Model report

Intro to Computer Programming
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Preamble: This report is not perfect, but it is pretty good. Please check the grading comments at the end.

Ski Jump

For our project, we wanted to see how the mass of a ski jumper could affect the distance they could achieve. We decided to analyze this problem since many skiers suffer from health issues such as eating disorders and anorexia because it is commonly believed that reducing bodyweight is an effective means of increasing flight distance. We wanted to test if this notion had any validity. To verify whether mass would alter the distance jumped, we chose to recreate the path followed by Wolfgang Loitzl at Whistler Olympic Park during the 2010 Winter Olympics. Using different masses, we compared the distances covered and then compared these to actual data from competitions. Our hypothesis was that a skier with a smaller mass would be able to cover more distance in the air despite having a slower takeoff speed. The reasoning is based on the analysis of the physics. The smaller the skier's mass, the larger the effect the drag and lift force have on the skier. During the takeoff the drag force acts opposite the skier's velocity and in the air both the drag and lift forces both act to reduce the skier's downward velocity, allowing for a greater distance even if the takeoff speed is reduced.

Description of Model

There are three phases that need to be analyzed to model the ski jump. The first is the descent along the ramp leading to the take-off called the inrun. This part of the descent is important because it allows us to get the initial take-off velocity, which plays a major role in the distance traveled. To determine the velocity along the descent and at take off, we will use Newton's laws and kinematics. In this phase, there are three main forces to consider: gravity, friction and air resistance. The weight of the skier will have different effects along different parts of the descent because the curvature of the slope is changing. This means that the friction will also be changing as the skier advances. The coefficient of kinetic friction between waxed skis and snow is $\mu_k = 0.05$ (Rasul 2007). Air resistance or drag is given by

$$D = 0.5\rho A C_d v^2 \quad (1)$$

where ρ is the density of air, which we calculated as $1.13 kg/m^3$ using the temperature, humidity, and elevation at the competition hill, A the effective surface area of the skier which will be approximated as their standing frontal area because of the aerodynamic takeoff position assumed, C_d the coefficient of drag and v the velocity of the skier.

By solving F = ma in order to get the acceleration and then using this acceleration in the kinematic formulas

$$v_f = v_i + a*\Delta t \quad (2)$$

$$d = d_i + v_i*\Delta t + a*\Delta t^2 \quad (3)$$

to get the velocity at the next point and the distance covered respectively, we can determine the takeoff velocity knowing what the takeoff position is.

The second phase is the jump phase, when the skier reaches the end of the ramp and propels himself in the air by jumping to gain extra velocity and height. We reasonable estimated that an Olympic skier would jump 40cm straight up in the air. Using kinematics, we determined what effect this would have on the x and y velocities and added them to the takeoff velocity.

Finally there is the phase that takes place in the air. In this phase of the jump, there are several key factors that affect the distance traveled. There is the initial takeoff velocity and altitude, the angle of attack (i.e. the angle between the skier and the velocity vector), the flight path angle (i.e. the angle between the skier and the hill), the effective surface area and the air density. Once again we use Newton's laws and kinematics to determine the acceleration, and from that the velocity and position. In the air, the forces in

play are the weight of the skier, the drag force and the lift force, the latter of which are both dependent on the air density, the surface area, the velocity and some coefficient.
The drag force is given by

$$D = 0.5\rho A C_d v^2 \tag{4}$$

The lift force is given by

$$L = 0.5\rho A C_L v^2 \tag{5}$$

where ρ is the density of air, which we will assume to be constant at this altitude and have a value of 1.17kg/m^3, A the effective surface area of the skier, C_d and C_L the coefficients of drag and lift respectively and v the velocity of the skier (Spathopoulos 2010).
The coefficients of drag and lift are dependent on the angle of attack α and are given by

$$C_L = -0.00025\alpha^2 + 0.0228\alpha - 0.092 \tag{6}$$

$$C_d = 0.0103\alpha \tag{7}$$

Description of Computational Method

The simulation of the ski jump was accomplished using numerical methods. As well, all constants used in the simulations were extrapolated from actual data, the physical model or reasonably approximated. The skier's height and weight are those of Wolfgang Loitzl. The air density was calculated using the average elevation of the hill along with the average humidity and temperature from the competition data we are simulating. The length, area and mass of the skis were calculated according to ski jumping regulations and dependent on the skier's height (Specifications For Competition Equipment and Commercial Markings). The actual geometry of the whistler hill is also present in the code as a method that returns the hill altitude as a function of a given x position (Certificate of Jumping Hill). This geometry was extrapolated from data describing the slope of the hill. After a time increment of 0.001s was chosen for the numerical method and initial conditions were set, the simulation was split into three phases: the takeoff, the jump, and flight.

