)}{(x+1)(x+1)(x-2)(x-3)} \frac{4}{15}$$

$$F(x) = \frac{15}{(x-1)8} + \frac{1}{-(x+1)4} + \frac{1}{2(x-3)} + \frac{4}{5}\frac{1}{(x-2)}$$

Ex-1 Construct divided difference table for the following data

x	4	5	8	11	16
y	120	112	180	210	420

x y 1^{st} diff

$x_0 = 4$ $y_0 = 120$

$[x_0, x_1] = \frac{112-120}{5-4} = -8$

$x_1 = 5$ $y_1 = 112$

$[x_1, x_2] = \frac{180-112}{8-5} = 22.67$

$x_2 = 8$ $y_2 = 180$

$[x_2, x_3] = \frac{210-180}{11-8} = 10$

$x_3 = 11$ $y_3 = 210$

$[x_3, x_4] = \frac{420-210}{16-11} = 42$

$x_4 = 16$ $y_4 = 420$

2^{nd} diff.

$[x_0, x_1, x_2] = \frac{22.67 - (-8)}{8 - 4} = 7.67$

$[x_0, x_1, x_2, x_3] = \frac{-2.11 - 7.67}{11 - 4} = 1.39$

$[x_1, x_2, x_3] = \frac{10 - 22.67}{11 - 5} = -2.11$

$[x_0, x_1, x_2, x_3, x_4] = \frac{0.55 + 1.3}{16 - 4}$

$\boxed{x_1 = 5}$

$[x_1, x_2, x_3, x_4] = \frac{4 - (2.11)}{16 - 5} = 0.16$

$[x_2, x_3, x_4] = \frac{42 - 10}{16 - 8} = 4$

* Newton Divided Difference formula (N.D.D.F.)

$$f(x) = y_0 + (x-x_0)[x_0, x_1] + (x-x_0)(x-x_1)[x_0, x_1, x_2]$$
$$+ (x-x_0)(x-x_1)(x-x_2)[x_0, x_1, x_2, x_3] +$$
$$\dots (x-x_0)(x-x_1)(x-x_2)\dots(x-x_{n-1})$$
$$[x_0, x_1, x_2 \dots x_n]$$

Ex-1 Using N.D.D.F. evaluate $f(8)$ for the following data

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

A
Q-33

⇒ Here, we need to find $f(x)$ when $x = 8$
first we prepare the divided diff. table

⇒ by N.D.D.F.

$$f(x) = y_0 + (x-x_0)[x_0, x_1] + (x-x_0)(x-x_1)$$
$$[x_0, x_1, x_2] + (x-x_0)(x-x_1)$$
$$(x-x_2)[x_0, x_1, x_2, x_3]$$

$$= 48 + (8-4)(52) + (8-4)(8-5)(15)$$
$$+ (8-4)(8-5)(8-7)(1)$$

$$\boxed{f(x) = 448}$$

x	y	1st	2nd	3rd	4th	5th
$x_0 = 4$	$y_0 = 48$					
		$[x_0, x_1] = 52$				
			$[x_0, x_1, x_2] = 15$			
$x_1 = 5$	$y_1 = 100$			$[x_0, x_1, x_2, x_3] = 1$		
		$[x_1, x_2] = 97$				
$x_2 = 7$	$y_2 = 294$		$[x_1, x_2, x_3] = 21$		$[x_0, x_1, x_2, x_3, x_4] = 0$	
		$[x_2, x_3] = 202$		$[x_1, x_2, x_3, x_4] = 1$		$[x_0, x_1, x_2, x_3, x_4, x_5] = 0$
$x_3 = 10$	$y_3 = 900$		$[x_2, x_3, x_4] = 27$		$[x_1, x_2, x_3, x_4, x_5] = 0$	
		$[x_3, x_4] = 310$		$[x_2, x_3, x_4, x_5] = 1$		
			$[x_3, x_4, x_5] = 33$			
$x_4 = 11$	$y_4 = 1210$					
		$[x_4, x_5] = 409$				
$x_5 = 13$	$y_5 = 2028$					

