Lecture 3-3

Tuples

Week 3 Friday

Miles Chen, PhD

Adapted from Chapter 12 of Think Python by Allen B Downey

Additional content on Dictionaries adapted from "Whirlwind Tour of Python" by Jake VanderPlas

Tuples are like lists in that they can contain objects of different types. Tuples are ordered and preserve their order.

Tuples are different from lists in that they are **immutable**. You cannot append to a tuple or modify values in a tuple.

Tuples are like lists in that they can contain objects of different types. Tuples are ordered and preserve their order.

Tuples are different from lists in that they are **immutable**. You cannot append to a tuple or modify values in a tuple.

```
In [1]: t = 0, 'apple', 2, 'cat', 'dog', 5, 6 # parentheses are not required to create a tuple
```

Tuples are like lists in that they can contain objects of different types. Tuples are ordered and preserve their order.

Tuples are different from lists in that they are **immutable**. You cannot append to a tuple or modify values in a tuple.

```
In [1]:    t = 0, 'apple', 2, 'cat', 'dog', 5, 6 # parentheses are not required to create a tuple
In [2]:    t
Out[2]:    (0, 'apple', 2, 'cat', 'dog', 5, 6)
```

Tuples are like lists in that they can contain objects of different types. Tuples are ordered and preserve their order.

Tuples are different from lists in that they are **immutable**. You cannot append to a tuple or modify values in a tuple.

```
In [1]:    t = 0, 'apple', 2, 'cat', 'dog', 5, 6 # parentheses are not required to create a tuple
In [2]:    t
Out[2]:    (0, 'apple', 2, 'cat', 'dog', 5, 6)
In [3]:    t = (0, 'apple', 2, 'cat', 'dog', 5, 6) # you can use parentheses
```

Tuples are like lists in that they can contain objects of different types. Tuples are ordered and preserve their order.

Tuples are different from lists in that they are **immutable**. You cannot append to a tuple or modify values in a tuple.

```
In [1]:    t = 0, 'apple', 2, 'cat', 'dog', 5, 6 # parentheses are not required to create a tuple
In [2]:    t
Out[2]:    (0, 'apple', 2, 'cat', 'dog', 5, 6)
In [3]:    t = (0, 'apple', 2, 'cat', 'dog', 5, 6) # you can use parentheses
In [4]:    t
Out[4]:    (0, 'apple', 2, 'cat', 'dog', 5, 6)
```

```
In [5]: t1 = "a",
```

```
In [5]: t1 = "a",
In [6]: t1
Out[6]: ('a',)
```

```
In [5]: t1 = "a",
In [6]: t1
Out[6]: ('a',)
In [7]: type(t1)
Out[7]: tuple
```

```
In [5]: t1 = "a",
In [6]:
Out[6]: ('a',)
In [7]:
         type(t1)
Out[7]:
         tuple
In [8]:
         len(t1)
Out[8]: 1
```

```
In [9]: t2 = ("a")
```

```
In [9]: t2 = ("a")
In [10]: type(t2)
Out[10]: str
```

```
In [9]: t2 = ("a")
In [10]: type(t2)
Out[10]: str
In [11]: t2 = ("a",)
```

```
In [9]: t2 = ("a")
In [10]: type(t2)
Out[10]: str
In [11]: t2 = ("a",)
In [12]: type(t2)
Out[12]: tuple
```

You can create an empty tuple with the tuple() function, similar to using the list() or dict() function

You can create an empty tuple with the tuple() function, similar to using the list() or dict() function

```
In [13]: t3 = tuple()
```

