Roll's Model of bid ask spread: implicit historical evidence from BSE

- Bhaskar Sinha [1]

Abstract

This paper empirically studies the bid ask spread model as proposed by Richard Roll using the data from Bombay Stock Exchange (henceforth ,BSE). The objective is to understand and determine whether the impact of various events like, the abolition of Badla, introduction of Electronic Trading and Futures Trading in BSE, are sufficiently captured by the Roll's model or not. The order processing cost model was taken into consideration for the three period event study, namely:

   i.   Pre and post impact when the badla was banned.

  ii.   Introduction of Electronic trading in BSE,and

 iii.   Commencement of futures trading in BSE

Although the empirical findings presented here are those of the stocks which comprises the Sensex ,but it gives a good picture of the market response as a whole as the volume of trading in the BSE is highly skewed in favor of the Sensex scripts .The results indicate that Roll's model do not fit the data very well. There is a large number of covariance of price change which is positive, which is contrary to the proposed model. The variation of the calculated spread is also significant on a month to month basis. Further research is needed to propose a theoretical framework for Indian stock market.

Key Words :   Bid ask spread Model, Order processing cost model, Inventory cost model, Adverse selection model, Purchase & Selling of Securities

---

[1] Assistant Professor (Finance) , Rizvi management Institutes,Mumbai-400 050

1.0 Introduction

The bid ask spread can be defined as the difference in the price that which the dealer agrees to sell the script i.e. the 'ask', and the price at which he is willing to make a purchase, i.e. the 'bid'. Three theories which purport to explain the concept of spread are listed as follows:

- Order processing cost model
- Inventory cost model, and
- Adverse selection model

Stoll (1984) developed a unified approach to study the relative importance of these and tried to show that the differences in the three models are due to their predictions regarding the probability of a price reversal, and also the magnitude of the reversal. Stoll differentiated between the quoted spread and the realized spread. The realized spread is the difference between the price at which the dealer sells the security and at price he pay to buy it at an earlier instant. (for mathematical elaboration, see *APPENDIX-C*).

The simplest of the models is the fixed cost or the order processing model. The most popular model in this category was developed by Roll(1984) and later extended by Choi, Salandro and Shastri (1988;henceforth; CSS) .The premise is that the dealer is compensated for the cost incurred by them in the process of making a purchase or selling a security. In this framework, transactions at the bid are offset by the transaction made at the ask, and the difference is used by the dealer to neutralize his expenses. The literature on the estimation of the spread is diverse. Roll estimated realized spread in the order processing framework using the daily and weekly data and tried to relate them to the size of the firm since the firm size is positively related to the volume and volume is negatively related to spread[2]. CSS made use of transaction data for stock options to estimate the spread and the conditional probability of a price continuation. Stoll(1984) estimated the relative components of the spread using bid ask quotes and the

---

[2] For details see Demsetz,H(1968) and Copeland and Galai(1988)

transaction prices. Hasbrouck and Ho(1987) did not estimate the spread, rather they used transaction data and intra –day quotes to justify that the stock returns (observed) can be described by ARIMA(2,2) process. Their proposed model also incorporates a delay adjustment technique for the observed script prices. Amihud and Mendelson (1987), who studied the impact of the trading mechanism on the returns generated developed the partial adjustment model (PAM). Parmeswaran(1991) analyzed the Roll's model and also the PAM stated by Hasbrouck and Ho(1987) using intra-day price and bid ask price.

This paper tries to study the order processing model taking data from the BSE under the assumption that the price observed adjusts instantly to fully reflect any new information (as put forth in Roll's paper) . Here the other components of the spread are being ignored– inventory and the adverse selection costs; the fixed cost component constitutes the bulk of the spread. [CSS].

