Entanglement from tensor networks on a trapped-ion QCCD quantum computer

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- Matrix product states (MPS)
- Simulating MPS with quantum computer
- Results

Matrix product states (MPS)

$$\sum_{\sigma_1\sigma_2\cdots\sigma_n} c_{\sigma_1\sigma_2\cdots\sigma_n} \left|\sigma_1\sigma_2\cdots\sigma_n\right\rangle$$

Matrix product states (MPS)

$$\sum_{\sigma_1\sigma_2\cdots\sigma_n} c_{\sigma_1\sigma_2\cdots\sigma_n}|\sigma_1\sigma_2\cdots\sigma_n\rangle$$

$$\sum_{\sigma_1\sigma_2\cdots\sigma_n} \mathrm{Tr}(V_{\sigma_1}V_{\sigma_2}\cdots V_{\sigma_n})|\sigma_1\sigma_2\cdots\sigma_n\rangle$$

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$$c_{\sigma_1\sigma_2\cdots\sigma_m\cdots\sigma_n} = \mathrm{Tr}\left(V_{\sigma_1}V_{\sigma_2}\cdots V_{\sigma_m}\cdots V_{\sigma_n}\right)$$

$$c_{\sigma_1\sigma_2\cdots\sigma'_m\cdots\sigma_n} = \mathrm{Tr}\left(V_{\sigma_1}V_{\sigma_2}\cdots V_{\sigma'_m}\cdots V_{\sigma_n}\right)$$

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Minimum matrix dimension:
entanglement in the system

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Exploding structure in the state

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$$\sum_{\sigma_1\sigma_2\cdots} LV_{\sigma_1}V_{\sigma_2}\cdots|\sigma_1\sigma_2\cdots\rangle$$

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Exploding structure in the state

Simulation of MPS

$$c_{\sigma_1\sigma_2\cdots} = LV_{\sigma_1}V_{\sigma_2}\cdots$$

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L as fictitious state

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$$\langle L_\beta|V_\sigma|L_\alpha\rangle$$

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V as operator on the fictitious space

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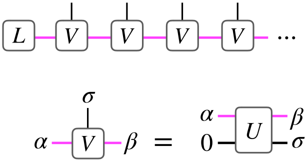
V as operator on the fictitious space

$$\langle\sigma_i|\langle L_\beta|U|L_\alpha\rangle|0_i\rangle$$

Embedding into the product space



Simulation of MPS

$$c_{\sigma_1\sigma_2\cdots} = L V_{\sigma_1} V_{\sigma_2} \cdots$$

L as fictitious state

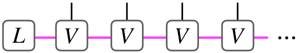
$$\langle L_\beta | V_\sigma | L_\alpha \rangle$$

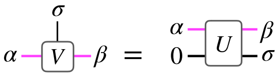
V as operator on the fictitious space

$$\langle \sigma_i | \langle L_\beta | U | L_\alpha \rangle | 0_i \rangle$$

Embedding into the product space

State $U|L_\alpha\rangle|0_i\rangle$ carries info about site i in the "physical subspace".





Simulation of MPS

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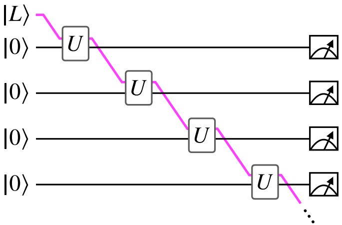

L as fictitious state

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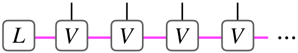

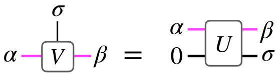

Simulation of MPS

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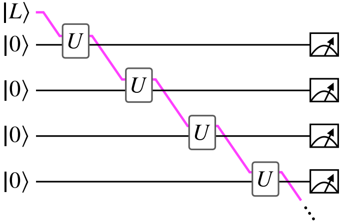
Embedding into the product space



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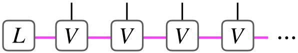

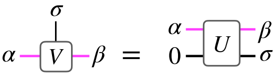

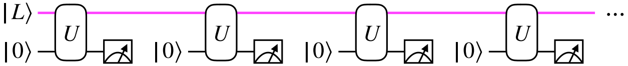

Entanglement spectrum

Eigenvalues of $\rho_{\mathrm{A}} = \mathrm{Tr}_{\mathrm{b}}(|\Psi\rangle\langle\Psi|)$

