1 Derivation

TODO

$$W(x,p) = \frac{\tanh(f)}{\pi\hbar}\exp\left(-\tanh(f)\left(\frac{m\omega x^2}{\hbar}+\frac{p^2}{m\omega\hbar}\right)\right)$$

where $f \equiv \frac{1}{2}\beta\hbar\omega$

2 Properties

2.1 x distribution

$$\begin{aligned}
p(x) &= \int \mathrm{d}p W \\
&= \frac{\tanh(f)}{\pi\hbar}\int \mathrm{d}p \exp\left(-\tanh(f)\left(\frac{m\omega x^2}{\hbar}+\frac{p^2}{m\omega\hbar}\right)\right) \\
&= \sqrt{\frac{m\omega\tanh(f)}{\pi\hbar}}\exp\left(-\tanh(f)\frac{m\omega x^2}{\hbar}\right)
\end{aligned}$$

In high temperature limit, this becomes

$$\begin{aligned}
p(x) &\approx \sqrt{\frac{m\omega f}{\pi\hbar}}\exp\left(-f\frac{m\omega x^2}{\hbar}\right) \\
&= \sqrt{\frac{m\omega^2\beta}{2\pi}}\exp\left(-\beta\frac{m\omega^2 x^2}{2}\right)
\end{aligned}$$

which is the classical Maxwell-Boltzmann distribution.

2.2 p distribution

$$\begin{aligned}
p(p) &= \int \mathrm{d}x W \\
&= \frac{\tanh(f)}{\pi\hbar}\int \mathrm{d}x \exp\left(-\tanh(f)\left(\frac{m\omega x^2}{\hbar}+\frac{p^2}{m\omega\hbar}\right)\right) \\
&= \sqrt{\frac{\tanh(f)}{\pi m\omega\hbar}}\exp\left(-\tanh(f)\frac{p^2}{m\omega\hbar}\right)
\end{aligned}$$

Similar to the x case, in high temperature limit, this becomes

$$\begin{aligned}
p(p) &\approx \sqrt{\frac{f}{\pi m\omega\hbar}}\exp\left(-f\frac{p^2}{m\omega\hbar}\right) \\
&= \sqrt{\frac{\beta}{2\pi m}}\exp\left(-\beta\frac{p^2}{2m}\right)
\end{aligned}$$

which is also the classical Maxwell-Boltzmann distribution.