
1 Matrix element

Wavefunction

$$\psi_n(x, z) = \sqrt{\frac{1}{2^n n! z \sqrt{\pi}}} e^{-(x/z)^2/2} H_n(x/z)$$

Define

$$H'_n(x) \equiv \frac{H_n(x)}{2^n}$$
$$\psi_n(x, z) = \sqrt{\frac{2^n}{n! z \sqrt{\pi}}} e^{-(x/z)^2/2} H'_n(x/z)$$

Matrix element

$$\begin{aligned} \langle n1, m1 | n2, m2 \rangle &= \int_{-\infty}^{\infty} \psi_{n1}(x, z1) \psi_{n2}(x, z1) \psi_{m1}(x, z2) \psi_{m2}(x, z2) dx \\ &= \int_{-\infty}^{\infty} dx \sqrt{\frac{2^{n1}}{n1! z1 \sqrt{\pi}}} e^{-(x/z1)^2/2} H'_{n1}(x/z1) \sqrt{\frac{2^{n2}}{n2! z1 \sqrt{\pi}}} e^{-(x/z1)^2/2} H'_{n2}(x/z1) \\ &\quad \sqrt{\frac{2^{m1}}{m1! z2 \sqrt{\pi}}} e^{-(x/z2)^2/2} H'_{m1}(x/z2) \sqrt{\frac{2^{m2}}{m2! z2 \sqrt{\pi}}} e^{-(x/z2)^2/2} H'_{m2}(x/z2) \\ &= \frac{1}{\pi z1 z2} \int_{-\infty}^{\infty} dx \sqrt{\frac{2^{n1+n2}}{n1! n2!}} e^{-(x/z1)^2} H'_{n1}(x/z1) H'_{n2}(x/z1) \\ &\quad \sqrt{\frac{2^{m1+m2}}{m1! m2!}} e^{-(x/z2)^2} H'_{m1}(x/z2) H'_{m2}(x/z2) \\ &= \frac{1}{\pi z1 z2} \sqrt{\frac{2^{n1+n2+m1+m2}}{n1! n2! m1! m2!}} \int_{-\infty}^{\infty} dx e^{-(x/z1)^2} H'_{n1}(x/z1) H'_{n2}(x/z1) \\ &\quad e^{-(x/z2)^2} H'_{m1}(x/z2) H'_{m2}(x/z2) \end{aligned}$$