Lecture 4-2

More NumPy

Week 5 Wednesday

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Based on Python Data Science Handbook by Jake VanderPlas

In [1]: import numpy as np

```
In [2]:
    x = np.arange(4)
    y = np.arange(100, 104)
    print(x)
    print(y)

[0 1 2 3]
    [100 101 102 103]
```

```
In [2]:
         x = np.arange(4)
         y = np.arange(100, 104)
         print(x)
         print(y)
          [0 1 2 3]
          [100 101 102 103]
In [3]:
         np.concatenate([x,y])
Out[3]: array([ 0, 1, 2, 3, 100, 101, 102, 103])
        np.concatenate has an argument for axis. The axes are 0-indexed.
In [4]:
         np.concatenate([x,y], axis = 0)
Out[4]: array([ 0, 1, 2, 3, 100, 101, 102, 103])
```

```
In [5]:
         np.concatenate([x,y], axis = 1) # throws an error
          AxisError
                                                     Traceback (most recent call last)
          <ipython-input-5-2cd32e4cabd5> in <module>
          ----> 1 np.concatenate([x,y], axis = 1) # throws an error
          < array function internals> in concatenate(*args, **kwargs)
          AxisError: axis 1 is out of bounds for array of dimension 1
In [6]:
          x.shape # you can't use axis with index 1, because axis index 1 does not exist
Out [6]: (4,)
In [7]:
          np.vstack([x,y]) # vstack will vertically stack unidimensional arrays
Out[7]: array([[0, 1, 2, 3],
                 [100, 101, 102, 103]])
```

```
In [8]: x.reshape(1,4)
```

Out[8]: array([[0, 1, 2, 3]])

```
In [8]: x.reshape(1,4)
Out[8]: array([[0, 1, 2, 3]])
In [9]: y.reshape(1,4)
Out[9]: array([[100, 101, 102, 103]])
```

```
In [8]: x.reshape(1,4)
Out[8]: array([[0, 1, 2, 3]])
In [9]: y.reshape(1,4)
Out[9]: array([[100, 101, 102, 103]])
In [10]: np.concatenate([x.reshape(1,4), y.reshape(1,4)], axis = 0)
Out[10]: array([[ 0,  1,  2,  3],
```

[100, 101, 102, 103]])

note that when I concatenate along axis 0 for a 2-dimensional array, it concatenates by rows. In a 2D array, index 0 is for rows, and index 1 is for columns.

```
In [8]:
           x.reshape(1,4)
 Out[8]: array([[0, 1, 2, 3]])
 In [9]:
           y.reshape(1,4)
 Out[9]:
          array([[100, 101, 102, 103]])
In [10]:
           np.concatenate([x.reshape(1,4), y.reshape(1,4)], axis = 0)
Out[10]: array([[ 0, 1, 2, 3],
                   [100, 101, 102, 103]])
         note that when I concatenate along axis 0 for a 2-dimensional array, it concatenates by
         rows. In a 2D array, index 0 is for rows, and index 1 is for columns.
In [11]:
           np.concatenate([x.reshape(1,4), y.reshape(1,4)], axis = 1)
Out[11]: array([[ 0, 1, 2, 3, 100, 101, 102, 103]])
```

```
In [12]:
    xm = np.arange(6).reshape((2,3))
    ym = np.arange(100,106,1).reshape((2,3))
    print(xm)
    print(ym)

    [[0 1 2]
      [3 4 5]]
    [[100 101 102]
      [103 104 105]]
```

Out[13]: (2, 3)

```
In [12]:
           xm = np.arange(6).reshape((2,3))
           ym = np.arange(100,106,1).reshape((2,3))
           print(xm)
           print(ym)
            [[0 1 2]
             [3 4 5]]
            [[100 101 102]
             [103 104 105]]
In [13]:
           xm.shape
Out[13]: (2, 3)
In [14]:
           ym.shape
Out[14]: (2, 3)
```

```
In [15]: print(np.concatenate([xm,ym])) # default behavior concatenates on axis 0

[[ 0 1 2]
       [ 3 4 5]
```

[100 101 102] [103 104 105]]

```
In [15]:
           print(np.concatenate([xm,ym])) # default behavior concatenates on axis 0
               3 4 5]
             [100 101 102]
             [103 104 105]]
In [16]:
           print(np.concatenate([xm,ym], axis = 0))
           # axes are reported as rows, then columns.
           # concatenating along axis 0 will concatenate along rows
               3 4 5]
             [100 101 102]
             [103 104 105]]
In [17]:
           print(np.concatenate([xm,ym], axis = 1))
           # concatenating along axis 1 will concatenate along columns
                0 1 2 100 101 102]
                3 4 5 103 104 105]]
```

You can always use vstack and hstack for 2D arrays.

```
In [20]:
    print(x)
    print(y)

[0 1 2 3]
    [100 101 102 103]
```

```
In [20]:
          print(x)
          print(y)
           [0 1 2 3]
           [100 101 102 103]
In [21]:
Out[21]: array([5, 6, 7, 8])
In [22]: x + y # elementwise addition
Out[22]: array([100, 102, 104, 106])
In [23]:
Out[23]: array([ 0, 101, 204, 309])
```

```
In [20]:
          print(x)
          print(y)
           [0 1 2 3]
           [100 101 102 103]
In [21]:
Out[21]: array([5, 6, 7, 8])
In [22]: x + y # elementwise addition
Out[22]: array([100, 102, 104, 106])
In [23]:
Out[23]: array([ 0, 101, 204, 309])
In [24]:
          np.sum(x * y)
Out[24]: 614
```

