A One-Way Quantum Computer

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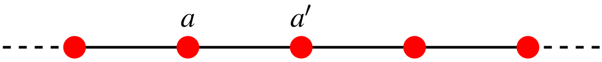
Ni Group

Feb. 12, 2021

- 1D Cluster state
  - Generation
  - Properties
- High dimensional cluster state
- Quantum circuit
- Gates and single qubit operations

1D Cluster State

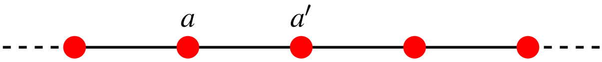


$$H = \sum_{a,a' \in \Gamma} \frac{1 + \sigma_z^{(a)}}{2} \frac{1 - \sigma_z^{(a')}}{2}$$

$$\Gamma = \left\{(a, a') | a' = a + 1\right\}$$

$$\mathcal{S} = \mathrm{e}^{\mathrm{i}\pi H}$$

1D Cluster State



$$|\phi_N\rangle = \mathcal{S}\bigotimes_a |+\rangle_a = \frac{1}{2^{N/2}}\bigotimes_a \left(|0\rangle_a \sigma_z^{a+1} + |1\rangle_a\right)$$

$$|\phi_2\rangle = \frac{1}{\sqrt{2}}(|0-\rangle + |1+\rangle)$$

$$|\phi_3\rangle = \frac{1}{\sqrt{2}}(|+0-\rangle - |-1+\rangle)$$

$$|\phi_4\rangle = \frac{1}{2}(|0-0-\rangle - |0+1+\rangle + |1+0-\rangle - |1-1+\rangle)$$

$$|\mathrm{GHZ}_N\rangle = \frac{1}{\sqrt{2}}\left(\bigotimes_a |0\rangle_a + \bigotimes_a |1\rangle_a\right)$$

1D Cluster State

- Maximum connectedness
  Ability to create Bell state by local measurements.
  Yes for both GHZ state and cluster state.
- Persistency
  Minimum local measurements to destroy all entanglements.
  GHZ: $P_e = 1$, cluster: $P_e = \lfloor N/2 \rfloor$

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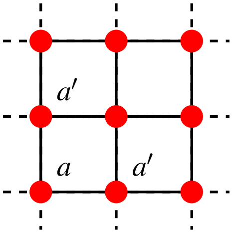
High Dimensional Cluster State



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$$\Gamma = \left\{ (a, a') | a' = a + \hat{e}_i \right\}$$

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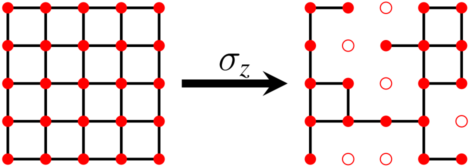

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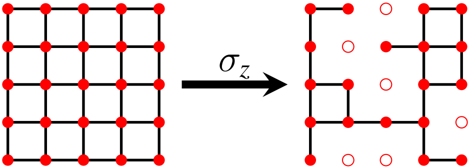

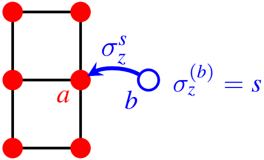

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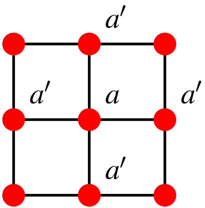

$$K_a = \sigma_x^{(a)} \bigotimes_{a' \in \Gamma'} \sigma_z^{(a')}$$

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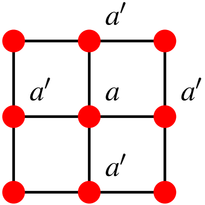

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- $\{K_a, \sigma_z^{(a)}\} = 0$
- $[K_a, K_b] = 0$
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- Independent
- Complete
- Equivalent definition of cluster state

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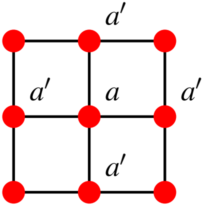

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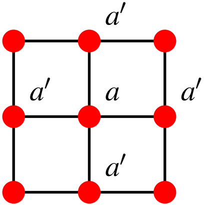

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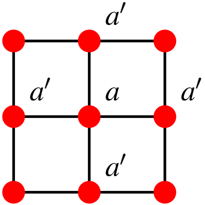

