Homework 1

ECE 6790 - Information Processing Models in Neural Systems

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Problem 1. INF Model

(a). Smallest Constant Current to generate a spike

Solution: We have,

$$\tau \frac{\mathrm{d}V(t)}{\mathrm{d}t} = -(V(t) - E_L) + I_e R_m$$

Taking Laplace transform on both sides, we get,

$$\tau(sV(s) - V(0)) = -V(s) + \frac{E_L}{s} + \frac{I_e R_m}{s}$$

Where $V(s)$ is the Laplace transform of $V(t)$.
At time $t = 0$, the Neuron is at Resting potential. In this case, $V(0) = V_{resting} = V_{reset} = E_L$. Thus,

$$\tau(sV(s) - E_L) = -V(s) + \frac{E_L}{s} + \frac{I_e R_m}{s}$$

Simplifying, we get,

$$V(s) = \frac{s(\tau E_L) + (E_L + I_e R_m)}{s^2(\tau) + s}$$

Let $k_1 = \tau E_L$ and $k_2 = E_L + I_e R_m$. Thus,

$$V(s) = \frac{s(k_1) + (k_2)}{s^2(\tau) + s}$$

Taking Inverse Laplace transform, we get,

$$\begin{aligned} V(t) &= \frac{k_1 - \tau k_2}{\tau} e^{-\frac{t}{\tau}} + k_2 \\ &= (E_L - E_L - I_e R_m) e^{-\frac{t}{\tau}} + (E_L + I_e R_m) \\ &= I_e R_m (1 - e^{-\frac{t}{\tau}}) + E_L \end{aligned}$$

For a spike to be induced, the Voltage induced at steady state should be atleast equal to threshold voltage. Thus, $V(\infty) = V_{thres}$.

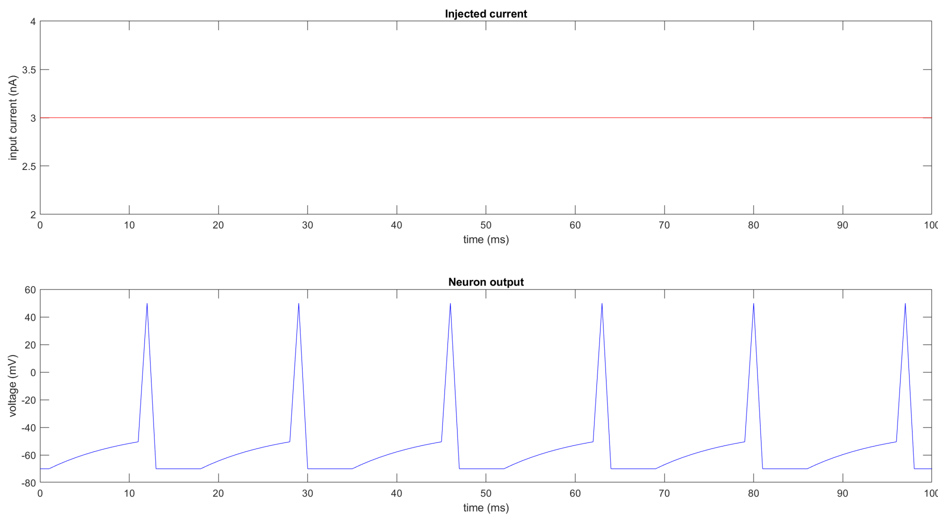
$$V(\infty) = V_{thres} = I_e R_m (1 - 0) + E_L$$

Plugging the value of the constants V_{thres}, I_e, R_m and E_L from the problem, we get,

