ECE 6786 / BMED 6786 / ECE 4813-MIS / BMED 4813-MIS: Medical Imaging Systems

Name (Last, First) ____

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Signature ____ Date__________

Homework 0

Instructions:

1) Print at least the first page of the homework assignment (this page you are currently reading), ***legibly*** write your name in the space provided (or type before printing) and sign indicating that your homework represents your own work and is copyrighted.
2) No other page of the homework should have your name (nor initials nor anything else that can identify you) – this is needed for a blind grading.
3) Please keep all the pages of your homework in order so that it is easy to follow your solutions and ***make sure all responses are legible***.

Problem 0.1 ____

What is $z = 5 + 3j$ in Euler notation $z = r_0 e^{j\varphi}$?

What is $z = 2.5e^{j(\pi/3)}$ in Cartesian $z = a + jb$ notation? Note that $i = j = (0,1)$ and is a “complex one.”

Problem 0.2 ____

a) Find Fourier transform of impulse function $\delta(t)$.

b) Find Fourier transform of the *rect* function:

$$rect(x) = \begin{cases} A_0 & \left| x \leq \dfrac{L_0}{2} \right| \\ & \\ 0 & \text{otherwise} \end{cases}$$

Note that the width of rect is L_0 (i.e., the width does not have to be equal to one).

c) Using Matlab, demonstrate that the Fourier transform of the *rect* function (above in part b) is the *sinc* function (as you derived). This means you will need to plot the Fourier transform (magnitude and phase) of rect(x) for some value of L_0 (and a sinc(…) function) and demonstrate that the they are the same. (Consider help, rectpuls.m, fft.m, fftshift.m, and abs.m functions in MATLAB). In Matlab, is the Fourier transform of a rect function a variable with real numbers? Is it consistent with the answer you obtained in part (b) of the question? Why or why not? Now shift the rect in the x-direction relative to zero by x_0 (i.e., it is no longer centered at x=0), and take the Fourier transform again. What changed? What did not change? Please summarize your observations in 2-3 short sentences. Include MATLAB code.
Hint: Generally, images consist of real numbers (amplitude) while Fourier transform – complex (written as a + jb or magnitude and phase, as in Problem 0.1). You should always plot magnitude and phase when the Fourier transform is taken.

d) Finally, take the Fourier transform of the *sinc* function – do you get a *rect* function or not? You are welcome to do it analytically too, but MATLAB only is ok. Discuss your findings if any.

Problem 0.3 ____

Give the formula for the Fourier transform of the "triangle" function g(x)

$$g(x) = \begin{cases} 1-|x| & |x| \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Hint: g(x) ~ rect(x) ∗ rect(x), where * denotes convolution and rect is the function generally defined in Problem 0.2. You could use MATLAB here as well, and possibly learn more about convolution. Regardless of how you arrived at your answer, be sure to explain it.

Problem 0.4: ____

Find the ***Fourier transforms*** of the following signals.

a) $s(x,y) = \cos[2\pi(u_o x + v_o y)]$

b) $s(x,y) = \sin[2\pi(u_o x + v_o y)]$

c) For part (a) find the Fourier transform ***both*** analytically and using MATLAB. Produce an image of s(x,y) with u_0=4, v_0=2, and x and y varying from -1 to 1. Then plot the Fourier spectrum and clearly label the axes of the image. Compare and discuss the results in (a), (b) and (c).

Problem 0.5 ____

The ***Nyquist*** sampling periods for 1-D band-limited signals $f(x)$ and $g(x)$ are Δ_f and Δ_g , respectively. Find the ***Nyquist*** sampling periods for the following signals:

(a) $f(x-x_o)$, where x_o is a given constant

(b) $f(x)+g(x)$

(c) $f(x) * f(x)$

(d) $f(x)g(x)$

(e) $|f(x)|$

Justify your answers, do not just state it. Provide an insightful example if needed.

Problem 0.6 ____

Write a MATLAB script file that executes the following steps:

(1) Load your favorite image/photograph into MATLAB. You will develop a script that will read a tiff or jpeg image that you select from your photo library or an image downloaded from the internet. Be considerate of the copyright laws. Also, your image needs to be between 256-512 pixels in each dimension – the image should not be too small or too big (for example, you could use image (a) in **Figure 0.6** – this set of images is also shown in slide 34 of lectures001_Math_2019).