You can create an empty tuple with the tuple() function, similar to using the list() or dict() function

```
In [13]: t3 = tuple()
In [14]: t3
Out[14]: ()
```

```
In [15]: tuple("hello")
Out[15]: ('h', 'e', 'l', 'l', 'o')
```

```
In [15]: tuple("hello")
Out[15]: ('h', 'e', 'l', 'l', 'o')
In [16]: tuple(range(5))
Out[16]: (0, 1, 2, 3, 4)
```

```
In [15]: tuple("hello")
Out[15]: ('h', 'e', 'l', 'l', 'o')
In [16]: tuple(range(5))
Out[16]: (0, 1, 2, 3, 4)
In [17]: tuple([1,4,7])
Out[17]: (1, 4, 7)
```

```
In [18]:
t = 0, 'apple', 2, 'cat', 'dog', 5, 6
```

```
In [18]: t = 0, 'apple', 2, 'cat', 'dog', 5, 6
In [19]: t[1]
Out[19]: 'apple'
```

```
In [18]:    t = 0, 'apple', 2, 'cat', 'dog', 5, 6
In [19]:    t[1]
Out[19]:    'apple'
In [20]:    t[2:5] # slicing
Out[20]:    (2, 'cat', 'dog')
```

```
In [21]:
    t = (0,'apple',2,'cat','dog',5,6) # tuple
    l = [0,'apple',2,'cat','dog',5,6] # list
```

```
In [21]:
           t = (0, 'apple', 2, 'cat', 'dog', 5, 6) # tuple
           1 = [0,'apple',2,'cat','dog',5,6] # List
In [22]:
           1[0] = 100 # we can change the value of the object at index 0
           print(1)
           [100, 'apple', 2, 'cat', 'dog', 5, 6]
In [23]:
           t[0] = 100 # trying to modify the value in a tuple is not allowed
           TypeError
                                                         Traceback (most recent call last)
           <ipvthon-input-23-1315e91aabf3> in <module>
           ----> 1 t[0] = 100 # trying to modify the value in a tuple is not allowed
           TypeError: 'tuple' object does not support item assignment
```

methods that modify lists in place (e.g. append, insert, pop, etc) do not work for tuples

methods that modify lists in place (e.g. append, insert, pop, etc) do not work for tuples

methods that modify lists in place (e.g. append, insert, pop, etc) do not work for tuples

Because tuples are immutable, you can't modify the elements. But you can replace one tuple with another:

Because tuples are immutable, you can't modify the elements. But you can replace one tuple with another:

```
In [26]: t = ("A",) + t[1:]
Out[26]: ('A', 'apple', 2, 'cat', 'dog', 5, 6)
```

This creates an entirely new tuple, unrelated to the other one.

```
In [27]: (0, 1, 2) < (0, 3, 4)
```

Out[27]: True

```
In [27]: (0, 1, 2) < (0, 3, 4)
Out[27]: True
In [28]: (0, 1, 20000000) < (0, 3, 4)
Out[28]: True</pre>
```

```
In [27]: (0, 1, 2) < (0, 3, 4)

Out[27]: True

In [28]: (0, 1, 2000000) < (0, 3, 4)

Out[28]: True

In [29]: (0, 500, 2) < (0, 3, 4)</pre>
Out[29]: False
```

Tuple assignment

A common and useful tuple idiom: You can switch value assignments via tuples

Tuple assignment

A common and useful tuple idiom: You can switch value assignments via tuples

```
In [30]:
# old option without tuples
a = 5
b = 1

temp = a
a = b
b = temp
print(a, b)
```

1 5

Tuple assignment

A common and useful tuple idiom: You can switch value assignments via tuples

```
In [30]:
            # old option without tuples
            b = 1
            temp = a
            a = b
            b = temp
            print(a, b)
            1 5
In [31]:
            # faster way with tuples
            b = 1
            b, a = a, b
            print(a, b)
```

You can take the results of a function and have the returned values assign to different elements in a tuple

You can take the results of a function and have the returned values assign to different elements in a tuple

```
In [32]:
    addr = "mileschen@stat.ucla.edu"
    uname, domain = addr.split("@")
```

You can take the results of a function and have the returned values assign to different elements in a tuple

```
In [32]: addr = "mileschen@stat.ucla.edu"
    uname, domain = addr.split("@")

In [33]: uname
Out[33]: 'mileschen'
```

You can take the results of a function and have the returned values assign to different elements in a tuple

```
In [32]: addr = "mileschen@stat.ucla.edu"
uname, domain = addr.split("@")

In [33]: uname

Out[33]: 'mileschen'

In [34]: domain

Out[34]: 'stat.ucla.edu'
```

