Credit allocation and bank competition: an IO approach

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Comments welcome

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Abstract: this compilation of work analyses oligopolistic model of bank competition when a foreign bank enters the market. The result suggest that the most cost efficient bank would have a larger share of the market (de posit). If both the local bank and the foreign bank have the same efficiency than they can evenly share the market. However if we use von-stakelberg approach where a foreign bank has entered a market where the domestic bank was already in operation , the credit allocation may not be efficient.

Introduction

Frexias and Rochet (1997) enlists four main functions of contemporary banking theory:

(1) Access to payment system.
(2) Transformation of short term deposits to long-maturity loan
(3) To manage risk
(4) To process information and monitor borrowers.

In Brief, Frexias and Rochet (1997), point out that bank transforms inputs (deposits) to output (in form of loans) as long as it has competitive advantage over the borrower. Moreover, as the banks provide costly services to the economic agents ; they have an incentive to compete for better market position . The factors which influence their decisions are the quality of loans, quality of deposits , interest rates and the quality of services.

The Monti-Klien (MK) model (1972) was the first model that used the IO approach to model the banking sector. The model considers a monopolistic bank with a cost function which captures the servicing of the loan and deposits. Bank determines the interest rate for deposit and loan that would maximize its profit. The decision variables are the amount of loan and the amount of deposit with the amount of equity as given. The MK model calculates the bank's profit and the sum of the intermediation margins on both loans and deposits. The MK model also suggested that the bank sets its interest rate such that the Learner's index is inversely proportional to the demand. The intuitive result was higher market power of the bank would lead to better intermediation margin. Additionally, the market of deposits and loans are interrelated through the cost function.

The Monti-Klien Model

The Monti-Klien model considers a single, monopolistic bank that chooses its outputs in order to maximize profits. The bank operates on the market for loans as well as on the market for deposits. The difference between the volume of loans L and the volume of deposits D of the bank can be borrowed (or lent, if negative) on an Interbank market. The interest rates on the loan market and deposit market are represented by r_L and r_D, respectively. The inverse demand function for loans is given by $r_L(L)$, with derivative $r`_L(L) < 0$, and the inverse supply function of deposits is $r_D(D)$, with derivative $r`_D(D) >$

0. The cost of managing an amount L of loans and an amount D of deposits is given by the convex management-cost function $C(L;D)$. Let r denote the exogenous interest rate on the interbank market, and α be the exogenous fraction of deposits that is required as capital reserve. Both r and α are set by the central bank.
The bank's decision problem is to maximize its profits $\Pi(L;D)$, i.e.

$$\pi(L,D)=(r_L(L)-r)L+(r(1-\alpha)-r_D(D))D-C(D,L)$$

Where, $\pi(L,D)$ is strictly concave. The first-order conditions are :

$$\frac{\partial\pi}{\partial L}=r_L{}'(L)L+r_L-r-C'_L(D,L)=0$$

$$\frac{\partial\pi}{\partial D}=-r_D{}'(D)D+r(1-\alpha)-r_D-C'_D(D,L)=0$$

Now relating the First order conditions to the elasticity of demand and loan respectively, the MK model derive the equalities between the Learner's indices (price-cost divided by price) and inverse elasticity. There adaptation of the IO to banking sector suggested the following outcomes:

(1) Intermediation margins are higher when banks have a higher market power.
(2) A monopolist bank would set his loan and deposit volumes such that the Lerner's indices equal inverse elasticity.
(3) When costs are separable, optimal deposit rate is independent of the deposit market.

MK model provide a very simplified approach to banking operations with a series of conclusions that can be confronted with empirical evidence (Freixas and Rochet, 1997). One of the more prominent researches that applied the IO model was that of Neuberger and Zimmerman (1990) where they tested the validity of the MK model in explaining the "California Rate mystery". The Mk model suggested an explanation based on the sellers concentration which was higher in California than in other parts of United States.

The original Klein-Monti model concentrates on the case of a single, monopolistic bank, which might apply in countries with only one (state) bank. But the situation of several banks is more interesting. In fact, as Molyneux et al. (1994) observe for the case of Europe, in many countries including India, the banking industry is very concentrated. This suggests that oligopoly models would be more relevant for the study than monopolistic model. We extend the Klein-Monti model to the case of more than one bank, In particular, towards a Cournot oligopoly, in which both the banks (local and foreign) are assumed to have a linear management-cost function. We assume that the foreign banks operating in LDC have access to cheaper funds due to their diverse exposure compared to the domestic bank.

