A One-Way Quantum Computer
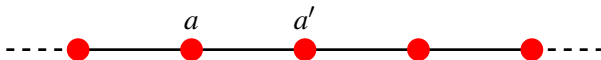
Yichao Yu

Ni Group

Feb. 15, 2021

- 1D Cluster state
  - Generation
  - Properties
- High dimensional cluster state
- Quantum circuit
- Gates and single qubit operations

1D Cluster State

$$H = \sum_{a,a' \in \Gamma} \frac{1 + \sigma_z^{(a)}}{2} \frac{1 - \sigma_z^{(a')}}{2}$$

$$\Gamma = \left\{ (a, a') | a' = a + 1 \right\}$$

$$\mathcal{S} = e^{i\pi H}$$

1D Cluster State

$$H = \sum_{a,a' \in \Gamma} \frac{1 + \sigma_z^{(a)}}{2} \frac{1 - \sigma_z^{(a')}}{2} \qquad H = \sum \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 1 & \\ & & & 0 \end{pmatrix}$$

$$\Gamma = \left\{ (a, a') | a' = a + 1 \right\}$$

$$\mathcal{S} = e^{i\pi H}$$

$$\mathcal{S} = \bigotimes \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & 1 \end{pmatrix}$$

1D Cluster State

$$|\phi_N\rangle = \mathcal{S} \bigotimes_a |+\rangle_a = \frac{1}{2^{N/2}} \bigotimes_a \left(|0\rangle_a \sigma_z^{a+1} + |1\rangle_a \right)$$

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$$|\text{GHZ}_N\rangle = \frac{1}{\sqrt{2}}\left(\bigotimes_a |0\rangle_a + \bigotimes_a |1\rangle_a\right)$$
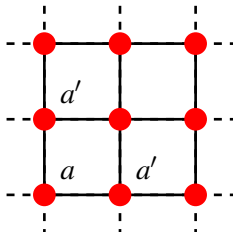
1D Cluster State

- Maximum connectedness
  Ability to create Bell state by local
  measurements.
  Yes for both GHZ state and cluster state.

- Persistency
  Minimum local measurements to destroy all
  entanglements.
  GHZ: $P_e = 1$, cluster: $P_e = \lfloor N/2 \rfloor$

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High Dimensional Cluster State

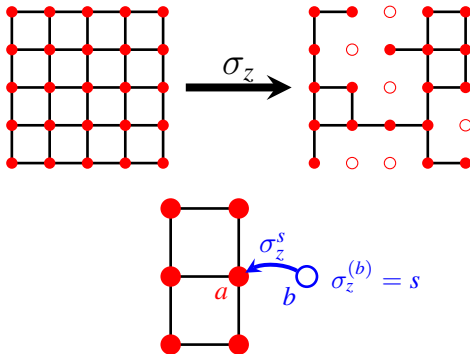$$H = \sum_{a,a' \in \Gamma} \frac{1 + \sigma_z^{(a)}}{2} \frac{1 - \sigma_z^{(a')}}{2}$$

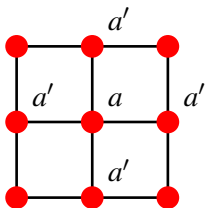$$\Gamma = \left\{ (a, a') | a' = a + \hat{e}_i \right\}$$

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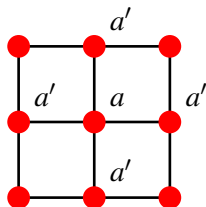$$[H, \sigma_z^{(a)}] = 0, \ [\mathcal{S}, \sigma_z^{(a)}] = 0$$

