Assignment 1 Report

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1 Introduction

This assignment is based on generating plots and the calculation of Linear Convolution and Circular Convolution using various techniques.

2 Generate plots for the sampled versions of the following signals:

The signals are sampled at two different intervals, $T = 0.001$ and $T = 0.1$, and plotted over the range $-5 \leq t \leq 5$. The mathematical expressions and plots of these signals are detailed below.

2.1 Signal 1: $x_1(t)$

$$x_1(t) = \frac{t}{1 + |t|}$$

The sampled plot are as follows:

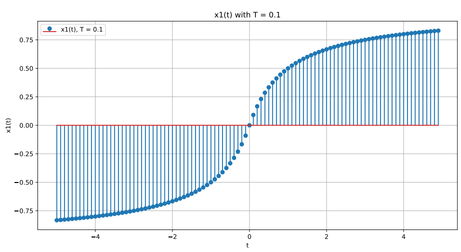


Figure 1: T = 0.1

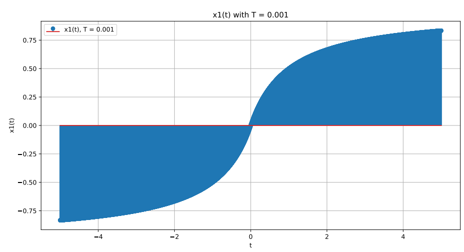


Figure 2: T = 0.001

2.2 Signal 2: $x_2(t)$

$$x_2(t) = \frac{t}{2}(1 + \text{erf}\left(\frac{t}{\sqrt{2}}\right))$$

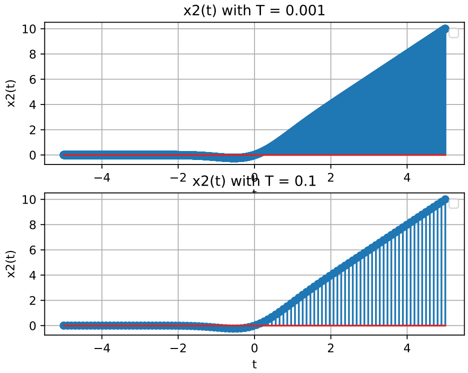


Figure 3: T = 0.1 and T = 0.001

2.3 Signal 3: $x_3(t)$

$$x_3(t) = \Re\left\{ \left(\frac{\alpha}{\pi}\right)^{1/4} \exp\left(-\frac{\frac{\alpha}{\pi}t^2}{2} + j\frac{\beta t^2}{2} + j\gamma t\right)\right\}$$

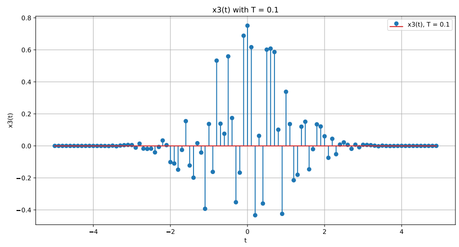


Figure 4: T = 0.1

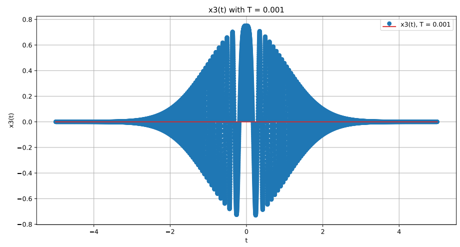


Figure 5: T = 0.001

3 Linear Convolution

Linear convolution is a fundamental operation in signal processing, commonly used for filtering and analyzing signals. There are various methods to perform linear convolution, each with its own advantages and computational complexities.