Model Synthesis

In this proposal, MK model is sought to be extended in order to study the entry of foreign banks in domestic market. The entry of foreign banks (entrant) is modeled as a Stackelberg mixed-motive game with entry barrier due to information asymmetry. In addition, credit risk and collateral as screening technology in the MK model are incorporated. As in the von Stackelberg duopoly game, the incumbent (here the domestic bank) commits to a particular level of deposits and loans. This is due to the fact that the incumbent bank had been operating for some time and hence has better knowledge about the volume of loan and deposits it can expect. The banks are assumed to be risk neutral.

Assumptions.

In this section we present the assumptions related to the proposed model. The assumptions are adapted from other existing literature related to the Mk based models().

[**Assumption 1**] Local government control entry of financial institution. Only two banks are allowed to act as financial intermediaries: a foreign bank (as a wholly own Subsidiary) and an incumbent bank. Let $i = F, L$, where f denotes the foreign bank and l is the local bank.

[**Assumption 2**] Each bank offers deposits (D) and Loans (L_i). Further, economic agents save through the banks only which offers a deposit rate R_D. The demand for deposit is an inverse function of R_D with $R_D{}'(D) > 0$. Also, the larger companies do not offer market based obligations. Here, $D = D_f + D_l$

[**Assumption 3**] projects can only be financed through loans with an interest rate R_L (where, $R_L(L)$ and $R_L{}'(L) < 0$) and, $L = L_f + L_l$.

[**Assumption 4**] the proportion of loan unpaid after their due date id θ_i where $i = F, L$

[**Assumption 5**] Interbank markets are assumed to be risk free with no exchange factor. The interest rate is R which is controlled by the central bank.

[**Assumption 6**] loans and the deposits have the same marginal costs C_i [D, L]. Due to constant cost there is no advantage for either bank related to economies of scope.

[**Assumption 7**] the banks have a fixed level of equity capital K_i and following the prudential norms (Basel) each bank maintains a risk-adjusted-capital ratio . $K_i \geq \gamma(1+\theta_i)L_i$

[**Assumption 8**] banks are assumed to be risk neutral.

The Objective Function

The objective function can be determined based on the assumptions stated in the previous section. We consider a two period framework where the banks have the following balance sheet.

Domestic bank (Incumbent)	
Loan (L_l)	Deposits (D_l)
Interbank position (M_l)	Equity (K_l)

Which changes, after a time period such that :

Domestic Bank (incumbent)	
$(1-\theta_l)L_l(1+R_L)$	$D_l(1+R_D)$

$M_l(1+R)$	$C_l(L_l+D_l)$

Similarly for the foreign bank , the balance sheet can be shown as :

Foreign Bank	
Loan (L_f)	Deposits (D_f)
Interbank position (M_f)	Equity (K_f) Parent bank fund (F)

Balance sheet at the end of the period would be :

Foreign Bank	
$(1-\theta_f)L_f(1+R_L)$	$D_f(1+R_D)$
$M_f(1+R)$	$C_f(L_f+D_f)$
	$(1+R_e)F$

The liquidation value of the banks at the end of the time period will be:

$$V_l=(1-\theta_l)L_l(1+R_L)+M_l(1+R)-D_l(1+R_D)-C_l(L_l+D_l)$$

and replacing M_l by $D_l+K_l-L_l$ in the above equation, we obtain the profit function :

$$\pi_l=V_l-K_l=[(R_l-R-C_l)-\theta_l(1+R_l)]L_l+RK_l+(R-R_D-C_l)D_l........(1)$$

Similarly in the case of foreign bank, we obtain:

$$\pi_f=V_f-K_f=[R_L(1-\theta_f)-R-C_f-\theta_f]L_f+RK_f+(R-R_D-C_f)D_f+(R-R_e)F......(2)$$

