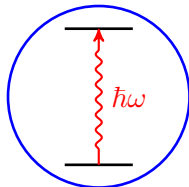


Lamb-Dicke regime/approximation

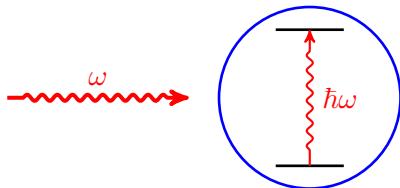
Yichao Yu

Journal Club

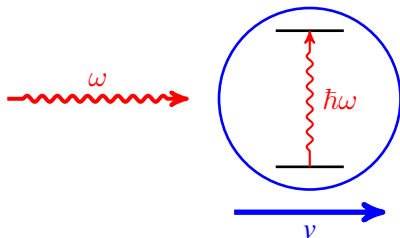
Doppler effect



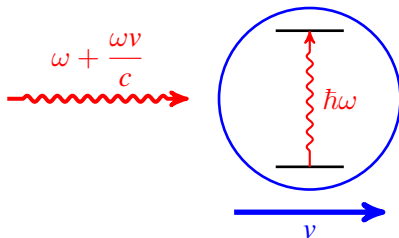
Doppler effect



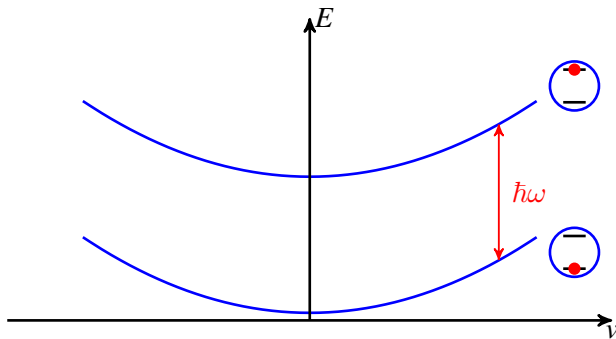
Doppler effect



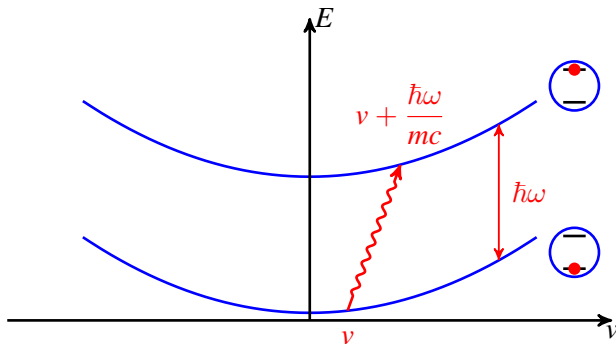
Doppler effect



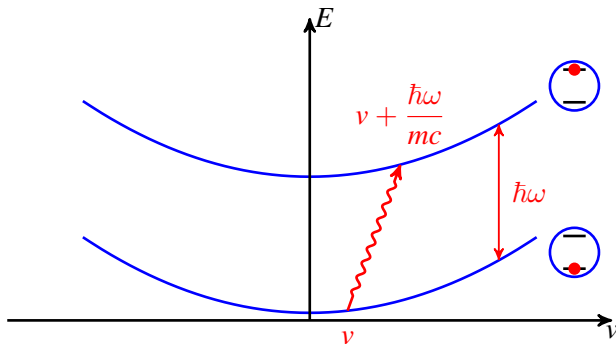
Doppler effect



Doppler effect

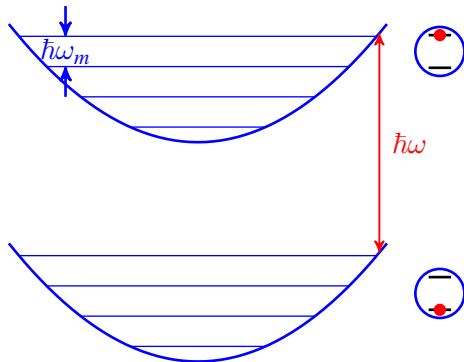


Doppler effect

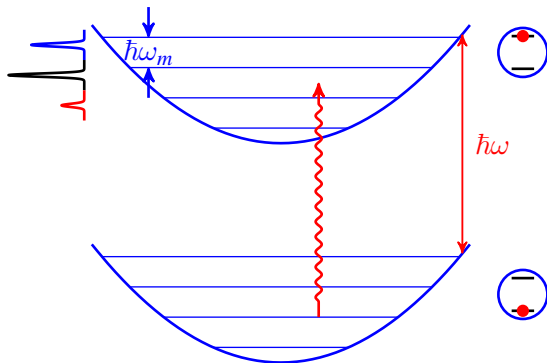


$$\omega + \frac{m}{2\hbar} \left(v + \frac{\hbar\omega}{mc} \right)^2 - \frac{mv^2}{2\hbar} = \omega + \frac{\omega v}{c} + \frac{\hbar\omega^2}{2mc^2}$$

