Q1. We have three foods: Apple pie, Burger, and Chicken
If the weather is sunny, we cook Apple pie.
If the weather is cloudy, we cook Burger.
- Build a Neural Network to model it.

Input: weather; sunny or cloudy

matrix representation $\begin{pmatrix}1\\0\end{pmatrix}$ or $\begin{pmatrix}0\\1\end{pmatrix}$

Output: food: Burger Apple pie Chicken

$\begin{pmatrix}0\\1\\0\end{pmatrix}$ $\begin{pmatrix}1\\0\\0\end{pmatrix}$ $\begin{pmatrix}0\\0\\1\end{pmatrix}$

Model The model should convert,

$$\Pi \begin{cases} \begin{pmatrix}1\\0\end{pmatrix} \longrightarrow \begin{pmatrix}1\\0\\0\end{pmatrix} \\ \begin{pmatrix}0\\1\end{pmatrix} \longrightarrow \begin{pmatrix}0\\1\\0\end{pmatrix} \end{cases} \text{and } \oslash$$

$3\times2 \quad 2\times1 \quad 3\times1$

$\boxed{\mathbb{M} * \Pi = \oslash}$. Now, we just need to find the matrix for the model.

Matrix dimensions $\Pi \longrightarrow 2\times1$, $\mathbb{M} \longrightarrow 3\times2$

$\oslash \longrightarrow 3\times1$

In general, $\mathbb{M} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}$. Now, we just need to find the matrix elements.

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Sunny weather — NN model — Apple pie food

$$\Downarrow$$

$$\begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$a_{11} = 1$, $a_{21} = 0$, and $a_{31} = 0$

So, $\mathbb{M} = \begin{pmatrix} 1 & a_{12} \\ 0 & a_{22} \\ 0 & a_{32} \end{pmatrix} * \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

Cloudy weather — burger

$$\Rightarrow \begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \Rightarrow a_{12} = 0,\ a_{22} = 1,\ a_{32} = 0$$

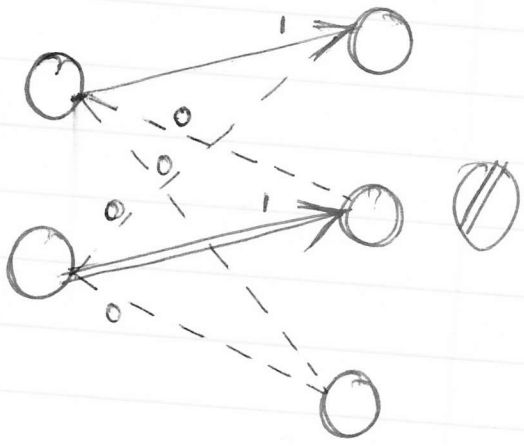
So the matrix for the NN is,

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

This NN looks as follows,

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} * \underline{\mathbb{I}} = \varnothing$$

$\mathbb{I}$



Q.2. Now, we cook a dish depending on what we cooked the day before. For instance,

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Apple pie	Burger	Chicken	Apple pie	Burger	Chicken	Apple pie

Since this is a recurring problem, we need to use recurring neural network.

$$\mathbb{M} * \mathbb{I} = \emptyset$$

Matrix dimensions

$\mathbb{I} \rightarrow 3 \times 1$

$\emptyset \rightarrow 3 \times 1$

$\mathbb{M} \rightarrow 3 \times 3$

So we just need to find $\mathbb{M}$ such that

$$\mathbb{M} * \begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}0\\1\\0\end{pmatrix} \rightarrow \mathbb{M} * \begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\1\end{pmatrix} \rightarrow \mathbb{M} * \begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix}.$$

In general, $\mathbb{M} = \begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{pmatrix}$

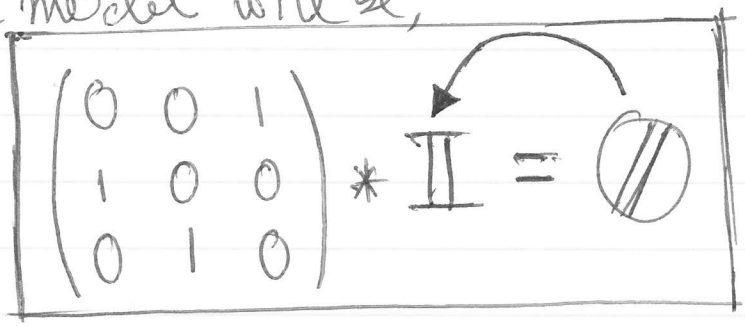
$$\begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{pmatrix} \begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}0\\1\\0\end{pmatrix} \Rightarrow \begin{pmatrix}a_{11}\\a_{12}\\a_{13}\end{pmatrix} = \begin{pmatrix}0\\1\\0\end{pmatrix} \begin{cases} a_{11} = 0 \\ a_{12} = 1 \\ a_{13} = 0 \end{cases}$$

