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Assignment - 2

1. Identify the details of Metaheuristics, its nature, advantage and disadvantage

- Metaheuristics:-

• A metaheuristic is a general solution method that provides both a general structure and strategy guidelines for developing a specific heuristic method to fit a particular kind of problem.

• Metaheuristic have become one of the most important techniques in the tool kit of OR practitioners.

Nature:-

• Metaheuristics is a general kind of solution method that orchestrates the interaction between local improvement procedures and higher level of strategies to create a process that is capable of escaping from local optima and performing a robust search of the feasible region.

• A key feature of the metaheuristic is its ability to escape from a local optimum.

• After reaching (or nearly) a local optimum, different metaheuristics execute this escape in different ways.

• However a common characteristic is that the trial solution that immediately follow a local optimum are allowed to be inferior to this local optimum.

Advantage:-

• The advantage of a well designed metaheuristics is that it tends to move relatively quickly towards very good solutions, so it provides a very efficient way of dealing with large and complicated problems.

Disadvantage:-

• The disadvantage is that there is no guarantee that the best solution found will be optimum solution or even a near optimal solution.

2. Identify the details of Tabu search algorithm

- • Initialization
• Start with a feasibility initial trial solution

Iteration:-

- use an appropriate local search procedure to define the feasible moves into the local neighbourhood of the current solution.
- Eliminate from consideration any move on the current tabu list unless the move would result in a better solution that the best trial solution found so far.
- Determine which of the remaining moves provides the best solution.
- Adopt this solution as the next trial solution, regardless of whether it is better or worse than the current trial solution.
- Update the tabu list to forbid cycling back to what had been the current trial solution
- If the tabu list already had been full, delete the oldest member of the Tabu list to provide more flexibility for future moves.

Stopping Rule:-

- Use any stopping criteria, such as fixed number of iterations, a fixed amount of CPU time, a fixed number of consecutive iterations without an improvement in the best objective function value.
- Also stop at any iteration values there are no feasible moves in the local neighbourhood of the current trial solution.
- Accept the best trial solution found at any iteration as the final solution.

3. What is operations Research & Explain the six phases of an operations Research study.

- Operations research is a scientific approach to decision making which seeks to determine how best to design and operate a system under conditions requiring the allocation of scarce resources.

The six phases of an operations research study are:-

• Formulating the problem:-

Before proceeding to find the solution of a problem, first of all, one must be able to formulate the problem in the form of an appropriate model.

The following information is required:-

(i) who has to take the decision?

(ii) what are the objectives?

(iii) what are the ranges of controlled variables?

(iv) what are the uncontrolled variables that may affect the possible solution.

(v) what are the restrictions (or) constraints on the variables?

• Constructing a Mathematical model:-

• It is concerned with the reformulation of the problem in an appropriate form which is concerned/convenient for analysis. The most suitable form for this purpose is to construct a mathematical model representing the system under study.

• It requires the identification of both static and dynamic structural elements. A mathematical model should include the following three variables and parameters

(i) Decision variables and parameters

(ii) Constraints (or) Restrictions.

(iii) Objective function

• Deriving the solution from the model.
- This phase is devoted to the computation of those values of decision variables that maximize (or minimize) the objective function
- Such solution is called optimum solution which is always in the best interest of the problems under consideration
• Testing the model and its solution (updating the model):-
- After completing the model, it is once again tested as a whole for errors if any.
- A model is said to be valid if it can provide a reliable production/predition of a system's performance.
• Controlling the solution:-
- This phase establishes controls over the solution with any degree of satisfaction.
- The model requires immediate modification as soon as the controlled variables (one or more) change significantly otherwise the model goes out of control.
• Implementing the solution:-
- Finally, the tested results of the model are implemented to work.
- This phase is primarily executed with the cooperation of OR experts and those who are responsible for managing and operating the systems.

4. Show how linear programming model can be formulated for the problem given below. The Apex Television company has to decide on the number of 27 and 20 inch sets to be produced at one of its factories. Market research indicates that at most 40 of the 27-inch sets & 10 of the 20-inch sets can be sold per month. The

maximum number of work-hours available is 500 per month. A 27-inch set requires 20 work-hours. Each 27 inch set a 20-inch set requires 10 work-hours. Each 27 inch set sold produces a profit of \$120 and each 20 inch set produces a profit of \$80. A wholesaler has agreed to purchase all the television sets produced if the numbers do not exceed the maxima indicated by the market research.

