System of Gear Fits

Backlash Tooth Thickness Allowances Tooth Thickness Tolerances

Principles

DIN 3967

Getriebe-Passystem; Flankenspiel, Zahndickenabmasse, Zahndickentoleranzen, Grundlagen

To facilitate use of this Standard, the calculation of tooth thickness allowances has been included as Appendix A. Information on converting the allowances for the various measuring methods is added in Appendix B.

The DIN backlash system of fits for gear pairs allows the limiting allowances of tooth thickness to be defined with attention given to all effects occurring in the operation of a gear transmission, and to all deviations throughout the gearing.

The system of fits therefore consists on the one hand of the allowances and tolerances of the gear teeth, referred to their prevailing mounting arrangements, and on the other hand of the allowances and tolerances of all the other components of the gear transmission in so far as they determine the position of the teeth relative to one another. These values defined for a reference temperature vary in operation through temperature changes in the upward or downward direction, through elastic deformation under load and possibly through swelling or contraction.

The system of fits is defined as a tooth thickness system of fits in the normal section on the reference cylinder, i. e. all allowances, tolerances and operationally induced alterations in the gear transmission are treated as tooth thickness alterations and require to be converted to the normal section.

The normal section was chosen because the production effort, i. e. the necessary tooth thickness tolerance in the normal section, is independent of the helix angle. The normal section was also chosen for metrological reasons, since the normal chordal tooth thickness and the base tangent length are measured in the normal section.

The calculation of the allowances however is made over the transverse section, since on the finished gear transmission the backlash is measured as circumferential backlash (see Appendix A).

The system of fits provides for safeguarding the minimum backlash and limiting the maximum backlash.

The reference basis of the system of fits is the zero-play condition at the nominal centre distance, with nominal addendum modification and with error-free components.

The necessary negative allowances of tooth thickness can be produced by an additional addendum modification in the negative direction Δx. This however is not taken into account in the nominal addendum modification.

Whether the weakening of the tooth thickness needs to be taken into consideration in calculations of load-carrying capacity is something which has to be decided for the case concerned. In any event this should be done whenever

$$\left|\frac{A_{sni}}{m_n}\right| > 0.005.$$

1 Other relevant Standards

DIN 3960 Definitions and parameters for cylindrical gears and cylindrical gear pairs with involute teeth

DIN 3961 Tolerances for cylindrical gear teeth; principles

DIN 3962 Part 1 Tolerances for cylindrical gear teeth; tolerances for deviations of individual parameters

DIN 3964 Centre distance allowances and shaft position tolerances of housings for cylindrical gear transmissions

DIN 3999 Symbols for gear teeth

2 Backlash

The backlash value says nothing about the quality of the gear teeth although, on the other hand, the different gear tooth qualities demand given tooth thickness allowances in order to ensure the requisite or permissible backlash.

The minimum backlash is determined by the upper allowances. However it does not correspond to the sum of the upper allowances because a whole series of factors alters the backlash (see Appendix A).

The maximum backlash is determined by the lower tooth thickness allowances which result from the upper allowances and the tooth thickness tolerances. This also does not correspond to the sum of the allowances because here again a series of factors alters the backlash.

2.1 Theoretical backlash

The theoretical backlash j_t results from the tooth thickness allowances converted to the transverse section and from the converted allowances of the centre distance.

$$j_t = -\frac{A_{sn\,1} + A_{sn\,2}}{\cos\beta} + A_a \cdot \frac{\tan\alpha_n}{\cos\beta} = -\Sigma A_{st} + \Delta j_a$$

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2.2 Acceptance backlash

The acceptance backlash is the backlash obtained with the unloaded gear transmission at reference temperature when one of the gears is rotated against the other. It is usually smaller than the theoretical backlash, since the backlash-reducing factors generally outweigh the factors tending to increase the backlash. Backlash-reducing factors are, for example, deviations in the gear teeth and also form and position deviations, see Appendix A.

2.3 Working backlash

The working backlash is the backlash resulting when the gear transmission is operating. It is not constant. During the starting up of the gear transmission in particular it is possible for the more rapid temperature rise of the gears compared with the housing to bring about larger changes in the working backlash. It is generally larger than acceptance backlash when the linear coefficient of expansion of the housing is greater than that of the gears. Shaft deflection and displacement also affect it.

3 Tooth thickness allowances and tooth thickness tolerances

Normally the tooth thickness allowances and tooth thickness tolerances can be found directly from Tables 1 and 2 on the basis of existing experience, such that, as a rule, the upper allowances for each gear should be at least as large (numerical value) as the lower allowance of the housing centre distance (without converting). If no empirical values are available for backlash and tooth thickness allowances, these must be calculated. A guide for this purpose will be found in Appendix A. The calculated values are usually rounded and then likewise taken from Tables 1 and 2. If exceptionally small amounts of backlash are necessary for functional reasons, calculation is indispensable.

Table 1. **Upper tooth thickness allowances A_{sne} in µm**

Reference diameter (mm)		Allowance series										
over	up to	a	ab	b	bc	c	cd	d	e	f	g	h
–	10	- 100	- 85	- 70	- 58	- 48	- 40	- 33	- 22	- 10	- 5	0
10	50	- 135	- 110	- 95	- 75	- 65	- 54	- 44	- 30	- 14	- 7	0
50	125	- 180	- 150	- 125	- 105	- 85	- 70	- 60	- 40	- 19	- 9	0
125	280	- 250	- 200	- 170	- 140	- 115	- 95	- 80	- 56	- 26	- 12	0
280	560	- 330	- 280	- 230	- 190	- 155	- 130	- 110	- 75	- 35	- 17	0
560	1 000	- 450	- 370	- 310	- 260	- 210	- 175	- 145	- 100	- 48	- 22	0
1000	1 600	- 600	- 500	- 420	- 340	- 290	- 240	- 200	- 135	- 64	- 30	0
1600	2 500	- 820	- 680	- 560	- 460	- 390	- 320	- 270	- 180	- 85	- 41	0
2500	4 000	- 1100	- 920	- 760	- 620	- 520	- 430	- 360	- 250	- 115	- 56	0
4000	6 300	- 1500	- 1250	- 1020	- 840	- 700	- 580	- 480	- 330	- 155	- 75	0
6300	10 000	- 2000	- 1650	- 1350	- 1150	- 940	- 780	- 640	- 450	- 210	- 100	0

Table 2. **Tooth thickness tolerances T_{sn} in µm**

Reference diameter (mm)		Tolerance series									
over	up to	21	22	23	24	25	26	27	28	29	30
–	10	3	5	8	12	20	30	50	80	130	200
10	50	5	8	12	20	30	50	80	130	200	300
50	125	6	10	16	25	40	60	100	160	250	400
125	280	8	12	20	30	50	80	130	200	300	500
280	560	10	16	25	40	60	100	160	250	400	600
560	1 000	12	20	30	50	80	130	200	300	500	800
1000	1 600	16	25	40	60	100	160	250	400	600	1000
1600	2 500	20	30	50	80	130	200	300	500	800	1300
2500	4 000	25	40	60	100	160	250	400	600	1000	1600
4000	6 300	30	50	80	130	200	300	500	800	1300	2000
6300	10 000	40	60	100	160	250	400	600	1000	1600	2400

3.1 Upper allowances

The upper allowances are to be taken from Table 1 independently of the reference diameter and the allowance series. Their choice is largely independent of the gear tooth quality. As a rule for transmissions of the same kind it is possible to choose the upper allowance for pinion and gear in all cases from a single allowance series; it is also permissible however to select values from different allowance series.

3.2 Lower allowances

The lower allowances are obtained by combining the upper allowances with the tooth thickness tolerances. Since the upper and lower allowances are always negative the amount of the tolerance has to be deducted from the upper allowance.

3.3 Tooth thickness tolerances

The tooth thickness tolerances are to be found from Table 2. Their choice is largely independent of the gear tooth quality and should be governed by the manufacturing facilities, although it should be borne in mind that the tooth thickness tolerance must be at least twice as large as the permissible tooth thickness fluctuation R_s according to DIN 3962 Part 1. If a maximum backlash has to be watched for functional reasons, calculation according to Appendix A will be necessary. Quite generally it should be noted that small tooth thickness tolerances unfavourably affect the maintaining of gear tooth quality, since they unnecessarily limit the correction possibilities during manufacture (see, for example, VDI/VDE 2608).

In order to distinguish them clearly from the gear tooth qualities, the tolerance series have been given the numbers 21 to 30. The preferred series are 24 to 27.

3.4 Information in drawings

The limiting allowances can be indicated in the drawing either directly or by means of an code designation, see DIN 3966 Part 1. The symbol consists of the number of the tooth thickness tolerance series and the letter symbol of the series for the upper tooth thickness allowance. Example: 27cd; this designation yields, for d = 100 mm for example, the limiting allowances $A_{sne} = -70\ \mu m$ and $A_{sni} = -170\ \mu m$.

4 Converting the tooth thickness allowances for the different test methods

The system of fits is referred to a theoretical value. This is the tooth thickness in the normal section which, however, is not directly measurable. Therefore indirect measurements are made by various methods, see DIN 3960.

For an error-free gear there are mathematical relationships connecting the different measured quantities. However, since the individual measured quantities are affected differently by the individual deviations of the gear teeth a purely mathematical conversion of the tooth thickness allowances does not necessarily guarantee the required backlash. Where adequate experience is available (e. g. in the case of tooth thickness tolerance zone 26e or coarser) the tooth thickness allowances can be converted directly into given test dimension allowances (e. g. base tangent length allowances) and these used for acceptance testing the gear. It may then happen, however, that acceptance testing by a different measuring method (e. g. pin dimension measurement) will show that the tolerance is not fully complied with.

With the closer tolerance zones it is therefore recommendable when calculating different test dimensions and their allowances to apply appropriate corrections which take account of the influence of the individual deviations on these test dimensions empirically (statistically). Guidance on determining correction values is given in Appendix B.

For calculating the allowance factors according to DIN 3960, October 1976 edition, Sections 4.1.3 and 5, the mean generating addendum modification coefficient x_{Em}, corresponding to the mean allowance should always be used.

5 Example

Length dimensions in mm

Helical gears			External	
			Pinion	Gear
Normal module		m_n	5	
Number of teeth		z	20	97
Standard basic rack tooth profile	Gear teeth		DIN 867	
	Tool		DIN 3972	
Helix angle		β	9° 53′ 49″ DIN 3978	
Flank direction			Left	Right
Reference diameter		d	101,511	492,326
Addendum modification coefficient to DIN 3992		x	+ 0,4000	+ 0,2389
Gear tooth quality			6	7
Facewidth		b	70	
Material			Pinion 16 MnCr 5 Gear 42 CrMo 4V	
Housing material			GG 22	
Housing centre distance		a	300 js 7	
Housing width			200	
Axial position accuracy class			5, $f_{\Sigma\beta} = f_{\Sigma\delta} = 0{,}02$	

Pinion hardened and ground, gear heat-treated and milled. It is assumed that it is known from experience that the upper allowances of tooth thickness of series cd are appropriate for this type of transmission.

