Rabi frequency conventions

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1 Introduction

This is simply a summary of the conventions for Rabi frequencies (factor of two's) and related quantities.

2 Two level system

For a two level system with Rabi frequency Ω and Detuning Δ, the Hamiltonian is,

$$H = \begin{pmatrix} \frac{\Delta}{2} & \frac{\Omega}{2} \\ \frac{\Omega^*}{2} & -\frac{\Delta}{2} \end{pmatrix} \tag{1}$$

The generalized Rabi frequency Ω_g is $\sqrt{|\Omega|^2 + \Delta^2}$ and this is the (angular) frequency the population is going to be oscillating at when evolving under this Hamiltonian.

In particular, when on-resonance $\Delta = 0$, the Rabi frequency Ω is the (angular) frequency the population oscillate at.

π-time is defined as the time it takes for the oscillation to reach maximum and is,

$$\tau_\pi = \frac{\pi}{\Omega_g} \tag{2}$$

The AC stark shift on the transition is

$$\begin{aligned} \delta_{\text{transition}} =& \Omega_g - \Delta \\ \approx& \frac{|\Omega|^2}{2\Delta} \quad (\text{for } \Omega \ll \Delta) \end{aligned} \tag{3}$$

The AC stark shift on each of the states is

$$\begin{aligned} \delta_{\text{state}} =& \pm \frac{1}{2}(\Omega_g - \Delta) \\ \approx& \pm \frac{|\Omega|^2}{4\Delta} \quad (\text{for } \Omega \ll \Delta) \end{aligned} \tag{4}$$

3 Three level system

The Raman Rabi frequency is $\Omega_R = \frac{\Omega_1 \Omega_2^*}{2\Delta}$ where Δ is the single photon detuning.

For the following 3-level Hamiltonian,

$$H_{3\mathrm{L}} = \begin{pmatrix} \frac{\delta}{2} & \frac{\Omega_1}{2} & \\ \frac{\Omega_1^*}{2} & -\Delta & \frac{\Omega_2}{2} \\ & \frac{\Omega_2^*}{2} & -\frac{\delta}{2} \end{pmatrix} \tag{5}$$

the effective Hamiltonian after eliminating the excited state up to the second order is,

$$\begin{aligned} H_{\mathrm{eff}} &= \begin{pmatrix} \frac{\delta}{2} + \frac{|\Omega_1|^2}{4\Delta} & \frac{\Omega_R}{2} \\ \frac{\Omega_R^*}{2} & -\frac{\delta}{2} + \frac{|\Omega_2|^2}{4\Delta} \end{pmatrix} \\ &= \begin{pmatrix} \frac{\delta}{2} + \frac{|\Omega_1|^2}{4\Delta} & \frac{\Omega_1 \Omega_2^*}{4\Delta} \\ \frac{\Omega_1^* \Omega_2}{4\Delta} & -\frac{\delta}{2} + \frac{|\Omega_2|^2}{4\Delta} \end{pmatrix} \end{aligned} \tag{6}$$