UE18MA251

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CSE, Sec-5

Linear Algebra

Assignment

1) $y = A + Bx + Cx^2$

Parabola passes through (1, 1) Hence

$1 = A + B + C$ — ①

Parabola passes through (2, -1)

$-1 = A + 2B + 4C$

Parabola passes through (3, 1) — ③

$1 = A + 3B + 9C$

In $Ax = b$ form

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$[A\,b] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & -1 \\ 1 & 3 & 9 & 1 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 2 & 8 & 0 \end{bmatrix} \quad R_3 \to R_3 - 2R_1$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$

$2c = 4$

$C = 2$

$B + 3C = 2$

$B = -2 - 6 = -8$

$A = 1 - B - C = 1 + 8 - 2 = 7$

∴ Equation of parabola is $y = 7 - 8x + 2x^2$

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2.) $$A = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 4 & 12 & 3 & -14 \\ -10 & -29 & -5 & 38 \\ 10 & 21 & 21 & -6 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$ $\quad l_{21} = 2$

$R_3 \to R_3 + 5R_1$ $\quad l_{31} = -5$

$R_4 \to R_4 - (+5)R_1$ $\quad l_{41} = 5$

$$A \sim \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -24 \\ 0 & -4 & 5 & 12 \\ 0 & -4 & 11 & -31 \end{bmatrix}$$

$R_3 \to R_3 - (-2)R_2$ $\quad l_{32} = -2$

$R_4 \to R_4 - (-2)R_2$ $\quad l_{42} = -2$

$$A \sim \begin{bmatrix} 2 & 5 & 2 & -8 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & +5 \\ 0 & 0 & 9 & 11 \end{bmatrix}$$ $R_4 \to R_4 - 3R_3$ $\quad l_{43} = 3$

$$A \sim \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & -4 \end{bmatrix} = 0$$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -5 & -2 & 1 & 0 \\ 5 & -2 & 3 & 1 \end{bmatrix}$$

$A = LU$

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3) $T(x,y,z) = [x+2y-z,\ y+z,\ x+y-2z]$

$T : R^3 \to R^3$

(i) standard basis for $R^3 = \{(1,0,0)\ (0,1,0)\ (0,0,1)\}$

$T = (1,0,0) = (1,0,1)$

$T = (0,1,0) = (2,1,1)$

$T = (0,0,1) = (-1,1,-2)$

$$T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

(ii) $$T = \left[\begin{array}{ccc|c} 1 & 2 & -1 & b_1 \\ 0 & 1 & 1 & b_2 \\ 1 & 1 & -2 & b_3 \end{array}\right]$$

$R_3 \to R_3 - R_1$

$$\sim \begin{bmatrix} 1 & 2 & -1 & b_1 \\ 0 & 1 & 1 & b_2 \\ 0 & -1 & -1 & b_3 - b_1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & b_1 \\ 0 & 1 & 1 & b_2 \\ 0 & 0 & 0 & b_2 + b_3 - b_1 \end{bmatrix}$$

rank = 2

Basis of $C(T) = \{(1,0,1)(2,1,1)\}$

Converting T to RREF

$R_1 \to R_1 - 2R_2$

$$T \sim \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Pivot variables

free variable

Expressing free variables in terms of pivot variables.

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$$Z = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$$

Basis of $N(T) = \{(3, -1, 1)\}$

Basis of $C(T^T) = \{(1, 2, -1), (0, 1, 1)\}$

Left null space $\rho(A) = \rho(A\ b)$ if $b_2 + b_3 - b_1 = 0$

$A^T y = 0$ $\quad b_2 + b_3 - b_1 = 0$

Basis of $N(T^T) = \{(-1, 1, 1)\}$

(iii) to find Eigenvalues and Eigen vectors of T

$$|T - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 2 & -1 \\ 0 & 1-\lambda & 1 \\ 1 & 1 & -2-\lambda \end{vmatrix} = 0$$

$\Rightarrow (1-\lambda)((1-\lambda)(-2-\lambda)-1) - 2(0-1) - 1(0-1+\lambda) = 0$