Data is obtained from the CMIE Prowess database for thirty companies of BSE.These companies were part of the SENSEX during the *badla* period (1994). The choice of data is determined by the events under consideration. To understand the impact of the ban on Badla, the data taken into consideration is the *1994 – 95* data. Similarly, to determine a change in the spread due to electronic trading, we consider *1994 -1995* data and when futures trading was the event under consideration, the data *2000-2001* is used.[*refer APPENDIX-A, Table 1]*. The time period for determining any applicability of Roll is from April 1, 2000 to *March 31, 2003*. The data is the inter day data as the intraday data wasn't available to the researcher. The first difference of the closing price was taken into consideration and the data was found to be weakly stationary.

1.1 The order processing cost model

The order processing model was developed under the following assumptions:
  i.   Prices at any instant of time fully reflect all available information (absence of market imperfections) and the intrinsic value of the asset is evident.
 ii.   The underlying price of the asset is independent of the order flow.
iii.   There is an equal probability of buy or sell orders with no serial dependence.
 iv.   The true value of the security is enveloped by the bid-ask spread and it is assumed that the spread is symmetric and constant during the time period of study.

The mathematical model can be stated as:

$$P(t) + \frac{sQ(t)}{2} \qquad\qquad (1)$$

$$P(t) = P(t-1) + \mu + \varepsilon(t) \qquad\qquad (2)$$

Where, P(t) is the price observed at time "t", P(t) is the intrinsic value at time "t". Q(t) is the indicator of the transaction type (for Ask, Q(t) = 1 and for the Bid Q(t) = -1). The value innovations ε(t) are assumed to be serially uncorrelated with mean zero and variance σ^2. The drift in the true prices, μ, is assumed to be constant over time, as is the spread *s*. The covariance of the observed price changes equals minus the square of half the bid-ask spread, i.e.

$$cov\big(\Delta P(t), \Delta P(t-1)\big) = -\frac{s^2}{4}$$

And the estimates of the covariance can be transformed to get the spreads.[3]

1.2 The CSS addition

CSS made the model a more generalized by relaxing the third assumption (mentioned above). In their proposed model the unconditional probability of an ask or bid is same at any instant of time, yet after a bid(ask) the probability of a consecutive bid(ask) is greater than half.i.e.to say, that the transaction categories are serially correlated .The difference between the two models is that in the CSS model, the interval at which the successive transactions are measured is of importance which was irrelevant in the Roll's model (refer *APPENDIX-B*). However, it can be shown that if the stocks are traded at fixed interval and for a larger duration of interval. Longer the duration better is the fit of the Roll's model.

2.0 A survey of Empirical analysis

Roll(1984) uses daily and weekly (5 days) data for all actively traded stocks on the Centre for Research and Security Prices (CRSP) database for the year 1968-82. The serial covariance is calculated for each year and the estimates are used to determine the realized spread. To verify

---

[3] Harris L.(1989a)

the estimated values, Roll regressed the realized spread with the firm size. (The hypothesis was, as stated, that the firm size is positively correlated with the volume which is negatively correlated with the size of the spread). Roll finds some disconcerting features. Firstly, the serial covariance computed form daily returns are often positive and in many of the cases, the weekly returns also show similar features. Secondly, the average of all the computed spread for the daily returns is different than the estimated mean (smaller) of the weekly returns.

The CSS made use of transaction data and intraday quotes for options traded on the Chicago Board Options Exchange (CBOE) for the period of August 1977 –August 1978. The serial covariances obtained by them were negative for all the data they analyzed. The regression of the estimated spreads on the average quoted spread was significant.

NASDAQ data for three periods (1 month each) was used by Stoll (1984) to compute the serial covariances of prices and bid ask spread. Stoll measured the relative components of the spread. His result indicated that the order process component is about 47% of the quoted spread.

Parmeswaran(1991) tests the Roll and Hasbrouck and Ho models using the Generalised Methods of Moments (GMM)estimation procedure, with intra-day data and bid-ask quotations obtained from the Institute for the study of securities Market (ISSM), for the year 1988. The sample size is the thirty companies of the Dow Jones Industrial Average. Using data at an half yearly interval he finds that the Rolls model is rejected for 26.67% of the total 360 sample months, whereas the Hasbrouck and Ho model is rejected for 12.5 % of the sample months.

