3. ~~MEAN / VARIANCE / STANDARD VARIANCE~~ :

Expectation or Expected Value or Average

$$\boxed{E(X) = \sum_{i=0}^{n} x_i p_i = x_1 p_1 + x_2 p_2 + \dots + x_i p_i}$$

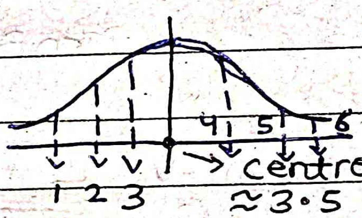
→ ~~throwing a die,~~

 $\{1,2,3,4,5,6\}$

X : nos of dots

X	1 (x_1)	2 (x_2)	3 (x_3)	4 (x_4)	5 (x_5)	6 (x_6)
$P(X=x)$	$\frac{1}{6}$ (p_1)	$\frac{1}{6}$ (p_2)	$\frac{1}{6}$ (p_3)	$\frac{1}{6}$ (p_4)	$\frac{1}{6}$ (p_5)	$\frac{1}{6}$ (p_6)

$$E[x] = \mu = x_1 p_1 + x_2 p_2 + \dots + x_6 p_6$$
$$\rightarrow (1)(1/6) + (2)(1/6) + (3)(1/6) + \dots + (6)(1/6)$$
$$\rightarrow 21/6 \rightarrow 3.5$$

die → Large nos of trials → 

Bell shaped

How ~~to~~ calculate ~~mean of~~ median ?

x	x_1	x_2	x_3	x_4		x_n
$P(X=x)$	P_1	P_2	P_3	P_4		P_n

odd points

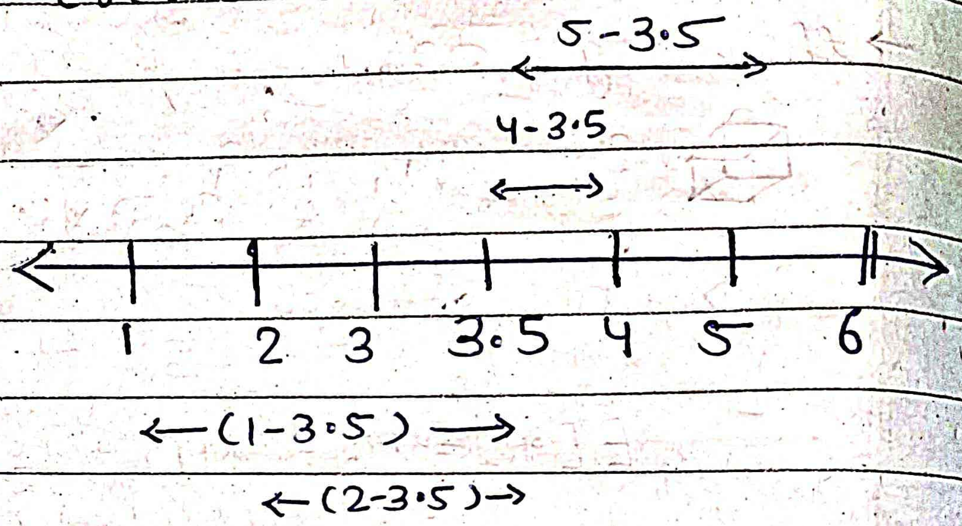
median = $\frac{(n+1)}{2}$ th term

if even

median → $\frac{n}{2}$, $\frac{n}{2}+1$ th term mean $\begin{pmatrix} \text{discrete} \\ \text{random} \\ \text{variable} \end{pmatrix} = \sum_{i=0}^{n} x_i p_i$

Variance : Method of dispersion (deviation).

$\left.\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}\right\} \Rightarrow 3.5$



due to negative values we make a formulae-

- mean $\rightarrow \dfrac{\Sigma f_i x_i}{\Sigma f_i}$

- var $\rightarrow \Sigma (f_i x_i^2)^2 - (\Sigma f_i x_i)^2$

- variation of every point about mean,

- var $\Rightarrow E[X^2] - [E[X]]^2$

or

$[E[X-\mu]^2]$

- $E(x^2) \Rightarrow x_0^2 p_0 + x_1^2 p_1 + x_2^2 p_2 + \cdots + x_n^2 p_n$

in case of throwing a dice,

$var(x) \Rightarrow (1)^2 \left(\frac{1}{6}\right) + (2)^2 \left(\frac{1}{6}\right) + \cdots + (6)^2 \left(\frac{1}{6}\right)$

(σ) standard deviation : RMS value $\rightarrow \sqrt{variance}$

$\because$ $var(x)$ cann't $\ominus$ve

discrete random variable (glossary) :

★ $\mu = E[X] = \sum_{i=0}^{n} x_i p_i$

★ variance $= E[X^2] - [E(X)]^2$

★ standard devi $= \sqrt{variance(x)}$

★ $\sum_{i=0}^{n} P[X = x_i] = 1$

Continuous random variable :

* $\int_{-\infty}^{\infty} f(x)\,dx = 1$ or $\int_{a}^{b} f(x)\,dx = 1$

$a \leq x \leq b$

* $E[x] = \int_a^b x \cdot f(x)\,dx$

* variance $= E[x^2] - [E[x]]^2$

$\rightarrow \int_a^b x^2 f(x)\,dx - \left[\int_a^b x f(x)\right]^2$

* standard variance $\Rightarrow \sqrt{\text{variance}}$ variance cann't be negative

Q) The random variable X takes on the values 1, 2 (or) 3 with probabilities $\frac{2+5P}{5}$, $\frac{1+3P}{5}$ and $\frac{1.5+2P}{5}$ respectively. The values of P and E(X) are respectively.