In [25]: np.dot(x,y) # 0 * 100 + 1 * 101 + 2 * 102 + 3 * 103

Out[25]: 614

```
In [25]: np.dot(x,y) # 0 * 100 + 1 * 101 + 2 * 102 + 3 * 103
```

Out[25]: 614

In [26]: x @ y # matrix multiplication

Out[26]: 614

```
In [27]:
    print(xm)
    print(ym)

[[0 1 2]
       [3 4 5]]
       [[100 101 102]
       [103 104 105]]
```

```
In [27]:
            print(xm)
            print(ym)
            [[0 1 2]
            [3 4 5]]
            [[100 101 102]
             [103 104 105]]
In [28]:
            xm + 5
Out[28]: array([[ 5, 6, 7], [ 8, 9, 10]])
In [29]:
           xm + ym # elementwise addition
           array([[100, 102, 104],
Out[29]:
```

[106, 108, 110]])

```
In [30]: print(xm)
print(ym)

[[0 1 2]
      [3 4 5]]
      [[100 101 102]
      [103 104 105]]
```

```
In [30]:
           print(xm)
           print(ym)
           [[0 1 2]
            [3 4 5]]
           [[100 101 102]
            [103 104 105]]
In [31]:
           xm * ym # element-wise multiplication
Out[31]: array([[ 0, 101, 204],
                   [309, 416, 525]])
In [32]:
           np.multiply(xm, ym) # element-wise multiplication
           array([[ 0, 101, 204],
Out[32]:
                   [309, 416, 525]])
```

```
In [33]: print(xm) print(ym)

[[0 1 2] [3 4 5]] [[100 101 102] [103 104 105]]
```

```
In [33]: print(xm)
print(ym)

[[0 1 2]
      [3 4 5]]
      [[100 101 102]
      [103 104 105]]

In [34]: np.dot(xm, ym.T)

Out[34]: array([[ 305, 314],
```

[1214, 1250]])

```
In [33]:
           print(xm)
           print(ym)
           [[0 1 2]
           [3 4 5]]
           [[100 101 102]
            [103 104 105]]
In [34]:
           np.dot(xm, ym.T)
Out[34]:
          array([[ 305, 314],
                  [1214, 1250]])
In [35]:
           xm.dot(ym.T)
Out[35]: array([[ 305, 314],
                  [1214, 1250]])
```

```
In [33]:
           print(xm)
           print(ym)
           [[0 1 2]
           [3 4 5]]
           [[100 101 102]
            [103 104 105]]
In [34]:
           np.dot(xm, ym.T)
Out[34]:
          array([[ 305, 314],
                  [1214, 1250]])
In [35]:
           xm.dot(ym.T)
Out[35]:
          array([[ 305, 314],
                  [1214, 1250]])
In [36]:
           xm @ ym.T
Out[36]:
          array([[ 305, 314],
                  [1214, 1250]])
```

```
In [37]:
           x = np.arange(4)
           print(x)
           [0 1 2 3]
In [38]:
           print(x + 4)
           [4 5 6 7]
In [39]:
           print(x - 5)
           [-5 -4 -3 -2]
In [40]:
           print(x * 2)
           [0 2 4 6]
```

```
In [41]: print(x / 2)
```