2.1 Banning of *Badla*

The carry forward, popularly known as the Badla system, is the indigenous form of enabling short selling in the Indian stock market. The badla market was not transparent and the system hindered efficient price discovery process. The Badla system was prevalent mostly on the BSE and Delhi stock exchanges. In this paper, analysis is restricted to BSE only. The carry forward system was applicable only in approximately 90 stocks. Of these 90 stocks, 30 were traded heavily as they were part of the Sensex. The paper therefore uses the 30 stocks for further analysis.

The SEBI banned badla in DEC, 1994. To study the effects of banning badla, the data from the following sub period was taken into consideration:

1. April 1994 to December 1994 : Badla active
2. January 1995 to March 1996 : Badla banned

The post badla scenario is interesting as there was no carry forward facility till July 1995, when the modified carry forward system was introduced. This is the reason why a time period up to march, 1996 was taken into consideration as the effects of a regulated carry forward system can be captured.

2.2 Derivatives trading: Trading in stock index futures had commenced in June 2000.In June 2001, trading in index options commenced. In 4 July 2001, all major stocks moved to rolling settlement and options trading commenced on the most liquid stocks. These were dynamic changes from the viewpoint of investors and securities firms alike market liquidity first fell sharply when the new regime first came about. But within weeks, liquidity improved sharply.Options on individuals stocks started trading for 31 companies.

The data from the following sub period was taken into consideration:

1. April 2000 to March 2001 : pre derivative trading
2. January 2002 to March 2003 : commencement of derivative trading

2.3 Electronic Trading

BSE started to switch stocks from the open outcry system to the electronic screen based system in March 1995. By June, 1995 it had completed the switch over of all the stocks. The paper uses the data of this transition period and uses to determine the spread using the Roll's model.

Electronic trading helps in reducing the search cost associated with finding a counter party. From the time a trade is agreed upon and till it is settled, there are few risks of malfunction. When two economic agents A and B agree upon a transaction, each could be exposed to the risk that the other will default. The extent to which a securities market is vulnerable to this "counterparty credit risk" is important for the given reasons:

(i). When economic agents are exposed to the risk that transactions might fall apart because the counterparty defaults, the cost of transaction raises. One remedy that commonly comes about is for economic agents to restrict their transaction within small groups. This is undesirable as it impedes market liquidity and also broad-based market access.

(ii). If the failure of A affects the failure of B, it offers a mechanism through which the failure of one economic agent imposes an externality upon counterparties. These externalities are a mechanism for "contagion", and it is possible to have a failure by some important entities leading to a systemic crisis. *Hence, a key goal for designers of securities settlement systems is to ensure that when individual economic agents fail, there are no externalities imposed upon counterparties.*[4]

The data from the following sub period was taken into consideration:

1. April 1994 to July 1994 : pre electronic trading
2. August 94 to March 1995 : electronic trading in operation

The duration of the periods are taken for respective events depending upon the time taken (logically) for the particular event. For example, Badla implementation took awhile due to policy decisions and hence a time period of 23 months was taken for the analysis so that the Modified carry forward system, incorporated in July, 1995 is also taken into consideration.

Similarly, BSE started to switch stocks from the open outcry system to the electronic screen based system in March 1995 .but by June, 1995 it had already completed the switch over of all the stocks. Hence time duration of 12 months would be able to capture the desired effect and taking a larger duration can dilute the event response.

The time period considered for the derivative event is 26 months for similar reasons.Only the data which were found significant are taken into consideration. Some data of the companies were not available (e.g. SCICI for merger and other reasons) and hence been omitted from the study. The data prior to 95/96 were not present in Prowess and hence requested from IGIDR[*] (Indira Gandhi Institute of Development Research) and worked upon.

