Transverse circular polarization in tightly focused beam

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1 Goal

Trying to accurately understand the origin of the transverse circular polarization on the side of the focus of a tighly focused beam.

2 Qualitative description

For a collimated light beam, the polarization vector is generally within the plane perpendicular to the wave propagation direction (since light is a transverse wave). However, for a focused beam, the "wave propagation direction" isn't very well defined anymore, which allows polarization parallel to the optical axis to occur. This "axial" polarization component could even lead to circular polarization that rotates in a plane parallel to the optical axis, especially near the focus.

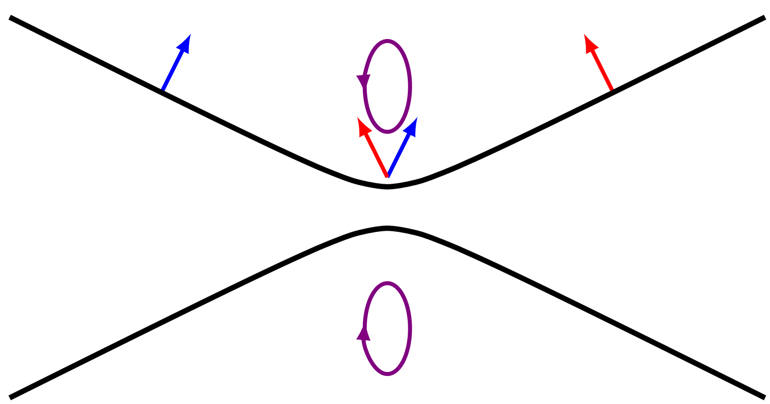

Figure 1: Transverse circular polarization near the focus of a tighly focused beam. The red and blue error shows the polarization vector on the same edge of a tightly focused beam before and after the focus.

We can qualitatively see this happening by looking at the field on the edge of the beam. The two edges have significantly different k vectors and therefore different polarization vectors as well. As shown in Fig. 1, the polarization on the two edges of the beam acquires an axial component due to the large angle between the k vector and the optical axis. While the two sides of the beam are generally far away from each other and their different polarization directions cause little problem, this is not the case anymore near the focus as the edge of the beam changes direction from converging to diverging and the polarization in that area (next to the focus in the focal plane) would have a polarization somewhere in between. This of course doesn't guarantee that there are any circular polarization or even axial polarization component, which would be the topic of the next section.

3 Semi-quantitative explanation

Full quantitative understanding of the transverse circular polarization near the focus of the beam requires a full calculation of the vector field. It is possible, however, to understand why such a polarization exists based on some continuity and symmetry considerations.

3.1 General idea

We can see from the far field (away from the focus) that the axial polarization does exist. We can therefore break down the proof of the existance of the transverse circular polarization in two steps.

1. Axial polarization also exist on the focal plane.
2. The axial polarization has to be out-of-phase with the transverse polarization.

These two would guarantee a transverse circular polarization.

3.2 Existance of axial polarization on the focal plane

$$\begin{aligned}\nabla \times \left(\nabla \times \vec{E}\right) =& -\nabla \times \frac{\partial \vec{B}}{\partial t} \\ =& -\frac{\partial \nabla \times \vec{B}}{\partial t} \\ =& -\frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}\end{aligned} \tag{1}$$

$$\begin{aligned}\nabla \times \left(\nabla \times \vec{E}\right) =& \nabla\left(\nabla \cdot \vec{E}\right) - \nabla^2 \vec{E} \\ =& -\nabla^2 \vec{E}\end{aligned} \tag{2}$$

$$\nabla^2 \vec{E} = \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} \tag{3}$$

Scalar wave equation for,

$$\nabla^2 u = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} \tag{4}$$

If the wave is monotonic, we can write it out as $u(\vec{r}, t) = u_0(\vec{r})\mathrm{e}^{\mathrm{i}\omega t}$, in which case the wave equation becomes,

$$\nabla^2 u_0 = k^2 u_0 \tag{5}$$

where $k \equiv \dfrac{\omega}{c}$. We can define the flux of the wave $\vec{j} \equiv u_0^* \nabla u_0 - u_0 \nabla u_0^*$, we have,

$$\begin{aligned}\nabla \cdot \vec{j} =& \nabla \cdot \left(u_0^* \nabla u_0 - u_0 \nabla u_0^*\right) \\ =& \nabla u_0^* \cdot \nabla u_0 - \nabla u_0 \cdot \nabla u_0^* + u_0^* \nabla^2 u_0 - u_0 \nabla^2 u_0^* \\ =& u_0^* k^2 u_0 - u_0 k^2 u_0^* \\ =& 0\end{aligned} \tag{6}$$

i.e. $\vec{j}$ is a continuous flow that cannot terminate in free space.