ANALYSIS ON FINAL OUTPUT OF DIRECT COMP PAPER

→ The aim here is to understand the structure of H_Q

→ The structure of H_Q proposed in paper, should satisfy

i) $H_Q^T B H_Q = Q$

ii) $[M_j(2)]^2 + [M_j(3)]^2 = M_j(4)$. $[\because x_j^2 + y_j^2 = m_j^T m_j]$ [2D]

after obtaining $M_j = H_Q H_S H_M \hat{M}_j$ $(\because M_j = H\hat{M}_j)$.

→ This analysis is based on microphone constraints only.

→ We already checked $H_Q^T B H_Q = Q$ condition, & it's working, but we need to check (ii) condition.

$\rightarrow M_j = H\hat{M}_j = H_a H_s H_m \hat{M}_j$ — ①

$\rightarrow S_i = (H^{-1})^T \hat{S}_i = (H_a^{-1})^T (H_s^{-1})^T (H_m^{-1})^T \hat{S}_i$ — ②

$\rightarrow H_s = \begin{bmatrix} I & h_s \\ 0 & \end{bmatrix}^{-1}$ & first element of $h_s = 0$.

Let $h_s = \begin{bmatrix} 0 \\ p \\ q \\ r \end{bmatrix}$

$\boxed{\begin{matrix}[h_s^T \hat{S}_i = 1] \\ \text{another condition on } h_s.\end{matrix}}$ → eqn ⓐ

$$\Rightarrow H_s = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & p \\ 0 & 0 & 1 & q \\ 0 & 0 & 0 & r \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -p/r & -q/r & 1/r \end{bmatrix}^T$$

$$H_s = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -p/r \\ 0 & 0 & 1 & -q/r \\ 0 & 0 & 0 & 1/r \end{bmatrix} \quad \text{— ③}$$

$\rightarrow H_m = \begin{bmatrix} h_M^T \\ 0 \quad I \end{bmatrix}$

$\boxed{\begin{matrix}[h_m^T \hat{M}_j = 1] \\ \text{Condition on } h_m\end{matrix}}$ → eqn ⓑ

Let $h_M = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$

$$\Rightarrow H_M = \begin{bmatrix} a & b & c & d \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{— ④}$$

$$H_Q = \begin{bmatrix} 1 & 0 \quad 0 & 0 \\ t & RK & 0 \\ t^Tt - Q_{11} & 2(t^TRK - [Q_{12} \ Q_{13}]) & 1 \end{bmatrix} \quad - (5)$$

→ Let $\hat{M}_j = \begin{bmatrix} \hat{M}_j(1) \\ \hat{M}_j(2) \\ \hat{M}_j(3) \\ \hat{M}_j(4) \end{bmatrix}$, where, $\hat{M}_j(4) = \left(\hat{M}_j(2)\right)^2 + \left(\hat{M}_j(3)\right)^2$ — (6)

→ Now consider equation (1)

$$M_j = H_Q H_S H_M \hat{M}_j$$

$$= H_Q \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -p/\gamma \\ 0 & 0 & 1 & -q/\gamma \\ 0 & 0 & 0 & 1/\gamma \end{bmatrix} \begin{bmatrix} a & b & c & d \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \hat{M}_j$$

$$= H_Q \cdot \begin{bmatrix} a & b & c & d \\ 0 & 1 & 0 & -p/\gamma \\ 0 & 0 & 1 & -q/\gamma \\ 0 & 0 & 0 & 1/\gamma \end{bmatrix} \begin{bmatrix} \hat{M}_j(1) \\ \hat{M}_j(2) \\ \hat{M}_j(3) \\ \hat{M}_j(4) \end{bmatrix}$$

$$= H_Q \cdot \begin{bmatrix} 1 \\ \hat{M}_j(2) - \frac{p}{\gamma}\left(\hat{M}_j(4)\right) \\ \hat{M}_j(3) - \frac{q}{\gamma}\left(\hat{M}_j(4)\right) \\ \frac{1}{\gamma}\left(\hat{M}_j(4)\right) \end{bmatrix}$$

$$
= \begin{bmatrix} 1 & 0\ 0 & 0 \\ t & RK & \begin{matrix}0\\0\end{matrix} \\ t^T t - Q_{11} & 2\left(t^T RK - \begin{bmatrix} Q_{12} & Q_{13} \end{bmatrix}\right) & 1 \end{bmatrix} \begin{bmatrix} 1 \\ \hat{M}_j(2) - \frac{p}{\gamma}\left(\hat{M}_j(4)\right) \\ \hat{M}_j(3) - \frac{q}{\gamma}\left(\hat{M}_j(4)\right) \\ \frac{1}{\gamma}\left(\hat{M}_j(4)\right) \end{bmatrix}
$$

