Report: Q3

August 30, 2020

1 Question 3

1.1 (a)

Let, after equalisation, the function

$$h(x) = \begin{cases} y_1 & 0 \le x \le a \\ y_2 & a < x \le 1 \end{cases}$$

Then, due to conservation of mass, $(y1) * a = \alpha$ and $y2(1-a) = 1-\alpha$ (Total mass of histogram - mass of $h1$).
Now, the mean intensity is (representing $h(x)$ to be transformed histogram)

$$\int_0^1 xh(x)dx$$

$$= \int_0^a xh(x)dx + \int_a^1 xh(x)dx$$

$$= \int_0^a x.y_1 dx + \int_a^1 x.y_2 dx$$

$$= \int_0^a x.\frac{\alpha}{a}dx + \int_a^1 x.\frac{1-\alpha}{1-a}dx$$

$$= \frac{\alpha}{2a}(a^2) + \frac{1-\alpha}{2(1-a)}(1-a^2)$$

$$= \frac{a\alpha + (1+a)(1-\alpha)}{2}$$

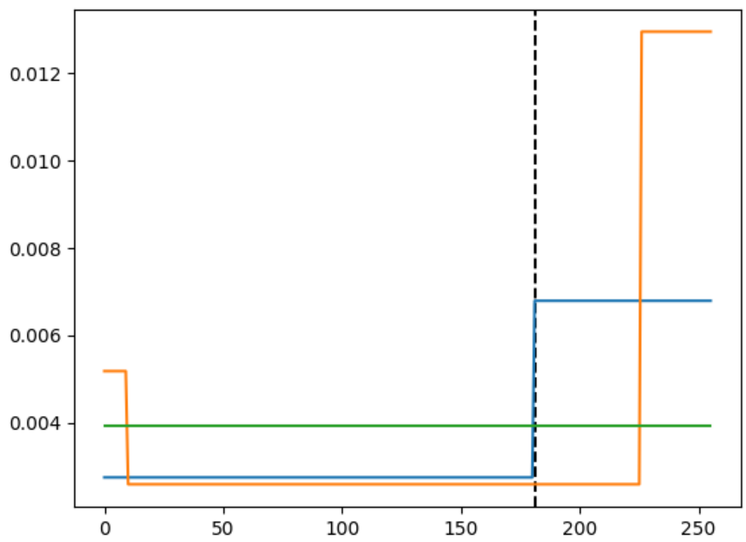
$$= \frac{1+a-\alpha}{2}$$

1.2 (b)

We are given that $\int_0^a h(x)dx = \frac{1}{2} = \alpha$ and $\int_0^1 xh(x)dx = a$.
This is not complete information to determine a. (Consider $h(x)$ with mean and median both $\frac{1}{4}$ or any arbitrary number).
So $ans = \frac{1+a-\frac{1}{2}}{2} = \frac{1}{4} + \frac{a}{2}$.

1.3 (c)

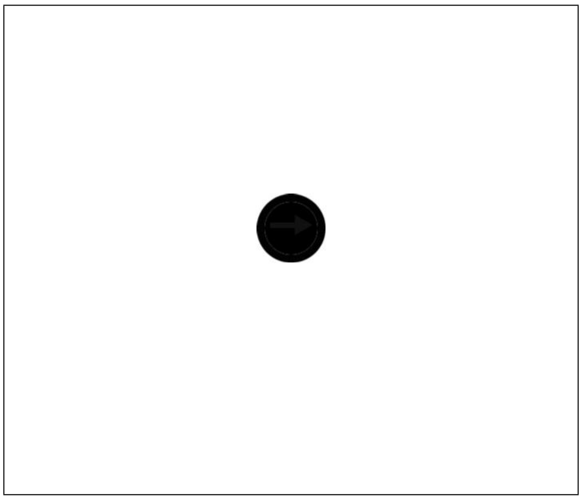
Consider the image, where the number of extreme intensities is large (Peaks at extremities in histogram). And one of the peaks is way bigger than the other one (Or vice versa).
So since the number of intensities towards one end (say the white one) is larger than the other one, the median point would be shifted towards that end.