$$\boxed{I_e = 2nA}$$

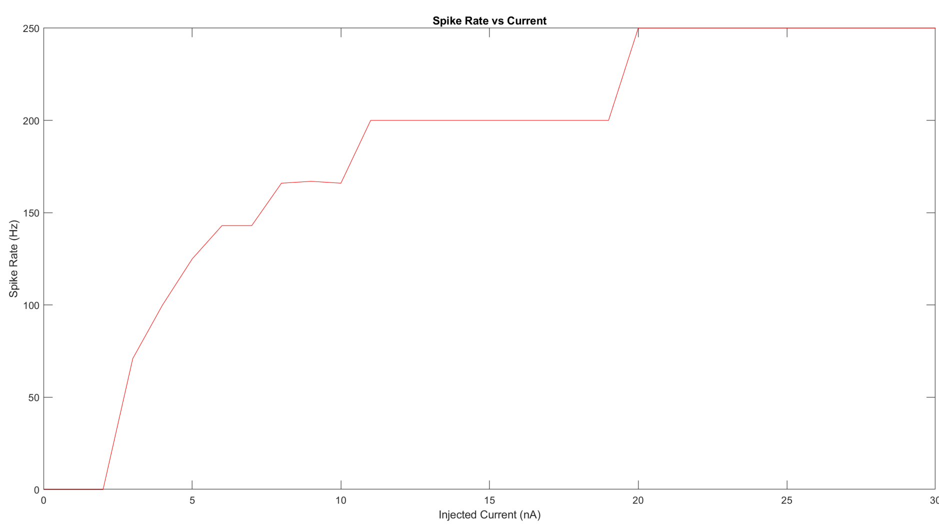
(b). Refractory Effect

Solution: The Voltage Trace of the potential responding to a high constant input with a 5ms refractory time is:

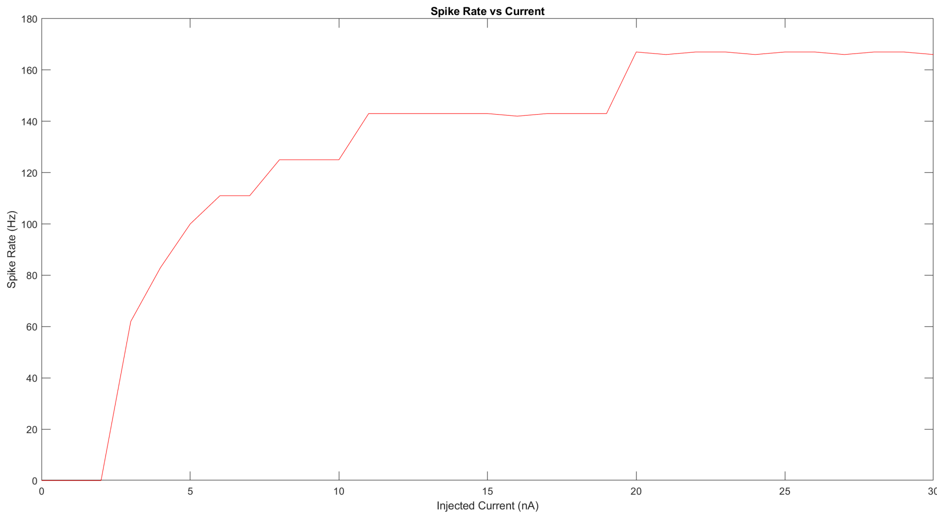


(c). Current Level vs Spike Rate

Solution: The plot of Spike rate vs Current Level for 2ms refractory period:



The plot of Spike rate vs Current Level for 4ms refractory period:



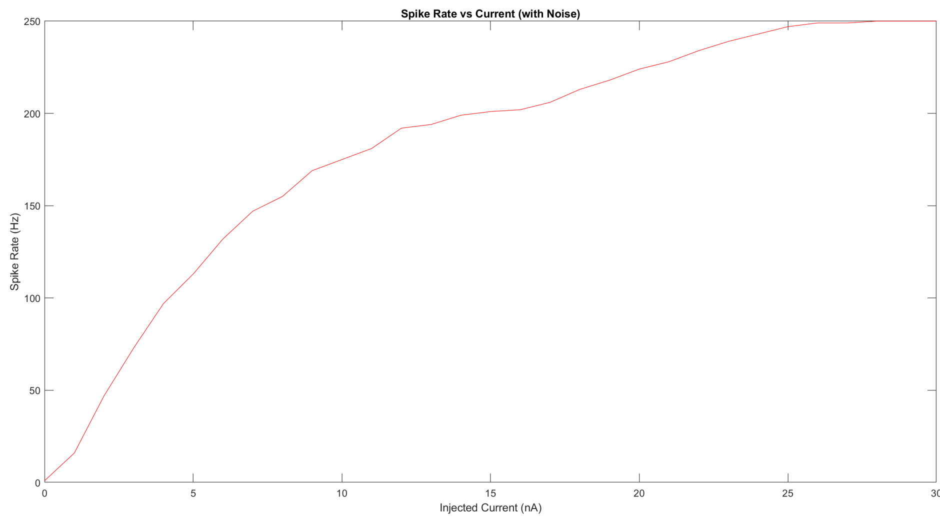
Determining Firing Rate

It is analytically possible to determine the highest possible firing rate from refractory period. The firing rate of a neuron is dependent on the time of synapse ($\tau_{synapse}$) and the refractory time ($\tau_{refractory}$). It is during the time between these two processes that the neuron cannot accept any new input and thus caps the firing rate of a neuron. Mathematically, the rate of firing is:

