Measuring the scrambling of quantum information

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- Scrambling of quantum information
- Out-of-time-order (OTO) correlator
- Measurement of OTO correlator
- Experimental realization with cavity QED system

Relaxation vs Scrambling

Relaxation

- Decay/leaking of information from a single qubit.
- Fast
  Time scale: τ

Scrambling

- Spreading of information to the whole system.
- Slow
  Time scale: $t_* = \tau \ln S$

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Out-of-time-order (OTO) correlator

$$F(t) \equiv \langle W_t^\dagger V^\dagger W_t V \rangle$$

$$W_t = U(-t) W U(t)$$
$$U(t) = \mathrm{e}^{-\mathrm{i}Ht}$$

- Interpretation: $F = \langle \psi_1 | \psi_2 \rangle$

  $|\psi_1\rangle = V W_t |\psi_0\rangle$
  $|\psi_2\rangle = W_t V |\psi_0\rangle$
- Choice of V and W?
- Scaling of F with system size.
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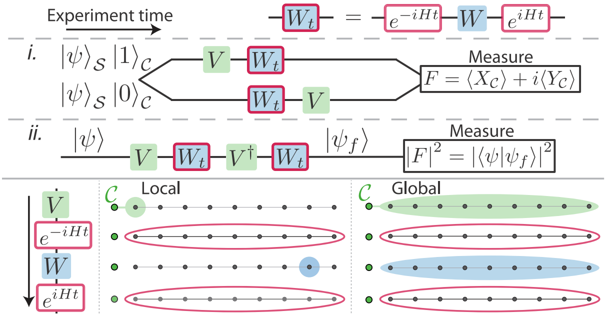

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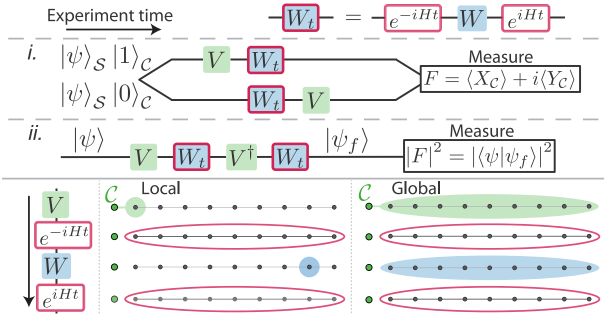

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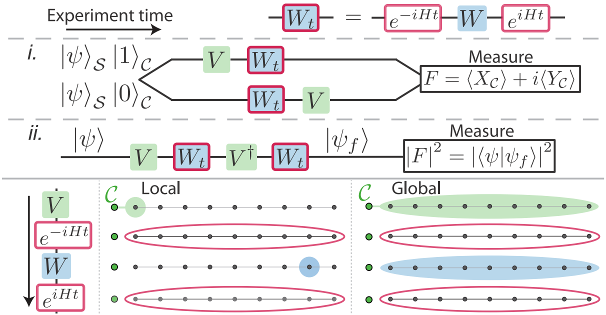
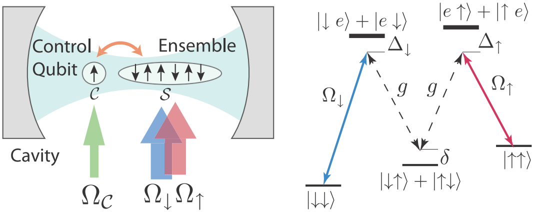


Time reversal: $U(-t)$

Controlled-V: $I_S \otimes |0\rangle\langle 0|_C + V_S \otimes |1\rangle\langle 1|_C$

Cavity QED implementation



$$H = \sum_{ij} J_{ij} s_i^x s_j^x + h.c.$$

$$J_{ij} = \sum_\alpha \frac{\Omega_\uparrow^*(r_i)\Omega_\downarrow(r_j)}{\Delta_\uparrow \Delta_\downarrow} \frac{g_\alpha(r_i) g_\alpha^*(r_j)}{\delta}$$

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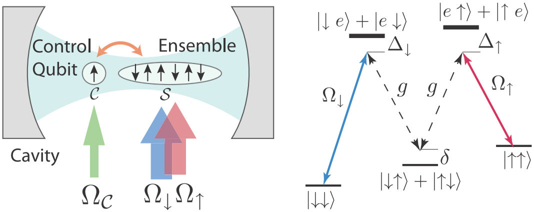
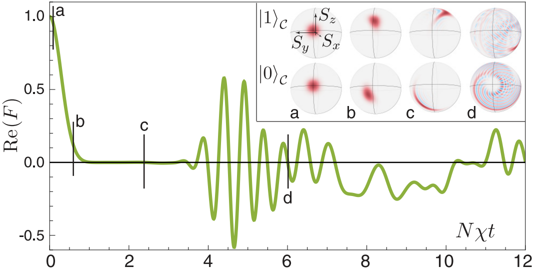


$$Z_\phi^C = I_S \otimes |0\rangle\langle 0|_C + \mathrm{e}^{-\mathrm{i}\phi S_z^S} \otimes |1\rangle\langle 1|_C$$

$$V = W = \mathrm{e}^{-\mathrm{i}\phi S_z^S}$$

S_x^2 dynamics

$$H = \sum_{ij} J_{ij} s_i^x s_j^x + h.c. \;\rightarrow\; H = JS_x^2$$



$S_x^2 + S_z$ dynamics

$$F(t) \equiv \langle W_t^\dagger V^\dagger W_t V \rangle \quad G(t) \equiv \langle V_t^\dagger V \rangle$$