According to Table 1 the upper allowances are selected as $A_{\text{sne 1}} = -70$ µm for the pinion and $A_{\text{sne 2}} = -130$ µm for the gear.

(These values are algebraically smaller than the lower allowance – 26 µm of the centre distance.)

For the particular application concerned, the observance of a functionally imposed maximum backlash is not necessary. To cater for hardening distortion, and also to keep the grinding cost low, the tolerance for the pinion is made comparatively large. For series 27 Table 2 gives $T_{\text{sn 1}} = 100$ µm and hence

$$A_{\text{sne 1}} = -70\ \mu\text{m} = -0{,}070\ \text{mm}$$
$$A_{\text{sni 1}} = -170\ \mu\text{m} = -0{,}170\ \text{mm}$$

(lower allowance = upper allowance minus tolerance).

Since the gear is milled, a tolerance of 100 µm (Table 2 series 26) is adequate. Consequently

$$A_{\text{sne 2}} = -130\ \mu\text{m} = -0{,}130\ \text{mm}$$
$$A_{\text{sni 2}} = -230\ \mu\text{m} = -0{,}230\ \text{mm}$$

The adoption of these tolerance zones, which meet the manufacturing requirements, means that the tooth thicknesses are not unacceptably weakened

$$\frac{|A_{\text{sni 2}}|}{m_n} = \frac{0{,}23}{5} < 0{,}05.$$

Since the tooth thickness fluctuation according to DIN 3962 Part 1 is allowed to be 14 µm for the pinion and 25 µm for the gear, the tolerances are correctly selected in this respect also (see Section 3.3).

From the nominal dimensions and the allowances the maximum, mean and minimum values of the tooth thickness s or the addendum modification x are calculated according to DIN 3960:

Length dimensions in mm

Number of teeth	z	20	97
Tooth thickness	$s_{n\,nenn}$	9,3099	8,7235
	$s_{n\,max}$	9,2399	8,5935
	$s_{n\,mittel}$	9,1899	8,5435
	$s_{n\,min}$	9,1399	8,4935
Addendum modification	x_{nenn}	+ 0,4000	+ 0,2389
	x_{max}	+ 0,3808	+ 0,2032
	x_{mittel}	+ 0,3670	+ 0,1894
	x_{min}	+ 0,3533	+ 0,1757

The above result in the following test dimensions with their allowances:

Base tangent length	W	39,619 ± 0,047	177,485 ± 0,047
Measured number of teeth	K	3	12
Allowance factor	A^*_W	0,940	0,940
Dimension over balls	M_{dK}	117,472 ± 0,099	507,604 ± 0,126
Dimension over rollers	M_{dR}		507,670 ± 0,126
Ball and roller diameter	D_M	9,297 ≈ 9	8,471 ≈ 9
Allowance factor	A^*_{Md}	1,988	2,524
Working distance with master gear	a''	129,314 ± 0,061	323,962 ± 0,066
Number of teeth of master gear (DIN 3970) [1]	z_L	30	30
Allowance factor	$A^*_{a''}$	1,218	1,325

[1]) The calculation is based on an addendum modification coefficient $x_L = +0.15$ and zero allowance of the master gear tooth thickness; for further information on this see VDI Code 2608.

The calculated allowances of the measured values are shown in Fig. 1; these are values for ideal geometry. For use in practical measurements they may need to be corrected, see Section 4 and Appendix B (Fig. B.1 and B.3).

The acceptance backlash may turn out to be smaller than the sum of the upper allowances, as dictated by the tolerance for the housing and other effects. It may, however, also turn out to be larger than the sum of the lower allowances, as dictated by the slope of the tooth and the housing tolerance, and further effects (see Appendix A).

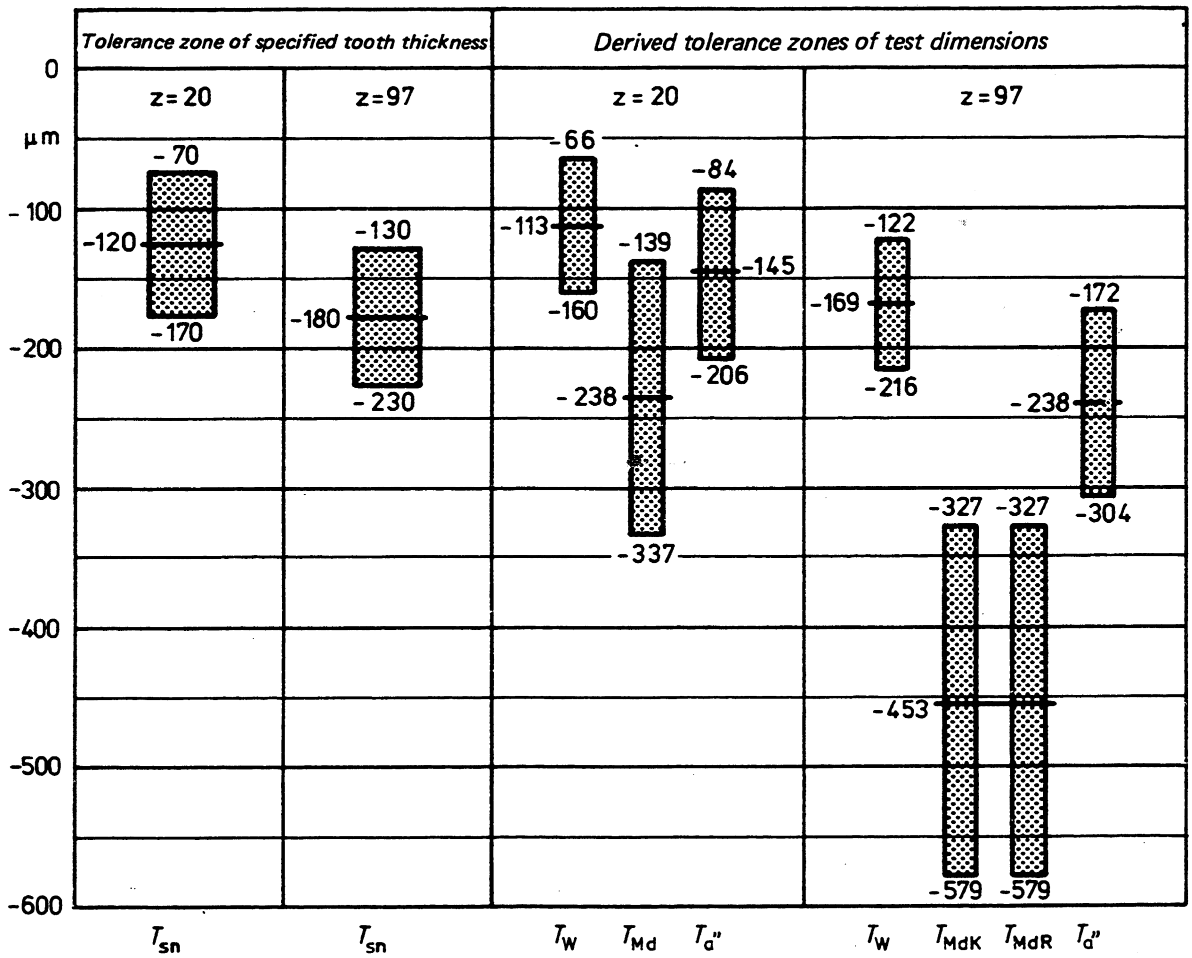


Figure 1. Tolerance zones of test dimensions after ideal-geometry conversion of tooth thickness tolerance zone

Appendix A
Calculation of tooth thickness allowances or backlash

Contents

A.1 General

A.1.1 Symbols and designations

a	Centre distance
b	Facewidth
d	Reference diameter
f_p	Individual pitch deviation
$f_{\Sigma\beta}$	Axial skew over length L_G
j_t	Theoretical backlash
j_{ta}	Acceptance backlash
j_{tw}	Working backlash
$j_{t\,max}$	Maximum circumferential backlash
$j_{t\,min}$	Minimum circumferential backlash
m	Module
w	Relative water absorption (relative volume expansion)
x_{mean}	Addendum modification coefficient with mean tooth thickness allowance
A_a	Centre distance allowance
A_{ae}	Upper centre distance allowance
A_{ai}	Lower centre distance allowance
A_{sne}	Upper allowance of tooth thickness in normal section
A_{sni}	Lower allowance of tooth thickness in normal section
A_{ste}	Upper allowance of tooth thickness in transverse section
A_{sti}	Lower allowance of tooth thickness in transverse section
F_f	Total profile deviation 1)
F_{pk}	Pitch span deviation over k pitches
F_r	Concentricity deviation
F_{β}	Tooth trace total deviation 1)
L_G	Separation of bearing centres of a shaft
Q_G	Swelling of housing
Q_R	Swelling of gears

1) Measured according to DIN 3960 in the transverse section tangential to the base cylinder.

R_s	Tooth thickness fluctuation
T	Tolerance
$T_{a''}$	Tolerance of two-flank working distance
$T_{R\bar{s}}$	Tolerance of normal chordal tooth thickness fluctuation
T_{sn}	Tooth thickness tolerance in the normal section
T_{Md}	Tolerance of diametral two-ball or two-roller measurement
T_{Mr}	Tolerance of radial single-ball or single-roller measurement
T_W	Base tangent length tolerance
α	Pressure angle
α_n	Normal pressure angle
α_t	Transverse pressure angle
α_G	Linear coefficient of expansion of housing
α_R	Linear coefficient of expansion of gears or gear rings
β	Helix angle
Δj_a	Backlash modification through centre distance tolerance
Δj_B	Backlash modification through form and dimension deviations of the components
Δj_E	Backlash modification through elasticity
Δj_F	Backlash modification through gear tooth individual deviations
Δj_Q	Backlash modification through swelling or contraction
Δj_ϑ	Backlash modification through temperature rise
$\Delta j_{\Sigma\beta}$	Backlash modification through non-parallelism of bore axes
$\Delta\vartheta_G$	Temperature difference of housing relative to 20 °C
$\Delta\vartheta_R$	Temperature difference of gears relative to 20 °C
ΣA_{sne}	Sum of upper allowances of tooth thickness of gear pair in the normal section
ΣA_{sni}	Sum of lower allowances of tooth thickness of gear pair in the normal section
ΣA_{ste}	Sum of upper allowances of tooth thickness of gear pair in the transverse section
ΣA_{sti}	Sum of lower allowances of tooth thickness of gear pair in the transverse section

Further subscripts:

1	For quantities on the smaller gear of a gear pair
2	For quantities on the larger gear of a gear pair
K	When measurement with balls
R	When measurement with rollers

A.1.2 Connection between backlash and allowances

In contrast with cylindrical fits the backlash arising with gear tooth fits cannot be calculated directly from the allowances, since various backlash-modifying factors are effective. Conversely, if a specific minimum or maximum backlash is required, this amount cannot simply be distributed over the allowances, but instead the backlash-modifying effects have to be taken into account in the calculation.

