$\rightarrow M_j = H\hat{M}_j = H_a H_s H_m \hat{M}_j$ — ①

$\rightarrow S_i = (H^{-1})^T \hat{S}_i = (H_a^{-1})^T (H_s^{-1})^T (H_m^{-1})^T \hat{S}_i$ — ②

$\rightarrow H_s = \begin{bmatrix} I & h_s \\ 0 & \end{bmatrix}^{-1}$ & first element of $h_s = 0$.

Let $h_s = \begin{bmatrix} 0 \\ p \\ q \\ r \end{bmatrix}$

$$\boxed{\begin{gathered} [h_s^T \hat{S}_i = 1] \\ \text{another condition on } h_s. \end{gathered}} \rightarrow \text{eqn (a)}$$

$$\Rightarrow H_s = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & p \\ 0 & 0 & 1 & q \\ 0 & 0 & 0 & r \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -p/r & -q/r & 1/r \end{bmatrix}^T$$

$$H_s = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -p/r \\ 0 & 0 & 1 & -q/r \\ 0 & 0 & 0 & 1/r \end{bmatrix} \text{ — ③}$$

$\rightarrow H_m = \begin{bmatrix} h_M^T \\ 0 \quad I \end{bmatrix}$

$$\boxed{\begin{gathered} [h_m^T \hat{M}_j = 1] \\ \text{Condition on } h_m \end{gathered}} \rightarrow \text{eqn (b)}$$

Let $h_M = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$

$$\Rightarrow H_M = \begin{bmatrix} a & b & c & d \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ — ④}$$

$$H_Q = \begin{bmatrix} 1 & 0 \; 0 & 0 \\ t & RK & 0 \\ t^T t - Q_{11} & 2(t^T RK - [Q_{12} \; Q_{13}]) & 1 \end{bmatrix} \quad -(5)$$

$\rightarrow$ Let $\hat{M}_j = \begin{bmatrix} \hat{M}_j(1) \\ \hat{M}_j(2) \\ \hat{M}_j(3) \\ \hat{M}_j(4) \end{bmatrix}$, where, $\hat{M}_j(4) = \left(\hat{M}_j(2)\right)^2 + \left(\hat{M}_j(3)\right)^2$ — (6)

$\rightarrow$ Now consider equation (1)

$$M_j = H_Q H_S H_M \hat{M}_j$$

$$= H_Q \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -p/r \\ 0 & 0 & 1 & -q/r \\ 0 & 0 & 0 & 1/r \end{bmatrix} \begin{bmatrix} a & b & c & d \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \hat{M}_j$$

$$= H_Q \cdot \begin{bmatrix} a & b & c & d \\ 0 & 1 & 0 & -p/r \\ 0 & 0 & 1 & -q/r \\ 0 & 0 & 0 & 1/r \end{bmatrix} \begin{bmatrix} \hat{M}_j(1) \\ \hat{M}_j(2) \\ \hat{M}_j(3) \\ \hat{M}_j(4) \end{bmatrix}$$

$$= H_Q \cdot \begin{bmatrix} 1 \\ \hat{M}_j(2) - \frac{p}{r}\left(\hat{M}_j(4)\right) \\ \hat{M}_j(3) - \frac{q}{r}\left(\hat{M}_j(4)\right) \\ \frac{1}{r}\left(\hat{M}_j(4)\right) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0\ 0 & 0 \\ t & RK & \begin{matrix}0\\0\end{matrix} \\ t^T t - Q_{11} & 2(t^T RK - [Q_{12}\ Q_{13}]) & 1 \end{bmatrix} \begin{bmatrix} \hat{M}_j(2) - \frac{p}{\gamma}(\hat{M}_j(4)) \\ \hat{M}_j(3) - \frac{q}{\gamma}(\hat{M}_j(4)) \\ \frac{1}{\gamma}(\hat{M}_j(4)) \end{bmatrix}$$

