Sideway circular polarization in tightly focused beam

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1 Goal

Trying to accurately understand the origin of the sideway circular polarization on the side of the focus of a tighly focused beam.

2 Qualitative description

For a collimated light beam, the polarization vector is generally within the plane perpendicular to the wave propagation direction (since light is a transverse wave). However, for a focused beam, the "wave propagation direction" isn't very well defined anymore, which allows polarization parallel to the optical axis to occur. This "axial" polarization component could even lead to circular polarization that rotates in a plane parallel to the optical axis, especially near the focus.

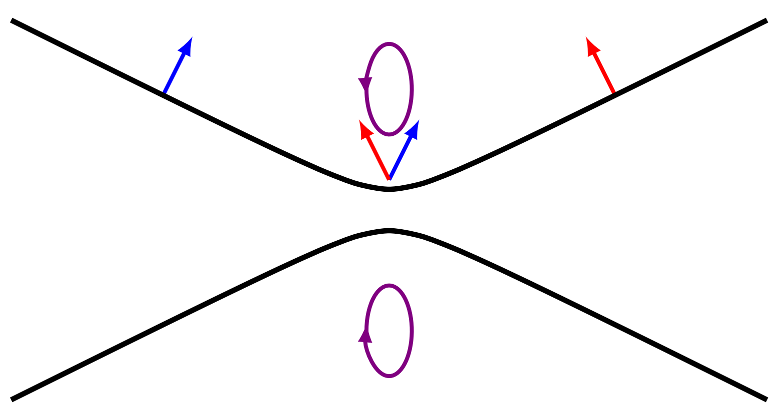

Figure 1: Sideway circular polarization near the focus of a tighly focused beam. The red and blue error shows the polarization vector on the same edge of a tightly focused beam before and after the focus.

We can qualitatively see this happening by looking at the field on the edge of the beam. The two edges have significantly different k vectors and therefore different polarization vectors as well. As shown in Fig. 1, the polarization on the two edges of the beam acquires an axial component due to the large angle between the k vector and the optical axis. While the two sides of the beam are generally far away from each other and their different polarization directions cause little problem, this is not the case anymore near the focus as the edge of the beam changes direction from converging to diverging and the polarization in that area (next to the focus in the focal plane) would have a polarization somewhere in between. This of course doesn't guarantee that there are any circular polarization or even axial polarization component, which would be the topic of the next section.

3 Semi-quantitative explanation

Full quantitative understanding of the sideway circular polarization near the focus of the beam requires a full calculation of the vector field. It is possible, however, to understand why such a polarization exists based on some continuity and symmetry considerations.

3.1 General idea

We can see from the far field (away from the focus) that the axial polarization does exist (Fig. 1). We can therefore break down the proof of the existance of the sideway circular polarization in two steps.

1. Show that the axial polarization also exist on the focal plane.
2. Prove that the axial polarization has to be out-of-phase with the sideway polarization.

These two would guarantee a sideway circular polarization.

3.2 Existance of axial polarization on the focal plane

Even though the three components of the electric field are related to one another, they do all satisfy the scalar wave equation on their own (Appendex A). This allows us to consider the propagation of each components independent of other ones.

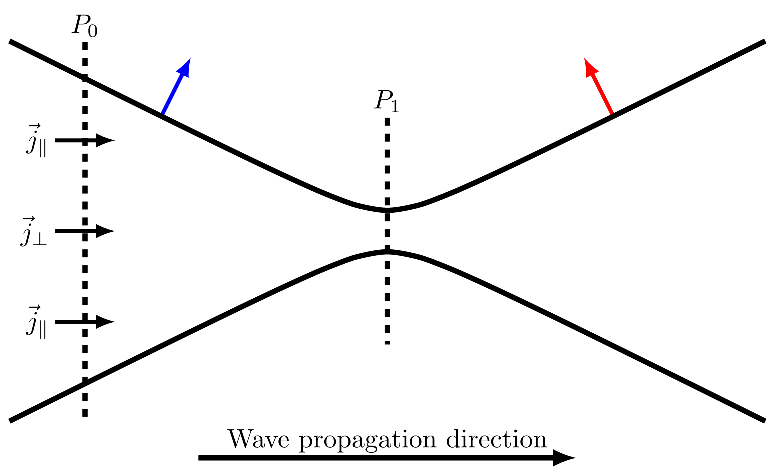

Figure 2: Scalar wave flux (Appendix B) for the transverse $\vec{j}_\perp$ and the longitudinal $\vec{j}_\parallel$ component of the field. In a plane far from the focus, P_0 we can verify that the flux for both components points to the right creating a net positive flux from left to right. For this reason, the net flux for both components should be non-null at all other parallel planes, including the focal plane P_1.

For a traveling wave from left to right (Fig. 2), we can consider the flux of this scalar wave (Appendix B) through a vertical plane. Since the flux is always in the direction of the beam propagation (i.e. phase gradient), the net flux for both components through a plane in the far field P_0 point to the right. This means that the flux for both fields, and therefore the value of both fields are non-null somewhere on the focal plane P_1. We can therefore reasonably expect that there are points on the focal plane where both fields are non-zero.

3.3 Phase between the transverse and longitudinal polarization on the focal plane

A Wave equation for electric field

From the Maxwell equation,

$$\begin{aligned}\nabla\times\left(\nabla\times\vec{E}\right)=&-\nabla\times\frac{\partial\vec{B}}{\partial t}\\=&-\frac{\partial\nabla\times\vec{B}}{\partial t}\\=&-\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}\end{aligned}\tag{1}$$

$$\begin{aligned}\nabla\times\left(\nabla\times\vec{E}\right)=&\nabla\left(\nabla\cdot\vec{E}\right)-\nabla^2\vec{E}\\=&-\nabla^2\vec{E}\end{aligned}\tag{2}$$

$$\nabla^2\vec{E}=\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}\tag{3}$$

Equation 3 contains no mixing between the different components of the electric field which shows that each compoments of the electric field all satisfy the scalar wave equation.

B Continuity equation for scalar wave

Consider a complex scalar wave $u(\vec{r},t)$, it satisfy the wave equation,

$$\nabla^2u=\frac{1}{c^2}\frac{\partial^2u}{\partial t^2}\tag{4}$$

If the wave is monotonic[1], we can write it out as $u(\vec{r},t)=u_0(\vec{r})\mathrm{e}^{\mathrm{i}\omega t}$, in which case the wave equation becomes,

$$\nabla^2u_0=k^2u_0\tag{5}$$

where the wave vector $k\equiv\frac{\omega}{c}$. We can now define the flux of the wave $\vec{j}\equiv u_0^*\nabla u_0-u_0\nabla u_0^*$, we have,

$$\begin{aligned}\nabla\cdot\vec{j}=&\nabla\cdot\left(u_0^*\nabla u_0-u_0\nabla u_0^*\right)\\=&\nabla u_0^*\cdot\nabla u_0-\nabla u_0\cdot\nabla u_0^*+u_0^*\nabla^2u_0-u_0\nabla^2u_0^*\\=&u_0^*k^2u_0-u_0k^2u_0^*\\=&0\end{aligned}\tag{6}$$

i.e. $\vec{j}$ is a continuous flow that cannot terminate in free space.
For a plane wave, this vector $\vec{j}$ points in the direction of the wave propagation. Generically, this points in the direction of the phase gradient (i.e. "local" propagation direction).

[1] We'll consider only a monotonic field to simplify the math but a time-dependenty continuity equation for a generic scalar wave also exists.