CS344: Introduction to Artificial Intelligence
(associated lab: CS386)

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Lecture 14-17: Predicate calculus, Prolog, Circuit verification

Models of human reasoning (1/2)

- Non-numerical
  - Non monotonic Logic
    - Negation by failure (*“innocent unless proven guilty”*)
    - Abduction (*P→Q AND Q gives P)*
  - Modal Logic
    - New operators beyond AND, OR, IMPLIES, Quantification etc.
  - Naïve Physics

Abduction Example

- **If**

  there is rain (P)
- **Then**

  there will be no picnic (Q)
- Abductive reasoning:

  Observation: There was no picnic(Q)

  Conclude : There was rain(P); *in absence of any other evidence*

Modeling human reasoning (2/2)

- Numerical
  - Fuzzy Logic
  - Probability Theory
    - Bayesian Decision Theory
  - Possibility Theory
  - Uncertainty Factor based on Dempster Shafer Evidence Theory (e.g. *yellow_eyes→jaundice; 0.3)*

Predicate calculus

Introduce through the "Himalayan Club Example"

Himalayan Club example

- Introduction through an example *(Zohar Manna, 1974):*
  - Problem: A, B and C belong to the Himalayan club. Every member in the club is either a mountain climber or a skier or both. A likes whatever B dislikes and dislikes whatever B likes. A likes rain and snow. No mountain climber likes rain. Every skier likes snow. *Is there a member who is a mountain climber and not a skier?*
- Given knowledge has:
  - Facts
  - Rules

Example contd.

- Let *mc* denote mountain climber and *sk* denotes skier. Knowledge representation in the given problem is as follows:
  1. *member(A)*
  2. *member(B)*
  3. *member(C)*
  4. $\forall x[member(x) \rightarrow (mc(x) \vee sk(x))]$
  5. $\forall x[mc(x) \rightarrow \sim like(x,rain)]$
  6. $\forall x[sk(x) \rightarrow like(x, snow)]$
  7. $\forall x[like(B, x) \rightarrow \sim like(A, x)]$
  8. $\forall x[\sim like(B, x) \rightarrow like(A, x)]$
  9. *like(A, rain)*
  10. *like(A, snow)*
  11. *Question:* $\exists x[member(x) \wedge mc(x) \wedge \sim sk(x)]$
- We have to infer the 11^{th} expression from the given 10.
- Done through Resolution Refutation.

Club example: Inferencing

1. *member(A)*
2. *member(B)*
3. *member(C)*
4. $\forall x[member(x) \rightarrow (mc(x) \vee sk(x))]$
   - Can be written as
   - $[member(x) \rightarrow (mc(x) \vee sk(x))]$
     $\sim member(x) \vee mc(x) \vee sk(x)$
5. $\forall x[sk(x) \rightarrow lk(x, snow)]$
   - $\sim sk(x) \vee lk(x, snow)$
6. $\forall x[mc(x) \rightarrow \sim lk(x, rain)]$
   - $\sim mc(x) \vee \sim lk(x, rain)$
7. $\forall x[like(A, x) \rightarrow \sim lk(B, x)]$
   - $\sim like(A, x) \vee \sim lk(B, x)$

8. $\forall x[\sim lk(A,x) \rightarrow lk(B,x)]$

– $lk(A,x) \vee lk(B,x)$

9. $lk(A, rain)$

10. $lk(A, snow)$

11. $\exists x[member(x) \wedge mc(x) \wedge \sim sk(x)]$

– Negate– $\forall x[\sim member(x) \vee \sim mc(x) \vee sk(x)]$

- Now standardize the variables apart which results in the following

1. $member(A)$
2. $member(B)$
3. $member(C)$
4. $\sim member(x_1) \vee mc(x_1) \vee sk(x_1)$
5. $\sim sk(x_2) \vee lk(x_2, snow)$
6. $\sim mc(x_3) \vee \sim lk(x_3, rain)$
7. $\sim like(A, x_4) \vee \sim lk(B, x_4)$
8. $lk(A, x_5) \vee lk(B, x_5)$
9. $lk(A, rain)$
10. $lk(A, snow)$
11. $\sim member(x_6) \vee \sim mc(x_6) \vee sk(x_6)$

$\sim like(A, x_4) \vee \sim lk(B, x_4)$ 7

$lk(A, snow)$ 10

12 $\sim lk(B, snow)$

$\sim sk(x_2) \vee lk(x_2, snow)$ 5

13 $\sim sk(B)$

$\sim member(x_1) \vee mc(x_1) \vee sk(x_1)$ 4

14 $\sim member(B) \vee mc(B)$

$member(B)$ 2

11 $\sim member(x_6) \vee \sim mc(x_6) \vee sk(x_6)$

$mc(B)$ 15

16 $\sim member(B) \vee sk(B)$

$\sim sk(B)$ 13

17 $\sim member(B)$

$member(B)$ 2

Well known examples in Predicate Calculus

- Man is mortal : rule

  $\forall x[man(x) \rightarrow mortal(x)]$

- shakespeare is a man

  man(shakespeare)

- To infer shakespeare is mortal

  mortal(shakespeare)

Predicate Calculus: origin

- Predicate calculus originated in language

Sentence

Subject Predicate

$\text{Grass} \; \underbrace{\text{is green}}_{\text{Predicate}}$

Subject

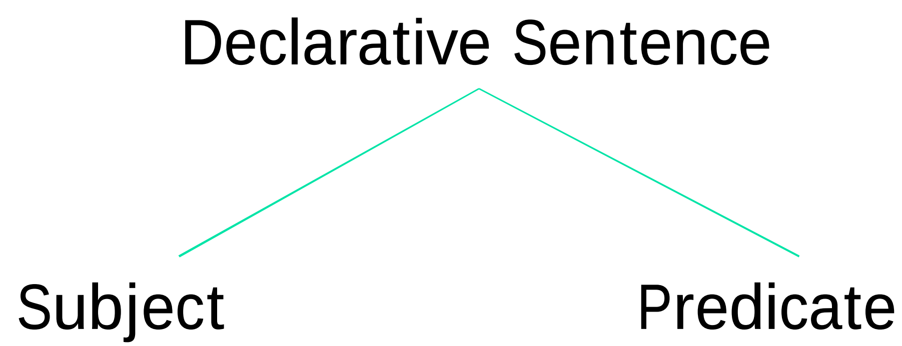
green(grass)

$$P : D \rightarrow \{T, F\}$$

D : Domain

Predicate Calculus: only for declarative sentences

- Is grass green? (Interrogative)
- Oh, grass is green! (Exclamatory)