$$M_j = \begin{bmatrix} 1 \\ t + RK \begin{bmatrix} \hat{M}_j(2) - \frac{p}{\gamma}(\hat{M}_j(4)) \\ \hat{M}_j(3) - \frac{q}{\gamma}(\hat{M}_j(4)) \end{bmatrix} \\ t^T t - Q_{11} + 2(t^T RK - [Q_{12}\ Q_{13}]) \begin{bmatrix} \hat{M}_j(2) - \frac{p}{\gamma}(\hat{M}_j(4)) \\ \hat{M}_j(3) - \frac{q}{\gamma}(\hat{M}_j(4)) \end{bmatrix} + \frac{1}{\gamma}(\hat{M}_j(4)) \end{bmatrix}$$

consider $K \begin{bmatrix} \hat{M}_j(2) - \frac{p}{\gamma}(\hat{M}_j(4)) \\ \hat{M}_j(3) - \frac{q}{\gamma}(\hat{M}_j(4)) \end{bmatrix}$ — (7)

where, $\hat{M}_j(4) = (\hat{M}_j(2))^2 + (\hat{M}_j(3))^2$

$$\hat{M}_j(4) = \begin{bmatrix} \hat{M}_j(2) & \hat{M}_j(3) \end{bmatrix} \begin{bmatrix} \hat{M}_j(2) \\ \hat{M}_j(3) \end{bmatrix} \quad \text{— (8)}$$

Eqn (7)

$\Downarrow$

$$K \left[\begin{bmatrix} \hat{M}_j(2) \\ \hat{M}_j(3) \end{bmatrix} - \begin{bmatrix} p/\gamma \\ q/\gamma \end{bmatrix} \hat{M}_j(4) \right]$$

Now sub. eqn (8)

$$\Rightarrow K\left[\begin{bmatrix}\hat{M}_j(2)\\ \hat{M}_j(3)\end{bmatrix} - \begin{bmatrix}p/\gamma\\ q/\gamma\end{bmatrix}\begin{bmatrix}\hat{M}_j(2) & \hat{M}_j(3)\end{bmatrix}\begin{bmatrix}\hat{M}_j(2)\\ \hat{M}_j(3)\end{bmatrix}\right]$$

$$= \underbrace{K\left[I - \begin{bmatrix}p/\gamma\\ q/\gamma\end{bmatrix}\begin{bmatrix}\hat{M}_j(2) & \hat{M}_j(3)\end{bmatrix}\right]}_{\text{Matrix "A"}}\begin{bmatrix}\hat{M}_j(2)\\ \hat{M}_j(3)\end{bmatrix}$$

$$\text{eqn (7)} = A\begin{bmatrix}\hat{M}_j(2)\\ \hat{M}_j(3)\end{bmatrix} \quad \text{—— (9)}$$

Now sub eqn (9) in M_j

$$M_j = \begin{bmatrix} 1 \\ t + RA\begin{bmatrix}\hat{M}_j(2)\\ \hat{M}_j(3)\end{bmatrix} \\ t^Tt - Q_{11} + 2t^TRA\begin{bmatrix}\hat{M}_j(2)\\ \hat{M}_j(3)\end{bmatrix} - 2\begin{bmatrix}Q_{12} & Q_{13}\end{bmatrix}K^{-1}A\begin{bmatrix}\hat{M}_j(2)\\ \hat{M}_j(3)\end{bmatrix} \\ + \frac{1}{\gamma}\left[(\hat{M}_j(2))^2 + (\hat{M}_j(3))^2\right] \end{bmatrix}$$

→ Now according to structure of M_j,

$$\begin{bmatrix} \hat{M}_j(2) & \hat{M}_j(3) \end{bmatrix} \begin{bmatrix} M_j(2) \\ M_j(3) \end{bmatrix} = M_j(4).$$

$$LHS = \left[t + RA \begin{bmatrix} \hat{M}_j(2) \\ \hat{M}_j(3) \end{bmatrix} \right]^T \left[t + RA \begin{bmatrix} \hat{M}_j(2) \\ \hat{M}_j(3) \end{bmatrix} \right]$$

$$= t^T t + t^T RA \begin{bmatrix} \hat{M}_j(2) \\ \hat{M}_j(3) \end{bmatrix} + \begin{bmatrix} \hat{M}_j(2) & \hat{M}_j(3) \end{bmatrix} A^T R^T t$$

