Lamb-Dicke regime/approximation

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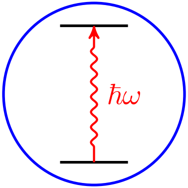
Journal Club

Doppler effect



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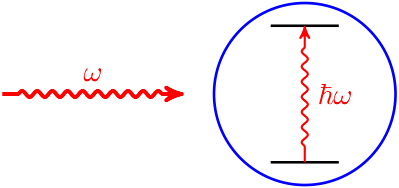

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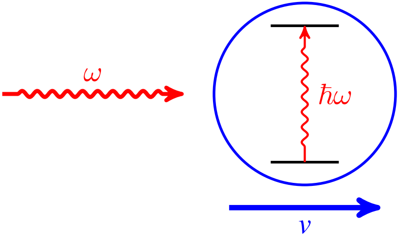

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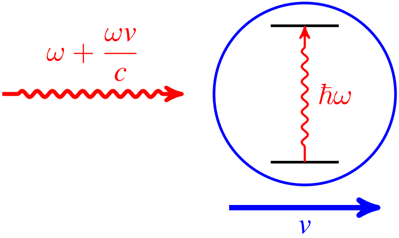

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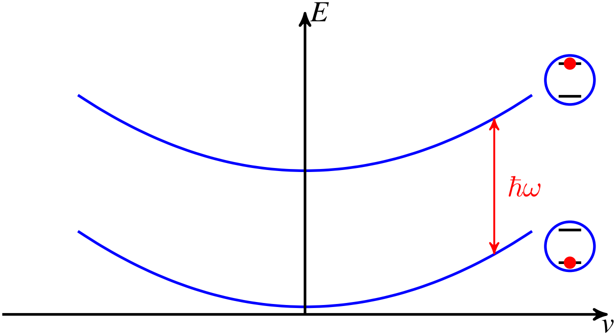

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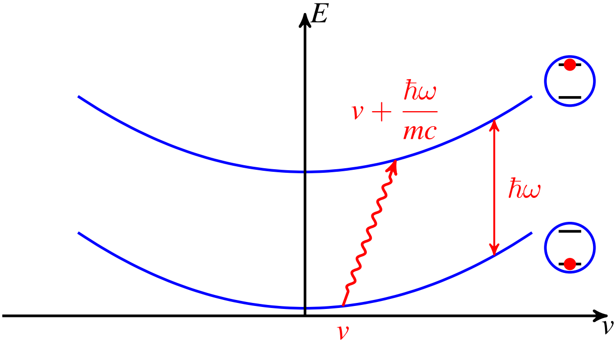

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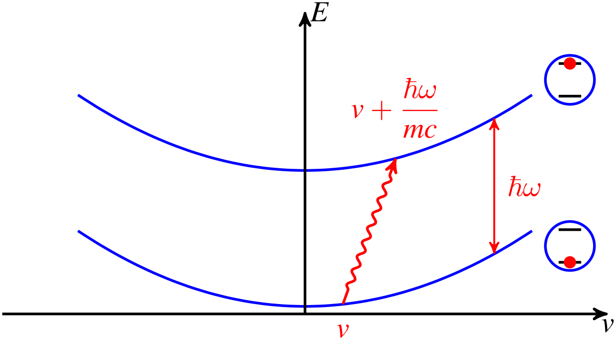
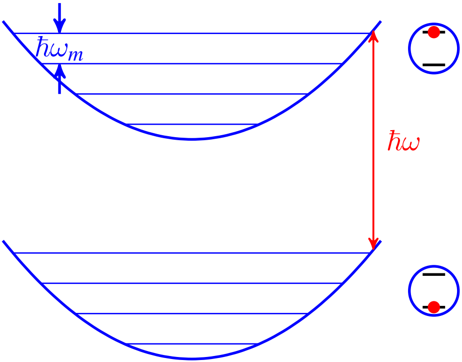


$$\omega + \frac{m}{2\hbar}\left(v + \frac{\hbar\omega}{mc}\right)^2 - \frac{mv^2}{2\hbar} = \omega + \frac{\omega v}{c} + \frac{\hbar\omega^2}{2mc^2}$$

Sideband



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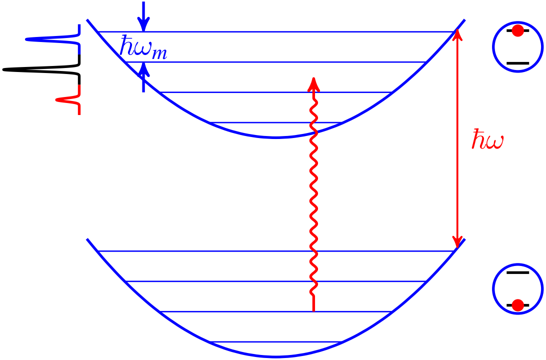
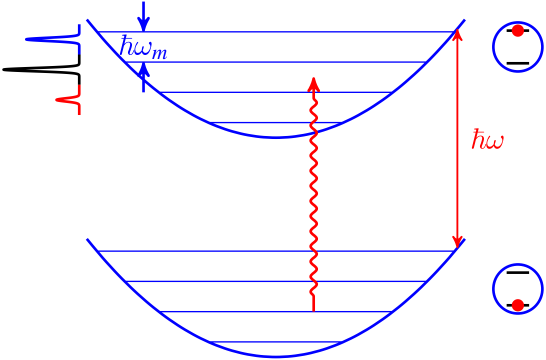