Equation (1) and (2) represent the objective function of the two players (banks) respectively. According to the stated assumptions, each player maximizes its objective function, subjected to the equity constraint:

$$K_i\geq\gamma(1+\theta_i)L_i$$

as mentioned in the assumption. Thus the constrained maximization problem can be represented, for each player respectively, as:

$$\Gamma_l=[(1-\theta_l)R_l(L)-R-\theta_l-C_l]L_l+RK_l+[R-R_D(D)-C_l]D_l-\lambda_l[K_l-\gamma(1+\theta_l)L_l]....(4)$$

$$\Gamma_f=[(1-\theta_f)R_L(L)-R-\theta-C_f]L_f+RK_f+[R-R_D(D)-C_f]D_f+(R-R_e)F-\lambda_f(K_f-\gamma(1+\theta_f)L_f]......(5)$$

where, λ_l and λ_f are the Lagrange's multiplier for the domestic bank and the foreign bank respectively. The value of the multipliers are positive (or zero) if the constraint is binding (not binding)

Solution to the Function

Given the Cournot conjectures, the players maximize their objective function (4) and (5) respectively. The first order condition for the banks:

$$\frac{\partial \Gamma_l (L_l, D_l)}{\partial L_l} = (1-\theta_l)(R_L + R_L' L) - R - \theta - C_l + \lambda_l [\gamma (1+\theta_l)] = 0 ....(6)$$

$$\frac{\partial \Gamma_l (L_l, D_l)}{\partial D_l} = R - C_l - R_D - R_D{}' D_l = 0 ....(7)$$

similarly for the foreign bank, the F.O.C will be :

$$\frac{\partial \Gamma_f (L_f, D_{f)}}{\partial L_f} = (1-\theta_f)(R_L + R_L' L) - R - \theta_f - C_f + (R - R_e) F' + \lambda_f [\gamma (1+\theta_f)] = 0 ....(8)$$

$$\frac{\partial \Gamma_f (L_f, D_f)}{\partial D_f} = R - C_f - R_D - R_D{}' D_f = 0 ....(9)$$

equilibrium condition when the constraint is non binding

The equilibrium condition for the loan market can be derived from the equation (7) and (9). Similarly the equilibrium conditions for the deposit market can be derived form the equations (6) and (8) respectively. But the banks generally lend less than the permissible limit i.e. they hold more than what is required under capital adequacy norm. an article in economist (1997) shows that thirteen banking system actually holds more than the stipulated capital. [1] Under such scenario, the constraint is not binding and hence the Lagrange's multiplier is zero. Under such conditions, the equilibrium in the market for deposits can be obtained by solving (6) and (8) . We obtain:

$$D_f{}^{FB} = D_l{}^{FB} + \frac{C_l - C_f}{R_D'} ....(10)$$

The result suggest that the most cost efficient bank would have a larger share of the market (de posit). If both the local bank and the foreign bank have the same efficiency than they can evenly share the market. Under such symmetric cost (i.e. $C_l = C_f = C^*$) we can determine the equilibrium of the deposit market is :

$$\frac{R - C^* - R_D}{R_D} = \frac{R_D{}' D}{2 \times R_D} ....(11)$$

this can be arranged as :

$$\frac{R - C^* - R_D}{R_D} = \frac{1}{2 \in} ....(12)$$

where, $\in$ is the elasticity of demand for the deposit market . The equation is similar to the Learner's indices; which is equal to the inverse of product of the elasticity and the number of banks (here, two). The index is positive.

[1] Argentina, Brazil, Chile ,Columbia, India, Indonesia, Japan, Malaysia, Mexico, south Korea, Taiwan, Thailand and United States. It can be seen most of the countries in the list are Emerging Market Economies.

The equilibrium for the other market (loans) can be obtained using the equations (7) and (9) . We consider the case when the banks don't take the restrictions on equity (both , $\lambda_l = \lambda_f = 0$) . The relationship shows:

$$L_f^{FB} = \frac{(1-\theta_l)L^{FB}{}_l}{(1-\theta_f)} - \frac{(C_l - C_f)}{R_L'(1-\theta_f)} + \frac{(\theta_f - \theta_l)(1+R_l)}{R_L'(1-\theta_f)} - \frac{(R-R_e)F'}{R_L'(1-\theta_f)} \text{.........(13)}$$

The steps are shown in the proof (I). It can be inferred from literature that $\theta_l \geq \theta_f$ and if the cost are symmetric ($C_l = C_f = C*$), then at equilibrium, foreign banks will lend more as compared to the local bank.