$\sum_{x=0}^{n} P(X=x) = 1$ $p = 1/20$

$E[x] \Rightarrow (1)\left(\frac{2+5(1/20)}{5}\right) + (2)\left(\frac{1+3(1/20)}{5}\right) + (3)\left(\frac{1.5+2(1/20)}{5}\right)$.

Q). A machine produces 0, 1 or 2 defective pieces in a day with associated probability of 1/6, 2/3 and 1/6 respectively. Then mean value & variance of defective pieces respectively is –

X	0	1	2
$P(X=x)$	1/6	2/3	1/6

$$\text{mean} \rightarrow \underbrace{(0)\left(\frac{1}{6}\right) + (1)\left(\frac{2}{3}\right) + (2)\left(\frac{1}{6}\right)}_{\rightarrow 1}$$

$$\text{var} \Rightarrow E[X^2] - [E[X]]^2$$

$$(0)^2\left(\frac{1}{6}\right) + (1)^2\left(\frac{2}{3}\right) + (2)^2\left(\frac{1}{6}\right) \Rightarrow \frac{4}{3}$$

$$\frac{4}{3} - 1 \rightarrow \frac{1}{3}$$

Q). A random variable X has probability density func. $f(x)$ as given below :

$$f(x) = \begin{cases} a + bx & \text{for } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

The expected value $E[X] = 2/3$, then $Pr[X < 0.5]$

0 [//////] 1

0.5

$P[X<0.5]$

$$\int_0^{0.5} (a+bx)\,dx$$

$$\int_0^1 x \cdot (a+bx)\,dx = \frac{2}{3}$$

$$\rightarrow 3a + 2b = 4 \quad (1)$$

If valid prob. density

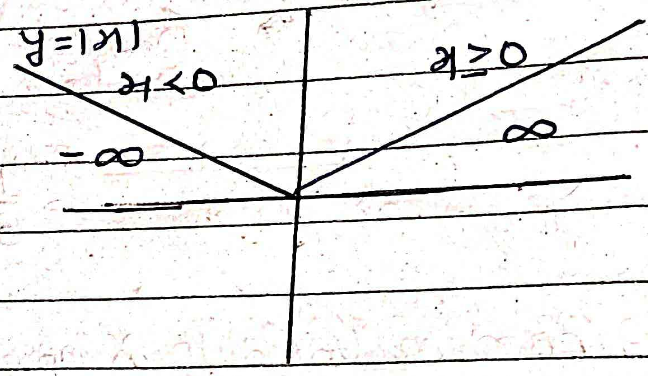
$$\int_0^1 (a+bx)\,dx = 1$$

$$2a + b = 2 \quad - (2)$$

Q). $P_x(x) = Me^{(-2|x|)} + Ne^{(-3|x|)}$ is the probability density function for the real random variable x, over the entire axis(x), M & N are both positive real nos. equn relating M & N = ?

$$f(x) = P_x(x) = Me^{(-2|x|)} + Ne^{(-3|x|)}$$

if funcⁿ is valid pdf $\int_{-\infty}^{\infty} (Me^{-2|x|} + Ne^{-3|x|})\, dx = 1$

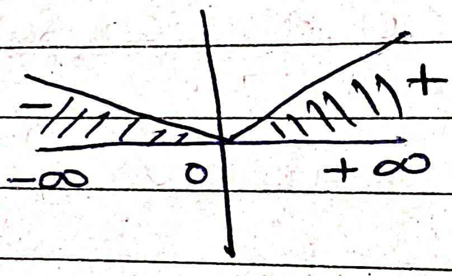


$$|x| = \begin{cases} x \;;\; x>0 \\ -x \;;\; x<0 \end{cases}$$

$x=0$, not differentiable

↓

∞ tangent can be drawn



$$\int_{-\infty}^{0} (Me^{-2(-x)} + Ne^{-3(-x)})\, dx + \int_{0}^{\infty} (Me^{-2(x)} + Ne^{-3(x)})\, dx$$

$$\left[\frac{Me^{2x}}{2} + \frac{Ne^{3x}}{3}\right]\Bigg|_{-\infty}^{0} + \left[\frac{Me^{-2x}}{2} + \frac{Ne^{-3x}}{3}\right]\Bigg|_{0}^{\infty}$$

$$\left(\left(\frac{M}{2} + \frac{N}{3}\right) + 0\right) + \left[-\left(-\frac{M}{2} - \frac{N}{3}\right)\right] = 1$$

$$M + \frac{2N}{3} = 1$$