A One-Way Quantum Computer

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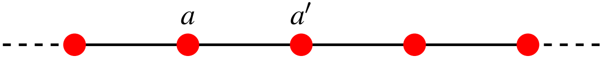
Ni Group

Feb. 12, 2021

- 1D Cluster state
  - Generation
  - Properties
- High dimensional cluster state
- Quantum circuit
- Gates and single qubit operations

1D Cluster State

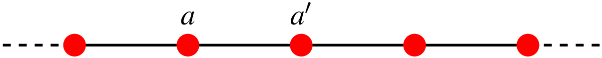


$$H = \sum_{a,a' \in \Gamma} \frac{1 + \sigma_z^{(a)}}{2} \frac{1 - \sigma_z^{(a')}}{2}$$

$$\Gamma = \left\{ (a, a') | a' = a + 1 \right\}$$

$$\mathcal{S} = \mathrm{e}^{\mathrm{i}\pi H}$$

1D Cluster State



$$|\phi_N\rangle = \mathcal{S} \bigotimes_a |+\rangle_a = \frac{1}{2^{N/2}} \bigotimes_a \left(|0\rangle_a \sigma_z^{a+1} + |1\rangle_a\right)$$

$$|\phi_2\rangle = \frac{1}{\sqrt{2}}(|0-\rangle + |1+\rangle)$$

$$|\phi_3\rangle = \frac{1}{\sqrt{2}}(|+0-\rangle - |-1+\rangle)$$

$$|\phi_4\rangle = \frac{1}{2}(|0-0-\rangle - |0+1+\rangle + |1+0-\rangle - |1-1+\rangle)$$

$$|\mathrm{GHZ}_N\rangle = \frac{1}{\sqrt{2}}\left(\bigotimes_a |0\rangle_a + \bigotimes_a |1\rangle_a\right)$$

1D Cluster State

- Maximum connectedness
  Ability to create Bell state by local measurements.
  Yes for both GHZ state and cluster state.
- Persistency
  Minimum local measurements to destroy all entanglements.
  GHZ: $P_e = 1$, cluster: $P_e = \lfloor N/2 \rfloor$

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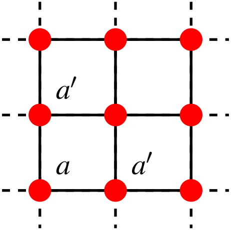
High Dimensional Cluster State



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$$\Gamma = \left\{ (a, a') | a' = a + \hat{e}_i \right\}$$

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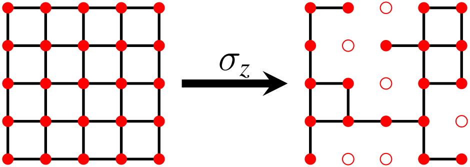

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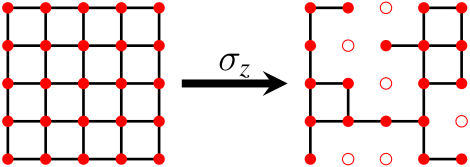

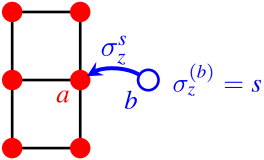

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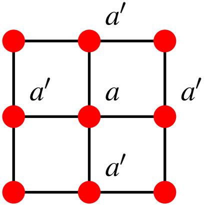

$$K_a = \sigma_x^{(a)} \bigotimes_{a' \in \Gamma'} \sigma_z^{(a')}$$

$$\Gamma' = \left\{ (a, a') | a' = a \pm \hat{e}_i \right\}$$

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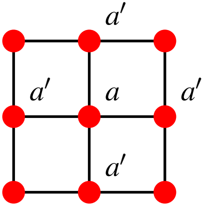

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- $\{K_a, \sigma_z^{(a)}\} = 0$
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- Independent
- Complete
- Equivalent definition of cluster state

