Biomedical Imaging

Project #1: Computed Tomography

In this project you will implement the following reconstruction algorithms discussed in class: 1) (simple) backprojection reconstruction, and 2) filtered backprojection reconstruction. As work on this project, you will make several discoveries – please report your findings by comparing the various results and drawing some conclusions.

You will apply the reconstruction techniques to two sets of sinograms. The first sinogram you will construct yourself from a given object. To do so, you will use a portion of the MATLAB code provided (CT_project_template.m). Knowing the object to be reconstructed should be helpful to check (i.e. debug) your code.

The imaging geometry is illustrated in Fig. 1, where the projection θ=0 is shown.

The reconstructed image should be 128 by 128 pixels with a pixel size of 2 by 2 mm. The number of radial samples is 128, with ΔR = 2 mm. The radial samples are symmetrically placed as shown above (for 16 of the 128 lines). Note that the radial samples are at

$$R = \pm\frac{\Delta R}{2}, \pm\frac{3\Delta R}{2}, \pm\frac{5\Delta R}{2}, \ldots$$

The angular sampling covers almost 180 degrees, i.e.,

$$\theta_k = \frac{\pi k}{N_\theta}, \quad k = 0,1,2,\ldots,N_\theta - 1,$$

where N_θ is the number of angular samples. Note the last angle is not at exactly π, but rather ($\pi - \pi/N_\theta$).

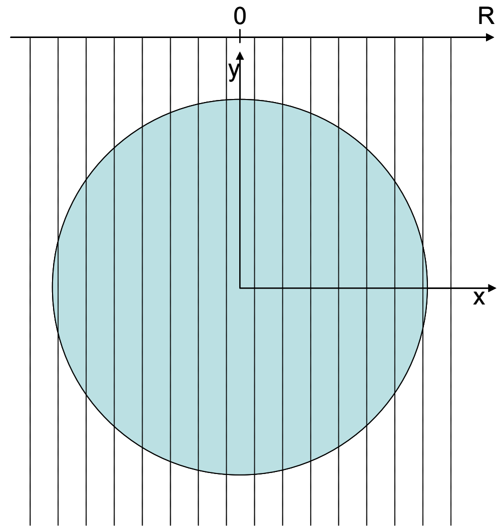

Fig. 1: Imaging geometry

A MATLAB code (CT_template.m) is provided for the known object. Within this well commented code, a disk phantom is created and the sinogram (MATLAB variable "sg1") is computed for this object. The output of this part of the program is shown below (Fig. 2). When you turn in the project, rename this file to CT_YourLastName.m

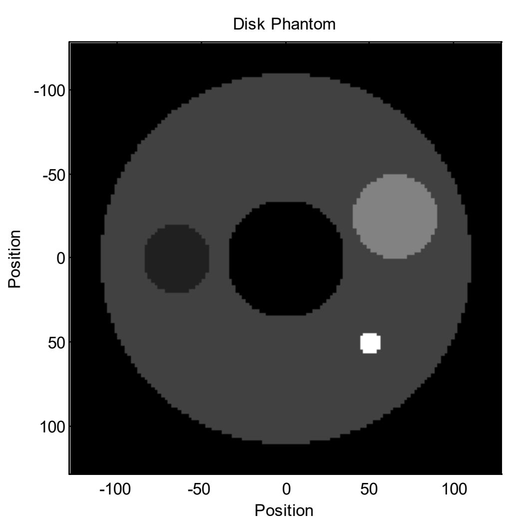

Fig. 2a: Disk phantom

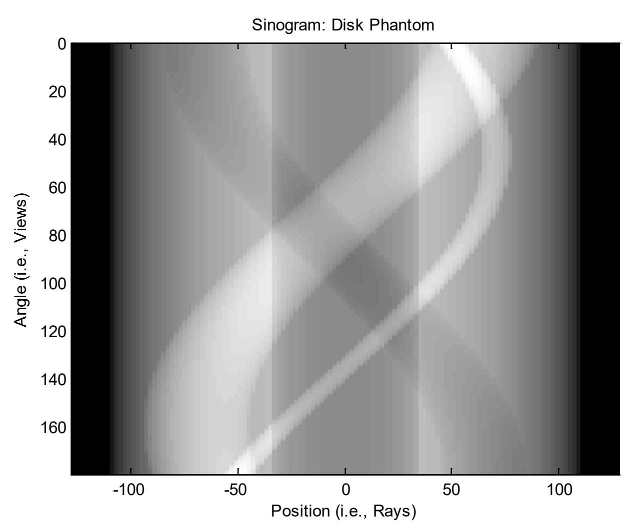

Fig. 2b: Sinogram of the phantom shown in Fig. 2a

The second sinogram (MATLAB file sinogram2.mat with variable sg2) contains projections of an unknown object. There are important changes in this sinogram – it was collected using 256 rays with 1 mm spacing and 540 views.