- Grass which is supple is green

$$\forall x(\mathrm{grass}(x)) \wedge \mathrm{supple}(x) \rightarrow \mathrm{green}(x))$$

Predicate Calculus: more expressive power than propositional calculus

- 2 is even and is divisible by 2: P1
- 4 is even and is divisible by 2: P2
- 6 is even and is divisible by 2: P3

Generalizing,

$$\forall x((Integer(x) \wedge even(x) \Rightarrow divides(2, x))$$

Predicate Calculus: finer than propositional calculus

1. Finer Granularity (Grass is green, ball is green, leaf is green (green(x)))
2. Succinct description for infinite number of statements which would need $\propto$ number of properties

3 place predicate
Example: x gives y to z   give(x,y,z)

4 place predicate
Example: x gives y to z through w   give(x,y,z,w)

Double causative in Hindi giving rise to higher place predicates

- जॉन ने खाना खाया
  John ne khana khaya
  John <CM> food ate
  John ate food
  eat(John, food)

- जॉन ने जैक को खाना खिलाया
  John ne Jack ko khana khilaya
  John <CM> Jack <CM> food fed
  John fed Jack
  eat(John, Jack, food)

- जॉन ने जैक को जिल के द्वारा खाना खिलाया
  John ne Jack ko Jill ke dvara khana khilaya
  John <CM> Jack <CM> Jill <CM> food made-to-eat
  John fed Jack through Jill
  eat(John, Jack, Jill, food)

PC primitive: N-ary Predicate

$$P(a_1, \ldots a_n)$$

$$P : D^n \rightarrow \{T, F\}$$

- Arguments of predicates can be variables and constants
- Ground instances : Predicate all whose arguments are constants

N-ary Functions

$$f : D^n \rightarrow D$$

president(India) : Pranab Mukherjee

- Constants & Variables : Zero-order objects
- Predicates & Functions : First-order objects

Prime minister of India is older than the president of India

older(prime_minister(India), president(India))

Operators

$$\wedge \vee \sim \oplus \forall \rightarrow \exists$$

- Universal Quantifier $\exists$
- Existential Quantifier

All men are mortal

$$\forall x[man(x) \rightarrow mortal(x)]$$

Some men are rich

$$\exists x[man(x) \wedge rich(x)]$$

Tautologies

$$\sim \forall x(p(x)) \rightarrow \exists x(\sim p(x))$$

$$\sim \exists x(p(x)) \rightarrow \forall x(\sim p(x))$$

- 2nd tautology in English:
  - *Not a single man in this village is educated implies all men in this village are uneducated*
- Tautologies are important instruments of logic, but uninteresting statements!

Inferencing: Forward Chaining

- $man(x) \rightarrow mortal(x)$
  - *Dropping the quantifier, implicitly Universal quantification assumed*
  - *man(shakespeare)*
- Goal mortal(shakespeare)
  - Found in one step
  - x = shakespeare, unification

Backward Chaining

- $man(x) \longrightarrow mortal(x)$
- Goal mortal(shakespeare)
  - x = shakespeare
  - Travel back over and hit the fact asserted
  - man(shakespeare)

Wh-Questions and Knowledge

what
where
who
when
which

Factoid / Declarative

how — procedural

why — Reasoning

Fixing Predicates

- Natural Sentences

<Subject> <verb> <object>

Verb(subject,object)

predicate(subject)

Examples

- Ram is a boy
  - Boy(Ram)?
  - Is_a(Ram,boy)?

- Ram Playes Football
  - Plays(Ram,football)?
  - Plays_football(Ram)?

Knowledge Representation of Complex Sentence

- *“In every city there is a thief who is beaten by every policeman in the city”*

Knowledge Representation of Complex Sentence

- *"In every city there is a thief who is beaten by every policeman in the city"*

$$\forall x[\text{city}(x) \rightarrow \{\exists y((\text{thief}(y) \wedge \text{lives\_in}(y,x)) \wedge \forall z(\text{policeman}(z,x) \rightarrow \text{beaten\_by}(z,y)))\}]$$

Insight into resolution

Resolution - Refutation

- $man(x) \rightarrow mortal(x)$
  - *Convert to clausal form*
  - $\sim man(shakespeare) \vee mortal(x)$
- Clauses in the knowledge base
  - $\sim man(shakespeare) \vee mortal(x)$
  - $man(shakespeare)$
  - $mortal(shakespeare)$

Resolution – Refutation contd

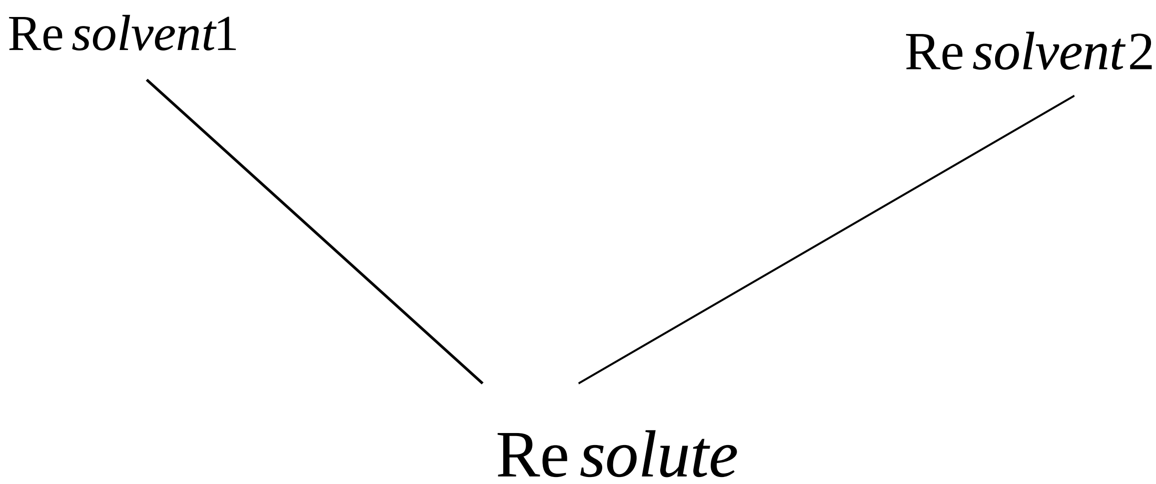
- *Negate the goal*
  - *~man(shakespeare)*
- Get a pair of resolvents

$\sim mortal(shakespeare)$ $\quad\quad$ $\sim man(shakespeare) \vee mortal(shakespeare)$

