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Tut 4

Independent Component Analysis.

Q 1. Mix Statistical independent resources.

→ Variance ⇒ $var(x) = \langle (x - \langle x \rangle)^2 \rangle$

$$= \langle x^2 \rangle - \langle x \rangle^2$$

$$= \left\langle \left(\sum_i w_i s_i \right)^2 \right\rangle - \left\langle \sum_i w_i s_i \right\rangle^2$$

$$= \left\langle \left(\sum_i w_i s_i \right) \left(\sum_i w_j s_j \right) \right\rangle - \left(\sum_i w_i \langle s_i \rangle \right) \left(\sum_j w_j \langle s_j \rangle \right)$$

$$= \left\langle \sum_{i,j} w_i w_j s_i s_j \right\rangle - \sum_{i,j} w_i w_j \langle s_i \rangle \langle s_j \rangle$$

$$= \sum_{i,j} w_i w_j \langle s_i, s_j \rangle - \sum_{i,j} w_i w_j \langle s_i \rangle \langle s_j \rangle$$

$$= \sum_i w_i w_j \left(\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle \right) + \sum_{i,j \; i \neq j} w_i w_j \left(\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle \right)$$

$$= \sum_i w_i^2 \left(\langle s_i s_j \rangle - \langle s_i \rangle^2 \right) + \sum_{i,j \; j \neq i} w_i w_j \langle \langle s_i \rangle \langle s_j \rangle - \langle s_i \rangle \langle s_j \rangle$$

s_i & s_j are statistically independent for $i \neq j \Rightarrow \langle s_i \rangle \langle s_j \rangle - \langle s_i \rangle \langle s_j \rangle = 0$

Also $var(s_i) = 1$

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$\therefore \quad var(x) = \sum_i w_i^2$

The mixture has unit variance,
to guarantee,

$$var(x) = 1$$

$$\therefore \sum_i w_i^2 = 1$$

For mixture to have unit variance,
constraint should be,

$$\sum_i w_i^2 = 1$$

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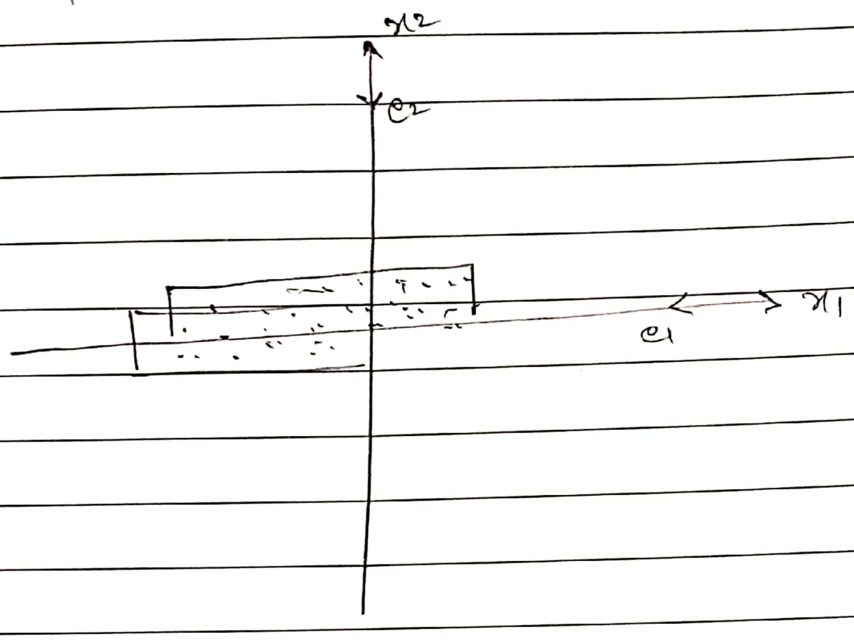
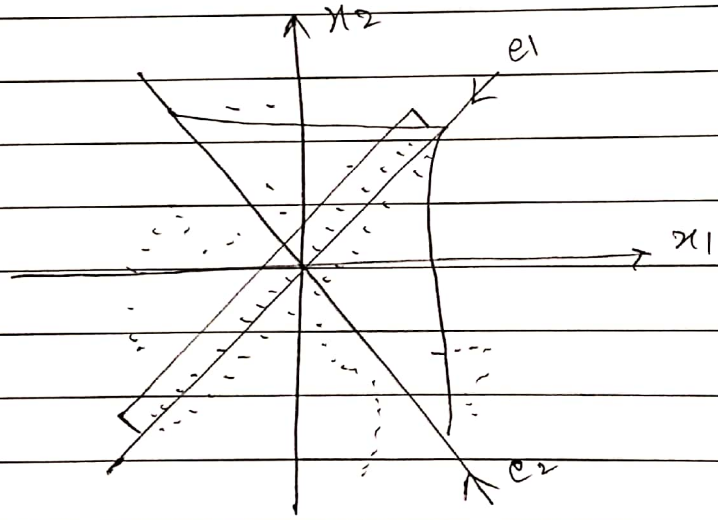
2. Guess Independent components of distribution from data

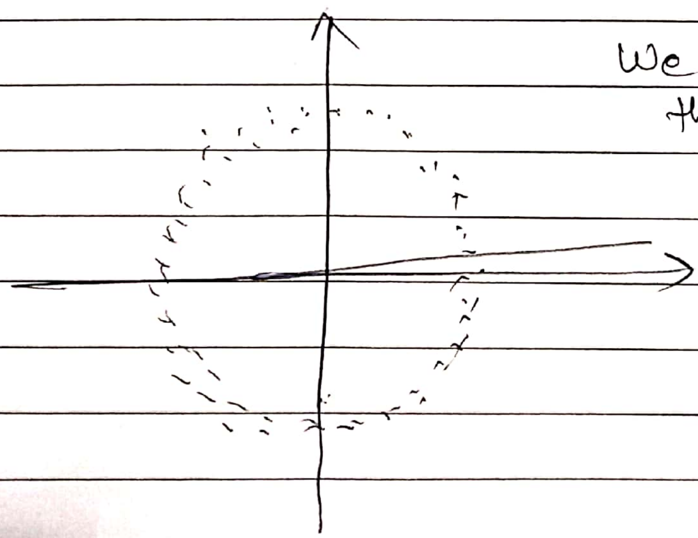
a)



b)



c)



We cannot separate this in independent components