Frequency: $\omega + n\omega_m$

Sideband



Frequency: $\omega + n\omega_m$

Strength: $\langle n|\mathrm{e}^{\mathrm{i}k\hat{x}}|n + \Delta n\rangle$

Lamb-Dicke parameter

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$$\hat{x}=\sqrt{\frac{\hbar}{2m\omega}}\left(a+a^\dagger\right)=z_0\left(a+a^\dagger\right)$$

$$k\hat{x}=\eta\left(a+a^\dagger\right)$$

$$\eta\equiv kz_0=k\sqrt{\frac{\hbar}{2m\omega}}$$

Lamb-Dicke parameter

$$\langle n|\mathrm{e}^{\mathrm{i}k\hat{x}}|n+\Delta n\rangle$$

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}\left(a + a^\dagger\right) = z_0\left(a + a^\dagger\right)$$

$$k\hat{x} = \eta\left(a + a^\dagger\right)$$

$$\eta \equiv kz_0 = k\sqrt{\frac{\hbar}{2m\omega}}$$

$$\eta = \frac{2\pi z_0}{\lambda}$$

$$\eta = \sqrt{\frac{\omega_R}{\omega_m}}$$

Sideband strength

$$\begin{aligned}&\langle n|\mathrm{e}^{\mathrm{i}k\hat{x}}|n+\Delta n\rangle\\=&\mathrm{e}^{-\eta^2/2}\eta^{\Delta n}\sqrt{\frac{n_-!}{n_+!}}\,L_{n_-}^{\Delta n}(\eta^2)\end{aligned}$$

$$n_-\equiv\min(n,n+\Delta n),\qquad n_+\equiv\max(n,n+\Delta n)$$

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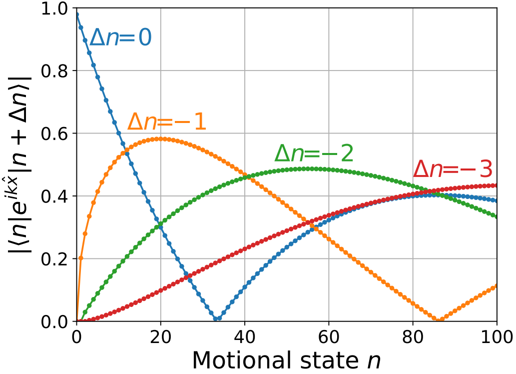
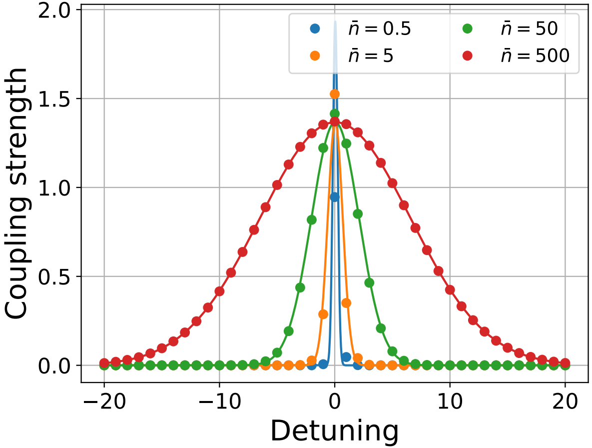


Sideband spectrum



Lamb-Dicke approximation/regime

$$e^{i\eta\left(a+a^\dagger\right)} = 1 + i\eta\left(a + a^\dagger\right) - \frac{\eta^2}{2}\left(a + a^\dagger\right)^2 + \mathcal{O}(\eta^3)$$

Lamb-Dicke approximation/regime

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When $\eta \ll 1$: $e^{i\eta\left(a+a^\dagger\right)} \approx 1 + i\eta\left(a + a^\dagger\right)$

Lamb-Dicke approximation/regime

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$$\text{When } \eta \ll 1: \quad e^{i\eta\left(a+a^\dagger\right)} \approx 1 + i\eta\left(a + a^\dagger\right)$$

