With 0 energy different between the ground and excited state, the Hamiltonian with a single drive of detuning δ_0 is

$$H = \Omega(\cos(\delta_0 t + \phi_0)\sigma_x + \sin(\delta_0 t + \phi_0)\sigma_y)$$

Schroedinger equation

$$\mathrm{i}\partial_t|\psi\rangle = H|\psi\rangle$$

Do transformation $|\psi\rangle = U|\psi'\rangle$, where $U = \mathrm{e}^{\mathrm{i}\sigma_z(\omega' t+\phi')}$

$$\begin{aligned}
\mathrm{i}\partial_t(U|\psi'\rangle) =& HU|\psi'\rangle \\
\mathrm{i}\partial_t|\psi'\rangle =& U^\dagger HU|\psi'\rangle - \mathrm{i}U^\dagger(\partial_t U)|\psi'\rangle \\
U^\dagger(\partial_t U) =& \mathrm{i}\sigma_z\omega' U^\dagger U \\
=& \mathrm{i}\sigma_z\omega' \\
\mathrm{i}\partial_t|\psi'\rangle =& U^\dagger HU|\psi'\rangle + \sigma_z\omega'|\psi'\rangle
\end{aligned}$$

For $U^\dagger HU$

$$\begin{aligned}
& \mathrm{e}^{-\mathrm{i}\sigma_z\varphi}(\cos\theta\sigma_x + \sin\theta\sigma_y)\mathrm{e}^{\mathrm{i}\sigma_z\varphi} \\
=& \mathrm{e}^{-\mathrm{i}\sigma_z\varphi}(\cos\theta\sigma_x + \sin\theta\sigma_y)(\cos\varphi + \mathrm{i}\sigma_z\sin\varphi) \\
=& \mathrm{e}^{-\mathrm{i}\sigma_z\varphi}(\sigma_x\cos\theta\cos\varphi - \sigma_x\sin\theta\sin\varphi + \sigma_y\cos\theta\sin\varphi + \sigma_y\sin\theta\cos\varphi) \\
=& \mathrm{e}^{-\mathrm{i}\sigma_z\varphi}(\sigma_x\cos(\theta+\varphi) + \sigma_y\sin(\theta+\varphi)) \\
=& (\cos\varphi - \mathrm{i}\sigma_z\sin\varphi)(\sigma_x\cos(\theta+\varphi) + \sigma_y\sin(\theta+\varphi)) \\
=& \sigma_x\cos\varphi\cos(\theta+\varphi) - \sigma_x\sin\varphi\sin(\theta+\varphi) + \sigma_y\sin\varphi\cos(\theta+\varphi) + \sigma_y\cos\varphi\sin(\theta+\varphi) \\
=& \sigma_x\cos(\theta+2\varphi) + \sigma_y\sin(\theta+2\varphi)
\end{aligned}$$

For $\omega' = -\dfrac{\delta_0}{2}$ and $\phi' = -\dfrac{\phi_0}{2}$

$$\begin{aligned}
U =& \exp\left(-\frac{\mathrm{i}\sigma_z}{2}(\delta_0 t + \phi_0)\right) \\
\mathrm{i}\partial_t|\psi'\rangle =& \left(\Omega\sigma_x - \frac{\delta_0}{2}\sigma_z\right)|\psi'\rangle
\end{aligned}$$

Integrate this equation

$$\begin{aligned}
|\psi'\rangle_t =& \exp\left(-\mathrm{i}\left(\Omega\sigma_x - \frac{\delta_0}{2}\sigma_z\right)t\right)|\psi'\rangle_0 \\
=& \exp\left(-\mathrm{i}t\sqrt{\Omega^2 + \frac{\delta_0^2}{4}}\frac{2\Omega\sigma_x - \delta_0\sigma_z}{\sqrt{4\Omega^2 + \delta_0^2}}\right)|\psi'\rangle_0
\end{aligned}$$