A.2 Backlash-modifying effects

A.2.1 Temperature rise

A change of backlash through temperature rise occurs not only when gears and housing are made of materials having different linear coefficients of expansion, but also due to the fact that, particularly when the gear transmission is started up, the temperature rise of gears and housing is different. It is often this component which is the largest of all. A temperature difference is equivalent to a change in the centre distance of the housing.

A.2.2 Centre distance tolerance of the housing

Through this tolerance the theoretical centre distance is reduced or increased, since according to DIN 3964 plus/minus tolerancing is used. In this way the backlash is reduced or increased.

A.2.3 Non-parallelism of bore axes in the housing

The non-parallelism of the bore axes in the housing may consist of axial inclination and axial skew. Axial inclination does not need to be taken into account because it is not allowed to exceed the centre distance tolerances and is thus covered by these. Axial skew is always backlash-reducing.

A.2.4 Gear tooth individual deviations

Gear tooth individual deviations may act differently at the circumference of the gear. In each case however a backlash reduction is effective at one or more points in the case of individual tooth trace, profile and pitch deviations and also with tooth thickness fluctuations. These deviations are in some cases inter-related, so that a summation of the maximum allowable values never occurs.

The concentricity deviation need not be taken into account if the tooth thickness is toleranced, because the tooth thickness, and correspondingly the tooth thickness fluctuation also, is referred via the reference circle to the gear axis.

A.2.5 Swelling or contraction

The swelling or contraction of plastics in damp air, water, hydrocarbons or other chemicals alters the backlash.

If the material has been suitably pretreated (pre-swelled) prior to machining, any subsequent alteration is generally so slight that it can be disregarded.

A.2.6 Position, form and dimension deviations of components

Mainly involved here are concentricity deviations of bearings (internal and external diameters) and of fixed or rotating parts mounted in one another. The deviations may accumulate or cancel, and do so cyclically in the case of moving (rotating) parts. Thus they have both a backlash-increasing and backlash-reducing effect. Tooth thickness alterations through form alterations resulting from the shrinking-on of toothed components have to be treated separately.

A.2.7 Elasticity

The effect of elasticity consists mainly of a displacement in the bearings and housings and deflection of the shafts and housing under load. It acts nearly always to increase

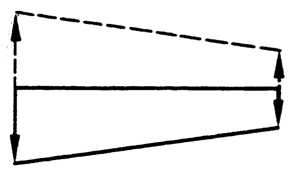
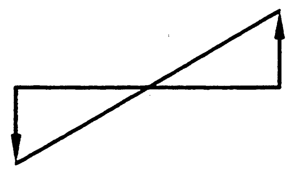
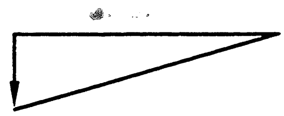
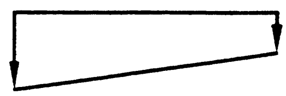
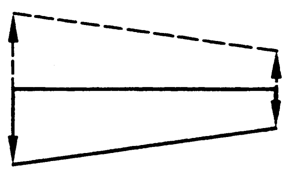
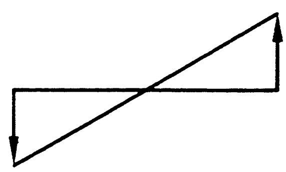
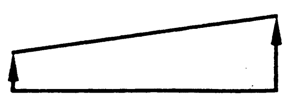
Effect	Symbol	Direction when calculating the upper lower allowances	Remarks
Temperature rise	Δj_{ϑ}		$\alpha_G > \alpha_R$ $\alpha_G < \alpha_R$
Centre distance tolerance	Δj_a		↑ for plus allowances ↓ for minus allowances
Non-parallelism of bores	$\Delta j_{\Sigma\beta}$		
Gear tooth individual deviations	Δj_F		
Swelling or contraction	Δj_Q		$Q_G > Q_R$ $Q_G < Q_R$
Form and dimension deviations of components	Δj_B		
Elasticity	Δj_E		

Figure A.1. Action of the backlash-modifying effects

↑ backlash-increasing +

↓ backlash-reducing –

backlash in the operating condition. When taken into account in the calculation it results in the acceptance backlash becoming smaller.

A.3 Action of the backlash-modifying effects

The action of the backlash-modifying effects is shown schematically in Fig. A.1. It differs according to whether the calculation has been made for minimum backlash or maximum backlash. In the case of minimum backlash each backlash reduction demands an increase in the amounts of the tooth thickness allowances. Since the allowances are always negative this is represented in the schematic by a downward-directed arrow. The worst case condition is considered each time. In the event of calculation for maximum backlash a different condition may arise and may result in a smaller or opposite effect.

A.4 Calculation of the backlash-modifying effects

Since it is necessary when fixing the tooth thickness allowances to start from a stipulated backlash (working backlash) the backlash-modifying effects are calculated as backlash modifications.

A.4.1 Backlash modification through temperature rise Δj_ϑ

The following applies with adequate accuracy:

$$\Delta j_\vartheta = a \cdot (\Delta\vartheta_G \cdot \alpha_G - \Delta\vartheta_R \cdot \alpha_R) \cdot 2 \cdot \frac{\tan\alpha_n}{\cos\beta} \tag{1}$$

If Δj_ϑ is positive, a backlash increase takes place, see also Section A.2.1. In the condition at rest a reduction of backlash can then arise, and this has to be separately calculated. This may be necessary both at the reference temperature of 20 °C and also at lower temperatures. In this case $\Delta\vartheta_G$ is equal to $\Delta\vartheta_R$.

A.4.2 Backlash modification through centre distance tolerance Δj_a

In the calculation the least favourable allowance has to be taken as the basis each time and given the appropriate sign. This means A_{ai} for the minimum backlash and A_{ae} for the maximum backlash in the case of external gear pairs, and A_{ae} for the minimum backlash and A_{ai} for the maximum backlash in the case of internal gear pairs.

$$\Delta j_a \approx 2 \cdot A_a \cdot \frac{\tan\alpha_n}{\cos\beta} \tag{2}$$

A.4.3 Backlash modification through non-parallelism of bore axes $\Delta j_{\Sigma\beta}$

The effect of axial skew is the same as that of tooth trace angle deviations. The backlash modification is calculated from

$$\Delta j_{\Sigma\beta} = - f_{\Sigma\beta} \cdot \frac{b}{L_G} \tag{3}$$

For calculating the maximum backlash the condition prevailing with perfectly parallel bore axes is the criterion. In this case $\Delta j_{\Sigma\beta} = 0$.

A.4.4 Backlash modification through gear tooth individual deviations Δj_F

The following are taken into consideration:

a) tooth trace deviations
b) profile deviations
c) individual pitch deviations

It is unlikely that all three deviations will be effective at the same time to their full value. The backlash-reducing component Δj_F is therefore calculated according to the error propagation law as follows

$$\Delta j_F = - \sqrt{\left(\frac{F_\beta}{\cos\alpha_t}\right)^2 + \left(\frac{F_f}{\cos\alpha_t}\right)^2 + f_p^2} \tag{4}$$

For the maximum backlash the least favourable case would arise if no deviations were present. With gears however this is never the case. At best the gears have a deviation which is equal to half the deviation permissible for their quality. For the maximum backlash therefore only 1/2 Δj_F is effective.

The dependence of the parameter Δj_F on the reference diameter and facewidth is negligible. For $\alpha = 20°$ therefore the rounded values for Δj_F can be taken from the Table A.1, only the dependence on module and gear tooth quality being taken into account.

Table A.1 **Rounded values of Δj_F in µm**

Module (mm)		Gear tooth quality											
over	up to	1	2	3	4	5	6	7	8	9	10	11	12
1	2	4	6	7	10	13	17	24	34	51	82	130	210
2	3,55	5	6	8	10	14	18	24	36	54	86	136	218
3,55	6	5	7	9	12	15	19	27	40	60	94	150	236
6	10	6	8	11	14	19	25	34	51	75	120	187	300
10	16	7	9	13	17	23	31	41	59	86	138	216	362
16	25	8	11	15	20	28	38	52	75	108	171	289	434
25	40	10	14	19	26	34	48	66	94	135	214	339	536

A.4.5 Backlash modification through swelling or contraction Δj_Q

The effect is the same as that of temperature rise. In the calculation the signs have to be watched: swelling is to be taken as positive and contraction as negative.

If both gears are of plastic material and if w is the relative water absorption (e. g. $w = 0.02 \triangleq 2$ percent by volume), then the relative linear expansion is approximately $^1/_3\, w$ and the following backlash modification arises:

$$\Delta j_Q = \left(\frac{1}{3} w \cdot a\right) \cdot 2 \frac{\tan \alpha_n}{\cos\beta} \qquad (5)$$

Reference values for the relative water absorption w can be found from the data published for the material concerned by the plastics manufacturers.

Similar considerations apply to the swelling and contraction of other components which influence the backlash.

A.4.6 Backlash modification through position, form and dimension deviations of components Δj_B

These act like centre distance deviations and are therefore calculated according to equation (2).

A.4.7 Backlash modification through elasticity Δj_E

This component depends on the service loading and has to be determined according to the design circumstances. First, the displacements of the gears, which affect the centre distance deviations, are calculated. The calculation of the backlash modification follows the lines of equation (2).

A.5 Calculation of sum of upper allowances ΣA_{ste} from minimum backlash $j_{t\,min}$ and the backlash-modifying effects

The calculation is based on the minimum backlash $j_{t\,min}$. This is the smallest circumferential backlash which must be present in the completed gear transmission in the least favourable operating condition.

When determining this backlash it should be borne in mind that all backlash-reducing effects are dealt with in the calculation. The minimum backlash can thus be kept small. This applies in particular to the coarser qualities in which the highest spots of the tooth flanks are present only at a few points and therefore wear more quickly than in the case of finer qualities.

The interaction of the different effects is shown in Fig. A.1. Some of these components however do not act simultaneously to the full extent. These have therefore to be allowed for according to the error propagation law. The sum of the upper allowances is first calculated in the transverse section.

$$\Sigma A_{ste} \leqq -\Big(j_{t\,min} - (\Delta j_\vartheta) - (\Delta j_Q) - (\Delta j_E) + \sqrt{\Delta j_a^2 + \Delta j_{\Sigma\beta}^2 + \Delta j_{F1}^2 + \Delta j_{F2}^2 + \Delta j_B^2}\Big) \qquad (6)$$

(For positive backlash the allowances are negative.)

The individual backlash modifications are to be inserted with the sign found for them.

For normal cases in general mechanical engineering $\Delta j_Q = \Delta j_E = \Delta j_B = 0$ can be adopted and the equation simplified accordingly as follows:

$$\Sigma A_{ste} \leqq -\Big(j_{t\,min} - (\Delta j_\vartheta) + \sqrt{\Delta j_a^2 + \Delta j_{\Sigma\beta}^2 + \Delta j_{F1}^2 + \Delta j_{F2}^2}\Big) \qquad (7)$$

A.5.1 Determining the upper allowances of tooth thickness in the normal section

The sum of the upper allowances in the transverse section calculated with equation (6) or (7) has to be converted to the normal section.

$$A_{sne\,1} + A_{sne\,2} = \Sigma A_{ste} \cdot \cos\beta \qquad (8)$$

It is immaterial how the sum of the allowances is distributed between the two gears, so long as the condition of equation (8) is complied with. Hence, for one of the gears the upper allowance 0 can also be adopted. The principle to be applied here is that impairment of the strength at the tooth root is to be avoided as far as possible.

