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Tutorial 5.

Evaluation & Measurement - Hypothesis Testing.

1) $H_0 : p = 0.7$

$H_1 : p \neq 0.7$

Level of significance $= \alpha = 0.10$

Test Statistic : Binomial Variable X with $P = 0.7$ &

$n = 15$

$X = 8$ & $np_0 = 15 \times 0.7 = 10.5$

$\therefore P = 2P(X \leq 8 \text{ when } p = 0.7)$

$= 2 \sum_{x=0}^{8} b(x; 15; 0.7)$

$= 2 \times 0.1311$ (From Binomial Prob. Table)

$= 0.2622$

$\therefore P > 0.10$ i.e. $P > \alpha$.

Don't Reject H_0.

Insufficient reason to doubt builder's claim

2) $H_0 : P = 0.6$

$H_1 : P > 0.6$

level of significance $= \alpha = 0.05$

$\Rightarrow x = 70, \quad n = 100, \quad p = 0.6$

$$\therefore z = \frac{x - np_0}{\sqrt{np_0q_0}}$$

$$\therefore z = \frac{70 - 100 \times 0.6}{\sqrt{100 \times 0.6 \times 0.4}}$$

$\therefore z = 2.04$

$p = P(z > 2.04)$

$P = 0.0207$ (From table)

As $P < \alpha$, reject H_0 & conclude. new drug is superior

Scanned with CamScanner

3) Let proportion of Mumbai Voters $\Rightarrow P_1$
& proportion of surrounding area residents $= P_2$

$$\hat{P}_1 = \frac{120}{200} = 0.6$$

$$\hat{P}_2 = \frac{240}{500} = 0.48$$

$$\alpha = 5\% = 0.05$$

$$\hat{P}_P = \frac{120+240}{200+500} = 0.514$$

Hypothesis

$H_0 : P_1 \le P_2$
$H_1 : P_1 > P_2$

$$Z = \frac{\hat{P}_1 - \hat{P}_2}{\sqrt{\hat{P}_P(1-\hat{P}_P)\left(\frac{1}{n_1} - \frac{1}{n_2}\right)}}$$

$$\therefore Z = \frac{0.6-0.48}{\sqrt{0.514(1-0.514)\left(\frac{1}{200}+\frac{1}{500}\right)}}$$

$$\therefore Z = 2.869$$
$$\therefore P = P(Z > 2.869) = 0.0044$$

As $P < \alpha$ reject H_0.
Conclusi $\Rightarrow$ prop. of Mumbai voters is higher than prop of surrounding area voters

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4) a) Null Hypothesis

$H_0 : p = 0.20$

Alternative Hypoth.

$H_1 = p > 0.20$

Critical region ⇒ right tail

b) $H_0 : \mu = 3$
$H_1 : \mu \neq 3$

Critical region in both tails.

c) $H_0 : p = 0.15$
$H_1 : p < 0.15$
Critical region ⇒ left tail

d) $H_0 : \mu = 500$
$H_1 : \mu > 500$
Critica reg ⇒ right tail

e) $H_0 : \mu = 15$
$H_1 : \mu \neq 15$

Critical reg ⇒ both tails.

$\mu_1 \Rightarrow$ Populal mean robust by Company A

$\mu_2 \Rightarrow$ Popul mean robu by Company B.

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

level of significa $= \alpha = 0.05$

$$\overline{x_1} = \frac{1}{n_1} \sum_{i=1}^{n_1} x_{1i}$$

$$= \frac{9.3 + 8.8 + 6.8 + 8.7 + 8.5 + 6.7 + 8.0 + 6.5 + 9.2 + 7.0}{10}$$

$$\therefore \overline{x_1} = 7.95$$

$$\overline{x_2} = \frac{1}{n_2} \sum_{i=1}^{n} x_{2i}$$

$$\overline{x_2} = 10.26$$

$$S_1^2 = \frac{1}{n_1 - 1}\left[\sum_{i=1}^{n_1} x_{1i}^2 - n_1 \overline{x_1}^2\right]$$

$$S_2^2 = \frac{10.865}{9} = 1.207$$

$$S_2^2 = \frac{1}{n_2 - 1}\left[\sum_{i=1}^{n_2} x_{2i}^2 - n_2 \overline{x_2}^2\right]$$

$$S_2^2 = \frac{2.924}{9} = 0.325$$

Simple variances are quite diff.
We cannot assum that population variances are equal.

So use ~~th~~ unpooled t-test.

Degree of freedom:-.

$$V = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{1}{n_1-1}\left(\frac{S_1^2}{n_1}\right)^2 + \frac{1}{n_2-1}\left(\frac{S_2^2}{n_2}\right)^2}$$

$$= \frac{\left(\frac{1.207}{10} + \frac{0.325}{10}\right)^2}{\frac{1}{10-1}\left(\frac{1.207}{10}\right)^2 + \frac{1}{10-1}\left(\frac{0.325}{10}\right)^2}$$

$V = 10.30$

$V \approx 10.$

Test statistics to test

$$T = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

$$\therefore T = \frac{7.95 - 10.26}{\sqrt{\frac{1.207}{10} + \frac{0.325}{10}}} = -5.90$$

Test is 2 sided, Value of test is double area under density curve of t-distrn with 10 degree of freed.

$|t| = |-5.90| = 5.90.$

$\therefore$ p-val $= 2 \cdot P(T \geq |t|) = 2 \cdot P(T \geq 5.90)$

$t_{0.0005}(10) = 4.587$ & $|t| = 5.9$ is even greater than $P(T \geq 5.90) < 0.0005$

$\therefore$ p-val < 0.001

$\therefore P < \alpha$,

We reject null hypothesis

& conclude, mean robustness of laptop is not same for both companies.