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Ex-2 Certain corresponding values of x & $\log_{10} x$ are given below : find $\log_{10} 310$ using suitable formula

x	300	304	305	307
$\log_{10} x$	2.4771	2.4829	2.4843	2.4871

$x = 310$ we need $\log_{10} 310$

x $\quad \log_{10} x$ $\quad 1^{st}$ $\quad 2^{nd}$ $\quad 3^{rd}$

$x_0 = 300 \quad y_0 = 2.4771$

$[x_0, x_1] = 0.00145$

$[x_0, x_1, x_2] = -0.00001$

$x_1 = 304 \quad y_1 = 2.4829$

$[x_0, x_1, x_2, x_3] = 0.00000142$

$[x_1, x_2] = 0.0014$

$[x_1, x_2, x_3] = 0$

$x_2 = 305 \quad y_2 = 2.4843$

$[x_2, x_3] = 0.0014$

$x_3 = 307 \quad y_3 = 2.4871$

$$f(x) = y_0 + (x - x_0)[x_0, x_1] + (x - x_0 - x_1)[x_0, x_1, x_2] + (x - x_0)(x - x_1)(x - x_2)[x_0, x_1, x_2, x_3]$$

$$= 2.4771 + (310 - 300)(0.00145) + (310 - 300)(310 - 304)(0.00001) + (310 - 300)(310 - 304)(310 - 309)(0.00000142)$$

$$\boxed{f(x) = 2.499}$$

Ex-3 determine $f(x)$ as a Polynomial in x for following data

x	-4	-1	0	2	5
$f(x)$	1245	33	5	9	1335

Ans. $3x^4 - 5x^3 + 6x^2 - 14x + 5$

Ex-4 N.D.D.F. $f(x)$

x	0	1	2	3	4	5	6
$f(x)$	1	14	15	5	65	6	19

x $f(x)$ 1^{st} 2^{nd} 3^{rd} 4^{th} 5^{th}

$x_0 = -4$ $y_0 = 1245$

$[x_0, x_1] = -404$

$[x_0, x_1, x_2] = 94$

$(x_0, x_1, x_2, x_4) = -14$

$x_1 = -1$ $y_1 = 33$

$[x_1, x_2] = -28$

$(x_0, x_1, x_2, x_3, x_4) = 3$

$[x_1, x_2, x_3] = 10$

$(x_1, x_2, x_3, x_4) = 13$

$x_2 = 0$ $y_2 = 5$

$[x_2, x_3] = +2$

$[x_2, x_3, x_4] = 88$

$x_3 = 2$ $y_3 = 9$

$[x_3, x_4] = 442$

$x_4 = 5$ $y_4 = 1335$

$f(x) = 1245 + (x+4)(-404) + (x+4)(x+1)$
$(94) + (x+4)(x+1)(x)(-14)$
$(x+4)(x+1)(x)(x-2)(3)$

$f(x) = 3x^4 - 5x^3 + 6x^2 - 14x + 5$

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Ex. 5

x

x	$f(x)$	1st	2nd	3rd	4th	5th
$x_0 = 0$	$y_0 = 1$					
		$(x_0, x_1) = 13$				
			$(x_0, x_1, x_2) = -6$			
$x_1 = 1$	$y_1 = 14$			$(x_0, x_1, x_2, x_3) = 1$		
		$(x_1, x_2) = 1$			$(x_0, x_1, x_2, x_4) = 0$	
			$(x_1, x_2, x_3) = -2$			$(x_0, x_1, x_2, x_3, x_4, x_5)$
$x_2 = 2$	$y_2 = 15$			$(x_1, x_2, x_3, x_4) = 1$		
		$(x_2, x_3) = 5$				
			$(x_2, x_3, x_4) = 2$		$(x_1, x_2, x_3, x_4) = 0$	
$x_3 = 4$	$y_3 = 5$			$(x_2, x_3, x_4, x_5) = 1$		
		$(x_3, x_4) = 1$				
			$(x_3, x_4, x_5) = 6$			
$x_4 = 5$	$y_4 = 6$					
		$(x_4, x_5) = 13$				
$x_5 = 6$	$y_5 = 19$					

$$f(x) = x^3 - 9x^2 + 21x + 1$$

Assi

$$\Delta f(x) = f(x+h) - f(x)$$

$$\Delta(\tan^{-1} x)$$

$$= \tan^{-1}(x+h) - \tan^{-1} x$$

$$= \tan^{-1}\left(\frac{x+h-x}{1+(x+h)(x)}\right)$$

$$= \tan^{-1}\left(\frac{h}{1+x^2+hx}\right)$$

①

Assignment - 2

Q-9

6] $\Delta^2(\sin(ax+b))$

$= \Delta(\Delta \sin(ax+b))$

$= \Delta(\sin(a(x+h)+b) - \sin(ax+b))$

$= \Delta(\sin(ax+ah+b) - \sin(ax+b))$

$= \Delta\left\{2\cos\left(ax+b+\frac{ah}{2}\right) \cdot \sin\left(\frac{ah}{2}\right)\right\}$