$\Rightarrow (1-\lambda)(-2-\lambda+2\lambda+\lambda^2-1) + 2 + 1 - \lambda = 0$

$= (1-\lambda)(\lambda^2+\lambda-3) + 3 - \lambda = 0$

$\Rightarrow 3\lambda - \lambda^3 = 0 \Rightarrow \lambda^3 = 3$

$\lambda = \pm\sqrt{3}$

$\lambda = 0$ or $\lambda = \sqrt{3}$ or $\lambda = -\sqrt{3}$

when $\lambda = 0$

$T - \lambda T = T$

~~Eigen vs~~ Eigenvector is nullspace of $T = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$ for $\lambda = 0$

then $\lambda = \sqrt{3}$

$$T - \lambda I = \begin{bmatrix} 1-\sqrt{3} & 2 & -1 \\ 0 & 1-\sqrt{3} & 1 \\ 1 & 1 & -2-\sqrt{3} \end{bmatrix}$$

$$R_3 \to R_3 - \left(\frac{1}{1-\sqrt{3}}\right) R_1$$

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$$\sim \begin{bmatrix} 1-\sqrt{3} & 2 & -1 \\ 0 & 1-\sqrt{3} & 1 \\ 0 & 1-\frac{2}{1-\sqrt{3}} & -2-\sqrt{3}+\frac{1}{1-\sqrt{3}} \end{bmatrix}$$

$$R_3 \to R_3 - \left(\frac{1-2/1-\sqrt{3}}{1-\sqrt{3}}\right) R_2$$

$$\sim \begin{bmatrix} 1-\sqrt{3} & 2 & -1 \\ 0 & 1-\sqrt{3} & 1 \\ 0 & 0 & -2-\sqrt{3}+\frac{1}{1-\sqrt{3}} - \frac{1-\sqrt{3}-2}{(1-\sqrt{3})^2} \end{bmatrix}$$

$$\sim \begin{bmatrix} 1-\sqrt{3} & 2 & -1 \\ 0 & 1-\sqrt{3} & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_1 \to R_1 - \left(\frac{2}{1-\sqrt{3}}\right) R_2$$

$$\sim \begin{bmatrix} 1-\sqrt{3} & 0 & -1-\frac{2}{1-\sqrt{3}} \\ 0 & 1-\sqrt{3} & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & \frac{-1+\sqrt{3}-2}{(1-\sqrt{3})^2} \\ 0 & 1 & \frac{1}{1-\sqrt{3}} \\ 0 & 0 & 0 \end{bmatrix}$$

$$\frac{-1+\sqrt{3}-2}{(1-\sqrt{3})^2} = \frac{-3+\sqrt{3}}{1+3-2\sqrt{3}} = \frac{-3+\sqrt{3}}{4-2\sqrt{3}} \times \frac{4+2\sqrt{3}}{4+2\sqrt{3}}$$

$$\Rightarrow \frac{-12-6\sqrt{3}+4\sqrt{3}+6}{16-12} = \frac{-6-2\sqrt{3}}{4}$$

$$= \frac{-3-\sqrt{3}}{2}$$

$$\frac{1}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}} = \frac{1+\sqrt{3}}{1-3} = \frac{-1-\sqrt{3}}{2}$$

$$\sim \begin{bmatrix} 1 & 0 & \frac{-3-\sqrt{3}}{2} \\ 0 & 1 & -1-\sqrt{3}/2 \\ 0 & 0 & 0 \end{bmatrix}$$

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Eigen vector when $\lambda = \sqrt{3}$ is $\begin{bmatrix} \frac{3+\sqrt{3}}{2} \\ \frac{1+\sqrt{3}}{2} \\ 1 \end{bmatrix}$

when $\lambda = \sqrt{3}$, $A - \lambda I = \begin{bmatrix} 1+\sqrt{3} & 2 & -1 \\ 0 & 1+\sqrt{3} & 1 \\ 1 & 1 & -2+\sqrt{3} \end{bmatrix}$