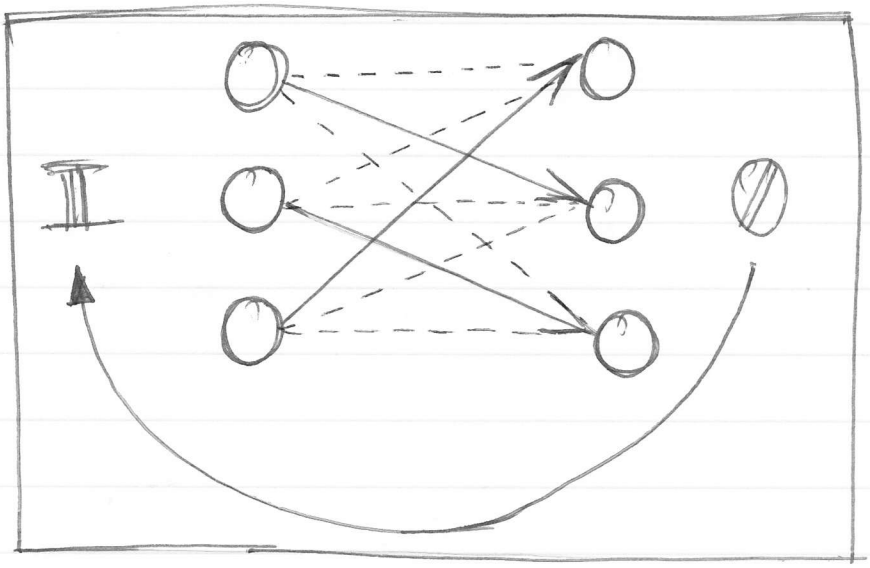
$$\begin{pmatrix} 0 & a_{21} & a_{31} \\ 1 & a_{22} & a_{32} \\ 0 & a_{23} & a_{33} \end{pmatrix} \begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\1\end{pmatrix} \Rightarrow \begin{pmatrix}a_{21}\\a_{22}\\a_{23}\end{pmatrix} = \begin{pmatrix}0\\0\\1\end{pmatrix} \begin{cases} a_{21} = 0 \\ a_{22} = 0 \\ a_{23} = 1 \end{cases}$$

$$\begin{pmatrix} 0 & 0 & a_{31} \\ 1 & 0 & a_{32} \\ 0 & 1 & a_{33} \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} a_{31} \\ a_{32} \\ a_{33} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{cases} a_{31} = 1 \\ a_{32} = 0 \\ a_{33} = 0 \end{cases}$$

So, $M = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

The final model will be,





Called recurrent neural network, very useful for sequential data.

- Stock prediction
- text generation
- voice recognition

Q3. As before, we cook Apple pie → Burger → Chicken, but if the weather is Sunny there is no cooking for that day and the dish from the previous day is served.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Apple pie	Apple pie	Burger	Chicken	Chicken	Apple pie	
Sunny	Cloudy	Cloudy	Sunny	Cloudy	Sunny	
Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday	

Now, we have two inputs,

i. weather: Sunny and Cloudy

$\begin{pmatrix}1\\0\end{pmatrix}$ $\begin{pmatrix}0\\1\end{pmatrix}$ and

ii. Dish from previous day: Apple pie, Burger, and Chicken

$\begin{pmatrix}1\\0\\0\end{pmatrix}$ $\begin{pmatrix}0\\1\\0\end{pmatrix}$ $\begin{pmatrix}0\\0\\1\end{pmatrix}$

Create food and weather matrices for matrix operations

mapping from previous day, given food for next day

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\\\hline 0&0&1\\1&0&0\\0&1&0\end{pmatrix} + \begin{pmatrix}1&0\\1&0\\1&0\\\hline 0&1\\0&1\\0&1\end{pmatrix} \longrightarrow \begin{pmatrix}\text{matrix for the same food}\\\hline \text{matrix for the next food}\end{pmatrix} + \begin{pmatrix}\text{the weather tells us should we cook today or cook tomorrow}\end{pmatrix}$$

food weather Concatinated

- weather matrix

$$\left(\begin{array}{cc} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ \hline 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{array}\right)\begin{pmatrix}1\\0\end{pmatrix} = \left(\begin{array}{c} 1 \\ 1 \\ 1 \\ \hline 0 \\ 0 \\ 0 \end{array}\right) \begin{array}{l} \text{Same day} \\ \\ \\ \text{next day} \end{array}$$

↑ Sunny

$$\left(\begin{array}{cc} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ \hline 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{array}\right)\begin{pmatrix}0\\1\end{pmatrix} = \left(\begin{array}{c} 0 \\ 0 \\ 0 \\ \hline 1 \\ 1 \\ 1 \end{array}\right) \begin{array}{l} \text{Same day} \\ \\ \\ \text{next day} \end{array}$$

↑ Cloudy

tells us if we need to cook today or we cooked from from the previous day

- food matrix give what is today's food and what is yesterday's food

+

weather matrix select from two food options depending on the current (today's) weather.

- e.g. yesterday we cooked an apple pie $\begin{pmatrix}1\\0\\0\end{pmatrix}$ and today the weather is rainy $\begin{pmatrix}0\\1\end{pmatrix}$

$$\left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right)\begin{pmatrix}1\\0\\0\end{pmatrix} = \left(\begin{array}{c} 1 \\ 0 \\ 0 \\ \hline 0 \\ 1 \\ 0 \end{array}\right) \begin{array}{l} \text{Apple pie} \text{ ——— Same food} \\ \\ \\ \text{Burger (next food)} \text{ — next food} \end{array}$$

↑ food matrix

↑ Apple pie from yesterday

$$\begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ \hline 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \hline 1 \\ 1 \\ 1 \end{pmatrix}$$

Same day

next day

↑ today's weather (Cloudy)

↑ weather matrix

→ $\begin{pmatrix} 1 \\ 0 \\ 0 \\ \hline 0 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \hline 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \hline 1 \\ 2 \\ 1 \end{pmatrix}$ Adding two resulting matrices

non-linear operation ↓ turns the largest number to '1' and rest of them to '0'

$\begin{pmatrix} 0 \\ 0 \\ 0 \\ \hline 0 \\ 1 \\ 0 \end{pmatrix}$ merge mapping $\begin{pmatrix} 0+0 \\ 0+1 \\ 0+0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ Burger

↑ top three values ↑ bottom three values

merge map matrix

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \begin{pmatrix} 0 \\ 0 \\ 0 \\ \hline 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

— In terms of neural network map