[0. 0.5 1. 1.5]

```
In [41]: print(x / 2)
        [0. 0.5 1. 1.5]
In [42]: print(-x)
        [ 0 -1 -2 -3]
```

```
In [41]: print(x / 2)
        [0. 0.5 1. 1.5]

In [42]: print(-x)
        [ 0 -1 -2 -3]

In [43]: print(x ** 2)
        [0 1 4 9]
```

```
In [41]:
           print(x / 2)
           [0. 0.5 1. 1.5]
In [42]:
           print(-x)
           [ 0 -1 -2 -3]
In [43]:
           print(x ** 2)
           [0 1 4 9]
In [44]:
           print(x % 2) # modulo division
           [0 1 0 1]
```

```
In [41]:
           print(x / 2)
           [0. 0.5 1. 1.5]
In [42]:
           print(-x)
           [ 0 -1 -2 -3]
In [43]:
           print(x ** 2)
           [0 1 4 9]
In [44]:
           print(x % 2) # modulo division
           [0 1 0 1]
In [45]:
           print(abs(x)) # abs
           [0 1 2 3]
```

```
In [46]:
           theta = np.linspace(0, np.pi, 5)
           print(theta)
           [0.
                       0.78539816 1.57079633 2.35619449 3.14159265]
In [47]:
           print(np.sin(theta))
           [0.00000000e+00 7.07106781e-01 1.00000000e+00 7.07106781e-01
            1.22464680e-16]
In [48]:
           print(np.cos(theta))
           [ 1.00000000e+00 7.07106781e-01 6.12323400e-17 -7.07106781e-01
            -1.00000000e+001
In [49]:
           print(np.tan(theta))
           [ 0.00000000e+00 1.00000000e+00 1.63312394e+16 -1.00000000e+00
            -1.22464680e-16]
```

```
In [50]:
           x = np.array([1, 10, 100])
           print(np.log(x)) # natural Log
           print(np.log10(x)) # common Log
           [0.
                2.30258509 4.60517019]
           [0. 1. 2.]
In [51]:
           y = np.arange(3)
           print(np.exp(y)) # e^y
           [1.
                2.71828183 7.3890561 ]
In [52]:
           print(np.exp2(y)) # 2^y
           [1. 2. 4.]
In [53]:
           print(np.power(3, y)) # power ^ y
           [1 3 9]
```

```
you can use sum()
or np.sum()
np.sum() is faster than sum, but doesn't always behave the same way
```

```
you can use sum()

or np.sum()

np.sum() is faster than sum, but doesn't always behave the same way

In [54]: 

x = np.arange(100)
print(x)

[ 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47
48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71
72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95
96 97 98 99]
```

```
you can use sum()
         or np.sum()
         np.sum() is faster than sum, but doesn't always behave the same way
In [54]:
          x = np.arange(100)
          print(x)
               1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
           24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47
           48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71
           72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95
           96 97 98 99]
In [55]:
          print(sum(x))
          4950
```

```
you can use sum()
         or np.sum()
         np.sum() is faster than sum, but doesn't always behave the same way
In [54]:
          x = np.arange(100)
           print(x)
                                        9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
            24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47
            48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71
            72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95
            96 97 98 99]
In [55]:
           print(sum(x))
           4950
In [56]:
           print(np.sum(x))
           4950
```

```
In [57]:
    big_array = np.random.rand(10000)
    %timeit sum(big_array)
    %timeit np.sum(big_array) # the np version is much faster
```

1.58 ms \pm 15.3 μ s per loop (mean \pm std. dev. of 7 runs, 1000 loops each) 5.44 μ s \pm 88 ns per loop (mean \pm std. dev. of 7 runs, 100000 loops each)

```
In [58]:
    print(min(big_array))
    print(max(big_array))
```

6.052560173452903e-05

0.9999830003841369

0.9999830003841369

```
In [58]:
           print(min(big_array))
           print(max(big_array))
            6.052560173452903e-05
            0.9999830003841369
In [59]:
           print(np.min(big_array))
           print(np.max(big array))
            6.052560173452903e-05
            0.9999830003841369
In [60]:
           %timeit min(big array)
           %timeit np.min(big_array) # the np version is much faster
           971 \mus \pm 4.35 \mus per loop (mean \pm std. dev. of 7 runs, 1000 loops each)
           4.22 \mus \pm 24.8 ns per loop (mean \pm std. dev. of 7 runs, 100000 loops each)
```

summaries for matrices

summaries for matrices

```
In [61]:
    np.random.seed(1)
# M = np.random.random((3, 4))
M = np.arange(12)
np.random.shuffle(M)
M = np.reshape(M, (3,4))
print(M)

[[ 2  3   4  10]
        [ 1  6  0  7]
        [ 11  9  8  5]]
```

summaries for matrices

summaries for matrices

```
In [61]:
           np.random.seed(1)
           \# M = np.random.random((3, 4))
           M = np.arange(12)
           np.random.shuffle(M)
           M = np.reshape(M, (3,4))
           print(M)
           [[ 2 3 4 10]
            [ 1 6 0 7]
             [11 9 8 5]]
In [62]:
           sum(M) # regular sum function
Out[62]: array([14, 18, 12, 22])
In [63]:
           np.sum(M) # np.sum function
Out[63]: 66
```

```
In [64]:
    print(M)

[[ 2  3  4 10]
       [ 1  6  0  7]
       [11  9  8  5]]
```

Out[65]: array([14, 18, 12, 22])

```
In [64]:
           print(M)
           [[ 2 3 4 10]
            [ 1 6 0 7]
            [11 9 8 5]]
In [65]:
           np.sum(M, axis = 0) # np.sum function with axis specified
           # matrices have two dimensions
           # 0 is rows, 1 is columns
           # np.sum axis = 0, will sum over rows, so you end up getting column totals
Out[65]: array([14, 18, 12, 22])
In [66]:
           np.sum(M, axis = 1)
Out[66]: array([19, 14, 33])
```

```
In [64]:
           print(M)
           [[ 2 3 4 10]
            [ 1 6 0 7]
            [11 9 8 5]]
In [65]:
           np.sum(M, axis = 0) # np.sum function with axis specified
           # matrices have two dimensions
           # 0 is rows, 1 is columns
           # np.sum axis = 0, will sum over rows, so you end up getting column totals
Out[65]: array([14, 18, 12, 22])
In [66]:
           np.sum(M, axis = 1)
Out[66]: array([19, 14, 33])
In [67]:
           np.min(M, axis = 0)
Out[67]: array([1, 3, 0, 5])
```

```
In [68]: print(M)

[[ 2 3 4 10]
      [ 1 6 0 7]
      [11 9 8 5]]
```

