Linear Algebra

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Matrix

Determinant of matrix,

0 Lower triangular or Upper triangular or Diagonal or

A ⇒

Scalar or Identity Matrix

$\det(A)$ = product of diagonal elements

0 $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ $(-1)^{i+j}|M|$

$i \rightarrow$ row, $j \rightarrow$ column

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

Q) find det of following matrices →

(1) $\begin{bmatrix} 0 & 1 & 0 & 2 \\ -1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \\ 1 & -2 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 1 & 0 & 2 \\ -1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \\ 1 & -2 & 0 & 1 \end{bmatrix}$

$$(-1)^{3+4} \begin{vmatrix} 0 & 1 & 0 \\ -1 & 1 & 1 \\ 1 & -2 & 0 \end{vmatrix}$$

$$(-1)(-1)^{1+2} \begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix} \Rightarrow -1$$

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(2) $\begin{bmatrix} 6 & -8 & 1 & 1 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & -1 \end{bmatrix}$ $(6)(2)(4)(-1)$

$\rightarrow -48$

(3) $\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 7 & 2 \\ 2 & 0 & 2 & 0 \\ 9 & 0 & 6 & 1 \end{bmatrix} \rightarrow 2 \begin{vmatrix} 1 & 7 & 2 \\ 0 & 2 & 0 \\ 0 & 6 & 1 \end{vmatrix}$

$\Rightarrow (2)(2) \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} \Rightarrow 4$

(4) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 100 & 1 & 0 & 0 \\ 100 & 200 & 1 & 0 \\ 100 & 200 & 300 & 1 \end{bmatrix} \Rightarrow 1$

(5) $\begin{bmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{bmatrix} \Rightarrow 1 \begin{vmatrix} 9 & 16 \\ 16 & 25 \end{vmatrix} - 4 \begin{vmatrix} 4 & 16 \\ 9 & 25 \end{vmatrix} + 9 \begin{vmatrix} 4 & 9 \\ 9 & 16 \end{vmatrix}$

$\Rightarrow -12$

(6) $\begin{bmatrix} 1 & 3 & 2 \\ 4 & 1 & 1 \\ 2 & 1 & 3 \end{bmatrix} \Rightarrow 1 \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} - 3 \begin{vmatrix} 4 & 1 \\ 2 & 3 \end{vmatrix} + 2 \begin{vmatrix} 4 & 1 \\ 2 & 1 \end{vmatrix}$

$\Rightarrow -28$

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Q) Given that $A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

the value of A^3 is —

power of matrix →

step(A)) make the characterstic equn

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2}$ $\quad |A - I\lambda| = 0$

↑ scalar quantity

$\lambda^2 - (\text{Trace})\lambda + \det A = 0$: characterstic Equn.

$\lambda^2 + (a+d)\lambda + (ad - bc) = 0$

↑ sum of diagonal element

$\lambda^2 - (-5+0)\lambda + (+6) = 0$

$\lambda^2 + 5\lambda + 6 = 0$

step(B)) $\lambda \xrightarrow{\text{replace}} A$

$A^2 + 5\lambda + 6 = 0$

$A^2 = -5A - 6$

↓ pre-multiply A

$A\,A^2 = -5AA - 6A$

$A^3 = -5A^2 - 6A$

$A^3 = -5(-5A - 6) - 6A$

$A^3 = 19A + 30$

A^3 in terms of A

Again multiply A,

$$A^4 = 19A^2 + 30A$$

$$A^4 = 19(-5A-6) + 30$$

$$A^5 =$$

$\left(A^4 \times A^5 = A^9\right)$

P) Consider the matrix :

$$J_6 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

which is obtained by reversing the order of the columns of the identity matrix I_6.

Let, $P = I_6 + \alpha J_6$ where α is an non-negative real number

Find α for which $\det(P) = 0$

$$P = I_6 + \alpha J_6$$

$\downarrow$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \alpha \\ 0 & 1 & 0 & 0 & \alpha & 0 \\ 0 & 0 & 1 & \alpha & 0 & 0 \\ 0 & 0 & \alpha & 1 & 0 & 0 \\ 0 & \alpha & 0 & 0 & 1 & 0 \\ \alpha & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

for $\alpha = 1$

$\det|P| = 0$

$$\begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

2-row repeat $\rightarrow$ det = 0

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Q) Let $P = [a_{ij}]$ be a 3×3 matrix and Let $Q = [b_{ij}]$, where $b_{ij} = 2^{i+j} a_{ij}$ for $1 \le i, j \le 3$, If det of P is 2, then det of matrix Q is —