$\sim man(shakespeare)$ $\quad\quad$ $\sim man(shakespeare)$

$\square$

Resolution Tree



Search in resolution

- Heuristics for Resolution Search
  - Goal Supported Strategy
    - Always start with the negated goal
  - Set of support strategy
    - Always one of the resolvents is the most recently produced resolute

Inferencing in Predicate Calculus

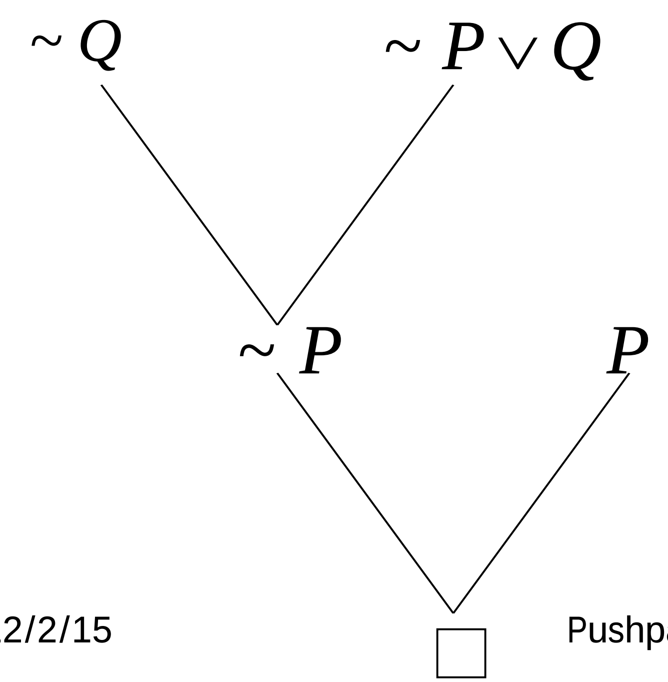
- Forward chaining
  - Given P, $P \rightarrow Q$, to infer Q
  - P, match *L.H.S* of
  - Assert Q from *R.H.S*
- Backward chaining
  - Q, Match *R.H.S* of $P \rightarrow Q$
  - assert P
  - Check if P exists
- Resolution – Refutation
  - Negate goal
  - Convert all pieces of knowledge into clausal form (disjunction of literals)
  - See if contradiction indicated by null clause $\square$ can be derived

1. P

2. $P \rightarrow Q$ converted to $\sim P \vee Q$

3. $\sim Q$

Draw the resolution tree (actually an inverted tree). Every node is a clausal form and branches are intermediate inference steps.



Theoretical basis of Resolution

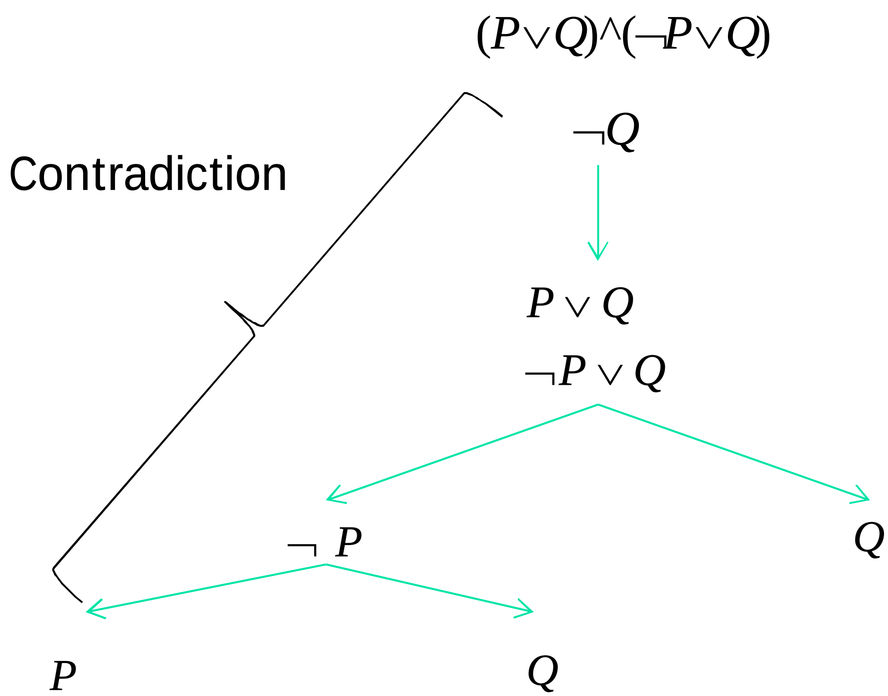
- Resolution is proof by contradiction
- ***resolvent1 .AND. resolvent2 => resolute*** is a tautology

$P \vee Q$ $\qquad\qquad\qquad$ $\neg P \vee Q$

Q

Tautologiness of Resolution

- Using Semantic Tree



Theoretical basis of Resolution (cont ...)

- Monotone Inference
  - Size of Knowledge Base goes on increasing as we proceed with resolution process since intermediate resolvents added to the knowledge base
- Non-monotone Inference
  - Size of Knowledge Base does not increase
  - Human beings use non-monotone inference

Interpretation in Logic

- Logical expressions or formulae are "FORMS" (placeholders) for whom <u>contents</u> are created through interpretation.
- Example:

$$\exists F\left[\left\{F(a)=b\right\}\wedge\forall x\left\{P(x)\rightarrow\left(F(x)=g\left(x,F\left(h(x)\right)\right)\right)\right\}\right]$$

- This is a Second Order Predicate Calculus formula.
- Quantification on 'F' which is a function.

Examples

- Interpretation:1

  $D=N$ (natural numbers)

  $a = 0$ and $b = 1$

  $x \in N$

  $P(x)$ stands for $x > 0$

  $g(m,n)$ stands for $(m \times n)$

  $h(x)$ stands for $(x - 1)$

- Above interpretation defines **Factorial**

Examples (contd.)

