The Big Square with the Missing Corner

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A method attributed to the Sulbasutras for obtaining an approximation for the square root of two is generalized to derive a series expansion whose coefficients are given by a recursion relation. The partial sums are shown to be equivalent to the succeeding iterations of the square root algorithm $r_{i+1} = \frac{1}{2}(r_i + \frac{N}{r_i})$ for $\sqrt{N}$. The relation with the so-called Pell's equation is shown. The method is generalized to find similar series expansions for obtaining rational approximations to square roots of other non-square integers.

The Sulbasutras are among the oldest known mathematical manuals and provide geometric rules for altar construction. They are dated to 800 BCE and were introduced to the West by German Indologist George Thibaut in 1875. They are most famous for the first ever known general statement of the so-called Pythagoras theorem. However, amongst the several interesting constructions and rules stated, they also contain the following rule for the approximate length of the diagonal of a square of unit length, viz., the square root of 2, stated in the Baudhayana Sulbasutra i.61 and i.62 (see, for example [1], pp. 188):

Increase the measure by its third and this third by its own fourth less the thirty-fourth part of that fourth. This measure is savisheshah *[i.e. excess].*

which translates to

$$\sqrt{2} \approx 1 + \frac{1}{3} + \frac{1}{3 \cdot 4} - \frac{1}{3 \cdot 4 \cdot 34}. \tag{1}$$

The Sanskrit word *savisheshah* in the original *sutra* says explicitly that the above value exceeds the actual value. No hint is given as to how it is derived, and several scholars have attempted to derive the possible method used. In this article we will revisit some of these methods and show how the most commonly accepted version today can be generalized to obtain an infinite series whose terms are related by a recursion relation. The successively better approximations obtained by keeping successive terms of this series are solutions to Pell's equation, as will also be shown. In the end it will be shown how to generalize this method to obtain the square roots of other numbers.

I. Thibaut's Reconstruction

In the same article in which Thibaut introduced the Sulbasutras to the Western world [4], he attempted also to explain how the authors of the Sulbasutras could have arrived at the expression for the square root of 2, stated in Eqn. (1). Although generally not accepted today, his method is worth recalling due to its historical significance.

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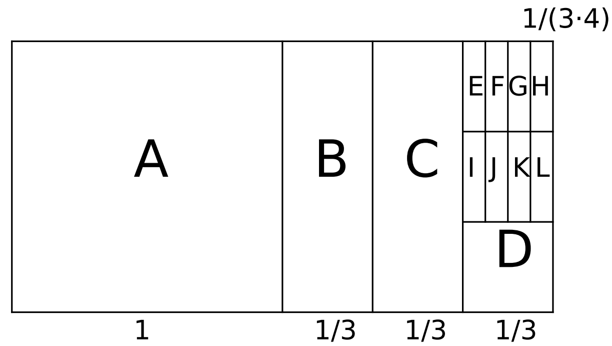

Fig. 1 Datta's proposed method for the $\sqrt{2}$ expression in the Sulbasutras.

According to Thibaut, the Sulbasutra authors noticed that the area of a square with side of length 17 is 289, while the area of the square formed by the hypotenuse of an isosceles right angle triangle with side 12 equals $2 \times 12^2 = 288$, which is quite close to 289. So, as a first approximation one can set $17^2 \approx 2 \times 12^2$, which gives $\sqrt{2} \approx 17/12$. This is actually an overestimate, since 17^2 exceeds 2×12^2. To obtain a better approximation, Thibaut proposed that the authors of the Sulbasutras again considered a square with side 17, and attempted to remove a strip of width x from two perpendicular sides, with x chosen such that the resulting square has an area of 288, i.e. 2×12^2. Then the length of the side of the new square is $17 - x$, and $(17 - x)^2 = 288 = 2 \times 12^2$, so that $\sqrt{2} = (17 - x)/12$.

