

M.Sc. IN HIGH-PERFORMANCE COMPUTING

5633 - NUMERICAL METHODS

ASSIGNMENT 0

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RULES

To submit, make a single tar-ball with all your code and a pdf of any written part you want to include. Submit this via `msc.tchpc.tcd.ie` or via email to `mmarina@maths.tcd.ie` by the end of **Wednesday November 15th**. Before uploading the solutions, please rename your tarball and the corresponding folder using the following convention: `5633_hw1_{uname}.tar`. Instead of R, you may/ use Matlab or Python for the numerical/plotting part. Late submissions without prior arrangement or a valid explanation will result in reduced marks.

QUESTION

1. (10p) Convert the following numbers into IEEE 754 single precision numbers:

(a) -45.0

(b) 0.085

and then use their binary representation to write these numbers in hexadecimal representation. The conversion table between decimal, binary and hexadecimal representation is given in Figure 1.

2. (20p) Write a R script that uses the bisection method to find the root of a given function on a given interval, and apply this program to find the roots of the functions below on the indicated intervals. Be careful to avoid unnecessary function calls in a single iteration. Use a relationship derived in class to determine *a priori* the number of steps necessary for the root to be accurate within 10^{-6}

(a) $x^3 - 2$, $[a, b] = [0, 2]$,

(b) $e^x - 2$, $[a, b] = [0, 1]$.

Apply Newton's method to find the root of the second function, $f(x) = e^x - 2$, for which the exact solution is $\alpha = \ln 2$. Perform the following experiments:

Binary	Hex	Decimal
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	8
1001	9	9
1010	A	10
1011	B	11
1100	C	12
1101	D	13
1110	E	14
1111	F	15

Figure 1: Conversion table between decimal, binary and hexadecimal representation.

(c) Compute the ratio

$$R_n = \frac{\alpha - x_{n+1}}{(\alpha - x_n)^2}$$

and observe whether or not it converges to the correct value as $n \rightarrow \infty$.

(d) Compute the modified ratio

$$R_n = \frac{\alpha - x_{n+1}}{(\alpha - x_n)^p},$$

for various $p \neq 2$, but near 2. What happens? Comment, in the light of the definition of order of convergence.

Apply the secant method to the same function, using x_0, x_1 equal to the end points of the given interval. Stop the iteration when the error as estimated by $|x_n - x_{n-1}|$ is less than 10^{-6} . Compare to your results for Newton and bisection method.

Have your codes print out the entire sequence of iterates to the following files bisection1.txt (for function a)), bisection2.txt, newton.txt, and secant.txt (for function b)). In the case of bisection method, the file should have following five columns:

$$n \quad x_n \quad b_{n-1} - a_{n-1} \quad a_n \quad b_n,$$

the file for the Newton's method should contain the following four columns:

$$n \quad x_n \quad \alpha - x_n \quad \log_{10}(\alpha - x_n),$$

and the file for the secant method should contain the following three columns:

$$n \quad x_n \quad \alpha - x_n.$$

3. (20p) Write an R code that does LU factorisation with partial pivoting for an arbitrary $n \times n$ matrix ($n < 100$). Use it to solve the systems of equations $Ax = b$:

$$(a) \quad A = \begin{bmatrix} 9 & 3 & 2 & 0 & 7 \\ 7 & 6 & 9 & 6 & 4 \\ 2 & 7 & 7 & 8 & 2 \\ 0 & 9 & 7 & 2 & 2 \\ 7 & 3 & 6 & 4 & 3 \end{bmatrix}, \quad b = [35, 58, 53, 37, 39]^T;$$

$$(b) \quad A = H_5, \quad b = [5.0, 3.550, 2.81428571428571, 2.34642857142857, 2.01746031746032]^T;$$

$$(c) \quad A = A_5, \quad b = [-4, -7, -6, -5, 16]^T,$$

where H_n and A_n denote the families of matrices parameterised by their dimension in the following way:

$$H_n = [h_{ij}], \quad h_{ij} = \frac{1}{i + j - 1},$$

$$A_n = [a_{ij}], \quad a_{ij} = \begin{cases} 1, & i = j; \\ 4, & i - j = 1; \\ -4, & i - j = -1; \\ 0, & \text{otherwise.} \end{cases}$$

In each case, have your code multiply out the L and U factors to check that the routine is working properly.