From Table 1 a suitable value has to be chosen consistent with the calculated sum $A_{sne\,1} + A_{sne\,2}$ or with empirical values. The tabulated values are the upper allowances of tooth thicknesses in the normal section and apply to all modules and all qualities. The selection should be made in such a manner that the amount of the sum of the selected allowances is at least as large as the amount of the sum calculated according to equations (6) to (8).

A.6 Calculation of sum of lower allowances ΣA_{sti} from maximum backlash $j_{t\,max}$ and the backlash-modifying effects

A.6.1 Definition

The sum of the lower allowances of tooth thickness of the gear pair in the transverse section is calculated from the maximum backlash which has not to be exceeded in the completed gear transmission when the backlash-modifying effects are operative. These are the same as apply to the minimum backlash (see Section A.5).

A.6.2 Maximum backlash $j_{t\,max}$

This is the largest circumferential backlash which may be present in the completed gear transmission in the least favourable operating condition.

When determining this backlash it must be borne in mind that all backlash-modifying effects are dealt with by the calculation. It should not be chosen too small. Narrowing of the backlash should be undertaken only if the function of the gear transmission demands this (actuator transmissions, transmissions for instrumentation purposes, gear transmissions with non-uniform drive or alternation of loading direction). In all other cases the only determining factor is a possible reduction of the root strength of the teeth through diminished tooth thickness; therefore it is often possible to determine the sum of the lower allowances without calculation.

A.6.3 Calculation

The interaction of the effects is shown in Fig. A.1. The rules which apply are the same as those for determining the upper allowances. Hence the following is obtained:

$$\Sigma A_{\text{sti}} = -\left(j_{\text{t max}} - (\Delta j_{\vartheta}) - (\Delta j_{\text{Q}}) - (\Delta j_{\text{E}}) \pm \sqrt{\left| -(\Delta j_{\text{a}})^2 + (\Delta j_{\Sigma\beta})^2 + \left(\frac{\Delta j_{\text{F1}}}{2}\right)^2 + \left(\frac{\Delta j_{\text{F2}}}{2}\right)^2 - (\Delta j_{\text{B}})^2 \right|} \right) \tag{9}$$

In the calculation the individual backlash modifications have to be inserted with the correct sign. If the value between the vertical strokes under the square root sign yields a negative figure, the minus sign should be used before the square root, otherwise the plus sign.

For normal cases in general mechanical engineering $\Delta j_{\text{Q}} = \Delta j_{\text{E}} = \Delta j_{\text{B}} = 0$ can be adopted and the equation thereby simplified as follows

$$\Sigma A_{\text{sti}} = -\left(j_{\text{t max}} - (\Delta j_{\vartheta}) \pm \sqrt{\left| -(\Delta j_{\text{a}})^2 + (\Delta j_{\Sigma\beta})^2 + \left(\frac{\Delta j_{\text{F1}}}{2}\right)^2 + \left(\frac{\Delta j_{\text{F2}}}{2}\right)^2 \right|} \right) \tag{10}$$

A.6.4 Determining the lower allowances of tooth thickness in the normal section

The sum of the lower allowances in the transverse section calculated by equation (9) is converted according to equation (8) to the normal section.

With equation (9) the case may arise in which the amount of the sum of the lower allowances is larger than the maximum backlash $j_{\text{t max}}$.

The allotment of the lower allowances should be made in such a way that, corresponding to their size, the two gears are given tolerances consistent with the production requirements.

From Table 2 tooth thickness tolerances are selected such that the sum of the tolerances for the two gears can be

$$T_1 + T_2 \leqq \Sigma A_{\text{sne}} - \Sigma A_{\text{sni}} \tag{11}$$

At the same time a check must be made to ensure that the tolerance is equal to at least twice the tolerance for the tooth thickness fluctuation

$$T \geqq 2\,R_{\text{s}} \tag{12}$$

The lower allowances then result from the upper allowances and the tolerances

$$A_{\text{sni}} = A_{\text{sne}} - T \tag{13}$$

If the upper and lower allowances are determined on the criterion of function, the case may arise in which the tooth thickness tolerance is no longer consistent with production requirements.

To arrive at a larger tolerance, the backlash-reducing components of the minimum backlash must be reduced and likewise the backlash-increasing components of the maximum backlash. In this case therefore a tooth thickness tolerance consistent with the production requirements frequently demands a higher accuracy class for the axial position; in other words it is necessary to weigh the cost of production of the gears against that of the housing.

If it is impossible or undesirable to increase component accuracy any further, a check should be made to establish whether, and if so at what cost, the working temperature can be influenced. Selective assembly of gear and mating gear also makes possible an increase in the tolerance, since in this case the tolerances do not add together in full, but instead overlap partially or completely.

A.7 Calculation of backlash from the tooth thickness allowances and the backlash-modifying effects

With prescribed allowances – e. g. in accordance with the system of fits of this Standard – the prospective acceptance or working backlash is calculated with the backlash-modifying effects taken into account.

$$j_{\text{t min}} = -\Sigma A_{\text{ste}} - \sqrt{\Delta j_{\text{a}}^2 + \Delta j_{\Sigma\beta}^2 + \Delta j_{\text{F1}}^2 + \Delta j_{\text{F2}}^2 + \Delta j_{\text{B}}^2} + (\Delta j_{\vartheta}) + (\Delta j_{\text{Q}}) + (\Delta j_{\text{E}}) \tag{14}$$

$$j_{\text{t max}} = -\Sigma A_{\text{sti}} \pm \sqrt{\left| -(\Delta j_{\text{a}})^2 + (\Delta j_{\Sigma\beta})^2 + \left(\frac{\Delta j_{\text{F1}}}{2}\right)^2 + \left(\frac{\Delta j_{\text{F2}}}{2}\right)^2 - (\Delta j_{\text{B}})^2 \right|} + (\Delta j_{\vartheta}) + (\Delta j_{\text{Q}}) + (\Delta j_{\text{E}}) \tag{15}$$

Attention should be paid to the remarks concerning Δj_{a} in Section A.4.2.

The individual backlash modifications are to be inserted with the signs determined for them. If the value in equation (15) between the vertical lines is negative, the plus sign in front of the square root sign should be used, otherwise the minus sign.

If the individual influencing factors are present as actual values, the equation changes as follows:

$$j_{\text{t min}} = -\Sigma A_{\text{ste}} + \Delta j_{\text{a}} + \Delta j_{\Sigma\beta} + \Delta j_{\text{B}} - \frac{F_{\beta 1}}{\cos\alpha_{\text{t}}} - \frac{F_{\beta 2}}{\cos\alpha_{\text{t}}} - \sqrt{\left(\frac{F_{\text{f1}}}{\cos\alpha_{\text{t}}}\right)^2 + f_{\text{p1}}^2} - \sqrt{\left(\frac{F_{\text{f2}}}{\cos\alpha_{\text{t}}}\right)^2 + f_{\text{p2}}^2} + \Delta j_{\vartheta} + \Delta j_{\text{Q}} + \Delta j_{\text{E}} \tag{16}$$

$$j_{\text{t max}} = -\Sigma A_{\text{sti}} + \Delta j_{\text{a}} + \Delta j_{\Sigma\beta} + \Delta j_{\text{B}} - \frac{F_{\beta 1}}{\cos\alpha_{\text{t}}} - \frac{F_{\beta 2}}{\cos\alpha_{\text{t}}} - \sqrt{\left(\frac{F_{\text{f1}}}{2\cdot\cos\alpha_{\text{t}}}\right)^2 + \left(\frac{f_{\text{p1}}}{2}\right)^2} - \sqrt{\left(\frac{F_{\text{f2}}}{2\cdot\cos\alpha_{\text{t}}}\right)^2 + \left(\frac{f_{\text{p2}}}{2}\right)^2} + \Delta j_{\vartheta} + \Delta j_{\text{Q}} + \Delta j_{\text{E}} \tag{17}$$

A.8 Allowance diagram of tooth thickness in the normal section

Fig. A.2 shows how the backlash and backlash modifications are constituted and how the sum of the allowances and tolerances of the two gears results therefrom. Since some components may act with both backlash-reducing and backlash-increasing effect, the diagram can only serve as an example and does not apply to all cases.

The starting point is zero allowance. Since both allowances are negative, the amounts of backlash (addition of the allowances) are also entered as negative values. The backlash modification through unequal temperature rise acts to reduce the backlash at $\Delta j_{\vartheta} < 0$ (lower part of the diagram). The upper allowance must therefore have