- Interpretation:2

  D={strings)

  $a = b = \lambda$

  $P(x)$ stands for “x is a non empty string”

  $g(m, n)$ stands for “append head of m to n”

  $h(x)$ stands for $tail(x)$
- Above interpretation defines “reversing a string”

Other examples

$$\exists P[\forall x \exists y P(x,y) \wedge \forall x \neg P(x,x) \wedge \forall x \forall y \forall z[(P(x,y) \wedge P(y,z)) \Rightarrow P(x,z)]]$$

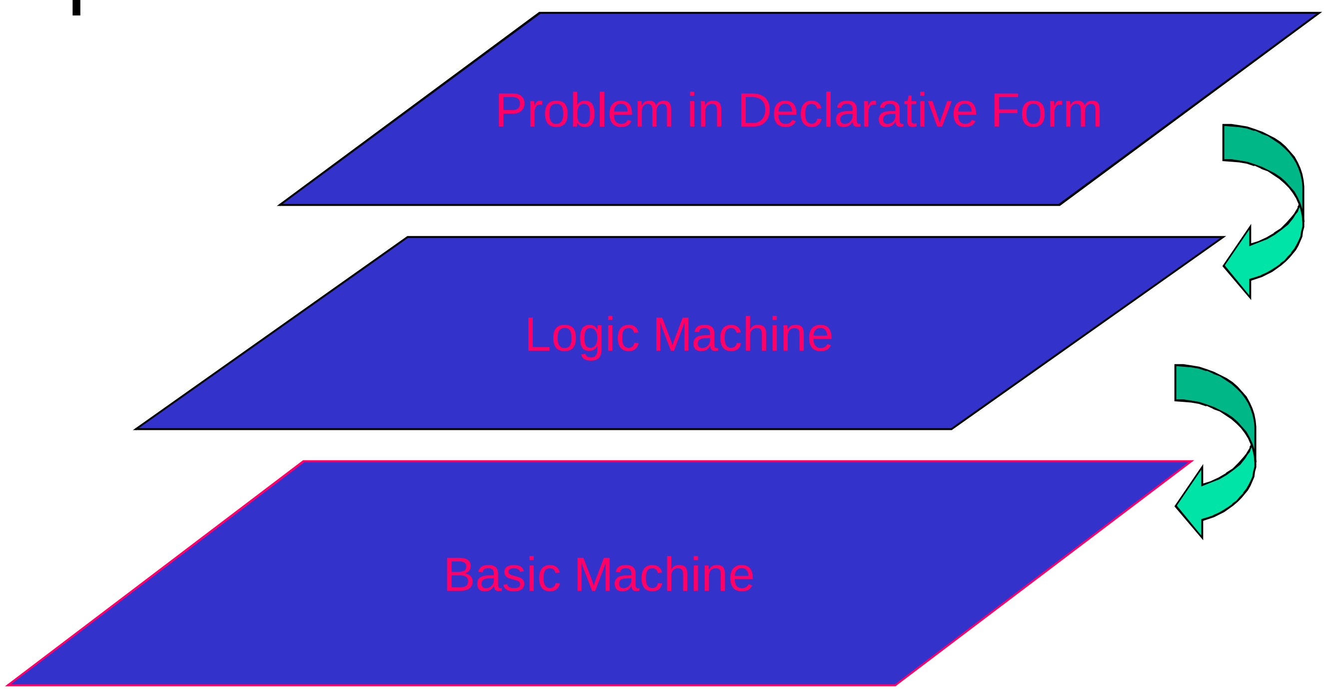
$$\forall x_1 x_2 x_3[\{P(x_1,x_1) \wedge P(x_2,x_2) \wedge P(x_3,x_3)\} \Rightarrow \{P(x_1,x_2) \vee P(x_1,x_3) \vee P(x_2,x_3)\}]$$

True in all domains of cardinality <=3

Prolog

Introduction

- PROgramming in LOGic
- Emphasis on *what* rather than *how*



A Typical Prolog program

Compute_length ([],0).

Compute_length ([Head|Tail], Length):-

Compute_length (Tail,Tail_length),

Length is Tail_length+1.

<u>High level explanation:</u>

The length of a list is 1 plus the length of the tail of the list, obtained by removing the first element of the list.

This is a declarative description of the computation.

Fundamentals

(absolute basics for writing Prolog Programs)

Facts

- *John likes Mary*
  - *like(john,mary)*
- Names of relationship and objects must begin with a lower-case letter.
- Relationship is written *first* (typically the *predicate* of the sentence).
- *Objects* are written separated by commas and are enclosed by a pair of round brackets.
- The full stop character '.' must come at the end of a fact.

More facts

Predicate	**Interpretation**
valuable(gold)	Gold is valuable.
owns(john,gold)	John owns gold.
father(john,mary)	John is the father of Mary
gives (john,book,mary)	John gives the book to Mary

Questions

- *Questions* based on facts
- Answered by *matching*

Two facts *match* if their predicates are same (spelt the same way) and the arguments each are same.

- If matched, prolog answers *yes*, else *no*.
- *No* does not mean falsity.

Prolog does *theorem proving*

- When a question is asked, prolog tries to match *transitively.*
- When no match is found, answer is *no.*
- This means *not provable* from the given facts.

Variables

- Always begin with a capital letter
  - *?- likes (john,X).*
  - *?- likes (john, Something).*
- But *not*
  - *?- likes (john,something)*

Example of usage of variable

Facts:

likes(john,flowers).

likes(john,mary).

likes(paul,mary).

Question:

?- likes(john,X)

Answer:

X=flowers and wait

;

mary

;

no

Conjunctions

- Use ‘,’ and pronounce it as *and*.
- Example
  - Facts:
    - likes(mary,food).
    - likes(mary,tea).
    - likes(john,tea).
    - likes(john,mary)
- ?-
    - likes(mary,X),likes(john,X).
    - Meaning *is anything liked by Mary also liked by John?*

Backtracking *(an inherent property of prolog programming)*

likes(mary,X),likes(john,X)

likes(mary,food)
likes(mary,tea)
likes(john,tea)
likes(john,mary)

1. First goal succeeds. *X=food*
2. Satisfy *likes(john,food)*

Backtracking *(continued)*

Returning to a marked place and trying to resatisfy is called *Backtracking*

likes(mary,X),likes(john,X)

likes(mary,food)
likes(mary,tea)
likes(john,tea)
likes(john,mary)

1. Second goal fails
2. Return to marked place
   and try to resatisfy the first goal

Backtracking *(continued)*

likes(mary,X),likes(john,X)

likes(mary,food)
likes(mary,tea)
likes(john,tea)
likes(john,mary)

1. First goal succeeds again, *X=tea*
2. Attempt to satisfy the *likes(john,tea)*

Backtracking *(continued)*

likes(mary,X),likes(john,X)

likes(mary,food)
likes(mary,tea)
likes(john,tea)
likes(john,mary)

1. Second goal also suceeds
2. Prolog notifies success and waits for a reply

Rules

- Statements about *objects* and their *relationships*
- Expess
  - *If-then conditions*
    - *I use an umbrella if there is a rain*
    - *use(i, umbrella) :- occur(rain).*
  - *Generalizations*
    - *All men are mortal*
    - *mortal(X) :- man(X).*
  - *Definitions*
    - *An animal is a bird if it has feathers*
    - *bird(X) :- animal(X), has_feather(X).*

Syntax

- **<head> :- <body>**
- Read **':-' as 'if'.**
- E.G.
  - *likes(john,X) :- likes(X,cricket).*
  - *"John likes X if X likes cricket".*
  - *i.e., "John likes anyone who likes cricket".*
- Rules always end with '.'.