How should x be chosen? Thibaut's hypothesis was that the Sulbasutra authors took away a strip of breadth x from one side of the square of length 17, so that the resulting area was $289 - 17x$, and after taking away a strip of breadth x from an adjoining side, they took the resulting area to be $289 - 34x$. Note that the two deductions result in the small corner square of side x (i.e. of area x^2) being deducted twice, so that the actual area of the new square should be $289 - 34x + x^2$. But since x is smaller than 1 (if it was not, the new square would have an area lesser than 288), they, according to Thibaut, neglected the x^2 term. Solving for x from the resulting linear equation, we get 577/408 as the new approximation to $\sqrt{2}$, i.e., the Sulba approximation of Eqn. (1).

Thibaut's reconstruction does not explain why the Sulba approximation is given as a sum of four different unit fractions instead of stating 577/408 directly. Below we will see another reconstruction which yields the exact Sulbasutra expression and which has the added advantage of being related to another *sutra* for finding the square root of a number by rearranging a rectangle into a square.

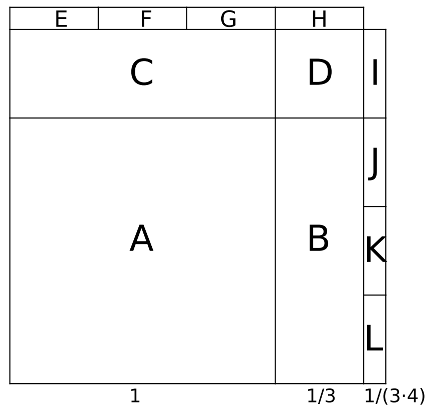

Fig. 2 Converting the rectangle of Fig. 1 into a square.

II. Datta's Reconstruction

This reconstruction, proposed by Datta *, goes as follows. Take a rectangle with sides of length 2 and 1 and divide this into smaller squares and rectangles as shown in Fig. 1. Rearrange these parts as shown in Fig. 2. This results in a square with a small square of side $1/12$ missing at the top right corner. Ignoring the missing corner, the length of the side of the big square is $1 + 1/3 + 1/(3 \times 4)$, which equals $17/12$ and which we call L_0:

$$L_0 = 1 + \frac{1}{3} + \frac{1}{3 \times 4} = \frac{17}{12}. \tag{2}$$

This is the value obtained by adding the first three terms of the Sulba expression i.e. $1 + \frac{1}{3} + \frac{1}{3\times4}$. It is clear that this value slightly exceeds the actual value of $\sqrt{2}$. To improve upon this, Datta proposed that the Vedic sages attempted to fill up the square gap of side $1/12$ by cutting off rectangular strips from the lower and left sides of the big square and fit these into the gap. To understand this, let us call the width of the missing square on the top right, which equals $1/12$, as $w_{initial}$. We will attempt to fill this gap by cutting a strip of width w_0 along the lower and left sides of the big square such that the total area removed fits into the square gap, as shown in Fig. 3 (not to scale).

The area removed when the lower horizontal strip is taken away is $L_0 w_0$, where $L_0 = 17/12$. The area removed when the left vertical strip is taken away would have been also $L_0 w_0$, but for the fact that a square of side w_0 at the bottom left corner is being removed twice. For the moment we ignore this and pretend that the area being removed is again $L_0 w_0$. Hence the total area removed is twice this, that is, $2L_0 w_0$. This is set equal to $w_{initial}^2$, the area of the missing corner at the top right. Solving for w_0 gives:

*[1], pp. 192-194.

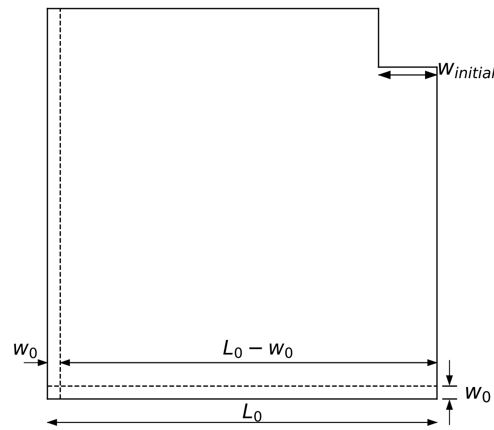

Fig. 3 w_0 **is chosen such that** $2L_0w_0 = w_{initial}^2$

$$w_0 = \frac{w_{initial}^2}{2L_0} = \frac{1}{3 \times 4 \times 34}, \tag{3}$$

where we have used $w_{initial} = 1/12$ and $L_0 = 17/12$. Since we have removed a square of area w_0^2 from the bottom left corner twice, we will now have a missing square of area w_0^2 in the top right corner. Hence the new situation will be exactly as that represented in Fig. 3, with L_0 replaced by $L_0 - w_0$ and $w_{initial}$ by w_0. This immediately allows us to repeat this method iteratively, as will be shown later.

