Assignment - 1

(1). Solve the initial basic feasible solution of transportation problem where cost-matrix is given below

Destination

		A	B	C	D	Supply
Origin	I	1	5	3	3	34
	II	3	3	1	2	15
	III	0	2	2	3	12
	IV	2	7	2	4	19
	Demand	21	25	17	17	

Sol^n Since supply and demand both are equal, this is a Minimization-balanced problem.

Using Vogel's Approximation method:

	A	B	C	D	Supply
I	1 (21)	5	3	3 (13)	~~34~~ 13
II	3	3 (13)	1	2 (2)	~~15~~ 13
III	0	2 (12)	2	3	~~12~~
IV	2	7	2 (17)	4 (2)	~~14~~ 2
	~~21~~	~~25~~	~~17~~	~~17~~ ~~15~~ 2	

Row penalties:

I	2	0	2	2	-
II	1	1	1	1	1
III	2	0	1	1	1
IV	0	2	3	-	-

Column penalties:

A	B	C	D
1	1	1	1
-	1	1	1
-	1	-	1
-	1	-	1
-	1	-	1

	A	B	C	D
I	1 21	5	3	3 13
II	3	3 13	1	2 2
III	0	2 12	2	3 [illegible]
IV	2	7	2 17	4 2

No. of allocations = $m+n-1 = 4+4-1 = 7$
Here it is a Non-degenerate BFS.

The Basic feasible solution is,
$= 1\times21 + 3\times13 + 3\times13 + 2\times2 + 12\times2 + 17\times2 + 4\times2$
$= 21+39+39+4+24+34+8$
$= 169$

(2). Develop an optimal solution to the following transportation problem.

Destination

		D1	D2	D3	D4	D5		
	O1	7	6	4	5	9	40	
origin	O2	8	5	6	7	8	30	Supply
	O3	6	8	9	6	5	20	
	O4	5	2	7	8	6	10	
		30	30	15	20	5	100 / 100	
		Demand						

Sol^n Since the supply and demand are equal it is a Minimization balanced problem.

Using Vogel's Approximation Method,

	D1	D2	D3	D4	D5						
O1	7 5	6	4 5	5 20	9	~~40~~ ~~25~~ 5	1	1	1	1	2
O2	8 10	5 20	6	7	8	~~30~~ 10	1	1	1	2	1
O3	6 15	8	9	6	5 5	~~20~~ 15	1	1	0	0	0
O4	5	2 10	7	8	6	10	3	-	-	-	-
	30	~~30~~ 20	15	20	5						
	1	3	2	1	1						
	1	1	2	1	3						
	1	1	2	1	-						
	1	1	-	1	-						
	1	-	-	1	-						

	D1	D2	D3	D4	D5
O1	7 5	6	4 15	5 20	9
O2	8 10	5 20	6	7	8
O3	6 15	8	9	6	5 5
O4	5	2 10	7	8	6

Basic feasible solution $= 7\times5 + 4\times15 + 5\times20 + 8\times10 + 5\times20 + 6\times15$
$+ 5\times5 + 2\times10$
$= 35 + 60 + 100 + 80 + 100 + 90 + 25 + 20$
$= \underline{\underline{510}}$

No. of allocations $= m+n-1 = 4+5-1 = 8$

Hence it is a non-degenerate BFS.

③. Identify different steps in Hungarian algorithm to solve an assignment problem.

ans.

Step ①: Construct a cost matrix, if the cost matrix is not a perfect square, add a row or column to make it so.

Step ②: Perform row subtraction by taking the minimum value in each row and subtract it with the other elements in the row.

step ③: Perform column subtraction by taking the minimum value in each column and subtract it with the other elements in the column.

step ④: If the number of jobs is equal to the number of persons then perform assignment. After assignment that is the cost of the assignment problem.

step ⑤: If the number of jobs is not equal to the number of persons then perform,

Ticking Procedure:

* Tick all the unassigned rows.
* If the unassigned row has an ~~assignment~~ zero then tick the corresponding column
* If the ticked column has an assignment then tick the corresponding row.
* Repeat the steps until no more ticking is possible.
* Draw lines through unticked rows and ticked columns.

Step ⑥: From the remaining matrix, find the value of θ such that it is the least value,

* Add θ if 2 lines pass through.
* Subtract θ if no lines pass through.
* No change if 1 line pass through

Step ⑦: Perform assignment such that one job should be assigned only to one person. If number of jobs is equal to number of persons compute minimum cost else repeat step 5 & step 6.

