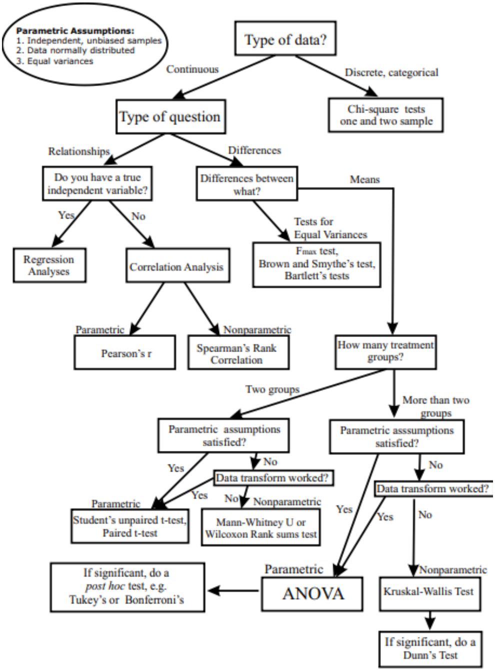
Statistical Data Modeling

Saumya Bhatnagar

March 15, 2020

Table of contents



1. $H_0 : \mu = 100$; $H_1 : \mu \neq 100$
2. if H_0 is true, how extreme is our sample?
3.
4.

Precision Recall tradeoff

content...

		CONDITION determined by "Gold Standard"			
	TOTAL POPULATION	CONDITION POS	CONDITION NEG	PREVALENCE $\frac{\text{CONDITION POS}}{\text{TOTAL POPULATION}}$	
TEST OUT-COME	TEST POS	True Pos TP	*Type I Error* False Pos FP	*Precision* Pos Predictive Value PPV = $\frac{\text{TP}}{\text{TEST P}}$	False Discovery Rate FDR = $\frac{\text{FP}}{\text{TEST P}}$
	TEST NEG	*Type II Error* False Neg FN	True Neg TN	False Omission Rate FOR = $\frac{\text{FN}}{\text{TEST N}}$	Neg Predictive Value NPV = $\frac{\text{TN}}{\text{TEST N}}$
	ACCURACY ACC ACC = $\frac{\text{TP + TN}}{\text{TOT POP}}$	*Sensitivity (SN), Recall* Total Pos Rate TPR TPR = $\frac{\text{TP}}{\text{CONDITION POS}}$	*Fall-Out* False Pos Rate FPR FPR = $\frac{\text{FP}}{\text{CONDITION NEG}}$	Pos Likelihood Ratio LR + LR + = $\frac{\text{TPR}}{\text{FPR}}$	Diagnostic Odds Ratio DOR DOR = $\frac{\text{LR +}}{\text{LR -}}$
		Miss Rate False Neg Rate FNR FNR = $\frac{\text{FN}}{\text{CONDITION POS}}$	*Specificity (SPC)* True Neg Rate TNR TNR = $\frac{\text{TN}}{\text{CONDITION NEG}}$	Neg Likelihood Ratio LR - LR - = $\frac{\text{TNR}}{\text{FNR}}$	

Type I Error: Reject H_0 when H_0 is true. H_0: usko bimari nahi hai

Prob of Type I Error = level of significance = α = 5% (generally)

Type II error: Not reject H_0 when H_0 is false

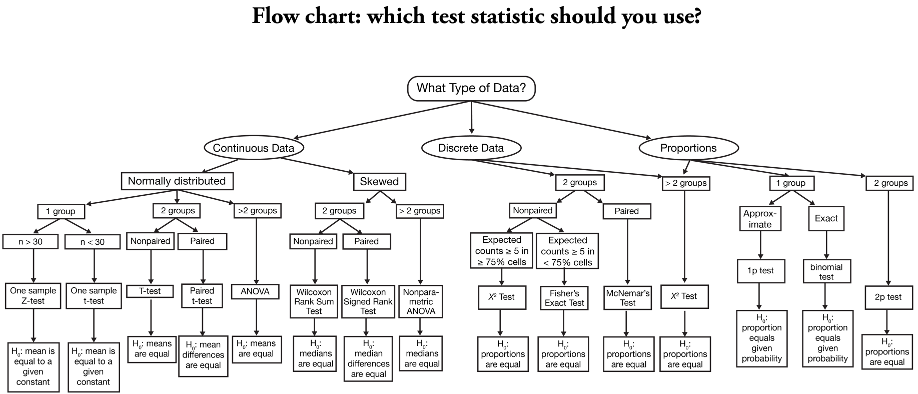
Prob of Type II Error = β and power of hypothesis test = 1-β

1-β = prob of rejecting a H_0 when H_0 is false

test statistics

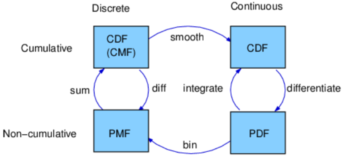
t-test, anova, chi-square, correlation test

Type of tests

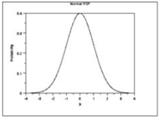


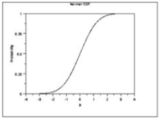
PMF, CMF, PDF, CDF

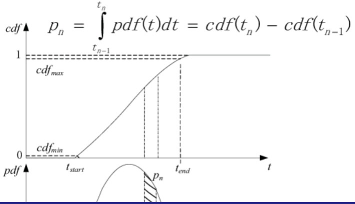
- A PMF, "f" returns the probability of an outcome: $f(x) = P(X = x)$



- Probability density function (p.d.f.) denote f
- Probability mass function (p.m.f.) denote f
  f(x)=P(X=x)
- Cumulative distribution function (c.d.f.) denote F
  F(x)=P(X≤x)