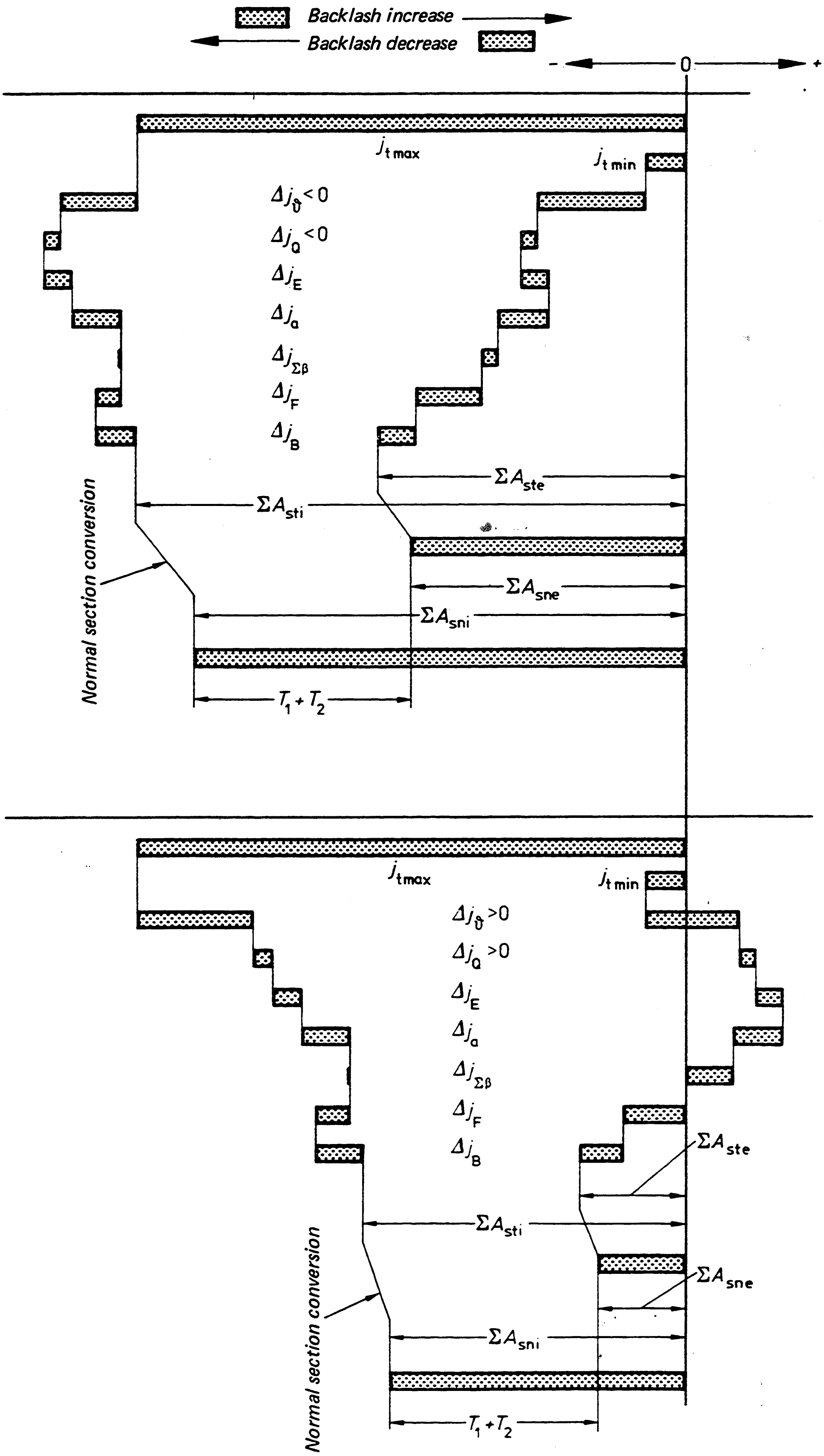


Figure A.2. Backlash-modifying effects and tooth thickness allowances under different conditions a larger absolute value than would be necessary for the minimum backlash only. The component Δj_ϑ therefore has to be indicated in the direction away from the zero line, like any other backlash reduction also; similarly backlash increases are directed towards the zero line.

After all components have been taken into account the totals of the upper and lower allowances in the transverse section result. The sums of the allowances in the normal section are smaller, corresponding to the helix angle. The difference of the two calculated sums is the sum of the tolerances of the two gears which is distributed with due consideration of production requirements. The sums of the allowances are distributed according to design considerations (tooth thickness).

The upper part of the diagram shows how the allowances develop when Δj_ϑ and Δj_Q are larger than 0 (e. g. in the case of steel gears in plastics housings).

A.9 Example for determining tooth thickness allowances

Same gear transmission data as in main part, Section 5, but also gear with gear tooth quality 6. Maximum temperature difference between housing and gears 20°C at 70 °C gear temperature, transmission temperature at full load: gears 90 °C, housing 80 °C. Coefficient of linear expansion of steel $\alpha_R = 11.5 \cdot 10^{-6}$, of cast iron $\alpha_G = 10 \cdot 10^{-6}$. Maximum backlash 300 µm.

A.9.1 Upper allowances

a) The minimum backlash $j_{t\,min}$ is taken as 20 µm.

b) According to equation (1) the backlash modification through temperature rise is

$$\Delta j_\vartheta = 300 \cdot [(50-20) \cdot 10 \cdot 10^{-6} - (70-20) \cdot 11{,}5 \cdot 10^{-6}] \cdot 2 \cdot \frac{\tan 20°}{\cos 9{,}8969°}$$
$$= -0{,}061 \text{ mm} \approx -60 \text{ µm}$$

c) According to equation (2) the backlash modification through centre distance tolerance is

$$\Delta j_a = 2 \cdot (-26) \cdot \frac{\tan 20°}{\cos 9{,}8969°} = -19 \text{ µm}$$

d) According to equation (3) the backlash modification through non-parallelism of the bore axes is

$$\Delta j_{\Sigma\beta} = -20 \cdot \frac{70}{200} = -7 \text{ µm}$$

e) Table A.1 gives $\Delta j_F = 19$ µm

f) Since steel and cast iron are not subject to swelling, $\Delta j_Q = 0$ is adopted.

g) For the backlash modification due to position, form and dimension deviations of the components, the following is assumed:

$\Delta j_B = -15$ µm.

h) The backlash modification through elasticity Δj_E is disregarded here because it is assumed that in this case it is not backlash-reducing, i. e. $\Delta j_E = 0$.

i) According to equation (6) the sum of the upper allowances in the transverse section is

$$\Sigma A_{ste} \leqq -\left(20-(-60)-0-0 + \sqrt{(-19)^2+(-7)^2+19^2+19^2+(-15)^2}\right)$$
$$= -117 \text{ µm}$$

j) The conversion to the normal section according to equation (8) yields

$$\Sigma A_{sne} = -117 \cdot \cos 9{,}8969° = -115 \text{ µm}$$

k) According to Table 1 the upper allowances are selected such that their sum amounts to at least 115 µm:

$A_{sne\,1} = -40$ µm (series e) $A_{sne\,2} = -75$ µm (series e)

A.9.2 Lower allowances

a) The maximum backlash $j_{t\,max}$ is taken as 300 µm.

b) According to equation (1) the backlash modification through temperature rise is

$$\Delta j_\vartheta = 300 \cdot [(80-20) \cdot 10 \cdot 10^{-6} - (90-20) \cdot 11{,}5 \cdot 10^{-6}] \cdot 2 \cdot \frac{\tan 20°}{\cos 9{,}8969°}$$
$$= -0{,}045 \text{ mm} = -45 \text{ µm}$$

c) According to equation (2) the backlash modification through centre distance tolerance is

$$\Delta j_a = 2 \cdot 26 \cdot \frac{\tan 20°}{\cos 9{,}8969°} = 19 \text{ µm.}$$

It therefore has the effect of increasing backlash.

d) According to equation (3) the backlash modification through non-parallelism of the bore axes is

$\Delta j_{\Sigma\beta} = 0$

e) According to Section A.4.4 and Table A.1 the backlash modification through gear tooth modification is:

$$\Delta j_F = \frac{1}{2} \cdot 19 = 9{,}5 \text{ µm}$$

f) Since steel and cast iron are not subject to swelling, $\Delta j_Q = 0$ is adopted.

g) For the backlash modification due to position, form and dimension deviations of the components, the following is assumed:

$\Delta j_B = 15$ µm.

h) The calculation of shaft deflection yields a backlash modification $\Delta j_E = 15$ µm.

i) According to equation (9) the sum of the lower allowances in the transverse section is

$$\Sigma A_{sti} = -\left(300-(-45)-0-15 \pm \sqrt{|-19^2+0^2+9{,}5^2+9{,}5^2-15^2|}\right)$$
$$= -310 \text{ µm}$$

The sum of the lower allowances may therefore amount to -310 µm if a maximum backlash of 300 µm is to be guaranteed.

j) The conversion to the normal section on the lines of equation (8) yields

$$\Sigma A_{sni} = - 310 \cdot \cos 9{,}8969° = - 305\ \mu m$$

k) According to equation (11) the sum of the tolerances for both gears is

$$T_1 + T_2 = |\Sigma A_{sni} - \Sigma A_{sne}| = |- 305 - (- 115)| = 190\ \mu m$$

From Table 2 the following are selected from tolerance series 26 for the pinion and gear

$$T_1 = 60\ \mu m \text{ and } T_2 = 100\ \mu m$$
$$T_1 + T_2 = 160\ \mu m$$

According to DIN 3962 Part 1 the tooth thickness fluctuation is allowed to be

$$R_{s\,1} = 14\ \mu m \qquad R_{s\,2} = 18\ \mu m$$

The tolerances are thus more than twice as large as the tooth thickness fluctuation, i. e. the condition according to equation (12) is fulfilled.

Equation (13) gives the lower allowances as $A_{sni\,1} = -40 - 60 = -100\ \mu m$ and $A_{sni\,2} = -75 - 100 = -175\ \mu m$. Fig. A. 3 shows the size of the backlash and backlash modifications.

A.9.3 Lower allowances without a specified maximum backlash

If no maximum backlash is specified, the tolerances are selected freely according to equation (12) and Table 2.

$$T_1 > 28\ \mu m \qquad T_2 > 36\ \mu m$$

On the basis of production requirements tolerance series 27 is chosen.

$$T_1 = 100\ \mu m \qquad T_2 = 160\ \mu m$$

From equation (13) the following are found

$$A_{sni\,1} = - 140\ \mu m \qquad A_{sni\,2} = - 235\ \mu m$$

A.9.4 Allowances under modified conditions

If instead of a grey iron housing a light metal housing with linear coefficient of expansion $\alpha = 24 \cdot 10^{-6}$ is used, the conditions are changed fundamentally. The backlash modification through temperature rise Δj_ϑ acts to increase backlash.

The worst condition for minimum backlash therefore occurs at reference temperature 20 °C. Here $\Delta j_\vartheta = 0$.

Hence the sum of the upper allowances in the transverse section is

$$\Sigma A_{ste} = -\left(20 + \sqrt{(-19)^2 + (-7)^2 + 19^2 + 19^2 + (-15)^2}\right)$$
$$= - 57\ \mu m$$

If the gear transmission is to be exposed to relatively low temperatures in the idle condition, as may be the case, for example, with vehicle transmissions, this circumstance must be taken into account, so that backlash is still present at this temperature. At a temperature of -30 °C, $\Delta\vartheta_R = \Delta\vartheta_G = 50$ °C will apply.

The backlash modification through temperature rise from -30 °C to $+20$ °C alone amounts to 138 µm. In this case therefore the minimum backlash must be made at least 140 µm, so that ΣA_{ste} is changed to -177 µm.

For calculation of the lower allowances there is a backlash modification through temperature rise

$$\Delta j_\vartheta = 300 \cdot (60 \cdot 24 \cdot 10^{-6} - 70 \cdot 11{,}5 \cdot 10^{-6}) \cdot 2 \cdot \frac{\tan 20°}{\cos 9{,}8969°}$$
$$= 0{,}141\ mm = 141\ \mu m$$

Hence, according to equation (9) the sum of the lower allowances is

$$\Sigma A_{sti} = -\left(300 - 141 - 15 \pm \sqrt{|-19^2 + 0^2 + 9{,}5^2 + 9{,}5^2 - 15^2|}\right)$$
$$= - 124\ \mu m$$

This however would make the sum of the lower allowances (-124 µm) larger than the sum of the upper allowances (-174 µm), so that there is no tolerance. For this case therefore the specification for the maximum backlash needs to be checked and a design modification undertaken if necessary.

A.9.5 Acceptance backlash

If checking of acceptance backlash is proposed, it must be borne in mind that when the gear transmission is cold then the minimum acceptance backlash must be larger than the minimum backlash by Δj_ϑ. In the example therefore 20 + 60 = 80 µm.

As regards the maximum acceptance backlash, the permissible maximum backlash can be exceeded in the cold condition by the corresponding Δj_ϑ although, of course, Δj_E must be taken into account. In the example therefore the maximum acceptance backlash is 300 + 45 − 15 = 330 µm.