After the rearrangement, the side of the big square is reduced by w_0. This is the next approximation to $\sqrt{2}$, which we call L_1:

$$L_1 = L_0 - w_0 = 1 + \frac{1}{3} + \frac{1}{3 \times 4} - \frac{1}{3 \times 4 \times 34}. \tag{4}$$

This is exactly the Sulba approximation for $\sqrt{2}$, it being explicitly mentioned by the term *savisheshah* that the above exceeds the actual value.

Note also that the shape shown in Fig. 2 has area 2, as it is simply a rearrangement of a rectangle of area 2. But this also equals the difference of the areas of square with sides L_0 and $w_{initial}$. Hence:

$$L_0^2 = 2 + w_{initial}^2 = 2 + \frac{1}{12^2}. \tag{5}$$

Similarly, after the rearrangement,

$$L_1^2 = 2 + w_0^2 = 2 + \frac{1}{(3 \times 4 \times 34)^2}. \tag{6}$$

Datta's proposed method of rearranging a rectangle into a square for finding the square root of 2 is related, although not exactly identical, to *sutra* i.54 from the Baudhayana Sulbasutra, which describes a generic method for transforming a rectangle into a square:

Translation (from [3]): *If you wish to turn a rectangle into a square, take the shorter side of the rectangle for the side of a square, divide the remainder into two parts and, inverting, join those two parts to two sides of the square.*

In general, the square root of any number N can be found by rearranging a rectangle with sides N and 1 into a square. The length of the square is then $\sqrt{N}$. Obviously for non-square numbers like 2, 3, 5 etc. the procedure cannot be exact and only approximations can be found.

III. Extension of Datta's Method

We saw that after the rearrangement, we again had a new square with a missing corner. This suggests that we can again cut away strips from the sides and attempt to fill in the corner to obtain an even better approximation to $\sqrt{2}$. In fact this can be done iteratively, as was noted by Henderson [3], to obtain an infinite series for $\sqrt{2}$. Let us illustrate the process for the next iteration. After the rearrangement, we have a square of length L_1 with a missing corner of side w_0. To fill in this corner, remove a strip, this time of width w_1, from the bottom and left edges of the big square. Neglecting again that w_1^2 is deducted twice, the total area of the strips is $2L_1w_1$, which should equal w_0^2, the area of the missing corner. This gives

$$w_1 = \frac{w_0^2}{2L_1}. \tag{7}$$

After rearranging, the new length of the square is L_2, where:

$$L_2 = L_1 - w_1. \tag{8}$$

But again, since we have removed w_1^2 twice, the rearrangement is not complete and a gap of area w_1^2 will still remain. Let us go through the arithmetic for getting w_1 and L_2. From Eqn. (4),

$$L_1 = 1 + \frac{1}{3} + \frac{1}{3 \cdot 4} - \frac{1}{3 \cdot 4 \cdot 34} = \frac{(3 \cdot 4 \cdot 34) + (4 \cdot 34) + 34 - 1}{3 \cdot 4 \cdot 34}. \tag{9}$$

Combining with Eqn. (3), we get:

$$w_1 = \frac{w_0^2}{2L_1} = \frac{1}{2} \cdot \frac{1}{3 \cdot 4 \cdot 34} \cdot \frac{1}{(3 \cdot 4 \cdot 34) + (4 \cdot 34) + 34 - 1}. \tag{10}$$

