Measuring the scrambling of quantum information

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Jan. 20, 2021

- Scrambling of quantum information
- Out-of-time-order (OTO) correlator
- Measurement of OTO correlator
- Experimental realization with cavity QED system

Relaxation vs Scrambling

Relaxation

- Decay/leaking of information from a single qubit.
- Fast
  Time scale: τ

Scrambling

- Spreading of information to the whole system.
- Slow
  Time scale: $t_* = \tau \ln S$

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$$F(t) \equiv \langle W_t^\dagger V^\dagger W_t V \rangle$$

$$W_t = U(-t) W U(t)$$
$$U(t) = \mathrm{e}^{-\mathrm{i}Ht}$$

- Interpretation: $F = \langle \psi_1 | \psi_2 \rangle$

  $|\psi_1\rangle = V W_t |\psi_0\rangle$
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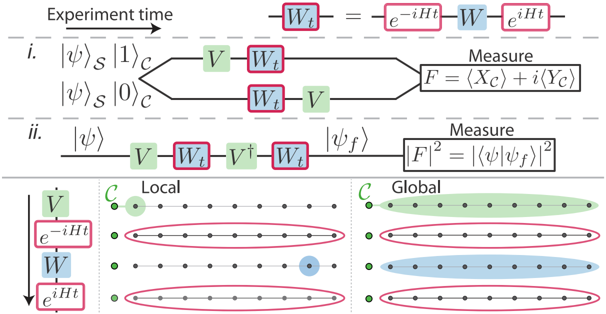

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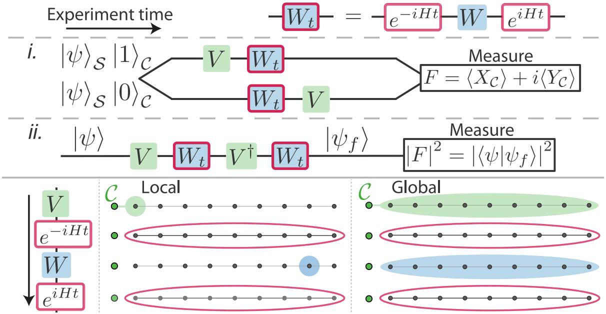

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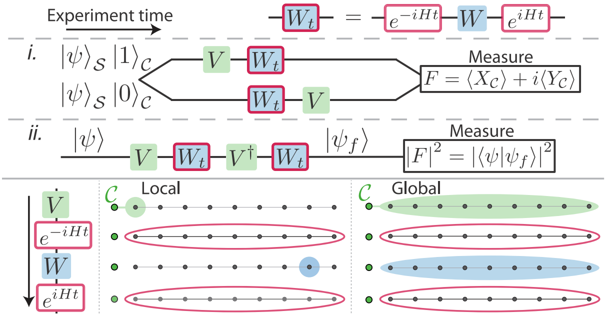
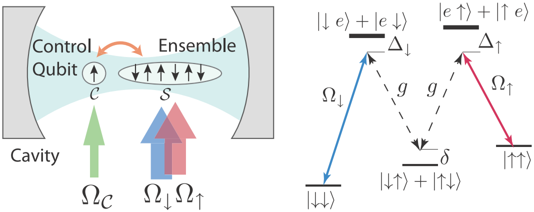


$$\text{Time reversal: } U(-t)$$

$$\text{Controlled-}V\text{: } I_S \otimes |0\rangle\langle 0|_C + V_S \otimes |1\rangle\langle 1|_C$$

Cavity QED implementation



$$H = \sum_{ij} J_{ij} s_i^x s_j^x + h.c.$$

$$J_{ij} = \sum_{\alpha} \frac{\Omega_{\uparrow}^*(r_i)\Omega_{\downarrow}(r_j)}{\Delta_{\uparrow}\Delta_{\downarrow}} \frac{g_{\alpha}(r_i)g_{\alpha}^*(r_j)}{\delta}$$

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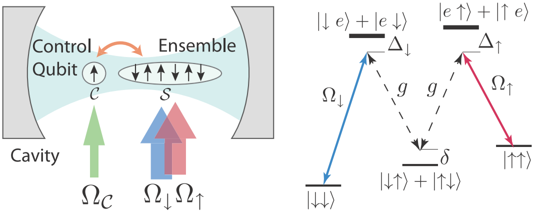
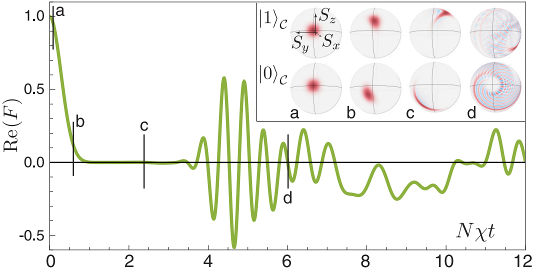


$$Z_\phi^C = I_S \otimes |0\rangle\langle 0|_C + \mathrm{e}^{-\mathrm{i}\phi S_z^S} \otimes |1\rangle\langle 1|_C$$

$$V = W = \mathrm{e}^{-\mathrm{i}\phi S_z^S}$$

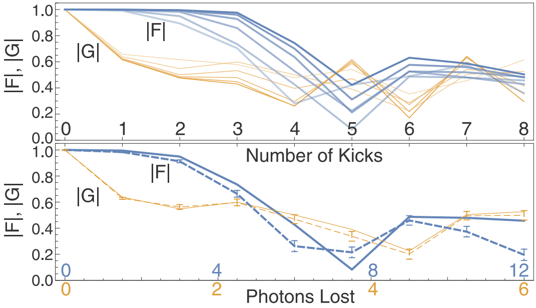
S_x^2 dynamics

$$H = \sum_{ij} J_{ij} s_i^x s_j^x + h.c. \;\rightarrow\; H = J S_x^2$$



$S_x^2 + S_z$ dynamics

$$F(t) \equiv \langle W_t^\dagger V^\dagger W_t V \rangle \quad G(t) \equiv \langle V_t^\dagger V \rangle$$



Questions

- The requirement on V and W.
  They should not commute with H (or F would be a constant 1).
  They can apparently either be local (operate on a single particle) or global (operate on more than one).
  The paper gives an argument for why it "works" for local operator but not for global operators.
  If V and W are identical local operators, it seems to me that it should reflet more about the relaxation of that particle rather than scrambling.
- How exactly is F measuring the scrambling.
- Kick-top Hamiltonian:
  How is the $\mathrm{e}^{-\mathrm{i}S_z}$ term reversed when doing time propagation.
- Why is time ordered correlator (G) relaxation.