(2) Compute the 2D Fourier Transform (using the fft2 command in MATLAB) to produce image similar to one displayed in panel (d) – this image displays only the absolute value or magnitude of the Fourier transform.

(3) Create a modified version of the Fourier Transform by forcing coefficients within certain regions to be zero (panels (e) and (f) in the figure below). This process is also called "masking" where you mask certain parts of the image. Select the regions carefully to achieve the desired outcome. You will need to use fftshift command in MATLAB – it is very useful here! Note that the (0,0) point in the Fourier transform image is in the center – confirm that it is as you apply the desired mask.

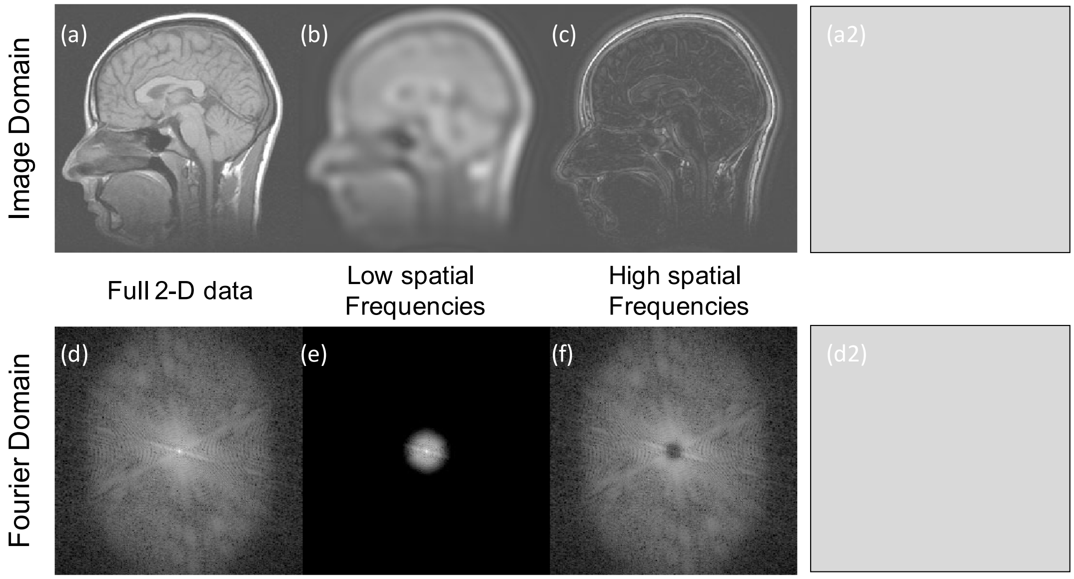

Figure 0.6

(4) Calculate the reconstructed images (panels (b) and (c) in Figure 0.6) using the modified Fourier Transform and plot the result. Here you may need to use the ifftshift function in MATLAB. Discuss the results.

(5) Go back to the original pair of images (a) and (d). Take the Fourier transform image (d) and add zeros on each side of the image. Specifically, make the dimensions of the image (d) 2x larger in each dimension (e.g., for an image with dimensions of 380 × 480 you will make an image with 760 by 960 dimensions) where the central part of the image contains values from original image (d). Now take the inverse Fourier transform (ifft2) of the new image (let's call it the (d2) panel) and produce image (a2). Compare (a) and (a2). Discuss your results and observations, commenting on anything *interesting* you notice.

(6) BONUS: the image (d) can be represented in Euler notation $z = r_0 e^{i\varphi}$ (see Problem 0.1). Modify the phase of the image (d) in any way you like (make it zero, or scale it by factor of 2, or whatever else you would like), and then take ifft2 of the resulting image (d3) – how does (a3) compare to (a)? Explain.

In your homework you should provide; a) commented copy of your MATLAB code, b) imaging results arranged as a 2 × 4 figure (or 2 × 5 if you did the bonus question).

Problem 0.7 ____

An organ with a tumor is imaged. In the resulting image, the organ has intensity I_o, and the tumor has intensity $I_t>I_o$. Which of the following methods can improve the local contrast if the organ is treated as background:

(a) Multiplying the image by a constant α.

(b) Subtracting a constant $0<I_s<I_o$ from the image.

As always, discuss your results.

Problem 0.8 ____

BONUS: Come up with a demonstration of aliasing using MATLAB. Provide a MATLAB script that will generate figures to demonstrate the phenomenon.