Out[69]: 3.452052529534663

Out[70]: array([4.49691252, 2.44948974, 3.26598632, 2.05480467])

```
In [68]:
           print(M)
           [[ 2 3 4 10]
            [ 1 6 0 7]
[11 9 8 5]]
In [69]:
           np.std(M)
Out[69]:
          3.452052529534663
In [70]:
           np.std(M, axis = 0)
Out[70]: array([4.49691252, 2.44948974, 3.26598632, 2.05480467])
In [71]:
           np.mean(M, axis = 1)
Out[71]: array([4.75, 3.5, 8.25])
```

```
In [72]:
           np.random.seed(1)
           A = np.ones(24)
           np.random.shuffle(A)
           A = np.reshape(A, (2, 3, 4)) # two sheets, 3 rows, 4 columns
           print(A)
            [[1. 1. 1. 1.]
              [1. 1. 1. 1.]
              [1. 1. 1. 1.]]
             [[1. 1. 1. 1.]
              [1. 1. 1. 1.]
              [1. 1. 1. 1.]]
In [73]:
           np.sum(A, axis = 0) # sum across "sheets"
Out[73]: array([[2., 2., 2., 2.],
                   [2., 2., 2., 2.],
                   [2., 2., 2., 2.]
```

```
In [72]:
           np.random.seed(1)
           A = np.ones(24)
           np.random.shuffle(A)
           A = np.reshape(A, (2, 3, 4)) # two sheets, 3 rows, 4 columns
           print(A)
           [[1. 1. 1. 1.]
              [1. 1. 1. 1.]
              [1. 1. 1. 1.]]
            [[1. 1. 1. 1.]
              [1. 1. 1. 1.]
              [1. 1. 1. 1.]]
In [73]:
           np.sum(A, axis = 0) # sum across "sheets"
Out[73]: array([[2., 2., 2., 2.],
                   [2., 2., 2., 2.],
                   [2., 2., 2., 2.]
In [74]:
           np.sum(A, axis = 1) # sum across rows
Out[74]: array([[3., 3., 3., 3.],
                   [3., 3., 3., 3.]
```

```
In [72]:
           np.random.seed(1)
           A = np.ones(24)
           np.random.shuffle(A)
           A = np.reshape(A, (2, 3, 4)) # two sheets, 3 rows, 4 columns
           print(A)
            [[1. 1. 1. 1.]
              [1. 1. 1. 1.]
              [1. 1. 1. 1.]]
             [[1. 1. 1. 1.]
              [1. 1. 1. 1.]
              [1. 1. 1. 1.]]
In [73]:
           np.sum(A, axis = 0) # sum across "sheets"
Out[73]: array([[2., 2., 2., 2.],
                   [2., 2., 2., 2.],
                   [2., 2., 2., 2.]]
In [74]:
           np.sum(A, axis = 1) # sum across rows
Out[74]: array([[3., 3., 3., 3.],
                   [3., 3., 3., 3.]
In [75]:
           np.sum(A, axis = 2) # sum across columns
```

Out[75]: array([[4., 4., 4.], [4., 4., 4.]])

```
In [76]:
    x = float("nan") # direct creation of nan
    print(x)
    print(type(x))

nan
    <class 'float'>
```

```
In [76]:
            x = float("nan") # direct creation of nan
            print(x)
            print(type(x))
            nan
            <class 'float'>
In [77]:
            y = float("inf") # y is the float representation of infinity
            print(y / y) # these calculations will yield a nan result
            print(y - y)
            nan
            nan
In [78]:
            np.sum([x, 2])
Out[78]:
            nan
```

```
In [76]:
            x = float("nan") # direct creation of nan
            print(x)
            print(type(x))
            nan
            <class 'float'>
In [77]:
            y = float("inf") # y is the float representation of infinity
            print(y / y) # these calculations will yield a nan result
            print(y - y)
            nan
            nan
In [78]:
            np.sum([x, 2])
Out[78]:
            nan
In [79]:
            np.nansum([x, 2]) # in R you have the option na.rm = TRUE
Out[79]:
```

The following table provides a list of useful aggregation functions available in NumPy:

| Function Name | NaN-safe Version | Description |
|----------------------|------------------|---|
| np.sum | np.nansum | Compute sum of elements |
| np.prod | np.nanprod | Compute product of elements |
| np.mean | np.nanmean | Compute mean of elements |
| np.std | np.nanstd | Compute standard deviation |
| np.var | np.nanvar | Compute variance |
| np.min | np.nanmin | Find minimum value |
| np.max | np.nanmax | Find maximum value |
| np.argmin | np.nanargmin | Find index of minimum value |
| np.argmax | np.nanargmax | Find index of maximum value |
| np.median | np.nanmedian | Compute median of elements |
| np.percentile | np.nanpercentile | Compute rank-based statistics of elements |
| np.any | N/A | Evaluate whether any elements are true |
| np.all | N/A | Evaluate whether all elements are true |

Broadcasting

This is a similar concept to recyling values in R, but only works when the dimensions are compatible

Broadcasting

This is a similar concept to recyling values in R, but only works when the dimensions are compatible

```
In [80]:
    a = np.array([1,2,3])
    b = np.array([4,5,6])
    print(a + b)
[5 7 9]
```

