1 Classical model

1.1 Energy distribution

Maxwell-Boltzmann distribution in a 3D harmonic trap.
Let $\varepsilon = \beta E$
PDF: $p(\varepsilon)\mathrm{d}\varepsilon = \frac{\varepsilon^2}{2}\mathrm{e}^{-\varepsilon}\mathrm{d}\varepsilon$
CDF (integrating PDF): $\mathbf{cdf}(\varepsilon) = 1 - \left(1 + \varepsilon + \frac{\varepsilon^2}{2}\right)\mathrm{e}^{-\varepsilon}$

1.2 Model of diabatic lowering

When the lowering is fast (diabatic) enough, assume the wavefunction (motion) of the particle doesn't change. Based on the virial theorem, half of the energy is potiential energy, which is lowered by the lowering factor, and the other half is kinetic energy, which does not change right after lowering so the final energy of the particle with initial energy E is $E' = \frac{\gamma + 1}{2}E$, where γ is the lowering factor.

1.3 Model of survival

Assume that the atom will leave the trap if its final energy is higher than the final trap depth.

1.4 The full experimental sequence

The atom starts in the trap with trap depth E_{trap} and initial energy E_0.
The trap depth is first ramped adabatically by a factor of α (to αE_{trap}) ($\alpha = 1$ if this step is skipped). The energy of the atom is $\sqrt{\alpha}E_0$ due to adabatic compression.
The trap depth is then diabatically lowered to γE_{trap}, the total energy of the atom is now $\frac{\alpha + \gamma}{2\sqrt{\alpha}}E_0$.
The atom will stay in the trap if

$$\frac{\alpha + \gamma}{2\sqrt{\alpha}}E_0 < \gamma E_{trap}$$

or

$$\varepsilon_0 < \frac{2\gamma\sqrt{\alpha}}{\alpha + \gamma}\varepsilon_{trap}$$

where ε_0 and ε_{trap} are the initial energy of the atom and the initial trap depth measured in unit of the initial temperature.
Therefore the total survival probability is,

$$p_0\mathbf{cdf}\left(\frac{2\gamma\sqrt{\alpha}}{(\alpha + \gamma)\tau}\right)$$

where p_0 models the overall survival probability and $\tau = \varepsilon_{trap}^{-1}$ is the temperature measured in trap depth.

2 Alternative classical model

Consider the survival probability in 1D and cube it to approximate the 3d survival probability. This is likely less accurate and might overestimate the energy since it increases the escape condition from energy larger than trap depth to energy in at least one axis larger than trap depth.

2.1 Energy distribution

Maxwell-Boltzmann distribution in a 1D harmonic trap.
Let $\varepsilon = \beta E$, where E in the degrees of freedoms in this dimension.
PDF: $p(\varepsilon)\mathrm{d}\varepsilon = \mathrm{e}^{-\varepsilon}\mathrm{d}\varepsilon$
CDF (integrating PDF): $\mathbf{cdf}(\varepsilon) = 1 - \mathrm{e}^{-\varepsilon}$

2.2 The full experimental sequence

The atom starts in the trap with 1D trap depth E_{trap} and initial 1D energy E_0.
The 1D trap depth is then diabatically lowered to γE_{trap}, the 1D energy of the atom is now $\dfrac{1+\gamma}{2}E_0$.
The atom will stay in the trap in this dimension if

$$\frac{1+\gamma}{2}E_0 < \gamma E_{trap}$$

or

$$\varepsilon_0 < \frac{2\gamma}{1+\gamma}\varepsilon_{trap}$$

where ε_0 and ε_{trap} are the initial 1D energy of the atom and the initial 1D trap depth measured in unit of the initial temperature.
Therefore the total survival probability is,

$$p_0\mathbf{cdf}\left(\frac{2\gamma}{(1+\gamma)\tau}\right)^3$$

where p_0 models the overall survival probability and $\tau = \varepsilon_{trap}^{-1}$ is the temperature measured in 1D trap depth.

3 A more quantum model

The classical models above ignores the fact that the state after the lowering does not have to be either trapped or not but it can have non-zero overlap with both the trapped and untrapped state. We can get a slightly better model using the coherent states of the initial and the final trap and calculate the survival rate using the overlap between the states.

3.1 Density matrix

The density matrix of the initial state is

$$\rho =\left(1 - \mathrm{e}^{-\varepsilon}\right)\sum_n |n\rangle \mathrm{e}^{-n\varepsilon}\langle n|$$

where $\varepsilon \equiv \hbar\omega\beta$.
We need to express this in the basis of coherent states. Since the coherent states form an overcomplete basis, there are multiple ways to express this. We can assume that the expression takes the form,

$$\rho = \int |\alpha\rangle f(|\alpha|)\langle\alpha|\mathrm{d}^2\alpha$$

i.e. there's only diagonal term. Multiply this by the eigen vectors of H,

$$\begin{aligned}\langle m|\rho|n\rangle &= \int \langle m|\alpha\rangle f(|\alpha|)\langle\alpha|n\rangle \mathrm{d}^2\alpha \\ &= \int \mathrm{e}^{-|\alpha|^2} \frac{\alpha^m}{\sqrt{m!}} f(|\alpha|) \frac{\alpha^{*n}}{\sqrt{n!}} \mathrm{d}^2\alpha \\ &= \int \mathrm{e}^{-|\alpha|^2} \frac{|\alpha|^{m+n}\mathrm{e}^{\mathrm{i}(m-n)\theta}}{\sqrt{m!n!}} f(|\alpha|)|\alpha|\mathrm{d}\theta\mathrm{d}|\alpha| \\ &= \pi\delta^{mn} \int \mathrm{e}^{-|\alpha|^2} \frac{|\alpha|^{2n}}{n!} f(|\alpha|)\mathrm{d}|\alpha|^2\end{aligned}$$

Compare to the expression in energy basis

$$\langle m|\rho|n\rangle = \delta^{mn}\left(1-\mathrm{e}^{-\varepsilon}\right)\mathrm{e}^{-n\varepsilon}$$

$$\left(1-\mathrm{e}^{-\varepsilon}\right)\mathrm{e}^{-n\varepsilon} = \pi \int \mathrm{e}^{-|\alpha|^2} \frac{|\alpha|^{2n}}{n!} f(|\alpha|)\mathrm{d}|\alpha|^2$$

Take a guess that $f(|\alpha|) = c_1\mathrm{e}^{c_2|\alpha|^2}$ and do the integral, we can get

$$f(|\alpha|) = \frac{\mathrm{e}^{\varepsilon}-1}{\pi} \exp\left(-|\alpha|^2(\mathrm{e}^{\varepsilon}-1)\right)$$