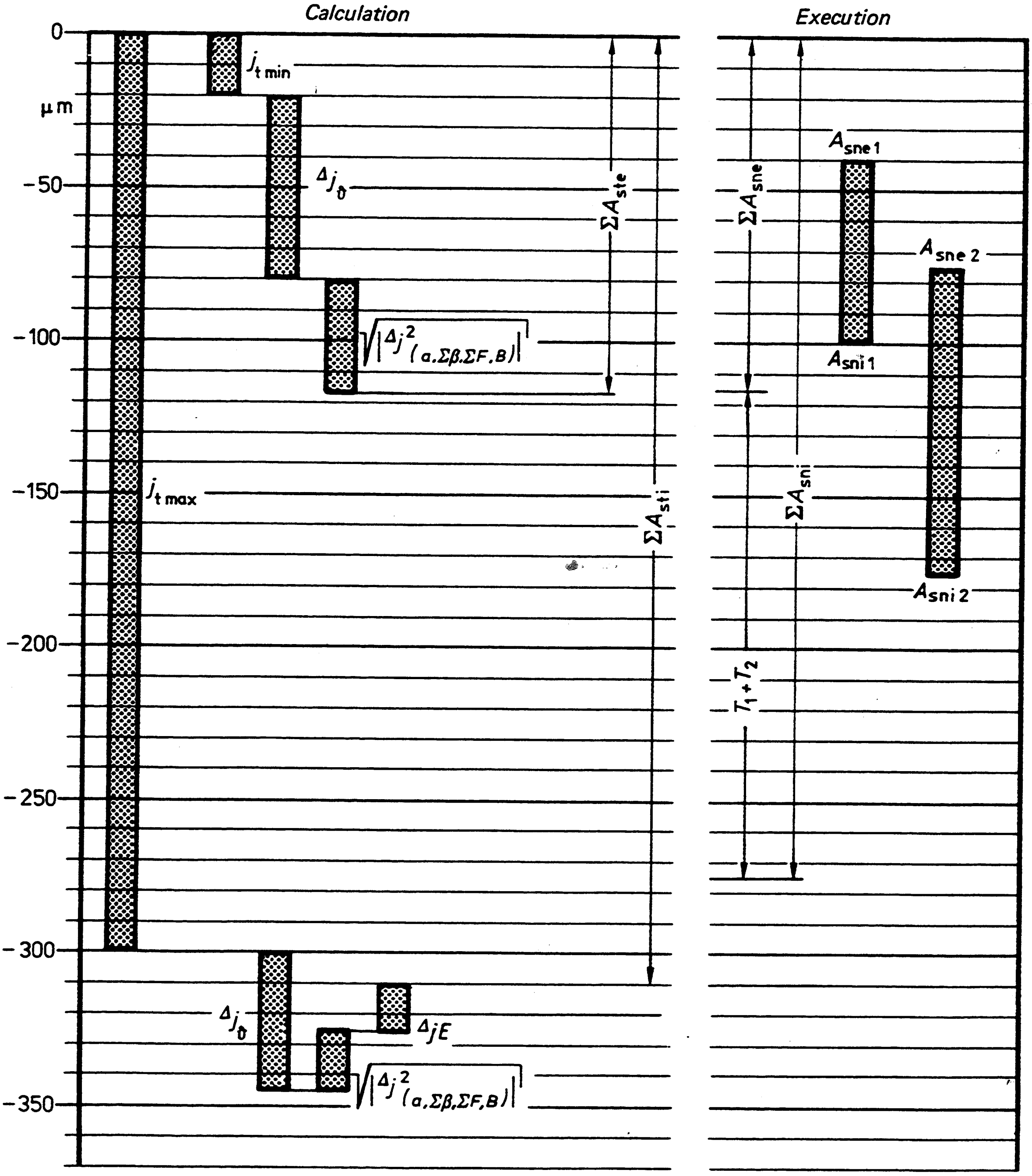


Figure A.3. Backlash, backlash modifications

Allowances and tolerances for the worked example (the components under the root sign of equations (7) and (9) have been combined to a single amount)

A.10 Example for determining the backlash to be expected

System of fits DIN 3967: pinion 27 cd

gear 26 cd

Centre distance allowances DIN 3964: ISO tolerance zone js7

Gear tooth quality DIN 3962: 6

For further data see Section A.9

A.10.1 Determining the theoretical backlash

1. The sum of the tooth thickness allowances is

$$\Sigma A_{sne} = -(70) + (-130) = -200\ \mu m$$

$$\Sigma A_{sni} = -(170) + (-230) = -400\ \mu m$$

2. Conversion to the transverse section according to equation (8) yields

$$\Sigma A_{ste} = \frac{-200}{\cos 9{,}8969°} = -203\ \mu m$$

$$\Sigma A_{sti} = \frac{-400}{\cos 9{,}8969°} = -406\ \mu m$$

3. According to equation (2) the backlash modification due to the centre distance allowances $A_{ai} = -26\ \mu m$ and $A_{ae} = +26\ \mu m$ is calculated as

$$\Delta j_{ai} = 2 \cdot (-26) \frac{\tan 20°}{\cos 9{,}8969°} = -19\ \mu m$$

$$\Delta j_{ae} = 2 \cdot (+26) \frac{\tan 20°}{\cos 9{,}8969°} = +19\ \mu m$$

4. According to Section 2.1 the theoretical backlash is

$$j_{t\,min} = -(-203) + (-19) = 184\ \mu m$$

$$j_{t\,max} = -(-406) + (19) = 425\ \mu m$$

A.10.2 Acceptance backlash

Δj_{ϑ}, Δj_Q, Δj_E are left out of consideration when determining the acceptance backlash.

A.10.2.1 Minimum backlash

1. According to Section A.9.1 the following apply

$$\Delta j_{\Sigma\beta} = -7\ \mu m$$

$$\Delta j_{F1} = \Delta j_{F2} = 19\ \mu m$$

$$\Delta j_B = -15\ \mu m$$

2. According to equation (14) therefore the minimum backlash is found as

$$j_{t\,min} = -(-203) - \sqrt{(-19)^2 + (-7)^2 + (19)^2 + (19)^2 + (-15)^2} + 0 = 166\ \mu m$$

A.10.2.2 Maximum backlash

1. Backlash modification due to non-parallelism of the bore axes according to Section A.4.3

$\Delta j_{\Sigma\beta} = 0$

2. According to Section A.9.2 the following apply

$$\Delta j_{F1} = \Delta j_{F2} = 19\ \mu m$$

$$\Delta j_B = 15\ \mu m$$

3. Therefore according to equation (15) the maximum acceptance backlash is

$$j_{t\,max} = -(-406) \pm \sqrt{\left| -(19)^2 + 0^2 + \left(\frac{19}{2}\right)^2 + \left(\frac{19}{2}\right)^2 - (15)^2 \right|} + 0 = 426\ \mu m$$

The theoretical backlash of $j_t = 184\ \mu m$ to $425\ \mu m$ is modified by the backlash-reducing effects to become $j_t = 166\ \mu m$ to $426\ \mu m$.

Appendix B

Conversion of allowances for the different measuring methods

Contents

Symbols and designations as in Appendix A

Additionally:

a''	Two-flank working distance
b	Facewidth
m	Module
s	Tooth thickness
$\bar{s}_y$	Normal chordal tooth thickness on the y cylinder
x	Addendum modification coefficient
z	Number of teeth
$A^*_{a''}$	Allowance factor of the two-flank working distance
A^*_{Md}	Allowance factor of diametral two-ball or two-roller dimension
A^*_{Mr}	Allowance factor of radial single-ball or single-roller dimension
F_β	Total tooth trace deviation
M_d	Diametral two-ball or two-roller dimension
M_{dK}	Diametral two-ball dimension
M_{dR}	Diametral two-roller dimension
M_r	Radial single-ball or single-roller dimension
R_s	Tooth thickness fluctuation
$R_{sa''}$	Tooth thickness fluctuation from two-flank working distance
$R_{s\bar{s}y}$	Tooth thickness fluctuation from chordal measurement on the y cylinder
R_{sMdK}	Tooth thickness fluctuation from two-ball measurement
R_{sMdR}	Tooth thickness fluctuation from two-roller measurement
R_{sW4}	Tooth thickness fluctuation from base tangent length over 4 measured teeth
$T_{a''}$	Tolerance of two-flank working distance
T_s	Tooth thickness tolerance
$T_{sa''}$	Tooth thickness tolerance for two-flank working distance
$T_{s\bar{s}y}$	Tooth thickness tolerance for the chordal measurement on the y cylinder
T_{sMdK}	Tooth thickness tolerance for two-ball measurement
T_{sMdR}	Tooth thickness tolerance for two-roller measurement
T_{sW}	Tooth thickness tolerance for the base tangent length
$T_{\bar{s}y}$	Tolerance of normal chordal tooth thickness on the y cylinder
T_{MdK}	Tolerance of the diametral two-ball dimension
T_{MdR}	Tolerance of the diametral two-roller dimension
T_W	Base tangent length tolerance
W_4	Base tangent length over 4 measured teeth
α_t	Transverse pressure angle
β	Helix angle
ψ	Tooth thickness half angle

B.1 Determining the tooth thickness from the tooth thickness angle

The tooth thickness can be measured by mechanically tracing the two flanks of a tooth in the V circle by means of a measuring pick-up (as null indication) in conjunction with an angle measuring instrument. The tooth thickness is found by converting the measured tooth thickness angle 2 ψ into radians, so that the tooth thickness is determined in accordance with the definition. The eccentricity of the gear teeth can be found, since the measurement is referenced to the mounting axis of the gear.

Since angle measuring facilities of the necessary accuracy are usually not available in industry, this method is not suitable for practical application and is only of significance for scientific investigations.

B.2 Measurement of the normal chordal tooth thickness

On the basis of a reference diameter, which can conveniently be the tip diameter, the normal chordal tooth thickness is measured in the normal section at a given depth, normally on the V circle. For this purpose it is necessary to determine beforehand, for the tooth to be measured, the tip circle radius with reference to the gear mounting. With the concentricity deviation of the tip circle taken into account and a sufficient number of measurements made on the gear circumference, the upper and lower actual allowances of the normal chordal tooth thickness are found.

Conversion of the tooth thickness allowances from the arc to the chord can normally be dispensed with. Where large numbers of teeth or virtual numbers of teeth are involved the conversion of the allowances from the reference circle to the V circle according to DIN 3960, October 1976 edition, Section 4.1.1 also yields negligible differences. Nevertheless to avoid obscurity the reference should always be to chordal measurement; the expression tooth thickness measurement should be avoided in this case.

