Lamb-Dicke regime/approximation

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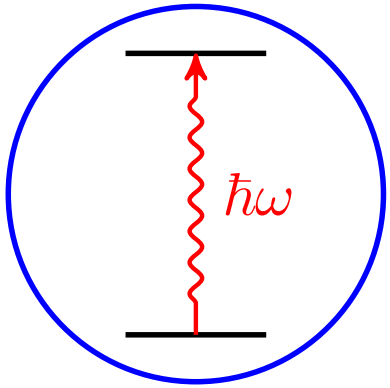
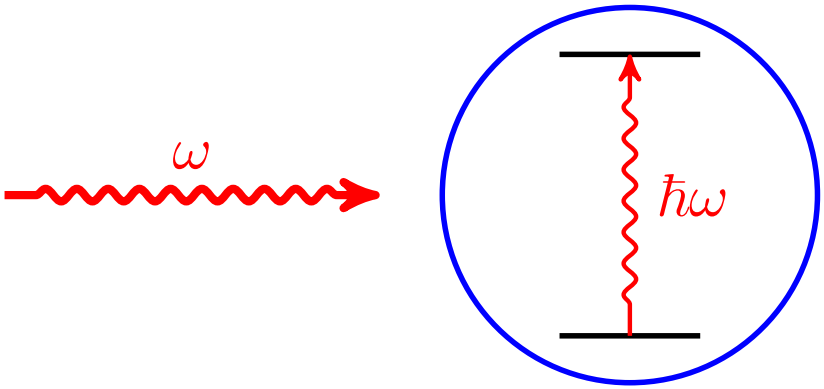
Journal Club