$$
M_j = \begin{bmatrix} 1 \\ t + RK \begin{bmatrix} \hat{M}_j(2) - \frac{p}{\gamma}\left(\hat{M}_j(4)\right) \\ \hat{M}_j(3) - \frac{q}{\gamma}\left(\hat{M}_j(4)\right) \end{bmatrix} \\ t^T t - Q_{11} + 2\left(t^T RK - \begin{bmatrix} Q_{12} & Q_{13} \end{bmatrix}\right) \begin{bmatrix} \hat{M}_j(2) - \frac{p}{\gamma}\left(\hat{M}_j(4)\right) \\ \hat{M}_j(3) - \frac{q}{\gamma}\left(\hat{M}_j(4)\right) \end{bmatrix} + \frac{1}{\gamma}\left(\hat{M}_j(4)\right) \end{bmatrix}
$$

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From the structure of M_j, we have

$$\underbrace{\begin{bmatrix} M_j(2) & M_j(3)\end{bmatrix}\begin{bmatrix} M_j(2)\\ M_j(3)\end{bmatrix}}_{LHS} = \underbrace{M_j(4)}_{RHS}.$$

$$LHS = \left[t + RK\begin{bmatrix} \hat{M}_j(2) - \frac{p}{\gamma}\left(\hat{M}_j(4)\right)\\ \hat{M}_j(3) - \frac{q}{\gamma}\left(\hat{M}_j(4)\right)\end{bmatrix}\right]^T \left[t + RK\begin{bmatrix} \hat{M}_j(2) - \frac{p}{\gamma}\left(\hat{M}_j(4)\right)\\ \hat{M}_j(3) - \frac{q}{\gamma}\left(\hat{M}_j(4)\right)\end{bmatrix}\right]$$

$$= t^T t + t^T RK\begin{bmatrix} \hat{M}_j(2) - \frac{p}{\gamma}\left(\hat{M}_j(4)\right)\\ \hat{M}_j(3) - \frac{q}{\gamma}\left(\hat{M}_j(4)\right)\end{bmatrix}$$

$$+ \begin{bmatrix} \hat{M}_j(2) - \frac{p}{\gamma}\left(\hat{M}_j(4)\right) & \hat{M}_j(3) - \frac{q}{\gamma}\left(\hat{M}_j(4)\right)\end{bmatrix} K^T R^T t \; +$$

$$\begin{bmatrix} \hat{M}_j(2) - \frac{p}{\gamma}\left(\hat{M}_j(4)\right) & \hat{M}_j(3) - \frac{q}{\gamma}\left(\hat{M}_j(4)\right)\end{bmatrix} \underbrace{K^T \cancel{R} K}_{\begin{bmatrix} Q_{22} & Q_{32}\\ Q_{23} & Q_{33}\end{bmatrix}} \begin{bmatrix} \hat{M}_j(2) - \frac{p}{\gamma}\left(\hat{M}_j(4)\right)\\ \hat{M}_j(3) - \frac{q}{\gamma}\left(\hat{M}_j(4)\right)\end{bmatrix}$$

$$RHS = t^T t - Q_{11} + 2\left(t^T R K - \begin{bmatrix} Q_{12} & Q_{13} \end{bmatrix}\right) \begin{bmatrix} \hat{M}_j(2) - \frac{p}{\gamma}\left(\hat{M}_j(4)\right) \\ \hat{M}_j(3) - \frac{q}{\gamma}\left(\hat{M}_j(4)\right) \end{bmatrix}$$

$$+ \frac{1}{\gamma}\left(\hat{M}_j(4)\right)$$

Now, RHS − LHS =

$$-Q_{11} + t^T R K \begin{bmatrix} \hat{M}_j(2) - \frac{p}{\gamma}\left(\hat{M}_j(4)\right) \\ \hat{M}_j(3) - \frac{q}{\gamma}\left(\hat{M}_j(4)\right) \end{bmatrix} - 2 \begin{bmatrix} Q_{12} & Q_{13} \end{bmatrix} \begin{bmatrix} \hat{M}_j(2) - \frac{p}{\gamma}\left(\hat{M}_j(4)\right) \\ \hat{M}_j(3) - \frac{q}{\gamma}\left(\hat{M}_j(4)\right) \end{bmatrix}$$

$\updownarrow$ same (cancel).

$$+ \frac{1}{\gamma}\left(\hat{M}_j(4)\right) + \begin{bmatrix} \hat{M}_j(2) - \frac{p}{\gamma}\left(\hat{M}_j(4)\right) & \hat{M}_j(3) - \frac{q}{\gamma}\left(\hat{M}_j(4)\right) \end{bmatrix} K^T R^T t$$

$$- \begin{bmatrix} \hat{M}_j(2) - \frac{p}{\gamma}\left(\hat{M}_j(4)\right) & \hat{M}_j(3) - \frac{q}{\gamma}\left(\hat{M}_j(4)\right) \end{bmatrix} \begin{bmatrix} Q_{22} & Q_{32} \\ Q_{23} & Q_{33} \end{bmatrix} \begin{bmatrix} \hat{M}_j(2) - \frac{p}{\gamma}\left(\hat{M}_j(4)\right) \\ \hat{M}_j(3) - \frac{q}{\gamma}\left(\hat{M}_j(4)\right) \end{bmatrix}$$