Brute Force Convolution

Brute force convolution involves the direct calculation of the convolution sum by iterating through each element of the input signals. For each output element, the sum of products of corresponding input elements is computed.

$$y[n] = \sum_k x[k]h[n-k] \tag{1}$$

Matrix Multiplication

Toeplitz matrix multiplication is a more efficient technique. It represents the convolution operation as a matrix-vector multiplication with a Toeplitz matrix constructed from one of the input signals.

$$y = Hx \tag{2}$$

$$H_{L+M-1 \times L-1} = \begin{bmatrix} a & 0 & \dots & 0 \\ b & a & \dots & 0 \\ c & b & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & c & 0 \end{bmatrix} \tag{3}$$

$$y = \begin{bmatrix} y[0] \\ y[1] \\ \vdots \\ y[L+M-1] \end{bmatrix}, \quad x_{L-1 \times 1} = \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[L-1] \end{bmatrix} \tag{4}$$

(L+M-1)-point DFT and IDFT Convolution

Discrete Fourier Transform (DFT) is a powerful algorithm for efficient convolution. By transforming the convolution operation to multiplication in the frequency domain using DFT, followed by the Inverse DFT (IDFT):

$$X[k] = \sum_{n=0}^{L+M-2} x[n] \cdot e^{-j \frac{2\pi}{L+M-1} kn}, \quad k = 0, 1, \dots, L+M-2 \tag{5}$$

$$x[n] = \frac{1}{L+M-1} \sum_{k=0}^{L+M-2} X[k] \cdot e^{j \frac{2\pi}{L+M-1} kn}, \quad k = 0, 1, \dots, L+M-2 \tag{6}$$

In conclusion, while brute force convolution is conceptually simple, Toeplitz matrix and FFT-based convolution methods provide more efficient alternatives, especially for handling larger input signals.

4 Circular Convolution

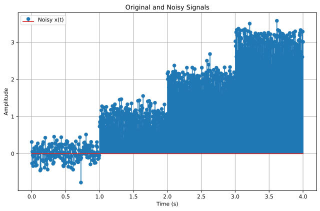
Circular convolution is a mathematical operation commonly used in signal processing. Unlike linear convolution, circular convolution assumes that the input sequences are periodic and wrap around at the edges.

Circulant Matrix

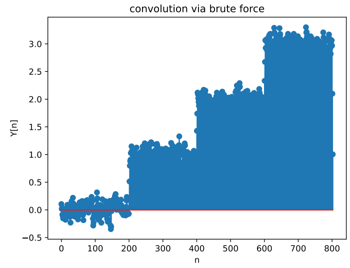
Circular convolution can be efficiently represented using a circulant matrix. Given two sequences $x[n]$ and $h[n]$, circular convolution $y[n]$ can be expressed as

$$y = Hx \tag{7}$$

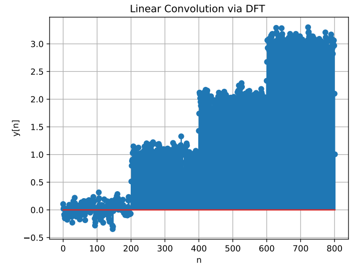
$$H_{L-1 \times L-1} = \begin{bmatrix} a & 0 & \dots & b \\ b & a & \dots & c \\ c & b & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & a \end{bmatrix}, \quad x_{L-1 \times 1} = \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[L-1] \end{bmatrix} \tag{8}$$