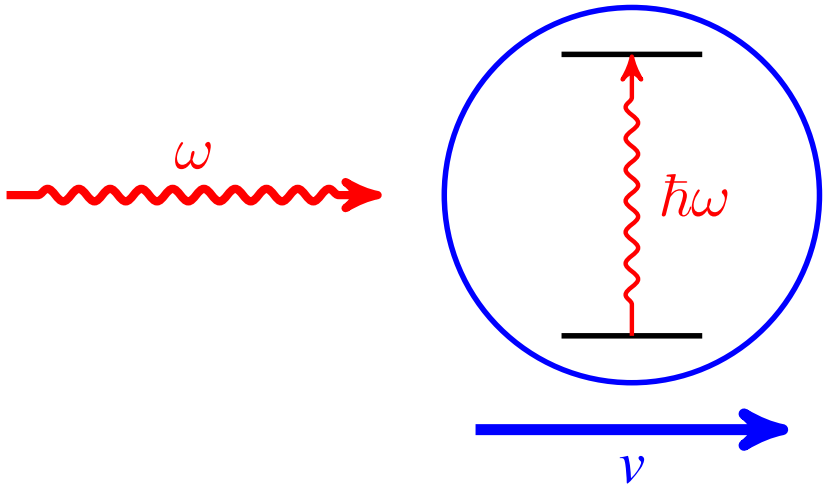
Doppler effect



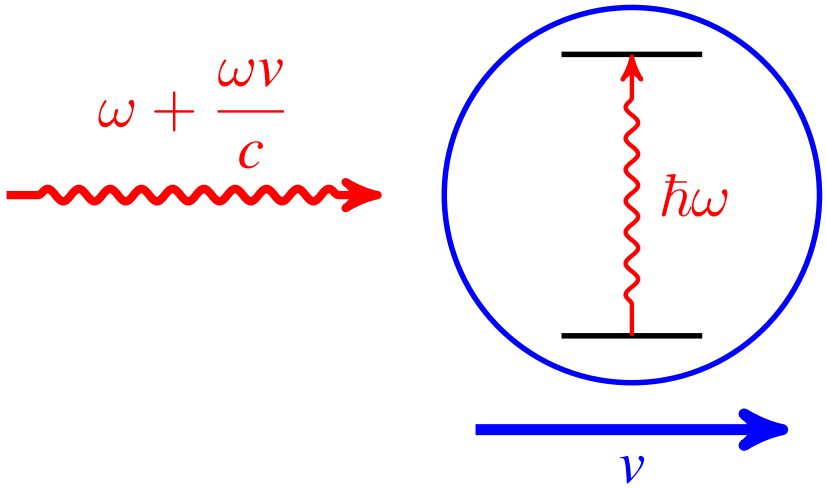
Doppler effect



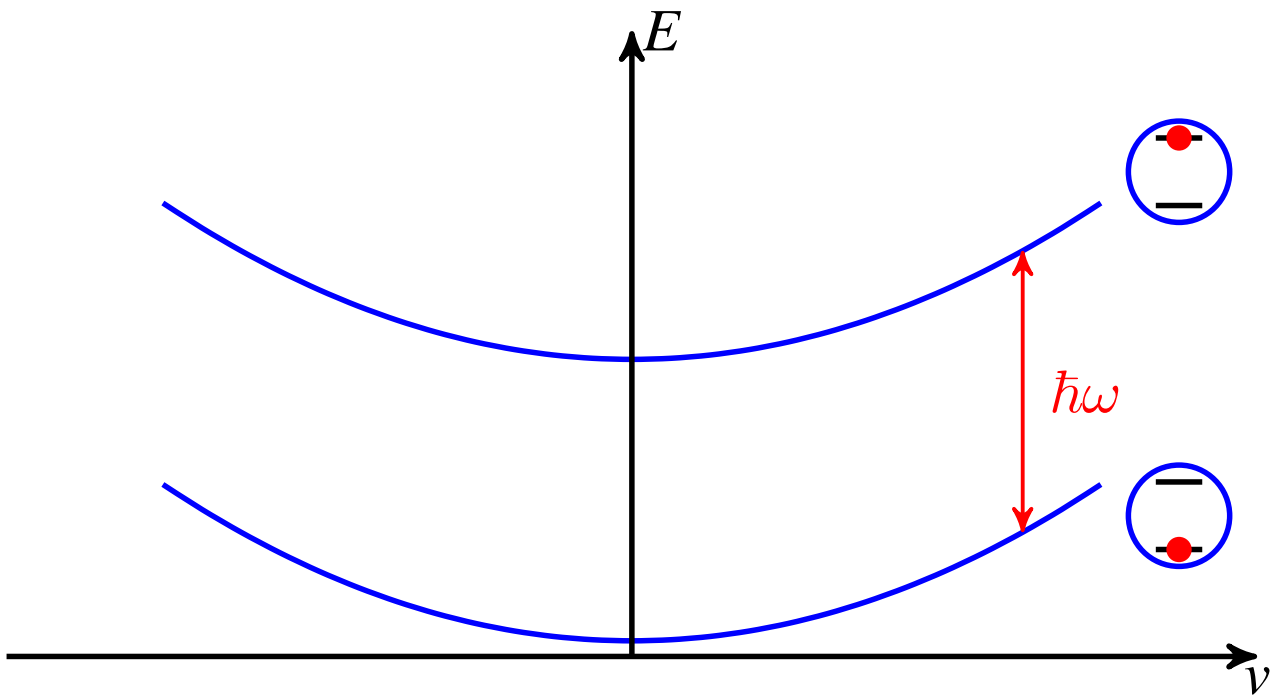