A simple calculation yields:

$$2 \times \{(3 \cdot 4 \cdot 34) + (4 \cdot 34) + 34 - 1\} = 1154 \tag{11}$$

so that we get for w_1 and L_2:

$$w_1 = \frac{1}{3 \cdot 4 \cdot 34 \cdot 1154}, \tag{12}$$

$$L_2 = 1 + \frac{1}{3} + \frac{1}{3 \cdot 4} - \frac{1}{3 \cdot 4 \cdot 34} - \frac{1}{3 \cdot 4 \cdot 34 \cdot 1154}, \tag{13}$$

which is the next approximation to $\sqrt{2}$. Again, we have:

$$L_2^2 = 2 + w_1^2 = 2 + \frac{1}{(3 \cdot 4 \cdot 34 \cdot 1154)^2}. \tag{14}$$

A. The Next Iteration

The next iteration is carried out in the same way. The length of the new square is

$$L_3 = L_2 - w_2, \tag{15}$$

where w_2 is

$$w_2 = \frac{1}{3 \cdot 4 \cdot 34 \cdot 1154 \cdot 1331714} \tag{16}$$

and the expression for L_3 is

$$\begin{aligned} L_3 = 1 + \frac{1}{3} \quad &+ \quad \frac{1}{3 \cdot 4} - \frac{1}{3 \cdot 4 \cdot 34} - \frac{1}{3 \cdot 4 \cdot 34 \cdot 1154} \\ &- \quad \frac{1}{3 \cdot 4 \cdot 34 \cdot 1154 \cdot 1331714}, \end{aligned} \tag{17}$$

which is the next iterative approximation to $\sqrt{2}$. As before, we have $L_3^2 = 2 + w_2^2$, that is,

$$L_3^2 = 2 + w_2^2 = 2 + \frac{1}{(3 \cdot 4 \cdot 34 \cdot 1154 \cdot 1331714)^2}. \tag{18}$$

B. The General Result

Thus, we get the sequence 3, 4, 34, 1154, 1331714, etc. Let us try to derive the general term of this sequence. In the previous sections, we saw that

$$\begin{aligned} 34 &= 2 \times \{(3 \times 4) + (4) + 1\} \\ 1154 &= 2 \times \{(3 \times 4 \times 34) + (4 \times 34) + (34) - 1\} \\ 1331714 &= 2 \times \{(3 \cdot 4 \cdot 34 \cdot 1154) + (4 \cdot 34 \cdot 1154) \\ & \quad +(34 \cdot 1154) - 1154 - 1\} \end{aligned} \tag{19}$$

etc. The pattern is clear. Denoting the sequence 3, 4, 34, 1154, etc. by x_1, x_2, x_3, etc. we have

$$\begin{aligned} x_n &= 2 \times \{\Pi_{i=1}^{n-1} x_i + \Pi_{i=2}^{n-1} x_i + \Pi_{i=3}^{n-1} x_i \\ &- \Pi_{i=4}^{n-1} x_i - \Pi_{i=5}^{n-1} x_i - \cdots \\ &- (x_{n-2} x_{n-1}) - x_{n-1} - 1\}. \end{aligned} \tag{20}$$

The first three terms in the curly brackets are added, while all the rest are subtracted.

A useful recursion relation can be derived from the above. Factoring out x_{n-1} from all terms on the right involving this term, and using the same relation as Eqn. (20) for x_{n-1}, it follows that

$$x_n = x_{n-1}^2 - 2. \tag{21}$$

The next two numbers can be shown to be 1773462177794 and 3145168096065837266706434, and so on. We thus have the following series expansion for $\sqrt{2}$:

$$\sqrt{2} = 1 + \frac{1}{x_1} + \frac{1}{x_1 x_2} - \frac{1}{x_1 x_2 x_3} - \frac{1}{x_1 x_2 x_3 x_4} - \cdots \tag{22}$$

with $x_1 = 3$, $x_2 = 4$, $x_3 = 34$, and the rest of the x_n related by Eqn. (21). From the expressions for L_1, L_2, L_3, (Eqns. (9), (13), (17)) etc., it also follows that

$$L_n = \frac{1}{2} \cdot \frac{x_{n+3}}{x_1 x_2 \cdots x_{n+2}}. \tag{23}$$

The above are the successive rational approximations for $\sqrt{2}$ given by the Sulbasutra algorithm. These are listed out in Table 1, along with their respective precisions.

