

Response of dressing rotation to amplitude noise

August 27, 2024

Dressing Hamiltonian

$$H = \Omega \sigma_x + \delta \sigma_z$$

Large Ω for the target ion and small Ω for the crosstalk ion.

The rotation should robustly turn the $|0\rangle$ and $|1\rangle$ states into the eigen state of the dressing Hamiltonian for both small and large Ω .

The two eigen states are

$$|\psi_{\pm}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \pm \sqrt{1 \pm 1/\sqrt{1 + (\Omega/\delta)^2}} \\ \sqrt{1 \mp 1/\sqrt{1 + (\Omega/\delta)^2}} \end{pmatrix}$$

The transformation needed is

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{1 + 1/\sqrt{1 + (\Omega/\delta)^2}} & -\sqrt{1 - 1/\sqrt{1 + (\Omega/\delta)^2}} \\ \sqrt{1 - 1/\sqrt{1 + (\Omega/\delta)^2}} & \sqrt{1 + 1/\sqrt{1 + (\Omega/\delta)^2}} \end{pmatrix}$$

For small Ω ,

$$\begin{aligned} U &\approx \begin{pmatrix} 1 & -\Omega/\delta/2 \\ \Omega/\delta/2 & 1 \end{pmatrix} \\ &= 1 - \frac{i\Omega}{2\delta} \sigma_y \end{aligned}$$

For large Ω , if $\Omega = \Omega_0(1 + \epsilon)$

$$\begin{aligned} &\sqrt{1 \pm 1/\sqrt{1 + (\Omega/\delta)^2}} \\ &\approx \sqrt{1 \pm 1/\sqrt{1 + (\Omega_0/\delta)^2}} - \frac{(\Omega_0/\delta)^2}{2(1 + (\Omega_0/\delta)^2)^{3/2} \sqrt{1 \pm 1/\sqrt{1 + (\Omega_0/\delta)^2}}} \epsilon \\ &\approx \sqrt{1 \pm 1/\sqrt{1 + (\Omega_0/\delta)^2}} - \frac{(\Omega_0/\delta) \sqrt{1 \mp 1/\sqrt{1 + (\Omega_0/\delta)^2}}}{2(1 + (\Omega_0/\delta)^2)} \epsilon \end{aligned}$$

$$\begin{aligned}
U &= \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{1+1/\sqrt{1+(\Omega/\delta)^2}} & -\sqrt{1-1/\sqrt{1+(\Omega/\delta)^2}} \\ \sqrt{1-1/\sqrt{1+(\Omega/\delta)^2}} & \sqrt{1+1/\sqrt{1+(\Omega/\delta)^2}} \end{pmatrix} \\
&\approx \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{1+1/\sqrt{1+(\Omega_0/\delta)^2}} & -\sqrt{1-1/\sqrt{1+(\Omega_0/\delta)^2}} \\ \sqrt{1-1/\sqrt{1+(\Omega_0/\delta)^2}} & \sqrt{1+1/\sqrt{1+(\Omega_0/\delta)^2}} \end{pmatrix} \\
&\quad + \frac{(\Omega_0/\delta)\varepsilon}{2\sqrt{2}(1+(\Omega_0/\delta)^2)} \begin{pmatrix} \sqrt{1-1/\sqrt{1+(\Omega_0/\delta)^2}} & -\sqrt{1+1/\sqrt{1+(\Omega_0/\delta)^2}} \\ \sqrt{1+1/\sqrt{1+(\Omega_0/\delta)^2}} & \sqrt{1-1/\sqrt{1+(\Omega_0/\delta)^2}} \end{pmatrix} \\
&= \sqrt{\frac{1+1/\sqrt{1+(\Omega_0/\delta)^2}}{2}} - \sqrt{\frac{1-1/\sqrt{1+(\Omega_0/\delta)^2}}{2}} i\sigma_y \\
&\quad + \frac{(\Omega_0/\delta)\sqrt{1-1/\sqrt{1+(\Omega_0/\delta)^2}}}{2\sqrt{2}(1+(\Omega_0/\delta)^2)} \varepsilon + \frac{(\Omega_0/\delta)\sqrt{1+1/\sqrt{1+(\Omega_0/\delta)^2}}}{2\sqrt{2}(1+(\Omega_0/\delta)^2)} i\sigma_y \varepsilon
\end{aligned}$$

The pulses should consists of rotations with the same Rabi frequency but may have variable detuning, phase and time. Note that only the xy component of the unitary matters. The z component and the global phase can be perfectly cancelled out by using the time reversal as the undress pulse sequence.

