Why do capacitors usually only have a small value

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In Lab 1 of PS3, there is a question about why capacitors usually has a small value. AFAIK, the student is supposed to relate this to the smallness of the electron charge in SI unit which I find to be problematic. This is a note that list all reasons I think such a question is problematic and some better answers that I can come up with.

1 Why such a explanation is problematic

To start, super-capacitors exist, and may have a capacitance of hundreds of F within a small volume. This alone suggests that this question is more about material properties that are not within the scope of this class.

Now about the expected answer. Electron charge is about $1.602 \times 10^{-19} C$, if we were to use this as an explanation for why C is such a large unit (which in turn means that $F = C/V$ is a large unit) we would expect capacitors to have a capacitance value on the order of 10^{-19}, which is wrong for at least two reasons,

1. They are not.
   In fact, due to parasitic capacitance, it is very hard to get stand alone capacitance smaller than few pF.
2. $1e$ is much smaller than the amount of charge we expect a normal capacitor to store.
   And I don't believe there is an obvious way for the students to do a correct estimation of how much charge the conductor in a capacitor can hold. (One is given below).

Therefore, while electron charge being small indeed put some limit on how much charge a normal capacitor can store. It is non-trivial to actual do so and certainly not for students in this class. This is not even mentioning (see below) that this limit isn't the limiting factors in capacitor manufacturing.

It is good to give students an idea of the magnitude of different quantities, electron charge, capacitor values, safe voltages, etc. However, it is better to do so with more direct arguments. Even if the argument related to electron charge is what limits the value of conventional capacitors, it's not, it shouldn't be used as an example if the student cannot fully understand the argument, and they can't. Doing so would encourage them to skip the logic and make incorrect connections between quantities, which IMHO is a bad habit that is very hard to correct later on.

2 The actual reason

2.1 TL;DR

It's complicated. Since units are arbitrarily defined, this question is essentially asking why the unit systems are designed in the way they are. Additionally, since the unit system was more-or-less defined based on the quantity that makes sense on human scale, we necessarily need to consider what is the correct human scale historically chosen. Below I present a few of the limiting factors based on the best I know of the unit system and material science. Note that I do not believe any of the following explanations are understandable for students taking PS3 (at this stage, or ever) and they are given mainly to show how complicated the issue is. It should hopefully also be an interesting read.

2.2 Limit from breakdown

In principle, this does not really limit the capacitor size since one can in principle always make a larger capacitor with lower breakdown voltage until running into manufacture limit, something I'll definitely **NOT** get into. Nevertheless, these are included here since the breakdown is the only thing we could remotely relate the electron charge value to.

2.2.1 The amount of charge one can store in a conductor

This is the closest limitation I (and Nick) could come up with that is related to electron charge.
The smallness of the electron charge means that a lot of electrons needs to be added or removed from a conductor to hold a large amount of charge. Since all the net charges are stored on the surface of the conductor in equilibrium, what we need is basically the maximum 2D charge density on a normal conductor. A very simple model that should give the right order of magnitude is to assume the metal can afford a single layer of electron or positive ion to stick to a neutral surface. Based on the density and atomic mass of copper, the density of copper atoms is roughly

$$\begin{aligned} n_{Cu} =& \rho_{Cu}/m_{Cu} \\ =& 8.96\mathrm{g}\cdot\mathrm{cm}^{-3}/63.546\mathrm{g}\cdot 6.02\cdot 10^{23} \\ =& 8.49\cdot 10^{22}\mathrm{cm}^{-3} \end{aligned}$$

Which corresponds to a surface charge density of

$$\begin{aligned} \sigma_{Cu} =& n_{Cu}^{2/3} \\ =& 1.93\cdot 10^{15}\mathrm{cm}^{-2} \end{aligned}$$

With our assumption of a single charge layer, this corresponds to a surface electric field of

$$\begin{aligned} E =& \frac{e\sigma_{Cu}}{\varepsilon_0} \\ =& 3.49\cdot 10^{11}\mathrm{V}\cdot\mathrm{m}^{-1} \end{aligned}$$

Any field higher than this will likely lead to field electron or ion emission. The number agrees in orders of magnitude with the typical field found in field ion or electron microscope which is "a few V/Å". (Note that the in field ion microscope the background gas is being ionized and not the material surface so the material can likely afford a stronger field)
(Note that the argument here does not fully apply to electrolytic capacitor since the charge is partially stored as ions)
This limits the energy density (since its proportional to field square) of a capacitor but it's very hard to convert this to a capacitance since it's strongly manufacture process dependent. However, it is possible to compare this to the other effect that limits the energy density, dielectric strength.

2.2.2 Dielectric strength

This is the max field that can be applied on a material before breakdown occurs. It depends strongly on the material so I do not have a good way to estimate the value. However, from a table on Wikipedia, the typical values are between 10^6 and $10^9\mathrm{V}\cdot\mathrm{m}^{-1}$, much smaller than the ion/electron emission threshold of conductors so this should be the limiting factor of the energy density in most (all) cases.

2.3 Actual reason, for real

As previously mentioned, the reason one typically get a small capacitor value in SI unit is, well, the SI unit itself. Since the size of a plate capacitor with $\varepsilon_r \approx 1$ is given by $\dfrac{\varepsilon_0 S}{d}$, this is really about why ε_0 has a tiny value in SI unit. As Nick pointed out, in order to understand this, we need to realize that the charge unit in SI is defined based on Ampere, which is defined based on force between current. Since $\dfrac{1}{\varepsilon_0}$ qualify the electric static force between a unit amount of charge, the reason capacitors are small is really because for the current that generate the “standard” (defined later) amount of force between two wires, the force between the charge delivered by that current in 1 second is large in SI unit.

We can directly calculate the ratio of the two forces with a time scale of $t = 1\mathrm{s}$ and length scale of $l = 1\mathrm{m}$

$$\begin{aligned}
F_{mag} =& \frac{\mu_0 I^2}{2\pi L} L \\
=& \frac{\mu_0 I^2}{2\pi} \\
F_{ele} =& \frac{Q^2}{4\pi\varepsilon_0 L^2} \\
=& \frac{I^2 t^2}{4\pi\varepsilon_0 L^2} \\
\frac{F_{ele}}{F_{mag}} =& \frac{I^2 t^2}{4\pi\varepsilon_0 L^2} \frac{2\pi}{\mu_0 I^2} \\
=& \frac{c^2 t^2}{2L^2} \\
=& \frac{c^2}{v^2}
\end{aligned}$$

where $v \equiv \sqrt{2}\dfrac{L}{t}$ is a human velocity scale. Here we clearly see that the electronic static interaction is much larger than magnetic interaction **for unit charge and current in SI unit** because the SI unit is defined with human length and time scale which gives a velocity scale much smaller than the speed of light. In fact, the “standard” force between two current carrying wires is already much smaller than 1N but it is not enough to make the corresponding electric static interaction small enough.

Note that the discussion above was based on the definition of Ampere. However, given that the standard force generated between wires is much smaller than 1N, it is likely that the rough magnitude of the Ampere unit was historically selected for other reasons, likely related to how current is generated back then. I could not find a good note about relation between early generator and current unit definition and since this strongly depends on the winding numbers and early permanent magnet I would stop speculating how it is selected.