Let the total no. of 27 inch TV be 'x'.

Let the total no. of 20 inch TV be ~~x_2~~ 'y'

	work hours	sales/month	profit
x 27 inch TV	20	1	120
y 20 inch TV	10	1	80

∴ Objective function $Z = 120x + 80y$

STC, $20x + 10y \leq 500$

$x + y \leq 50$

$x \geq 0$.

$y \geq 0$.

$x \leq 40$.

$y \leq 10$.

5. Explain the following terms with examples.

(i) Feasible solution :-

Any set of variables $X = \{x_1, x_2, k, x_{n+m}\}$ is called a feasible solution of LPP if it satisfies the set of constraints and non restrictions also

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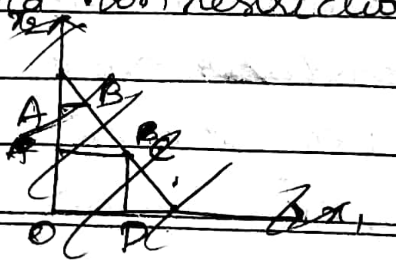

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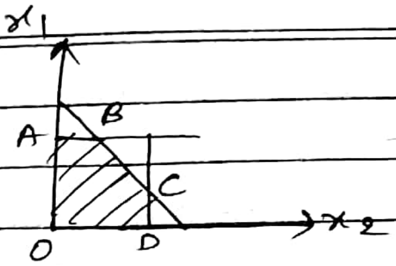

The points O, A, B, C, D all lie under feasible region and hence it is a feasible solutions.

(ii) Feasible region:- The set of all feasible solutions is a feasible region. It is a region in which all constraints and non-negativity conditions hold good.
Ex:- The region shaded in the above graph is a feasible region.

(iii) Infeasible region:- It is the proposed solution to an optimization problem that does not satisfy all the constraints is an Infeasible solution.
Ex:- The empty feasibility region.

(iv) Optimal solution:- A feasible solution that maximizes or minimizes the value of the objective function is an optimal solution.
Ex:- Max $Z = 36$ is an optimal solution.
An optimal solution is one among all feasible solutions.

(v) CPF solution:- Corner point Feasible solution is a feasible solution that doesn't lie on any line segment connecting two other feasible solution.

(vi) Unbounded solution:-
If the value of the objective function can be increased or decreased indefinitely, then the solution is called an unbounded solution.

6. Identify the meaning of the following terms with respect to an LPP.

(i) Feasible solutions:- A set of variables $\{x_1, x_2, x_3 \ldots x_n\}$ is called a feasible solution if these variables satisfy constraints are non negative.

(ii) Infeasible solutions:- It is the proposed solution to an optimization problem that does not satisfy all the constraints.

(iii) Feasible Region:- It is a region in which all constraints and non-negativity conditions hold good.

(iv) Optimal solution:- A ~~is~~ solution is called optimal if it maximizes or minimizes the value of the objective function.

(v) CPF solution:- It is a feasible solution that doesn't lie on any line segment connecting two other feasible solution.

(vi) Unbounded solution:- If the value of the objective function can be increased or decreased indefinitely then the solution is called an unbounded solution.

(vii) Slack variables:- It is a variable added to the LHS of a less than or equal to constraint to convert the constraint into equality. The value of this variable can usually be interpreted as amount of unused resources.

(viii) Surplus variables:- A variable subtracted from the LHS of a greater than or equal to constraint to convert the constraint into an equality. The value of this variable can be interpreted as the amount of over and above some required minimum level.

(ix) Artificial variables:- A variable that has no physical meaning in terms of the original linear program problem but serves merely to enable a basic feasible solution to be used for starting the simplex method.

7. Solve the LPP using simplex method:

Minimize $Z = x_2 - 3x_3 + 2x_5$

STC, $x_1 + 3x_2 - x_3 + 2x_5 = 7$

$-2x_2 + 4x_3 + x_4 = 12$

$-4x_2 + 3x_3 + 8x_5 + x_6 = 10$

and $x_1, x_2, \ldots, x_6 \geq 0$.

Standard form:- $\max Z' = -x_2 + 3x_3 - 2x_5 + 0x_1 + 0x_4 + 0x_6$.