Let $\Omega' = \sqrt{\Omega^2 + \dfrac{\delta_0^2}{4}}$

$$\begin{aligned}
|\psi'\rangle_t =& \exp\left(-\mathrm{i}\Omega' t\frac{2\Omega\sigma_x - \delta_0\sigma_z}{2\Omega'}\right)|\psi'\rangle_0 \\
=& \left(\cos\Omega' t - \mathrm{i}\frac{2\Omega\sigma_x - \delta_0\sigma_z}{2\Omega'}\sin\Omega' t\right)|\psi'\rangle_0
\end{aligned}$$

Transform back to the original basis,

$$
\begin{aligned}
&|\psi\rangle_t \\
=&U_t|\psi'\rangle_t \\
=&U_t\left(\cos\Omega' t-\mathrm{i}\frac{2\Omega\sigma_x-\delta_0\sigma_z}{2\Omega'}\sin\Omega' t\right)U_0^\dagger|\psi\rangle_0 \\
=&\exp\left(-\frac{\mathrm{i}\sigma_z}{2}(\delta_0 t+\phi_0)\right)\left(\cos\Omega' t-\mathrm{i}\frac{2\Omega\sigma_x-\delta_0\sigma_z}{2\Omega'}\sin\Omega' t\right)\exp\left(\frac{\mathrm{i}\sigma_z}{2}\phi_0\right)|\psi\rangle_0 \\
=&\exp\left(-\frac{\mathrm{i}\delta_0 t}{2}\sigma_z\right)\exp\left(-\frac{\mathrm{i}\sigma_z}{2}\phi_0\right)\left(\cos\Omega' t-\mathrm{i}\frac{2\Omega\sigma_x-\delta_0\sigma_z}{2\Omega'}\sin\Omega' t\right)\exp\left(\frac{\mathrm{i}\sigma_z}{2}\phi_0\right)|\psi\rangle_0 \\
=&\exp\left(-\frac{\mathrm{i}\delta_0 t}{2}\sigma_z\right)\left(\cos\Omega' t-\mathrm{i}\frac{2\Omega(\sigma_x\cos\phi_0+\sigma_y\sin\phi_0)-\delta_0\sigma_z}{2\Omega'}\sin\Omega' t\right)|\psi\rangle_0 \\
=&\begin{pmatrix}\mathrm{e}^{-\mathrm{i}\delta_0 t/2} & 0\\ 0 & \mathrm{e}^{\mathrm{i}\delta_0 t/2}\end{pmatrix}\begin{pmatrix}\cos\Omega' t+\frac{\mathrm{i}\delta_0}{2\Omega'}\sin\Omega' t & -\mathrm{i}\frac{\Omega}{\Omega'}\mathrm{e}^{-\mathrm{i}\phi_0}\sin\Omega' t\\ -\mathrm{i}\frac{\Omega}{\Omega'}\mathrm{e}^{\mathrm{i}\phi_0}\sin\Omega' t & \cos\Omega' t-\frac{\mathrm{i}\delta_0}{2\Omega'}\sin\Omega' t\end{pmatrix}|\psi\rangle_0 \\
=&\begin{pmatrix}\mathrm{e}^{-\mathrm{i}\delta_0 t/2}\left(\cos\Omega' t+\frac{\mathrm{i}\delta_0}{2\Omega'}\sin\Omega' t\right) & -\mathrm{i}\frac{\Omega}{\Omega'}\mathrm{e}^{-\mathrm{i}(\delta_0 t/2+\phi_0)}\sin\Omega' t\\ -\mathrm{i}\frac{\Omega}{\Omega'}\mathrm{e}^{\mathrm{i}(\delta_0 t/2+\phi_0)}\sin\Omega' t & \mathrm{e}^{\mathrm{i}\delta_0 t/2}\left(\cos\Omega' t-\frac{\mathrm{i}\delta_0}{2\Omega'}\sin\Omega' t\right)\end{pmatrix}|\psi\rangle_0
\end{aligned}
$$