Confident interval with binomial confidence interval with measurement error

December 11, 2024

1 Setup

There are two possible states to be detected, 0 and 1, and the same two possible measurement outcome. Let p_{ij} $(i, j \in \{0, 1\})$ be the probability of measuring i when the actual state is j. With N trials of measurements, 0 was detected n_0 times and 1 was detected n_1 times. The goal is to esimate the probability p of the input being in 1 as well as the confidence interval for this estimation.

2 Without measurement

For a measurement that is useful we should have $p_{ij} \approx \delta_{ij}$. If $p_{ij} = \delta_{ij}$, i.e. no measurement error, we use the Wilson score interval,

$$p_\pm = \frac{1}{1 + z^2/N}\left(\hat{p} + \frac{z^2}{N} \pm \frac{z}{2N}\sqrt{4N\hat{p}(1 - \hat{p}) + z^2}\right)$$

where z is standard normal interval half-width corresponding to the desired confidence and $\hat{p} \equiv n_1/N$ is the measured probability of 1.