1 Single rotation

$$\begin{aligned}
&\exp(i(X(\cos\varphi\sigma_x + \sin\varphi\sigma_y) + Z\sigma_z)) \\
&= \cos\left(\sqrt{X^2 + Z^2}\right) + i\sin\left(\sqrt{X^2 + Z^2}\right) \frac{X\cos\varphi}{\sqrt{X^2 + Z^2}}\sigma_x \\
&\quad + i\sin\left(\sqrt{X^2 + Z^2}\right) \frac{X\sin\varphi}{\sqrt{X^2 + Z^2}}\sigma_y + i\sin\left(\sqrt{X^2 + Z^2}\right) \frac{Z}{\sqrt{X^2 + Z^2}}\sigma_z
\end{aligned}$$

xy component

$$\begin{aligned}
a_1 &= \sqrt{\frac{X^2 \cos^2(\sqrt{X^2 + Z^2}) + Z^2}{X^2 + Z^2}} \\
x_1 &= -i \frac{\sin(\sqrt{X^2 + Z^2})X}{\sqrt{X^2 \cos^2(\sqrt{X^2 + Z^2}) + Z^2}} \left(\cos(\sqrt{X^2 + Z^2}) \cos\varphi + \frac{Z \sin(\sqrt{X^2 + Z^2}) \sin\varphi}{\sqrt{X^2 + Z^2}} \right) \\
y_1 &= i \frac{\sin(\sqrt{X^2 + Z^2})X}{\sqrt{X^2 \cos^2(\sqrt{X^2 + Z^2}) + Z^2}} \left(\cos(\sqrt{X^2 + Z^2}) \sin\varphi + \frac{Z \sin(\sqrt{X^2 + Z^2}) \cos\varphi}{\sqrt{X^2 + Z^2}} \right)
\end{aligned}$$

$$\begin{aligned}
& \cos\left(\sqrt{X^2 + Z^2}\right) \cos \varphi + \frac{Z \sin\left(\sqrt{X^2 + Z^2}\right) \sin \varphi}{\sqrt{X^2 + Z^2}} = 0 \\
& -\sin\left(\sqrt{X^2 + Z^2}\right) \cos \varphi + \frac{Z \cos\left(\sqrt{X^2 + Z^2}\right) \sin \varphi}{\sqrt{X^2 + Z^2}} - \frac{Z \sin\left(\sqrt{X^2 + Z^2}\right) \sin \varphi}{X^2 + Z^2} = 0 \\
& \cos\left(\sqrt{X^2 + Z^2}\right) \cos \varphi = -\frac{Z \sin\left(\sqrt{X^2 + Z^2}\right)}{\sqrt{X^2 + Z^2}} \sin \varphi \\
& \sin\left(\sqrt{X^2 + Z^2}\right) \cos \varphi = \frac{Z \cos\left(\sqrt{X^2 + Z^2}\right) \sin \varphi}{\sqrt{X^2 + Z^2}} - \frac{Z \sin\left(\sqrt{X^2 + Z^2}\right) \sin \varphi}{X^2 + Z^2} \\
& -\frac{Z \sin^2\left(\sqrt{X^2 + Z^2}\right)}{\sqrt{X^2 + Z^2}} = \frac{Z \cos^2\left(\sqrt{X^2 + Z^2}\right)}{\sqrt{X^2 + Z^2}} - \frac{Z \sin\left(\sqrt{X^2 + Z^2}\right) \cos\left(\sqrt{X^2 + Z^2}\right)}{X^2 + Z^2} \\
& \sin\left(\sqrt{X^2 + Z^2}\right) \cos\left(\sqrt{X^2 + Z^2}\right) = \sqrt{X^2 + Z^2}
\end{aligned}$$

Can't satisfy.

1.1 Small power limit

$$\begin{aligned}
& \exp(i(X(\cos \varphi \sigma_x + \sin \varphi \sigma_y) + Z \sigma_z)) \\
& \approx \cos Z + i \sin Z \sigma_z + iX \frac{\sin Z}{Z} (\cos \varphi \sigma_x + \sin \varphi \sigma_y)
\end{aligned}$$