To the first order,

$$\begin{aligned}
\langle n|e^{i\eta\left(a+a^\dagger\right)}|n\rangle \approx& 1 \\
\langle n+1|e^{i\eta\left(a+a^\dagger\right)}|n\rangle \approx& i\eta\sqrt{n} \\
\langle n-1|e^{i\eta\left(a+a^\dagger\right)}|n\rangle \approx& i\eta\sqrt{n+1}
\end{aligned}$$

Lamb-Dicke approximation/regime

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$$\text{When } \eta \ll 1: \quad e^{i\eta\left(a+a^\dagger\right)} \approx 1 + i\eta\left(a + a^\dagger\right)$$

To the second order,

$$\langle n|e^{i\eta\left(a+a^\dagger\right)}|n\rangle \approx 1 - \frac{\eta^2(2n+1)}{2}$$

Lamb-Dicke approximation / regime

$$\langle n|\mathrm{e}^{\mathrm{i}\eta\left(a+a^\dagger\right)}|n\rangle \approx 1 - \frac{\eta^2(2n+1)}{2}$$

Lamb-Dicke approximation / regime

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Wavefunction spread,

$$\langle n|x^2|n\rangle = z_0^2(2n+1)$$

Lamb-Dicke approximation / regime

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Wavefunction spread,

$$\langle n|x^2|n\rangle = z_0^2(2n+1)$$

$$\eta_{eff} \equiv \eta\sqrt{2n+1}$$

Scattering / optical pumping heating

Probability of remaining in n: $P_0 \approx 1 - \eta_{eff}^2/2$

Scattering / optical pumping heating

Probability of remaining in n: $P_0 \approx 1 - \eta_{eff}^2/2$

Average energy gain,

$$
\begin{aligned}
\bar{n}' =& \langle n | \mathrm{e}^{\mathrm{i}\eta\left(a+a^\dagger\right)} a^\dagger a \mathrm{e}^{-\mathrm{i}\eta\left(a+a^\dagger\right)} | n \rangle \\
=& \langle n | \mathrm{e}^{\mathrm{i}\eta a^\dagger} \mathrm{e}^{\mathrm{i}\eta a} a^\dagger a \mathrm{e}^{-\mathrm{i}\eta a} \mathrm{e}^{-\mathrm{i}\eta a^\dagger} | n \rangle \\
=& \langle n | \mathrm{e}^{\mathrm{i}\eta a^\dagger} \left(a^\dagger \mathrm{e}^{\mathrm{i}\eta a} + \mathrm{i}\eta \mathrm{e}^{\mathrm{i}\eta a} \right) \mathrm{e}^{-\mathrm{i}\eta a} \left(\mathrm{e}^{-\mathrm{i}\eta a^\dagger} a - \mathrm{i}\eta \mathrm{e}^{-\mathrm{i}\eta a^\dagger} \right) | n \rangle \\
=& \langle n | (a^\dagger + \mathrm{i}\eta)(a - \mathrm{i}\eta) | n \rangle \\
=& n + \eta^2
\end{aligned}
$$

Scattering / optical pumping heating

Questions

- Is energy gain always a constant?
- Is there a Δn upper bound on when $\langle n|\mathrm{e}^{\mathrm{i}\eta\left(a+a^\dagger\right)}|n+\Delta n\rangle \neq 0$?

Scattering / optical pumping heating

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Coherence between n, momentum distribution.

$$
\begin{aligned}
&\langle\alpha|e^{i\eta\left(a+a^\dagger\right)}a^\dagger a e^{-i\eta\left(a+a^\dagger\right)}|\alpha\rangle\\
=&\langle\alpha|(a^\dagger+i\eta)(a-i\eta)|\alpha\rangle\\
=&|\alpha-i\eta|^2
\end{aligned}
$$

Scattering / optical pumping heating

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Coherence between n, momentum distribution.

$$\begin{aligned}&\mathrm{e}^{\mathrm{i}\eta\left(a+a^\dagger\right)}a^\dagger a\mathrm{e}^{-\mathrm{i}\eta\left(a+a^\dagger\right)}\\=&(a^\dagger+\mathrm{i}\eta)(a-\mathrm{i}\eta)\\=&n+\eta^2+pk/m\omega\end{aligned}$$

Sideband spectroscopy

For thermal distribution

$$
\begin{aligned}
p_n &\propto \left(\frac{\bar{n}}{\bar{n}+1}\right)^n \\
h_{blue} &= \sum_n p_n \sin^2\left(\Omega_{n,n+1} t/2\right) \\
h_{red} &= \sum_n p_n \sin^2\left(\Omega_{n,n-1} t/2\right) \\
&= \sum_n p_n \sin^2\left(\Omega_{n-1,n} t/2\right) \\
&= \frac{\bar{n}}{\bar{n}+1} \sum_n p_n \sin^2\left(\Omega_{n,n+1} t/2\right) \\
&= \frac{\bar{n}}{\bar{n}+1} h_{blue}
\end{aligned}
$$