Another Example

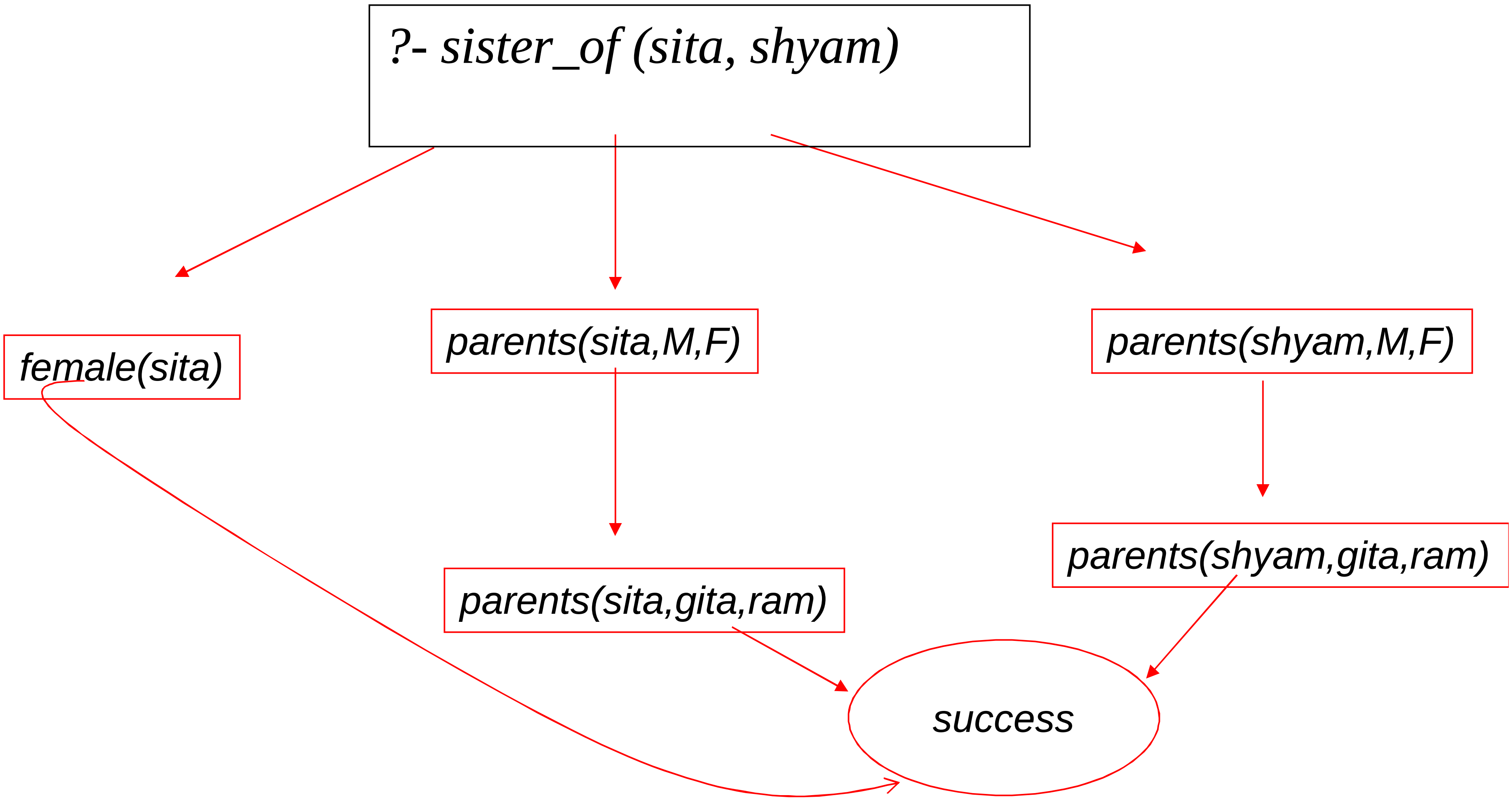
sister_of (X,Y):- female (X),
parents (X, M, F),
parents (Y, M, F).