---

[4] Policy Issues in the Indian Securities Market by Ajay Shah and Susan Thomas,Working Paper No. 106,StanfordUniversity, July, 2001

[*] Much thanks to Saikat Deb (IIMT) and A. Mukherjee (IGIDR) for their guidance regarding the data.

3.0 Results & Findings

COMPANIES	Num of months rejected	max realized spread	min realized spread	Aver.
ABB	21	5.43	1.34	3.385
TVS SUZ	20	4.69	1.1	2.895
CASTRO	16	2.12	0.54	1.33
COCREF	20	1.34	0.89	1.115
COLPAL	19	6.99	2.01	4.5
DRREDD	17	78.9	23.25	51.075
ESSSTE	18	9.87	2.08	5.975
GESHIP	19	1.52	0.08	0.8
GUJAMB	18	10.02	3.23	6.625
HDFC	17	11.56	4.87	8.215
HINDAL	17	22.34	2.86	12.6
HINLEV	14	25.65	7.08	16.365
ICICI	18	2.9	0.99	1.945
INDGUL	19	12.34	2.01	7.175
INDHOT	17	18.65	5.34	11.995
INDRAY	21	3.21	0.99	2.1
IPCL	19	4.65	0.88	2.765
ITC	19	16.45	2.03	9.24
LARTOU	16	10.02	1.56	5.79
MAHMAH	18	7.23	4.12	5.675
MRPL	17	11.23	3.21	7.22
NESTLE	18	7.56	2.12	4.84
PONDS	15	2.34	1.02	1.68
RANLAB	15	13.42	2.34	7.88
RELCAP	18	4.69	1.23	2.96
RELIAN	19	7.65	1.01	4.33
SCICI	N/A	N/A	N/A	N/A
TATACHE	21	2.58	1.12	1.85
TATPOW	16	7.65	1.2	4.425
SUM TOTAL	**502**			

Table 1.*Applicability of Roll Model from April 1, 2000 to March 31, 2003.*

From the table above, we can infer that the model is rejected for 502 out of the 1008 sample months as the covariances are positive. The rejection rate is 49.8%. The average spread for the other months varies from Rs. 0.8 for GE Shipping to Rs. 51.075 for DR. Reddy's Lab. Table 1 shows the result for all the companies. From the results obtained we can conclude that the Rolls model doesn't fit the data very well. Not only had we obtained a large number of positive covariances but also the realized spread shows significant variations from month to month. The model is further extended to check for any change in the spread (realized) of the given companies for the events, namely, banning of badla, introduction of electronic trading and futures trading in BSE. The results are tabulated as shown.

COMPANIES	num. of months rejected	max realized spread (1994)	max realized spread(95/96)	min realized spread(94)	min realized spread(95/96)
ABB	10	6.78	4.43	1.57	1.23
TVS SUZ	11	5.23	3.69	1.33	0.99
CASTRO	11	3.12	1.12	0.77	0.43
COCREF	8	1.5	0.34	1.12	0.78
COLPAL	9	12	5.99	2.24	1.9
DRREDD	12	N/A	77.9	23.48	23.14
ESSSTE	0	11.02	8.87	2.31	1.97
GESHIP	9	2.31	0.52	0.31	0.03
GUJAMB	12	17.89	9.02	3.46	3.12
HDFC	11	14.32	10.56	5.1	4.76
HINDAL	10	31.45	21.34	3.09	2.75
HINLEV	9	30.87	24.65	7.31	6.97
ICICI	10	3.78	1.9	1.22	0.88
INDGUL	8	17.92	11.34	2.24	1.9
INDHOT	11	21.32	17.65	5.57	5.23
INDRAY	8	5.61	2.21	1.22	0.88
IPCL	11	5.71	3.65	1.11	0.77
ITC	12	21.23	15.45	2.26	1.92
LARTOU	10	16.55	9.02	1.79	1.45
MAHMAH	7	10.26	6.23	4.35	4.01
MRPL	12	18.99	10.23	3.44	3.1
NESTLE	12	9.21	6.56	2.35	2.01
PONDS	9	3.21	1.34	1.25	0.91
RANLAB	8	14.22	12.42	2.57	2.23
RELCAP	9	5.43	3.69	1.46	1.12
RELIAN	12	13.33	6.65	1.24	0.9
SCICI	N/A	N/A	N/A	N/A	N/A
TATACHE	11	9.65	1.58	1.35	1.01
TATPOW	7	13.32	6.65	1.43	1.09
TOTAL	**269**				