Proposition 1 *: under the conditions (i) when banks chooses loan such that the equity constraint is not binding,(ii) default risk for the foreign banks are similar or less than their local counter part and, (iii) both the banks have the same cost , then :*
[a] at the equilibrium in the deposit market both the banks have the same deposit.
[b] Foreign banks give more loans compared to the local bank.

Proof: the proof for [a] can be obtained form equation (10) when the cost is symmetric. For [b] , consider the following scenario:

Case I : $\theta_l = \theta_f$, i.e. the bank faces the same credit risk . we find that $R_L' < 0$

as $R_L' < 0$.

Case II : when , $\theta_l > \theta_f$ we need to show that $L_f^{FB} > L_l^{FB}$.

From the equation (13) , we obtain :

$$R_L' \underbrace{(L_l^{FB}(1-\theta_l) - L_f^{FB}(1-\theta_f))} = (\theta_l - \theta_f)(1+R_L) + (R-R_e)F' \text{.....(14)}$$

the right hand side of the equation (14) is positive as R>R $_e$ and also $\theta_l > \theta_f$. Hence the term in the bracket (highlighted above) must be negative. Hence , by rearranging the terms , we obtain:

$$\frac{L_l^{FB}}{L_f^{FB}} < \frac{(1-\theta_f)}{(1-\theta_l)}$$

or ,

$L_l^{FB} < L_f^{FB}$ as $\theta_f < \theta_l$

proposition 1 suggest that , given our assumptions , *when both the banks start their operation at the same time* then a foreign bank would be better of in credit allocation, while they would be able to share the deposit market. the reason is two folds:

(i) Access to less costly international funds from the parent bank.
(ii) Better credit risk assessment skills due to their international exposure and thus the credit risk for local banks are more compared to the foreign banks.

Winton (1997) suggested that the above result is valid only when the foreign banks are able to overcome the information, cultural and political barrier of operating in a foreign region. Thus only when a foreign bank has overcome the initial difficulties and have similar branching network (size) then they would be in better position to allocate credit (Proposition 1).

Second order conditions

This section describes the stability conditions for equilibrium and the reaction function of each bank I both the market. In order to get the global maxima for the banks the second partial derivative must be negative, when calculated at the optimal values.

For stability, each market must satisfy:

$$\frac{\partial^2 \prod_l}{\partial D_l^{\,2}}\frac{\partial^2 \prod_f}{\partial D_f^{\,2}}-\frac{\partial^2 \prod_l}{\partial D_l \partial D_f}\frac{\partial^2 \prod_f}{\partial D_l \partial D_f}>0$$

$$\frac{\partial^2 \prod_l}{\partial L_l^{\,2}}\frac{\partial^2 \prod_f}{\partial L_f^{\,2}}-\frac{\partial^2 \prod_l}{\partial L_l \partial L_f}\frac{\partial^2 \prod_f}{\partial L_l \partial L_f}>0$$

for each of the banks. The equilibrium for both the markets are stable when the direct effect is more than the cross effect. In our analysis the stability condition is met as the second term is negative as the marginal profit for one of the player decreases as other player receives more deposits (or allocate more loans).

the slope of the reaction function for both the banks can be obtained from the differentiation of the equation (7) and (9) respectively for the inverse demand function. Similarly, the differentiation of the equation (8) and (10) would give the slope of the reaction function for loans in each case.

The slope of the inverse demand function can be obtained using the implicit function theorem (Simon, Blume, page 339) .we get the expression:

$$S_l^{\,D} = -\frac{\partial^2 \prod_l}{\partial D_l^{\,2}} \Big/ \frac{\partial^2 \prod_l}{\partial D_l \partial D_f} < 0$$

$$= \frac{-(2R_D{}' + R_D{}'' D_l)}{(R_D{}' + R_D{}'' D_l)} \text{........(15)}$$