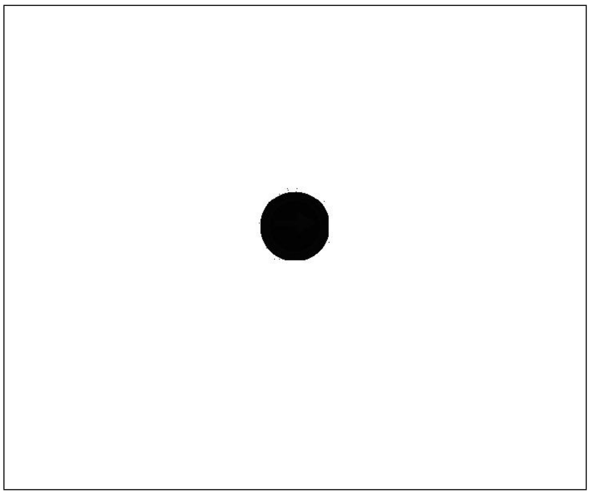


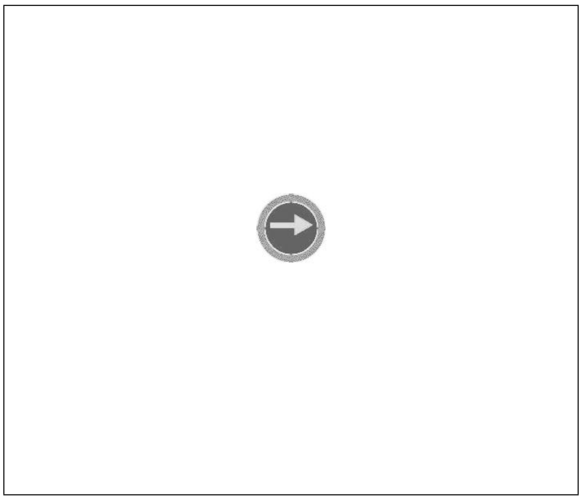
Consider this example. The dotted line is the median (shifted towards white extreme). The blue line is the histogram after equalisation on both. The green line is the histogram after normal equalisation.
Now let us suppose a scenario, where there is some important information hidden in the dark intensities, but the white intensities are irrelevant.
But since there is very less contrast in the dark intensities, we cannot see the information.
So we naturally want most contrast stretch in the dark intensities, to properly distinguish between the dark ones.
Now notice that if we do normal equalisation, the amount of black extreme intensities that get distributed is smaller than that in the case of green one. So there is more contrast stretch in this the median case.
The reason behind this is: Since there are more intensities towards one extreme, the median is shifted towards that. Now we want more contrast stretch in the other extreme intensities. So if we do median equalisation, we get a larger contrast stretch there (since it has more region in its interval).

1.4 (d)

The correct image (corresponding to the above scenario:







The top left image is the original image
Top right image is normally histogram equalised image
Last image is median equalised image
We have very less dark intensities, but the information is within those intensities. So, as predicted earlier, the third method gives the best results.

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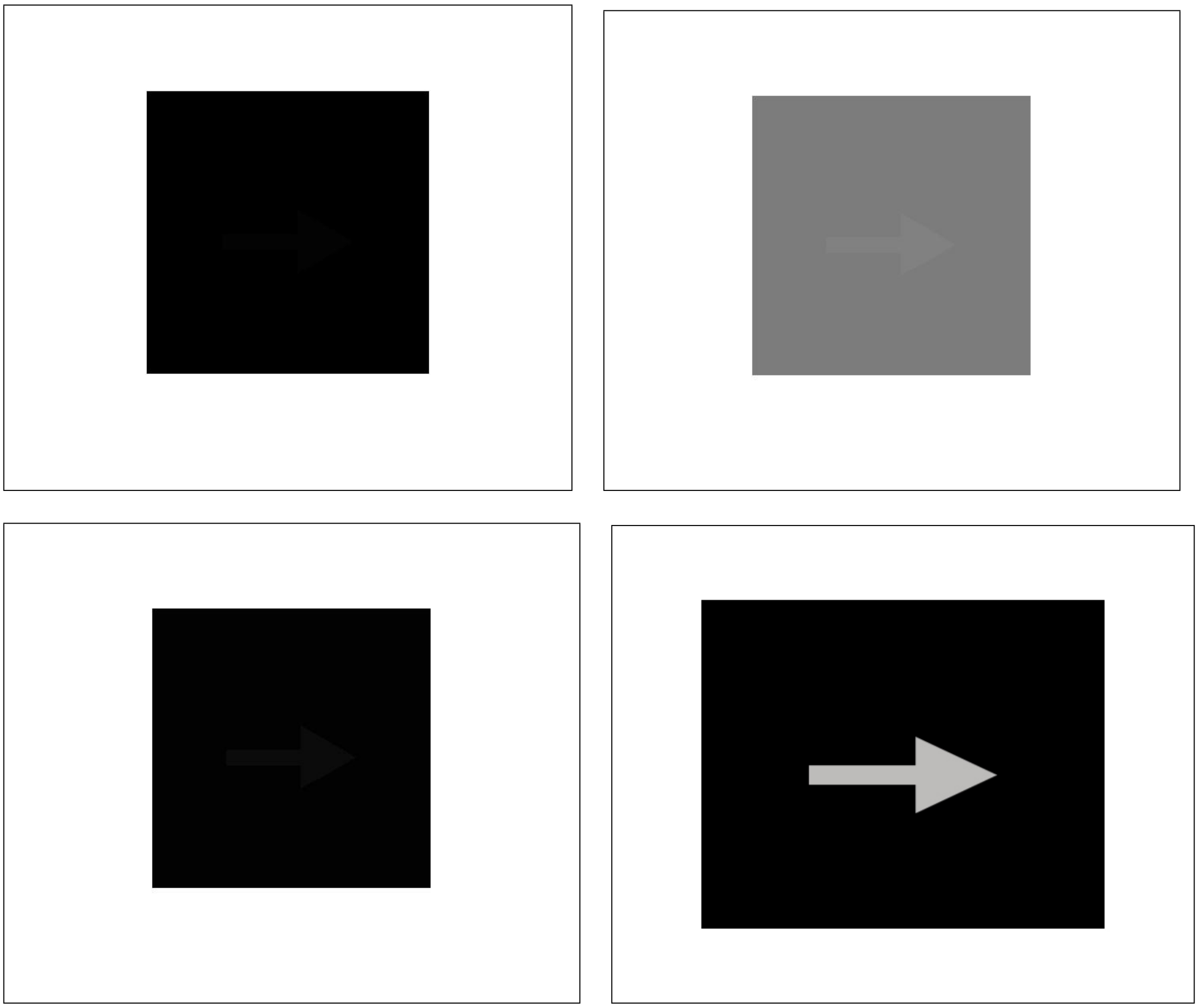
Notice that the number of dark intesities should be less, or else:



The top left image is the original image
Top right image is normally histogram equalised image
Bottom left image is median equalised image
Bottom right image shows the hidden information
So when, the information is hidden inside the intensities whose frequency is larger, then median method doesn't work.

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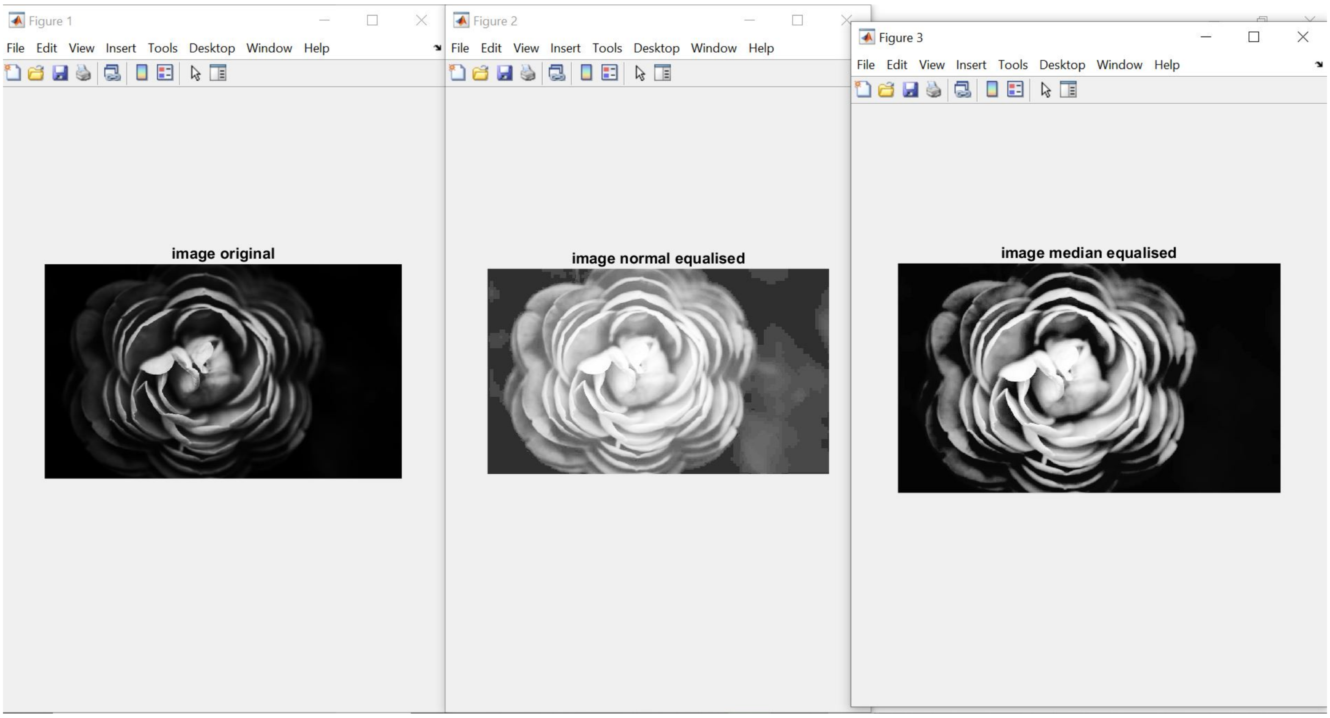
In case of a real image like this,



In this image, there are more black intensities, and normal equalisation dissolves the white into all of the black.
But median equalisation separates them, and equalises dark ones and white ones separately, which stretches the white and black intensity ranges separately, still keeping the original colors more-or-less intact. (And since white ones are lesser, they are stretched more)