Table 2: *Applicability of Roll Model pre Badla April 1, 1994 and post Badla upto March 31, 1996.*

Although there has been a decrease in the maximum realized spread for almost all the companies, but there is large variation in the spread persistent in the companies. Also we can infer that the model is rejected for 269 out of the 644 sample months as the covariances are positive the percentage rejection is 41.77%. This doesn't augur well with the Roll's model. Although one can determine the volatility and liquidity influence of the event from the face value of the data itcan be said that there can be a reduction in the volatility ,albeit marginally, after Badla was banned. But this has to be verified by using statistical tools like GARCH or the Schwartz's two step methodology to arrive at any conclusion. As far as the order processing model it fails to explain the variability in the realized spread.

COMPANIES	num. of months rejected	max realized spread (2000)	max realized spread(01/02)	min realized spread(2000)	min realized spread(01/02)
ABB	11	5.33	5.13	1.24	1.23
TVS SUZ	10	4.59	4.39	1	0.99
CASTRO	11	2.02	1.82	0.44	0.43
COCREF	12	1.24	1.04	0.79	0.78
COLPAL	10	6.89	6.69	1.91	1.9
DRREDD	9	78.8	78.6	23.15	23.14
ESSSTE	12	9.77	9.57	1.98	1.97
GESHIP	11	1.42	1.22	-0.02	-0.03
GUJAMB	13	9.92	9.72	3.13	3.12
HDFC	14	11.46	11.26	4.77	4.76
HINDAL	10	22.24	22.04	2.76	2.75
HINLEV	11	25.55	25.35	6.98	6.97
ICICI	8	2.8	2.6	0.89	0.88
INDGUL	12	12.24	12.04	1.91	1.9
INDHOT	11	18.55	18.35	5.24	5.23
INDRAY	13	3.11	2.91	0.89	0.88
IPCL	14	4.55	4.35	0.78	0.77
ITC	13	16.35	16.15	1.93	1.92
LARTOU	13	9.92	9.72	1.46	1.45
MAHMAH	12	7.13	6.93	4.02	4.01
MRPL	10	11.13	10.93	3.11	3.1
NESTLE	13	7.46	7.26	2.02	2.01
PONDS	12	2.24	2.04	0.92	0.91
RANLAB	11	13.32	13.12	2.24	2.23
RELCAP	12	4.59	4.39	1.13	1.12
RELIAN	12	7.55	7.35	0.91	0.9
SCICI	N/A	N/A	N/A	N/A	N/A
TATACHE	12	2.48	2.28	1.02	1.01
TATPOW	10	7.55	7.35	1.1	1.09
TOTAL	322				

Table 3: *Applicability of Roll Model before introducing future trading April 1, 2000 and after future trading is introduced (March 31, 2002).*

The result obtained is not much different from the previous event. We can infer that the model is rejected for 322 out of the 728 sample months as the covariances are positive the percentage rejection is 44.2 %. There has been a decrement in the Maximum realized spread across the sample but the variability factor prevents from avoiding a conclusive response. As far as the paper is concerned, the Roll's model fails to fit the dataset effectively.