$x_1 + 3x_2 - x_3 + 2x_5 = 7$

$-2x_2 + 4x_3 + x_4 = 12$

$-4x_2 + 3x_3 + 8x_5 + x_6 = 10$ $\quad x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$

	CB	B.V	x_1	x_2	x_3	x_4	x_5	x_6	RHS	$\theta = \frac{RHS}{\text{coeff of EV}}$
	$c_j \rightarrow$		0	-1	-3	0	-2	0		
	0	x_1	1	3	-1	0	2	0	7	—
	0	x_4	0	-2	$\boxed{4}$	1	0	0	12	3
	0	x_6	0	-4	3	0	8	1	10	$10/3 = 3.33$
		$c_j - z_j \rightarrow$	0	-1	3	0	-2	0	$Z'_{max} = 0$	$x_2 = x_3 = x_5 = 0$
$x_1 = x_1 + x_3$	0	x_1	1	$\boxed{5/2}$	0	1/4	2	0	10	4
$x_3 = x_4/4$	3	x_3	0	-1/2	1	1/4	0	0	3	$3/-1/2 = $ —
$x_6 = x_6 - 3x_3$	0	x_6	0	-5/2	0	-3/4	8	1	1	$1/-5/2 = $ —
		$c_j - z_j \rightarrow$	0	1/2	0	-3/4	-2	0	$Z'_{max} = 9$	$x_2 = x_4 = x_5 = 0$
$x_2 = \frac{2}{5} x_1$	-1	x_2	2/5	1	0	1/10	4/5	0	4	NA
$x_3 = x_3 + \frac{1}{2} x_2$	3	x_3	1/5	0	1	3/10	2/5	0	5	
$x_6 = x_6 + \frac{5}{2} x_2$	0	x_6	1	0	0	-1/2	10	1	11	
		$c_j - z_j \rightarrow$	-1/5	0	0	-4/5	-12/5	0	$Z'_{max} = 11$	$x_1 = x_4 = x_5 = 0$

optimality reached

$\therefore$ optimum solution, $x_2 = 4$, $x_5 = 0$, $x_3 = 5$, $x_6 = 11$.

$Z'_{max} = 11 \quad \therefore \min Z = -11$

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8. Show all the basic solutions of the following equations identifying in each case the basic & non-basic variables.

$2x_1 + x_2 + 4x_3 = 11$

$3x_1 + x_2 + 5x_3 = 14$

→ All the basic solutions can be found using algebraic method, by fixing 2 of the variables to zero and finding the 3rd variable.

Non Basic variables	Basic variables	feasibility
$x_1 = 0$	$x_2 = -1$	NO
	$x_3 = 3$	
$x_2 = 0$	$x_1 = 0.5$	YES
	$x_3 = 2.5$	
$x_3 = 0$	$x_1 = 3$	Yes
	$x_2 = 5$	

9. Interpret the concept of graphical method to solve the following LPP.

Max $Z = 3x_1 + 5x_2$

STC $x_1 \leq 4$

$2x_2 \leq 12$

$3x_1 + 2x_2 \leq 18$

and $x_1, x_2 \geq 0$.

→ $x_1 = 4$ (line parallel to y axis)

$2x_2 = 12$ (line parallel to x axis)

$3x_1 + 2x_2 = 18$

	x_1	x_2
$3x_1 + 2x_2 = 18$	0	9
	6	0

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~~from the graph, the feasible region is~~
from the graph, the feasible region is OABCD.

point B, $2x_1 = 12$ $x_2 = 6$

$3x_1 + 2x_2 = 18$

$\Rightarrow 3x_1 + 2(6) = 18$ $\boxed{B(2,6)}$

$\Rightarrow 3x_1 = 6$

$x_1 = 2$

point C, $x_1 = 4$ $3x_1 + 2x_2 = 18$

$\Rightarrow 3(4) + 2x_2 = 18$ $\boxed{C(4,3)}$

$2x_2 = 6$

$\boxed{x_2 = 3}$

Corner points	$Z = 3x_1 + 5x_2$
O(0,0)	$Z = 3(0) + 5(0) = 0$
A(0,6)	$Z = 3(0) + 5(6) = 30$
B(2,6)	$Z = 3(2) + 5(6) = 36$
C(4,3)	$Z = 3(4) + 5(3) = 27$
D(4,0)	$Z = 3(4) + 5(0) = 12$

Max Z = Max (0, 30, 36, 27, 12).

Max $Z = 36$. optimum solution

$x_1 = 2$ $x_2 = 6$

10. Use graphical method to solve the LPP.

Maximize $Z = 5x_1 + 4x_2$

S.T.C $6x_1 + 4x_2 \leq 24$.