B.3 Measurement of the working distance with the master gear

Apart from the concentricity deviation of the gear teeth, this measurement also covers the tooth trace deviation up to the width of the master gear. The tooth trace deviation however has already been taken into account in calculating the tooth thickness allowances. Consequently before converting it is necessary to apply a plus correction to the permissible allowances by the amount of the tooth trace deviation. The criterion for the lower allowance is F_β (Appendix A, Section A.4.4). The upper allowance is displaced according to the master gear width. If this is larger than or equal to the width of the working gear all tooth trace deviations will be covered. If it is smaller an allowance in proportion to the two widths is sufficiently accurate. The mean measured value a'' and the allowance factor $A^*_{a''}$ are to be calculated as in DIN 3960, October 1976 edition, Section 5.

B.4 Measurement of base tangent length

This measurement does not cover the eccentricity of the gear teeth relative to the gear mounting. Pitch deviations over k teeth however enter into the measurement. Hence although theoretically the tooth thickness tolerance could be converted with a factor for the base tangent length tolerance, it is necessary here to add a correction value. The same applies to the base tangent length fluctuation.

B.5 Measurement of dimension over two rollers or balls

Here, too, the eccentricity of the gear teeth is not covered. Therefore in order to be sure that the tooth thickness is not exceeded at any point on the gear, it is necessary in this case also to add a correction value to the conversion factor. It must be borne in mind however that where relatively large allowances amounting practically to an additional addendum modification are involved, the allowance factor A^*_{Md} may alter to such an extent that it can no longer be disregarded. It is therefore appropriate to calculate it for the mean value of the tooth thickness allowances and not for the nominal dimension of the tooth thickness (see DIN 3960, October 1976 edition, Section 5), similarly the measurement over balls or rollers M_d.

B.6 Single-ball measurement and single-roller measurement

If these measurements are made radially from a reference diameter or from centring elements which can be equated with the gear mounting, they also cover eccentricities and are thus equatable with the chordal measurement. The mean measured value M_r and the allowance factor A_{Mr} are to be calculated as in DIN 3960, October 1976 edition, Section 5.

B.7 Determining the correction values

Although the influences which make correction values necessary are known, it is not possible to make any general statements. It is therefore recommended that measurements be made in the various production areas and the correction values determined from the actual values obtained by the different measuring methods.

B.7.1 Theoretical position of the tolerance zones

As an example, Fig. B.1 shows the tolerance zones for a given gear.

This illustration shows (right) how different in position and size are the allowances and tolerances of the measured values although on an ideal-geometry basis they express the same tooth thickness production tolerance.

B.7.2 Actually determined position of allowances

The results of measurements on five gears (milled, shaved, case hardened) on all teeth are compared in Fig. B.2. This shows both the fluctuations of the individual measured values in one and the same gear and also the differences of the gears one from another. All the measured values were converted to tooth thickness values and presented as such. Of all the measured values of each measuring method the weighted average was determined and presented each time.

The results of the measurements on all five gears were combined (right-hand side of illustration) the offset of the mean value of a particular type of measurement relative to the mean value of the normal chordal tooth thickness measurement of the gear concerned being shown each time. For the fluctuation of the values the weighted average was again determined. From these the correction value for the tolerance centre of each method of measurement can be determined.

Example:

With gear 1 the fluctuation of the measured values of base tangent length ranges from + 11 µm to – 27 µm. The weighted average is – 10 µm. The offset relative to the weighted average of the normal chordal tooth thickness is – 7 µm. For this value the fluctuation for this offset is between + 10 µm and – 7 µm for all five gears (see Fig. B.2, right). The weighted average of this fluctuation is + 1 µm. The mean offset relative to the normal chordal tooth thickness measurement is thus in this case + 1 µm.

Apart from this however the measurements on the individual gears yield fluctuations differing widely in magnitude, according to measuring method. It is therefore necessary to alter the size of the tolerance if it is to be guaranteed that all tooth thicknesses are within tolerance when checking is carried out by way of only individual measured values (and not over the whole circumference).

B.7.3 Actual allowances of normal chordal tooth thickness

In the example the fluctuation of the measured values on one and the same gear ranges up to 55 µm, which points to a concentricity deviation. If the maximum value is in tolerance, the minimum backlash is guaranteed.

B.7.4 Actual allowances of working distance

The fluctuation of the measured values of the gear, converted to the tooth thickness, confirms the concentricity deviation. If the maximum value of the working distance is within the theoretically calculated tolerance the minimum backlash is guaranteed. Compared with the chordal measurement however the mean value of the measurements is offset by + 10 µm. The cause of this lies in the profile and tooth trace deviations. These however have already been taken into account in the calculation of the tooth thickness allowances. The upper allowance of the working distance measurement can thus be displaced relative to the normal chordal tooth thickness measurement by the amount determined, without this endangering the stipulated minimum backlash. The lower allowance must however be displaced by the amount $\frac{F_\beta}{2 \cdot \cos\alpha_t}$.

B.7.5 Actual allowances of the base tangent length measurement

Although the concentricity deviation does not enter into this measurement the fluctuation of the measured values is larger than with the two-ball and two-roller measurement. This is due to the fact that the pitch-span deviations influence the measurement. These are not taken into account in the calculation of the allowances of tooth thickness. If the allowances of the chordal tooth thickness measurement were to be converted purely theoretically as allowances for the base tangent length measurement (Fig. B.1), neither the minimum backlash nor the maximum backlash would be guaranteed if this tolerance were to be utilized to the full. The tolerance must therefore be reduced not only by the concentricity tolerance but also by the pitch-span tolerance. The error propagation law can of course also be applied here. The mean value of the measurements is not displaced compared with the chordal measurement. Since the width of bearing on the tooth is larger than with the chordal measurement a tooth trace form deviation could exert influence. This however is taken into account in the calculation of the allowances. Hence if an offset occurs the tolerance zone can be displaced accordingly.

Although in general the base tangent length measurement is subject to the smallest measuring errors, compared with the other measuring methods, and ought for this reason to be the preferred measurement, it is also the method which involves reducing the theoretical tolerance by the largest amount in order to guarantee the backlash in the same way as with the chordal measurement. This security of compliance with the minimum and maximum backlash however is bought at the cost of making the tolerance small, so that in some cases objections may be made against gears which are found to be serviceable, as regards the functional conditions, in the two-flank working test. This affects in particular shaved gears which may have relatively large pitch deviations which are not always allocatable to the concentricity deviations (see Fig. B.2, gears 3 and 5, also the limits shown by dashed lines in Fig. B.3).

B.7.6 Actual allowances of the two-ball measurement

The fluctuation of the measured values converted to the tooth thickness is significantly smaller than is the case with the foregoing measured values. This shows that the concentricity deviation determined is attributable to eccentricity and not to out-of-roundness, since eccentricity does not enter into the result. If the theoretically calculated tolerance were to be utilized, neither the minimum backlash nor the maximum backlash would be guaranteed any more. The tolerance has therefore to be reduced by the amount of the permissible concentricity deviation. The mean value of the measurements is slightly displaced compared with the chordal measurement. The reason for this may lie in the fact that only point contact is made with the flanks, and also in the fact that the measurement circle differs and in this way profile deviations become effective. For comparative measurements it is therefore expedient to carry out all measurements at approximately the same points on the flanks, i. e. to determine the ball and roller diameters approximately for the measurement points of the base tangent length measurement, and to calculate the normal chordal tooth thicknesses also for this.

B.7.7 Actual allowances of the two-roller measurement

Here, the same remarks as in Section B.7.6 apply. It is of course possible for tooth trace form deviations to have an effect. This makes the measurement "thicker" than with the two-ball measurement. The tolerance can be displaced by the offset compared with the chordal measurement, since the causes of this have already been taken into account in the calculation of the allowances.

The tolerance must however likewise be reduced by the amount of the permissible concentricity deviation.

B.8 Tooth thickness fluctuations

Tooth thickness fluctuations are often measured as fluctuations of the base tangent length and in some cases also as fluctuations of the two-ball or two-roller dimension. The tolerances stated in DIN 3962 Part 1 apply to the fluctuation of the tooth thicknesses. For other measurements these values have to be converted. For this conversion the same applies as for converting the tooth thicknesses themselves. Use of the theoretical allowance factors alone can thus lead to errors. Therefore so long as no firm conversion figures are available it is recommended when using the tolerances to the full that a check should be carried out by measuring the normal chordal tooth thicknesses or the working distance with a master gear in order to avoid difficulties at acceptance.

B.9 Reliability of results

The relationships of the different measuring methods shown here by way of an example can only apply qualitatively. For quantitative statements not only is the number of measurements too small, but also it is not permissible to transfer conclusions from one size of gear to others, or from one method of manufacture to another. It is therefore necessary to have a large number of measurement series for the widely differing areas before statistically affirmed correction values can be stated for the conversion. The tolerance modifications of the given tooth thickness tolerances, as described in Sections B.7.4 to B.7.7, should however be carried out where possible. On the one hand this will lead to the backlash effectively obtained being largely the same with all measuring methods, and on the other hand the necessary production effort also. The modifications of Fig. B.1 necessary from the evaluation of Fig. B.2 (right) are shown in Fig. B.3.

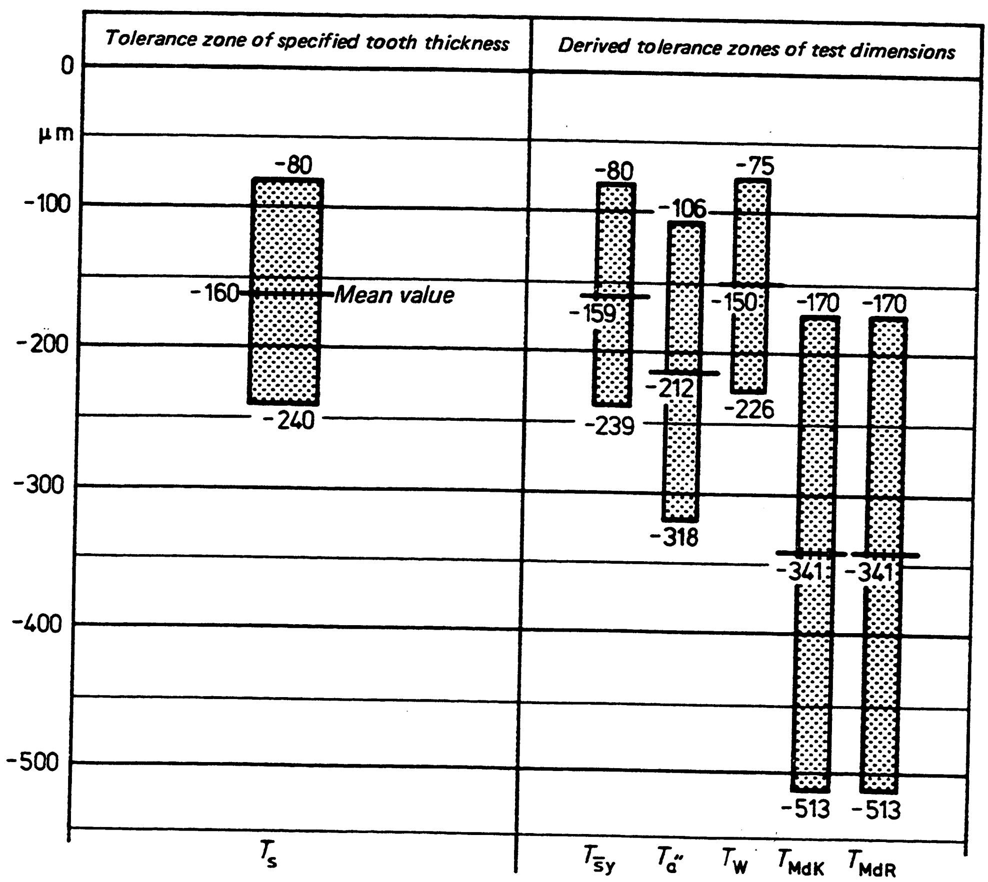


Figure B.1. Tolerance zones of test dimensions after ideal-geometry conversion of tooth thickness tolerance zone

Example:

m = 4.25 mm; z = 29, $\beta = 0°$; b = 22 mm; x = 0.2063. The zero line corresponds with zero-backlash design of the gear transmission, tooth thickness for this s = 7.314 mm. The mean allowance – 160 µm corresponds to Δ_x = – 0.0517, the mean tooth thickness is then s = 7.154 mm corresponding to x = 0.1546.

