# Effect of a slight change in polarization on light shift

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## 1   Goal

Vector and tensor light shift from an optical tweezer can cause coupling between different angular momentum states that were not allowed without it. This effect can be different if the tweezer polarization changes between site.

While it's fairly easy to control when there's only vector light shift, just use linear polarization, the effect can be much more complex when tensor light shift terms. It's of course always possible to turn off all of the coupling with perfect $\pi$ polarization (since it doesn't break rotational symmetry around $z$ and therefore doesn't couple different $m_F$ states) but non-$\pi$ linear polarization could still have nontrivial effects in the experiment.

Questions to answer,

1. Can the solution for vector light shift, i.e. using pure linear polarization, eliminate all the unwanted coupling in the presence of tensor light shift (section 2).

2. For a single tweezer, can the effect from different source, e.g. circularity of polarization, angle of polarization, angle of B field, cancel out each other and to what level, i.e. is it possible to achieve "optimal" result without $\pi$ polarization (section 3).

3. If yes, is it possible to have local optimal (e.g. cancellation of some terms at a condition that doesn't cancel all terms), and if that's the case, what is the best way to align everything.

4. In general, what is the condition for the coupling to the wrong state being insensitive to certain type of change in the polarization. E.g. is there a magic certain polarization that is at least first order insensitive to arbitrary small angle rotation (i.e. linear polarization when there's only vector shift).

We'll mainly (only) consider the state mixing for a single state. This is fine since the system we want to study here, $^{171}$Yb neutral, has a $J = 0, I = 1/2$ ground state that should have a very small (if any) vector shift. Any existing vector shift should also be factored into a slightly tweaked excited state vector shift (at least to the first order).

We'll also mostly consider the coupling for spontaneous emission. For darkness of state during OP, one could almost always adjust the OP polarization to cancel out any unwanted coupling. With the ground state setting the quantization axis, the presence of unwanted coupling/decay path is simply proportional to any non-diagonal element in the Hamiltonian.

## 2 Linear polarization

Assuming the polarization of the tweezer is linear and selecting the quantization axis for the ground state, we'd like to see what coupling could be caused by a linear polarization that is not pure-$\pi$. As mentioned above, this can only be caused by the tensor term. Tensor shift hamiltonian ignoring constant terms that causes no coupling,

$$H_t \propto \left\{ \vec{u}^* \cdot \vec{F}, \vec{u} \cdot \vec{F} \right\} \tag{1}$$

For linear polarization $\vec{u} = \vec{u}^*$

$$H_t \propto 2\left( \vec{u} \cdot \vec{F} \right)^2 \tag{2}$$

If the polarization is very closed to $\pi$ but off by a small angle $\theta$ in the $x$ direction (can also be any other direction in the $x$-$y$ plane without loss of generality)

$$\vec{u} = \cos\theta \hat{z} + \sin\theta \hat{x} \tag{3}$$

$$\begin{aligned}
H_t &\propto 2(\cos\theta F_z + \sin\theta F_x)^2 \\
&= 2\left( \cos^2\theta F_z^2 + \sin\theta\cos\theta(F_z F_x + F_x F_z) + \sin^2\theta F_x^2 \right) \\
&\approx 2\left( \cos^2\theta F_z^2 + \frac{1}{2}\sin\theta\cos\theta(F_z F_+ + F_z F_- + F_+ F_z + F_- F_z) \right) \\
&= 2\left( \cos^2\theta m_F^2 + \frac{1}{2}\sin\theta\cos\theta((2m_F+1)F_+ + (2m_F-1)F_-) \right) \\
&= 2\cos^2\theta m_F^2 + \sin\theta\cos\theta((2m_F+1)F_+ + (2m_F-1)F_-)
\end{aligned} \tag{4}$$

Here we've ignored the second order term (in $\theta$) and calculated the matrix element in the $z$ basis.

The result shows that the tensor light shift can cause a first order coupling between the neighboring $m_F$ state when the polarization is not $\pi$ even if it remains linear. This is different from vector light shift which only have non-trivial effect for circular polarization.

