Lec: VI. Hypothesis Testing

A statistical hypothesis test is a method of statistical inference used to decide whether the data at hand sufficiently support a particular hypothesis.
Hypothesis testing allows us to make probablistic statements about population parameter.

ie lays → green fav in market (profit ↑↑)

↓ replace with

purple How to analyze profit?

Sell some Quantity in market & Analyze.

Suited for small firms, but what for big companies. If purple lays didn't work they catch huge loss

↓ solution

① Null Hypothesis : Null hypothesis is hypothesis of no difference.

Thus, we shall persume there's no significant diffc b/w observed value and expected value

Then we shall test whether this hypothesis is satisfied by data or not. If hypothesis is not approved the difference is considered to be significant.

If the hypothesis approved then the difference would be described as due to sampling fluctuations.

denoted by : H_0

② Alternative Hypothesis : Any hypothesis which is complementary to Null-hypo.

ie If we want to test null hypothesis that the population has a specified mean μ_0 (say)

$H_0 : \mu = \mu_0$ then the alternative hypothesis could be

$H_1 : \mu \neq \mu_0$ (ie $\mu > \mu_0$ or $\mu < \mu_0$)

NULL Hypothesis,

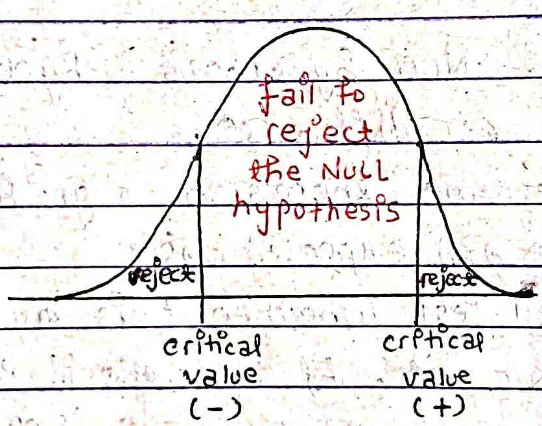
green Lays (profit) = purple Lays (profit)

alt. hypothesis,

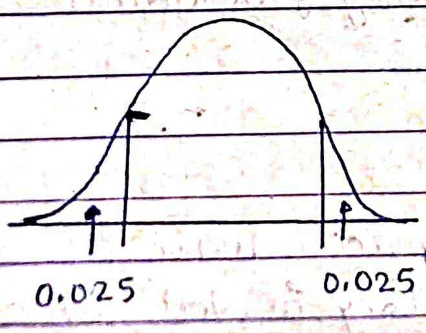
green Lays (profit) > or < purple Lays (profit)

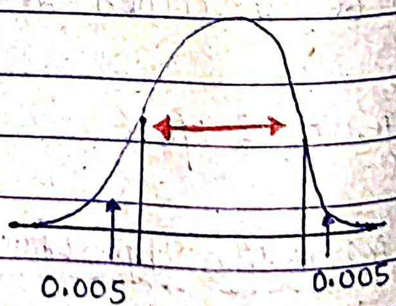
★ **Significance Level:** denote as α (alpha), is a predetermined threshold used in hypothesis testing to determine whether the null-hypothesis should be reject or not.

It represents the probability of rejecting the null hypothesis when it's actually true also known as Type 1 error.



ie, 5% significance ($\alpha\downarrow$) 1% significance:





$\alpha\uparrow \longleftrightarrow$ NULL hypothesis area shrinks

hence, fail's even after it's true.

→ Test of significance of large samples ($n \geq 30$):

Normal distribution is the limiting case of binomial distribution when n is large enough.

For Normal distribution 5% of the items lie outside $\mu \pm 1.96\sigma$ while only 1% of the items lie outside $\mu \pm 2.58\sigma$

★ Z-test: $Z = \frac{x - \mu}{\sigma}$ where Z: standard normal variate.
x: observed nos of successes.

first we find value of Z –

(i) ⓐ if $|Z| < 1.96$ difference b/w the observed & expected nos of successes is not significant at 5% level of significance

ⓑ if $|Z| > 1.96$, if diff. is significant at 5% level of significance.

(ii) similar with 2.58 at 1% level of significance.

The prob. of sucess is p for normal distribution, we know that 99.7% of it's members lie within a range $\pm 3\sigma$ ie, $3\sqrt{npq}$ on either side of mean np, so that only 0.3% of the members lie outside range.

Again, only 5% of the members of normal dist. lie outside the range $\pm 2\sigma$ ie, $2\sqrt{npq}$.

Sampling distribution of the proportion:

A simple sample of n items as drawn from the population. If each individual in sample has a chance p for sucess & q for failure

- mean $= np$
- std. dev. $= \sqrt{npq}$

Let us consider the proportion of sucess then,

- mean proportion of success $\Rightarrow \frac{np}{n} \Rightarrow p$
- standard error propn. of sucess $\Rightarrow \frac{\sqrt{npq}}{n} \Rightarrow \sqrt{\frac{pq}{n}}$

Steps involved in hypothesis testing:

(a) Rejection region approach –

(i) formulate a null & alternative hypothesis.

ie, $H_0 : \mu = 6\text{min}$

$H_1 : \mu > 6\text{min}$

(ii) select a significance level (This is the prob. of rejecting Null hypothesis when it's actually true, usually set at 0.05 or 0.01)

(iii) Check assumptions (ie, distributions)

(iv) decide which test is appropriate (Z-test, T-test, Chi-square test, ANOVA)

(v) Select the relevant test statistic. (z-test → z-score, t-test → t-value)

(vi) Conduct the test.
(vii) Reject or not reject null hypothesis
(viii) Interpret the result.