$R_3 \to R_3 - \left(\frac{1}{1+\sqrt{3}}\right) R_1$

$\sim \begin{bmatrix} 1+\sqrt{3} & 2 & -1 \\ 0 & 1+\sqrt{3} & 1 \\ 0 & 1-\frac{2}{1+\sqrt{3}} & -2+\sqrt{3}+\frac{1}{1+\sqrt{3}} \end{bmatrix}$

$\sim \begin{bmatrix} 1+\sqrt{3} & 2 & -1 \\ 0 & 1+\sqrt{3} & 1 \\ 0 & 0 & -2+\sqrt{3}+\frac{1}{1+\sqrt{3}} - \frac{1-\frac{2}{1+\sqrt{3}}}{1+\sqrt{3}} \end{bmatrix}$

$R_1 \to R_1 - \left(\frac{2}{1+\sqrt{3}}\right) R_2$

$\sim \begin{bmatrix} 1+\sqrt{3} & 0 & -1-\frac{2}{1+\sqrt{3}} \\ 0 & 1+\sqrt{3} & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 0 & \frac{-1}{1+\sqrt{3}} - \frac{2}{(1+\sqrt{3})^2} \\ 0 & 1 & 1/1+\sqrt{3} \\ 0 & 0 & 0 \end{bmatrix}$

$\frac{-1}{1+\sqrt{3}} - \frac{2}{(1+\sqrt{3})^2} = \frac{-1-\sqrt{3}-2}{(1+\sqrt{3})^2} = \frac{3-\sqrt{3}}{1+3+2\sqrt{3}} = \frac{3-\sqrt{3}}{4+2\sqrt{3}} \times \frac{4-2\sqrt{3}}{4-4\sqrt{3}}$

$= \frac{-6+2\sqrt{3}}{4}$

$= \frac{-3+\sqrt{3}}{2}$

$\frac{1}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}} = \frac{1-\sqrt{3}}{1-3} = \frac{1-\sqrt{3}}{-2}$

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$$\sim \begin{bmatrix} 1 & 0 & \frac{-3+\sqrt{3}}{2} \\ 0 & 1 & \frac{1-\sqrt{3}}{-2} \\ 0 & 0 & 0 \end{bmatrix}$$

Eigenvector (null space) for $\lambda = \sqrt{3}$ is

$$\underline{\left(\frac{3-\sqrt{3}}{2}, \frac{1-\sqrt{3}}{2}, 1\right)}$$

(iv) T = QR decomposition

$$q_1 = \frac{a}{\|a\|} = \left[\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right]$$

$$q_2 = \frac{B}{\|B\|} = \left[\frac{1}{2} \times \sqrt{\frac{2}{3}}, \frac{\sqrt{2}}{\sqrt{3}}, \frac{1}{2} \times \frac{\sqrt{2}}{\sqrt{3}}\right]$$

$$B = b - (q_1^T b)\, q_1$$

$$= \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1.5 \\ 0 \\ 1.5 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 \\ 1 \\ -1/2 \end{bmatrix}$$

$$= \left[\frac{1}{\sqrt{6}}, \sqrt{\frac{2}{3}}, \frac{-1}{\sqrt{6}}\right]$$

$$\|B\| = \sqrt{3/2} = \sqrt{3}/\sqrt{2}$$

$$q_3 = \frac{c}{\|c\|} \quad C = c - (q^T c) q_1 - (q_2^T c)\, q_2$$

$$= \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} - \begin{bmatrix} -1/2 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} -1/2 \\ 1 \\ -1 \end{bmatrix}$$

$$q_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$q_3 = (0, 0, 0)$$

$$R = \begin{bmatrix} q_1^T a & q_1^T b & q_1^T c \\ 0 & q_2^T b & q_2^T c \\ 0 & 0 & q_3^T c \end{bmatrix}$$

$$q^T a = \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 1/\sqrt{2} + \frac{1}{\sqrt{2}} = \sqrt{2}$$