We saw this when we talked about functions. You can have functions return multiple values in the form of a tuple

```
def my_divide(x, y):
    integer = x // y
    remainder = x % y
    return integer, remainder
```

```
In [35]: def my_divide(x, y):
    integer = x // y
    remainder = x % y
    return integer, remainder
In [36]: a, b = my_divide(23, 5)
```

```
In [35]:    def my_divide(x, y):
        integer = x // y
        remainder = x % y
        return integer, remainder

In [36]:    a, b = my_divide(23, 5)

In [37]:    a, b

Out[37]:    (4, 3)
```

```
In [35]:
            def my_divide(x, y):
                integer = x // y
                remainder = x \% y
                return integer, remainder
In [36]:
            a, b = my divide(23, 5)
In [37]:
            a, b
Out[37]: (4, 3)
In [38]:
            divmod(23, 5) # divmod() is a built-in function that does exactly this.
           (4, 3)
Out[38]:
```

```
In [39]:
t = 0, 'apple', 2, 'cat', 'dog', 5, 6
```

```
In [39]: t = 0, 'apple', 2, 'cat', 'dog', 5, 6
In [40]: t.index('dog')
Out[40]: 4
```

```
In [39]: t = 0, 'apple', 2, 'cat', 'dog', 5, 6
In [40]: t.index('dog')
Out[40]: 4
In [41]: t.count(5)
Out[41]: 1
```

Functions that support tuples and other iterables as inputs

Even though tuples only have two methods, there are several functions that support tuples (and other iterables like lists, dicts, strings) as inputs

- len()
- sum()
- sorted()
- min()
- max()

None of these functions affect the list or tuple itself.

```
In [42]:
some_digits = (4,2,7,9,2,5,3) # a tuple of numbers
some_words = ['dog','apple','cat','hat','hand'] # this is a list
```

```
In [42]: some_digits = (4,2,7,9,2,5,3) # a tuple of numbers
    some_words = ['dog','apple','cat','hat','hand'] # this is a list
In [43]: len(some_digits)
```

Out[43]: 7

```
In [42]: some_digits = (4,2,7,9,2,5,3) # a tuple of numbers
    some_words = ['dog','apple','cat','hat','hand'] # this is a list

In [43]: len(some_digits)

Out[43]: 7

In [44]: sum(some_digits)
```

Out[44]: 32

```
In [42]:
           some digits = (4,2,7,9,2,5,3) # a tuple of numbers
           some words = ['dog','apple','cat','hat','hand'] # this is a list
In [43]:
           len(some_digits)
Out[43]: 7
In [44]:
           sum(some_digits)
Out[44]: 32
In [45]:
           sum(some words) # won't work on strings
                                                        Traceback (most recent call last)
           TypeError
           <ipython-input-45-7ad2d781cfbf> in <module>
           ----> 1 sum(some_words) # won't work on strings
           TypeError: unsupported operand type(s) for +: 'int' and 'str'
```

```
In [46]:

sorted(some_digits) # sorts the tuple, but does not affect the list or tuple itself.

# contrast to list.sort() which will sort the list in place

# but the object returned is a list
```

Out[46]: [2, 2, 3, 4, 5, 7, 9]

```
In [46]: sorted(some_digits) # sorts the tuple, but does not affect the list or tuple itself.
# contrast to list.sort() which will sort the list in place
# but the object returned is a list

Out[46]: [2, 2, 3, 4, 5, 7, 9]

In [47]: print(some_digits) # just to show the list is unchanged

(4, 2, 7, 9, 2, 5, 3)
```

```
In [46]:
           sorted(some digits) # sorts the tuple, but does not affect the list or tuple itself.
           # contrast to list.sort() which will sort the list in place
            # but the object returned is a list
Out[46]: [2, 2, 3, 4, 5, 7, 9]
In [47]:
           print(some digits) # just to show the list is unchanged
            (4, 2, 7, 9, 2, 5, 3)
In [48]:
            sorted(some_words) # when applied to a list of strings, it will alphabetize them
Out[48]: ['apple', 'cat', 'dog', 'hand', 'hat']
In [49]:
           min(some_digits)
```