Doppler effect



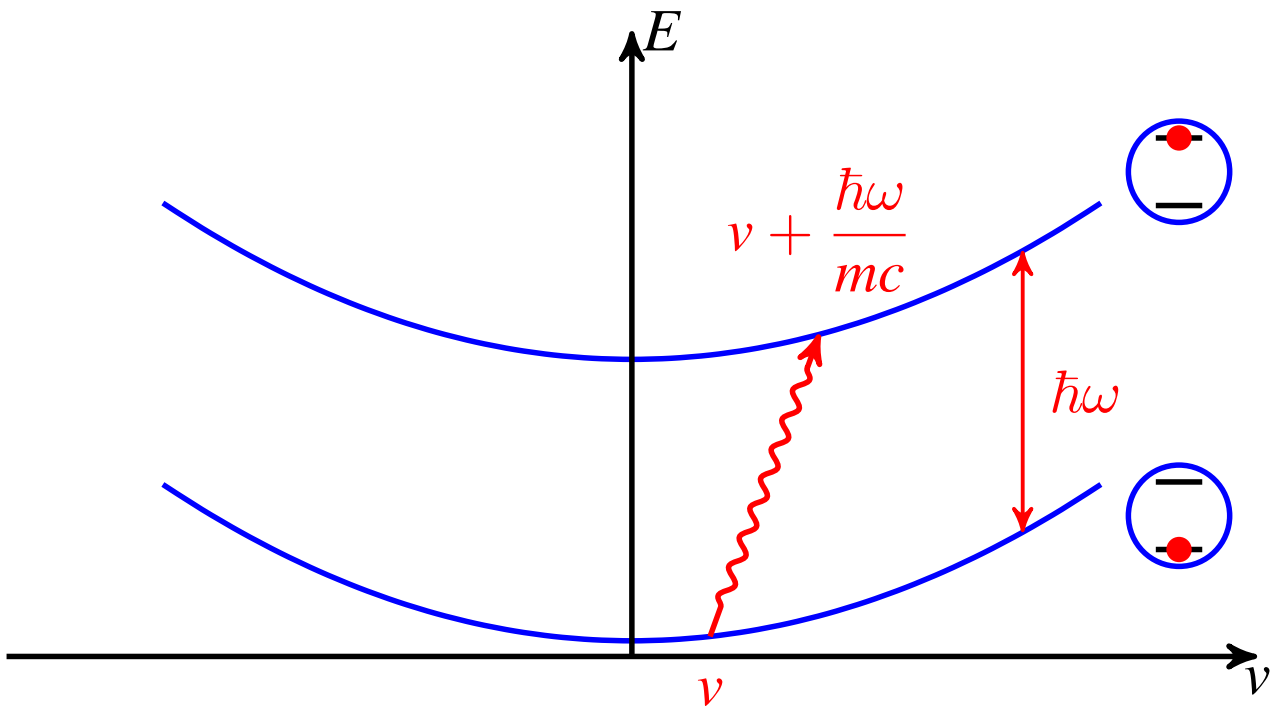
Doppler effect



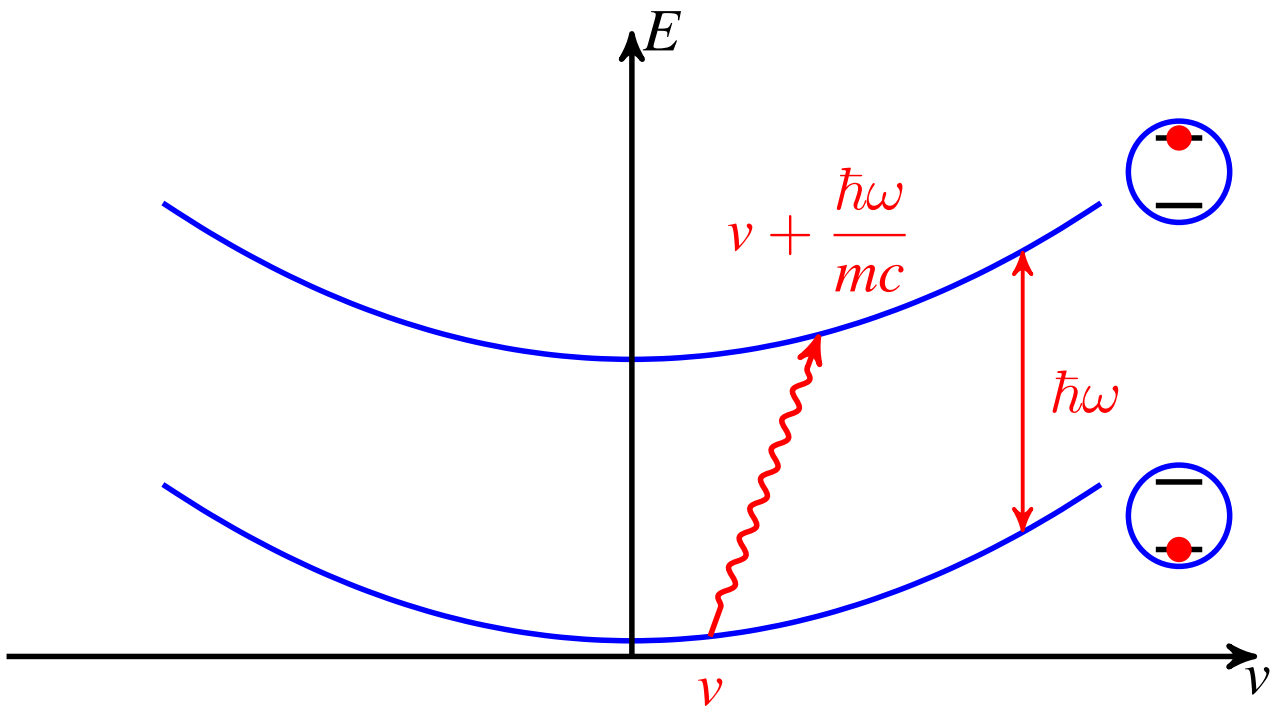
