2. Heat Diffusion Equation using FDTD

Heat diffusion equation is given by $\phi_t = \phi_{xx}$, which when been approximated using FDTD gives us –

$\phi(i, j+1) = r\{\phi(i+1, j) - 2\phi(i, j) + \phi(i-1, j)\}$, where $r = \dfrac{\Delta t}{(\Delta x)^2}$ →eq. 1

Taking value of r = $\frac{1}{2}$, we can reduced eq. 1 to → $\phi(i, j+1) = \frac{1}{2}\{\phi(i+1, j) + \phi(i-1, j)\}$ →eq. 2

Given boundary conditions →$\phi(0, t) = \phi(1, t) = 0$, for $t > 0$ →eq. 3

And initial condition →$\phi(x, 0) = 100$ →eq. 4

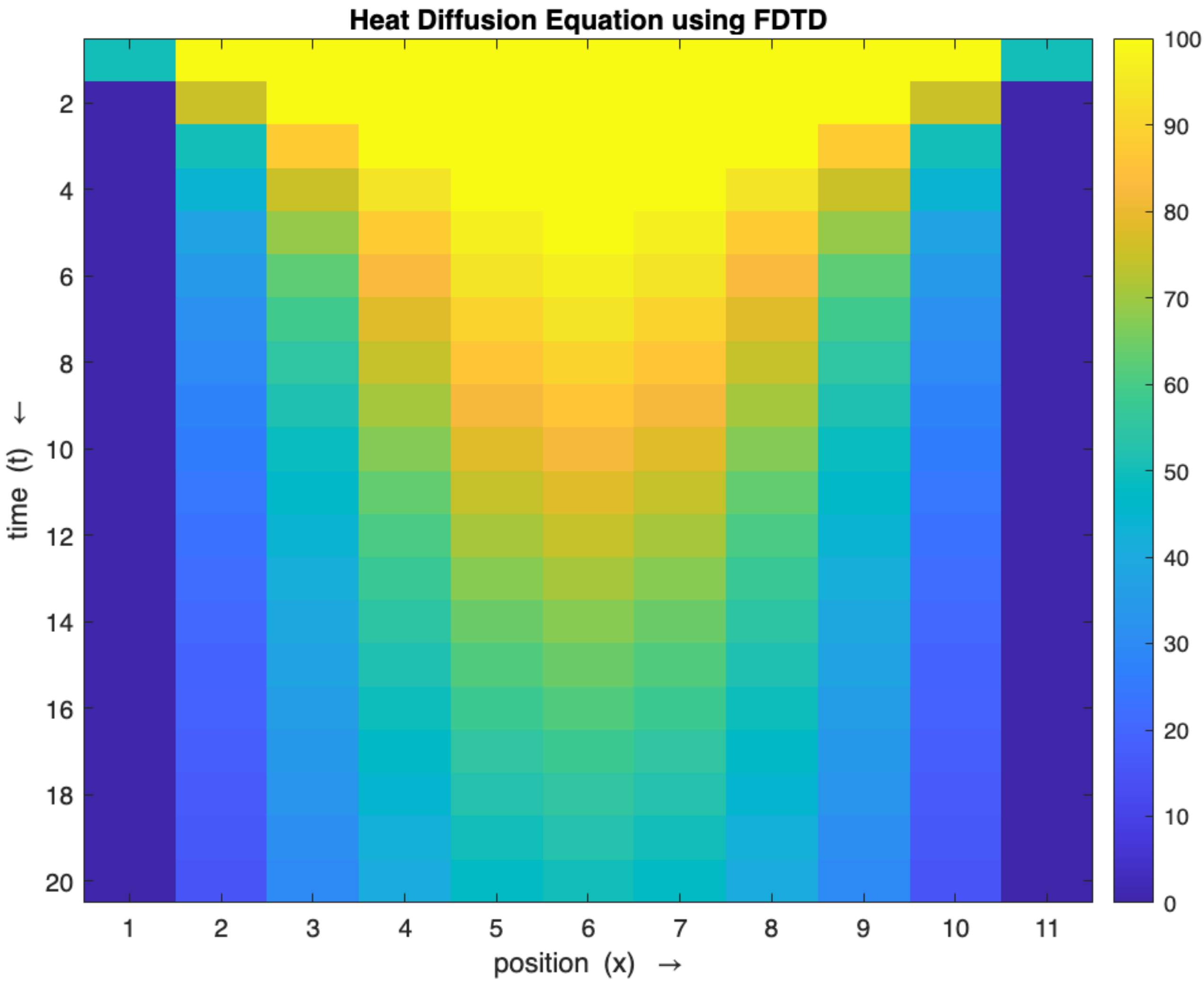
```
del_x = 0.1;
r = 0.5;
del_t = r * del_x^2;

out_init = zeros(round(del_x / del_t), 7); % initializing grid
out_init(:,  1) = 0; % boundary conditions, eq. 3
out_init(1, 2:end) = 100; % initial condition, eq. 4

% finite differencing, eq. 2
out_init(1, 1) = 50;
for i = 2:size(out_init, 1)
    for j = 2:size(out_init, 2)-1
        out_init(i, j) = (out_init(i - 1, j + 1) + out_init(i - 1, j - 1)) * 0.5;
    end
    out_init(i, end) = out_init(i, end-2);
end

out_final = [out_init, fliplr(out_init(:, 1:4))];

figure;
imagesc(out_final);
title("Heat Diffusion Equation using FDTD");
xlabel("position (x) \rightarrow");
ylabel("time (t) \leftarrow");
colorbar;
```



Key takeaways –

1. `imagesc` plots a graphical representation of a matrix, representing color intensities for matrix values.
2. `fliplr` flips a matrix left to right.
3. We can stack columns to a matrix just by the notation [existing matrix, new column or group of columns].
4. Code for Experiment no. 2 can be downloaded from https://github.com/bhargawananbhuyan/computational-electromagnetics/raw/main/exp2.mlx