$x_1 + x_2 \leq 6$.

$-x_1 + x_2 \leq 1$

and $x_1 \leq 2$, $x_1, x_2 \geq 0$.

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$6x_1 + 4x_2 = 24$

$x_1 + x_2 = 6$

$-x_1 + x_2 = 1$

$x_1 = 2$ (line parallel to y-axis)

	x_1	x_2
$6x_1+4x_2=24$	0	6
	4	0
$x_1+x_2=6$	0	6
	6	0
$-x_1+x_2=1$	0	1
	-1	0

From the graph, the feasible region is OABC.

point B, $-x_1 + x_2 = 1 \longrightarrow (1)$

$6x_1 + 4x_2 = 24 \longrightarrow (2)$

eq^n (1) × 4 and eq (3) − eq (2)

$-4x_1 + \cancel{4x_2} = 4$

$\underline{-6x_1 \underset{(+)}{+} \cancel{4x_2} = \underset{(-)}{24}}$ B(2, 3)

$-10x_1 = -20$

$x_1 = 2 \Rightarrow x_2 = 3$

Cornerpoints	$Z = 5x_1 + 4x_2$
O(0,0)	$Z = 5(0)+4(0) = 0$
A(0,1)	$Z = 5(0)+4(1) = 4$
B(2,3)	$Z = 5(2)+4(3) = 22$
C(2,0)	$Z = 5(2)+4(0) = 10$