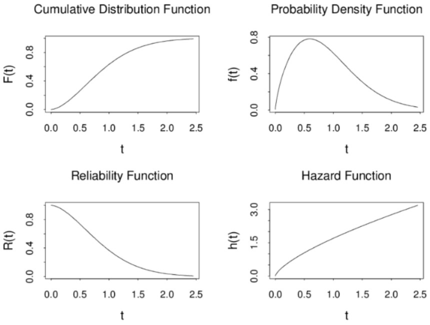


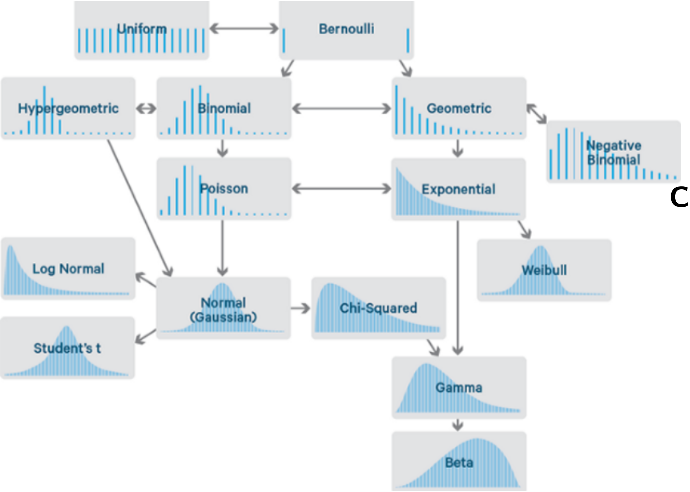


- Reliability function & Hazard Function

PMF, CMF, PDF, CDF

- Reliability function & Hazard Function





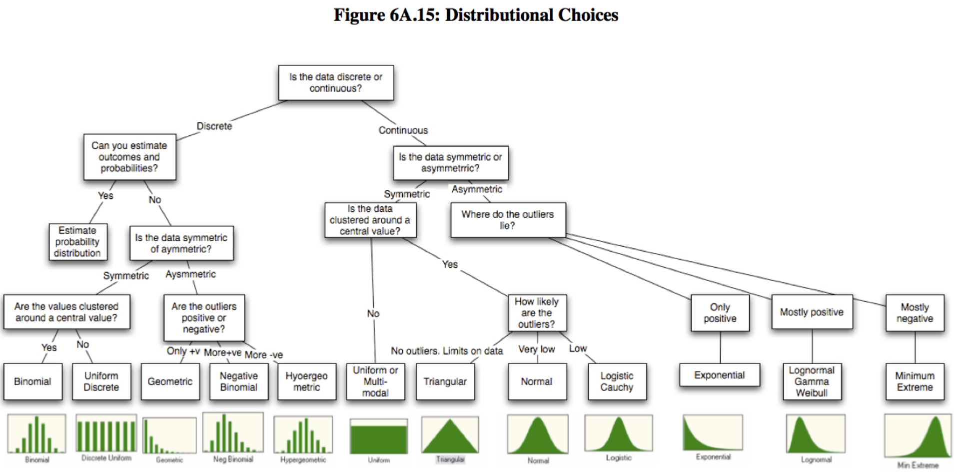
Discrete

1. Uniform Discrete or Rectangular
2. Binomial
3. Hypergeometric
4. Poisson
5. Geometric

Continuous

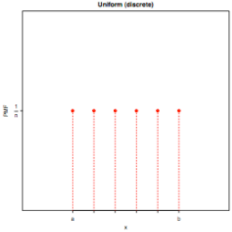
1. Uniform
2. Normal/Gaussian
3. Student's T
4. chi-squared
5. Exponential
6. Beta
7. Triangular
8. Gamma

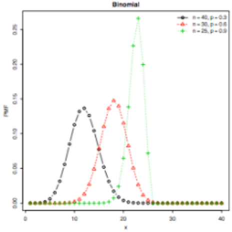
Distributions

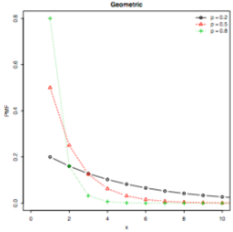


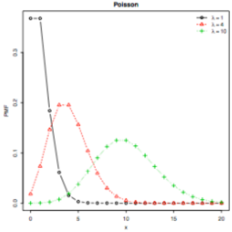
1.1 Discrete Distributions

		CDF/CMF	PMF	Expected Val of RV	Var of RV	
	Notation[1]	$F_X(x)$	$f_X(x)$	$\mathbb{E}[X]$	$\mathbb{V}[X]$	$M_X(s)$
Uniform	$\text{Unif}\{a,\ldots,b\}$	$\begin{cases} 0 & x<a \\ \frac{\lfloor x\rfloor - a+1}{b-a} & a\le x\le b \\ 1 & x>b \end{cases}$	$\frac{I(a<x<b)}{b-a+1}$	$\frac{a+b}{2}$	$\frac{(b-a+1)^2-1}{12}$	$\frac{e^{as}-e^{-(b+1)s}}{s(b-a)}$
Bernoulli	$\text{Bern}(p)$	$(1-p)^{1-x}$	$p^x(1-p)^{1-x}$	p	$p(1-p)$	$1-p+pe^s$
Binomial	$\text{Bin}(n,p)$	$I_{1-p}(n-x,x+1)$	$\binom{n}{x}p^x(1-p)^{n-x}$	np	$np(1-p)$	$(1-p+pe^s)^n$
Multinomial	$\text{Mult}(n,p)$		$\frac{n!}{x_1!\ldots x_k!}p_1^{x_1}\cdots p_k^{x_k} \quad \sum_{i=1}^k x_i = n$	np_i	$np_i(1-p_i)$	$\left(\sum_{i=0}^k p_i e^{s_i}\right)^n$
Hypergeometric	$\text{Hyp}(N,m,n)$	$\approx \Phi\left(\frac{x-np}{\sqrt{np(1-p)}}\right)$	$\frac{\binom{m}{x}\binom{m-x}{n-x}}{\binom{N}{x}}$	$\frac{nm}{N}$	$\frac{nm(N-n)(N-m)}{N^2(N-1)}$	
Negative Binomial	$\text{NBin}(n,p)$	$I_p(r,x+1)$	$\binom{x+r-1}{r-1}p^r(1-p)^x$	$r\frac{1-p}{p}$	$r\frac{1-p}{p^2}$	$\left(\frac{p}{1-(1-p)e^s}\right)^r$
Geometric	$\text{Geo}(p)$	$1-(1-p)^x \quad x\in\mathbb{N}^+$	$p(1-p)^{x-1} \quad x\in\mathbb{N}^+$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{p}{1-(1-p)e^s}$
Poisson	$\text{Po}(\lambda)$	$e^{-\lambda}\sum_{i=0}^{x}\frac{\lambda^i}{i!}$	$\frac{\lambda^x e^{-\lambda}}{x!}$	λ	λ	$e^{\lambda(e^s-1)}$