$P = [a_{ij}]$ 3×3 det(P) = 2

$Q = [b_{ij}]$

$b_{ij} = 2^{i+j} a_{ij}$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = |P| = 2$$

$$Q \rightarrow \begin{bmatrix} 2^2 a_{11} & 2^3 a_{12} & 2^4 a_{13} \\ 2^3 a_{21} & 2^4 a_{22} & 2^5 a_{23} \\ 2^4 a_{31} & 2^5 a_{32} & 2^6 a_{33} \end{bmatrix}$$

$$Q \rightarrow 2^2\, 2^3\, 2^4 \begin{bmatrix} a_{11} & 2a_{12} & 2^2 a_{13} \\ a_{21} & 2a_{22} & 2^2 a_{23} \\ a_{31} & 2a_{32} & 2^2 a_{33} \end{bmatrix}$$

$$Q \rightarrow (2^2)(2^3)(2^4)(2)(2^2) \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \rightarrow 2$$

$\rightarrow 2^{13}$

Q. Adjoint of a Matrix :

$$A = \begin{array}{c} \begin{bmatrix} 1 & 2 & -1 \\ 0 & 2 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{array}{l} - R_1 \\ - R_2 \\ - R_3 \end{array} \\ \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \end{array}$$

R3

R1 (repeat) -

R2

$$\begin{bmatrix} 2 & 3 & 0 & 2 \\ -1 & 0 & 1 & -1 \\ 2 & -1 & 1 & 2 \\ 2 & 3 & 0 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} +3 & 3 & -2 \\ 1 & +1 & 3 \\ 8 & -3 & 2 \end{bmatrix}$$

Q. Let M be a 3×3 matrix satisfying:

$$M\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, M\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \text{ and } M\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 12 \end{bmatrix}$$

Then the sum of diagonal entries of M is —

$$M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

solve equns.

↓

sum of diagonal ⇒ $a+e+i$.

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \times \begin{bmatrix} j & k & l \\ m & n & o \\ p & q & r \end{bmatrix}$$

⇓

$$\begin{bmatrix} (aj+bm+cp) & (ak+bn+cq) & (al+bo+cr) \\ (dj+em+fp) & (dk+en+fq) & (dl+eo+fr) \\ (gj+hm+ip) & (gk+hn+iq) & (gl+ho+ir) \end{bmatrix}$$

$A \cdot B$

$m \times n \quad n \times p$

equal

dimension of AB

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Let, ABCD be $n \times n$ matrices, each with non-zero determinant $ABCD = I$ then $B^{-1} = ?$

A, B, C, D are non-singular matrix

- inverse of matrix, $A^{-1} = \dfrac{\operatorname{adj} A}{|A|}$, $|A| \neq 0$
  (non-singular)
  $\downarrow$ means
  A^{-1} exist.

- if $\det(A) = 0 \rightarrow$ singular matrix
  A^{-1} doesn't exist.

- $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2}$ $\quad A^{-1} = \dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

- A, B, C, D are non-singular matrix

  * $(ABCD)^{-1} = D^{-1}C^{-1}B^{-1}A^{-1}$
  * $(A^{-1})^{-1} = A$ $\quad$ * $(KA)^{-1} = \dfrac{1}{K}A^{-1}$

$ABCD = I$

both side inverse

$(ABCD)^{-1} = (I)^{-1} \rightarrow I$

$\downarrow$

$D^{-1}C^{-1}B^{-1}A^{-1} = I$

$\downarrow$

$C \cdot \underbrace{\underbrace{D \cdot D^{-1}}_{I}\, C^{-1}}_{I} B^{-1}A^{-1} = ICD$

$\downarrow$

$B^{-1}AA^{-1} = ICDA$

$B^{-1} = CDA$.

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Q). Let P is a 3×3 matrix such that $P^T = 2P + I$, whe
P^T is the transpose of P and I is the 3×3 identi
matrix, then there exist a column matrix

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(A) $PX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (B) $PX = 2X$

(C) $PX = X$ (D) $PX = -X$

$$P^T = 2P + I$$

$$(P^T)^T = 2P^T + I^T$$

$$P = 2(2P + I) + I$$

$$P = 4P + 3I$$

$$-3P = 3I$$

$$P = -I \rightarrow PX = -X$$

Q). Let $M^4 = I$ (where $I \rightarrow$ identity matrix) and $M \neq I$
$M^2 \neq I$ & $M^3 \neq I$. Then for any natural number k,
M^{-1} equals —

$$M^4 = I$$

both sides multiply by M^{-1}.

$$M^4 M^{-1} = M^{-1}$$

$$M^3 = M^{-1}$$

$$M^8 = I$$

$$M^{-1} M^8 = M^{-1}$$

$$M^7 = M^{-1}$$

$$M^{12} = I$$

$$M^{-1} M^{12} = M^{-1} I$$

$$M^{-1} = M^{11}$$

$M^3, M^7, M^{11} \rightarrow M^{4K+3}$

form