7.3. a) $y[n] = 2.5y[n-1] - y[n-2] + x[n] - 5x[n-1] + 6x[n-2]$

$$Y(z) = 2.5\,Y(z)z^{-1} - Y(z)z^{-2} + X(z) - 5X(z).z^{-1} + 6X(z)z^{-2}$$

$$Y(z)[1 - 2.5z^{-1} + z^{-2}] = X(z)[1 - 5z^{-1} + 6z^{-2}]$$

$$\therefore \frac{Y(z)}{X(z)} = \frac{1-5z^{-1}+6z^{-2}}{1-2.5z^{-1}+z^{-2}}$$

Zeros are root of $1 - 5z^{-1} + 6z^{-2} = 0$

$$1 - 3z^{-1} - 2z^{-1} + 6z^{-2} = 0$$

$$(1-3z^{-1}) - 2z^{-1}(1-3z^{-1}) = 0$$

$$(1-3z^{-1})(1-2z^{-1}) = 0$$

$$z^{-1} = \frac{1}{3}, \quad z^{-1} = \frac{1}{2}$$

$$\text{or } z = 3, \quad z = 2$$

$\therefore$ zeros: $z = 3,\ z = 2$

Poles are roots of $1 - 2.5z^{-1} + z^{-2} = 0$

$$z^{-1} = \frac{2.5 \pm \sqrt{6.25-4}}{2}$$

$$= \frac{2.5 \pm \sqrt{2.25}}{2}$$

$$= \frac{2.5+1.5}{2}, \quad \frac{2.5-1.5}{2}$$

$$\therefore z^{-1} = 2, \quad z^{-1} = 1/2$$

$$\text{or } z = 1/2 \quad \Rightarrow z = 2$$

Poles: $z = 1/2,\ z = 2$

Now we note that $z = 2$ is both a pole and a zero.

b). $$\frac{Y(z)}{X(z)} = \frac{(z-3)\cancel{(z-2)}}{(z-1/2)\cancel{(z-2)}} = \frac{z-3}{z-1/2} = \frac{1-3z^{-1}}{1-(1/2)z^{-1}}$$

$$Y(z) - Y(z).\frac{1}{2}z^{-1} = X(z) - 3X(z)z^{-1}$$

Taking inverse z-transform,

$$y[n] - \frac{1}{2}y[n-1] = x[n] - 3x[n-1] \longrightarrow (1)$$

Clearly eqn (1) denotes a reduced order-difference eqn.

c) $$\frac{Y(z)}{X(z)} = \frac{z-3}{z-1/2}$$

$$\therefore H(z) = \frac{z-3}{z-1/2} = \cancel{\frac{z-1/2-5/2}{z-1/2} = 1-\frac{5}{2}\cdot\frac{1}{(z-1/2)}} \; \cancel{= 1-\frac{5}{2}}$$

$$= \frac{z}{z-1/2} - \frac{3.z.z^{-1}}{z-1/2}$$

$$h[n] = (1/2)^n u[n] - 3\,(1/2)^n u[n-1]$$

$$\left[\because a^n u[n] \xrightarrow[\text{trans-form}]{z-} \frac{z}{z-a}\right] \quad |z|>|a|$$

also

$$-a^n u[-n-1] \xrightarrow[\text{transform}]{z-} \frac{z}{z-a}\Big] \quad |z|<|a|$$

But we need to choose a causal system.

$\therefore |z|>|a|$ includes $|z| = \infty$,

We use $a^n u[n] \longrightarrow \frac{z}{z-a}$ $\quad |z|>|a|$.

$\therefore$ reqd impulse response

$$h[n] = \left(\frac{1}{2}\right)^n u[n] - 3(1/2)^n u[n-1]$$

which matches with the output of impz() fn.