## 3 General polarization

Tensor term for generic polarization,

$$\begin{aligned}
H_t &\propto \left( u_x^* F_x + u_y^* F_y + u_z^* F_z \right)\left( u_x F_x + u_y F_y + u_z F_z \right) + \left( u_x F_x + u_y F_y + u_z F_z \right)\left( u_x^* F_x + u_y^* F_y + u_z^* F_z \right) \\
&= 2|u_x|^2 F_x^2 + 2|u_y|^2 F_y^2 + 2|u_z|^2 F_z^2 \\
&\quad + 2\mathrm{Re}\left( u_x u_y^* \right)(F_x F_y + F_y F_x) + 2\mathrm{Re}(u_x u_z^*)(F_x F_z + F_z F_x) + 2\mathrm{Re}(u_y u_z^*)(F_y F_z + F_z F_y) \\
&= |u_x|^2 \frac{(F_+ + F_-)^2}{2} - |u_y|^2 \frac{(F_+ - F_-)^2}{2} + 2|u_z|^2 F_z^2 \\
&\quad + \frac{\mathrm{Re}\left( u_x u_y^* \right)}{2i}((F_+ + F_-)(F_+ - F_-) + (F_+ - F_-)(F_+ + F_-)) \\
&\quad + \mathrm{Re}(u_x u_z^*)((F_+ + F_-)F_z + F_z(F_+ + F_-)) - i\mathrm{Re}(u_y u_z^*)((F_+ - F_-)F_z + F_z(F_+ - F_-)) \\
&= \frac{\left( |u_x|^2 - |u_y|^2 \right)\left( F_-^2 + F_+^2 \right) + \left( |u_x|^2 + |u_y|^2 \right)(F_+ F_- + F_- F_+)}{2} + 2|u_z|^2 F_z^2 + \frac{\mathrm{Re}\left( u_x u_y^* \right)}{i}\left( F_+^2 + F_-^2 \right) \\
&\quad + \mathrm{Re}(u_x u_z^*)((F_+ + F_-)F_z + F_z(F_+ + F_-)) - i\mathrm{Re}(u_y u_z^*)((F_+ - F_-)F_z + F_z(F_+ - F_-))
\end{aligned} \tag{5}$$

Ignoring all the diagonal terms in $z$ basis calculate the matrix element,

$$H'_t = \left(\frac{|u_x|^2 - |u_y|^2}{2} - \mathrm{iRe}\big(u_x u_y^*\big)\right)\big(F_+^2 + F_-^2\big)$$
$$+ \,(\mathrm{Re}(u_x u_z^*) - \mathrm{iRe}(u_y u_z^*))(2m_F + 1)F_+ + (\mathrm{Re}(u_x u_z^*) + \mathrm{iRe}(u_y u_z^*))(2m_F - 1)F_- \tag{6}$$

We can see that unless $\dfrac{|u_x|^2 - |u_y|^2}{2}$, $\mathrm{Re}\big(u_x u_y^*\big)$, $\mathrm{Re}(u_x u_z^*)$ and $\mathrm{Re}(u_y u_z^*)$ are all zero the effect of the tensor shift will not be zero (which requires $u_x = u_y = 0$ or pure $\pi$ polarization).

Moreover, since the term that contains a single $F_+$ or $F_-$ contains $m_F$ dependent pre-factor, they cannot be fully cancelled by the vector term which only contains terms that are proportional to $F_+$ or $F_-$ without $m_F$ dependent pre-factor. It is possible, however, for the effect to be cancelled for the stretched state since they are coupled by only a single $F_\pm$ term and the coefficient can in principle be cancelled by the vector term. For this to work in the experiment, the second order (i.e. $F_+^2$ and $F_-^2$) terms from the tensor effect still need to be zero (the vector term cannot cancel them out). This requires both $\dfrac{|u_x|^2 - |u_y|^2}{2}$ and $\mathrm{Re}\big(u_x u_y^*\big)$ to be 0, or in another word $u_x = \pm i u_y$.

The vector term

$$H_v \propto \mathrm{i}(\vec{u} \times \vec{u}^*) \cdot \vec{F}$$
$$= 2\mathrm{Im}\big(u_y^* u_z\big)F_x + 2\mathrm{Im}(u_z^* u_x)F_y + 2\mathrm{Im}(u_x^* u_y)F_z$$
$$= \mathrm{Im}\big(u_y^* u_z\big)(F_+ + F_-) - \mathrm{iIm}(u_z^* u_x)(F_+ - F_-) + 2\mathrm{Im}(u_x^* u_y)F_z \tag{7}$$
$$= \big(\mathrm{Im}\big(u_y^* u_z\big) - \mathrm{iIm}(u_z^* u_x)\big)F_+ + \big(\mathrm{Im}\big(u_y^* u_z\big) + \mathrm{iIm}(u_z^* u_x)\big)F_- + 2\mathrm{Im}(u_x^* u_y)F_z$$

Assuming $u_x = \pm i u_y$, define $\mathcal{R} \equiv \mathrm{Re}(u_x u_z^*)$, $\mathcal{I} \equiv \mathrm{Im}(u_x u_z^*)$

$$\mathrm{Re}(u_y u_z^*) = \pm\,\mathcal{I} \tag{8}$$
$$\mathrm{Im}\big(u_y^* u_z\big) = \pm\,\mathcal{R} \tag{9}$$

Off diagonal terms,

$$H'_v = \pm\,(\mathcal{R} \mp \mathrm{i}\mathcal{I})F_+ \pm (\mathcal{R} \pm \mathrm{i}\mathcal{I})F_- \tag{10}$$
$$H'_t = (\mathcal{R} \mp \mathrm{i}\mathcal{I})(2m_F + 1)F_+ + (\mathcal{R} \pm \mathrm{i}\mathcal{I})(2m_F - 1)F_- \tag{11}$$

The coefficient in front of the $F_+$ and $F_-$ terms are the same (or at least have the same polarization dependency). This means that the effect of the two cannot be cancelled out by adjusting the polarization (of the tweezer) but may be cancelled out by selecting the right wavelength. It also means that unless the wavelength is magic (for this particular coupling), the only good coupling for a single tweezer is $\pi$.