December 2, 2024

1 Pulse sequence with three rotations

1. $\sigma_1 \equiv \sigma_x \cos\theta_1 + \sigma_y \sin\theta_1$ by angle $2\psi_1$
2. $\sigma_2 \equiv \sigma_x$ by angle $2\psi_2$
3. $\sigma_3 \equiv \sigma_x \cos\theta_3 + \sigma_y \sin\theta_3$ by angle $2\psi_3$

Full rotation.

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\begin{aligned}
U =& \exp(\mathrm{i}\psi_1\sigma_1)\exp(\mathrm{i}\psi_2\sigma_2)\exp(\mathrm{i}\psi_3\sigma_3)\\
=&(\cos\psi_1 + \mathrm{i}\sin\psi_1\sigma_1)(\cos\psi_2 + \mathrm{i}\sin\psi_2\sigma_2)(\cos\psi_3 + \mathrm{i}\sin\psi_3\sigma_3)\\
=&(\cos\psi_1 + \mathrm{i}\sin\psi_1\cos\theta_1\sigma_x + \mathrm{i}\sin\psi_1\sin\theta_1\sigma_y)(\cos\psi_2 + \mathrm{i}\sin\psi_2\sigma_x)\\
&(\cos\psi_3 + \mathrm{i}\sin\psi_3\cos\theta_3\sigma_x + \mathrm{i}\sin\psi_3\sin\theta_3\sigma_y)\\
=&((\cos\psi_1\cos\psi_2 - \sin\psi_1\cos\theta_1\sin\psi_2) + \mathrm{i}(\cos\psi_1\sin\psi_2 + \sin\psi_1\cos\theta_1\cos\psi_2)\sigma_x\\
&+\mathrm{i}\sin\psi_1\sin\theta_1\cos\psi_2\sigma_y + \mathrm{i}\sin\psi_1\sin\theta_1\sin\psi_2\sigma_z)\\
&(\cos\psi_3 + \mathrm{i}\sin\psi_3\cos\theta_3\sigma_x + \mathrm{i}\sin\psi_3\sin\theta_3\sigma_y)\\
=&(\cos\psi_1\cos\psi_2 - \sin\psi_1\cos\theta_1\sin\psi_2)\cos\psi_3 - (\cos\psi_1\sin\psi_2 + \sin\psi_1\cos\theta_1\cos\psi_2)\sin\psi_3\cos\theta_3\\
&-\sin\psi_1\sin\theta_1\cos\psi_2\sin\psi_3\sin\theta_3\\
&+\mathrm{i}(\cos\psi_1\cos\psi_2 - \sin\psi_1\cos\theta_1\sin\psi_2)\sin\psi_3\cos\theta_3\sigma_x\\
&+\mathrm{i}(\cos\psi_1\sin\psi_2 + \sin\psi_1\cos\theta_1\cos\psi_2)\cos\psi_3\sigma_x\\
&+\mathrm{i}\sin\psi_1\sin\theta_1\sin\psi_2\sin\psi_3\sin\theta_3\sigma_x\\
&+\mathrm{i}(\cos\psi_1\cos\psi_2 - \sin\psi_1\cos\theta_1\sin\psi_2)\sin\psi_3\sin\theta_3\sigma_y\\
&+\mathrm{i}\sin\psi_1\sin\theta_1\cos\psi_2\cos\psi_3\sigma_y\\
&-\mathrm{i}\sin\psi_1\sin\theta_1\sin\psi_2\sin\psi_3\cos\theta_3\sigma_y\\
&-\mathrm{i}(\cos\psi_1\sin\psi_2 + \sin\psi_1\cos\theta_1\cos\psi_2)\sin\psi_3\sin\theta_3\sigma_z\\
&+\mathrm{i}\sin\psi_1\sin\theta_1\cos\psi_2\sin\psi_3\cos\theta_3\sigma_z\\
&+\mathrm{i}\sin\psi_1\sin\theta_1\sin\psi_2\cos\psi_3\sigma_z
\end{aligned}
$$

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\begin{aligned}
&= \cos\psi_1 \cos\psi_2 \cos\psi_3 \\
&\quad - \sin\psi_1 \sin\psi_2 \cos\psi_3 \cos\theta_1 \\
&\quad - \cos\psi_1 \sin\psi_2 \sin\psi_3 \cos\theta_3 \\
&\quad - \sin\psi_1 \cos\psi_2 \sin\psi_3 \cos(\theta_1 - \theta_3) \\
&\quad + \mathrm{i}(\cos\psi_1 \cos\psi_2 - \sin\psi_1 \cos\theta_1 \sin\psi_2) \sin\psi_3 \cos\theta_3 \sigma_x \\
&\quad + \mathrm{i}(\cos\psi_1 \sin\psi_2 + \sin\psi_1 \cos\theta_1 \cos\psi_2) \cos\psi_3 \sigma_x \\
&\quad + \mathrm{i} \sin\psi_1 \sin\theta_1 \sin\psi_2 \sin\psi_3 \sin\theta_3 \sigma_x \\
&\quad + \mathrm{i}(\cos\psi_1 \cos\psi_2 - \sin\psi_1 \cos\theta_1 \sin\psi_2) \sin\psi_3 \sin\theta_3 \sigma_y \\
&\quad + \mathrm{i} \sin\psi_1 \sin\theta_1 \cos\psi_2 \cos\psi_3 \sigma_y \\
&\quad - \mathrm{i} \sin\psi_1 \sin\theta_1 \sin\psi_2 \sin\psi_3 \cos\theta_3 \sigma_y \\
&\quad - \mathrm{i} \cos\psi_1 \sin\psi_2 \sin\psi_3 \sin\theta_3 \sigma_z \\
&\quad + \mathrm{i} \sin\psi_1 \sin\psi_2 \cos\psi_3 \sin\theta_1 \sigma_z \\
&\quad + \mathrm{i} \sin\psi_1 \cos\psi_2 \sin\psi_3 \sin(\theta_1 - \theta_3) \sigma_z
\end{aligned}
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