Asymmetry in population distribution with respect to detuning caused by EIT/coherent scattering

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1 Introduction

While simulating a simple three-level system with a Raman transition coupling two ground states and an excited state with a finite lifetime, I noticed that there's a difference in the dynamic and the final/steady-state population distribution when the two-photon detuning changes sign.

This effect cannot be reproduced in a two-level system if we assume the two ground states scatters independently, even if we include the difference in the scattering rate of the two states caused by the slight difference in the single photon detuning when a non-zero two photon detuning. It could be reproduced, OTOH, if the initial state of the scattering is assumed to be a superposition of the two ground states. This difference is of course very important since it's also where EIT comes from.

While the result of the simulation is pretty clear and with a 2-by-2 density matrix it shouldn't be that difficult to write out the full master equation and solve it directly, I do want to understand the origin of this asymmetry better and here are some of the approaches I can think of to understand this phenomenon.

2 System description

Hamiltonian,

$$H = \frac{1}{2}(\delta\sigma_z + \Omega\sigma_x) \tag{1}$$

For scattering, we'll assume that the state $|\psi_s\rangle \equiv \frac{|0\rangle + |1\rangle}{\sqrt{2}}$ scatters at a rate of γ with a 50/50 branching ratio back to the $|0\rangle$ and $|1\rangle$ states.

3 Steady state solution in the eigen basis of the Hamiltonian

The eigen states of the Hamiltonian is,

$$|\psi_1\rangle = \begin{pmatrix} -\sqrt{\frac{1}{2} - \frac{\delta}{2\sqrt{\delta^2+\Omega^2}}} \\ \sqrt{\frac{1}{2} + \frac{\delta}{2\sqrt{\delta^2+\Omega^2}}} \end{pmatrix} \tag{2}$$

$$|\psi_2\rangle = \begin{pmatrix} \sqrt{\frac{1}{2} + \frac{\delta}{2\sqrt{\delta^2+\Omega^2}}} \\ \sqrt{\frac{1}{2} - \frac{\delta}{2\sqrt{\delta^2+\Omega^2}}} \end{pmatrix} \tag{3}$$

Since there is no coupling between these two states and there's no coherence in the scattering final state (because of the 50/50 branching ratio) we can complete ignore any coherence between these two states and simply treat this using a scattering rate equation. The overlap between these two states and the scattering state $|\psi_s\rangle$ is,

$$\langle\psi_1|\psi_s\rangle = \frac{1}{2}\left(\sqrt{1+\frac{\delta}{\sqrt{\delta^2+\Omega^2}}} - \sqrt{1-\frac{\delta}{\sqrt{\delta^2+\Omega^2}}}\right) \tag{4}$$

$$\langle\psi_2|\psi_s\rangle = \frac{1}{2}\left(\sqrt{1+\frac{\delta}{\sqrt{\delta^2+\Omega^2}}} + \sqrt{1-\frac{\delta}{\sqrt{\delta^2+\Omega^2}}}\right) \tag{5}$$

Scattering rates,

$$\begin{aligned} \gamma_1 =& \gamma|\langle\psi_1|\psi_s\rangle|^2 \\ =& \frac{\gamma}{4}\left(\sqrt{1+\frac{\delta}{\sqrt{\delta^2+\Omega^2}}} - \sqrt{1-\frac{\delta}{\sqrt{\delta^2+\Omega^2}}}\right)^2 \\ =& \frac{\gamma}{4}\left(1+\frac{\delta}{\sqrt{\delta^2+\Omega^2}} + 1 - \frac{\delta}{\sqrt{\delta^2+\Omega^2}} - 2\sqrt{1+\frac{\delta}{\sqrt{\delta^2+\Omega^2}}}\sqrt{1-\frac{\delta}{\sqrt{\delta^2+\Omega^2}}}\right) \\ =& \frac{\gamma}{2}\left(1-\frac{\Omega}{\sqrt{\delta^2+\Omega^2}}\right) \end{aligned} \tag{6}$$

$$\begin{aligned} \gamma_2 =& \gamma|\langle\psi_2|\psi_s\rangle|^2 \\ =& \frac{\gamma}{2}\left(1+\frac{\Omega}{\sqrt{\delta^2+\Omega^2}}\right) \end{aligned} \tag{7}$$

Since the branching ratio for the scattering is still 50/50 in this basis, the population ratio at steady state is the inverse of the scattering rate ratio of the two states.

