Mølmer-Sørensen detuning and red-blue asymmetry error

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In the laser frame (rotating at the average frequency of the red and blue tones),

$$
\begin{aligned}
H(t) =& \frac{\Omega(t)}{2} \sum_{j=1,2} \sum_k \eta_{jk} \Big((1+\varepsilon_j) \Big(a_k \mathrm{e}^{-\mathrm{i}\theta_k(t)} \sigma_+^j + a_k^\dagger \mathrm{e}^{\mathrm{i}\theta_k(t)} \sigma_-^j \Big) \\
&+ (1-\varepsilon_j) \Big(a_k^\dagger \mathrm{e}^{\mathrm{i}\theta_k(t)} \sigma_+^j + a_k \mathrm{e}^{-\mathrm{i}\theta_k(t)} \sigma_-^j \Big) \Big) - \frac{\Delta_j}{2} \sigma_z^j \\
=& \frac{\Omega(t)}{2} \sum_{j=1,2} \sum_k \eta_{jk} \Big(\sigma_x^j \Big(a_k \mathrm{e}^{-\mathrm{i}\theta_k(t)} + a_k^\dagger \mathrm{e}^{\mathrm{i}\theta_k(t)} \Big) + \mathrm{i}\varepsilon_j \sigma_y^j \Big(a_k \mathrm{e}^{-\mathrm{i}\theta_k(t)} - a_k^\dagger \mathrm{e}^{\mathrm{i}\theta_k(t)} \Big) \Big) - \frac{\Delta_j}{2} \sigma_z^j
\end{aligned}
$$

First order in Magnus expansion,

$$
\begin{aligned}
M_1(\tau) =& \int_0^\tau H(t) \mathrm{d}t \\
=& \frac{1}{2} \sum_{j=1,2} \sum_k \eta_{jk} \Big(\sigma_x^j \Big(a_k \alpha_k(t) + a_k^\dagger \alpha_k^*(t) \Big) + \mathrm{i}\varepsilon_j \sigma_y^j \Big(a_k \alpha_k(t) - a_k^\dagger \alpha_k^*(t) \Big) \Big) - \frac{\Delta_j t}{2} \sigma_z^j
\end{aligned}
$$

where

$$
\alpha_k(t) \equiv \Omega(t) \mathrm{e}^{-\mathrm{i}\theta_k(t)}
$$

similar to the ideal case. Assuming closure of all α_k, the none trivial part of this term would also vanish other than the detuning.