Raman between $F = 1/2$ states with dipole transition

September 7, 2023

1 Introduction

As a special case of the general effective Hamiltonian caused by second order perturbation from dipole transition, if the ground states have total angular momentum $\frac{1}{2}$, the only allowed terms in the effective Hamiltonian would be scalar and vector shifts due the the limited degrees of freedoms.[1] This is a special case that is particularly easy to show and we'll calculate the effective Hamiltonian of such a system in this note.

2 Calculation

We can calculate the effective Hamiltonian in any reference frame of choice so we can pick a reference frame where the decomposition of the polarization is the easist to handle. We'll pick the polarization plane[2] as the x-y plane (*for a single beam this means the z axis is along the k vector of the beam*), which means that the polarization will only have $\sigma^\pm$ components.[3]

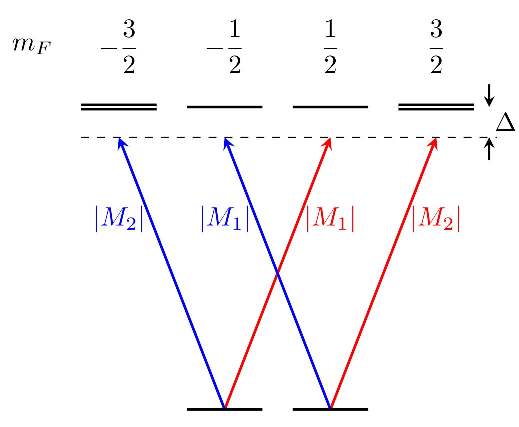

Figure 1: Coupling to the excited state with $\sigma^\pm$ polarizations and single-photon detuning Δ. The double-lined excited states only exists if the exited states have $F = 3/2$. The allowed coupling between the ground and excited states are drawn in red for σ^+ and blue for σ^-. The absolute value of the matrix elements are labeled next to the arrow. Note that there are only two distinct values $|M_1|$ and $|M_2|$ from symmetry. For excited state with $F = 1/2$, we would have $M_2 = 0$.

[1] This applies even if the transition inducing the shift is not a dipole transition but we'll limit the calculation here to that from dipole transitions.

[2] which must exist for monotonic light with pure polarization

[3] This choice is also motivated by the general knowledge that the effective B field from the vector shift would be along the z axis given this choice.

The energy diagram of the system is shown in Fig. 1, where we can clearly see that the only effect from the light is causing light shift on the states, as the two ground states never couple to the same excited state in this frame. Therefore, we just need to calculate the light shift on the two ground states. The energy shift for the two ground states are,

$$E_{1/2} = \frac{\left|\Omega_{1/2\to 3/2}\right|^2 + \left|\Omega_{1/2\to -1/2}\right|^2}{4\Delta} \tag{1}$$

$$E_{-1/2} = \frac{\left|\Omega_{-1/2\to -3/2}\right|^2 + \left|\Omega_{-1/2\to 1/2}\right|^2}{4\Delta} \tag{2}$$

Ignoring the global energy shift (i.e. scalar shift), the effective Hamiltonian on the ground state is proportional to σ_z, and more specifically,

$$H_{\text{eff}} = \frac{E_{1/2} - E_{-1/2}}{2}\sigma_z \tag{3}$$

To obtain the effective Hamiltonian in a different frame, one can simply replace the σ_z matrix with a Pauli matrix along the appropriate axis in the desired frame.

We can further simplify the expression for the effective Hamiltonian (eq. 3) to show the dependency on the polarization and power more directly. Using the matrix elements labeled in fig. 1, and assuming the $\sigma^\pm$ polarization have amplitude $A_\pm$, we can rewrite the effective Hamiltonian

$$\begin{aligned} H_{\text{eff}} =& \frac{\sigma_z}{2}\left(E_{1/2} - E_{-1/2}\right) \\ =& \frac{\sigma_z}{8\Delta}\left(\left|\Omega_{1/2\to 3/2}\right|^2 + \left|\Omega_{1/2\to -1/2}\right|^2 - \left|\Omega_{-1/2\to -3/2}\right|^2 - \left|\Omega_{-1/2\to 1/2}\right|^2\right) \\ =& \frac{\sigma_z}{8\Delta}\left(|M_2|^2|A_+|^2 + |M_1|^2|A_-|^2 - |M_1|^2|A_+|^2 - |M_2|^2|A_-|^2\right) \\ =& \frac{\sigma_z}{8\Delta}\left(|M_2|^2 - |M_1|^2\right)\left(|A_+|^2 - |A_-|^2\right) \end{aligned} \tag{4}$$

Note that the effective Hamiltonian is proportional to the difference in the amplitude of σ^+ and σ^- polarization. The effect is 0 for linear polarization where $|A_+| = |A_-|$ and is maximized (at a given total power, i.e. a fixed $|A_+|^2 + |A_-|^2$) for circular polarization where either $A_+ = 0$ or $A_- = 0$.