$= 2\sin\left(\frac{ah}{2}\right) \cdot \Delta\cos\left(ax+b+\frac{ah}{2}\right)$

$= 2\sin\left(\frac{ah}{2}\right)\left[\cos\left(a(x+h)+b+\frac{ah}{2}\right) - \cos\left(ax+b+\frac{ah}{2}\right)\right]$

$= 2\sin\left(\frac{ah}{2}\right)\left[\cos\left(ax+ah+b+\frac{ah}{2}\right) - \cos\left(ax+ah+\frac{ah}{2}\right)\right]$

$= 2\sin\left(\frac{ah}{2}\right)\left[-2\sin(ax+b+ah) \cdot \sin\left(\frac{ah}{2}\right)\right]$

$\Delta^2\sin(ax+b) = -4\sin^2\left(\frac{ah}{2}\right)\sin(ax+b+ah)$

$\Delta^n\sin(ax+b) = (-1)^{n-1}2^n\sin^n\left(\frac{ah}{2}\right)\sin(ax+b+a(n-1)h))$

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2] $\Delta^n e^x$

$\Rightarrow \Delta^{n-1}(\Delta e^x)$

$\Rightarrow \Delta^{n-1}[e^{x+h} - e^x]$

$\Rightarrow \Delta^{n-1}[e^x(e^h - 1)]$

1] $\Delta \tan^{-1} x$

$= \tan^{-1}(x+h) - \tan^{-1} x$

$= \tan^{-1}\left(\dfrac{x+h-x}{1+(x+h)(x)}\right)$

$= \tan^{-1}\left(\dfrac{h}{1+x^2+hx}\right)$

3] $\Delta\left(\dfrac{1}{f(x)}\right)$

$= \dfrac{1}{f(x+h)} - \dfrac{1}{f(x)}$

$= \dfrac{f(x) - f(x+h)}{f(x)f(x+h)}$

4] $\Delta\left(\dfrac{f(x)}{g(x)}\right)$

$= \left(\dfrac{f(x+h)}{g(x+h)} - \dfrac{f(x)}{g(x)}\right)$

$= \dfrac{f(x+h)\,g(x) - f(x)\,g(x+h)}{g(x)\,g(x+h)}$

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7] $\Delta^{10}[(1-ax)(1-bx^2)(1-cx^3)(1-dx^4)]$

$\Rightarrow (1-bx^2-ax+abx^3)(1-dx^4-cx^3+cdx^7)$

$\Rightarrow 1-dx^4-cx^3+cdx^7-bx^2+bdx^6+bcx^5-bcdx^9$
$-ax+adx^5+acx^7 \cancel{+abcd\,x^{10}}\ acdx^8$
$+abx^3-abdx^7-abcx^6 \boxed{+abcd\,x^{10}}$

$\Rightarrow f(x)$

$\Delta^{10}(f(x)) = 10!\,\cancel{f(x)}\,(abcd)$
6!

8] $\Delta^4[(1-x)(1-2x)(1-3x)(1-4x)]$

$\Rightarrow (1-2x-x+2x^2)(1-4x-3x+12x^2)$
$\Rightarrow (1-4x-3x+12x^2-2x+8x^2+24x^3$
$-x+4x^2+3x^2-12x^3+2x^2-8x^3-6x^3$
$+24x^4 \quad \Rightarrow f(x)$

$\Delta^4(f(x)) = 4!\,\cancel{f(x)}\,(x^4)$
(24)

Q-10 all in classwork book

Q-11 Using gregary - Newton forward formula $f(1.6) = ?$

x	1	1.4	1.8	2.2
$f(x)$	3.49	4.82	5.96	6.5

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$x = 1.6$, $x_0 = 1.4$, $h = 0.4$