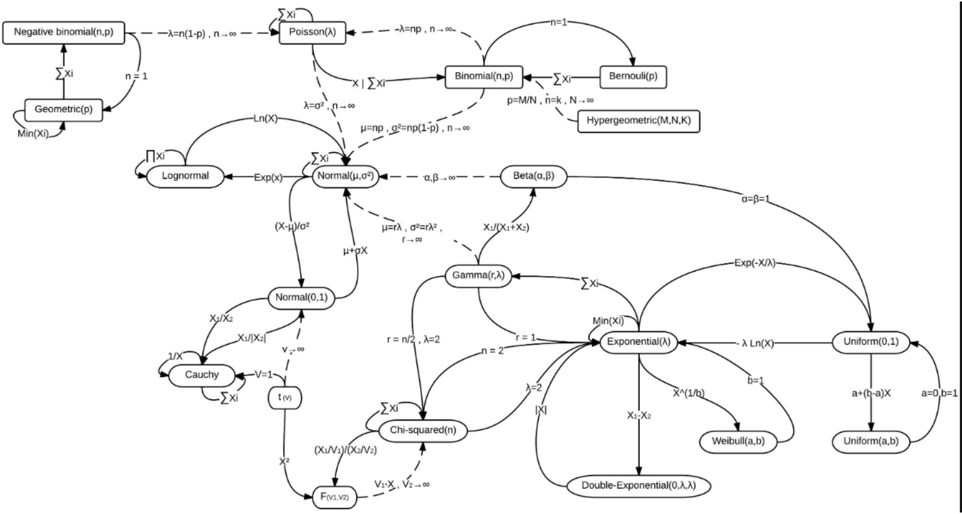




1.2 Continuous Distributions

	Notation	$F_X(x)$	$f_X(x)$	$\mathbb{E}[X]$	$\mathbb{V}[X]$	$M_X(s)$
Uniform	$\text{Unif}(a,b)$	$\begin{cases} 0 & x<a \\ \frac{x-a}{b-a} & a<x<b \\ 1 & x>b \end{cases}$	$\dfrac{I(a<x<b)}{b-a}$	$\dfrac{a+b}{2}$	$\dfrac{(b-a)^2}{12}$	$\dfrac{e^{sb}-e^{sa}}{s(b-a)}$
Normal	$\mathcal{N}(\mu,\sigma^2)$	$\Phi(x)=\int_{-\infty}^{x}\phi(t)\,dt$	$\phi(x)=\dfrac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\dfrac{(x-\mu)^2}{2\sigma^2}\right\}$	μ	σ^2	$\exp\left\{\mu s+\dfrac{\sigma^2 s^2}{2}\right\}$
Log-Normal	$\ln\mathcal{N}(\mu,\sigma^2)$	$\dfrac{1}{2}+\dfrac{1}{2}\text{erf}\left[\dfrac{\ln x-\mu}{\sqrt{2\sigma^2}}\right]$	$\dfrac{1}{x\sqrt{2\pi\sigma^2}}\exp\left\{-\dfrac{(\ln x-\mu)^2}{2\sigma^2}\right\}$	$e^{\mu+\sigma^2/2}$	$(e^{\sigma^2}-1)e^{2\mu+\sigma^2}$	
Multivariate Normal	$\text{MVN}(\mu,\Sigma)$		$(2\pi)^{-k/2}\|\Sigma\|^{-1/2}e^{-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)}$	μ	Σ	$\exp\left\{\mu^T s+\dfrac{1}{2}s^T\Sigma s\right\}$
Student's t	$\text{Student}(\nu)$	$I_x\left(\dfrac{\nu}{2},\dfrac{\nu}{2}\right)$	$\dfrac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\Gamma\left(\frac{\nu}{2}\right)}\left(1+\dfrac{x^2}{\nu}\right)^{-(\nu+1)/2}$	0	0	
Chi-square	χ_k^2	$\dfrac{1}{\Gamma(k/2)}\gamma\left(\dfrac{k}{2},\dfrac{x}{2}\right)$	$\dfrac{1}{2^{k/2}\Gamma(k/2)}x^{k/2}e^{-x/2}$	k	$2k$	$(1-2s)^{-k/2}\ s<1/2$
F	$\text{F}(d_1,d_2)$	$I_{\frac{d_1x}{d_1x+d_2}}\left(\dfrac{d_1}{2},\dfrac{d_1}{2}\right)$	$\dfrac{\sqrt{\frac{(d_1x)^{d_1}d_2^{d_2}}{(d_1x+d_2)^{d_1+d_2}}}}{x\text{B}\left(\frac{d_1}{2},\frac{d_1}{2}\right)}$	$\dfrac{d_2}{d_2-2}$	$\dfrac{2d_2^2(d_1+d_2-2)}{d_1(d_2-2)^2(d_2-4)}$	
Exponential	$\text{Exp}(\beta)$	$1-e^{-x/\beta}$	$\dfrac{1}{\beta}e^{-x/\beta}$	β	β^2	$\dfrac{1}{1-\beta s}\ (s<1/\beta)$
Gamma	$\text{Gamma}(\alpha,\beta)$	$\dfrac{\gamma(\alpha,x/\beta)}{\Gamma(\alpha)}$	$\dfrac{1}{\Gamma(\alpha)\beta^\alpha}x^{\alpha-1}e^{-x/\beta}$	$\alpha\beta$	$\alpha\beta^2$	$\left(\dfrac{1}{1-\beta s}\right)^\alpha\ (s<1/\beta)$
Inverse Gamma	$\text{InvGamma}(\alpha,\beta)$	$\dfrac{\Gamma\left(\alpha,\frac{\beta}{x}\right)}{\Gamma(\alpha)}$	$\dfrac{\beta^\alpha}{\Gamma(\alpha)}x^{-\alpha-1}e^{-\beta/x}$	$\dfrac{\beta}{\alpha-1}\ \alpha>1$	$\dfrac{\beta^2}{(\alpha-1)^2(\alpha-2)^2}\ \alpha>2$	$\dfrac{2(-\beta s)^{\alpha/2}}{\Gamma(\alpha)}K_\alpha\left(\sqrt{-4\beta s}\right)$
Dirichlet	$\text{Dir}(\alpha)$		$\dfrac{\Gamma\left(\sum_{i=1}^k\alpha_i\right)}{\prod_{i=1}^k\Gamma(\alpha_i)}\prod_{i=1}^k x_i^{\alpha_i-1}$	$\dfrac{\alpha_i}{\sum_{i=1}^k\alpha_i}$	$\dfrac{\mathbb{E}[X_i](1-\mathbb{E}[X_i])}{\sum_{i=1}^k\alpha_i+1}$	
Beta	$\text{Beta}(\alpha,\beta)$	$I_x(\alpha,\beta)$	$\dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}$	$\dfrac{\alpha}{\alpha+\beta}$	$\dfrac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	$1+\sum_{k=1}^{\infty}\left(\prod_{r=0}^{k-1}\dfrac{\alpha+r}{\alpha+\beta+r}\right)\dfrac{s^k}{k!}$

Distributions



relation btw various dist

Bernoulli and Binomial: Bernoulli Distribution is a special case of Binomial Distribution with a single trial.
Poisson and Binomial: Poisson Distribution is a limiting case of binomial distribution under the following conditions:
The number of trials is indefinitely large or $\lim_{x\to\infty}$. The probability of success for each trial is same and indefinitely small or $\lim_{x\to 0}$. np $= \lambda$, is finite.
Normal and Binomial: Normal distribution is another limiting form of binomial distribution under the following conditions:
The number of trials is indefinitely large, $\lim_{n\to\infty}$. Both p and q are not indefinitely small. **Normal and Poisson Distribution**: The normal distribution is also a limiting case of Poisson distribution with the parameter $\lim_{\lambda\to\infty}$.