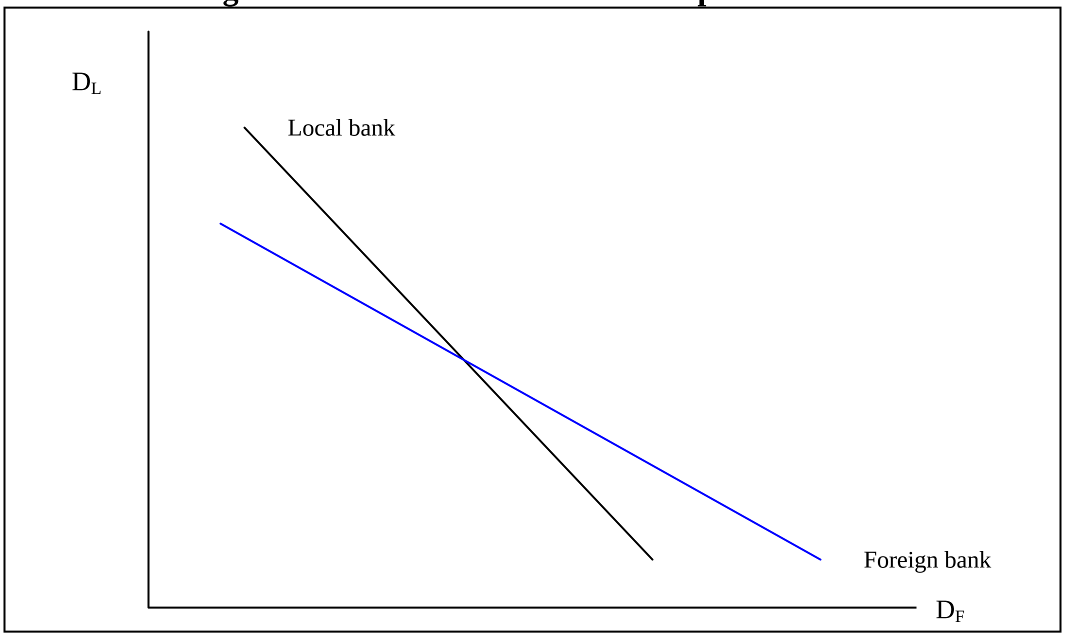
and for foreign bank the slope is :

$$S_f^{\,D} = -\frac{\partial^2 \prod_f}{\partial D_l \partial D_f} \Big/ \frac{\partial^2 \prod_f}{\partial D_f^{\,2}} < 0$$

$$= \frac{(R_D{}' + R_D{}'' D_f)}{-(2R_D{}' + R_D{}'' D_f)} \text{........(16)}$$

The slope of the reaction function of the local bank is steeper than that of the foreign bank. For a linear demand line, the reaction function would be as shown in the figure 1.

Figure1. Reaction functions in the deposit market



(I) first order condition for the equilibrium of Loan markets

$$\frac{\partial \Gamma_l(L_l, D_l)}{\partial L_l} = (1-\theta_l)(R_L + R_L' L) - R - \theta - C_l + \lambda_l[\gamma(1+\theta_l)] = 0........(6)$$

$$\frac{\partial \Gamma_f(L_f, D_f)}{\partial L_f} = (1-\theta_f)(R_L + R_L' L) - R - \theta_f - C_f + (R - R_e)F' + \lambda_f[\gamma(1+\theta_f)] = 0........(8)$$

$$\lambda_l = \lambda_f = 0$$

the equation then can be expressed as :

$$(1-\theta_l)(R_L + R_L' L_l) - R - \theta_l - C_l = (1-\theta_f)(R_L + R_L' L_f) - R - \theta_f - C_f + (R - R_e)F'$$

rearranging the terms , we obtain :

$$R_L'(1-\theta_f)L^{FB}{}_f = R_L'(1-\theta_l)L^{FB}{}_l - (C_l - C_f) + (\theta_f - \theta_l)(1+R_l) - (R - R_e)F'$$

which further gives:

$$L_f{}^{FB} = \frac{R_L'(1-\theta_l)L^{FB}{}_l}{R_L'(1-\theta_f)} - \frac{(C_l - C_f)}{R_L'(1-\theta_f)} + \frac{(\theta_f - \theta_l)(1+R_l)}{R_L'(1-\theta_f)} - \frac{(R-R_e)F'}{R_L'(1-\theta_f)}$$

or

$$L_f{}^{FB} = \frac{(1-\theta_l)L^{FB}{}_l}{(1-\theta_f)} - \frac{(C_l - C_f)}{R_L'(1-\theta_f)} + \frac{(\theta_f - \theta_l)(1+R_l)}{R_L'(1-\theta_f)} - \frac{(R-R_e)F'}{R_L'(1-\theta_f)}$$

QED.