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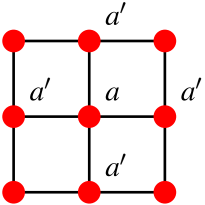

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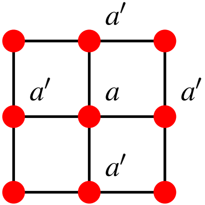

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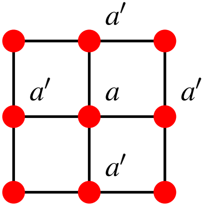

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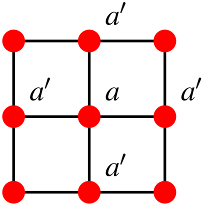

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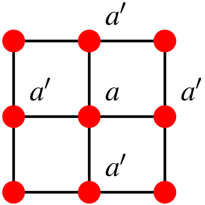

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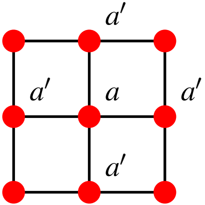

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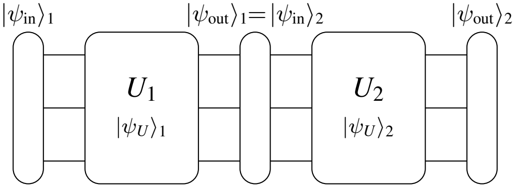
Quantum Circuit on Cluster State



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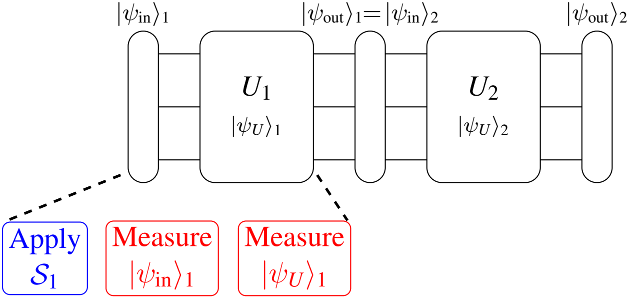

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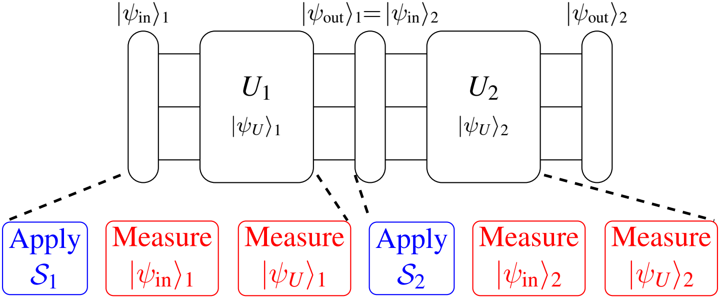

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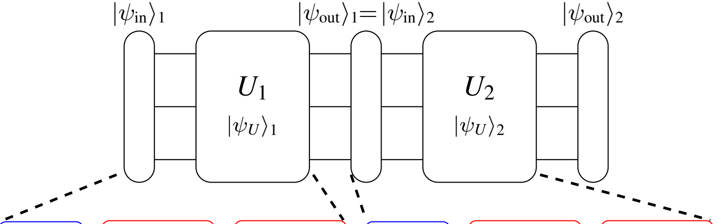

Apply $\mathcal{S}_1$	Measure $|\psi_{\text{in}}\rangle_1$	Measure $|\psi_U\rangle_1$	Apply $\mathcal{S}_2$	Measure $|\psi_{\text{in}}\rangle_2$	Measure $|\psi_U\rangle_2$

Apply $\mathcal{S}_1$	Apply $\mathcal{S}_2$	Measure $|\psi_{\text{in}}\rangle_1$	Measure $|\psi_U\rangle_1$	Measure $|\psi_{\text{in}}\rangle_2$	Measure $|\psi_U\rangle_2$

Gates and single qubit operations

Questions?