Q. The owner of a small machine shop has four machines available to assign for the jobs. Five jobs are offered to assign, with the expected profits in hundreds of rupees for each machine on each job being as follows:

Machines		j1	j2	j3	j4	j5
	A	6.2	7.8	*	10.1	8.2
	B	7.0	8.4	6.5	7.5	6.0
	C	8.7	9.2	11.1	7.0	8.2
	D	*	6.4	8.7	7.7	8.0

* indicates that machine A and D cannot perform the jobs 3 and 1 respectively. Solve the assignment of jobs to machines that will result in the maximum profit.

Sol: The given matrix has 4 jobs and 5 machines. So, it is an unbalanced matrix. Also the profit has to be maximized.

The given matrix has to be converted to minimization
UnBalanced.

	1	2	3	4	5
A	4.9	3.3	*	1.0	2.9
B	4.1	2.7	4.6	3.6	5.1
C	2.4	1.9	0	4.1	2.9
D	*	4.7	2.4	3.4	3.1
~~dummy~~					

(Subtract every value with the max. profit i.e. 11.1)

Now, converting the minimization unbalanced to minimization balanced.

	1	2	3	4	5	min
A	4.9	3.3	*	1.0	2.9	1.0
B	4.1	2.7	4.6	3.6	5.1	2.7
C	2.4	1.9	0	4.1	2.9	0
D	*	4.7	2.4	3.4	3.1	2.4
dummy	0	0	0	0	0	0

Row Subtraction,

	1	2	3	4	5	
~~A~~	~~3.9~~	~~2.3~~	~~*~~	$\boxed{0}$	~~1.9~~	
~~B~~	~~1.4~~	$\boxed{0}$	~~1.9~~	~~0.9~~	~~2.4~~	
C	2.4	1.9	$\boxed{0}$	4.1	2.9	✓
D	*	2.3	~~0~~	1.0	0.7	✓
dummy	$\boxed{0}$	~~0~~	~~0~~	~~0~~	~~0~~	

$\theta = 0.7$.

	1	2	3	4	5
A	3.9	2.3	*	$\boxed{0}$	1.9
B	1.4	$\boxed{0}$	2.6	2.9	2.4
C	1.7	1.2	$\boxed{0}$	3.4	2.2
D	*	1.6	Ø	0.3	$\boxed{0}$
dummy	$\boxed{0}$	Ø	Ø	Ø	Ø

The job assignments are, A → 4 = 10.1
B → 2 = 8.4
C → 3 = 11.1
D → 5 = 8.0
37.6

⑤ Four jobs are to be done on four different machines. The cost (in rupees) of producing ith job on the jth machine is given below:

Machines

		M_1	M_2	M_3	M_4
Jobs	J_1	15	11	13	15
	J_2	17	12	12	13
	J_3	14	15	10	14
	J_4	16	13	11	17

Sol^n Row Subtraction,

	M_1	M_2	M_3	M_4
J1	4	0	2	4
J2	5	0	0	1
J3	4	5	0	4
J4	5	2	0	6
Min	4	0	0	1

(Subtract each value with its row minimum)

Column subtraction,

	M_1	M_2	M_3	M_4
J_1	$\cancel{0}$	$\boxed{0}$	2	3
J_2	1	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$
J_3	$\boxed{0}$	5	$\cancel{0}$	3
J_4	1	2	$\boxed{0}$	5

Job assignments are,

$J_1 \rightarrow M_2 = 11$
$J_2 \rightarrow M_4 = 13$
$J_3 \rightarrow M_1 = 14$
$J_4 \rightarrow M_4 = 11$

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(6). Identify basic characteristics of two persons, zero sum game. For the game having following pay off table. Develop an optimal strategy for each player by successively eliminating dominated strategies. Indicate the order in which you eliminate strategies.

Player - 2

		1	2	3
	1	1	2	0
Player - 1	2	2	-3	-2
	3	0	3	-1

ans. Basic characteristics of a two person zero sum game are:
* Only two persons/players participate in the game.
* Each player has a finite number of strategies to use.
* Each specific strategy results in a pay off.
* Total payoff to the two players at the end of each play

is zero.

	1	2	3	
1	1	2	$\boxed{0}$	0
2	$\boxed{2}$	-3	-2	-3
3	0	$\boxed{3}$	-1	-1
	$\boxed{2}$	$\boxed{3}$	$\boxed{0}$	

* 0 is the saddle point
* Optimal strategy ~~strategy~~ for player A is 1
* Optimal strategy for player 2 is 3
* Value of the game is 0 (i.e, it is a fair game, No winner, no loser)

⑦ Identify the details of solving simple games in game theory.

ans. A game can be of 2 types:

(i) Games with saddle point

(ii) Games without saddle point

(i) Games with saddle point:

* Select the minimum value in the row and maximum value in the column.
* Wherever both these values will intersect that will be the saddle point.
* Hence saddle point will give us the values of the game and the corresponding strategies will be the pure strategy of the game player.