n	x_n	L_n	Precision
1	3	$\frac{4}{3}$	10^{-1}
2	4	$\frac{17}{12}$	10^{-3}
3	34	$\frac{577}{408}$	10^{-6}
4	1154	$\frac{665857}{470832}$	10^{-12}
5	1331714	$\frac{886731088897}{627013566048}$	10^{-24}

Table 1 Successive rational approximations to $\sqrt{2}$ using the Sulbasutra method and their precision.

Similarly, from the expressions for w_1, w_2, (Eqns. (10), (16)) etc., it follows that

$$w_n = \frac{1}{x_1 x_2 \cdots x_{n+3}}, \tag{24}$$

and finally, using that fact that the difference between the area of the square of side L_n and the square gap of side w_{n-1} equals 2, it follows that

$$L_n^2 - w_{n-1}^2 = 2. \tag{25}$$

IV. Relation to Pell's equation

The Pell's equation is a family of equations given by:

$$Nx^2 + 1 = y^2, \tag{26}$$

where N is a positive integer and integral solutions in x and y are sought. (The "Pell's equation" was solved by Brahmagupta (seventh century CE), Bhaskara and Jayadeva (both 12th century CE) with the chakravala algorithm.) The history of the "Pell's equation", erroneously named after John Pell (17th cent. CE) by Leonhard Euler, goes back to the Hindu mathematicians Brahmagupta (seventh century CE), Bhaskara and Jayadeva (both 12th century CE), who found a general algorithm for finding the solutions, which is called the chakravala algorithm. They applied the algorithm and found the solutions even to the case of $N = 61$, which is especially difficult. Centuries later, Pierre de Fermat (17th cent.) posed exactly this as a challenge problem to the mathematicians of his time. For a more detailed exposition of the history and method of the chakravala algorithm, see [2].

The approximations obtained by the Sulbasutra method have a close connection to Pell's equation for $N = 2$. From Eqn. (23), we have $L_n = Y_n/X_n$, where $Y_n = \frac{1}{2}x_{n+3}$ and $X_n = x_1 x_2 \cdots x_{n+2}$. Using Eqns. (23) and (24) in Eqn. (25), it follows that

$$2X_n^2 + 1 = Y_n^2, \tag{27}$$

which is the Pell's equation for $N = 2$. In addition, using Eq. (21) and (27), the following relation can be derived in addition:

$$x_{n+4} = 2 + 8x_1^2 x_2^2 \cdots x_{n+2}^2. \tag{28}$$

Brahmagupta used solutions to the so-called Pell's equation to find rational approximations to the square roots of non-square integers. As an interesting aside, mention may be made of the Pell numbers and the Pell-Lucas numbers. The Pell numbers form a sequence defined by $p_n = 2p_{n-1} + p_{n-2}$, with $p_0 = 0$ and $p_1 = 1$, whereas the Pell-Lucas numbers are also defined as $q_n = 2q_{n-1} + q_{n-2}$, but with $q_0 = q_1 = 2$. Each term of the Pell-Lucas sequence divided by 2, and each term of the Pell sequence, have the property that they satisfy, respectively, Eqn. (26) for $N = 2$. Moreover, as we have just shown, the numerators and denominators of the Sulbasutra approximations also satisfy this equation. Thus it follows that they will also be found in these sequences. For example, $q_4/2 = 17$ and $p_4 = 12$, and as we have seen, this is a solution to Eqn. (26) for $N = 2$, and these numbers also form the Sulbasutra approximation L_0 given in Eqn. (2). Similarly, $q_8/2 = 577$ and $p_8 = 408$, which are found in the next approximation L_1 given by Eqn. (4), which equals $577/408$. Similarly, $q_{16}/2 = 665857$ and $p_{16} = 470832$, which equal the next approximation L_2 given by Eqn. (13), viz., $665857/470832$, etc.