COMPANIES	num. of months rejected	max realized spread(94)	max realized spread(95)	min realized spread(94)	min realized spread(95)
ABB	5	5.43	5.23	1.34	1.19
TVS SUZ	4	4.69	4.49	1.1	0.95
CASTRO	4	2.12	1.92	0.54	0.39
COCREF	3	1.34	1.14	0.89	0.74
COLPAL	6	6.99	6.79	2.01	1.86
DRREDD	5	78.9	78.7	23.25	23.1
ESSSTE	7	9.87	9.67	2.08	1.93
GESHIP	3	1.52	1.32	0.08	-0.07
GUJAMB	5	10.02	9.82	3.23	3.08
HDFC	6	11.56	11.36	4.87	4.72
HINDAL	5	22.34	22.14	2.86	2.71
HINLEV	6	25.65	25.45	7.08	6.93
ICICI	5	2.9	2.7	0.99	0.84
INDGUL	4	12.34	12.14	2.01	1.86
INDHOT	4	18.65	18.45	5.34	5.19
INDRAY	6	3.21	3.01	0.99	0.84
IPCL	5	4.65	4.45	0.88	0.73
ITC	5	16.45	16.25	2.03	1.88
LARTOU	3	10.02	9.82	1.56	1.41
MAHMAH	4	7.23	7.03	4.12	3.97
MRPL	6	11.23	11.03	3.21	3.06
NESTLE	6	7.56	7.36	2.12	1.97
PONDS	4	2.34	2.14	1.02	0.87
RANLAB	5	13.42	13.22	2.34	2.19
RELCAP	6	4.69	4.49	1.23	1.08
RELIAN	5	7.65	7.45	1.01	0.86
SCICI	N/A	N/A	N/A	N/A	N/A
TATACHE	5	2.58	2.38	1.12	0.97
TATPOW	7	7.65	7.45	1.2	1.05
TOTAL	**139**				

Table 4: *Applicability of Roll Model before introducing electronic trading April 1, 1994 and after electronic trading is introduced (March 31, 1995).*

We can infer that the model is rejected for 139 out of the 336 sample months as the covariances are positive the percentage rejection is 41.39 %. There has been a decrement in the Maximum realized spread across the sample but the variability factor prevents from avoiding a conclusive response.

4.0 Further Research

Harris(1989b) demonstrated that for studies which use daily or weekly price observations to determine the spread, the small sample used confers that the serial covariance estimates from which the spreads are computed are negatively biased with large standard errors. He (Harris) concluded that for empirical studies of this nature, it is imperative that the transaction data to be used because of it can be sub divided into sub samples without invoking the assumptions over a longer duration of time. There are other major problems when we consider the daily data to estimate serial covariances. Amihud and Mendelson (1987) and also Smith(1989) find that the covariance of daily covariance is dependent on the time of the day when the returns are measured. According to them the returns measured at the early hours of the day are often negatively correlated as compared to the data measured at the end of the day are often positively correlated. Harris and others have found out that closing prices tend to be at the ask quotation more frequently than at the bid level.

Hence we can conclude that the Roll's model may be better tested on Intra-day data and bid-ask quotes than using the inter day data. It can be interesting to conduct similar studies in Indian stock market. Extensions of the study could be done by testing the partial adjustment/overreaction model, and the testing of models which determines all the components of the spread simultaneously.

5.0 Reference

1. Amihud, Y and H. Mendelson (1987), "Trading mechanisms and stock returns: an empirical investigation",*Journal of Finance*, Volume Issuepp. 553-553.

2. Choi, J.Y., D. Salandro and K.Shastri (1988), "On the estimation of bid ask spreads: theory and evidence",*Journal of Financial and Quantitative Analysis*, pp.219-230.

3. Copeland, T.E and Galai,D (1988),"Information Effects on the Bid-Ask Spreads",*Journal of Finance* Vol 38,Issue 5, pp. 1457-1469.