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$$q_2^T b = \begin{bmatrix} 1/\sqrt{6} & \sqrt{2/3} & -1/\sqrt{6} \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \frac{2}{\sqrt{6}} + \sqrt{\frac{2}{3}} \frac{-1}{\sqrt{6}}$$

$$= 3/\sqrt{6}$$

$$q_3^T c = 0$$

$$q_1^T b = \frac{3}{\sqrt{2}}, \quad q_1^T c = -3/\sqrt{2}$$

$$q_2^T c = 3/\sqrt{6}$$

$$R = \begin{bmatrix} \sqrt{2} & 3/\sqrt{2} & -3/\sqrt{2} \\ 0 & 3/\sqrt{6} & 3/\sqrt{6} \\ 0 & 0 & 0 \end{bmatrix}$$

$$T = QR$$

$$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 0 \\ 0 & \sqrt{2/3} & 0 \\ 1/\sqrt{2} & -1/\sqrt{6} & 0 \end{bmatrix} \begin{bmatrix} \sqrt{2} & 3/\sqrt{2} & -3/\sqrt{2} \\ 0 & 3/\sqrt{6} & 3/\sqrt{6} \\ 0 & 0 & 0 \end{bmatrix}$$

4.)

x	-4	1	2	3
y	4	6	10	8

$$y = c + dx$$
$$4 = c + d(-4)$$
$$6 = c + d(1)$$
$$10 = c + d(2)$$
$$8 = c + d(3)$$

$$\begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

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to find $\vec{x}$, $A^T A \vec{x} = A^T b$

$$\vec{x} = (A^T A)^{-1} A^T b$$

$$A^T A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -4 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 20 & 30 \end{bmatrix}$$

$$(A^T A)^{-1} = \frac{1}{116} \begin{bmatrix} 30 & -2 \\ -20 & 4 \end{bmatrix}$$

$$(A^T A)^{-1} A^T = \frac{1}{116} \begin{bmatrix} 30 & -2 \\ -20 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ -4 & 1 & 2 & 3 \end{bmatrix}$$

$$= \frac{1}{116} \begin{bmatrix} 38 & 28 & 26 & -24 \\ -18 & 2 & 6 & 10 \end{bmatrix}$$

$$(A^T A)^{-1} A^T b = \frac{1}{116} \begin{bmatrix} 38 & 28 & 26 & -24 \\ -18 & 2 & 6 & 10 \end{bmatrix} \begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

$$= \frac{1}{116} \begin{bmatrix} 772 \\ 80 \end{bmatrix}$$

$$= \begin{bmatrix} 193/29 \\ 20/29 \end{bmatrix} = \begin{bmatrix} c \\ d \end{bmatrix}$$

∴ Equation of the best straight line is,

$$y = \frac{193}{29} + \frac{20}{29} x$$

5.) $Q = A (A^T A)^{-1} A^T$

$P = I - Q$

Equation of plane $x_1 + x_2 + 3x_3 + 4x_4 = 0$

$$A = \begin{bmatrix} 1 \\ 1 \\ 3 \\ 4 \end{bmatrix}$$

$A^T A = [1\ 1\ 3\ 4]\begin{bmatrix}1\\1\\3\\4\end{bmatrix} = 1+1+9+16 = 27$

$(A^T A)^{-1} = \frac{1}{27}$

$Q = \frac{1}{27}\begin{bmatrix}1\\1\\3\\4\end{bmatrix}[1\ 1\ 3\ 4] \qquad 1+1+9+16=27$

$(A^T A)^{-1} = 1/27$

$$Q = \frac{1}{27}\begin{bmatrix}1\\1\\3\\4\end{bmatrix}[1\ 1\ 3\ 4] = \frac{1}{27}\begin{bmatrix}1 & 1 & 3 & 4\\ 1 & 1 & 3 & 4\\ 3 & 3 & 9 & 12\\ 4 & 4 & 12 & 16\end{bmatrix}$$

$$P = I - Q = \frac{1}{27}\begin{bmatrix}27 & 0 & 0 & 0\\ 0 & 27 & 0 & 0\\ 0 & 0 & 27 & 0\\ 0 & 0 & 0 & 27\end{bmatrix} - \frac{1}{27}\begin{bmatrix} \end{bmatrix}$$

