Measuring the scrambling of quantum information

Yichao Yu

Ni Group

Jan. 20, 2021

- Scrambling of quantum information
- Out-of-time-order (OTO) correlator
- Measurement of OTO correlator
- Experimental realization with cavity QED system

Relaxiation vs Scrambling

Relaxiation

- Decay/leaking of information from a single qubit.

- Fast
  Time scale: τ

Scrambling

- Spreading of information to the whole system.

- Slow
  Time scale: $t_* = \tau \ln S$

Relaxiation vs Scrambling

Relaxiation	**Scrambling**
- Decay/leaking of information from a single qubit.	- Spreading of information to the whole system.
- Fast Time scale: τ	- Slow Time scale: $t_* = \tau \ln S$

Relaxiation vs Scrambling

Relaxiation

- Decay/leaking of information from a single qubit.
- Fast
  Time scale: τ

Scrambling

- Spreading of information to the whole system.
- Slow
  Time scale: $t_* = \tau \ln S$

Relaxiation vs Scrambling

Relaxation

- Decay/leaking of information from a single qubit.
- Fast
  Time scale: τ

Scrambling

- Spreading of information to the whole system.
- Slow
  Time scale: $t_* = \tau \ln S$

Out-of-time-order (OTO) correlator

$$F(t) \equiv \langle W_t^\dagger V^\dagger W_t V \rangle$$

$$W_t = U(-t)WU(t)$$
$$U(t) = e^{-iHt}$$

- Interpretation: $F = \langle \psi_1 | \psi_2 \rangle$
  $|\psi_1\rangle = W_t V |\psi_0\rangle$
  $|\psi_2\rangle = V W_t |\psi_0\rangle$
- Choice of V and W?
- Scaling of F with system size.
- Relation with scrambling time?

Out-of-time-order (OTO) correlator

$$F(t) \equiv \langle W_t^\dagger V^\dagger W_t V \rangle$$

$$W_t = U(-t)WU(t)$$
$$U(t) = e^{-iHt}$$

- Interpretation: $F = \langle \psi_1 | \psi_2 \rangle$

  $|\psi_1\rangle = W_t V |\psi_0\rangle$
  $|\psi_2\rangle = V W_t |\psi_0\rangle$

- Choice of V and W?

- Scaling of F with system size.

- Relation with scrambling time?

Out-of-time-order (OTO) correlator

$$F(t) \equiv \langle W_t^\dagger V^\dagger W_t V \rangle$$

$$W_t = U(-t)WU(t)$$
$$U(t) = e^{-iHt}$$

- Interpretation: $F = \langle \psi_1 | \psi_2 \rangle$

  $$|\psi_1\rangle = W_t V |\psi_0\rangle$$
  $$|\psi_2\rangle = V W_t |\psi_0\rangle$$

- Choice of V and W?

- Scaling of F with system size.

- Relation with scrambling time?

Out-of-time-order (OTO) correlator

$$F(t) \equiv \langle W_t^\dagger V^\dagger W_t V \rangle$$

$$W_t = U(-t)WU(t)$$
$$U(t) = e^{-iHt}$$

- Interpretation: $F = \langle \psi_1 | \psi_2 \rangle$

  $|\psi_1\rangle = W_t V |\psi_0\rangle$
  $|\psi_2\rangle = V W_t |\psi_0\rangle$

- Choice of V and W?
- Scaling of F with system size.
- Relation with scrambling time?

Out-of-time-order (OTO) correlator

$$F(t) \equiv \langle W_t^\dagger V^\dagger W_t V \rangle$$

$$W_t = U(-t)WU(t)$$
$$U(t) = e^{-iHt}$$

- Interpretation: $F = \langle \psi_1 | \psi_2 \rangle$

  $$|\psi_1\rangle = W_t V |\psi_0\rangle$$
 $$|\psi_2\rangle = V W_t |\psi_0\rangle$$
- Choice of V and W?
- Scaling of F with system size.
- Relation with scrambling time?

Out-of-time-order (OTO) correlator

$$F(t) \equiv \langle W_t^\dagger V^\dagger W_t V \rangle$$

$$W_t = U(-t)WU(t)$$
$$U(t) = e^{-iHt}$$

- Interpretation: $F = \langle \psi_1 | \psi_2 \rangle$

  $|\psi_1\rangle = W_t V |\psi_0\rangle$
  $|\psi_2\rangle = V W_t |\psi_0\rangle$
- Choice of V and W?
- Scaling of F with system size.
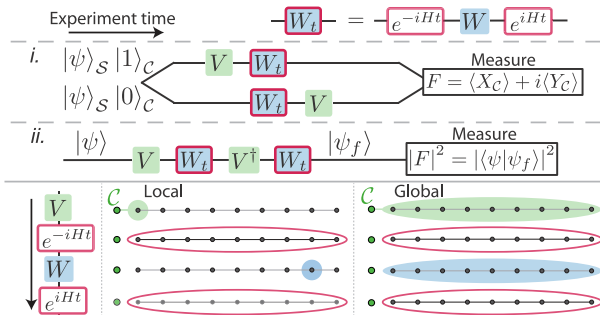- Relation with scrambling time?