X is a sister of Y is
X is a female and
X and Y have same parents

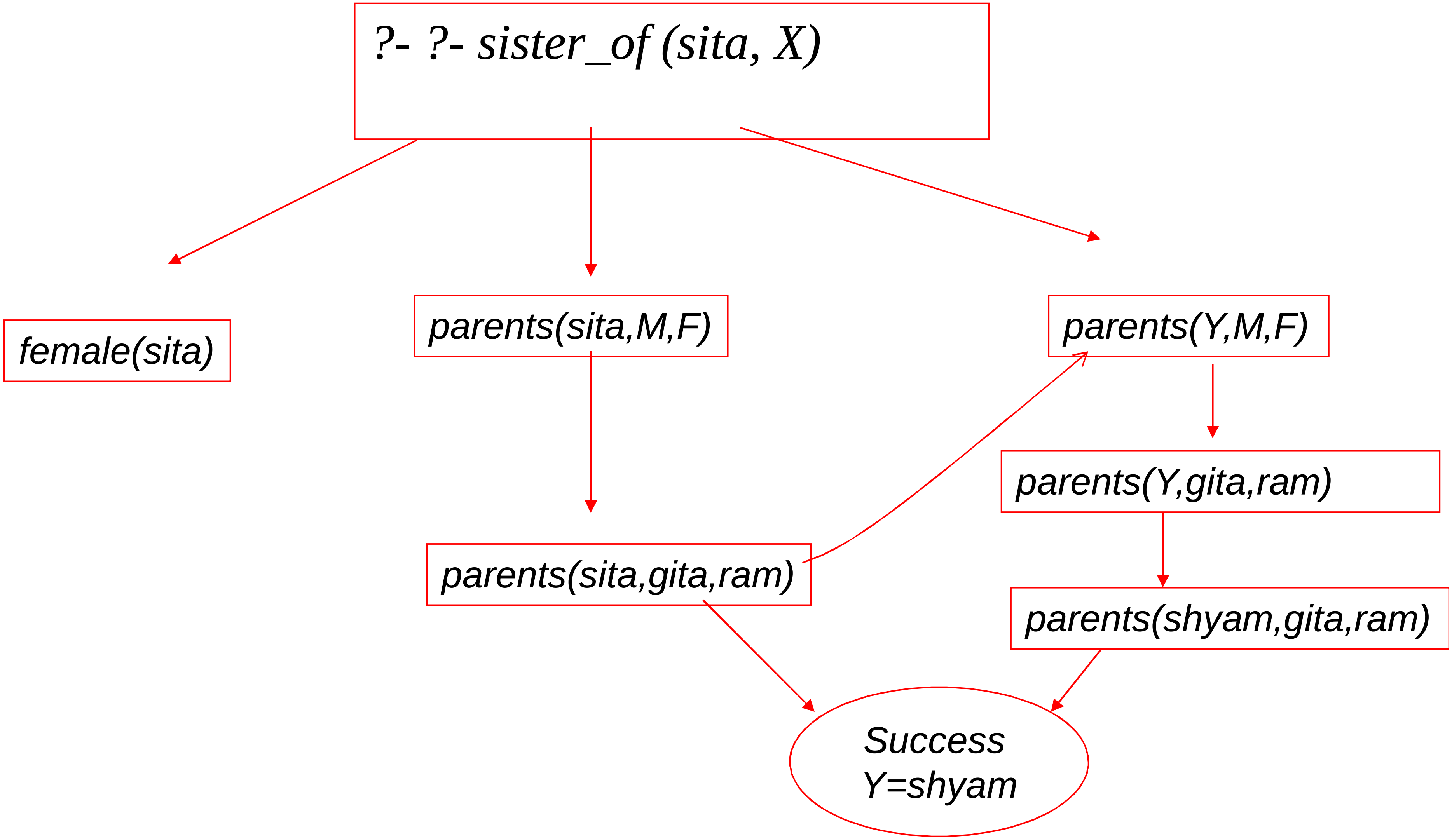
Question Answering in presence of *rules*

- Facts
  - male (ram).
  - male (shyam).
  - female (sita).
  - female (gita).
  - parents (shyam, gita, ram).
  - parents (sita, gita, ram).

Question Answering: Y/N type: *is sita the sister of shyam?*



Question Answering: wh-type: *whose sister is sita?*



Rules

- Statements about *objects* and their *relationships*
- Express
  - *If-then conditions*
    - *I use an umbrella if there is a rain*
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Make and Break

Fundamental to Prolog

Prolog examples using making and breaking lists

```
%incrementing the elements of a list to produce another list
incr1([],[]).
incr1([H1|T1],[H2|T2]) :- H2 is H1+1, incr1(T1,T2).

%appending two lists; (append(L1,L2,L3) is a built is
function in Prolog)
append1([],L,L).
append1([H|L1],L2,[H|L3]):- append1(L1,L2,L3).


%reverse of a list (reverse(L1,L2) is a built in function
reverse1([],[]).
reverse1([H|T],L):- reverse1(T,L1),append1(L1,[H],L).
```

Remove duplicates

Problem: to remove duplicates from a list

```
1. rem_dup([],[]).
2. rem_dup([H|T],L) :- member(H,T), !, rem_dup(T,L).
3. rem_dup([H|T],[H|L1]) :- rem_dup(T,L1).
```

Note: The cut ! in the second clause needed, since after succeeding at member(H,T), the expression no. 3 clause should not be tried even if rem_dup(T,L) fails, which prolog will otherwise do.

Member (membership in a list)

```
member(X,[X|_]).
member(X,[_|L]):- member(X,L).
```

Union (lists contain unique elements)

```
union([],Z,Z).
union([X|Y],Z,W):-
  member(X,Z),!,union(Y,Z,W).
union([X|Y],Z,[X|W]):- union(Y,Z,W).
```

Intersection (lists contain unique elements)

```
intersection([],Z,[]).
intersection([X|Y],Z,[X|W]):-
    member(X,Z),!,intersection(Y,Z,W).
intersection([X|Y],Z,W):-
    intersection(Y,Z,W).
```

XOR

```
%xor of two lists
xor(L1,L2,L3):-
   diff1(L1,L2,X),diff1(L2,L1,Y),append(X,Y,L3).

%diff(P,Q,R) returns true if R is is P-Q

diff1([],Q,[]).
diff1([H|T],Q,R):- member(H,Q),!,diff1(T,Q,R).
diff1([H|T1],Q,[H|T2]):- diff1(T1,Q,T2).
```

Prolog Programs are close to Natural Language

Important Prolog Predicate:

member(e, L) / true if e is an element of list L*

member(e,[e|L1). / e is member of any list which it starts*

*member(e,[_|L1]):- member(e,L1) /*otherwise e is member of a list if the tail of the list contains e*

Contrast this with:

P.T.O.

Prolog Programs are close to Natural Language, C programs are not

```
For (i=0;i<length(L);i++){
   if (e==a[i])
         break(); /*e found in a[]
}
If (i<length(L){
   success(e,a); /*print location where e appears in
         a[]/*
else
   failure();
}
```

What is *i* doing here? Is it natural to our thinking?

Machine should ascend to the level of man

- A prolog program is an example of reduced man-machine gap, unlike a C program
- That said, a very large number of programs far outnumbering prolog programs gets written in C
- The demand of practicality many times incompatible with the elegance of ideality
- But the ideal should nevertheless be striven for

Prolog Program Flow, BackTracking and Cut

Controlling the program flow

Prolog’s computation

- **Depth First Search**
  - **Pursues a goal till the end**
- **Conditional AND; *falsity* of any goal prevents satisfaction of further clauses.**
- **Conditional OR; *satisfaction* of any goal prevents further clauses being evaluated.**

Control flow (top level)

Given

g:- a, b, c. (1)

g:- d, e, f; p. (2)

If prolog cannot satisfy (1), control will automatically fall through to (2).

Control Flow within a rule

Taking (1),

g:- a, b, c.

If *a* succeeds, prolog will try to satisfy *b,* succeding which *c* will be tried.

For ANDed clauses, control flows forward till the '.', iff the current clause is *true.*

For ORed clauses, control flows forward till the '.', iff the current clause evaluates to *false.*

What happens on failure

- **REDO the immediately preceding goal.**

Fundamental Principle of prolog programming

- **Always place the more general rule AFTER a specific rule.**

CUT

- **Cut tells the system that**

IF YOU HAVE COME THIS FAR

DO NOT BACKTRACK

EVEN IF YOU FAIL SUBSEQUENTLY.

