

## $h_s$ & $h_m$ : DIFFERENT CASES & OUTPUTS

→ In section 2.3 of direct computation paper, we compute

$H_s$  &  $H_m$  as follows

$$H_m = \begin{bmatrix} h_m^T \\ 0 \quad I \end{bmatrix} ; H_s = \begin{bmatrix} I & h_s \\ 0 & \end{bmatrix}$$

$$\Downarrow$$

$$h_m^T \hat{M}_j = 1$$

To compute  $h_m$

$$\Downarrow$$

$$h_s^T \hat{S}_i = 1$$

To compute  $h_s$

→ In paper, It is stated that, to remain the first row of  ~~$H_m$~~  unchanged, first element of  $h_s$  has to be zero.  
because,  $H_m \hat{M}_j$

$$S_i^T M_j = \hat{S}_i^T H^{-1} H \hat{M}_j$$

$$= \hat{S}_i^T H_m^{-1} \underline{H_s^{-1}} H_o^{-1} \cancel{H_o} H_s \underline{H_m} \hat{M}_j$$

$$\begin{bmatrix} I & h_s \\ 0 & \end{bmatrix} \dots \begin{bmatrix} 1 & 1 & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

$\therefore$  First element has to be zero, to avoid changing 1st row of  $H_m \hat{M}_j$

$\rightarrow$  But there is also other observation, i.e., last element of  $h_m$  should be zero as well to ~~remain~~ avoid changing column of  $\hat{S}_i^T$ , because

$$\hat{S}_i^T M_j = \hat{S}_i^T H^{-1} H \hat{M}_j$$

$$= \hat{S}_i^T H_m^{-1} \underline{H_s^{-1}} H_o^{-1} \cancel{H_o} H_s \underline{H_m} \hat{M}_j$$

$$\begin{bmatrix} \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \dots \begin{bmatrix} h_m^T \\ 0 & I \end{bmatrix}$$

$\therefore$  Last element of  $h_m^T$  has to be '0' to avoid changing last column of  $\hat{S}_i^T$ .

$\rightarrow$  on this observation, I considered different cases

& compared their o/p's. & those cases ~~are~~ are:

i) Including  $h_s$  constraint & excluding  $h_m$  constraint  
(the one in paper)

ii) Incl.  $h_s$ , Incl.  $h_m$

iii) excl.  $h_s$ , Incl.  $h_m$

iv) excl.  $h_s$ , excl.  $h_m$ .

→ The o/p's for these cases are based on "direct-comp-paper-script1", where we calculate the total relative error, part-by-part rel. errors w.r.t true EDMs, & following are the observations.

① 1 Source, 4 virtual sources, 8 Mics

$$S = 340^\circ [3 \ 2]$$

$$M = 340^\circ [1 \ 6; 2 \ 5; 4 \ 5; 4 \ 4; 5 \ 1; 6 \ 6; 7 \ 1; 7 \ 3]$$

$$T_i = [2; 2; 2; 2; 2].$$

Cases	Total Rel. error	mic-mic rel. error	Source-mic rel. error	Source-Source Rel. error
S X	0.4158	0.7500	0.5814	$2.8 \times 10^{-15}$
S M	0.4257	0.6806	0.5826	0.1441
X M	24.1669	0.9881	33.2367	11.8152
X X	24.1672	0.9881	33.2371	11.8154

Rounded ones - lower values.

& In ~~(X M)~~ (X M) & (X X) case,  $Q_{mid}$  is not PSD; so not valid.

② 1 Source, 4 V.S, 12 Mics

$$S = 340^\circ [3 \ 2]$$

$$M = 340^\circ [1 \ 6; 2 \ 5; 4 \ 5; 4 \ 4; 5 \ 1; 6 \ 6; 7 \ 1; 7 \ 3; 1 \ 1; 2 \ 2; 5 \ 7; 7 \ 5]$$

$$T_i = [2; 2; 2; 2; 2].$$



Cases	Total Rel. error	Mic-Mic Rel. error	Source-Mic part Rel. error	Source-Source
S X	0.4853	0.7500	0.6043	$3 \times 10^{-15}$
S M	0.4973	0.6764	0.6086	0.1965

(X M), (X X) → Invalid cases.

③ 1 Source, 4 V.S, 16 Mics

Same S, VS, M as in case ② + 4 Mics  
 $[1\ 4; 2\ 7; 5\ 5; 6\ 3]$

Cases	Total Rel. error	Mic-Mic	Source-Mic	Source-Source
S X	0.5088	0.7500	0.5981	$1.2 \times 10^{-15}$
S M	0.5146	0.6789	0.6010	0.1911

(X, M), (X X) → Invalid cases

Conclusion:

if (S X), (S M) are the only valid cases, & for any no. of mics, Total Rel. error is less for (S X) [the one proposed in paper], but (S M) is estimating the Mic-mic part of the EDM well (less Rel. error).