Doppler effect



Doppler effect

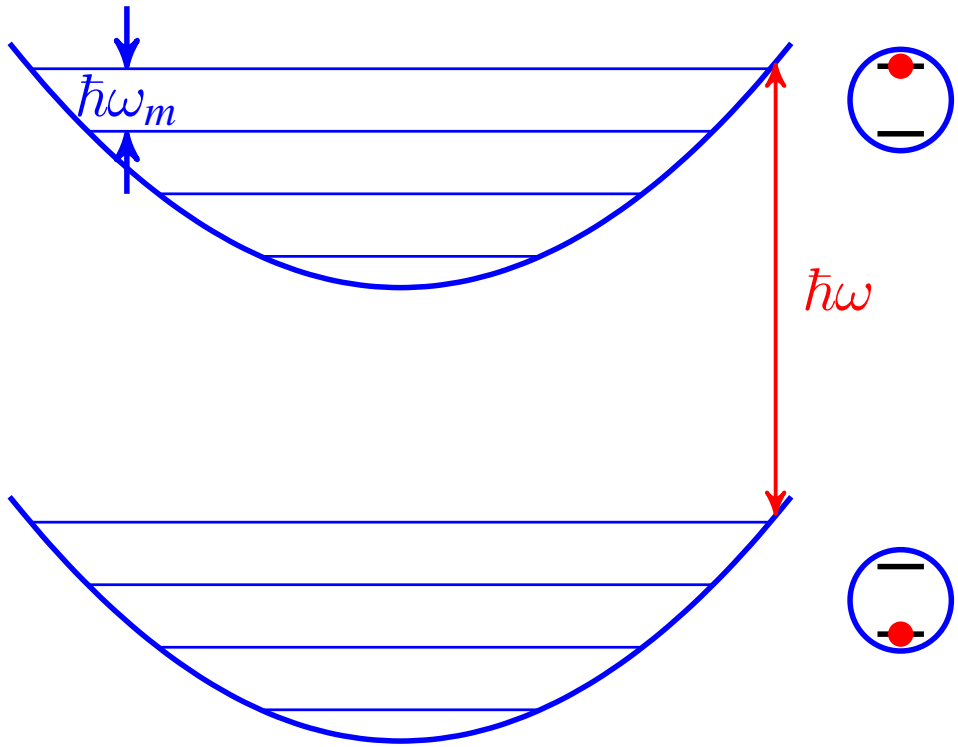


Doppler effect

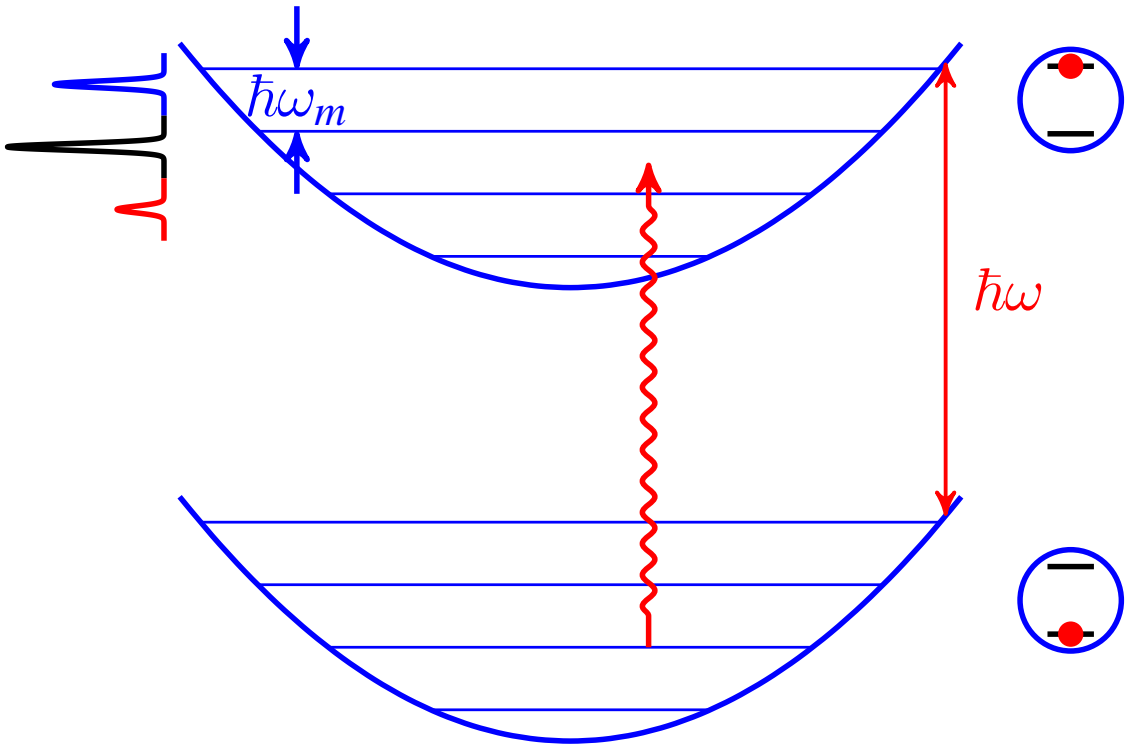


$$\omega + \frac{m}{2\hbar}\left(v + \frac{\hbar\omega}{mc}\right)^2 - \frac{mv^2}{2\hbar} = \omega + \frac{\omega v}{c} + \frac{\hbar\omega^2}{2mc^2}$$