V. Relation to the Square Root Algorithm

A well-known algorithm for iteratively obtaining rational approximations to the square root of a number N consists of starting with an initial guess, r_0, for the square root, and successively replacing it as follows:

$$r_{i+1} = \frac{1}{2} \cdot \left(r_i + \frac{N}{r_i} \right). \tag{29}$$

The algorithm works because if $r_i < N$, then $N/r_i > N$, and vice versa, so that at each successive iteration r_i and N/r_i approach closer and closer to each other; in other words r_i approaches the square root of N.

The Sulbasutra method has a close connection to this method. As we have seen, the Sulbasutra method gives us successively $\frac{17}{12}$, $\frac{577}{408}$, etc. as the rational approximations to $\sqrt{2}$. Similarly, starting with $r_1 = \frac{4}{3}$, we get, from Eqn. (29), $r_2 = \frac{17}{12}$ as the next approximation, $r_3 = \frac{577}{408}$ as the next approximation, etc. Thus, the square root algorithm and the Sulbasutra method give exactly the same results for the successive approximations. This surprising result can be proven as follows. Starting from $\frac{4}{3}$ as the starting guess, we see that both the methods give $\frac{17}{12}$ at the next iteration. Let us assume that the n-th step the result obtained by the square root algorithm and the Sulbasutra method are identical, i.e., both yield L_n, which is defined in Eq. (23). At the next iteration, the square root algorithm gives us

$$L'_{n+1} = \left(\frac{1}{2x_1x_2\cdots x_{n+3}}\right)\left[\frac{x_{n+3}^2}{2} + 4(x_1x_2\cdots x_{n+2})^2\right]. \tag{30}$$

From Eqn. (27), we get $4(x_1x_2\cdots x_{n+2})^2 = \frac{x_{n+3}^2}{2} - 2$, using which the term in the square bracket can be written as $x_{n+3}^2 - 2$ which, from Eqn. (21), equals x_{n+4}. Thus we have

$$L'_{n+1} = \frac{x_{n+4}}{2x_1x_2\cdots x_{n+3}}, \tag{31}$$

which from Eqn. (23) can be seen to be the same as L_{n+1}, the next approximation from the Sulbasutra method. Thus, by induction, the Sulbasutra method and the square root algorithm give identical results if we start with $1 + \frac{1}{3}$ as the initial guess.

As a historical note, it may be mentioned that, according to Otto Neugebauer's conjecture, the square root algorithm of Eqn. (29) was already known to the Babylonians and was used by them for obtaining the approximation to the square root of 2 as given on the Babylonian clay tablet YBC 7289 [6]. Thus it has become a widespread practice to credit the above algorithm to the Babylonians. However, there is no direct evidence for such a claim, as pointed out by Fowler and Robson [7].

VI. Extensions

As pointed by Henderson [3], the method described in this article of finding the square root by rearranging a rectangle into a square can be used to find successive rational approximations for the square root of any positive integer that can be expressed as the difference of the squares of two rational numbers. In the present case, from Fig. 2 and Eqn. (5) we have

$$2 = \left(\frac{17}{12}\right)^2 - \left(\frac{1}{12}\right)^2, \tag{32}$$

from which the the process of rearrangement was iteratively carried out to obtain better and better approximations to $\sqrt{2}$. Similarly, the square root of 5 can be obtained by this method as well, since it can be written as the difference of the squares of two rational numbers:

$$5 = \left(\frac{9}{4}\right)^2 - \left(\frac{1}{4}\right)^2. \tag{33}$$

As an example, let us try to find the square root of 5. The first step is provided by *sutra* i.54 which directs us to rearrange a rectangle of sides 1 and 5 into a square. This can be done by dividing the rectangle into five squares each of side 1, and rearranging four of them, which we call A, B, C and D, and dividing the fifth into 4 smaller equal strips, which we call E, F, G and H, and rearranging them as shown in Fig. 4. This gives us the initial approximation for $\sqrt{5}$ as $2 + 1/4$. It can