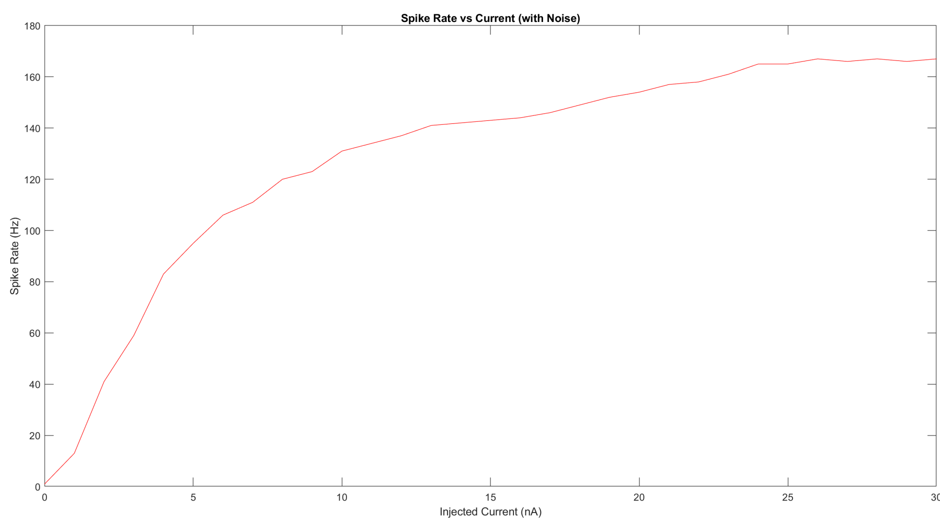
$$\textit{Firing Rate} \; = \frac{1}{\tau_{synapse} + \tau_{refractory}}$$

(d). Current Level vs Spike Rate with Noise

Solution: The plot of Spike rate vs Current (with Noise) Level for 2ms refractory period:

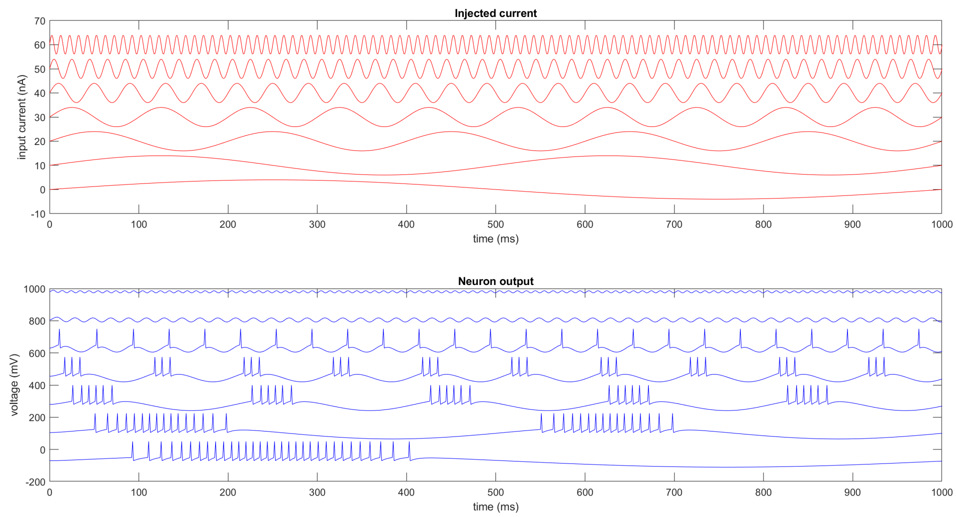


The plot of Spike rate vs Current (with Noise) Level for 4ms refractory period:

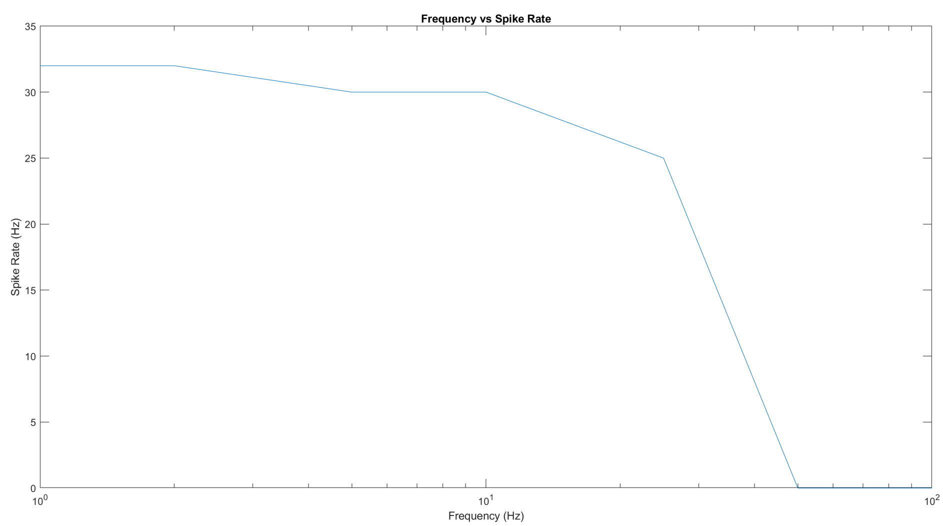


(e). Neural Response to Time-Varying Current

Solution: The voltage trace corresponding to time-varying inputs is:



The corresponding plot of Spike rate vs Frequency is:



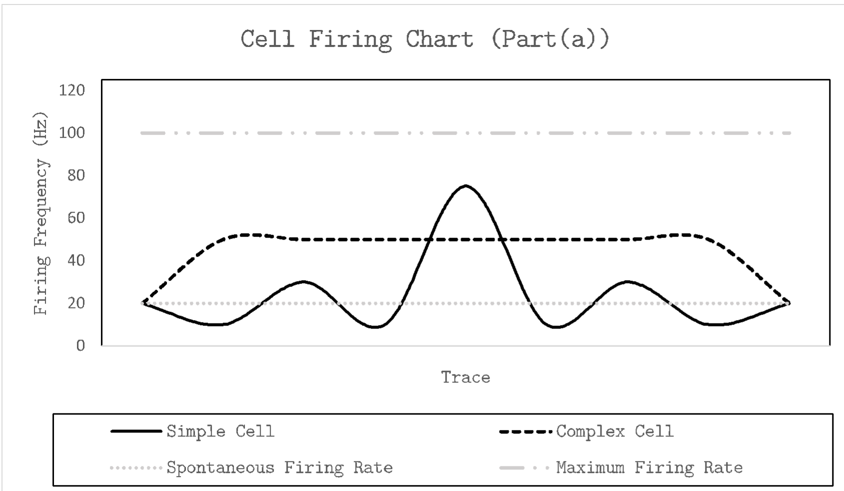
Physical Interpretation

The behavior of the Neuron is similar to that of a low pass filter (LPF) wherein low-frequency inputs are allowed to pass through while high filter inputs are rejected.

Problem 2. Classical Models for V1 Cells

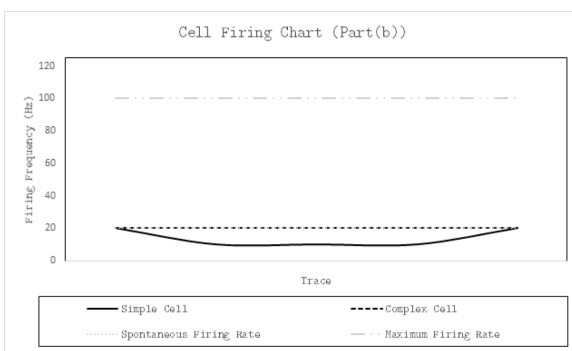
(a). Pattern (a)

Solution:



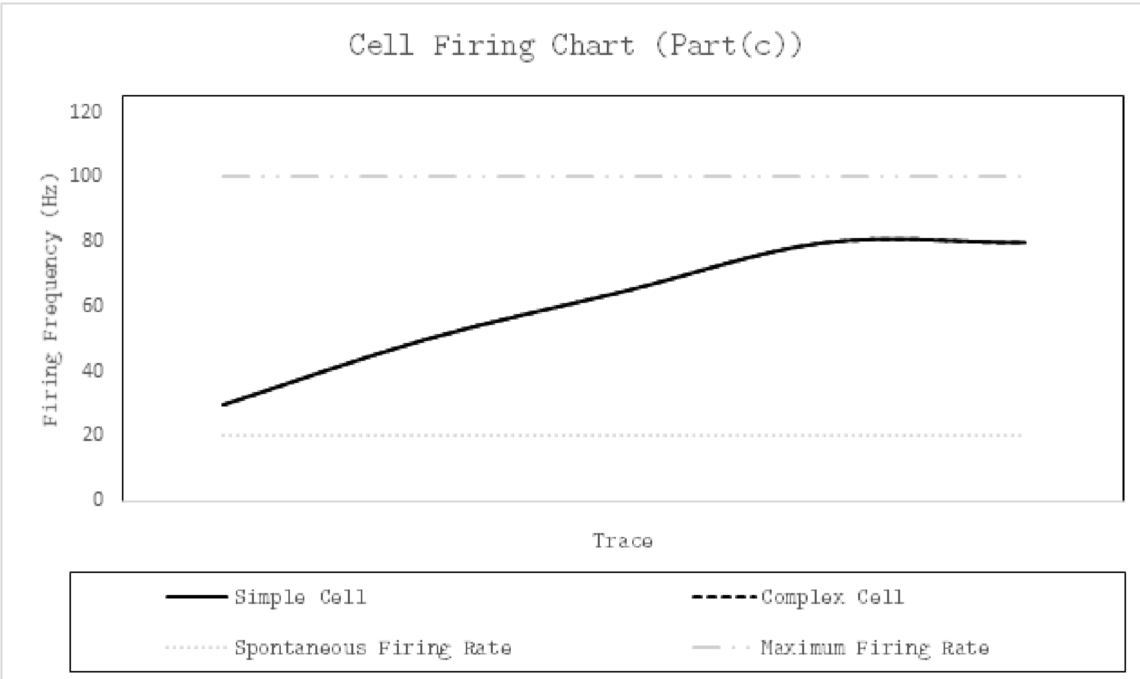
(b). Pattern (b)

Solution: In this case, the complex cell shows no response (spontaneous firing) as the orientation is opposite to the desired orientation. That is,k $v_1^T x = 0$ and $v_2^T x = 0$ resulting in net zero effect.



(c). Pattern (c)

Solution: The firing response for light bars of varying length is similar for both simple and complex cells.



(d). Spatial Invariance

Solution: From the firing charts produced by subparts (a) and (b), it is evident that Complex cells do not respond differently to a change of translation of a signal. That is, the location of light bar in the cell do not change it's firing response. Thus, spatial invariance means the complex cell respond exactly as they would for a sample in different position in time as long as the shape and form remains the same.