Rules of the Game:

You will work on the project independently (yes, you can do it). Prepare a written report by answering the questions as requested. Your report will have graphs and images and will be submitted through Canvas. In addition, you will also submit a copy of your final ".m" file. In the report, name your plots and images, and label the axes (unlike on figures here), and write a few sentences commenting on what you did, which equation you utilized, and the appearance of the images. *Please say something interesting!!!*

Working with other students is generally OK if the intent is to provide you with environment where you can discuss questions, etc. Please do not split up the problems and then share solutions. You are expected to do your own work, but when you get stuck, you may ask anyone in the class or the instructor for help. Copying solutions from anyone will be considered a violation of the rules.

If you are having problems, please ask the instructor after class or schedule a meeting by email.

Answer the following questions using sinogram (sg1) from a known object. Before you begin, however, produce the 2 images shown in Fig. 2 by running a part of the provided MATLAB code. The first figure should be an image of the object (circular inclusions in an otherwise homogeneous background), and the second image should be the corresponding sinogram. These images in your report will be used to understand the dynamic range of your grayscale images. (0 points but must be done, 100 points penalty if not done)

A) What is the 0^{th} moment of each projection as a function of the projection angle? That is, what is the area under the curve for each projection as a function of the projection angle? Explain why this result occurs. *(1 figure, 5 points)*

B) Produce an image reconstructed using simple backprojection technique, i.e., without filtering (these images are also called laminograms). One easy way to do this in MATLAB is to backproject at $\theta = 0$ to a temporary image and then rotate that image to the appropriate angle of backprojection. Superimpose these temporary images to generate the final image. *(1 figure, 10 points)*

C) Filter the sinogram using the Ram-Lak filter (and then you can try other filters). Now filter each projection and plot the projection at θ =~ 45 degrees (see template) before and after filtering. Please verify that the projection is real valued (if there are small imaginary components, please set them to zero by using only the real part). Produce the filtered sinogram and display it. Then produce the complete reconstructed image by backprojecting the filtered sinogram. Watch for pixels with negative values – there may be a few on the periphery, and if so, set these negative pixels to zero. Generate an image. Please describe any artifacts that you see in the images and their source. Describe any observed effects due to trying other filters. *(3 figures: 1 plot (showing $\theta = 45°$ projection without and with filtering) and 2 images (1 sinogram and 1 reconstructed image), 15 points)*

D) Plot the profile through the image along the y = –7 mm line (note that this line will intersect 3 inclusions – see Fig. 2a for line position) for both simple and filtered backprojection methods (you may need to normalize the profiles to ensure the amplitudes are on the same scale) and compare them with the original profile. Thus there should be 3 curves in this one figure: the original profile (a cross-sectional line through "phantom"), and two reconstructed profiles (one without filtering and one with filtering). Comment on any inaccuracies or artifacts. Using these plots and images from parts B and C, contrast the spatial resolution versus noise characteristics in these sets of reconstructed images. *(1 figure, 10 points)*

E) Downsample the original sinogram by a factor of 8 (i.e. $N_\theta/8$ angular samples) and reconstruct the filtered backprojection image (use code developed in part C). Comment on any inaccuracies and artifacts. *(1 figure, 10 points)*

Answer the following questions using sinogram from an unknown object.

F) Display the sinogram from the unknown object and examine it. *(1 figure, 0 points but must be done, 100 points penalty if not done)* Can you guess the object?

G) Reconstruct object using both simple and filtered backprojection techniques – can you guess object now? *(2 figures, 20 points)*

<u>What to submit:</u>

There should be a report submitted containing the figures described above with your written answers to the questions. You should also submit your individual, documented MATLAB code. When your code is run (and it will be), it should produce the same images as in your report.

Good luck!