2 Two rotations

$$\begin{aligned}
& \exp(i(X_1(\cos \varphi_1 \sigma_x + \sin \varphi_1 \sigma_y) + Z_1 \sigma_z)) \exp(i(X_2(\cos \varphi_2 \sigma_x + \sin \varphi_2 \sigma_y) + Z_2 \sigma_z)) \\
&= \left(\cos \left(\sqrt{X_1^2 + Z_1^2} \right) + i \sin \left(\sqrt{X_1^2 + Z_1^2} \right) \frac{X_1 \cos \varphi_1}{\sqrt{X_1^2 + Z_1^2}} \sigma_x \right. \\
&\quad \left. + i \sin \left(\sqrt{X_1^2 + Z_1^2} \right) \frac{X_1 \sin \varphi_1}{\sqrt{X_1^2 + Z_1^2}} \sigma_y + i \sin \left(\sqrt{X_1^2 + Z_1^2} \right) \frac{Z_1}{\sqrt{X_1^2 + Z_1^2}} \sigma_z \right) \\
&\quad \left(\cos \left(\sqrt{X_2^2 + Z_2^2} \right) + i \sin \left(\sqrt{X_2^2 + Z_2^2} \right) \frac{X_2 \cos \varphi_2}{\sqrt{X_2^2 + Z_2^2}} \sigma_x \right. \\
&\quad \left. + i \sin \left(\sqrt{X_2^2 + Z_2^2} \right) \frac{X_2 \sin \varphi_2}{\sqrt{X_2^2 + Z_2^2}} \sigma_y + i \sin \left(\sqrt{X_2^2 + Z_2^2} \right) \frac{Z_2}{\sqrt{X_2^2 + Z_2^2}} \sigma_z \right) \\
&= \cos \left(\sqrt{X_1^2 + Z_1^2} \right) \cos \left(\sqrt{X_2^2 + Z_2^2} \right) - \frac{\sin \left(\sqrt{X_1^2 + Z_1^2} \right) \sin \left(\sqrt{X_2^2 + Z_2^2} \right) Z_1 Z_2}{\sqrt{X_1^2 + Z_1^2} \sqrt{X_2^2 + Z_2^2}} \\
&\quad - \sin \left(\sqrt{X_1^2 + Z_1^2} \right) \sin \left(\sqrt{X_2^2 + Z_2^2} \right) \frac{X_1 X_2 \cos(\varphi_1 - \varphi_2)}{\sqrt{X_1^2 + Z_1^2} \sqrt{X_2^2 + Z_2^2}} \\
&\quad + i \sin \left(\sqrt{X_1^2 + Z_1^2} \right) \frac{X_1 \cos \varphi_1}{\sqrt{X_1^2 + Z_1^2}} \cos \left(\sqrt{X_2^2 + Z_2^2} \right) \sigma_x + i \cos \left(\sqrt{X_1^2 + Z_1^2} \right) \sin \left(\sqrt{X_2^2 + Z_2^2} \right) \frac{X_2 \cos \varphi_2}{\sqrt{X_2^2 + Z_2^2}} \sigma_x \\
&\quad + i \frac{\sin \left(\sqrt{X_1^2 + Z_1^2} \right) \sin \left(\sqrt{X_2^2 + Z_2^2} \right) (Z_1 X_2 \sin \varphi_2 - X_1 Z_2 \sin \varphi_1)}{\sqrt{X_1^2 + Z_1^2} \sqrt{X_2^2 + Z_2^2}} \sigma_x \\
&\quad + i \sin \left(\sqrt{X_1^2 + Z_1^2} \right) \frac{X_1 \sin \varphi_1}{\sqrt{X_1^2 + Z_1^2}} \cos \left(\sqrt{X_2^2 + Z_2^2} \right) \sigma_y - i \sin \left(\sqrt{X_1^2 + Z_1^2} \right) \frac{Z_1}{\sqrt{X_1^2 + Z_1^2}} \sin \left(\sqrt{X_2^2 + Z_2^2} \right) \frac{X_2 \cos \varphi_2}{\sqrt{X_2^2 + Z_2^2}} \\
&\quad + i \cos \left(\sqrt{X_1^2 + Z_1^2} \right) \sin \left(\sqrt{X_2^2 + Z_2^2} \right) \frac{X_2 \sin \varphi_2}{\sqrt{X_2^2 + Z_2^2}} \sigma_y + i \sin \left(\sqrt{X_1^2 + Z_1^2} \right) \frac{X_1 \cos \varphi_1}{\sqrt{X_1^2 + Z_1^2}} \sin \left(\sqrt{X_2^2 + Z_2^2} \right) \frac{Z_2}{\sqrt{X_2^2 + Z_2^2}} \\
&\quad + i \sin \left(\sqrt{X_1^2 + Z_1^2} \right) \frac{Z_1}{\sqrt{X_1^2 + Z_1^2}} \cos \left(\sqrt{X_2^2 + Z_2^2} \right) \sigma_z + i \sin \left(\sqrt{X_1^2 + Z_1^2} \right) \frac{X_1 \sin \varphi_1}{\sqrt{X_1^2 + Z_1^2}} \sin \left(\sqrt{X_2^2 + Z_2^2} \right) \frac{X_2 \cos \varphi_2}{\sqrt{X_2^2 + Z_2^2}} \\
&\quad - i \sin \left(\sqrt{X_1^2 + Z_1^2} \right) \frac{X_1 \cos \varphi_1}{\sqrt{X_1^2 + Z_1^2}} \sin \left(\sqrt{X_2^2 + Z_2^2} \right) \frac{X_2 \sin \varphi_2}{\sqrt{X_2^2 + Z_2^2}} \sigma_z + i \cos \left(\sqrt{X_1^2 + Z_1^2} \right) \sin \left(\sqrt{X_2^2 + Z_2^2} \right) \frac{Z_2}{\sqrt{X_2^2 + Z_2^2}}
\end{aligned}$$