$$P = \frac{1}{27}\begin{bmatrix}26 & -1 & -3 & -4\\ -1 & 26 & -3 & -4\\ -3 & -3 & 18 & -12\\ -4 & -4 & -12 & 11\end{bmatrix}$$

b) $A = \begin{bmatrix}a & 2 & 2\\ 2 & a & 2\\ 2 & 2 & a\end{bmatrix}$

$R_2 \to R_2 - \left(\frac{2}{a}\right)R_1, \quad R_3 \to R_3 - \left(\frac{2}{a}\right)R_1$

$$\sim \begin{bmatrix}a & 2 & 2\\ 0 & \frac{a^2-4}{a} & \frac{2a-4}{a}\\ 0 & \frac{2a-4}{a} & \frac{a^2-4}{a}\end{bmatrix}$$

$R_3 \to R_3 - \left(\frac{2a-4}{a^2-4}\right)R_2$

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$$\sim \begin{bmatrix} a & 2 & 2 \\ 0 & \frac{a^2-4}{a} & \frac{2a-4}{a} \\ 0 & 0 & \frac{a^2-4}{a} - \left(\frac{2a-4}{a^2-4}\right)\left(\frac{2a-4}{a}\right) \end{bmatrix}$$

$\therefore a > 0$, $\frac{a^2-4}{a} > 0$

$\Rightarrow a > 2$

$$\frac{a^2-4}{a} - \left(\frac{(2a-4)^2}{a(a^2-4)}\right) > 0$$

$(a^2-4) > (2a-4)^2$

$a^4 + \cancel{16} - 8a^2 > 4a^2 + \cancel{16} - 16a$

$a^3 - 12a + 16 > 0$

$\Rightarrow a > 2$ and $a > -4$ (since $a > 0$)

Taking the intersection of all ranges

$a > 2$

or $a \in (2, \infty)$ is interval in which A is positive definite

$f = x^T A x = 2x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_2x_3$ — ①

$$= \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{12} & a_{22} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= [a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \quad a_{12}x_1 + a_{22}x_2 + a_{23}x_3 \quad a_{13}x_1 + a_{23}x_2 + a_{33}x_3] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$\Rightarrow a_{11}x_1^2 + a_{12}x_1x_2 + a_{13}x_1x_3 + a_{12}x_1x_2 + a_{22}x_2^2 + a_{23}x_2x_3 + a_{13}x_1x_3 + a_{23}x_2x_3 + a_{33}x_3^2$

$= a_{11}x_1^2 + 2a_{12}x_1x_2 + a_{22}x_2^2 + 2a_{23}x_2x_3 + a_{33}x_3^2 + 2a_{13}x_1x_3$

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Comparing the Coefficients of ① and ②

$a_{11} = 2 \quad a_{12} = -1$
$a_{22} = 2 \quad a_{23} = -1$
$a_{33} = 2 \quad a_{13} = 0$

$\therefore$ The required matrix is:

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

7) $A = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix}$

$$A^T A = \begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix}$$

To find Eigen values $|ATA - \lambda I| = 0$

$$\begin{vmatrix} 81-\lambda & -27 \\ -27 & 9-\lambda \end{vmatrix} = 0$$

$$\lambda^2 - 90\lambda = 0$$

$$= \lambda = 0 \text{ or } \lambda = 90$$

Eigen vectors, for $\lambda = 0$

$$\begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 81 & -27 \\ 0 & 0 \end{bmatrix}$$

Eigen vector is $k_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

for $\lambda = 90$

$$A - 90I = \begin{bmatrix} -9 & -27 \\ -27 & -81 \end{bmatrix}$$

Eigen vector is $k_2 = [\cancel{8}\ 1\ 3]$

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$v_1 = \frac{x_1}{\|x_1\|} = \begin{vmatrix} -3/\sqrt{10} \\ 1/\sqrt{10} \end{vmatrix}$ $\quad v_2 = \frac{x_2}{\|x_2\|} = \begin{bmatrix} 1/\sqrt{10} \\ 3/\sqrt{10} \end{bmatrix}$