Sideband

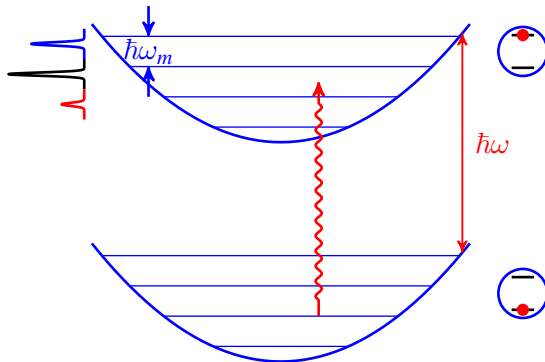


Sideband



Frequency: $\omega + n\omega_m$

Sideband



Frequency: $\omega + n\omega_m$

Strength: $\langle n | e^{ik\hat{x}} | n + \Delta n \rangle$

Lamb-Dicke parameter

$$\langle n | e^{ik\hat{x}} | n + \Delta n \rangle$$

Lamb-Dicke parameter

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$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}} (a + a^\dagger) = z_0 (a + a^\dagger)$$

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$$k\hat{x} = \eta (a + a^\dagger)$$

$$\eta \equiv kz_0 = k\sqrt{\frac{\hbar}{2m\omega}}$$

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$$\eta \equiv kz_0 = k\sqrt{\frac{\hbar}{2m\omega}}$$

$$\eta = \frac{2\pi z_0}{\lambda}$$

$$\eta = \sqrt{\frac{\omega_R}{\omega_m}}$$

Sideband strength

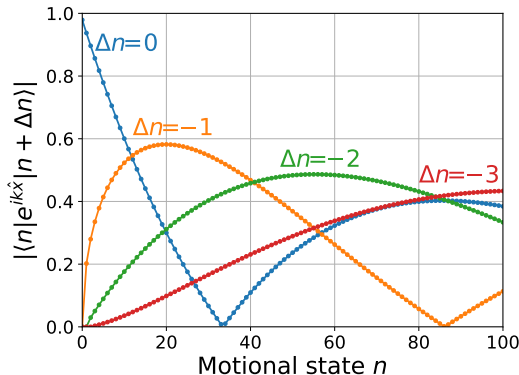
$$\begin{aligned} & \langle n | e^{ik\hat{x}} | n + \Delta n \rangle \\ &= e^{-\eta^2/2} \eta^{\Delta n} \sqrt{\frac{n_-!}{n_+!}} L_{n_-}^{\Delta n}(\eta^2) \end{aligned}$$

$$n_- \equiv \min(n, n + \Delta n), \quad n_+ \equiv \max(n, n + \Delta n)$$

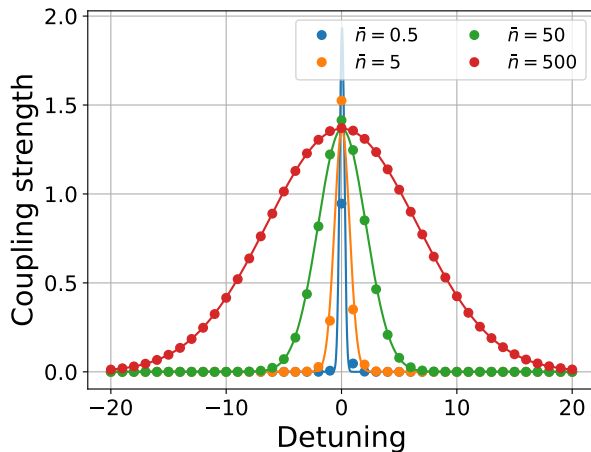
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Sideband spectrum



Lamb-Dicke approximation/regime

$$e^{i\eta(a+a^\dagger)} = 1 + i\eta(a + a^\dagger) - \frac{\eta^2}{2}(a + a^\dagger)^2 + \mathcal{O}(\eta^3)$$

Lamb-Dicke approximation/regime

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To the first order,

$$\langle n | e^{i\eta(a+a^\dagger)} | n \rangle \approx 1$$

$$\langle n+1 | e^{i\eta(a+a^\dagger)} | n \rangle \approx i\eta\sqrt{n}$$

$$\langle n-1 | e^{i\eta(a+a^\dagger)} | n \rangle \approx i\eta\sqrt{n+1}$$

Lamb-Dicke approximation/regime

$$e^{i\eta(a+a^\dagger)} = 1 + i\eta(a + a^\dagger) - \frac{\eta^2}{2}(a + a^\dagger)^2 + \mathcal{O}(\eta^3)$$

$$\text{When } \eta \ll 1: e^{i\eta(a+a^\dagger)} \approx 1 + i\eta(a + a^\dagger)$$

To the second order,

$$\langle n | e^{i\eta(a+a^\dagger)} | n \rangle \approx 1 - \frac{\eta^2(2n+1)}{2}$$

Lamb-Dicke approximation/regime

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Wavefunction spread,

$$\langle n | x^2 | n \rangle = z_0^2(2n+1)$$

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$$\eta_{eff} \equiv \eta\sqrt{2n+1}$$