(ii) Games without saddle point:

* Depending on the size of the game we have various methods to solve the game.
* For a 2×2 game, we can use arithmetic or algebraic method.

→ Arithmetic method:

* Find out the difference of the first row and write it next to the second row and vice versa.
* Similarly we will find the difference of the first column elements and write it next to the second column & vice versa.
* Find the probability for all values. This gives us the optimal solution and also value of the game.

→ Algebraic method:

* Selecting one strategy for a player we will find the profit equation and then we will find the profit equation for the second strategy and finally comparing them, we will obtain the optimal solutions and value of the game.

* For a 2×m and n×2 game, we can solve the game graphically.
* For a 3×3 game we will use the shortcut method or dominance rules if possible. Sometimes it is impossible to reduce a 3×3 game, the shortcut method comes into action in this case.
* Dominance rules are/can also be applied for Bigger games.

8. Identify the various variations in solving games, with examples.

ans. For games without saddle point:
For a 2×2 matrix games can be solved by either arithmetic or algebraic method.

Algebraic method:
* Select one strategy of a player A and find the profit of other player and then select others strategy and find the profit.
* By comparing the profits, we will get the probabilities and from that we get the optimal solution and value of the game.

Ex:

			q_1	q_2
			B	
			H	T
p_1	A	H	1	-1
p_2		T	-1	1

When player A fixes head as his strategy, profit for
$B = q_1 - q_2$

When player A fixes tail as his strategy, profit for
$B = -q_1 + q_2$

Comparing,

$$q_1 - q_2 = -q_1 + q_2$$
$$q_1 + q_1 = q_2 + q_2$$
$$\cancel{2}q_1 = \cancel{2}q_2$$
$$q_1 = q_2 \quad \text{--- (i)}$$

Also we know that, sum of probabilities for any player must be 1,

$$q_1 + q_2 = 1$$
$$q_1 + q_1 = 1 \quad (\text{from } (i))$$
$$2q_1 = 1$$
$$\therefore q_1 = 1/2$$
$$\therefore q_2 = 1/2 \quad (\text{from } (i))$$

When player B fixes head as his strategy, profit for player A $= p_1 - p_2$

When player B fixes tail as his strategy, profit for player A $= -p1 + p2$

Comparing,

$$p_1 - p_2 = -p_1 + p_2$$
$$p_1 + p_1 = p_2 + p_2$$
$$\cancel{2}p_1 = \cancel{2}p_2$$
$$p_1 = p_2 \quad - (i)$$

Also we know that, sum of probabilities must be 1 for any player,

$$p_1 + p_2 = 1$$
$$p_1 + p_1 = 1 \quad (\text{from } (i))$$
$$2p_1 = 1$$
$$p_1 = 1/2$$
$$\therefore p_2 = 1/2 \quad (\text{from } (i))$$

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optimal strategy for player A = $(1/2, 1/2)$
optimal strategy for player B = $(1/2, 1/2)$
Value of the game, $V = 1\times 1/2 - 1\times 1/2 = 0$ (fair game)

Arithmetic method:

* For the first strategy, find the row difference and similarly find out the 2nd row difference, write the first row difference in next to the second row and the second row difference next to the first row.
* Repeat the same for columns and obtain the probability and find the optimal solution and value of the game.

$q_1 = 2/4$ $q_2 = 2/4$

B

			H	T	
$p_1 = 2/4$	A	H	1	-1	2
$q_1 = 2/4$		T	-1	1	2
			2	2	

* The optimal strategy for player A = $(1/2, 1/2)$
* The optimal strategy for player B = $(1/2, 1/2)$
* Value of the game, $V = 1\times 1/2 - 1\times 1/2 = 0$ (fair game)

Dominance Rules:

For a game bigger than 2×2 we can use dominance rule to reduce it to 2×2.

(i) Row dominance: Delete a row in which all the elements are having minimum values compare to the other rows.

(ii) Column dominance: Delete the column in which all the elements are having greater values compared to any other column i.e., delete a column which is dominant.

(iii) Row Average : Find the average of 2 rows and compare it with the 3rd row. Delete a row if average is greater than the row values.

(iv) Column Average : Find the average of 2 columns and compare it with the values of 3rd column. Delete a column if average is lesser than the compared values.