‘CUT’ WRITTEN AS ‘!’ ALWAYS SUCCEEDS.

Fail

- This predicate always fails.
- *Cut* and *Fail* combination is used to produce negation.
- Since the LHS of the neck cannot contain any operator, $A \rightarrow \sim B$ is implemented as

B :- A, !, Fail.

Prolog and Himalayan Club example

- *(Zohar Manna, 1974):*
  - Problem: A, B and C belong to the Himalayan club. Every member in the club is either a mountain climber or a skier or both. A likes whatever B dislikes and dislikes whatever B likes. A likes rain and snow. No mountain climber likes rain. Every skier likes snow. *Is there a member who is a mountain climber and not a skier?*
- Given knowledge has:
  - Facts
  - Rules

A syntactically wrong prolog program!

1. belong(a).
2. belong(b).
3. belong(c).
4. mc(X);sk(X) :- belong(X) /* X is a mountain climber or skier or both if X is a member; operators NOT allowed in the head of a horn clause; hence wrong*/
5. like(X, snow) :- sk(X). /*all skiers like snow*/
6. \+like(X, rain) :- mc(X). /*no mountain climber likes rain; \+ is the not operator; negation by failure; wrong clause*/
7. \+like(a, X) :- like(b,X). /* a dislikes whatever b likes*/
8. like(a, X) :- \+like(b,X). /* a likes whatever b dislikes*/
9. like(a,rain).
10. like(a,snow).

?- belong(X),mc(X),\+sk(X).

Correct (?) Prolog Program

```
belong(a).
belong(b).
belong(c).
belong(X):-\+mc(X),\+sk(X), !, fail.
belong(X).
like(a,rain).
like(a,snow).
like(a,X) :- \+ like(b,X).
like(b,X) :- like(a,X),!,fail.
like(b,X).
mc(X):-like(X,rain),!,fail.
mc(X).
sk(X):- \+like(X,snow),!,fail.
sk(X).
g(X):-belong(X),mc(X),\+sk(X),!. /*without this cut, Prolog will look for the next
    answer on being given ';' and return 'c' which is wrong*/
```

Himalayan club problem: working version

```
belong(a).
belong(b).
belong(c).

belong(X):-notmc(X),notsk(X),!, fail. /*contraposition to have horn clause
belong(X).

like(a,rain).
like(a,snow).
like(a,X) :- dislike(b,X).
like(b,X) :- like(a,X),!,fail.
like(b,X).

mc(X):-like(X,rain),!,fail.
mc(X).
notsk(X):- dislike(X,snow). /*contraposition to have horn clause
notmc(X):- mc(X),!,fail.
notmc(X).

dislike(P,Q):- like(P,Q),!,fail.
dislike(P,Q).

g(X):-belong(X),mc(X),notsk(X),!.
```

Circuit verification

Circuit Verification

- Does the circuit meet the specs?
- Are there faults?
- are they locatable?

Example : 2-bit full adder

C1	X2	X1	Y	C2
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

X_1, X_2: inputs; C_1: prev. carry; C_2: next carry; Y: output

K-Map

Y

c1 \ x2x1	00	01	11	10
0	0	1	0	1
1	1	0	1	0

$$Y = C1(\overline{X1 \oplus X2}) + \overline{C1}(X1 \oplus X2)$$

$$= (C1 \oplus (X1 \oplus X2))$$

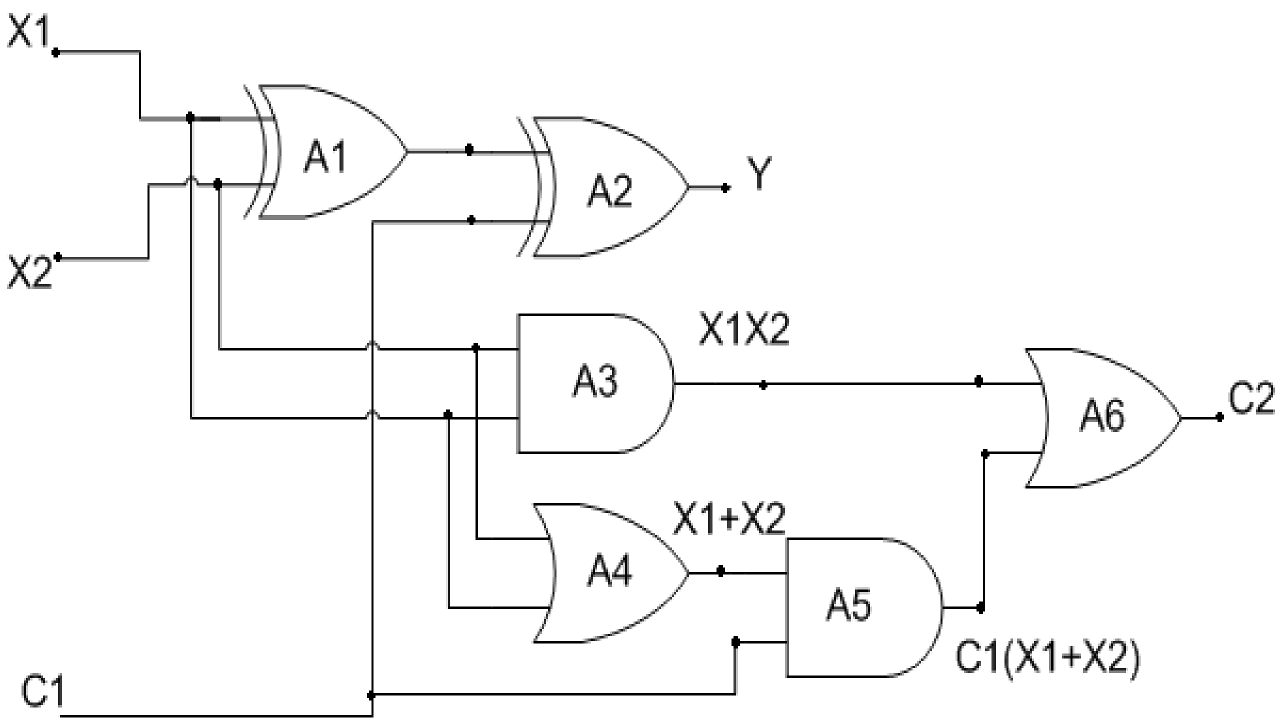
K-Map (contd..)