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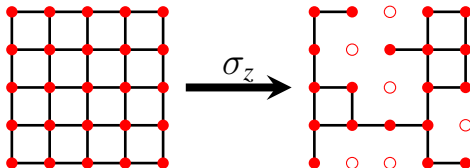
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$$\Gamma' = \left\{ (a, a') \mid a' = a \pm \hat{e}_i \right\}$$
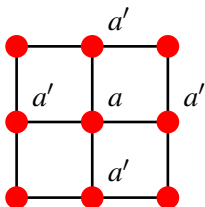
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- $[K_a, K_b] = 0$
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  - Apply full $\mathcal{S}$ and measure σ_z's on removed sites
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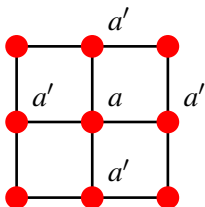 - Eigenstates of all K_a
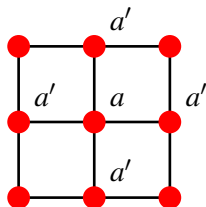
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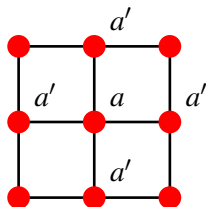
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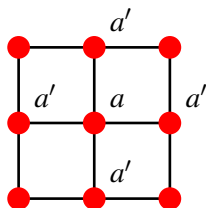
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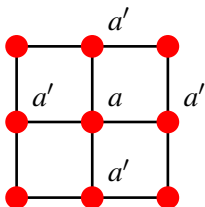
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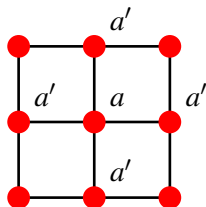
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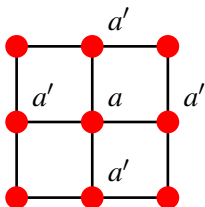
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Quantum Circuit on Cluster State
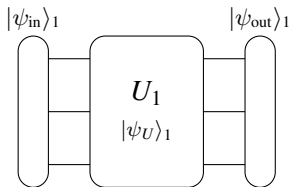
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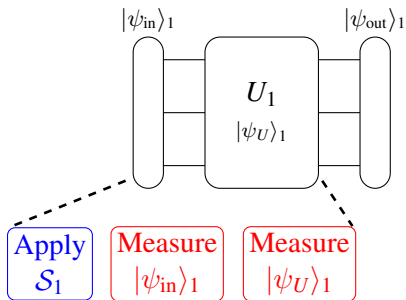
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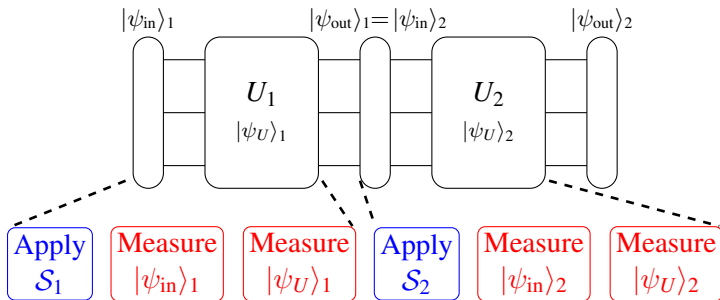
Quantum Circuit on Cluster State

Gates and Single Qubit Operations (Single Qubit)

Propagation: Measure σ_x
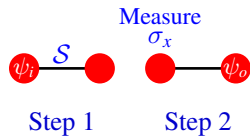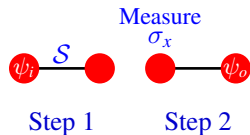
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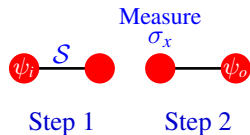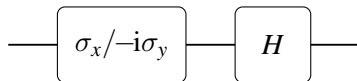Propagation: Measure σ_x

$\psi_i \quad \mathcal{S}$

Step 1

Gates and Single Qubit Operations (Single Qubit)

Propagation: Measure σ_x

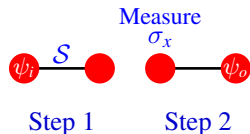# Gates and Single Qubit Operations (Single Qubit)

Propagation: Measure σ_x

Step 1   Step 2

$$\mathcal{S}(a|0\rangle_1 + b|1\rangle_1)|+\rangle_2 = |+\rangle_1(a|-\rangle_2 + b|+\rangle_2)$$
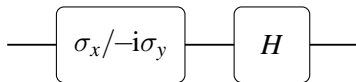$$+ |-\rangle_1(a|-\rangle_2 - b|+\rangle_2)$$

Gates and Single Qubit Operations (Single Qubit)

