UMA205: Introduction to Algebraic Structures

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Lecture 01: Wed 03 Jan '24

0 The course

Instructor: Prof. Arvind Ayyer
Office: X-15
Office hours: TBD
Lecture hours: MWF 11:00–11:50
Tutorial hours: Tue 9:00–9:50

80% attendance is mandatory.

Prerequisites: UMA101 and UMA102 **Texts:** Several

- *Analysis I*, Terence Tao.

Grading

(20%) Quizzes on alternate Tuesdays, worst dropped. No makeup quizzes, but if a quiz is missed for a medical reason (with certificate), that quiz will be dropped.

(30%) Midterm

(50%) Final

Homeworks after every class, ungraded. Exams are closed book and closed notes, with no electronic devices allowed.

Aims of the Course

- Deal with formal mathematical structures.
- Learning the axiomatic method.
- See how more complicated structures arise from simpler ones.

1 Peano's Axioms

We try to formulate two fundamental quantities: 0 and the successor function $n \mapsto n_{++}$.

(P1) 0 is a natural number.

(P2) If n is a natural number, so is n_{++}.

(P3) 0 is not the successor of any natural number.

(P4) Different natural numbers have different successors.

(P5) (Principle of mathematical induction) Let $P(n)$ be any "property" for a natural number n. Suppose that $P(0)$ is true, and that $P(n_{++})$ is true whenever $P(n)$ is true. Then P is true for all natural numbers.

Denote the set of natural numbers by $\mathbb{N}$. Note that $\mathbb{N}$ is itself infinite, but all of its elements are finite.

Proof. 0 is finite. If n is finite, then n_{++} is finite. Thus, by induction, all natural numbers are finite. (But wtf is a finite number?) $\square$

Remarks.

- There exist number systems which admit infinite numbers. For example, cardinals, ordinals, etc.
- This way of thinking is *axiomatic*, but not constructive.