Beta and Binomial

Prior and Posterior

1. Conjugate prior for binomial
$x|p \sim Bin(n, p); p \sim Beta(a, b)[prior]$
$f(p|X = k) = f(X = k|p)f(p)/f(X = k)$ [use bayes]
replace with beta and bin
to learn more

$p|X \sim Beta(a + X, b + n - x)$

Poisson event per unit time
how many calls do you get in a day
The number of printing errors at each page of the book
Num of metro arrivals in t time
The number of arrivals reported in an area on a day.
The number of soldiers killed by horse-kick per year
Air conditioners in a lifetime

exponential time per event
What about the interval of time btw the calls
Num of pages before until x num of printing errors
Length of time btw metro arrivals,
Length of time between arrivals at a gas station
Num of years btw horse-kick deaths in the army
The life of an Air Conditioner

Standard Uniform Density

parameters a = 0 and b = 1, so the PDF for standard uniform density is given by:

$$f(x) = \begin{cases} 1, & 0 <= x <= 1 \\ 0, & otherwise \end{cases} \tag{1}$$

Normal Distribution

Standard Normal Distribution $\mu = 0$; $\sigma = 1$
The 68-95-99.7 rule: Given a normally distributed random variable: $P(\mu - \sigma \leq X \leq \mu + \sigma) \approx .68$ =>68% of samples fall within 1 SD of the mean
$P(\mu - 2\sigma \leq X \leq \mu + 2\sigma) \approx .95$
$P(\mu - 3\sigma \leq X \leq \mu + 3\sigma) \approx .997$
characteristics of Normal distribution:

1. Mean = median = mode
2. The distribution curve is bell-shaped and symmetrical about the line x=μ.
3. The total AUC = 1.
4. Exactly half of the values are to the left of the center and the other half to the right.

Student's t distribution

characteristics

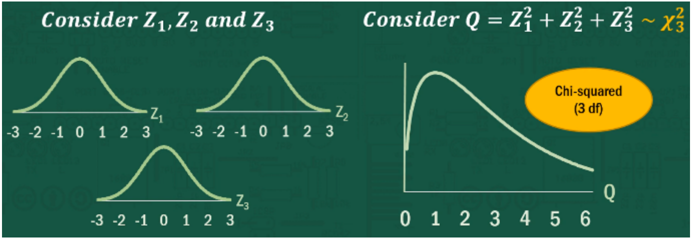
1. Underlying dist is Normal
2. Pop dist is unknown
3. sample size is too small for CLT to apply

$z \sim \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \longleftrightarrow t_{n-1} \sim \frac{\bar{x}-\mu}{s/\sqrt{n}}$ **t test measures**

1. to test hypothesized population mean $\frac{\bar{x}-\mu}{s/\sqrt{n}}$
2. regression $\frac{b-\beta}{SE(b)}$ (uses Std error, as pop std dev is known)
3. 2 sampled t-test: assessing the diff btw 2 pop $\frac{(\bar{x}_1-\mu_1)-(\bar{x}_2-\mu_2)}{\sqrt{\frac{s_1^2}{\sqrt{n_1}}+\frac{s_1^2}{\sqrt{n_1}}}}$

Chi-squared dist

comes directly from a normal dist (square of selection from standard Normal Distribution) so sample size should be large enough (>5) s.t. CLT applies
for k degrees of freedom: $\chi_k^2 = \sum_{i=1}^{k} Z_i^2$
$\chi = \sum \frac{(obs-exp)^2}{exp}$ with DF = (row-1)(col-1)



Binomial, bernoulli, hyper-geometric

Repeat Bernoulli n times and it's Binomial.
Hypergeometric is Binomial without replacement
the properties of a Binomial Distribution are

1. Each trial is independent.
2. There are only two possible outcomes per trial.
3. A total number of n identical trials are conducted.
4. The probability of success and failure is same for all trials. (Trials are identical.)

Hypergeometric

1. Discrete
2. equivalent to Binomial, without replacement
3. N = total population
   m=total items of interest in population
   n=sample size
4. region bounded by 0 and m

Characteristics of Poisson distribution:

1. Event are not Independent.
2.

Poisson

1. Discrete
2. events in fixed region of opportunity (or time interval, t)
3. region bounded by 0 and ∞

Characteristics of Poisson distribution:

1. Independent event.
2. The probability of success over a short interval must equal the probability of success over a longer interval.
3. The probability of success in an interval approaches zero as the interval becomes smaller.
4. The rate at which event occurs is constant (λ)

Poisson RV, $X =$ number of events in t.
mean number of events in t, $\mu = \lambda$*t
The PMF of X: $P(X = x) = e^{-\mu} * \frac{\mu^x}{x!}$

Exponential Dist

inverse of Poisson: rate parameter or mean for poisson $= \lambda$ and mean for expo $= \beta = 1/\lambda$
Exponential distribution is widely used for survival analysis.
Memoryless-ness:
events must occur at constant rate
events must be independent of each other
probab of event occurring in first min = probab of the event occuring in (t+1)min
probab of first visitor on website in first min = p
probab of first visitor on website in second min = (1-p)p
probab of first visitor on website within third min = $(1\text{-}p)^2$p
Each minute graph dropping $\rightarrow$ exponential decay

Memorylessness

Memoryless property: $P(X >= s + t | X >= s) = P(X >= t)$

$P(X >= s) = 1 - CDF = 1 - P(X <= s) = e^{-\lambda s}$

$P(X >= s + t | X >= s) = \frac{P(X>=s+t, X>=s)}{P(X>=s)}$

$P(X >= s + t | X >= s) = \frac{P(X>=s+t)}{P(X>=s)}$

$P(X >= s + t | X >= s) = \frac{e^{-\lambda(s+t)}}{e^{-\lambda s}}$

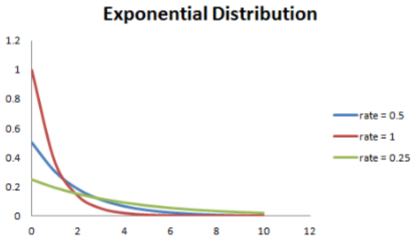
$P(X >= s + t | X >= s) = e^{-\lambda t}$

Memoryless property:

$E(X | X > a) = a + E(X - a | X > a)$

$E(X | X > a) = a + 1/\lambda$

failure rate of any device at time t, given that it has survived up to t; $\lambda = \frac{1}{\beta} > 0$
area under the density curve

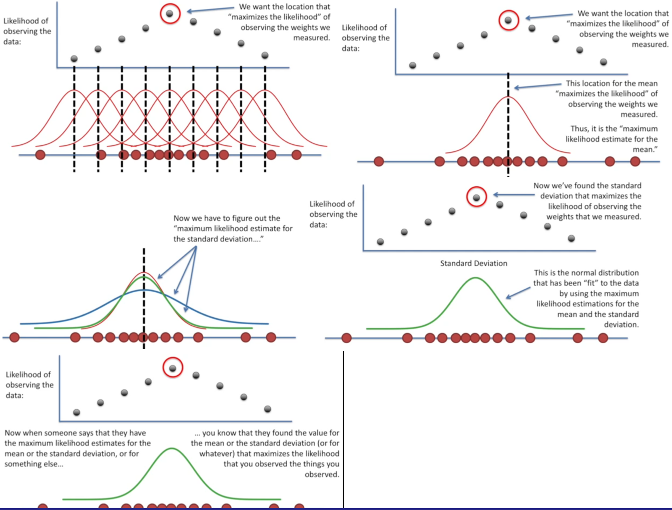


to the left of x $P\{X \leq x\} = 1 - e^{-\lambda x}$
to the right of x $P\{X > x\} = e^{-\lambda x}$
$P\{x1 < X \leq x2\} = e^{-\lambda x1} - e^{-\lambda x2}$

