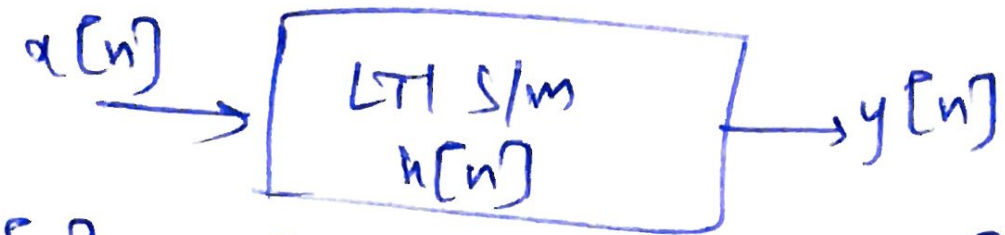
6.2 a Given an LTI System



Output $y[n] = x[n] * h[n]$ $\left[\because x[n] = e^{\pm j\omega n}\right]$

$$= \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

$$= \sum_{k=-\infty}^{\infty} h[k]\, e^{+j\omega(n-k)}$$

$$= \sum_{k=-\infty}^{\infty} h[k]\, e^{+j\omega n} e^{-j\omega k}$$

$$= e^{j\omega n} \sum_{k=-\infty}^{\infty} h[k] e^{-j\omega k}$$

$$= e^{j\omega n} H(e^{j\omega}).$$

6-2b Given a discrete time LTI system

$$y[n] = x[n] * h[n]$$

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} (x[n] * h[n]) e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x[k] h[n-k] e^{-j\omega n}$$

$$= \sum_{k=-\infty}^{\infty} x[k] \sum_{n=-\infty}^{\infty} h[n-k] e^{-j\omega n}$$

$$= \sum_{k=-\infty}^{\infty} x[k] \cdot e^{-jk\omega} H(e^{j\omega})$$

$$= H(e^{j\omega}) \sum_{k=-\infty}^{\infty} x[k] e^{-jk\omega}$$

$$= H(e^{j\omega}) X(e^{j\omega})$$