Ex:

	1	2	3	4
~~1~~	~~5~~	~~10~~	~~9~~	~~6~~
2	6	7	8	1
3	8	7	15	1
4	3	4	-1	4

	1	2	3	4
~~2~~	~~6~~	~~7~~	~~8~~	~~1~~
3	8	7	15	1
4	3	4	-1	4

	1	2	3	4
3	8	~~7~~	15	1
4	3	~~4~~	-1	4

	1	3	4
3	~~8~~	15	1
4	~~3~~	-1	4

	3	4		
3	15	1	5	$p_1 = 5/19$
4	-1	4	14	$p_2 = 14/19$
	3	16		
	$q_1 = 3/19$	$q_2 = 16/19$.		

*Optimal strategy for player A = $(0, 0, 5/19, 14/19)$
*optimal strategy for player B = $(0, 0, 3/19, 16/19)$
*Value of the game, $V = 15 \times 15/19 + 1 \times 14/19$

$$= \frac{75-14}{19}$$

$$= \frac{61}{19} = 3.21 \text{ (A wins, B loses)}$$

⑨ The transportation costs per truck load of cement (in hundreds of rupees) from each plant to each project site are as follows:

Project sites

		1	2	3	4	
	1	2	3	11	7	6
Factories	2	1	0	6	1	1	Supply
	3	5	8	15	9	10
		7	5	3	2	17

Demand.

~~Sol: Since the supply~~
Determine an optimal distribution for the company so as to minimize the total transportation cost.

Solⁿ

	1	2	3	4	
1	2	3 5	11	7	~~6~~ 1
2	1	0	6	1 1	1
3	5 6	8	15 3	9 1	~~10~~ 4
	~~7~~ 6	~~5~~	3	~~2~~ 1	

1	1	5
1	–	–
3	3	4

1	3	5	6
3	5	4	2
3	–	4	2

	1	2	3	4
1	2 1	3 5	11	7
2	1	0	6	1 1
3	5 6	8	15 3	9 1

Basic feasible solution $= 2\times1+3\times5+1\times1+5\times6+15\times3+9\times1$
$= 2+15+1+30+45+9$
$= 102.$

Optimality check:
No. of allocations $= m+n-1 = 3+4-1 = 6.$
∴ The solution is non-degenerate BFS.

Using MODI method.

$v_1=5$ $v_2=3$ $v_3=15$ $v_4=9$

		1	2	3	4
$u_1=-3$	1	2 1	3 5	11 -1	7 +
$u_2=-8$	2	1 +	0 +	6 +θ $\boxed{-1}$	1 -θ 1
$u_3=0$	3	5 6	8 8 +	15 3 -θ	19 +θ 1

$\theta_{max} = 1$

		$V_1=5$	$V_2=6$	$V_3=15$	$V_4=9$
		1	2	3	4
$U_1=-3$	1	2 1 $-\theta$	3 5	11 $+\theta$ -1	7 +
$U_2=-9$	2	1 +	0 +	6 1	1 +
$U_3=0.$	3	5 6 $+\theta$	8 +	15 $-\theta$ 2	9 2

$\theta_{max} = 1$.

		$V_1=5$	$V_2=7$	$V_3=15$	$V_4=9$
		1	2	3	4
$U_1=-4$	1	2 +	3 5	11 1	7 +
$U_2=-9$	2	1 +	0 +	6 1	1 +
$U_3=0.$	3	5 7	8 +	15 1	9 2

All the cells here are trying to increase the cost. Hence this is the optimal allocation.

Optimal solution $= 3\times5 + 11\times1 + 6\times1 + 7\times5 + 15\times1 + 9\times2$

$= 15 + 11 + 6 + 35 + 15 + 18$

$= \underline{\underline{100}}$

(18). Solve the game whose payoff matrix to the player A is given below :

B

A	I	II	III
I	1	7	2
II	6	2	7
III	5	2	6.

Sol^n Applying Dominance rule,

	I	II	III
I	1	7	2
II	6	2	7
~~III~~	~~5~~	~~2~~	~~6~~

($\because$ 3rd row $\leq$ 2nd row)

	I	II	III
I	1	7	2
II	6	2	7

$(\because (2,7) \geq (1,6))$

	I	II		
I	1	7	4	$p_1 = 4/10$
II	6	2	6	$p_2 = 6/10$
	5	5		
	$q_1 = 5/10$	$q_2 = 5/10$		

*Optimal strategy for player A = $(2/5, 3/5, 0)$

*Optimal strategy for player B = $(1/2, 1/2, 0)$

*Value of the game, $V = 1 \times 2/5 + 6 \times 3/5 = 2/5 + 18/5$

$= 4$ (A wins, B loses)