E0 225: Design and Analysis of Algorithms

Naman Mishra

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Contents

The course

MS Teams:

Lecture 1.
Wednesday
August 07

Coverage

- Algorithmic paradigms
- Flows and applications
- NP-completeness and reductions
- Approximation algorithms
- Randomized algorithms
- Linear programming

The last three are dependent on time.

References

- *Algorithm Design* by Jon Kleinberg and Éva Tardos
- *Introduction to Algorithms* by Cormen, Leiserson, Rivest, and Stein
- *Algorithms* by Jeff Erickson

Prerequisites

At least one UG course in algorithms/data structures. MaTheMaTiCal maTUrItY.

Evaluation

(50%) 2 midterms: 3rd/4th week of September and 1st/2nd week of November

(40%) Final: end of November

(10%) 7–8 assignments

Chapter I

Introduction

Question I.1. ***Input:*** *A list of n numbers $a_1, a_2, \ldots, a_n$ and an integer $k \leq n$.*
Output: *The k smallest numbers in the list (in any order).*

- Sort: $O(n \log n)$
- Iterative min-max: Find the minimum, remove it, repeat k times. This takes $O(kn)$ time.
- Min heap: building a heap takes $O(n)$ time, each extract-min takes $O(\log n)$ time, so the total time is $O(n + k \log n)$.
- Max heap: build a heap from the first k elements in $O(k)$ time. For each remaining element, compare it with the maximum element in the heap. If it is smaller, replace the maximum element with the new element. This takes $O(k + (n-k)\log k)$ time. Worst case is $O(n \log n)$ when $k = n/2$.
- k-selection: Find the kth smallest element in $O(n)$ time. Partition the list around the kth element. This takes $O(n)$ time.

Thus we have the following new question:

Question I.2. ***Input:*** *A list of n numbers $a_1, a_2, \ldots, a_n$ and an integer $k \leq n$.*
Output: *The kth smallest number in the list.*

Solution. Select the median of the list in $O(n)$ time. Use this as the pivot to partition the list. Recurse on the appropriate half.

$$T(n) = n + T(n/2) = O(n)$$ ■

This of course requires finding the median in $O(n)$ time.

Question I.3. ***Input:*** *A list of n numbers $a_1, a_2, \ldots, a_n$ and an integer $k \leq n$.*
Output: *The median element.*

Solution. ■