Broadcasting

This is a similar concept to recyling values in R, but only works when the dimensions are compatible

```
In [80]:
           a = np.array([1,2,3])
           b = np.array([4,5,6])
           print(a + b)
           [5 7 9]
In [81]:
           c = np.array([7,8])
           print(a + c) # doesn't work
           ValueError
                                                       Traceback (most recent call last)
           <ipython-input-81-7215ac8feb02> in <module>
                 1 c = np.array([7,8])
           ----> 2 print(a + c) # doesn't work
           ValueError: operands could not be broadcast together with shapes (3,) (2,)
```

In [82]: print(a)

[1 2 3]

```
In [82]: print(a)
        [1 2 3]

In [83]: e = np.ones([3,3])
        print(e)

        [[1. 1. 1.]
        [1. 1. 1.]
        [1. 1. 1.]]
```

```
In [82]:
           print(a)
           [1 2 3]
In [83]:
           e = np.ones([3,3])
           print(e)
           [[1. 1. 1.]
            [1. 1. 1.]
             [1. 1. 1.]]
In [84]:
           print(e + a) # the array a gets 'broadcast' across all three rows
           [[2. 3. 4.]
            [2. 3. 4.]
            [2. 3. 4.]]
```

```
In [82]:
           print(a)
            [1 2 3]
In [83]:
           e = np.ones([3,3])
            print(e)
            [[1. 1. 1.]
             [1. 1. 1.]
             [1. 1. 1.]]
In [84]:
           print(e + a) # the array a gets 'broadcast' across all three rows
            [[2. 3. 4.]
             [2. 3. 4.]
             [2. 3. 4.]]
In [85]:
           print(a.reshape([3,1])) # we reshape a to be a 3x1 array
            [[1]
             [2]
             [3]]
```

```
In [82]:
            print(a)
            [1 2 3]
In [83]:
           e = np.ones([3,3])
            print(e)
            [[1. 1. 1.]
             [1. 1. 1.]
             [1. 1. 1.]]
In [84]:
            print(e + a) # the array a gets 'broadcast' across all three rows
            [[2. 3. 4.]
             [2. 3. 4.]
             [2. 3. 4.]]
In [85]:
            print(a.reshape([3,1])) # we reshape a to be a 3x1 array
            [[1]
             [2]
             [3]]
In [86]:
            print(e + a.reshape([3,1])) # the reshaped array is broadcast across columns
            [[2. 2. 2.]
             [3. 3. 3.]
             [4. 4. 4.]]
```

```
In [87]:
    d = np.vstack([a,b]) # we stack the arrays a and b vertically
    print(d)

[[1 2 3]
    [4 5 6]]
```

In [90]: print(c)

[7 8]

```
In [90]: print(c)

[7 8]

In [91]: print(d)

[[1 2 3]
  [4 5 6]]
```

```
In [90]:
           print(c)
           [7 8]
In [91]:
           print(d)
           [[1 2 3]
            [4 5 6]]
In [92]:
           print(d + c) # c does not have compatible dimensions
           ValueError
                                                      Traceback (most recent call last)
           <ipython-input-92-8c651d5d46fc> in <module>
           ----> 1 print(d + c) # c does not have compatible dimensions
           ValueError: operands could not be broadcast together with shapes (2,3) (2,)
```

```
In [90]:
           print(c)
           [7 8]
In [91]:
           print(d)
           [[1 2 3]
            [4 5 6]]
In [92]:
           print(d + c) # c does not have compatible dimensions
           ValueError
                                                        Traceback (most recent call last)
           <ipython-input-92-8c651d5d46fc> in <module>
           ----> 1 print(d + c) # c does not have compatible dimensions
           ValueError: operands could not be broadcast together with shapes (2,3) (2,)
In [93]:
           print(d + c.reshape([2,1])) # after we reshape c to be a column, we can broadcast it
           [[ 8 9 10]
            [12 13 14]]
```

[9]]

[0 1 2 3 4 5 6 7 8 9 10]

```
In [94]:
          e = np.arange(10).reshape((10, 1))
          f = np.arange(11)
          print(e)
          print(f)
           [[0]]
            [1]
            [2]
            [3]
            [4]
            [5]
            [6]
            [7]
            [8]
            [9]]
           [0 1 2 3 4 5 6 7 8 9 10]
In [95]:
          print(e + f) ## e and f are broadcast into compatible matrices and then added
                             5 6 7 8 9 10]
                   3 4 5 6 7 8 9 10 11]
                3 4 5 6 7 8 9 10 11 12]
                4 5 6 7 8 9 10 11 12 13]
                5 6 7 8 9 10 11 12 13 14]
                         9 10 11 12 13 14 15]
                      9 10 11 12 13 14 15 16]
                8 9 10 11 12 13 14 15 16 17]
                9 10 11 12 13 14 15 16 17 18]
            [ 9 10 11 12 13 14 15 16 17 18 19]]
```

