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Tut 6 Machine learning I

1) a) $P(H) = \lambda$

$\therefore P(T) = 1-\lambda$

P (H at k+1 th toss) = P(T at k toss and H at k+1)

$= (1-\lambda)^k \lambda$

b) Let M $\Rightarrow$ no. of toss req. to get first head

& let $S = E[M]$

Tosses are independ & expect is additive

$S = \lambda \times 1 + (1-\lambda) \times (S+1)$

$\therefore \cancel{S} = \cancel{\lambda} + \cancel{S} + 1 - \lambda S - \cancel{\lambda}$

$\therefore S\lambda = 1$

$\therefore S = \frac{1}{\lambda}$

2) X is a random Variable.

a) Var of X = $Var(X) = E[(X - E[X])^2]$

To prove:- $Var(X) = E[X^2] - E[X]^2$

$Var(X) = E[(X - E[X])^2]$

$= E[X^2 - 2E[X] + E[X]^2]$

$= E[X^2] - 2E[XE[X]] + E[X]^2$

$= E[X^2] - 2E[X]^2 + E[X]^2$

$= E[X^2] - E[X]^2$

b) $E[X]=0$ & $E[X^2]=1$

To find i) Var of X

2) iff $Y=a+bX$, $Var(Y)=?$

① $Var(X) = E[X^2] - E[X]^2$

$= 1-0^2$

$\therefore Var(X)=1$

② $Y=a+bX$

$E[Y^2] = E[(a+bX)^2]$

$= E[a^2+2abX+b^2X^2]$

$= a^2 + 2abE[X] + b^2E[X^2]$

$= a^2+2ab(0)+b^2(1)$

$\therefore E[Y^2] = \underline{a^2+b^2}$

$E[Y] = E[a+bX] = a+bE[X]$

$= a+b(0)$

$E[Y] = a$

$Var(Y) = E[Y^2] - E[Y]^2 = a^2+b^2-a^2$

$= \underline{b^2}$

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3) . Let A be event that " Aku predicts → given horse is winning "

Let ~A be event → Aku predicts given horse is not winning horse

Similarly, let B be event that given horse wins & ~B → given horse does not win.

a) $P(B) = P(B, A) + P(B, \sim A)$

$= P(B|A)P(A) + P(B|\sim A)P(\sim A)$

$= 0.99 \times 10^{-5} + (1 - 0.9999) \times (1 - 10^{-5})$

$\therefore P(B) \approx 1.99 \times 10^{-5}$ —— (I)

b) Prob. that Aku predicts Black beauty is winni

$P(A|B) = \frac{P(A, B)}{P(B)} = \frac{P(A|B)P(A)}{P(B)}$

$= \frac{0.99 \times 10^{-5}}{1.99 \times 10^{-5}}$ —— (from I)

$\therefore P(A|B) = 0.497.$