Q). A sample of 900 days is taken from metrological records of certain district & 100 of them are found to be foggy, what are the probable limits to the percentage of foggy days in the district?

$n = 900$, nos of foggy days $= 100$

proportion of foggy days in the sample $(p) = \frac{100}{900}$

$$q = 1 - \frac{1}{9} \Rightarrow \frac{8}{9}$$

The standard error for proprn of foggy days in district $\Rightarrow \sqrt{\frac{pq}{n}} \rightarrow \sqrt{\frac{1}{9} \times \frac{8}{9} \times \frac{1}{900}} \Rightarrow 0.0105$

Limit for propr of foggy days. $\rightarrow$ point estimate $\pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$$\rightarrow \frac{1}{9} \pm 3 \times 0.0105$$

$$\rightarrow (0.08, 0.1325) \text{ or}$$
$$\rightarrow 8 \text{ to } 13.5\%$$

Q). A random sample of 500 pineapples was taken from a large consigment & 65 were found to be bad? what is the percentage on bad pineapple in the consignment?

$n = 500$, nos of bad pineapple in sample $= 65$

propn. of bad pine apple in sample $\Rightarrow p = \frac{65}{500} \Rightarrow 0.$

$q = 1 - p \Rightarrow 0.87$

S.E of proportion $= \sqrt{\frac{pq}{n}} \Rightarrow \sqrt{\frac{0.13 \times 0.87}{500}} \Rightarrow 0.015$

Thus, Limit for the propn. of bad pineapple in consigment $\rightarrow 0.13 \pm 3 \times 0.015$

$\rightarrow (0.085, 0.175)$

or 8.5 to 17.5%

- **Examples on hypothesis :**

Q): In a locality containing 18000 families, a sample of 840 families was selected at random of these 840 families, 206 families were found to have a monthly income of rs. 10000 or less within what limits would you place your estimate?

total nos of sample $\Rightarrow 840$

nos of families having family income 10,000 or le

$\Rightarrow 206$

proportion of sucess of the sample $\Rightarrow p \Rightarrow \frac{206}{840} \rightarrow \frac{103}{420}$

$q = 1 - p \Rightarrow 317/420$

S.E of proportion $\Rightarrow \sqrt{\frac{pq}{n}} \Rightarrow \sqrt{\frac{103}{420} \times \frac{317}{420} \times \frac{1}{840}}$

$\Rightarrow 0.015$

We have that limits, $\frac{103}{420} \pm 3 \times 0.015$

$\Rightarrow 0.20, 0.29$ or

20 to 29%.

Q). In a sample of 1000 people in maharastra, 540 are vegetarian and the others non-vegan Can we assume that both vegan & non-vegan are equally popular in this state at 1% level of significance?

We are given $n = 1000$

nos of vegan $= 540$

Null hypothesis → both vegan & Non-vegan are equal population in the state.

$p = \frac{500}{1000} \rightarrow 1/2 \rightarrow q = 1/2$

The test statics is → $z = \frac{x - \mu}{\sqrt{npq}}$

$$\frac{540 - 500}{\sqrt{1000 \times \frac{1}{2} \times \frac{1}{2}}} \Rightarrow \frac{40 \times 2}{\sqrt{1000}} \Rightarrow \frac{80}{10\sqrt{10}}$$

or 2.532

$2.532 < 2.58$ (Null hypothesis, accepted)

We may conclude both vegan & Non-vegan are equal popular in maharastra

Q). A dealer takes 100 samples from consignment of 1000 items of a certain goods and finds that there are 50 items are grade I worth rupees 5 per thousand, 30 item of grade II worth rupees 4 per thousand, and 20 item of grade III worth 3 per thousand

Within what limits should the value of the consignment be fixed

Total number of sample $n = 100$

for items of grade I, $p = \frac{50}{100} \rightarrow 1/2$

$q = 1/2$

for item of grade II, $p = \frac{30}{100} \rightarrow 3/10$

$q = 7/10$

for item of grade III, $p = \frac{20}{100} \rightarrow 2/10$

$q = 4/5$

Hence, the standard error for —

grade I, $\sqrt{\frac{pq}{n}} \Rightarrow \sqrt{\frac{1}{2} \times \frac{1}{2} \times \frac{1}{100}} \Rightarrow 0.05$

grade II, 0.046 III, 0.04

Lower & upper limit percentage of three grades —

Limits	I	II	III
Upper	$0.50 - 3 \times 0.05 \Rightarrow 0.35 \Rightarrow 35\%$	16.2%	0%
lower	$0.50 + 3 \times 0.05 \Rightarrow 0.65 \Rightarrow 65\%$	43.8%	32%

highest → 35 × 5 + 16.2 × 4 + 8 × 3

Q). A sample of 400 male students in found to have a mean height of 168 cm. Can it be reasonable regarded as a sample from a large population with a mean height of 167.8 cm and S.E 3.25 cm ?

Under H_0, the test static is –

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$\bar{x} \to 168$ $n \to 400$

$\mu \to 167.8$ cm

$\sigma \to 3.25$

$Z \Rightarrow 1.25$

Since, $|Z| < 3$ we conclude that the data don't provide any evidence against null hypothesis which may be accepted at 0.3% Level of significance