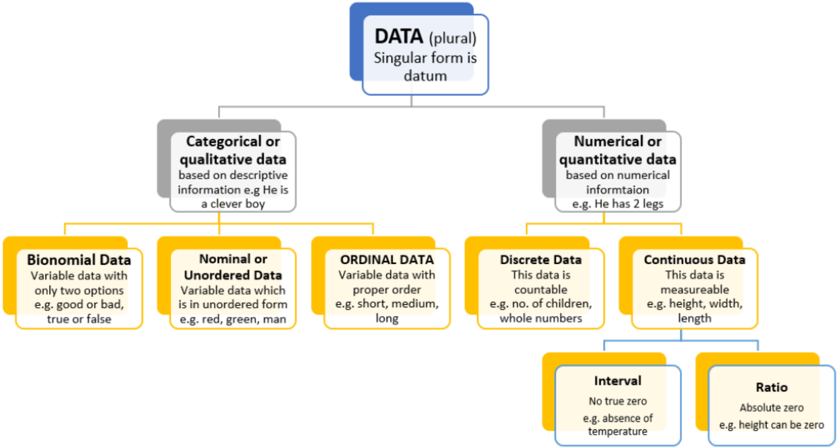
to read

https://www.quora.com/What-is-the-relation-between-standard-normal-and-gamma-distribution
https://stats.stackexchange.com/questions/37461/the-relationship-between-the-gamma-distribution-and-the-normal-distribution

Max Likelihood

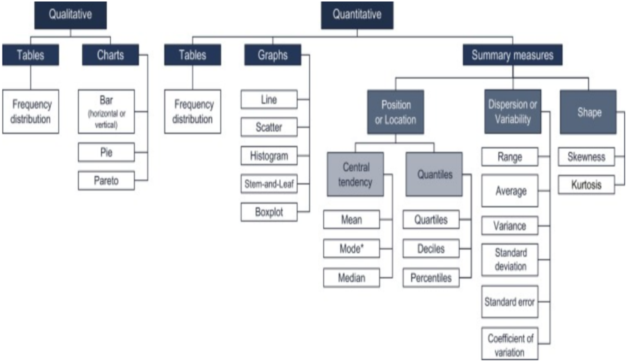


Stats Flow


DATA (plural)
Singular form is datum
Categorical or qualitative data
based on descriptive information e.g He is a clever boy
Numerical or quantitative data
based on numerical informtaion
e.g. He has 2 legs
Bionomial Data
Variable data with only two options
e.g. good or bad, true or false
Nominal or Unordered Data
Variable data which is in unordered form
e.g. red, green, man
ORDINAL DATA
Variable data with proper order
e.g. short, medium, long
Discrete Data
This data is countable
e.g. no. of children, whole numbers
Continuous Data
This data is measureable
e.g. height, width, length
Interval
No true zero
e.g. absence of temperature
Ratio
Absolute zero
e.g. height can be zero

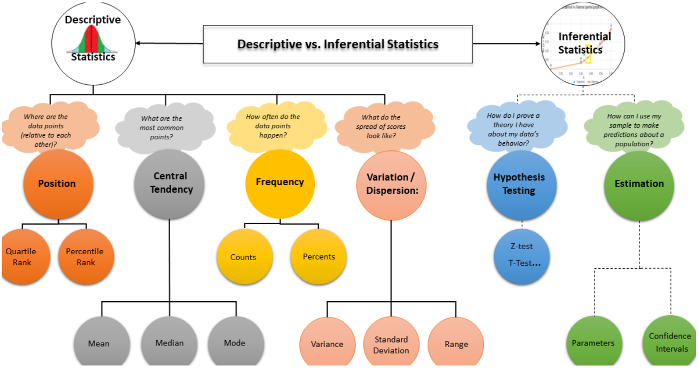
Types of Analysis

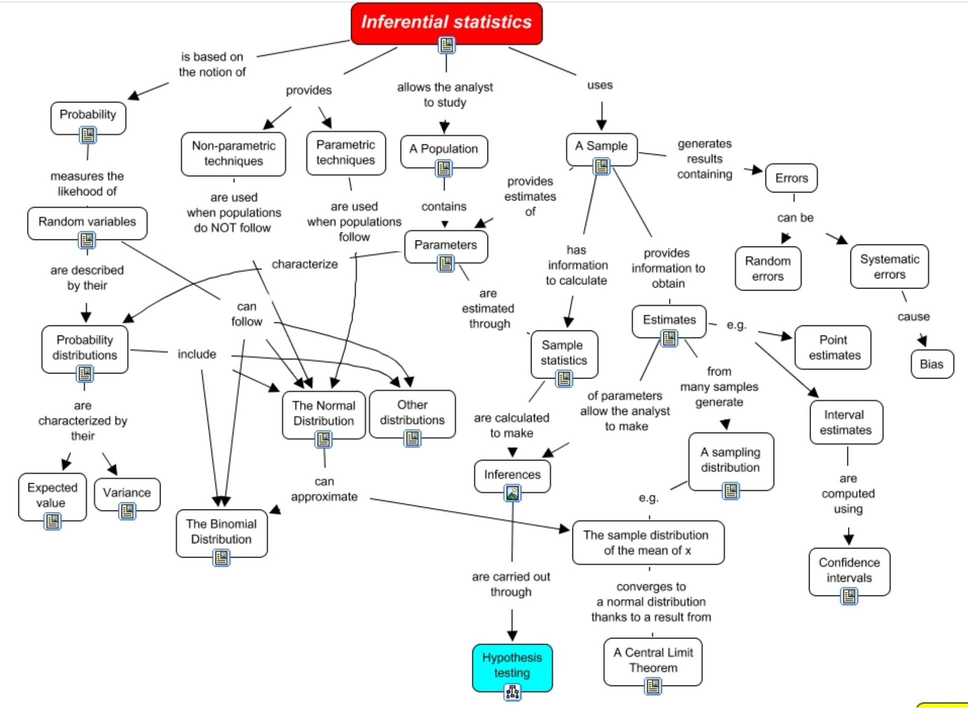
- Qualitative Analysis/Non-Statistical Analysis gives generic information (uses text, sound and other forms of media).
- Quantitative Analysis/Statistical Analysis: collecting and interpreting data.



Types of Statistics

- Descriptive Statistics: provides descriptions of the population.
- Inferential Statistics makes inferences and predictions from sample to generalize a population.




