Entanglement from tensor networks on a trapped-ion QCCD quantum computer

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- Matrix product states (MPS)
- Simulating MPS with quantum computer
- Results

Matrix product states (MPS)

$$\sum_{\sigma_1\sigma_2\cdots\sigma_n} c_{\sigma_1\sigma_2\cdots\sigma_n} |\sigma_1\sigma_2\cdots\sigma_n\rangle$$

Matrix product states (MPS)

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$$\sum_{\sigma_1 \sigma_2 \cdots \sigma_n} \mathrm{Tr}(V_{\sigma_1} V_{\sigma_2} \cdots V_{\sigma_n}) |\sigma_1 \sigma_2 \cdots \sigma_n\rangle$$

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$$c_{\sigma_1 \sigma_2 \cdots \sigma_m \cdots \sigma_n} = \mathrm{Tr}(V_{\sigma_1} V_{\sigma_2} \cdots V_{\sigma_m} \cdots V_{\sigma_n})$$

$$c_{\sigma_1 \sigma_2 \cdots \sigma_m' \cdots \sigma_n} = \mathrm{Tr}(V_{\sigma_1} V_{\sigma_2} \cdots V_{\sigma_m'} \cdots V_{\sigma_n})$$

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Minimum matrix dimension:
entanglement in the system

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Minimum matrix dimension: entanglement in the system

Exploding structure in the state

Matrix product states (MPS)

$$\sum_{\sigma_1 \sigma_2 \cdots \sigma_n} c_{\sigma_1 \sigma_2 \cdots \sigma_n} |\sigma_1 \sigma_2 \cdots \sigma_n\rangle$$

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Minimum matrix dimension: entanglement in the system

$$c_{\sigma_1 \sigma_2 \cdots \sigma_m \cdots \sigma_n} = \text{Tr}(V_{\sigma_1} V_{\sigma_2} \cdots V_{\sigma_m} \cdots V_{\sigma_n})$$

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Exploding structure in the state

$$\sum_{\sigma_1 \sigma_2 \cdots} L V_{\sigma_1} V_{\sigma_2} \cdots |\sigma_1 \sigma_2 \cdots \rangle$$

Simulation of MPS

$$c_{\sigma_1 \sigma_2 \cdots} = L V_{\sigma_1} V_{\sigma_2} \cdots$$

Simulation of MPS

L as fictitious state

$$c_{\sigma_1\sigma_2\cdots} = L V_{\sigma_1} V_{\sigma_2} \cdots$$

Simulation of MPS

$$c_{\sigma_1 \sigma_2 \ldots} = L V_{\sigma_1} V_{\sigma_2} \cdots$$

$$\langle L_\beta | V_\sigma | L_\alpha \rangle$$

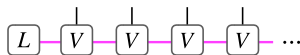L as fictitious state

V as operator on the fictitious space

Simulation of MPS

$$c_{\sigma_1 \sigma_2 \cdots} = L V_{\sigma_1} V_{\sigma_2} \cdots$$

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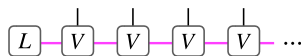$$\langle \sigma_i | \langle L_\beta | U | L_\alpha \rangle | 0_i \rangle$$

L as fictitious state

V as operator on the fictitious space

Embedding into the product space

Simulation of MPS

$$c_{\sigma_1\sigma_2\cdots} = LV_{\sigma_1}V_{\sigma_2}\cdots$$

$$\langle L_\beta|V_\sigma|L_\alpha\rangle$$

$$\langle\sigma_i|\langle L_\beta|U|L_\alpha\rangle|0_i\rangle$$

L as fictitious state

V as operator on the fictitious space

Embedding into the product space

State $U|L_\alpha\rangle|0_i\rangle$ carries info about site i in the "physical subspace".
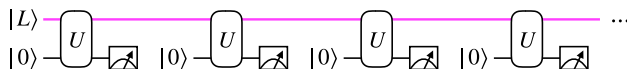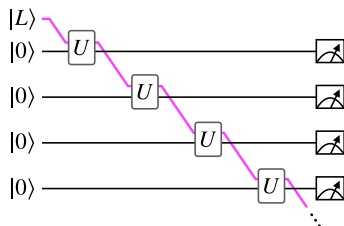
Simulation of MPS

$$c_{\sigma_1 \sigma_2 \cdots} = L V_{\sigma_1} V_{\sigma_2} \cdots$$

$$\langle L_\beta | V_\sigma | L_\alpha \rangle$$

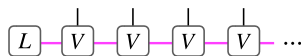$$\langle \sigma_i | \langle L_\beta | U | L_\alpha \rangle | 0_i \rangle$$

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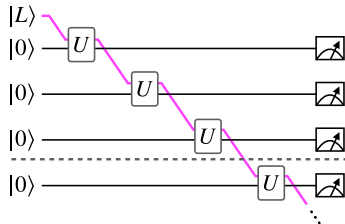
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Embedding into the product space

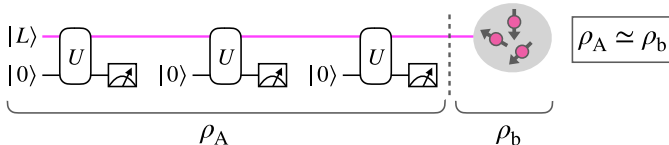State $U|L_\alpha\rangle|0_i\rangle$ carries info about site i in the "physical subspace".
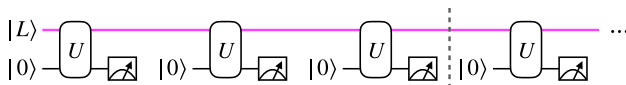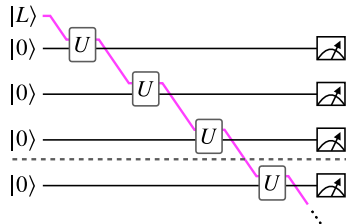
Entanglement spectrum

Eigenvalues of $\rho_A = \text{Tr}_B(|\Psi\rangle\langle\Psi|)$

Entanglement spectrum

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Entanglement spectrum

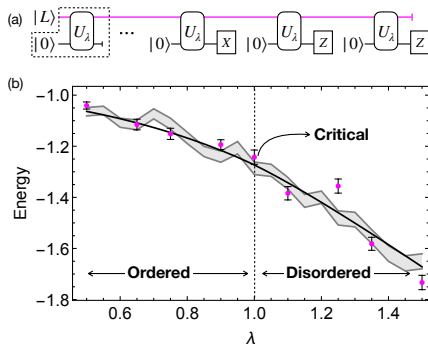Eigenvalues of $\rho_A = \text{Tr}_B(|\Psi\rangle\langle\Psi|)$

$\rho_A \simeq \rho_b$

Results

(a)

(b)

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