Sideband

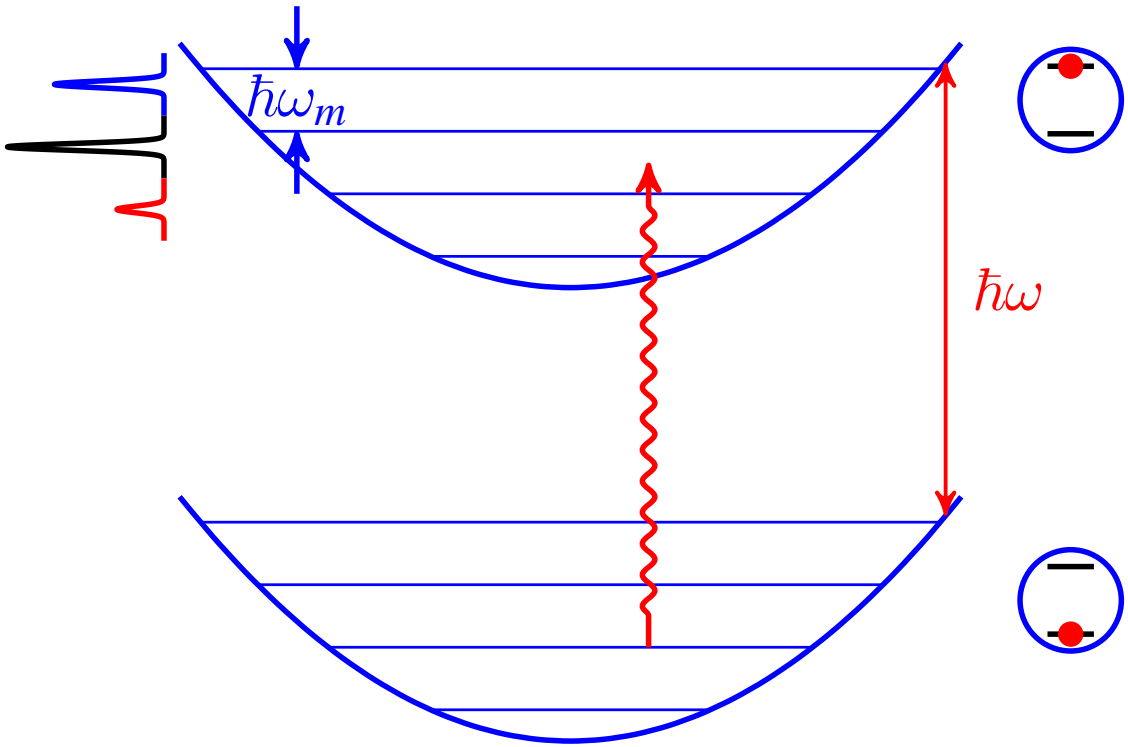


Sideband



Frequency: $\omega + n\omega_m$

Sideband



Frequency: $\omega + n\omega_m$

Strength: $\langle n|e^{ik\hat{x}}|n + \Delta n\rangle$

Lamb-Dicke parameter

$$\langle n|\mathrm{e}^{\mathrm{i}k\hat{x}}|n+\Delta n\rangle$$

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$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}\left(a + a^{\dagger}\right) = z_0\left(a + a^{\dagger}\right)$$

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$$\hat{x}=\sqrt{\frac{\hbar}{2m\omega}}\left(a+a^{\dagger}\right)=z_0\left(a+a^{\dagger}\right)$$

$$k\hat{x}=\eta\left(a+a^{\dagger}\right)$$

$$\eta\equiv kz_0=k\sqrt{\frac{\hbar}{2m\omega}}$$

Lamb-Dicke parameter

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$$k\hat{x} = \eta\left(a + a^\dagger\right)$$

$$\eta \equiv kz_0 = k\sqrt{\frac{\hbar}{2m\omega}}$$

$$\eta = \frac{2\pi z_0}{\lambda}$$

$$\eta = \sqrt{\frac{\omega_R}{\omega_m}}$$

Sideband strength

$$\langle n|e^{ik\hat{x}}|n+\Delta n\rangle$$

$$=e^{-\eta^2/2}\eta^{\Delta n}\sqrt{\frac{n_-!}{n_+!}}\,L_{n_-}^{\Delta n}(\eta^2)$$

$$n_- \equiv \min(n, n+\Delta n), \qquad n_+ \equiv \max(n, n+\Delta n)$$

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