[0 9 18 27 36 45 54 63 72 81 90]]

```
In [96]:
           print(e * f) ## e and f are broadcast into compatible matrices and then multiplied element-wise
                0 0 0 0 0 0 0 0 0 0 0]
1 2 3 4 5 6 7 8 9 10]
              0 2 4 6 8 10 12 14 16 18 20]
                3 6 9 12 15 18 21 24 27 30]
                4 8 12 16 20 24 28 32 36 40]
                5 10 15 20 25 30 35 40 45 50]
            [ 0 6 12 18 24 30 36 42 48 54 60]
            [ 0 7 14 21 28 35 42 49 56 63 70]
            [ 0 8 16 24 32 40 48 56 64 72 80]
            [ 0 9 18 27 36 45 54 63 72 81 90]]
In [97]:
           print(d)
           [[1 2 3]
            [4 5 6]]
```

```
In [96]:
          print(e * f) ## e and f are broadcast into compatible matrices and then multiplied element-wise
                0 0 0 0 0 0 0 0 0 0 0]
1 2 3 4 5 6 7 8 9 10]
               2 4 6 8 10 12 14 16 18 20]
               3 6 9 12 15 18 21 24 27 30]
             0 4 8 12 16 20 24 28 32 36 40]
               5 10 15 20 25 30 35 40 45 50]
            [ 0 6 12 18 24 30 36 42 48 54 60]
            [ 0 7 14 21 28 35 42 49 56 63 70]
            [ 0 8 16 24 32 40 48 56 64 72 80]
            [ 0 9 18 27 36 45 54 63 72 81 90]]
In [97]:
          print(d)
          [[1 2 3]
           [4 5 6]]
In [98]:
          d.reshape((1,6)) + d.reshape((6,1))
          array([[ 2, 3, 4, 5, 6, 7],
Out[98]:
                  [3, 4, 5, 6, 7, 8],
                  [4, 5, 6, 7, 8, 9],
                  [5, 6, 7, 8, 9, 10],
                  [6, 7, 8, 9, 10, 11],
                  [7, 8, 9, 10, 11, 12]])
```

```
In [99]:
           x = np.arange(6)
           print(x)
           [0 1 2 3 4 5]
In [100]:
           print(x < 3)</pre>
           [ True True False False False]
In [101]:
           print(x >= 3)
           [False False True True]
In [102]:
           print(x == 3)
           [False False True False False]
```

```
In [103]: # the results can then be used to subset
print(x[x >= 3])
[3 4 5]
```

```
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print(x[x >= 3])

[3 4 5]

In [104]: np.sum(x >= 3) # True = 1, False = 0, so sum counts how many are true
```

Out[104]: 3

```
In [103]: # the results can then be used to subset
print(x[x >= 3])

[3 4 5]

In [104]: np.sum(x >= 3) # True = 1, False = 0, so sum counts how many are true

Out[104]: 3

In [105]: np.mean(x >= 3) # finds the proportion that is True
```

Out[105]: 0.5

```
In [103]:
            # the results can then be used to subset
             print(x[x >= 3])
             [3 4 5]
In [104]:
             np.sum(x \ge 3) # True = 1, False = 0, so sum counts how many are true
Out[104]: 3
In [105]:
             np.mean(x \ge 3) # finds the proportion that is True
Out[105]:
            0.5
In [106]:
             print(\sim(x == 3)) # use the tilde for negation of boolean values
             [ True True True False True True]
```

```
In [107]: print(~x == 3) # be careful if you leave off parenthesis
```

[False False False False False]

Out[108]: array([-1, -2, -3, -4, -5, -6], dtype=int32)

```
In [109]:
          y = np.arange(12).reshape([3,4])
           print(y)
           [[ 0 1 2 3]
           [ 4 5 6 7]
            [ 8 9 10 11]]
In [110]:
           print(y >= 6)
           [[False False False]
            [False False True True]
            [ True True True]]
In [111]:
           np.sum(y >= 6)
Out[111]: 6
```

```
In [109]:
           y = np.arange(12).reshape([3,4])
           print(y)
            [[0 1 2 3]
            [ 4 5 6 7]
             [8 9 10 11]]
In [110]:
           print(y >= 6)
            [[False False False]
             [False False True True]
             [ True True True ]
In [111]:
           np.sum(y >= 6)
Out[111]: 6
In [112]:
           np.sum(y >= 6, axis = 0) # you can perform sums and other aggregate functions axis-wise on the bool
Out[112]: array([1, 1, 2, 2])
```

```
In [109]:
           y = np.arange(12).reshape([3,4])
           print(y)
            [[0 1 2 3]
             [ 4 5 6 7]
             [8 9 10 11]]
In [110]:
           print(y >= 6)
            [[False False False]
             [False False True True]
             [ True True True ]]
In [111]:
           np.sum(y >= 6)
Out[111]: 6
In [112]:
           np.sum(y >= 6, axis = 0) # you can perform sums and other aggregate functions axis-wise on the bool
Out[112]: array([1, 1, 2, 2])
In [113]:
           np.sum(y >= 6, axis = 1)
Out[113]: array([0, 2, 4])
```

```
In [114]:
    a = np.array([True, True, False, False])
    b = np.array([True, False, True, False])
    print(a)
    print(b)

[ True True False False]
    [ True False True False]
```

```
In [114]:    a = np.array([True, True, False, False])
    b = np.array([True, False, True, False])
    print(a)
    print(b)

    [ True True False False]
    [ True False True False]

In [115]:    print(a & b) # bitwise and
    [ True False False False]

In [116]:    print(a | b) # bitwise or
    [ True True True False]
```

```
In [114]:
            a = np.array([True, True, False, False])
            b = np.array([True, False, True, False])
            print(a)
            print(b)
             [ True True False False]
             [ True False True False]
In [115]:
            print(a & b) # bitwise and
            [ True False False False]
In [116]:
            print(a | b) # bitwise or
            [ True True False]
In [117]:
            print(a ^ b) # bitwise xor (exclusive or)
            [False True True False]
```

```
In [118]: print(~a) # bitwise not
```