$$+ \begin{bmatrix} \hat{M}_j(2) & \hat{M}_j(3) \end{bmatrix} A^T A \begin{bmatrix} \hat{M}_j(2) \\ \hat{M}_j(3) \end{bmatrix}$$

$$RHS = t^T t - Q_{11} + 2 t^T RA \begin{bmatrix} \hat{M}_j(2) \\ \hat{M}_j(3) \end{bmatrix} - 2 \begin{bmatrix} Q_{12} & Q_{13} \end{bmatrix} K^{-1} A \begin{bmatrix} \hat{M}_j(2) \\ \hat{M}_j(3) \end{bmatrix}$$

$$+ \frac{1}{\gamma} \left[(\hat{M}_j(2))^2 + (\hat{M}_j(3))^2 \right]$$

$$RHS - LHS = -Q_{11} + t^T RA \begin{bmatrix} \hat{M}_j(2) \\ \hat{M}_j(3) \end{bmatrix} - \begin{bmatrix} \hat{M}_j(2) & \hat{M}_j(3) \end{bmatrix} A^T R^T t$$

$$- 2 \begin{bmatrix} Q_{12} & Q_{13} \end{bmatrix} K^{-1} A \begin{bmatrix} \hat{M}_j(2) \\ \hat{M}_j(3) \end{bmatrix} + \frac{1}{\gamma} \left[(\hat{M}_j(2))^2 + (\hat{M}_j(3))^2 \right]$$

$$- \begin{bmatrix} \hat{M}_j(2) & \hat{m}(3) \end{bmatrix} A^T A \begin{bmatrix} \hat{M}_j(2) \\ \hat{M}_j(3) \end{bmatrix}$$

$$= -Q_{11} - 2 \begin{bmatrix} Q_{12} & Q_{13} \end{bmatrix} K^{-1} A \begin{bmatrix} \hat{M}_j(2) \\ \hat{M}_j(3) \end{bmatrix}$$

$$+ \frac{1}{\gamma} \begin{bmatrix} \hat{M}_j(2) & \hat{M}_j(3) \end{bmatrix} \begin{bmatrix} \hat{M}_j(2) \\ \hat{M}_j(3) \end{bmatrix} - \begin{bmatrix} \hat{m}_j(2) & \hat{M}_j(3) \end{bmatrix} A^T A \begin{bmatrix} \hat{M}_j(2) \\ \hat{M}_j(3) \end{bmatrix}$$

$$= -Q_{11} - 2\begin{bmatrix} Q_{12} & Q_{13} \end{bmatrix}\begin{bmatrix} \hat{M}_j(2) \\ \hat{M}_j(3) \end{bmatrix} + 2\begin{bmatrix} Q_{12} & Q_{13} \end{bmatrix}\begin{bmatrix} p/\gamma \\ q/\gamma \end{bmatrix}\left[(\hat{M}_j(2))^2 + (\hat{M}_j(3))^2\right]$$

$$+ \frac{1}{\gamma}\left[(\hat{M}_j(2))^2 + (\hat{M}_j(3))^2\right]$$

$$- \begin{bmatrix} \hat{M}_j(2) & \hat{M}_j(3) \end{bmatrix}\left[I - \begin{bmatrix} \hat{M}_j(2) \\ \hat{M}_j(3) \end{bmatrix}\begin{bmatrix} p/\gamma & q/\gamma \end{bmatrix}\right]\begin{bmatrix} Q_{22} & Q_{32} \\ Q_{23} & Q_{33} \end{bmatrix}\left[I - \begin{bmatrix} p/\gamma & q/\gamma \end{bmatrix}\begin{bmatrix} \hat{M}_j(2) \\ \hat{M}_j(3) \end{bmatrix}\right]\begin{bmatrix} \hat{M}_j(2) \\ \hat{M}_j(3) \end{bmatrix}$$

$$\left[\because A = K\left[I - \begin{bmatrix} p/\gamma & q/\gamma \end{bmatrix}\begin{bmatrix} \hat{M}_j(2) \\ \hat{M}_j(3) \end{bmatrix}\right]\right.$$

which is not '0' in general.