2.1 Small power limit

$$\begin{aligned}
& \exp(i(X_1(\cos \varphi_1 \sigma_x + \sin \varphi_1 \sigma_y) + Z_1 \sigma_z)) \exp(i(X_2(\cos \varphi_2 \sigma_x + \sin \varphi_2 \sigma_y) + Z_2 \sigma_z)) \\
& \approx \left(\cos Z_1 + i \sin Z_1 \sigma_z + i X_1 \frac{\sin Z_1}{Z_1} (\cos \varphi_1 \sigma_x + \sin \varphi_1 \sigma_y) \right) \\
& \quad \left(\cos Z_2 + i \sin Z_2 \sigma_z + i X_2 \frac{\sin Z_2}{Z_2} (\cos \varphi_2 \sigma_x + \sin \varphi_2 \sigma_y) \right) \\
& \approx \cos(Z_1 + Z_2) + i \sin(Z_1 + Z_2) \sigma_z \\
& \quad + i X_2 \frac{\sin Z_2}{Z_2} (\cos(\varphi_2 - Z_1) \sigma_x + \sin(\varphi_2 - Z_1) \sigma_y) \\
& \quad + i X_1 \frac{\sin Z_1}{Z_1} (\cos(\varphi_1 + Z_2) \sigma_x + \sin(\varphi_1 + Z_2) \sigma_y)
\end{aligned}$$

$$\begin{aligned}
& \exp(i(X_1(\cos \varphi_1 \sigma_x + \sin \varphi_1 \sigma_y) + Z_1 \sigma_z)) \exp(i(X_2(\cos \varphi_2 \sigma_x + \sin \varphi_2 \sigma_y) + Z_2 \sigma_z)) \\
& \exp(i(X_3(\cos \varphi_3 \sigma_x + \sin \varphi_3 \sigma_y) + Z_3 \sigma_z)) \\
& \approx \left(\cos Z_1 + i \sin Z_1 \sigma_z + i X_1 \frac{\sin Z_1}{Z_1} (\cos \varphi_1 \sigma_x + \sin \varphi_1 \sigma_y) \right) \\
& \quad \left(\cos Z_2 + i \sin Z_2 \sigma_z + i X_2 \frac{\sin Z_2}{Z_2} (\cos \varphi_2 \sigma_x + \sin \varphi_2 \sigma_y) \right) \\
& \quad \left(\cos Z_3 + i \sin Z_3 \sigma_z + i X_3 \frac{\sin Z_3}{Z_3} (\cos \varphi_3 \sigma_x + \sin \varphi_3 \sigma_y) \right) \\
& \approx (\cos(Z_1 + Z_2) + i \sin(Z_1 + Z_2) \sigma_z) (\cos Z_3 + i \sin Z_3 \sigma_z) \\
& \quad + i X_3 \frac{\sin Z_3}{Z_3} (\cos(Z_1 + Z_2) + i \sin(Z_1 + Z_2) \sigma_z) (\cos \varphi_3 \sigma_x + \sin \varphi_3 \sigma_y) \\
& \quad + i X_2 \frac{\sin Z_2}{Z_2} (\cos(\varphi_2 - Z_1) \sigma_x + \sin(\varphi_2 - Z_1) \sigma_y) (\cos Z_3 + i \sin Z_3 \sigma_z) \\
& \quad + i X_1 \frac{\sin Z_1}{Z_1} (\cos(\varphi_1 + Z_2) \sigma_x + \sin(\varphi_1 + Z_2) \sigma_y) (\cos Z_3 + i \sin Z_3 \sigma_z) \\
& \approx \cos(Z_1 + Z_2 + Z_3) + i \sin(Z_1 + Z_2 + Z_3) \sigma_z \\
& \quad + i X_3 \frac{\sin Z_3}{Z_3} (\cos(Z_1 + Z_2) + i \sin(Z_1 + Z_2) \sigma_z) (\cos \varphi_3 \sigma_x + \sin \varphi_3 \sigma_y) \\
& \quad + i X_2 \frac{\sin Z_2}{Z_2} (\cos(\varphi_2 - Z_1) \sigma_x + \sin(\varphi_2 - Z_1) \sigma_y) (\cos Z_3 + i \sin Z_3 \sigma_z) \\
& \quad + i X_1 \frac{\sin Z_1}{Z_1} (\cos(\varphi_1 + Z_2) \sigma_x + \sin(\varphi_1 + Z_2) \sigma_y) (\cos Z_3 + i \sin Z_3 \sigma_z)
\end{aligned}$$