$P = \frac{x - x_0}{h} = \frac{1.6 - 1.4}{0.4} = +0.5$

x $f(x)$ Δ Δ^2 Δ^3

~~$x_{-2} = 1$ $y_{-2} = 3.49$~~

~~Δ~~

~~$x_{-1} = 1.4$ $y_{-1} = 4.82$~~

~~$x_0 = 1.8$ $y_0 = 5.96$~~

~~$x_1 = 2.2$ $y_1 = 6.5$~~

x $f(x)$ Δ Δ^2 Δ^3

$x_{-1} = 1$ $y_{-1} = 3.49$

$\Delta y_{-1} = 1.33$

$x_0 = 1.4$ $y_0 = 4.82$ $\Delta^2 y_{-1} = -0.19$

$\Delta y_0 = 1.14$ $\Delta^3 y_{-1} = -0.41$

$x_1 = 1.8$ $y_1 = 5.96$ $\Delta^2 y_0 = -0.6$

$\Delta y_1 = 0.54$

$x_2 = 2.2$ $y_2 = 6.5$

$$y_x = y_0 + P\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_{-1} + \frac{P(P-1)(P+1)}{3!}\Delta^3 y_{-1}$$

$$f(1.6) = 4.82 + (0.5)(1.14) + (0.5)\frac{(0.5-1)}{2!}(0.19)$$

$$+ (0.5)\frac{(0.5-1)(0.5+1)}{6}(-0.41)$$

$f(1.6) = 4.82 + 0.57 + 0.02375 + 0.0256$

$\boxed{f(1.6) = 0.57}$

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5

Q-12

D	80	85	90	95	100
A	5026	5674	6362	7088	7854

QBF. 105

$p = \frac{105-100}{5} = 1$

$D = x$ | $A = y$ | Δ | Δ^2 | Δ^3 | Δ^4

$x_{-4} = 80$ | $y_{-4} = 5026$

$\Delta y_{-4} = 648$

$x_{-3} = 85$ | $y_{-3} = 5674$

$\Delta^2 y_{-4} = 40$

$\Delta y_{-3} = 688$

$\Delta^3 y_{-4} = -2$

$x_{-2} = 90$ | $y_{-2} = 6362$

$\Delta^2 y_{-3} = 38$

$\Delta^4 y_{-4} = 4$

$\Delta y_{-2} = 726$

$\Delta^3 y_{-3} = 2$

$x_{-1} = 95$ | $y_{-1} = 7088$

$\Delta^2 y_{-2} = 40$

$\Delta y_{-1} = 766$

$x_0 = 100$ | $y_0 = 7854$

$$A_{105}\, y_p = y_0 + p\Delta y_0 + \frac{p(p+1)}{2!}\Delta^2 y_{-1} + \frac{p(p+1)(p-1)}{3!}\Delta^3 y_{-1}$$

$$+ \frac{p(p+1)(p-1)(p+2)}{4!}\Delta^4 y_{-2}$$

$$= 7854 + (1)(766) + \frac{(1)(1+1)^2(80)}{2} + \frac{(1)(2)(3)(2)}{6}$$

$$+ (1)\frac{(2)(3)}{6}(2) + \frac{(1)(2)(3)(4)(40)}{24}$$

$$\boxed{y_p = 8666}$$

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Q- ~~10~~ 14, 17, 18 ~~R~~
Class-book

Q-18

Wages in R.S	0-10	10-20	20-30	30-40
Frequency	9	30	35	42

10 small

$x = 15 \quad x_0 = 10 \quad P = \frac{5}{10} = 0.5$

$h = 10$

< 15

< R.S.	10	20	30	40
Frequency	9	39	74	116

x	Y	Δ	Δ^2	Δ^3
$x_0 = 10$	$y_0 = 9$			
		$\Delta y_0 = 30$		
$x_1 = 20$	$y_1 = 39$		$\Delta^2 y_0 = 5$	
		$\Delta y_1 = 35$		$\Delta^3 y_0 = 2$
$x_2 = 30$	$y_2 = 74$		$\Delta^2 y_1 = 7$	
		$\Delta y_2 = 42$		
$x_4 = 40$	$y_3 = 116$			

$$y_p = y_0 + P\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!}\Delta^3 y_0$$