$$\begin{aligned} \frac{p_{\psi_1}}{p_{\psi_2}} =& \frac{\gamma_2}{\gamma_1} \\ =& \frac{\sqrt{\delta^2+\Omega^2}+\Omega}{\sqrt{\delta^2+\Omega^2}-\Omega} \end{aligned} \tag{8}$$

$$p_{\psi_1} = \frac{\sqrt{\delta^2+\Omega^2}+\Omega}{2\sqrt{\delta^2+\Omega^2}} \tag{9}$$

$$p_{\psi_2} = \frac{\sqrt{\delta^2+\Omega^2}-\Omega}{2\sqrt{\delta^2+\Omega^2}} \tag{10}$$

$$\tag{11}$$

The density matrix

$$
\begin{aligned}
\rho =&|\psi_1\rangle p_{\psi_1}\langle\psi_1| + |\psi_2\rangle p_{\psi_2}\langle\psi_2| \\
=&\begin{pmatrix} -\sqrt{\frac{1}{2}-\frac{\delta}{2\sqrt{\delta^2+\Omega^2}}} & \sqrt{\frac{1}{2}+\frac{\delta}{2\sqrt{\delta^2+\Omega^2}}} \end{pmatrix} \frac{\sqrt{\delta^2+\Omega^2}+\Omega}{2\sqrt{\delta^2+\Omega^2}} \begin{pmatrix} -\sqrt{\frac{1}{2}-\frac{\delta}{2\sqrt{\delta^2+\Omega^2}}} \\ \sqrt{\frac{1}{2}+\frac{\delta}{2\sqrt{\delta^2+\Omega^2}}} \end{pmatrix} + \\
&\begin{pmatrix} \sqrt{\frac{1}{2}+\frac{\delta}{2\sqrt{\delta^2+\Omega^2}}} & \sqrt{\frac{1}{2}-\frac{\delta}{2\sqrt{\delta^2+\Omega^2}}} \end{pmatrix} \frac{\sqrt{\delta^2+\Omega^2}-\Omega}{2\sqrt{\delta^2+\Omega^2}} \begin{pmatrix} \sqrt{\frac{1}{2}+\frac{\delta}{2\sqrt{\delta^2+\Omega^2}}} \\ \sqrt{\frac{1}{2}-\frac{\delta}{2\sqrt{\delta^2+\Omega^2}}} \end{pmatrix} \\
=&\begin{pmatrix} \sqrt{\delta^2+\Omega^2}-\delta & -\Omega \\ -\Omega & \sqrt{\delta^2+\Omega^2}+\delta \end{pmatrix} \frac{\sqrt{\delta^2+\Omega^2}+\Omega}{4(\delta^2+\Omega^2)} + \\
&\begin{pmatrix} \sqrt{\delta^2+\Omega^2}+\delta & \Omega \\ \Omega & \sqrt{\delta^2+\Omega^2}-\delta \end{pmatrix} \frac{\sqrt{\delta^2+\Omega^2}-\Omega}{4(\delta^2+\Omega^2)} \\
=&\begin{pmatrix} \left(\sqrt{\delta^2+\Omega^2}-\delta\right)\left(\sqrt{\delta^2+\Omega^2}+\Omega\right) & -\Omega\left(\sqrt{\delta^2+\Omega^2}+\Omega\right) \\ -\Omega\left(\sqrt{\delta^2+\Omega^2}+\Omega\right) & \left(\sqrt{\delta^2+\Omega^2}+\delta\right)\left(\sqrt{\delta^2+\Omega^2}+\Omega\right) \end{pmatrix} \frac{1}{4(\delta^2+\Omega^2)} + \\
&\begin{pmatrix} \left(\sqrt{\delta^2+\Omega^2}+\delta\right)\left(\sqrt{\delta^2+\Omega^2}-\Omega\right) & \Omega\left(\sqrt{\delta^2+\Omega^2}-\Omega\right) \\ \Omega\left(\sqrt{\delta^2+\Omega^2}-\Omega\right) & \left(\sqrt{\delta^2+\Omega^2}-\delta\right)\left(\sqrt{\delta^2+\Omega^2}-\Omega\right) \end{pmatrix} \frac{1}{4(\delta^2+\Omega^2)} \\
=&\begin{pmatrix} \delta^2+\Omega^2-\delta\Omega & -\Omega^2 \\ -\Omega^2 & \delta^2+\Omega^2+\delta\Omega \end{pmatrix} \frac{1}{2(\delta^2+\Omega^2)}
\end{aligned} \tag{12}
$$

In another word, the population of $|0\rangle$ and $|1\rangle$

$$p_0 = \frac{1}{2}\left(1-\frac{\delta\Omega}{\delta^2+\Omega^2}\right) \tag{13}$$

$$p_1 = \frac{1}{2}\left(1+\frac{\delta\Omega}{\delta^2+\Omega^2}\right) \tag{14}$$

showing the asymmetry between the two states when the detuning changes sign.

We can compare this with the numerical simulation result 1 showing very good agreement.

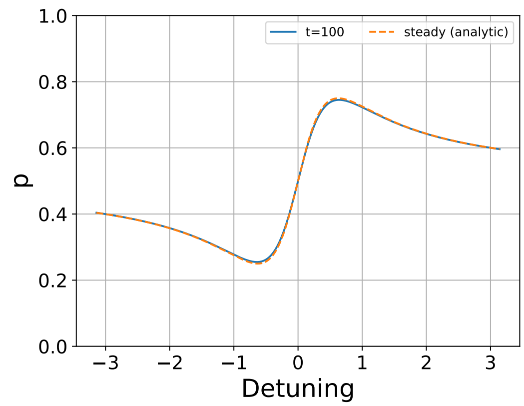

Figure 1: Comparison between the analytic steady state population of $|1\rangle$ and numerical result from master equation simulation.