A One-Way Quantum Computer

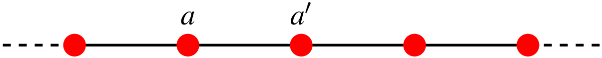
Yichao Yu

Ni Group

Feb. 15, 2021

- 1D Cluster state
  - Generation
  - Properties
- High dimensional cluster state
- Quantum circuit
- Gates and single qubit operations

1D Cluster State



$$H = \sum_{a,a' \in \Gamma} \frac{1 + \sigma_z^{(a)}}{2} \frac{1 - \sigma_z^{(a')}}{2}$$

$$\Gamma = \left\{ (a, a') | a' = a + 1 \right\}$$

$$\mathcal{S} = \mathrm{e}^{\mathrm{i}\pi H}$$

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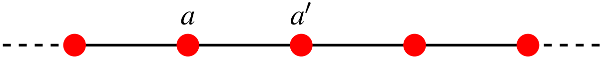

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$$H = \sum \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 1 & \\ & & & 0 \end{pmatrix}$$

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$$\mathcal{S} = \bigotimes \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & 1 \end{pmatrix}$$

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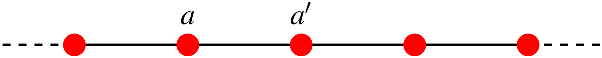

$$|\phi_N\rangle = \mathcal{S}\bigotimes_a |+\rangle_a = \frac{1}{2^{N/2}}\bigotimes_a \left(|0\rangle_a \sigma_z^{a+1} + |1\rangle_a\right)$$

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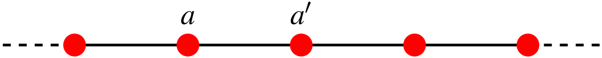

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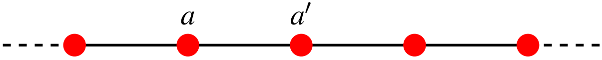

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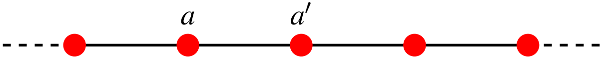

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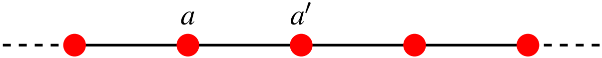

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$$|\mathrm{GHZ}_N\rangle = \frac{1}{\sqrt{2}}\left(\bigotimes_a |0\rangle_a + \bigotimes_a |1\rangle_a\right)$$

1D Cluster State

- Maximum connectedness
  Ability to create Bell state by local measurements.
  Yes for both GHZ state and cluster state.
- Persistency
  Minimum local measurements to destroy all entanglements.
  GHZ: $P_e = 1$, cluster: $P_e = \lfloor N/2 \rfloor$

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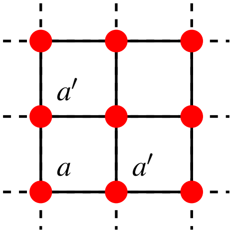
High Dimensional Cluster State



$$H=\sum_{a,a'\in\Gamma}\frac{1+\sigma_z^{(a)}}{2}\frac{1-\sigma_z^{(a')}}{2}$$

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$$[H, \sigma_z^{(a)}] = 0, \;\; [\mathcal{S}, \sigma_z^{(a)}] = 0$$

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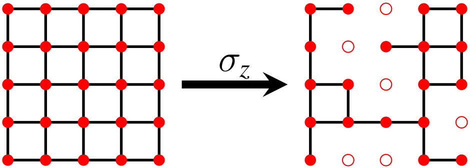

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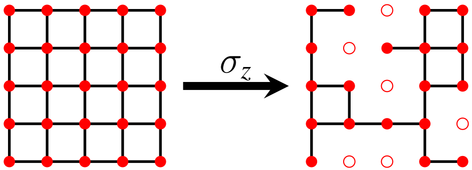

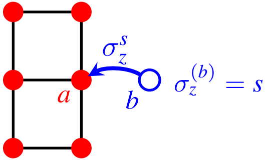

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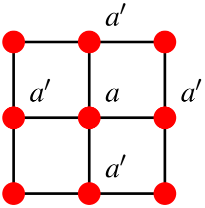

$$K_a = \sigma_x^{(a)} \bigotimes_{a' \in \Gamma'} \sigma_z^{(a')}$$

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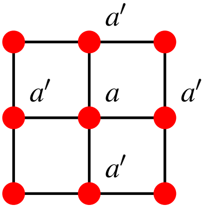

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- Complete
- Equivalent definitions of cluster state
  - Apply full $\mathcal{S}$ and measure σ_z's on removed sites
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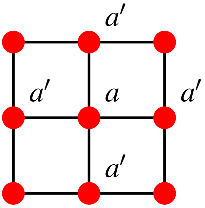

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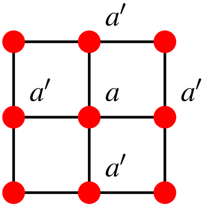