Out[49]: 2

```
In [46]:
            sorted(some_digits) # sorts the tuple, but does not affect the list or tuple itself.
            # contrast to list.sort() which will sort the list in place
            # but the object returned is a list
Out[46]: [2, 2, 3, 4, 5, 7, 9]
In [47]:
           print(some digits) # just to show the list is unchanged
            (4, 2, 7, 9, 2, 5, 3)
In [48]:
            sorted(some_words) # when applied to a list of strings, it will alphabetize them
Out[48]: ['apple', 'cat', 'dog', 'hand', 'hat']
In [49]:
           min(some digits)
Out[49]: 2
In [50]:
           max(some words) # max returns the last word if alphabetized,
            # min will return the first in an alphabetized list
Out[50]:
            'hat'
```

```
In [51]:
    dict_num = {1: "a", 2: "b", 3: "c"}
    dict_alpha = {"a":1, "b":2, "c":3}
```

```
In [51]: dict_num = {1: "a", 2: "b", 3: "c"}
dict_alpha = {"a":1, "b":2, "c":3}
In [52]: len(dict_num) # number of items in the dictionary
```

Out[52]: 3

```
In [51]: dict_num = {1: "a", 2: "b", 3: "c"}
dict_alpha = {"a":1, "b":2, "c":3}

In [52]: len(dict_num) # number of items in the dictionary

Out[52]: 3

In [53]: sorted(dict_num) # a list of the keys sorted
```

Out[53]: [1, 2, 3]

```
In [51]:
           dict_num = {1: "a", 2: "b", 3: "c"}
           dict_alpha = {"a":1, "b":2, "c":3}
In [52]:
           len(dict_num) # number of items in the dictionary
Out[52]: 3
In [53]:
           sorted(dict_num) # a list of the keys sorted
Out[53]: [1, 2, 3]
In [54]:
           sorted(dict_alpha)
Out[54]: ['a', 'b', 'c']
```

```
In [51]:
           dict_num = {1: "a", 2: "b", 3: "c"}
           dict_alpha = {"a":1, "b":2, "c":3}
In [52]:
           len(dict_num) # number of items in the dictionary
Out[52]: 3
In [53]:
           sorted(dict_num) # a list of the keys sorted
Out[53]: [1, 2, 3]
In [54]:
           sorted(dict_alpha)
Out[54]: ['a', 'b', 'c']
In [55]:
           max(dict_num) # the "maximum" key
Out[55]: 3
```

Math operators and lists, tuples, strings

multiplication generally duplicates

addition generally appends

behaviors across lists, tuples, and strings are similar

Math operators and lists, tuples, strings

multiplication generally duplicates

addition generally appends

behaviors across lists, tuples, and strings are similar

```
In [56]:
L1 = ['a','b','c']
L2 = ['d','e','f']
```

Math operators and lists, tuples, strings

multiplication generally duplicates

addition generally appends

behaviors across lists, tuples, and strings are similar

```
In [56]:
L1 = ['a','b','c']
L2 = ['d','e','f']

In [57]:
L1 * 2 # multiplication extends duplicates

Out[57]: ['a', 'b', 'c', 'a', 'b', 'c']
```

Math operators and lists, tuples, strings

multiplication generally duplicates

addition generally appends

behaviors across lists, tuples, and strings are similar

```
In [56]: L1 = ['a','b','c']
L2 = ['d','e','f']

In [57]: L1 * 2 # multiplication extends duplicates

Out[57]: ['a', 'b', 'c', 'a', 'b', 'c']

In [58]: L1 + L2 # addition appends list objects

Out[58]: ['a', 'b', 'c', 'd', 'e', 'f']
```

```
In [59]:

T1 = ('a','b','c')

T2 = ('d','e','f')
```

Variable-length argument tuples

Functions can take a variable number of arguments. A parameter name that begins with * gathers arguments into a tuple.

The gather parameter can have any name you like, but args is conventional.

Variable-length argument tuples

Functions can take a variable number of arguments. A parameter name that begins with * gathers arguments into a tuple.

The gather parameter can have any name you like, but args is conventional.

```
In [64]:
    def printall(*args):
        print(args)
```

Variable-length argument tuples

Functions can take a variable number of arguments. A parameter name that begins with * gathers arguments into a tuple.