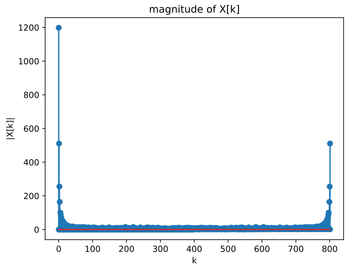
(a) Original and Noisy Signals



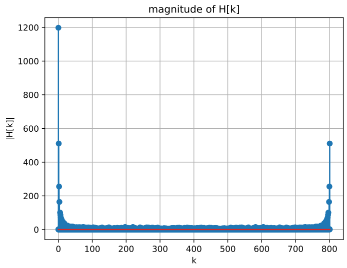
(b) Convolution via Brute Force



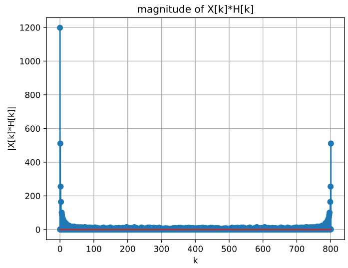
(c) Linear Convolution via DFT



(d) Magnitude of X_K



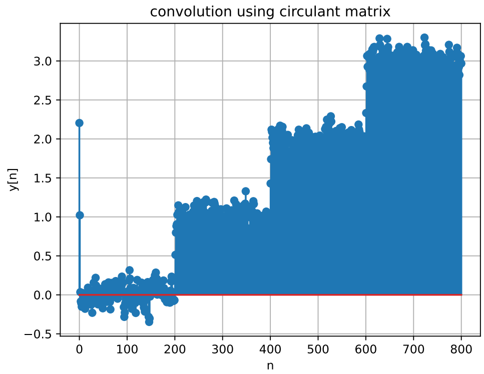
(e) Magnitude of H_k



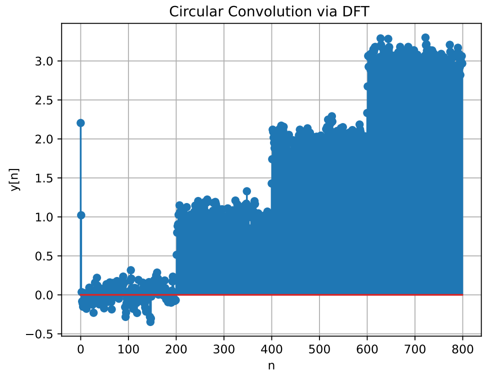
(f) Multi

N-point DFT and IDFT Convolution

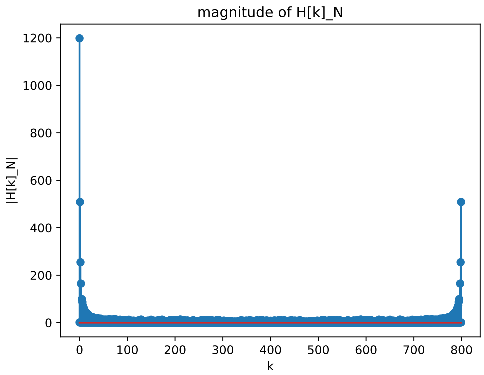
This is similar to the (L+M-1)-point DFT with a change that only N-point DFT and IDFT are computed instead of the full-length signal, where $N = \max(L, M)$ (M denotes input signal length and L denotes filter length).



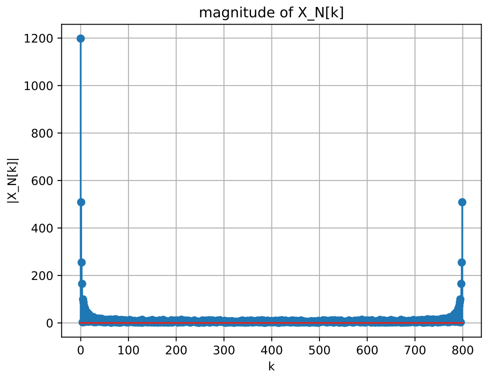
(a) Convolution using Circulant Matrix



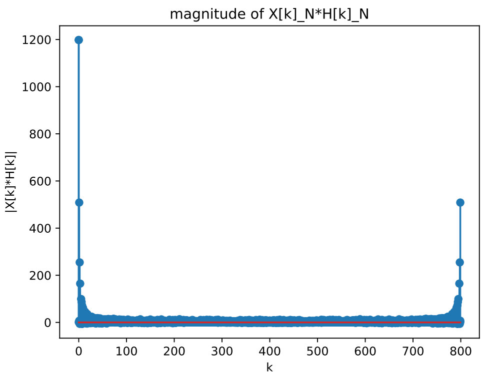
(b) Circular Convolution via DFT



(c) Magnitude of $H[k]_N$



(d) Magnitude of $X[k]_N$



(e) Magnitude of $X[k]_N \times H[k]_N$