[False False True True]

Out[119]: True

Out[120]: False

fancy indexing

Regular lists in python do not support fancy indexing, but NumPy does!

fancy indexing

Regular lists in python do not support fancy indexing, but NumPy does!

```
In [121]:
    np.random.seed(1)
    x = np.random.randint(100, size = 10)
    print(x)

[37 12 72 9 75 5 79 64 16 1]
```

fancy indexing

Regular lists in python do not support fancy indexing, but NumPy does!

```
In [123]:
    a = [1, 4, 7]
    b = [2, 3, 8]
    ind = np.vstack([a,b])
    print(ind)

[[1 4 7]
    [2 3 8]]
```

```
In [123]:
            a = [1, 4, 7]
            b = [2, 3, 8]
            ind = np.vstack([a,b])
            print(ind)
            [[1 4 7]
             [2 3 8]]
In [124]:
            print(x[ind])
            [[12 75 64]
             [72 9 16]]
In [125]:
            X = np.arange(12).reshape((3, 4))
            print(X)
            [[ 0 1 2 3]
             [ 4 5 6 7]
             [ 8 9 10 11]]
```

```
In [123]:
            a = [1, 4, 7]
            b = [2, 3, 8]
            ind = np.vstack([a,b])
            print(ind)
            [[1 4 7]
             [2 3 8]]
In [124]:
            print(x[ind])
            [[12 75 64]
             [72 9 16]]
In [125]:
            X = np.arange(12).reshape((3, 4))
            print(X)
            [[ 0 1 2 3]
             [ 4 5 6 7]
             [ 8 9 10 11]]
In [126]:
            row = np.array([0, 1, 2])
            col = np.array([2, 1, 3])
            X[row, col]
Out[126]: array([ 2, 5, 11])
```

- np.sort()
- np.argsort() gives the indexes of the values to have the proper sorting

- np.sort()
- np.argsort() gives the indexes of the values to have the proper sorting

```
In [127]:
    np.random.seed(2)
    x = np.arange(5)
    np.random.shuffle(x)
    print(x)
```

[2 4 1 3 0]

- np.sort()
- np.argsort() gives the indexes of the values to have the proper sorting

- np.sort()
- np.argsort() gives the indexes of the values to have the proper sorting

```
In [127]:
             np.random.seed(2)
             x = np.arange(5)
             np.random.shuffle(x)
             print(x)
             [2 4 1 3 0]
In [128]:
             x.sort() # sorts x in place
             print(x)
             [0 1 2 3 4]
In [129]:
             y = np.array([5, 2, 1, 4])
             print(y)
             print(y.argsort())
             [5 2 1 4]
             [2 1 3 0]
```

- np.sort()
- np.argsort() gives the indexes of the values to have the proper sorting

```
In [127]:
             np.random.seed(2)
             x = np.arange(5)
             np.random.shuffle(x)
             print(x)
             [2 4 1 3 0]
In [128]:
             x.sort() # sorts x in place
             print(x)
             [0 1 2 3 4]
In [129]:
             y = np.array([5, 2, 1, 4])
             print(y)
             print(y.argsort())
             [5 2 1 4]
             [2 1 3 0]
In [130]:
             d = y.argsort()
             y[d]
Out[130]: array([1, 2, 4, 5])
```

```
In [131]:
            np.random.seed(1)
            X = np.random.randint(0, 10, (4, 6))
            print(X)
             [[5 8 9 5 0 0]
              [1 7 6 9 2 4]
              [5 2 4 2 4 7]
              [7 9 1 7 0 6]]
In [132]:
            # sort each column of X
            # np.sort returns a copy of X after sorted. It does not modify X
            np.sort(X, axis=0)
Out[132]: array([[1, 2, 1, 2, 0, 0],
                    [5, 7, 4, 5, 0, 4],
                    [5, 8, 6, 7, 2, 6],
                    [7, 9, 9, 9, 4, 7]])
```

[0, 1, 6, 7, 7, 9]])

```
In [131]:
            np.random.seed(1)
            X = np.random.randint(0, 10, (4, 6))
            print(X)
            [[5 8 9 5 0 0]
             [1 7 6 9 2 4]
              [5 2 4 2 4 7]
              [7 9 1 7 0 6]]
In [132]:
            # sort each column of X
            # np.sort returns a copy of X after sorted. It does not modify X
            np.sort(X, axis=0)
Out[132]: array([[1, 2, 1, 2, 0, 0],
                    [5, 7, 4, 5, 0, 4],
                    [5, 8, 6, 7, 2, 6],
                    [7, 9, 9, 9, 4, 7]])
In [133]:
            # sort each row of X
            np.sort(X, axis=1)
Out[133]: array([[0, 0, 5, 5, 8, 9],
                    [1, 2, 4, 6, 7, 9],
                    [2, 2, 4, 4, 5, 7],
```