The gather parameter can have any name you like, but args is conventional.

```
In [66]:
    def print_lines(*args):
        for element in args:
            print(element)
```

```
In [66]:    def print_lines(*args):
        for element in args:
            print(element)
In [67]:    print_lines("hi", "goodbye")

hi
goodbye
```

```
In [66]:
            def print_lines(*args):
                for element in args:
                    print(element)
In [67]:
            print_lines("hi", "goodbye")
            hi
            goodbye
In [68]:
            print_lines(1, 5, 7, 9, 10)
            1
5
7
            9
            10
```

```
def my_divide(x, y):
    integer = x // y
    remainder = x % y
    return integer, remainder
```

```
In [69]:
    def my_divide(x, y):
        integer = x // y
        remainder = x % y
        return integer, remainder

In [70]:
    t = (23, 5)
```

```
In [69]:
           def my divide(x, y):
               integer = x // y
               remainder = x \% y
               return integer, remainder
In [70]:
          t = (23, 5)
In [71]:
           my divide(t)
                                                        Traceback (most recent call last)
           TypeError
           <ipython-input-71-ea0afdae4ba1> in <module>
           ----> 1 my divide(t)
           TypeError: my_divide() missing 1 required positional argument: 'y'
```

```
In [69]:
           def my divide(x, y):
               integer = x // y
               remainder = x \% y
               return integer, remainder
In [70]:
          t = (23, 5)
In [71]:
           my divide(t)
                                                        Traceback (most recent call last)
           TypeError
           <ipython-input-71-ea0afdae4ba1> in <module>
           ----> 1 my divide(t)
           TypeError: my_divide() missing 1 required positional argument: 'v'
In [72]:
           my divide(*t)
Out[72]: (4, 3)
```

Lists, Tuples and Iterators

zip() is a built-in function that takes two or more sequences and interleaves them. The name of the function refers to a zipper, which interleaves two rows of teeth.

Lists, Tuples and Iterators

zip() is a built-in function that takes two or more sequences and interleaves them. The name of the function refers to a zipper, which interleaves two rows of teeth.

```
In [73]:
    s = 'abc'
    t = 0, 1, 2
    zip(s, t)
```

Out[73]: <zip at 0x1bbe6b81388>

Lists, Tuples and Iterators

zip() is a built-in function that takes two or more sequences and interleaves them. The name of the function refers to a zipper, which interleaves two rows of teeth.

A zip object is a kind of iterator, which is any object that iterates through a sequence. Iterators are similar to lists in some ways, but unlike lists, you can't use an index to select an element from an iterator.

It will return a list of tuples

It will return a list of tuples

```
In [75]: list(zip(s,t))
```

Out[75]: [('a', 0), ('b', 1), ('c', 2)]

It will return a list of tuples

```
In [75]: list(zip(s,t))
```

If the sequences are not the same length, the result has the length of the shorter one.