Inferential statistics
is based on the notion of
provides
allows the analyst to study
uses
Probability
measures the likehood of
Random variables
are described by their
Probability distributions
include
are characterized by their
Expected value
Variance
Non-parametric techniques
are used when populations do NOT follow
Parametric techniques
are used when populations follow
characterize
can follow
The Normal Distribution
Other distributions
can approximate
The Binomial Distribution
A Population
contains
Parameters
are estimated through
provides estimates of
A Sample
has information to calculate
Sample statistics
are calculated to make
Inferences
are carried out through
Hypothesis testing
provides information to obtain
Estimates
of parameters allow the analyst to make
from many samples generate
A sampling distribution
e.g.
The sample distribution of the mean of x
converges to a normal distribution thanks to a result from
A Central Limit Theorem
e.g.
Point estimates
Interval estimates
are computed using
Confidence intervals
generates results containing
Errors
can be
Random errors
Systematic errors
cause
Bias

Contingency Table and Probabilities

Joint, Marginal and Conditional

- Joint probabilities for rain and wind:

	no wind	some wind	strong wind	storm
no rain	0.1	0.2	0.05	0.01
light rain	0.05	0.1	0.15	0.04
heavy rain	0.05	0.1	0.1	0.05

- Marginalize to get simple probabilities:
  - $P(\text{no wind}) = 0.1 + 0.05 + 0.05 = 0.2$
  - $P(\text{light rain}) = 0.05 + 0.1 + 0.15 + 0.04 = 0.34$
- Combine to get conditional probabilities:
  - $P(\text{no wind}|\text{light rain}) = \frac{0.05}{0.34} = 0.147$
  - $P(\text{light rain}|\text{no wind}) = \frac{0.05}{0.2} = 0.25$

Central Limit Theorem(CLT)

CLT: as n $\uparrow$, the distribution of sample mean or sum approaches a Normal Dist

Law of large num: as a sample size grows, its mean gets closer to the average of the whole population.

(LLN) is a theorem that describes the result of performing the same experiment a large number of times

In a financial context, the law of large numbers indicates that a large entity which is growing rapidly cannot maintain that growth pace forever.

law of averages: the supposed principle that future events are likely to turn out so that they balance any past deviation from a presumed average.

The law of averages is a lay term used to express a belief that outcomes of a random event will "even out" within a small sample

Variance, Standard Deviation

- Variable and Random Variable (RV)
- Parameter and Hyper-parameter
- Mean, Median, Mode
- mode sucks for small samples
- Range, IQR
- Standard Deviation (σ): Measure of how spread out the data is from its mean.
- Variance (σ^2): It describes how much a random variable differs from its expected value. It entails computing squares of deviations. The average of the squared differences from the Mean.
  1. Deviation is the difference bw each element from the mean.
  2. Population Variance = avg of squared deviations
  3. Sample Variance = avg of squared differences from the mean

EXPECTED VALUE

Discrete random variable $E(X) = \sum_x x\, p_x(x)$

- Provided $\sum_x |x|\, p_x(x) < \infty$. If the sum diverges, the expected value does not exist. **For the jar full of numbered balls**
- A ball is selected at random; all balls are equally likely to be chosen $P(X = x_i) = \frac{1}{N}$.
- Say n_1 balls have value v_1, and n_2 balls have value v_2, and $\ldots n_n$ balls have value v_n. Unique values are v_i, for $i = 1, \ldots, n$. Note $n_1 + \cdots + n_n = N$, and $P(X = v_j) = \frac{n_j}{N}$. $E(X) = \frac{\sum_{i=1}^{N} x_i}{N}$

Continuous random variable $E(X) = \int_{-\infty}^{\infty} x\, f_x(x)\, dx$

- Provided $\int_{-\infty}^{\infty} |x|\, f_x(x)\, dx < \infty$. If the integral diverges, the expected value does not exist.

Sometimes the expected value does not exist

Need $\int_{-\infty}^{\infty} |x| \, f_x(x) \, dx < \infty$

For the Cauchy distribution, $f(x) = \frac{1}{\pi(1+x^2)}$.

$$\begin{aligned}
E(|X|) &= \int_{-\infty}^{\infty} |x| \frac{1}{\pi(1+x^2)} \, dx \\
&= 2 \int_{0}^{\infty} \frac{x}{\pi(1+x^2)} \, dx \\
& \quad u = 1 + x^2, \;\; du = 2x \, dx \\
&= \frac{1}{\pi} \int_{1}^{\infty} \frac{1}{u} \, du \\
&= \ln u \big|_1^{\infty} \\
&= \infty - 0 = \infty
\end{aligned}$$

=> an integral "equals" infinity, it is unbounded above.

For a RV X with PDF $\rho(x)$. The variance($\mathbb{V}$) and the standard deviation(σ_X) of X, are defined by
Variance $\sigma^2 = (1/n)\sum_{i=1}^{n}(x_i - \mu)^2$

$$\mathbb{V} = \mathbb{E}\left[(X - \mathbb{E}[X])^2\right] = \mathbb{E}[X^2] - \mathbb{E}[X]^2 = \int_D (x - \mathbb{E})^2 \, dP.$$

$$\mathbb{V} = \int_D x^2 \, dP - \mathbb{E}^2.$$

$$\sigma_X = \sqrt{V[X]} \qquad = \sqrt{E[X^2] - E[X]^2}$$

$$\mathbb{V} = \sqrt{\int_D x^2 \rho(x)\, dx - \left(\int_D x\rho(x)\, dx\right)^2}.$$

If one interprets the PDF ($\rho(x)$) as the density of a rod at location (x), then:
The mean, ($\mu = \int x\rho(x)\,dx$), gives the center of mass of the rod.
The variance, ($V = \int (x-\mu)^2\rho(x)\,dx$), gives the moment of inertia about the line ($x = \mu$).
The standard deviation, ($\sigma = \sqrt{V}$), gives the radius of gyration about the line ($x = \mu$).

Std error vs std deviation

std error = std dev of the means
std dev quantifies the variation within a set of measurements
std error quantifies variation in the means from multiple sets of measurements
take a sample and get the mean and std dev
take multiple sets of samples and get their means and std dev
plot the means of the various samples
std dev of this plot of means is std error

coeff of variation

$CV = \frac{sd}{\bar{x}}, where \bar{x} =$sample mean
$x = [1, 2, 3] => \bar{x} = 2$ and $S_x = 1 => CV(x) = 1/2$
$y = [101, 102, 103] => \bar{y} = 102$ and $S_y = 1 => CV(y) = 1/102$
Higher the CV means higher fluctuations in the dataset

skewness and kurtosis

https://www.thoughtco.com/what-is-kurtosis-3126241

skewness

mode skewness = $\frac{mean-mode}{stddev}$

for small dataset, use below:

in skewed data: mode = 3(median) - 2(mean)

median skewness = $\frac{3(mean-median)}{stddev}$

$$skewness = \begin{cases} approx\_symmetric, & -0.5 <= x <= 0.5 \\ moderately\_skewed, & 0.5 < |x| < 1 \\ highly\_skewed, & |x| > 1 \end{cases} \quad (2)$$