Measurement of OTO correlator

Given H, F, W, measure $F(t) \equiv \langle W_t^\dagger V^\dagger W_t V \rangle$
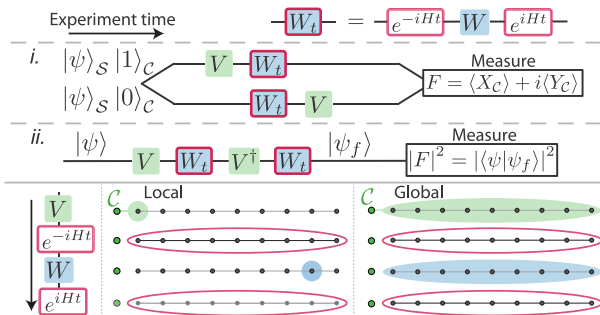
Measurement of OTO correlator

Given H, F, W, measure $F(t) \equiv \langle W_t^\dagger V^\dagger W_t V \rangle$
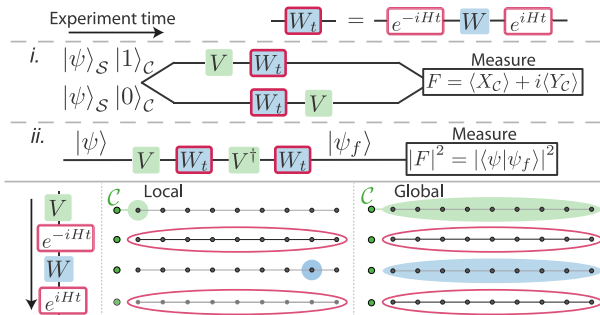
Measurement of OTO correlator

Given H, F, W, measure $F(t) \equiv \langle W_t^\dagger V^\dagger W_t V \rangle$

Time reversal: $U(-t)$
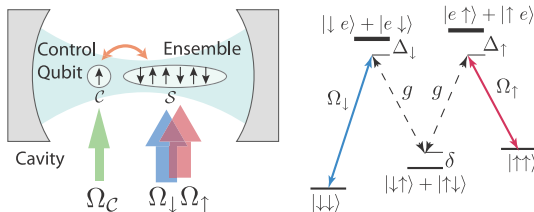
Measurement of OTO correlator

Given H, F, W, measure $F(t) \equiv \langle W_t^\dagger V^\dagger W_t V \rangle$

Time reversal: $U(-t)$

Controlled-V: $I_S \otimes |0\rangle\langle 0|_C + V_S \otimes |1\rangle\langle 1|_C$
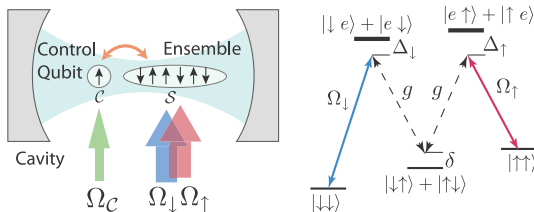
Cavity QED implementation

$$H = \sum_{ij} J_{ij} s_i^x s_j^x + h.c.$$

$$J_{ij} = \sum_\alpha \frac{\Omega_\uparrow^*(r_i)\Omega_\downarrow(r_j)}{\Delta_\uparrow \Delta_\downarrow} \frac{g_\alpha(r_i)g_\alpha^*(r_j)}{\delta}$$

Cavity QED implementation

$$Z_\phi^C = I_S \otimes |0\rangle\langle 0|_C + \mathrm{e}^{-\mathrm{i}\phi S_z^s} \otimes |1\rangle\langle 1|_C$$

$$V = W = \mathrm{e}^{-\mathrm{i}\phi S_z^s}$$

S_x^2 dynamics

$$H = \sum_{ij} J_{ij} s_i^x s_j^x + h.c. \;\rightarrow\; H = J S_x^2$$

$$F(t) \equiv \langle W_t^\dagger V^\dagger W_t V \rangle \quad G(t) \equiv \langle V_t^\dagger V \rangle$$