It will return a list of tuples

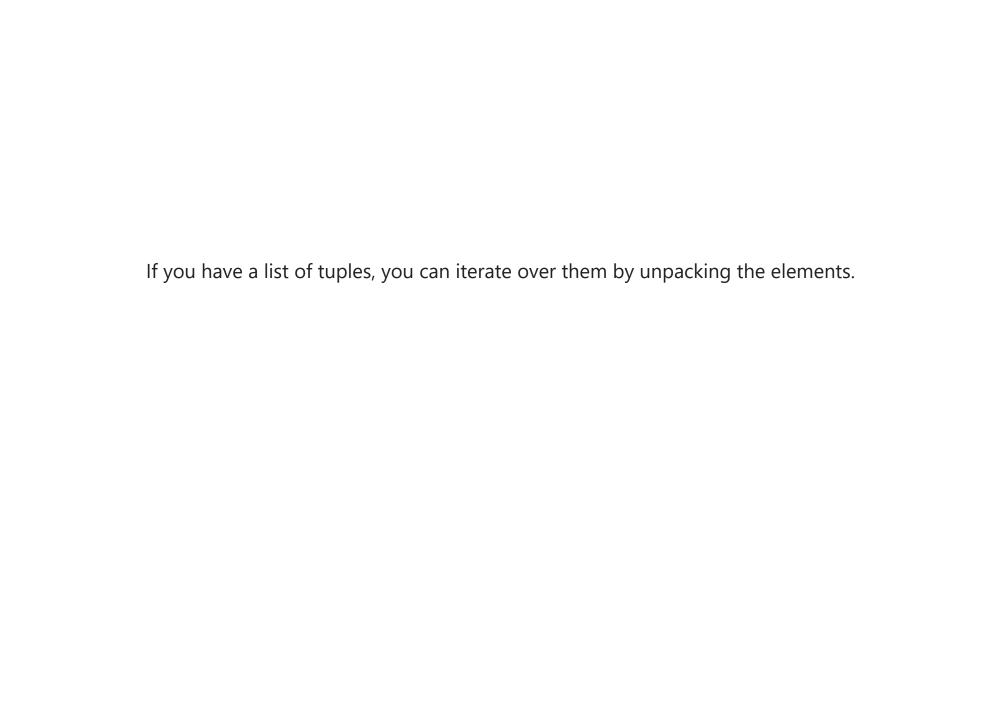
```
In [75]: list(zip(s,t))
```

Out[75]: [('a', 0), ('b', 1), ('c', 2)]

If the sequences are not the same length, the result has the length of the shorter one.

```
In [76]:
list(zip("Anne" , "Elk" ))
```

Out[76]: [('A', 'E'), ('n', 'l'), ('n', 'k')]



If you have a list of tuples, you can iterate over them by unpacking the elements.

```
In [77]:
    t = [('a', 0), ('b', 1), ('c', 2)]
    for letter, number in t:
        print(number, letter)

0 a
    1 b
    2 c
```

```
In [78]:
    def has_match(t1, t2):
        for x, y in zip(t1, t2):
            if x == y:
                return True
        return False
```

```
In [78]: def has_match(t1, t2):
    for x, y in zip(t1, t2):
        if x == y:
            return True
    return False
In [79]: has_match(('a', 'b', 'c'), ('d', 'e', 'f'))
```

Out[79]: False

```
In [78]:
            def has match(t1, t2):
                for x, y in zip(t1, t2):
                    if x == y:
                       return True
                return False
In [79]:
            has_match(('a', 'b', 'c'), ('d', 'e', 'f'))
Out[79]:
            False
In [80]:
            has_match(('a', 'b', 'c'), ('d', 'e', 'c'))
Out[80]:
            True
In [81]:
            has_match(('a', 'b', 'c', 'd'), ('d', 'c', 'b', 'a'))
Out[81]:
           False
```

enumerate()

The built-in function enumerate is useful. It takes an iterable and returns an iterator of the index paired with the elements You can think of enumerate() as zipping a range object of the same length with the iterable.

enumerate()

The built-in function enumerate is useful. It takes an iterable and returns an iterator of the index paired with the elements You can think of enumerate() as zipping a range object of the same length with the iterable.

```
In [82]: enumerate("morning")
Out[82]: <enumerate at 0x1bbe6b4c598>
```

enumerate()

The built-in function enumerate is useful. It takes an iterable and returns an iterator of the index paired with the elements You can think of enumerate() as zipping a range object of the same length with the iterable.

enumerate()

The built-in function enumerate is useful. It takes an iterable and returns an iterator of the index paired with the elements You can think of enumerate() as zipping a range object of the same length with the iterable.

```
In [82]:
           enumerate("morning")
Out[82]:
            <enumerate at 0x1bbe6b4c598>
In [83]:
           for index, value in enumerate("morning"):
               print(index, value)
            0 m
            1 o
            3 n
            4 i
            5 n
            6 g
In [84]:
           list(enumerate(['a','b','c','d']))
Out[84]:
           [(0, 'a'), (1, 'b'), (2, 'c'), (3, 'd')]
```

enumerate()