[0, 1, 6, 7, 7, 9]])

```
In [131]:
            np.random.seed(1)
            X = np.random.randint(0, 10, (4, 6))
            print(X)
            [[5 8 9 5 0 0]
             [1 7 6 9 2 4]
              [5 2 4 2 4 7]
              [7 9 1 7 0 6]]
In [132]:
            # sort each column of X
            # np.sort returns a copy of X after sorted. It does not modify X
            np.sort(X, axis=0)
Out[132]: array([[1, 2, 1, 2, 0, 0],
                    [5, 7, 4, 5, 0, 4],
                    [5, 8, 6, 7, 2, 6],
                    [7, 9, 9, 9, 4, 7]])
In [133]:
            # sort each row of X
            np.sort(X, axis=1)
Out[133]: array([[0, 0, 5, 5, 8, 9],
                    [1, 2, 4, 6, 7, 9],
                    [2, 2, 4, 4, 5, 7],
```

In [134]: x[0,:] # selecting a row

Out[134]: array([5, 8, 9, 5, 0, 0])

[0, 1, 6, 7, 7, 9]])

```
In [131]:
            np.random.seed(1)
            X = np.random.randint(0, 10, (4, 6))
            print(X)
            [[5 8 9 5 0 0]
             [1 7 6 9 2 4]
              [5 2 4 2 4 7]
              [7 9 1 7 0 6]]
In [132]:
            # sort each column of X
            # np.sort returns a copy of X after sorted. It does not modify X
            np.sort(X, axis=0)
Out[132]: array([[1, 2, 1, 2, 0, 0],
                    [5, 7, 4, 5, 0, 4],
                    [5, 8, 6, 7, 2, 6],
                    [7, 9, 9, 9, 4, 7]])
In [133]:
            # sort each row of X
            np.sort(X, axis=1)
Out[133]: array([[0, 0, 5, 5, 8, 9],
                    [1, 2, 4, 6, 7, 9],
                    [2, 2, 4, 4, 5, 7],
```

[0, 1, 6, 7, 7, 9]])

```
In [131]:
            np.random.seed(1)
            X = np.random.randint(0, 10, (4, 6))
            print(X)
            [[5 8 9 5 0 0]
             [1 7 6 9 2 4]
              [5 2 4 2 4 7]
              [7 9 1 7 0 6]]
In [132]:
            # sort each column of X
            # np.sort returns a copy of X after sorted. It does not modify X
            np.sort(X, axis=0)
Out[132]: array([[1, 2, 1, 2, 0, 0],
                    [5, 7, 4, 5, 0, 4],
                    [5, 8, 6, 7, 2, 6],
                    [7, 9, 9, 9, 4, 7]])
In [133]:
            # sort each row of X
            np.sort(X, axis=1)
Out[133]: array([[0, 0, 5, 5, 8, 9],
                    [1, 2, 4, 6, 7, 9],
                    [2, 2, 4, 4, 5, 7],
```

```
In [134]: X[0,:] # selecting a row

Out[134]: array([5, 8, 9, 5, 0, 0])
In [135]: print(X)

        [[5 8 9 5 0 0]
        [1 7 6 9 2 4]
        [5 2 4 2 4 7]
        [7 9 1 7 0 6]]

In [136]: X[:,1].argsort() # the argsort for the column index 1

Out[136]: array([2, 1, 0, 3], dtype=int64)
```

[0, 1, 6, 7, 7, 9]])

```
In [131]:
            np.random.seed(1)
            X = np.random.randint(0, 10, (4, 6))
            print(X)
            [[5 8 9 5 0 0]
             [1 7 6 9 2 4]
              [5 2 4 2 4 7]
              [7 9 1 7 0 6]]
In [132]:
            # sort each column of X
            # np.sort returns a copy of X after sorted. It does not modify X
            np.sort(X, axis=0)
Out[132]: array([[1, 2, 1, 2, 0, 0],
                    [5, 7, 4, 5, 0, 4],
                    [5, 8, 6, 7, 2, 6],
                    [7, 9, 9, 9, 4, 7]])
In [133]:
            # sort each row of X
            np.sort(X, axis=1)
Out[133]: array([[0, 0, 5, 5, 8, 9],
                    [1, 2, 4, 6, 7, 9],
                    [2, 2, 4, 4, 5, 7],
```

```
In [134]:
           X[0,:] # selecting a row
Out[134]: array([5, 8, 9, 5, 0, 0])
In [135]:
            print(X)
            [[5 8 9 5 0 0]
             [1 7 6 9 2 4]
             [5 2 4 2 4 7]
             [7 9 1 7 0 6]]
In [136]:
            X[:,1].argsort() # the argsort for the column index 1
Out[136]: array([2, 1, 0, 3], dtype=int64)
In [137]:
            print(X[ X[:,1].argsort() , : ]) # 'subset' X by the argsort to arrange X by the column
            [[5 2 4 2 4 7]
             [1 7 6 9 2 4]
             [5 8 9 5 0 0]
             [7 9 1 7 0 6]]
```