C2

$c1 \backslash x2x1$	00	01	11	10
0	0	0	1	0
1	0	1	1	1

$$C2 = X2X1 + C1(X1 + X2)$$

Circuit



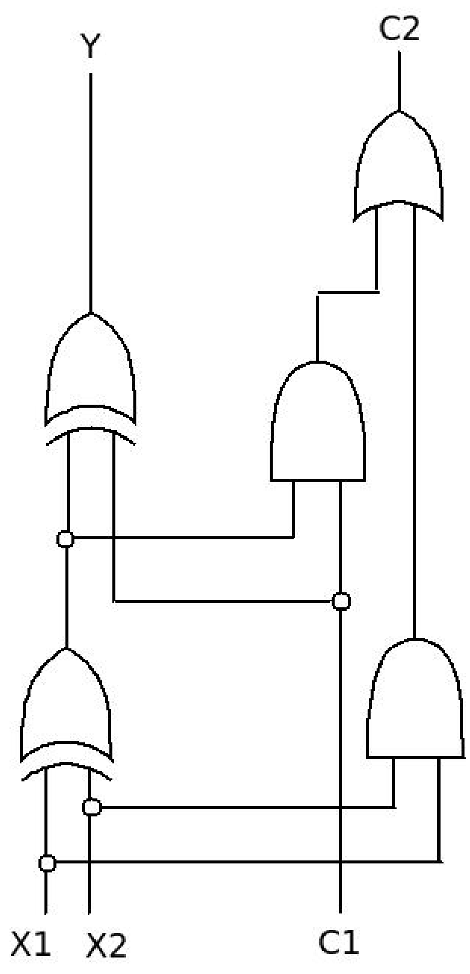
Verification

- First task (most difficult)
  - Building blocks : predicates
  - Circuit observation : Assertion on terminals

Predicates & Functions

Function–1	signal(t)	t is a terminal ; signal takes the value 0 or 1
Function–2	type(x)	x is a circuit element; type(x) takes the value AND, OR, NOT, XOR
Predicate – 3	connected(t1,t2)	t1 is an output terminal and t2 is an input terminal
Function-3	In(n,x)	n^{th} input of ckt element x
Function-4	Out(x)	Output of ckt element x

Alternate Full Adder Circuit



Functions

- type(X) : takes values AND, OR NOT and XOR, where X is a gate.
- in(n, X) : the value of signal at the n^{th} input of gate X.
- out(X) : output of gate X.
- signal(t) : state at terminal t = 1/0

Predicates

- connected(t1,t2): true, if terminal t1 and t2 are connected

General Properties

- Commutativity:

  $\forall t_1,t_2$ [connected$(t_1,t_2) \rightarrow$ connected(t_2,t_1)]

- By definition of connection:

  $\forall t_1,t_2$ [connected$(t_1,t_2) \rightarrow$ { signal(t_1) = signal(t_1)}]

Gate properties

1. OR definition:

$$\forall X\,[\{type(X) = OR\} \equiv \\ \{(out(X) = 1) \equiv \exists y\,(in(y, X) = 1)\}]$$

2. AND definition:

$$\forall X\,[\{type(X) = AND\} \equiv \\ \{(out(X) = 1) \equiv \forall y\,(in(y, X) = 1)\}]$$

Gate properties contd...

1. XOR definition:

$$\forall X\,[\{type(X) = XOR\} \equiv \{(out(X) = 1) \equiv (in(1, X) \neq in(2, X))\}]$$

2. NOT definition:

$$\forall X\,[\{type(X) = NOT\} \equiv \{out(X) \neq in(1, X)\} \wedge (no\_of\_input(X) = 1)]$$

Some necessary functions

a. no_of_input(x), takes integer values
b. Count_ls(x), returns *no. of 1s* in the input of X

$$\forall X\, [\{type(X) = XOR\} \equiv$$
$$\{(out(X) = 1) \equiv odd((count\_ls(X))\}]$$

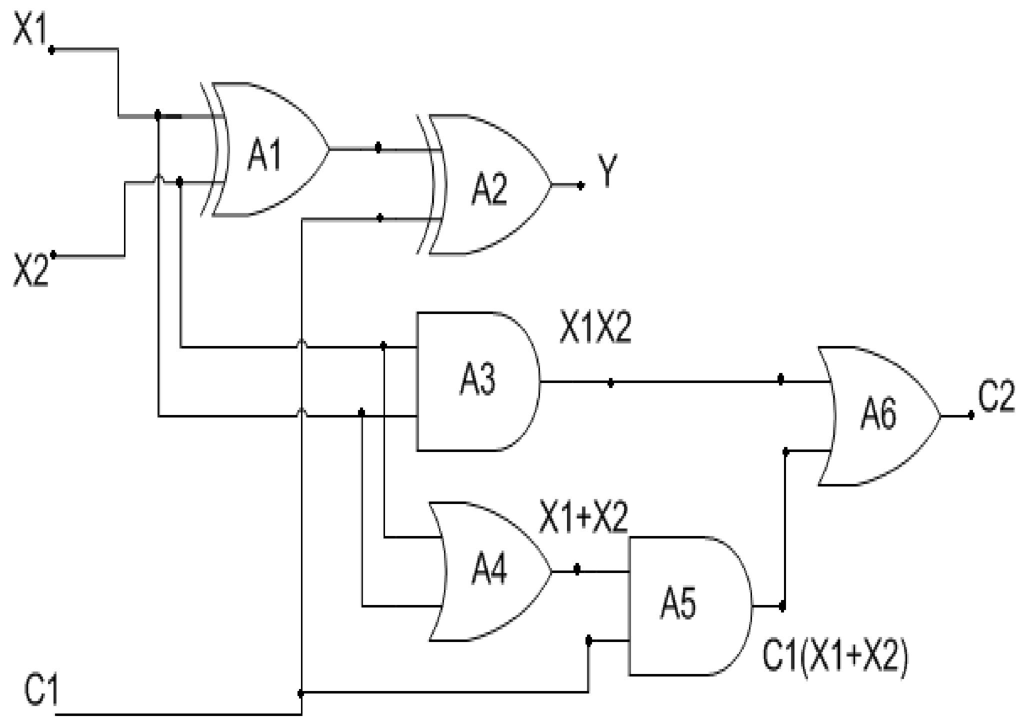
Circuit specific properties

- Connectivity:

connected(x_1, in(1,A_1))
connected(x_1, in(2, A_1))
connected(out(A_1), in(1, A_2))
connected(c1, in(2, A_2))
connected(y, out(A_2)) ...

- Circuit elements:

type(A_1) = XOR,
type(A_2) = XOR,
type(A_3) = AND ...



Circuit