$$\min_{x \in \mathbb{R}^d} \quad f(x) \qquad f, f_i \text{ are convex functions}$$

$$f_i(x) \leq 0 \quad i = 1, \dots, m$$

(P) $$a_i^T x = b_i \quad j = 1, \dots, n$$

$$\mathcal{L}(x, \lambda, \mu) = f(x) + \sum_{i=1}^{m} \lambda_i f_i(x) + \sum_{j=1}^{n} \mu_j (a_j^T x - b_j)$$

(1) $$\nabla f(x) + \sum_{i=1}^{m} \lambda_i \nabla f_i(x) + \sum_{j=1}^{n} \mu_j a_j = 0$$

(2) $$\lambda_i f_i(x) = 0 \qquad \lambda_i \geq 0$$

(3) $$f_i(x) \leq 0 \qquad a_j^T x = b_j$$

$$i = 1, \dots, m \qquad j = 1, \dots, m$$

If for any $x^*$ there exists $\lambda^*, \mu^*$ such that $(x^*, \lambda^*, \mu^*)$ satisfy ①–③ then $x^*$ is a K.K.T. point of (P).

If $x^*$ is a K.K.T. point then it is global minimum of (P)

Distance of $z$ from

$w^T x + b = 0$

$$\boxed{\frac{|w^T z + b|}{\|w\|}}$$

Generalization error from Perceptron

$$E_{\mathcal{D} \sim p^{(N)}} R\left(h_{\mathcal{D}}^{(P)}\right)$$

$$\leq E_{\mathcal{D} \sim p^{(N+1)}} \frac{\min\left(M(\mathcal{D}), \frac{R^2(D)}{\gamma^2(\mathcal{D})}\right)}{N+1}$$

$$\mathcal{D} = \left\{ (x^{(i)}, y^{(i)}) \right\}_{i=1}^{N}$$

$$\min_i \frac{y_i (w^T x^{(i)} + b)}{\|w\|} = \gamma(w)$$

$$\max_{w, b, \gamma} \gamma$$

$$\frac{y_i (w^T x^{(i)} + b)}{\|w\|} \geq \gamma \quad i = 1, -, N$$

If $(w^*, b^*)$ solves the problem then $(s w^*, s b^*)$ $s > 0$ also solves the problem.

We choose $s$ such that $r \|w\| = 1$

$$\max_{w,b} \frac{1}{\|w\|}$$

$$y^{(i)}(w^T x^{(i)} + b) \geq 1$$

<u>Equivalent problem</u>

$$\min_{w,b} \frac{1}{2} \|w\|^2$$

$$y^{(i)}(w^T x^{(i)} + b) \geq 1$$

$i = 1, \dots, N$

$$\mathcal{L}(w, b, \lambda)$$

$$= \frac{1}{2}\|w\|^2 - \sum_{i=1}^{N} \lambda_i \left\{ y^{(i)} \left( w^T x^{(i)} + b \right) - 1 \right\}$$

$$\nabla_w \mathcal{L} = 0 = w - \sum_{i=1}^{N} \lambda_i y^{(i)} x^{(i)} = 0$$

$$\nabla_b \mathcal{L} = 0 \qquad - \sum_{i=1}^{N} \lambda_i y^{(i)} = 0$$

<u>K. K. T.</u>

$$w = \sum_{i=1}^{N} \lambda_i y^{(i)} x^{(i)},$$

$$\lambda_i \geq 0, \quad \lambda_i \left\{ y^{(i)} \left( w^T x^{(i)} + b \right) - 1 \right\} = 0$$

$$y^{(i)} \left( w^T x^{(i)} + b \right) \geq 1$$

$$i = 1, \ldots, N$$

If $\lambda_i > 0 \Rightarrow y^{(i)}(w^T x^{(i)} + b) = 1$

If $y^{(i)}(w^T x^{(i)} + b) > 1$

$\Rightarrow \lambda_i = 0$

$w^T x^{(i)} + b \geq 1$ $\quad y^{(i)} = 1$

$w^T x^{(i)} + b \leq -1$ $\quad y^{(i)} = -1$

$\lambda_i > 0$ $\quad x^{(i)}$ is a support vector.