kurtosis: same mean or sd but diff peakedness

higher peaked => higher kurtosis

Moments

I moment: $\frac{\sum x}{n}$ => **mean** => considered as values from 0
second moment: $\frac{\sum x^2}{n}$ =>values further from 0 will be higher, so instead we take centralized
second (centralized) moment: $\frac{\sum (x-\mu)^2}{n}$ => variance
third (centralized) moment: $\frac{1}{n}\frac{\sum (x-\mu)^3}{\sigma^3} -> $ skew
but since we don't have population mean, we have sample mean, we adjust the above value with degrees of freedom
II (centralized) moment: $\frac{\sum (x-\bar{x})^2}{n-1}$ => **variance**
III (centralized) moment: $\frac{n}{(n-1)(n-2)}\frac{\sum (x-\bar{x})^3}{s^3}$ => **skew**
IV moment: $\frac{n(n+1)}{(n-1)(n-2)(n-3)}\frac{\sum (x-\bar{x})^4}{s^4} - \frac{3(n-1)^2}{(n-2)(n-3)}$ => **kurtosis**

confounding variables

$\uparrow$ variance, can introduce bias

control variable vs control group

control group $\rightarrow$ if difficult to identify or control all potential confounding variables

positive confounding: overestimate the effect

negative confounding:

- underestimate the effect
- observed association is biased towards NULL

To reduce impact of confounding variables, bias can be eliminated with random samples

Real vs Empirical Conversion

$1/2 =? \ 10/20 =? \ 100/200 =? \ 1000/2000$

same empirical conversion rate but different real conv rate because of uncertainty

A = Control group; B = Test group

Types of AB Testing I

Split Testing: divide the traffic to the two new versions
A/A testing (dummy experiment): split traffic to same version, still if difference, that would imply the way users are assigned is impacting the results
Hypothesis testing - Frequentist Approach: Uses χ^2 test?

- p-value
- confidence intervals
- needs a fixed sample size in advance
  1. statistical power (how often will you recognize a successful effect): typically 80%
  2. significance level (how often will you observe positive result, however, there's none): typically 5%

Types of AB Testing II

Bayesian AB testing

	Hypothesis Testing	Bayesian A/B Testing
Knowledge of Baseline Performance	Required	Not Required
Intuitiveness	Less, as p-value is a convoluted term	More, as we directly calculate the probability of A being better than B
Sample size	Pre-defined	No need to pre-define
Peeking at the data while the test runs	Not allowed	Allowed (with caution)
Quick to make decisions	Less, as it has more restrictive assumptions on distributions	More, as it has less restrictive assumptions
Representing uncertainty	Confidence Interval (again, a convoluted interpretation which is often misunderstood)	Highest Posterior Density Region – highly intuitive interpretation
Declaring a winner	When sample size is reached and p-value is below a certain threshold	When either "probability to be best" goes above a threshold or the expected loss is below a threshold (in which case a "tie" can be declared between multiple variations)

When can AB test fail

1. in the case of a **referral program**, The referrer and Referee could be split across test and control groups causing spillover on the control or variant group
2. Novelty effects: Prompts and CTA tend to exhibit novelty effects, if not measuring their performance over the long term using a holdout a wrong attribution and/or customer fatigue can happen.
3. What-if scenarios: If you are looking to understand the impact of **not having launched a product**, for instance a subscription offering on a website. A/B test wouldn't be the right fit.

How long to run the experiment

A p-value measures the probability of observing a difference between the two variants at least as extreme as what we actually observed, given that there is no difference between the variants. Once the p-value achieves statistical significance or we've seen enough data, the experiment is over.

Assumptions in Frequentist Approach for AB tests:

- after obtaining infinite samples, empirical conversion approaches real conversion
- data is iid (independent and identically distributed)
- difference btw empirical conv rates for A and B is due to randomness (H0: A and B are same)
- conv of both versions come from the same distributions (same parameters)

When can Frequentist approach fail

1. We need to know how much of the data we need to collect for the test before starting the test.
2. We can't evaluate the result in real-time as we go, instead we need to wait to make any decision until we collect a full of the planned data size.
3. The test result is not intuitively understandable especially for those without a statistical background. (What is P-value again?)
4. The test result can be read as black and white, either it is statistically significant or not. This makes it hard to figure out what to do especially when not statistically significant

In case of small improvements of variants I

- there are scenarios where we want to stick with the null hypothesis when the treatment variant is marginally better than the control. If the treatment requires a lot of engineering maintenance or causes a disruption to the user experience, the costs of implementing the new variant might outweigh the small benefits
- In scenarios similar to the one of the slightly better model, Bayesian methodology is appealing because it is more willing to accept variants that provide small improvements. Over the next few years, as we perform hundreds of experiments on the same handful of key business metrics, these marginal gains will accumulate on top of each other. Crucially, since we

In case of small improvements of variants II

conclude an experiment once we are confident it will provide at least a small improvement, we can iterate more quickly and run more experiments over all.

- By accepting variants that offer a small improvement, Bayesian A/B testing asserts that the false positive rate — the proportion of times we accept the treatment when the treatment is not actually better — is not very important. While this may be shocking to some statisticians, we agree with this sentiment because not all false positives are created equal. Choosing variant B when its conversion rate is 10% and the conversion rate for variant A is 10.1% is a very different mistake than choosing variant B when the conversion

In case of small improvements of variants III

rates are 10% for B and 15% for A. Yet, under frequentist methodology, these would both count as a single false positive.

- Instead, Bayesian A/B testing focuses on the average magnitude of wrong decisions over the course of many experiments. It limits the average amount by which your decisions actually make the product worse, thereby providing guarantees about the long run improvement of a metric. We believe that these types of guarantees are much more relevant to Convoy's use case than the false positive guarantees made by frequentist procedures.

In case of small improvements of variants IV

Bayesian

In experiments where the improvement of the new variant is small, Bayesian methodology is more willing to accept the new variant. By using Bayesian A/B testing over the course of many experiments, we can accumulate the gains from many incremental improvements. Bayesian A/B testing accomplishes this without sacrificing reliability by controlling the magnitude of our bad decisions instead of the false positive rate.

Type I (false positive) and Type II (false negative) Errors

1. FP (REJECT a TRUE hypothesis): the sample population and AB test results → the challenger will increase conversion rates (so you reject the null hypothesis of both rates)
   In reality (for the whole population of visitors), the challenger will NOT increase conversion rates for the overall population
2. FN(**NOT** REJECT a FALSE hypo): the sample population and AB test results → the challenger will not increase conversion rates (so you do not have enough evidence to reject the null hypothesis about the equality of both rates)
   In reality, the challenger will increase conversion rates.