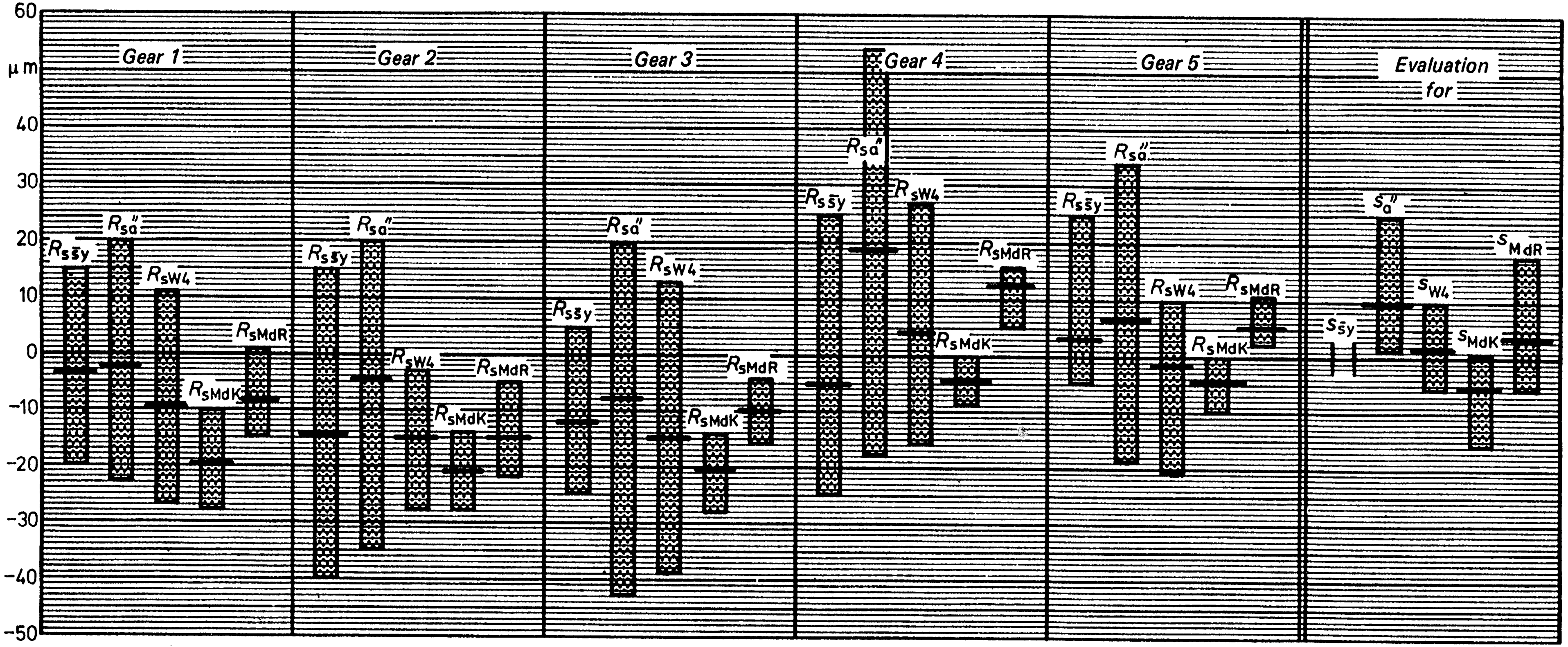


Figure B.2. Tooth thickness fluctuations R_s converted from measurements on five gears of the same kind

The contact diameter was approximately the same with all measurements because the measuring ball and roller diameter and the depth of measurement for the chordal measurement for 4 teeth as the selected measuring number were matched as far as possible to the base tangent length.

The starting point of the tooth thickness scale has been placed at the middle of the tolerance zone (– 160 µm) (Fig. B.1).

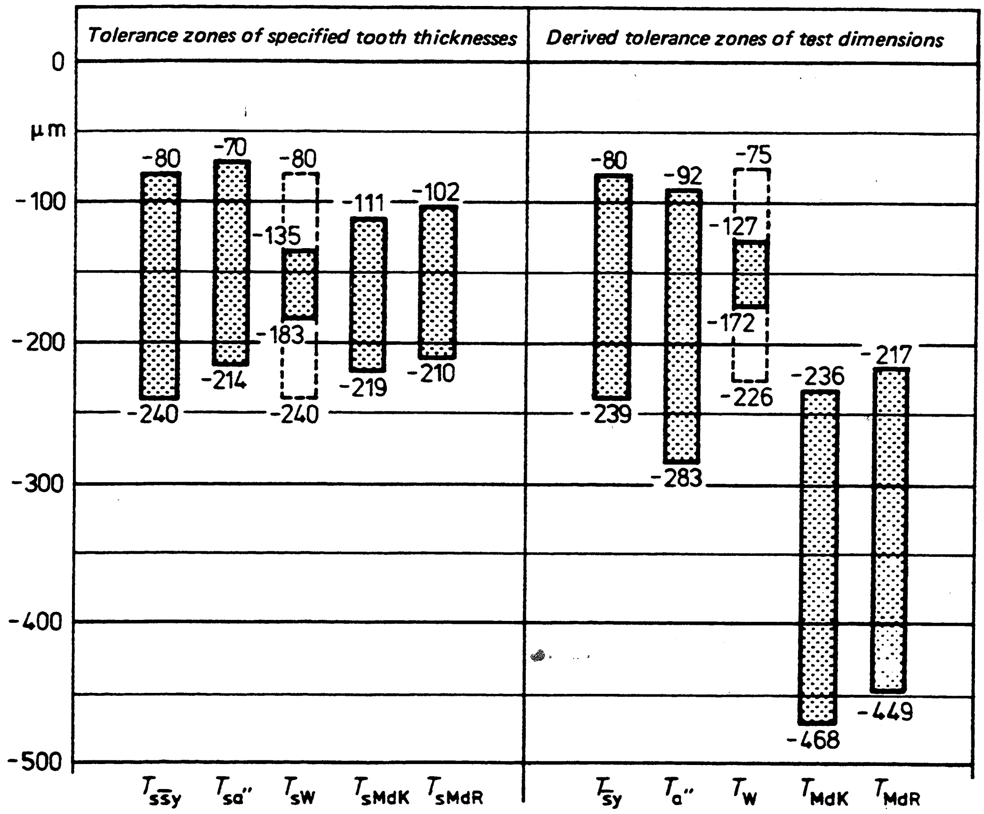


Figure B.3. Altered tolerance zones of tooth thickness and resulting tolerance zones of the test dimensions

On the basis of the unmodified tooth thickness allowances of Fig. B.1 for the chordal measurement the tooth thickness tolerance zones for the other test dimensions (apart from the base tangent length measurement, cf. Section B.7.5) were calculated for position and size according to the evaluation in Fig. B.2 (right), see upper illustration (left), and the allowances of the test dimensions determined therefrom. These tolerance zones result in approximately equal production effort and the same effective backlash.

The limits shown by dashed lines of the tooth thickness tolerance for the base tangent length (T_{sW}) and base tangent length tolerance (T_W) are needed for shaved gear teeth (see Section 7.5 and Explanations).

Explanations

With this Standard the system of fits for securing the minimum backlash and limiting the maximum backlash is placed on a broader footing compared with DIN 3963 and DIN 3967 dating from 1953. It is also intended to stimulate the collection of practical experience, so that further general pronouncements can be made on the system of gear tooth fits.

The proposed system of fits is regarded as a general basis. Empirical values for specific systems of fits involving given gear transmission categories could not be laid down in a uniform system by the responsible committee because with differing diameters no uniform minimum backlash can be stated. It is left to the user to establish a selection system or reference values for the backlash concerned.

To simplify use of this Standard the main text is followed in Appendix A by information on the calculation of tooth thickness allowances and in Appendix B by information on the conversion of allowances for the different measuring methods. The values of the upper tooth thickness allowances and the tooth thickness tolerances should be taken from Tables 1 and 2. Ten tolerance series based on the preferred number series R 10 are given and allocated to the reference diameter (see Table 2). The progression from one tolerance series to the next is 1.6. The allowance series (Table 1) are based on a progression of $\sqrt[16]{20}$, certain series near the zero line being omitted. Within an allowance series the upper tooth thickness allowances have a progression of $\sqrt[10]{20}$. The calculated values have been rounded according to ISO/R 286. The previously standardized allocation of tooth thickness tolerances to gear tooth quality has thus been dropped. It has also not been considered expedient to adopt coding of the tolerance values with multiples of the individual pitch deviation f_{pt} as contained in the International Standard ISO 1328 – 1975.

When values obtained from experience for the backlash or tooth thickness allowances are available or when for functional reasons no exact determination of backlash is necessary, the accurate calculation of the tooth thickness allowances is superfluous. Otherwise, the calculation can be carried out according to Appendix A. The information in Appendix B on the relationships between the values obtained with different measuring methods should be considered additionally because in industry the tooth thickness is not measured as defined, but instead has to be superseded by indirect tooth thickness measurements using different methods. The examples given in Appendix B are not directly transferable to other gears. Assuming a sufficiently large number of measurement series for the most diverse gears and manufacturing methods there is a possibility that the offset of the tolerances for measurements by way of working distance, base tangent length or the two-ball distance can be kept the same relative to the normal chordal tooth thickness. It is also possible that the tolerance for the base tangent length need not be so severely restricted as indicated in Section 5 of Appendix B, since in the case of the five gears measured the base tangent length is not always within the calculated reduced tolerance, yet despite this the gears are within tolerance (see limits shown by dashed lines in Fig. B.3). In any case it is recommendable to determine correction values by measurements, so that the desired backlash is complied with despite the different influence exerted on the measured variables by the individual deviations of the teeth.

The parameters, symbols and designations of this Standard have been redefined in conformity with DIN 3960, DIN 3998 and DIN 3999.