1 Simplifies Optical Bloch Equation for Sideband Cooling Simulation.

Rabi frequency between state m and n (assume to be real since the phase is not important for sidband cooling.): Ω_{mn}
Pumping rate from state n to m: Γ_{mn}

Diagonal terms,

$$\frac{\partial \rho_{mm}}{\partial t} = -\rho_{mm}\sum_k \Gamma_{km} + \sum_k \rho_{kk}\Gamma_{mk} + \mathrm{i}\sum_k (\rho_{mk}\Omega_{km} - \Omega_{mk}\rho_{km})$$

Off-diagnal terms,

$$\frac{\partial \rho_{mn}}{\partial t} = -\frac{\rho_{mn}}{2}\sum_k (\Gamma_{km} + \Gamma_{kn}) + \mathrm{i}\sum_k (\rho_{mk}\Omega_{kn} - \Omega_{mk}\rho_{kn})$$

When only one sideband is driven,

$$\Omega_{mn} = \Omega_m \delta_{m,n-\Delta} + \Omega_n \delta_{n,m-\Delta}$$

where $Delta$ include both the change in vibrational level and internal level.
With $p_n = \rho nn$, the equations becomes,

$$\begin{aligned}
\frac{\partial p_m}{\partial t} =& \sum_k (p_k\Gamma_{mk} - p_m\Gamma_{km}) + \mathrm{i}(\rho_{m,m-\Delta}\Omega_{m-\Delta} - \Omega_m\rho_{m+\Delta,m} + \rho_{m,m+\Delta}\Omega_m - \Omega_{m-\Delta}\rho_{m-\Delta,m}) \\
\frac{\partial \rho_{m,m+\Delta}}{\partial t} =& -\frac{\rho_{m,m+\Delta}}{2}\sum_k (\Gamma_{km} + \Gamma_{k,m+\Delta}) + \mathrm{i}\Omega_m(\rho_{mm} - \rho_{m+\Delta,m+\Delta})
\end{aligned}$$

ρ_{mn}'s with $|m-n| \neq 0, \Delta$ are ignored since they are 0. In particular, since Δ includes change of internal levels, elements with $|m-n| \geqslant 2\Delta$ does not exist.