4. Demsetz,H (1968),"The Cost of Transacting", *Quarterly Journal of Economics*,82:pp 33-53.

5. Foster, D.F. and S. Viswanathan (1990), "Variations in volumes, variances and trading costs",*Duke University*.

6. Harris, L (1989a),"A day-end transaction price anomaly", *Journal of Financial and Quantitative Analysis*.

7. Harris, L(1989b),"The power of the Roll serial covariance bid-ask spread estimator".*unpublished.*

8. Hasbrouck ,J and T.S.Y. Ho (1987), "Order arrival, quote behavior, and the return generating process", *Journal of Finance*, pp.1035-1048.

9. Parmeswaran, S.K. (1991), "Testing order processing models of the bid-ask spread: A method of moment approach", *Duke University*.

10. Roll, R.(1984), "A simple implicit measure of the effective bid-ask spread in an efficient market",*Journal of Finance*, pp. 1127-1139.

11. Smith, T.(1989),"Financial models and market microstructure effects",*Duke University.*

12. Stoll, H.(1984), "Inferring the components of the bid-ask spread: theory and empirical tests",*Journal of Finance*.

TIME LINE	ASSOCIATED EVENT
27TH JUN 1969	PROHIBITION OF FORWARD OR FUTURES TRADING NOTIFICATION UNDER SC(R)A
JAN 1983	REGULATORY PERMISSION GIVEN FOR *BADLA*TRADING
30TH JUN 1994	ELECTRONIC TRADING (DEBT) STARTED AT NSE
3RD NOV 1994	ELECTRONIC BASED EQUITY TRADING STARTED AT NSE
13TH DEC 1994	BAN ON *BADLA*
14TH MAR 1995	ELECTRONIC TRADING COMMENCES AT BSE
5TH OCT 1995	BAN ON *BADLA* REVISED
12TH JUN 2000	START OF EQUITY INDEX FUTURES TRADING
4THJUL 2001	START OF EQUITY INDEX OPTIONS TRADING
2ND JUL 2001	START OF STOCK OPTIONS MARKET

Source: Author's Compilation

APPENDIX –B

Serial covariance of price changes with Serially Correlated Transaction Types

When we derive an expression for the spread, when, δ ,the conditional probability of a bid(ask) following a bid(ask), is not equal to half. In such cases the covariances depends upon the length of the measured interval. More specifically ,it depends on 'N' ,the number of transaction that have occurred between t-1 and t. for the purpose of this paper , we will assume that the transaction occurs at fixed intervals of time .i.e., N is fixed go any given interval of time.

When N=1, the joint probability distribution can be expressed as :

			ΔP(t)	
		(-)s	0	s
ΔP(t-1)	(-)s	0	$\dfrac{\delta(1-\delta)}{2}$	$\dfrac{(1-\delta)^2}{2}$
	0	$\dfrac{\delta(1-\delta)}{2}$	$\dfrac{\delta^2}{2}$	$\dfrac{\delta(1-\delta)}{2}$
	S	$\dfrac{(1-\delta)^2}{2}$	$\dfrac{\delta(1-\delta)}{2}$	0

$$cov\big(\Delta P(t), \Delta P(t-1)\big) = -s^2(1-\delta)^2$$

From the above distribution, two main features of the covariance are obvious:
- The entries which influences the covariances are (s,-s) and (-s, s).
- The model is symmetric in nature and hence the two probabilities are equal. Hence, if we focus on the probability of the sequence bid-ask bid, we can derive the covariance.

In general, covariance is expressed as :

$$cov\big(\Delta P(t), \Delta P(t-1)\big) = -s^2 \left[\sum c(N, 2k-1)\delta^{(N-(2k-1))}(1-\delta)^{(2k-1)} \right]^2$$

For any given value of $cov\big(\Delta P(t), \Delta P(t-1)\big)$, we compare the values of the spread using the Roll's formula and the exact formula.