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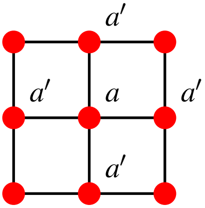

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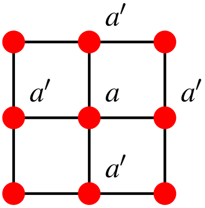

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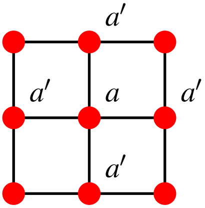

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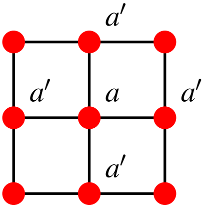

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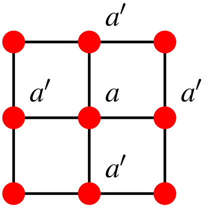

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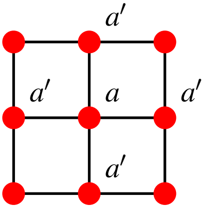

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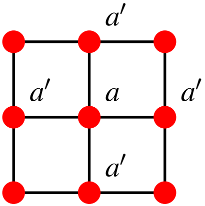

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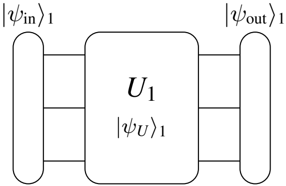
Quantum Circuit on Cluster State



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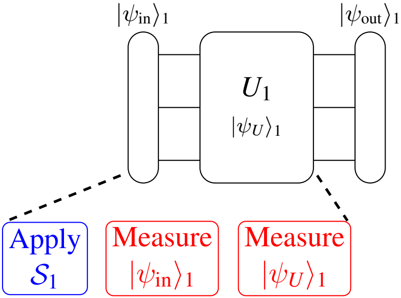

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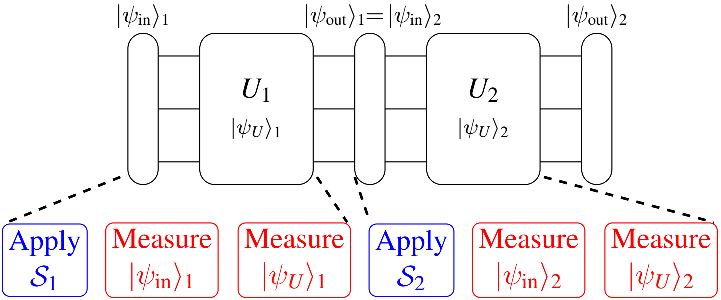

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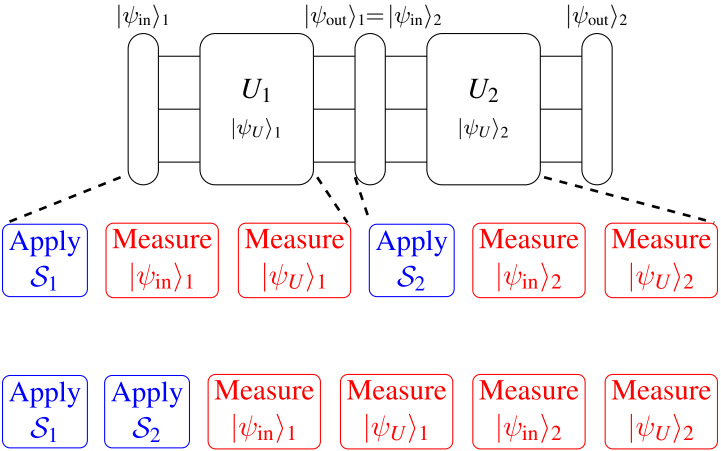

Gates and Single Qubit Operations

Single qubit propagation: Measure σ_x

$$\begin{aligned}\mathcal{S}(a|0\rangle_1 + b|1\rangle_1)|+\rangle_2 =& |+\rangle_1(a|-\rangle_2 + b|+\rangle_2)\\ &+ |-\rangle_1(a|-\rangle_2 - b|+\rangle_2)\end{aligned}$$

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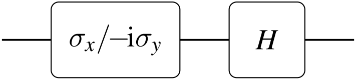

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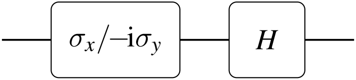
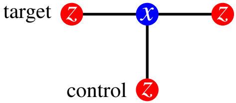


Single qubit rotation: Measure $\sigma_x \cos\theta + \sigma_y \sin\theta$

Gates and Single Qubit Operations

CNOT



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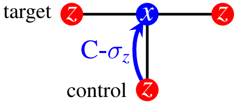

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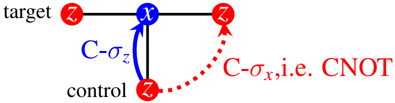

Questions?