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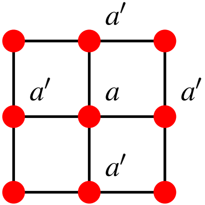

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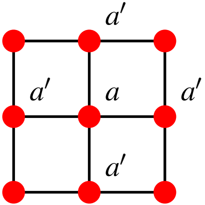

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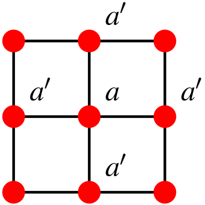

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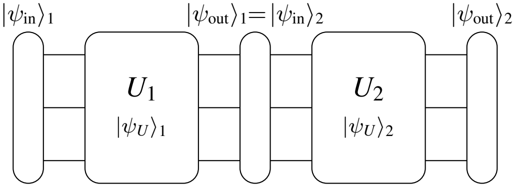
Quantum Circuit on Cluster State



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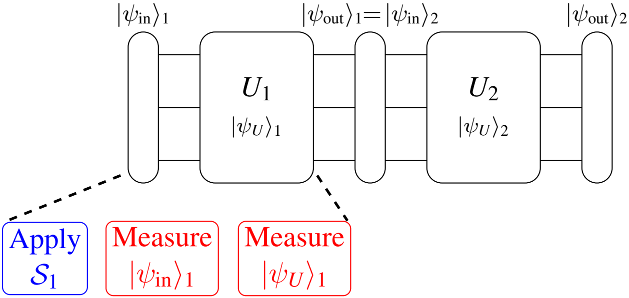

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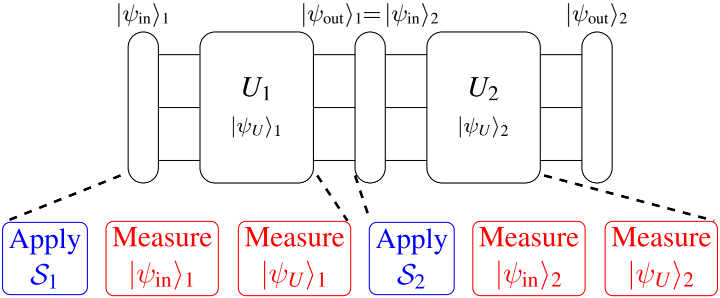

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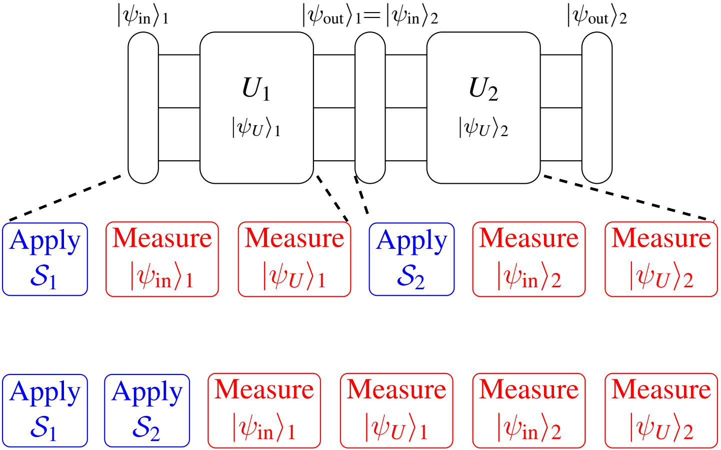

Gates and Single Qubit Operations

Single qubit propagation: Measure σ_x

$$\begin{aligned}\mathcal{S}(a|0\rangle_1 + b|1\rangle_1)|+\rangle_2 = & |+\rangle_1(a|-\rangle_2 + b|+\rangle_2) \\ & + |-\rangle_1(a|-\rangle_2 - b|+\rangle_2)\end{aligned}$$

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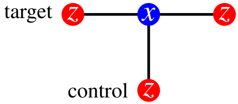
Single qubit rotation: Measure $\sigma_x \cos\theta + \sigma_y \sin\theta$

Gates and Single Qubit Operations

CNOT



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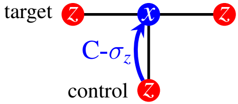

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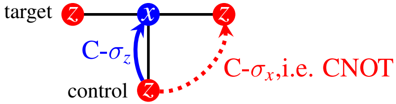

Questions?