$$= 9 + (0.5)(30) + \frac{(0.5)(0.5-1)(5)}{2} + \frac{(0.5)(0.5-1)(0.5-2)2}{6}$$

$$= 9 + 15 + (-0.625) + 0.125$$

$$\boxed{y_p = 23.5}$$

7

Q-16 Newton's Backward Interpolation for degree two

x	0	1	2	3	4	5	6	7
y	1	2	4	7	11	16	22	29

x	y	Δ	Δ^2	Δ^3
$x_0 = 0$	$y_0 = 1$			
		$\Delta y_0 = 1$		
$x_1 = 1$	$y_1 = 2$		$\Delta^2 y_0 = 1$	
		$\Delta y_1 = 2$		$\Delta^3 y_0 = 0$
$x_2 = 2$	$y_2 = 4$		$\Delta^2 y_1 = 1$	
		$\Delta y_2 = 3$		$\Delta^3 y_1 = 0$
$x_3 = 3$	$y_3 = 7$		$\Delta^2 y_2 = 1$	
		$\Delta y_3 = 4$		$\Delta^3 y_2 = 0$
$x_4 = 4$	$y_4 = 11$		$\Delta^2 y_3 = 1$	
		$\Delta y_4 = 5$		$\Delta^3 y_3 = 0$
$x_5 = 5$	$y_5 = 16$		$\Delta^2 y_4 = 1$	
		$\Delta y_5 = 6$		$\Delta^3 y_4 = 0$
$x_6 = 6$	$y_6 = 22$		$\Delta^2 y_5 = 1$	
		$\Delta y_6 = 7$		
$x_7 = x_7$	$y_7 = 29$			

$$P = \frac{x - 0}{1}$$

$$\underline{P = x}$$

$$y(x) = y_0 + P\,\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!}\Delta^3 y_0 + \dots$$

$$= 1 + x(1) + \frac{x(x-1)}{2!}(1) + 0$$

$$= 1 + x + \frac{x^2 - x}{2}$$

$$= 1 + x + x^2/2 - x/2$$

$$\boxed{y(x) = \frac{1}{2}(x^2 + x + 2)}$$

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Q-16 $\frac{1}{5}$ N.B.F. Polynomial of degree '3'
(3, 6) (4, 24) (5, 60) (6, 120)

x	3	4	5	6
y	6	24	60	120

P = x-

x	y	Δ	Δ^2	Δ^3
$x_0 = 3$	$y_0 = 6$			
		$\Delta y_0 = 18$		
$x_1 = 4$	$y_1 = 24$		$\Delta^2 y_0 = 18$	
		$\Delta y_1 = 36$		$\Delta^3 y_0 = 6$
$x_2 = 5$	$y_2 = 60$		$\Delta^2 y_1 = 24$	
		$\Delta y_2 = 60$		
$x_3 = 6$	$y_3 = 120$			

8/17/1

$$y(x) = y_0 + P\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!}\Delta^3 y_0$$

$$= 6 + (x-3)(18) + \frac{(x-3)(x-4)}{\cancel{2}}(\overset{9}{\cancel{18}})$$

$$+ \frac{(x-3)(x-4)(x-5)}{\cancel{6}}(\cancel{6})$$

$$= 6 + 18x - 54 + 9(x^2 - \cancel{4x} - \cancel{3x} + 12) \quad -6x \quad 108$$

$$+ (x^2 - 4x - 3x - 12)(x-5)$$

$$= 6 + 18x - 54 + 9x^2 - 6x + 108 + x^3 - 7x^2 - 12x$$

$$- 5x^2 + 20x + 15x + 60$$

$$y(x) = 120 + 35x - 3x^2 + x^3$$

Q-8 (1) Forward diffrence operator :-
The forward diff. operator is denoted by Δ and is defined as

$$\Delta f(x) = f(x+h) - f(x)$$

h is known as the Interval of differencing

(2) Backward difference operator :-
The backward diff operator is denoted by ∇ and is defined as

$$\nabla f(x) = f(x) - f(x-h)$$

(3) Central Difference operator :-
The central diff. operator is denoted by δ and is difined as

$$\delta f(x) = f\left(x+\frac{h}{2}\right) - f\left(x-\frac{h}{2}\right)$$

(4) Averaging operator :- The averaging is denoted by μ and is defined as

$$\mu f(x) = \frac{f\left(x+\frac{h}{2}\right) + f\left(x-\frac{h}{2}\right)}{2}$$