Since x_1 and x_2 are orthogonal

$$V = [v_1 \cdot v_2] = \begin{bmatrix} -3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{bmatrix}$$

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$$\Sigma = \begin{bmatrix} \sqrt{\lambda_1} & 0 \\ 0 & \sqrt{\lambda_2} \\ 0 & 0 \end{bmatrix} \text{ where } \sqrt{\lambda_1} > \sqrt{\lambda_2}$$

$$= \begin{bmatrix} \sqrt{90} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$ where $\sigma_1 = \sqrt{\lambda_1}$, $\sigma_2 = \sqrt{\lambda_2}$
σ_1 and σ_2 are singular values of A

$\sigma_1 = \sqrt{90}$, $\sigma_2 = 0$

$u_1 = \frac{A v_1}{\sigma_1}$ if and only if $\sigma_1 \neq 0$

eigen values of AA^T are 90, 0 and 0

for $\sigma_1 = \sqrt{90}$, $u_1 = \frac{A v_1}{\sigma_1} = \frac{1}{90} \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix} \begin{bmatrix} -3/\sqrt{10} \\ 1/\sqrt{10} \end{bmatrix}$

$$u_1 = \begin{bmatrix} 1/3 \\ -2/3 \\ -2/3 \end{bmatrix}$$

for $\sigma_2 = 0$, formula (1) cannot be used
wRT u_2 and u_3 are orthogonal vectors associated with eigen values of AA^T
find the eigen ~~values~~ vectors of AA^T w.r.t $\lambda = 0$

$(AA^T - 0I)x = 0$

$$\begin{bmatrix} 10 & -20 & -20 \\ -20 & 40 & 40 \\ -20 & 40 & 40 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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To find null space

$$\begin{bmatrix} 10 & -20 & -20 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$y = \begin{bmatrix} -20 \\ 1 \\ 0 \end{bmatrix} \qquad z = \begin{bmatrix} -20 \\ 0 \\ 1 \end{bmatrix}$$

$$\text{or } \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \qquad \text{or } \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore$ $x_2 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ and $x_3 = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ are eigen vectors.

of AA^T when $\lambda = 0$

x_2 and x_3 are orthogonal to u_1

To find u_2 and u_3, apply Gram schmidt process on x_2 and x_3

To find u_2, $u_2 = \frac{x}{\|x_2\|} = \begin{bmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \\ 0 \end{bmatrix}$

To find u_3

find vector $\perp^r$ to both u_1 and u_2

$$C = x_3 - (u_1 - x_3)u_1 - (u_2^T x_3)u_2$$

$$= \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} - (0)\begin{pmatrix} 1/3 \\ -2/3 \\ -4/3 \end{pmatrix} - \left[\left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0\right)\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}\right] \begin{bmatrix} 2/5 \\ 1/\sqrt{5} \\ 0 \end{bmatrix}$$

$$= \begin{pmatrix} 2/5 \\ -4/5 \\ 1 \end{pmatrix}$$

$$u_3 = \frac{C}{\|C\|} = \begin{bmatrix} 2/3\sqrt{5} \\ -4/3\sqrt{5} \\ \sqrt{5}/3 \end{bmatrix}$$

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$$U = \begin{bmatrix} | & | & | \\ v_1 & v_2 & v_3 \\ | & | & | \end{bmatrix} = \begin{bmatrix} 1/3 & 2/\sqrt{5} & 2/3\sqrt{5} \\ -2/3 & 1/\sqrt{5} & -4/3\sqrt{5} \\ -2/3 & 0 & \sqrt{5}/3 \end{bmatrix}$$

$$A = U \Sigma V^T$$

$$= \begin{bmatrix} 1/3 & 2/\sqrt{5} & 2/3\sqrt{5} \\ -2/3 & 1/\sqrt{5} & -4/3\sqrt{5} \\ -2/3 & 0 & \sqrt{5}/3 \end{bmatrix} \begin{bmatrix} \sqrt{20} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{bmatrix}$$