Simulation of systems with more than one drive frequencies

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1 Goal

The trick of moving to a rotating frame is only exact when the system experiences drives of a single frequency. In experiments, there are usually more than one drive frequencies and I'd like to see how exactly those can be directly calculated and whether the approximation we use (assuming the shift/effective Hamiltonian from the different drives adds linearly) makes sense or when do they break down.

2 Two-level system with two drive frequencies

We'll go directly into the rotation from one of the drives,

$$H = \frac{1}{2}\begin{pmatrix} \Delta & \Omega_1 + \Omega_2 \mathrm{e}^{\mathrm{i}\delta t} \\ \Omega_1 + \Omega_2 \mathrm{e}^{-\mathrm{i}\delta t} & -\Delta \end{pmatrix}$$

We can diagnolize the time independent part of the Hamiltonian. The unitary transformation that diagonalize the time independent part is,

$$U_1 = \begin{pmatrix} -\sqrt{\frac{1}{2} - \frac{\Delta}{2\sqrt{\Delta^2 + \Omega_1^2}}} & \sqrt{\frac{1}{2} + \frac{\Delta}{2\sqrt{\Delta^2 + \Omega_1^2}}} \\ \sqrt{\frac{1}{2} + \frac{\Delta}{2\sqrt{\Delta^2 + \Omega_1^2}}} & \sqrt{\frac{1}{2} - \frac{\Delta}{2\sqrt{\Delta^2 + \Omega_1^2}}} \end{pmatrix}$$

$$\begin{aligned} H_1 =& UHU^\dagger \\ =& \begin{pmatrix} -\frac{\sqrt{\Delta^2 + \Omega_1^2}}{2} - \frac{\Omega_1 \Omega_2 \cos \delta t}{2\sqrt{\Delta^2 + \Omega_1^2}} & \frac{\Delta \Omega_2 \cos \delta t}{2\sqrt{\Delta^2 + \Omega_1^2}} - \mathrm{i}\frac{\Omega_2 \sin \delta t}{2} \\ \frac{\Delta \Omega_2 \cos \delta t}{2\sqrt{\Delta^2 + \Omega_1^2}} + \mathrm{i}\frac{\Omega_2 \sin \delta t}{2} & \frac{\sqrt{\Delta^2 + \Omega_1^2}}{2} + \frac{\Omega_1 \Omega_2 \cos \delta t}{2\sqrt{\Delta^2 + \Omega_1^2}} \end{pmatrix} \end{aligned}$$