optimum solution :- max Z = max (0, 4, 22, 10) = 22,

$x_1 = 2 \quad x_2 = 3$

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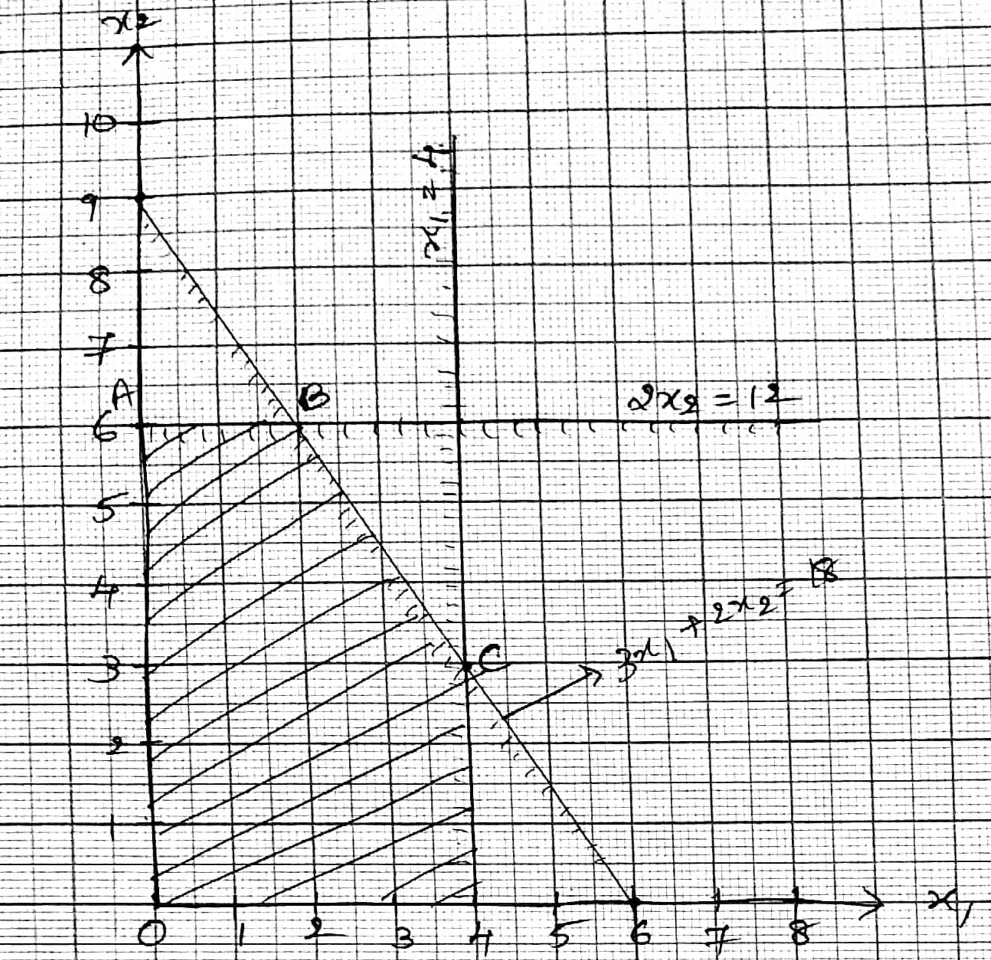
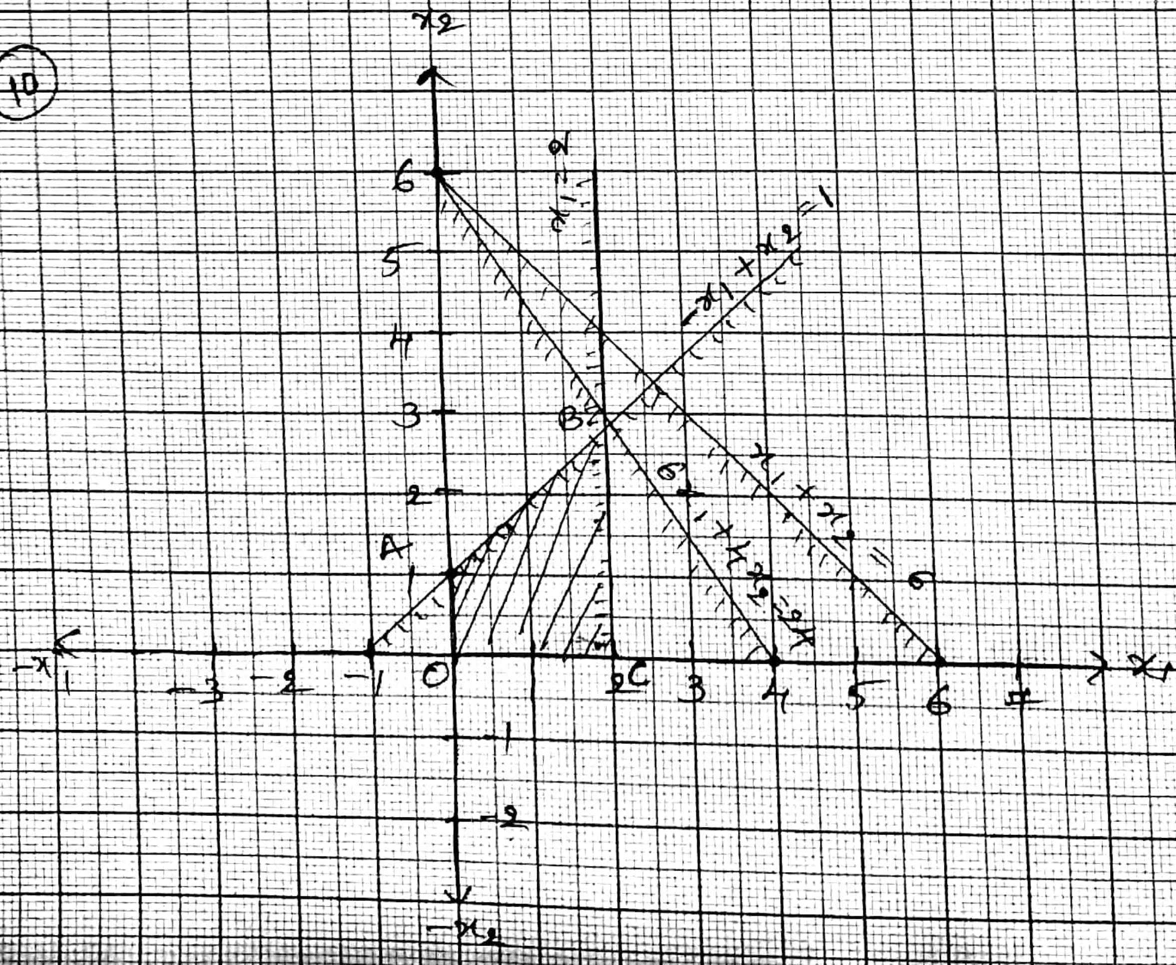


x_2
10
9
8
7
6
5
4
3
2
1
A
B
C
$2x_2 = 12$
$x_1 = 4$
$3x_1 + 2x_2 = 18$
0
1
2
3
4
5
6
7
8
x_1

10



x_2
6
5
4
3
2
1
$x_1 = 2$
$-x_1 + x_2 = 1$
B
A
C
O
$-x_1$
-3
-2
-1
3
4
5
6
7
x_1
-1
-2
$-x_2$