Problem 3. Mach Band

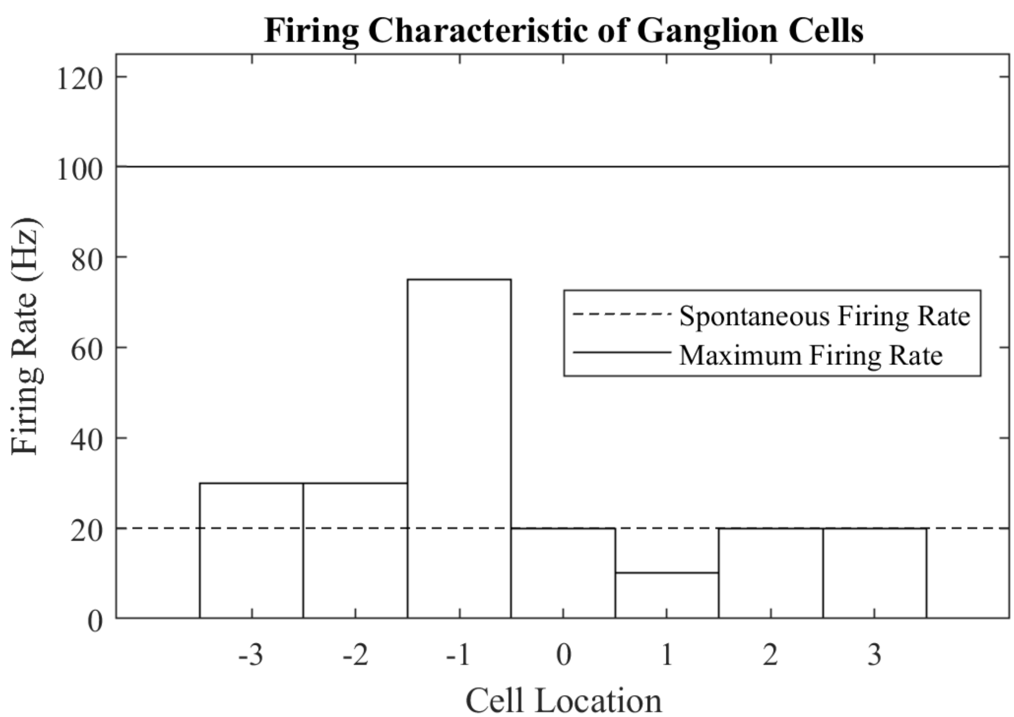
(a). Human Visual System vs Illusion

Solution: Perceptually, the image appears to be a collection of two color blocks separated by an edge. The actual image has a relatively smooth transition of color. The human visual system (HVS) fails at recognizing these small linear gradients and perceives the image as a collection of block of two colors.

Such an response by the HVS can be attributed to the phenomenon of lateral inhibition. Lateral inhibition is the capacity of an excited neuron to reduce the activity of its neighbors. This phenomenon helps increase sharpness and contrast. As the eye senses the image, the difference in contrast between the left and the mid gradient is large enough to trigger an edge detection resulting in the said perception.

(b). Ganglion Cells

Solution:



(c). Mach Band Illusion

Solution: Observing the response furnished in the question above, it is evident that at the firing rate abruptly drops between positions -1 (light) and 0(central). Such an sudden change in frequency is highly characteristic of edges. This effect causes the triggering of the edge detection system in the Human Visual System and subsequent Mach band illusion.

Codes

```
%% (b)Refractory Period
for count=2:length(t);

    if tcounter <= tref                    % reset voltage if spike just occurred
        v(count) = Vre;                    % after spike, cell will spend one sample at reset voltage before integration continues
        tcounter = tcounter + 1;
    else
        dvdt = ((El-v(count-1))/R + Iin(count))/C;    % otherwise, evaluate ode using first order Euler method
        v(count) = v(count-1) + dvdt*DT;
    end

    if(v(count) >= Vth)                    % check for threshold
        v(count) = Vsp;                    % if necessary, generate a spike
        tcounter = 0;
    end

end
%% (c)Varying Constant Current Inputs
Iin = 3*ones(1,length(t));
I = [];
S = [];
for i = 1:31
    I = [I; (i-1)*ones(1,length(t))];
end
%%% Simulate cell %%%
for i = 1:31
    Iin = I(i, :);
    spike = 0;
    for count=2:length(t);

    if tcounter <= tref                    % reset voltage if spike just occurred
        v(count) = Vre;                    % after spike, cell will spend one sample at reset voltage before integration continues
        tcounter = tcounter + 1;
    else
        dvdt = ((El-v(count-1))/R + Iin(count))/C;    % otherwise, evaluate ode using first order Euler method
        v(count) = v(count-1) + dvdt*DT;
    end

    if(v(count) >= Vth)                    % check for threshold
        v(count) = Vsp;                    % if necessary, generate a spike
        tcounter = 0;
        spike = spike + 1;
    end

    end
```

```
    S = [S; spike];
end
%% (d) Code modification for adding noise to part (c)
for i = 1:31
    I = [I; (i-1)*ones(1,length(t)) + 3*randn(1,length(t))];
end
%% (e) Modification for time varying inputs
amp = 4;        %4nA amplitude
freq = [1, 2, 5, 10, 25, 50, 100];       %Frequencies
I = [];
for f = freq
    I = [I, amp*sin(2*pi*f*t)];
end
%%% Simulate cell %%%
for col = 1:7
    Iin = I(:, col);
    spike = 0;
%     v = zeros(length(t),1);          % voltage trace
    for count=2:length(t)

        if((v(count-1) == Vsp))                     % reset voltage if spike just occurred
            v(count) = Vre;                         % after spike, cell will spend one sample at reset voltage before integration continues
        else
            dvdt = ((El-v(count-1))/R + Iin(count))/C;    % otherwise, evaluate ode using first order Euler method
            v(count) = v(count-1) + dvdt*DT*1000;
        end

        if(v(count) >= Vth)                         % check for threshold
            v(count) = Vsp;                         % if necessary, generate a spike
            spike = spike + 1;
        end

    end
    V = [V, v];
    spikes = [spikes, spike];
end
```