→ consider the eqn $\hat{M}_j'^{T} Q \hat{M}_j' = 0$, — (7)

where, $\hat{M}_j' = H_s H_m \hat{M}_j$; $Q = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & -\frac{1}{2} \\ Q_{12} & Q_{22} & Q_{23} & 0 \\ Q_{13} & Q_{23} & Q_{33} & 0 \\ -\frac{1}{2} & 0 & 0 & 0 \end{bmatrix}$

$\Downarrow$ From previous computations

$$\begin{bmatrix} 1 \\ \hat{M}_j(2) - \frac{p}{\gamma}\left(\hat{M}_j(4)\right) \\ \hat{M}_j(3) - \frac{q}{\gamma}\left(\hat{M}_j(4)\right) \\ \frac{1}{\gamma}\left(\hat{M}_j(4)\right) \end{bmatrix}$$

Eqn (7) $\Rightarrow LHS = \hat{M}_j'^{T} Q \hat{M}_j'$

$$= \begin{bmatrix} 1 & \hat{M}_j(2) - \frac{p}{\gamma}\left(\hat{M}_j(4)\right) & \hat{M}_j(3) - \frac{q}{\gamma}\left(\hat{M}_j(4)\right) & \frac{1}{\gamma}\left(\hat{M}_j(4)\right) \end{bmatrix} \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & -\frac{1}{2} \\ Q_{12} & Q_{22} & Q_{23} & 0 \\ Q_{13} & Q_{23} & Q_{33} & 0 \\ -\frac{1}{2} & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} Q_{11} + \left[\hat{M}_j(2) - \frac{p}{\gamma}\left(\hat{M}_j(4)\right) \quad \hat{M}_j(3) - \frac{q}{\gamma}\left(\hat{M}_j(4)\right)\right] \begin{bmatrix} Q_{12} \\ Q_{13} \end{bmatrix} - \frac{1}{2\gamma}\left(\hat{M}_j(4)\right) \\ \left[\begin{bmatrix} Q_{12} \\ Q_{13} \end{bmatrix} + \left[\left[\hat{M}_j(2) - \frac{p}{\gamma}\left(\hat{M}_j(4)\right) \quad \hat{M}_j(3) - \frac{q}{\gamma}\left(\hat{M}_j(4)\right)\right] \begin{bmatrix} Q_{22} & Q_{23} \\ Q_{23} & Q_{33} \end{bmatrix}\right]\right]^T \\ -\frac{1}{2} \end{bmatrix} \odot \hat{M}_j'$$

$\odot$ → dot product

$\Leftarrow$ $A^T A = A \odot A$

$A \rightarrow$ vector

$$\Rightarrow Q_{11} + \begin{bmatrix} \hat{M}_j(2) - \frac{p}{\gamma}\left(\hat{M}_j(4)\right) & \hat{M}_j(3) - \frac{q}{\gamma}\left(\hat{M}_j(4)\right) \end{bmatrix} \begin{bmatrix} Q_{12} \\ Q_{13} \end{bmatrix} - \frac{1}{2\gamma}\left(\hat{M}_j(4)\right)$$

$$+ \begin{bmatrix} Q_{12} & Q_{13} \end{bmatrix} \begin{bmatrix} \hat{M}_j(2) - \frac{p}{\gamma}\left(\hat{M}_j(4)\right) \\ \hat{M}_j(3) - \frac{q}{\gamma}\left(\hat{M}_j(4)\right) \end{bmatrix}$$

$$+ \begin{bmatrix} \hat{M}_j(2) - \frac{p}{\gamma}\left(\hat{M}_j(4)\right) & \hat{M}_j(3) - \frac{q}{\gamma}\left(\hat{M}_j(4)\right) \end{bmatrix} \begin{bmatrix} Q_{22} & Q_{23} \\ Q_{23} & Q_{33} \end{bmatrix} \begin{bmatrix} \hat{M}_j(2) - \frac{p}{\gamma}\left(\hat{M}_j(4)\right) \\ \hat{M}_j(3) - \frac{q}{\gamma}\left(\hat{M}_j(4)\right) \end{bmatrix}$$

$$- \frac{1}{2\gamma}\left(\hat{M}_j(4)\right). = 0$$

→ Substitute this in RHS−LHS.

$\Rightarrow$ RHS−LHS $\underline{\underline{=0}}$.

therefore, condition (i) & (ii) both are satisfied

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