Construct the test: Avoid FP: Assume significance level @0.05
Avoid FN: The more sample, the more probability of rejecting of the false hypothesis (more power).

p_value

probab of data given hypothesis

p-value is the probab of getting a sample as extreme as ours, given H_0 is true

if H0 is true, how extreme is our sample?

either reject H_0 or do not reject H_0 because there's less evidence to reject it (given α level of significance)

Hypothesis in AB testing: H0 = B is better

Better performance of B is statistically significant: B is better

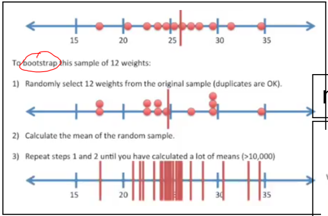
Better performance of B is **not** statistically significant: Need more data

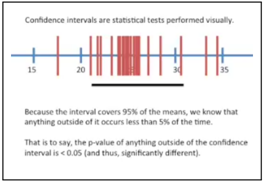
importance of results?

calculate confidence intervals to capture uncertainty of experiment

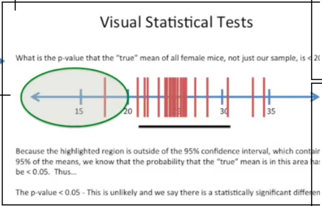
http://www.statisticshowto.com/what-is-statistical-significance/

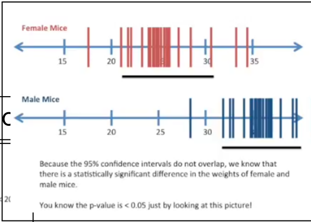
confidence interval

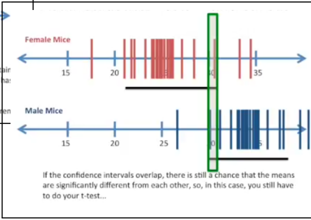




nvjfvjnfvnkevnkervnmkd c







Statistical power vs Significance level

content...

Define sample size in advance to avoid FN

content...

Problem with significance based testing

- not intuitive
- $\uparrow$ sample size $\rightarrow$ experiment run time
- p-value reaches significance very early on due to novelty effect
- statistical significance is not valid stopping criteria
- depends on arbitrary parameters (95% conf, 0.05 p-val)
- confidence interval: contains true parameter with 95% probability
  not 95% probability the true parameter falls within the interval

Multiple Null Hypothesis test

1-(1-0.05)^{n}

t-statistics vs z-statistics

when population std dev is known then we use z-statistics, if unknown then we use t-statistics
t-statistics assumes that underlying distribution is normal
t-distribution is bell curved, defined by it's DF (degrees of freedom)
Measure of extremeness, z=$\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$
$\uparrow$ z value, more likely to reject H_0

Frequentist vs Bayesian I

Frequentist Approach
treats random events probabilistically and doesn't quantify the uncertain
usually used for AB Testing when no uncertainty involved such as fair C
Option B is better than A with p-value of 0.03
otherwise need more data
assumes iid
wait till data has been collected
using only data from your current experiment
frequentist methods assume that you repeat your experiment many, man
take a sample, measure and average their height to produce a point esti

Going Bayesian

Update prior beliefs with new beliefs

Bayes Theorem

$P(H|E) = \frac{P(E|H) * P(H)}{P(E)}$

Posterior $\propto$ likelihood x Prior

where

Posterior probab of hypothesis given the evidence $= P(H|E)$

Likelihood of evidence if hypothesis is true $= P(E|H)$

Prior Probab of Hypothesis = P(H)

Prior Probab that Evidence is True = P(E)

Bayesian AB Testing I

Given data:
From sample: CVR_A and CVR_B
From prior population: CVR_A (historical data)
strong prior distribution

Non-overlapping populations

$$P(CVR_A, CVR_B|data) = P(CVR_A|data) * P(CVR_B|data)$$
$$P(CVR_A, CVR_B|data) = \frac{P(data|CVR_A)P(A) * P(data|CVR_B)P(B)}{P(data)P(data)}$$

Success/failure output, Likelihood follows Binomial Dist

$$P(views, clicks|CVR) = \binom{views}{clicks} CVR^{clicks}(1 - CVR)^{views-clicks}$$

Prior probab follows Beta Dist

Bayesian AB Testing II

$$P(CVR_A) = \frac{(1-CVR_A)^{b-1} CVR_A^{a-1}}{B(a,b)}$$

Binomial Likelihood implies conjugate Beta

$\rightarrow$ Beta Posterior $\rightarrow$Prior and Posterior are of same family

$$P(CVR|views, clicks) = \frac{P(views,clicks|CVR)P(CVR)}{P(views,clicks)}$$

where, CVR = Conversion Rate

Bayesian AB Testing.. contd...

Posterior Probab

$Beta(CVR_A, a + clicks_A, b + views_A - clicks_A)$

Choosing a, b for Beta dist

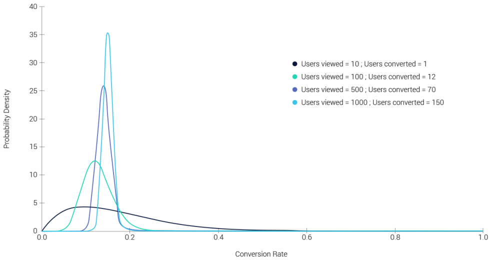
Uninformative prior: Randomly choose a and b
Informative prior: Decide a and b after running the experiment multiple times and already analyzed CVR

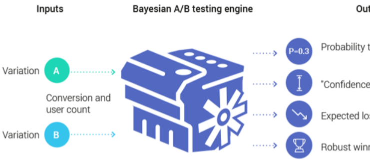
get average height bayesian example I

1. Initially, the Bayesian statistician has some basic prior knowledge which is being assumed: for example, that the average height is somewhere between 50cm and 250cm.
2. Then, the Bayesian begins to measure heights of specific citizens, and with each measurement updates the distribution to become a bit more “bell-shaped” around the average height measured so far. As more data is collected, the “bell” becomes sharper and more concentrated around the measured average height.

get average height bayesian example II

3. For Bayesians, probabilities are fundamentally related to their knowledge about an event. This means, for example, that in a Bayesian view, we can meaningfully talk about the probability that the true conversion rate lies in a given range, and that probability codifies our knowledge of the value based on prior information and/or available data.





Deterministic vs Stochastic

content...

Random Walks

content...

Monte Carlo

content...

Gibbs Sampler

content...

How to choose the method of predictive modelling. algorithms
https://www.quora.com/What-is-the-difference-between-supervised-and-unsupervised-learning-algorithms
https://blog.udacity.com/2016/04/5-skills-you-need-to-become-a-machine-learning-engineer.html
https://towardsdatascience.com/how-to-build-a-data-science-portfolio-5f566517c79c
https://www.youtube.com/watch?v=qv6UVOQ0F44
https://en.wikipedia.org/wiki/F-test
numpy matplotlib
http://slideplayer.com/slide/6260251/
http://www.sfu.ca/ ber1/iat802/pdfs/When
https://www.youtube.com/watch?v=RlhnNbPZC0A

Thank You!