In the first phase, Euler's method is used with a changing frame of reference after each iteration. For each increment of time, the acceleration is calculated using Newton's laws (F=ma). From this, the new velocity and position of the skier are then determined using kinematic equations. In these calculation both acceleration and velocity are determined in the direction of the hill. The new x and y position of the skier are extrapolated from the change in distance along the slope of the hill (according the frame of reference). The acceleration, velocity, and change in distance after the next time increment will be in the direction of the slope at the new position point. In this way the skier's takeoff along the takeoff slope is simulated. The geometry of the slope is given in Figure 1.

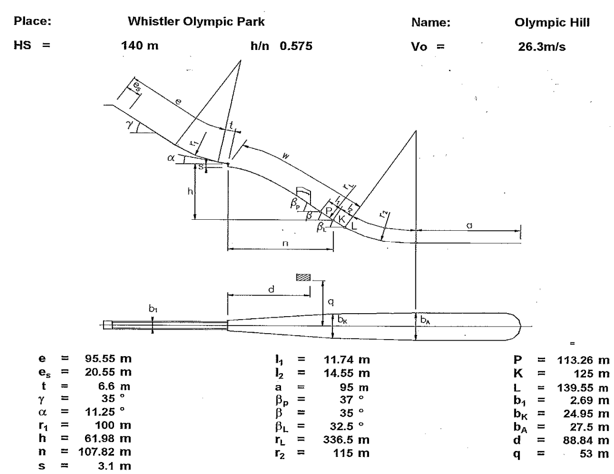

Figure 1. Whistler slope details.

The second phase of the simulation is the jump (which is considered instantaneous for our purposes) of the skier at the takeoff point. The jump at the takeoff point is simulated by increasing the x and y components of the skier's velocity. This increase in velocity is calculated using Newton's laws. The initial velocity of an athlete able to jump 0.4 meters straight up was calculated (2*9.81*0.4)^0.5 and this increase in velocity was applied perpendicular to the slope at the point of takeoff.

The third phase of the simulation is the skier's flight in the air. Euler's method again is used in this phase along with Newton's laws and kinematics. First the x and y components of acceleration are calculated according to the forces acting on the skier (force due to gravity, drag, and lift). Both the drag and the lift forces are calculated as magnitudes then applied in the proper direction (drag force is in the opposite direction of the skier's velocity, and lift force is perpendicular to the velocity). Each of these two forces is dependent on air density, the projected area of the skier and his skis in the direction of the force, a coefficient and the velocity of the skier. As mentioned, the drag and lift coefficient are calculated as functions of the angle of attack (angle of the skier relative to the angle of his velocity vector). After the acceleration calculation is complete, the x and y components of the velocity are updated as well as the x and y positions of the skier using kinematic equations. Time is then incremented and the loop conducting Euler's method iterates again as long as the y position of the skier is higher than the hill by a third of his height. This value (a third of his height) is the approximate distance from his center of mass to the bottom of his skis when in his stance during the flight. If this does not hold true, Euler's method terminates and the results of the simulation are displayed. The resulting distance of the skier's jump is calculated as the distance from the takeoff to the landing point.

Results

When running our simulation, we used the 2010 Olympic champion Wolfgang Loitzl as a model, taking his physical attributes for our parameters. His height is 1.80m and his weight is 63kg and with the added equipment we estimated his total weight to be 70.22kg. Using these values, the simulation determined the takeoff velocity to be 26.62 m/s and the final distance jumped to be 134.38 m. The trajectory of the jumper is shown in Figure 2. When compared to Loitzl's actual results from his Olympic run in 2010, we can see that our calculated values are very close and within reasonable errors. The takeoff velocity only had a percent error of 1.21% from the actual value of 26.3 m/s and the distance jumped had