Propagation: Measure σ_x

Measure
σ_x

ψ_i — $\mathcal{S}$ — ● ● — ψ_o

Step 1   Step 2
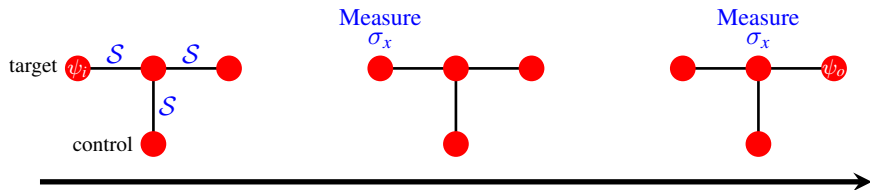
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$$\boxed{\sigma_x/\!-i\sigma_y} \quad \boxed{H}$$

Gates and Single Qubit Operations (Single Qubit)

Propagation: Measure σ_x

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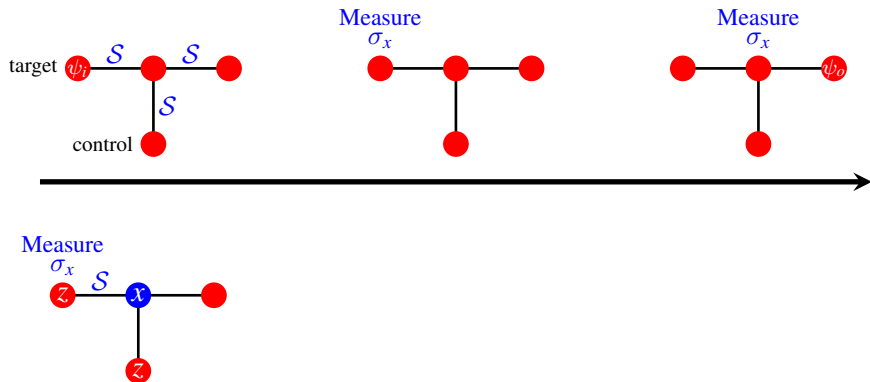

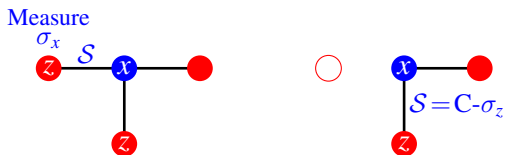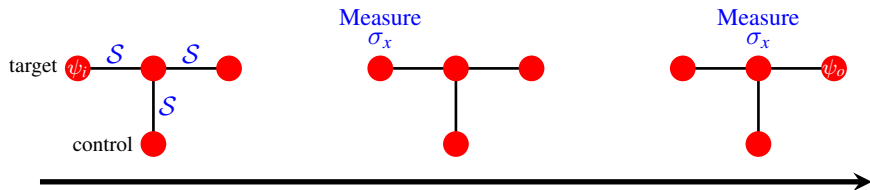Rotation: Measure $\sigma_x \cos\theta + \sigma_y \sin\theta$

Gates and Single Qubit Operations (CNOT)
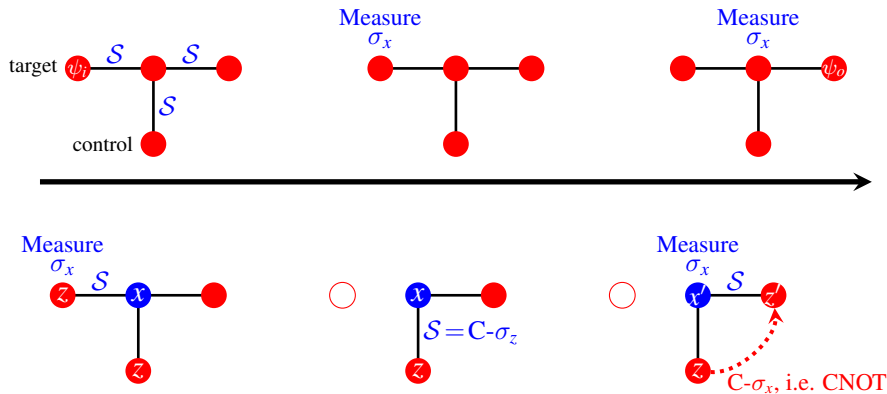
Gates and Single Qubit Operations (CNOT)

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Gates and Single Qubit Operations (CNOT)

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Questions?