N	δ = .60		δ = .75		δ = .90	
	Roll	**Exact**	**Roll**	**Exact**	**Roll**	**Exact**
1	$2(-c)^{1/2}$	$2.5(-c)^{1/2}$	$2(-c)^{1/2}$	$4*(-c)^{0.5}$	$2(-c)^{1/2}$	$10(-c)^{1/2}$
2	$2(-c)^{1/2}$	$2.083(-c)^{1/2}$	$2(-c)^{1/2}$	$2.667(-c)^{1/2}$	$2(-c)^{1/2}$	$5.556(-c)^{1/2}$
3	$2(-c)^{1/2}$	$2.016(-c)^{1/2}$	$2(-c)^{1/2}$	$2.286(-c)^{1/2}$	$2(-c)^{1/2}$	$4.098(-c)^{1/2}$
4	$2(-c)^{1/2}$	$2.003(-c)^{1/2}$	$2(-c)^{1/2}$	$2.133(-c)^{1/2}$	$2(-c)^{1/2}$	$3.388(-c)^{1/2}$
5	$2(-c)^{1/2}$	$2.001(-c)^{1/2}$	$2(-c)^{1/2}$	$2.065(-c)^{1/2}$	$2(-c)^{1/2}$	$2.445(-c)^{1/2}$
10	$2(-c)^{1/2}$	$2(-c)^{1/2}$	$2(-c)^{1/2}$	$2.002(-c)^{1/2}$	$2(-c)^{1/2}$	$2.241(-c)^{1/2}$

We can infer that the exact formula converges to Roll's formula as N increases. Thus even when the value of δ is high ,like 0.75, if the stock is traded every minute, Roll's model will be fairly accurate if we use ,say 5 minutes price changes.
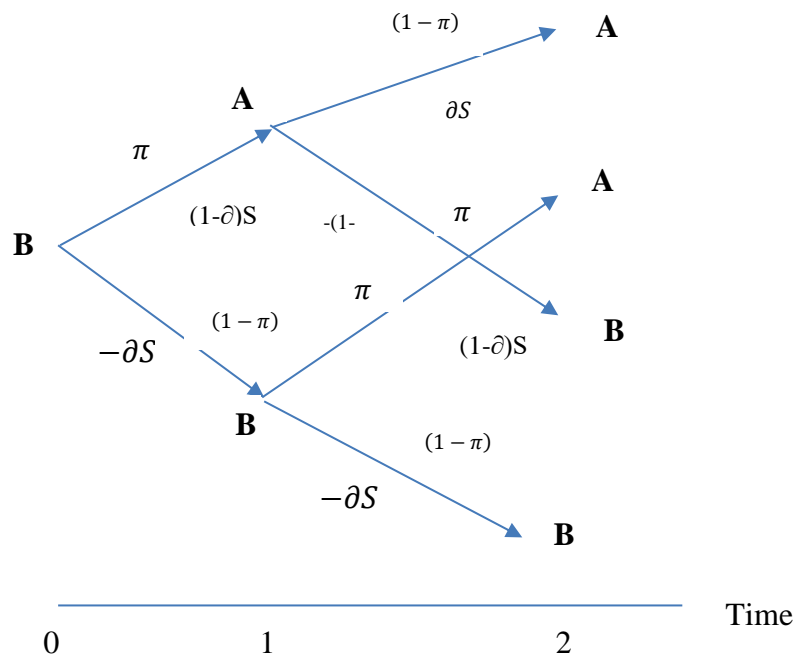
APPENDIX –C

Exhibit 2:Sequences of transaction prices starting at the bid price (B).A is the ask price; π is the probability of price reversals;$(1-\partial)$is the amount of a price reversal as a fraction of the spread (S).

Hence, (from the Exhibit 2) the covariance is given by:

$$cov_T \equiv cov(\Delta P_t, \Delta P_{t+1}) = S^2[\partial^2(1-2\pi) - \pi^2(1-2\partial)]$$