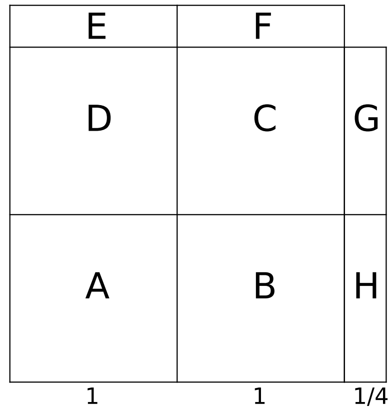

Fig. 4 The Sulbasutra method for finding the approximation to $\sqrt{5}$ by rearranging a rectangle of sides 1 and 5.

be an exercise for the reader to verify that the series expansion for $\sqrt{5}$ based on the Sulbasutra method is be found to be:

$$
\begin{aligned}
\sqrt{5} \quad = \quad & 2 + \frac{1}{4} - \frac{1}{4 \cdot 18} - \frac{1}{4 \cdot 18 \cdot 322} \\
& - \frac{1}{4 \cdot 18 \cdot 322 \cdot 103682} - \cdots ,
\end{aligned} \tag{34}
$$

where the terms of the sequence 4, 18, 322, 103682, $\cdots$ from the third term onwards are again related by Eqn. (21), and the numerators and denominators of the successive solutions satisfy the so-called Pell's equation $5X^2 + 1 = Y^2$.

VII. Conclusions

In his article we introduced the Sulbasutra expression for a rational approximation to $\sqrt{2}$, and outlined various proposals how this value could have been arrived at. We demonstrated how the commonly accepted proposal, namely, the one proposed by Datta, could be extended to any order to generate an infinite series for the square root of 2. We showed the existence of a recursion relation connecting the terms of the series, and also the remarkable result that the rational approximations thus obtained are identical to the successive iterations of the square root algorithm. We further showed that the rational approximations satisfy the so-called Pell's equation with coefficient 2. Finally, we pointed out how this method could be applied to find rational approximations to square roots of other integers.

We conclude with a few historical observations on the Sulbasutras. Thibaut's 1875 translation of the Sulbasutras created an uproar in the Western world, as it showed that, if the Sulbasutras predated the ancient Greeks, the hitherto accepted belief of ancient Greece being the originator of mathematics would be no longer tenable. Scholars such as the extremely influential Moritz Cantor countered by initially assigning a date as late as 140 CE to the Sulbasutras, and thereby inferring that they were derived from Greek mathematics. However, when it became undeniable that the Sulbasutras predated Greek mathematics, he tried to point to Egypt instead as being a possible influence on Greek mathematics [9]. With the advent of Neugebauer's research in the 1930s, the focus shifted entirely to Babylonian mathematics. The hesitation to consider the Sulbasutras as a possible influence on early Greek mathematics was absent in the case of Babylonian mathematics, as the following comment by Neugebauer shows: *"What is called Pythagorean in the Greek tradition had better be called Babylonian..."* [8]. Moreover, the proclivity of Western scholars to locate Babylonian mathematics as a "precursor" to classical Greek mathematics and Western civilization has been pointed out by scholars such as Robson [5], which also could lead to a certain kind of bias when it comes to Babylonian mathematics.

The Sulbasutras are one of the oldest known documents in the history of mathematics. In 1978, A Seidenberg [9] undertook a detailed investigation of Babylonian mathematics, early Greek mathematics and the Sulbasutras, and rejecting the hypothesis that the Sulbasutras were influenced by Babylonian mathematics, came to the conclusion that *"A common source for the Pythagorean and Vedic mathematics [i.e. the mathematics contained in the Sulbasutras] is to be sought either in the Vedic mathematics or in an older mathematics very much like it. ... What was this older, common source like? I think its mathematics was very much like what we see in the Sulbasutras."*

The Sulbasutras thus appear to have played a more significant role in the history of mathematics than has been usually supposed. Unfortunately, as Seidenberg puts it, *"they have never taken the position in the history of mathematics that they deserve"* [9]. Nevertheless, it is remarkable that even after so many centuries, they remain a fertile source of interesting mathematical ideas.

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