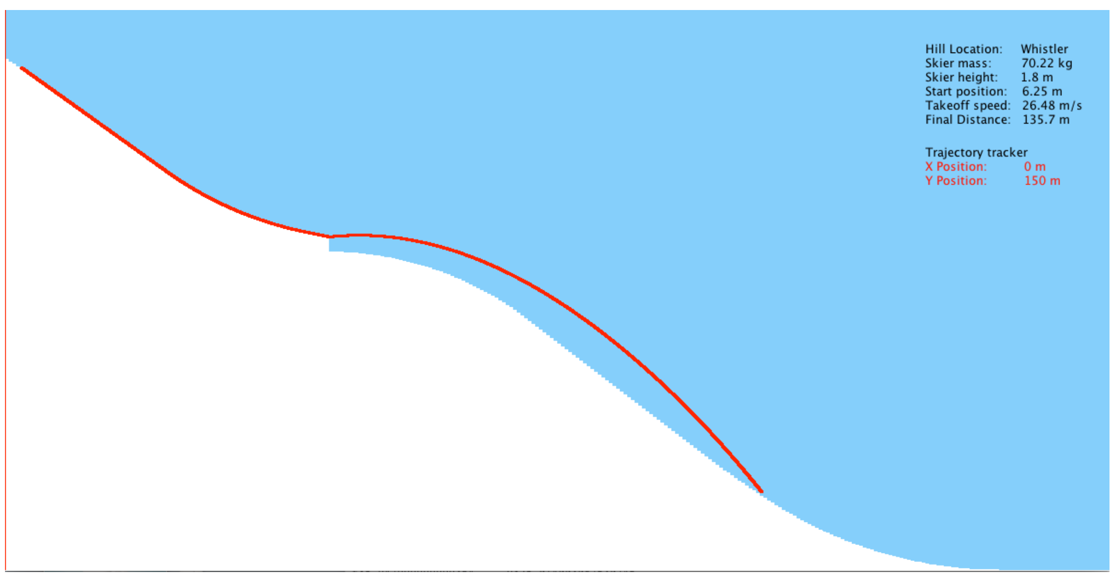

Figure 2. Trajectory of the jump.

a percent error of 2.62% from his jump of 138m (Team Ski Jumping - Complete Event - Vancouver 2010 Winter Olympic Games). The fact that our results are so close to the accepted data confirms that the numerical method we adopted and the approximations we made were justified. Having validated our code using recorded data, we were then able to test our hypothesis. Using different values of mass above and below our test value of 70.22 kg, we saw that a lower mass clearly allowed the skier to cover more distance and the opposite for a heavier skier. From our code, we determined that a skier with a mass 5kg lower than our model would cover 0.56 m more than him. However, a skier with a mass 5kg greater would cover 0.48m less.

Discussion

Our results show a distinct, but not extremely large difference in the distance covered, as opposed to an experiment done by Schmolzer and Muller, where they determined that "the mass profoundly influences the jump length and the velocity of motion along the flight path: The jump length l was only 106.2 m at m=75 kg and increased to 125.7m at m=55 kg" (Schmolzer, Muller 2002). This leads us to believe that the actual effect lies somewhere in between these results. Their model probably did not take into account certain parameters and places too much importance on mass, since Antonin Hajek who weighs 58 kg (5 less than Loitzl) was able to jump 135 m (3 less than Loitzl) (Team Ski Jumping - Complete Event - Vancouver 2010 Winter Olympic Games).

References

Muller, W. "The Physics of Ski Jumping." *CERN* (2005): p269-278. Web. 11 Apr. 2013.
Rasul, Tabraiz. "Coefficients of Friction for Snow." *The Physics Factbook*. n.p., 2007. Web. 11 Apr. 2013.
Schmolzer B., Muller W. "The Importance of Being Light: Aerodynamic Forces and Weight In Ski Jumping." *Journal of Biomechanics* 35 (2002): p1-11. Web. 11 Apr. 2013.
Spathopoulos, V. "A Simple Model For Ski Jump Flight Mechanics Used as a Tool for Teaching Aircraft Gliding Flight." *Journal of Science & Technology* (2010): p33-39. Web. 11 Apr. 2013.
"Certificate of Jumping Hill." *International Ski Federation*, 27 Dec. 2008. Web. 11 Apr. 2013.
"Final Results." *Whistler Olympic Park*. n.p., 22 Feb. 2010. Web. 11 Apr. 2013.
Olympic Vancouver. "Team Ski Jumping - Complete Event - Vancouver 2010 Winter Olympic Games." Online Video clip. *Youtube*. Youtube, 16 Mar. 2010. Web. 11 Apr. 2013.
"Specifications For Competition Equipment and Commercial Markings." *International Ski Federation,* Jul. 2012. Web. 11 Apr. 2013.

Grading Comments:

- Intro:
  - Nicely set the table, the problem is original, anchored with context and the question is clearly stated.
- Model:
  - You mention using kinematics equations, but the acceleration is not constant. It should be made explicit that these will only be used over short time intervals.
  - The model used to determine the orientation of the normal from the ramp is not clearly stated. Apart from the figure, the details of the geometry are kept very vague.
  - It is not clear to me what directions you assume for the drag and lift forces? Are they define with respect to the ground or to the velocity vector? You describe it late in the numerical method section, but really it should be explained in this section.
  - Also, what happens to the angle of attack? Is it fixed or does it vary during the jump?
  - Use an equation editor!
- Numerical method:
  - Nicely described method.
  - Could summarize what Euler's method refers to.
  - Still not sure how you handle the angle of attack.
- Results:
  - The results section could be expanded. Not a lot of results presented…
  - Nice that you clearly state the parameters used.
  - Data are poorly presented. Data are at the center of your report, you should think of a better way to convey the information. A plot showing variation of distance as a function of the mass of the jumper would be much more convincing than checking a +/- 5 kg effect. A table summarizing the data could also be a good option.
  - Great that you test your code against known values.
- Discussion:
  - Nice wrap up with a clearly stated answer to your original question.
  - Why didn't you run your simulation with the value used in by *Schmolzer and Muller*? That would have boosted your discussion. As is, it looks incomplete.
- References:
  - Good use of references.