1 Simplifies Optical Bloch Equation for Sideband Cooling Simulation.

Rabi frequency between state m and n (assume to be real since the phase is not important for sidband cooling.): Ω_{mn}
Pumping rate from state n to m: Γ_{mn}

Diagonal terms,

$$\frac{\mathrm{d}\rho_{mm}}{\mathrm{d}t} = -\rho_{mm}\sum_k \Gamma_{km} + \sum_k \rho_{kk}\Gamma_{mk} + \mathrm{i}\sum_k \left(\rho_{mk}\Omega_{km} - \Omega_{mk}\rho_{km}\right)$$

Off-diagnal terms,

$$\frac{\mathrm{d}\rho_{mn}}{\mathrm{d}t} = -\frac{\rho_{mn}}{2}\sum_k \left(\Gamma_{km} + \Gamma_{kn}\right) + \mathrm{i}\sum_k \left(\rho_{mk}\Omega_{kn} - \Omega_{mk}\rho_{kn}\right)$$

When only one sideband is driven,

$$\Omega_{mn} = \Omega_m\delta_{m,n-\Delta} + \Omega_n\delta_{n,m-\Delta}$$

where $Delta$ include both the change in vibrational level and internal level.
Define $p_n = \rho_{nn}$, the equations becomes,

$$\begin{aligned}
\frac{\mathrm{d}p_m}{\mathrm{d}t} =& \sum_k \left(p_k\Gamma_{mk} - p_m\Gamma_{km}\right) + \mathrm{i}(\rho_{m,m-\Delta}\Omega_{m-\Delta} - \Omega_m\rho_{m+\Delta,m} + \rho_{m,m+\Delta}\Omega_m - \Omega_{m-\Delta}\rho_{m-\Delta,m}) \\
\frac{\mathrm{d}\rho_{m,m+\Delta}}{\mathrm{d}t} =& -\frac{\rho_{m,m+\Delta}}{2}\sum_k \left(\Gamma_{km} + \Gamma_{k,m+\Delta}\right) + \mathrm{i}\Omega_m(p_m - p_{m+\Delta})
\end{aligned}$$

ρ_{mn}'s with $|m-n| \neq 0, \Delta$ are ignored since they are 0. In particular, since Δ includes change of internal levels, elements with $|m-n| \geqslant 2\Delta$ does not exist.

Define $q_n = \mathrm{i}\rho_{n,n+\Delta}$,

$$\begin{aligned}
\rho_{n,n+\Delta} =& -\mathrm{i}q_n \\
\rho_{n+\Delta,n} =& \mathrm{i}q_n^*
\end{aligned}$$

$$\begin{aligned}
\frac{\mathrm{d}p_m}{\mathrm{d}t} =& \sum_k \left(p_k\Gamma_{mk} - p_m\Gamma_{km}\right) + \mathrm{i}\left(\mathrm{i}q_{m-\Delta}^*\Omega_{m-\Delta} - \mathrm{i}\Omega_m q_m^* - \mathrm{i}q_m\Omega_m + \mathrm{i}\Omega_{m-\Delta}q_{m-\Delta}\right) \\
-\mathrm{i}\frac{\mathrm{d}q_m}{\mathrm{d}t} =& \mathrm{i}\frac{q_m}{2}\sum_k \left(\Gamma_{km} + \Gamma_{k,m+\Delta}\right) + \mathrm{i}\Omega_m(p_m - p_{m+\Delta}) \\
\frac{\mathrm{d}p_m}{\mathrm{d}t} =& \sum_k \left(p_k\Gamma_{mk} - p_m\Gamma_{km}\right) + \Omega_m(q_m^* + q_m) - \Omega_{m-\Delta}\left(q_{m-\Delta}^* + q_{m-\Delta}\right) \\
\frac{\mathrm{d}q_m}{\mathrm{d}t} =& -\frac{q_m}{2}\sum_k \left(\Gamma_{km} + \Gamma_{k,m+\Delta}\right) + \Omega_m(p_{m+\Delta} - p_m)
\end{aligned}$$

For a process starting with $q_m = 0$, q_m will always remain real.

$$\begin{aligned}
\frac{\mathrm{d}p_m}{\mathrm{d}t} =& \sum_k p_k\Gamma_{mk} - p_m\Gamma_m + 2\Omega_m q_m - 2\Omega_{m-\Delta}q_{m-\Delta} \\
\frac{\mathrm{d}q_m}{\mathrm{d}t} =& -\frac{q_m}{2}(\Gamma_m + \Gamma_{m+\Delta}) + \Omega_m(p_{m+\Delta} - p_m)
\end{aligned}$$

where $\Gamma_m \equiv \sum_k \Gamma_{km}$ is the decay rate of state m.

After writing the two internal states (a and b) explicitly,

$$\begin{aligned}
\frac{\mathrm{d}p_m^a}{\mathrm{d}t} &= \sum_{\alpha=a,b;k} p_k^\alpha \Gamma_{mk}^{a\alpha} - p_m^a \Gamma_m^a + 2\Omega_m q_m \\
\frac{\mathrm{d}p_m^b}{\mathrm{d}t} &= \sum_{\alpha=a,b;k} p_k^\alpha \Gamma_{mk}^{b\alpha} - p_m^b \Gamma_m^b - 2\Omega_{m-\delta} q_{m-\delta} \\
\frac{\mathrm{d}q_m}{\mathrm{d}t} &= -\frac{q_m}{2}\left(\Gamma_m^a + \Gamma_{m+\delta}^b\right) + \Omega_m\left(p_{m+\delta}^b - p_m^a\right)
\end{aligned}$$