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# 1 Simplifies Optical Bloch Equation for Sideband Cooling Simulation.

Rabi frequency between state  $m$  and  $n$  (assume to be real since the phase is not important for sidband cooling.):  $\Omega_{mn}$

Pumping rate from state  $n$  to  $m$ :  $\Gamma_{mn}$

Diagonal terms,

$$\frac{d\rho_{mm}}{dt} = -\rho_{mm} \sum_k \Gamma_{km} + \sum_k \rho_{kk} \Gamma_{mk} + i \sum_k (\rho_{mk} \Omega_{km} - \Omega_{mk} \rho_{km})$$

Off-diagonal terms,

$$\frac{d\rho_{mn}}{dt} = -\frac{\rho_{mn}}{2} \sum_k (\Gamma_{km} + \Gamma_{kn}) + i \sum_k (\rho_{mk} \Omega_{kn} - \Omega_{mk} \rho_{kn})$$

When only one sideband is driven,

$$\Omega_{mn} = \Omega_m \delta_{m,n-\Delta} + \Omega_n \delta_{n,m-\Delta}$$

where  $\Delta$  include both the change in vibrational level and internal level.

Define  $p_n = \rho_{nn}$ , the equations becomes,

$$\begin{aligned} \frac{dp_m}{dt} &= \sum_k (p_k \Gamma_{mk} - p_m \Gamma_{km}) + i(\rho_{m,m-\Delta} \Omega_{m-\Delta} - \Omega_m \rho_{m+\Delta,m} + \rho_{m,m+\Delta} \Omega_m - \Omega_{m-\Delta} \rho_{m-\Delta,m}) \\ \frac{d\rho_{m,m+\Delta}}{dt} &= -\frac{\rho_{m,m+\Delta}}{2} \sum_k (\Gamma_{km} + \Gamma_{k,m+\Delta}) + i\Omega_m (p_m - p_{m+\Delta}) \end{aligned}$$

$\rho_{mn}$ 's with  $|m-n| \neq 0, \Delta$  are ignored since they are 0. In particular, since  $\Delta$  includes change of internal levels, elements with  $|m-n| \geq 2\Delta$  does not exist.

Define  $q_n = i\rho_{n,n+\Delta}$ ,

$$\begin{aligned} \rho_{n,n+\Delta} &= -iq_n \\ \rho_{n+\Delta,n} &= iq_n^* \end{aligned}$$

$$\begin{aligned} \frac{dp_m}{dt} &= \sum_k (p_k \Gamma_{mk} - p_m \Gamma_{km}) + i(iq_{m-\Delta}^* \Omega_{m-\Delta} - i\Omega_m q_m^* - iq_m \Omega_m + i\Omega_{m-\Delta} q_{m-\Delta}) \\ -i\frac{dq_m}{dt} &= i\frac{q_m}{2} \sum_k (\Gamma_{km} + \Gamma_{k,m+\Delta}) + i\Omega_m (p_m - p_{m+\Delta}) \\ \frac{dp_m}{dt} &= \sum_k (p_k \Gamma_{mk} - p_m \Gamma_{km}) + \Omega_m (q_m^* + q_m) - \Omega_{m-\Delta} (q_{m-\Delta}^* + q_{m-\Delta}) \\ \frac{dq_m}{dt} &= -\frac{q_m}{2} \sum_k (\Gamma_{km} + \Gamma_{k,m+\Delta}) + \Omega_m (p_{m+\Delta} - p_m) \end{aligned}$$

For a process starting with  $q_m = 0$ ,  $q_m$  will always remain real.

$$\begin{aligned} \frac{dp_m}{dt} &= \sum_k p_k \Gamma_{mk} - p_m \Gamma_m + 2\Omega_m q_m - 2\Omega_{m-\Delta} q_{m-\Delta} \\ \frac{dq_m}{dt} &= -\frac{q_m}{2} (\Gamma_m + \Gamma_{m+\Delta}) + \Omega_m (p_{m+\Delta} - p_m) \end{aligned}$$

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where  $\Gamma_m \equiv \sum_k \Gamma_{km}$  is the decay rate of state  $m$ .

After writing the two internal states ( $a$  and  $b$ ) explicitly and adding a third state ( $c$ ) to take into account  $m_F$  pumping,

$$\begin{aligned}\frac{dp_m^a}{dt} &= \sum_{\alpha=a,b,c;k} p_k^\alpha \Gamma_{mk}^{a\alpha} - p_m^a \Gamma_m^a + 2\Omega_m q_m \\ \frac{dp_m^b}{dt} &= \sum_{\alpha=a,b,c;k} p_k^\alpha \Gamma_{mk}^{b\alpha} - p_m^b \Gamma_m^b - 2\Omega_{m-\delta} q_{m-\delta} \\ \frac{dp_m^c}{dt} &= \sum_{\alpha=a,b,c;k} p_k^\alpha \Gamma_{mk}^{c\alpha} - p_m^c \Gamma_m^c \\ \frac{dq_m}{dt} &= -\frac{q_m}{2} (\Gamma_m^a + \Gamma_{m+\delta}^b) + \Omega_m (p_{m+\delta}^b - p_m^a)\end{aligned}$$

For sodium, states  $a$ ,  $b$  and  $c$  corresponds to  $|F=1, m_F=1\rangle$ ,  $|F=2, m_F=2\rangle$  and  $|F=2, m_F=1\rangle$ .

When the trapping frequency is much smaller than the line width,  $\Gamma_m^\alpha$  is independent with  $m$  and is proportional to the corresponding pumping power.  $\Gamma_{mn}^{\alpha\beta}$  can be written as,

$$\begin{aligned}\Gamma_{mn}^{\alpha\beta} &= \Gamma^{\alpha\beta} \gamma_{mn} \\ &= \begin{pmatrix} \Gamma^{aa} & \Gamma^{ab} & \Gamma^{ac} \\ \Gamma^{ba} & \Gamma^{bb} & \Gamma^{bc} \\ \Gamma^{ca} & \Gamma^{cb} & \Gamma^{cc} \end{pmatrix} \gamma_{mn} \\ &= \begin{pmatrix} \Gamma^a B^{aa} & \Gamma^b B^{ab} & \Gamma^c B^{ac} \\ \Gamma^a B^{ba} & \Gamma^b B^{bb} & \Gamma^c B^{bc} \\ \Gamma^a B^{ca} & \Gamma^b B^{cb} & \Gamma^c B^{cc} \end{pmatrix} \gamma_{mn} \\ &= \begin{pmatrix} \Gamma_1 B^{aa} & \varepsilon \Gamma_2 B^{ab} & \Gamma_2 B^{ac} \\ \Gamma_1 B^{ba} & \varepsilon \Gamma_2 B^{bb} & \Gamma_2 B^{bc} \\ \Gamma_1 B^{ca} & \varepsilon \Gamma_2 B^{cb} & \Gamma_2 B^{cc} \end{pmatrix} \gamma_{mn}\end{aligned}$$

where  $B^{\alpha\beta}$  is the branching ratio,  $\Gamma_1$  is the  $F$  pumping power ( $(1,1) \rightarrow (2,2)'$  light),  $\Gamma_2$  is the  $m_F$  pumping power ( $(2,1) \rightarrow (2,2)'$  light) and  $\varepsilon$  is the off-resonance coupling of the  $m_F$  pumping light to the  $(2,2) \rightarrow (2,3)'$  transition.