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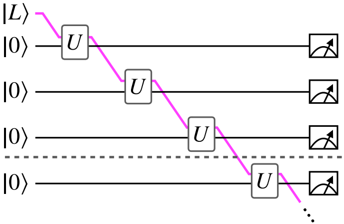
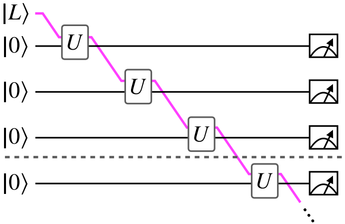
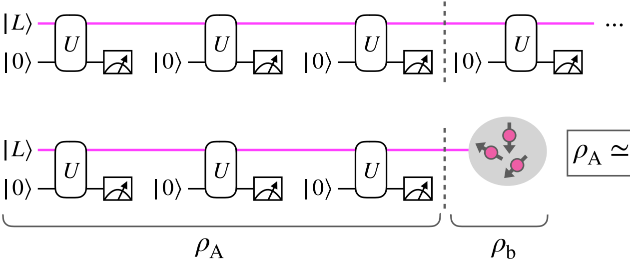


Entanglement spectrum

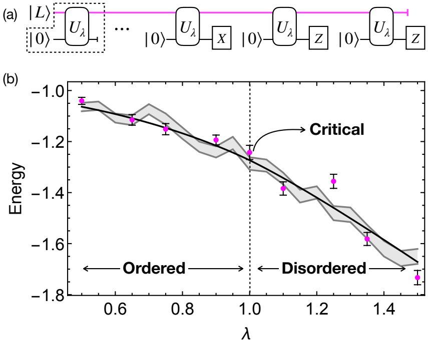
Eigenvalues of $\rho_A = \mathrm{Tr}_b(|\Psi\rangle\langle\Psi|)$





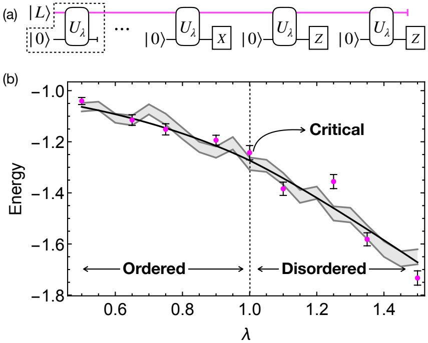
$\boxed{\rho_A \simeq \rho_b}$

Results



(a)
$|L\rangle$
$|0\rangle$
U_λ
U_λ
U_λ
U_λ
$|0\rangle$
$|0\rangle$
$|0\rangle$
X
Z
Z
(b)
Energy
−1.0
−1.2
−1.4
−1.6
−1.8
0.6
0.8
1.0
1.2
1.4
λ
Critical
Ordered
Disordered

Results



(a)
$|L\rangle$
$|0\rangle$
U_λ
$\cdots$
X
Z
(b)
Energy
−1.0
−1.2
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λ
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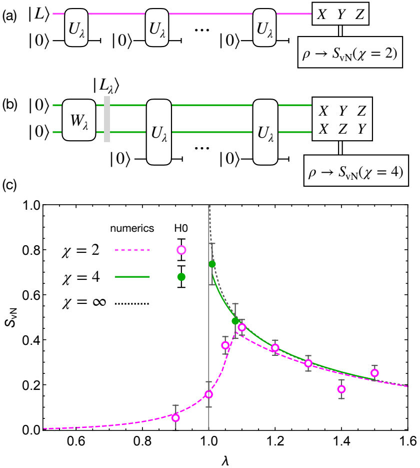

(a)
$|L\rangle$
$|0\rangle$
U_λ
$\cdots$
$X\ Y\ Z$
$\rho \to S_{vN}(\chi = 2)$
(b)
$|L_\lambda\rangle$
W_λ
$X\ Y\ Z$
$X\ Z\ Y$
$\rho \to S_{vN}(\chi = 4)$
(c)
numerics
H0
$\chi = 2$
$\chi = 4$
$\chi = \infty$
S_{vN}
0.0
0.2
0.4
0.6
0.8
1.0
0.6
0.8
1.0
1.2
1.4
1.6
λ