UMC202: Introduction to Numerical Analysis

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Contents

Lecture 01: Introduction

Wed 02 Aug '23

1 The Course

1.1 Grading

Absolute grading. 90 ± 2 marks out of 100 for an A+.

- **Final exam:** 50%.
- **Midterm exam:** 30%.
- **Assignments:** 20%.

1.2 References

- Numerical Analysis by Richard L. Burden and J. Douglas Faires
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2 Introduction

Solving *algebraic* systems of equations numerically.

(i) $F(x) = 0$.

(ii) $F_j(x_1, \ldots, x_n) = 0$ where $j \in \{1, \ldots, m\}$.

(iii) $y' = f(t, y)$ with $y(t_0) = y_0$.

(iv) $y'' + ay' + by = 0$ with either $y(t_0) = y_0, y'(t_0) = y_1$ or $y(t_0) = y_0, y(t_1) = y_1$.

We'll do interpolation, root-finding techniques, differential equations with initial conditions, etc.

3 Single Variable Root-Finding

Given a continuous function $F : \mathbb{R} \to \mathbb{R}$, we want to find x such that $F(x) = 0$.

Let x^* be a solution to $F(x) = 0$. We algorithmically generate a sequence $\{x_n\}$ that tends to x^*.

The algorithm: Find two points a and b such that $F(a)F(b) < 0$. By the intermidiate value theorem, there exists $x^* \in (a, b)$ such that $F(x^*) = 0$. We can perform a binary search to close in on x^*.

Remarks. This only works if such points a and b exist. The graph of F could be tangent to the x-axis, as in $x \mapsto x^2$.

3.1 Fixed point

We can rewrite $F(x) = 0$ as $x = g(x)$, where $g(x) = x + F(x)$. Finding a root of F is equivalent to finding a fixed point of g.