The built-in function enumerate is useful. It takes an iterable and returns an iterator of the index paired with the elements You can think of enumerate() as zipping a range object of the same length with the iterable.

```
In [82]:
           enumerate("morning")
Out[82]:
            <enumerate at 0x1bbe6b4c598>
In [83]:
           for index, value in enumerate("morning"):
               print(index, value)
            0 m
            1 o
            2 r
            3 n
            4 i
            5 n
            6 g
In [84]:
           list(enumerate(['a','b','c','d']))
           [(0, 'a'), (1, 'b'), (2, 'c'), (3, 'd')]
Out[84]:
In [85]:
           list(zip(range(4),['a','b','c','d'])) # zipping a range object with a list
Out[85]: [(0, 'a'), (1, 'b'), (2, 'c'), (3, 'd')]
```

```
In [86]: d = {'a':0, 'b':1, 'c':2}
```

```
In [86]: d = {'a':0, 'b':1, 'c':2}
In [87]: d.items()
Out[87]: dict_items([('a', 0), ('b', 1), ('c', 2)])
```

```
In [86]: d = {'a':0, 'b':1, 'c':2}
In [87]: d.items()
Out[87]: dict_items([('a', 0), ('b', 1), ('c', 2)])
In [88]: for key, value in d.items():
    print(key, value)
    a 0
    b 1
    c 2
```

```
In [89]: d = dict(enumerate("efg"))
```

```
In [89]: d = dict(enumerate("efg"))
In [90]: d
Out[90]: {0: 'e', 1: 'f', 2: 'g'}
```

```
In [89]: d = dict(enumerate("efg"))
In [90]: d
Out[90]: {0: 'e', 1: 'f', 2: 'g'}
```

Swap the keys and elements in a dictionary

swapped[value] = key

```
In [89]:
           d = dict(enumerate("efg"))
In [90]:
Out[90]: {0: 'e', 1: 'f', 2: 'g'}
         Swap the keys and elements in a dictionary
In [91]:
           swapped = {}
           for key, value in d.items():
              swapped[value] = key
In [92]:
           swapped
Out[92]: {'e': 0, 'f': 1, 'g': 2}
In [93]:
           dict(zip("efg", range(3)))
Out[93]: {'e': 0, 'f': 1, 'g': 2}
```

We can create dictionaries out of sequences of tuples and with zip objects

We can create dictionaries out of sequences of tuples and with zip objects

```
In [94]:
    1 = [('z', 25), ('y', 24), ('x', 23)]
    dict(1)
```

Out[94]: {'z': 25, 'y': 24, 'x': 23}

We can create dictionaries out of sequences of tuples and with zip objects

Tuples as dictionary keys

Because tuples are immutable, they can be used as keys in a dictionary

For example, there might be a 2D function that is very expensive to compute for coordinates. You can create a dictionary that will store all of the values that have been calculated for each 2D pair.

Let's say you have a function: $f(x,y)=x^2+2y$

Tuples as dictionary keys

Because tuples are immutable, they can be used as keys in a dictionary

For example, there might be a 2D function that is very expensive to compute for coordinates. You can create a dictionary that will store all of the values that have been calculated for each 2D pair.

Let's say you have a function: $f(x,y) = x^2 + 2y$

```
In [96]:
    # this dictionary contains values that are known solutions
    known = {(0, 0): 0}
```

Tuples as dictionary keys

Because tuples are immutable, they can be used as keys in a dictionary

For example, there might be a 2D function that is very expensive to compute for coordinates. You can create a dictionary that will store all of the values that have been calculated for each 2D pair.

Let's say you have a function: $f(x,y)=x^2+2y$

```
In [96]: # this dictionary contains values that are known solutions
known = {(0, 0): 0}

In [97]: 
    def f(x, y):
        t = (x,y)
        if t in known:
            print("value already exists dictionary")
            return known[t]
        print("value must be calculated")
        res = x ** 2 + 2 * y
        known[t] = res
        return res
```

In [98]: f(0, 0)

value already exists dictionary

Out[98]: 0

```
In [98]: f(0, 0)
     value already exists dictionary
Out[98]: 0
In [99]: f(1, 2)
     value must be calculated
Out[99]: 5
```

```
In [98]:
           f(0, 0)
           value already exists dictionary
Out[98]: 0
 In [99]:
           f(1, 2)
           value must be calculated
Out[99]: 5
In [100]:
           f(1, 2)
           value already exists dictionary
Out[100]: 5
```

```
In [98]:
           f(0, 0)
           value already exists dictionary
Out[98]: 0
 In [99]:
           f(1, 2)
           value must be calculated
Out[99]: 5
In [100]:
           f(1, 2)
           value already exists dictionary
Out[100]:
In [101]:
           known
Out[101]: {(0, 0): 0, (1, 2): 5}
```