

Solve all Multiplication problems using Bit Pair recoding Method while preparing for final exam.

①

Module - 4 Arithmetic
Solved Problems

1. Perform following operation on 5 bit signed numbers using 2's complement representation. Also indicate whether overflow has occurred.

a) $(-9) + (-7)$

Take 2's complement.

$$9 = 01001 \xrightarrow{2c} -9 = \begin{array}{r} 10110 \\ + 1 \\ \hline 10111 \end{array} \quad \begin{array}{l} f=00111, 2'c \\ -7 = 11000 \\ \quad \quad \quad + 1 \\ \hline 11001 \end{array}$$

$$\begin{array}{r} 10111 \\ + 11001 \\ \hline 110000 \end{array}$$

$$= \underline{\underline{-16}}$$

b) $(+7) - (-8)$

$$+7 = 00111$$

$$-8 = 01000$$

$$\begin{array}{r} 2'c \\ 10111 \\ + 1 \\ \hline 11000 \end{array}$$

$$(-8) \cdot \underline{\underline{11000}}$$

As subtraction operation take 2'c of multiplier again

$$\begin{array}{r} 11000 \\ \downarrow 2'c \\ 00111 \\ + 1 \\ \hline 01000 \end{array}$$

So,

$$\begin{array}{r} 00111 \\ + 01000 \\ \hline 01111 \end{array}$$

$$= \underline{\underline{+15}}$$

Problems

(2)

1. Perform Multiplication of -13 and +09 using Booth's Algorithm.

Sol. (-13) = Multiplicand

$$2^C = 10011$$

+09 = Multiplier.

Booth recoding of Multiplier

$$+9 = \underbrace{01001}_{+1-10+1-1} \text{ Add zero to LSB.}$$

$$\begin{array}{r}
 & 10011 \\
 \times & +1-10+1-1 \\
 \hline
 \end{array}$$

① 0 0 0 0 0 0 1 1 0 1
 0 0 0 0 0 0 0 0 |
 0 0 0 1 1 0 1 |
 1 1 1 1 1 0 0 1 1 |
 0 0 0 0 0 0 0 0 |
 0 0 0 1 1 0 1 |
 1 1 0 0 1 1 |
 \hline
 ① 1 1 1 0 0 0 1 0 1 1

↓ 2'c
 ↓ 2'c
 ↓ 2'c
 ↓ 2'c

for -1 Multiplier
 take 2's complement
 of Multiplier.
 $(-13) = 10011$
 $\downarrow 2'c$
 01100
 $\hline 01101$

0001110100
 $+ 1$
 $\hline 10001110101 = \underline{\underline{-117}}$

2. Perform Signed Multiplication of numbers (-12) and (-11) using Booth recoding.

Sol. -12 $\xrightarrow{2'c} 01100 \xrightarrow{2'c} \begin{smallmatrix} 10011 \\ + 1 \\ \hline 10100 \end{smallmatrix}$

-11 $\rightarrow 01011 \xrightarrow{2'c} \begin{smallmatrix} 10100 \\ + 1 \\ \hline 11011 \end{smallmatrix}$

(-11) Multiplier

(3)

Booth recoding

$$\begin{array}{r} 10101 \\ -1+1-1+1-1 \\ \hline \end{array} \leftarrow \text{add } 0.$$

$$\begin{array}{r} 10100 \\ \times -1+1-1+1-1 \\ \hline 00000001100 \\ 111110100 \\ 00001100 \\ 1110100 \\ 001100 \\ \hline \end{array}$$

$$\begin{array}{r} 10 \\ 0010000100 \\ \hline \end{array}$$

Ignore

$$= \underline{\underline{+132}}$$

for -1, take 2's complement of multiplicand

$$\begin{array}{r} 10100 \\ \downarrow 2^c \\ 01d'1 \\ + 1 \\ \hline 01100 \end{array}$$

(5) Perform $14 * -8$ using Booth's Algorithm

$$14 \rightarrow 01110 \text{ (Multiplicand)}$$

$$-8 \rightarrow 01000 \xrightarrow{2^c} \begin{array}{r} 10111 \\ + 11000 \\ \hline 01100 \end{array} \rightarrow \text{Booth recoding}$$

$$\begin{array}{r} 01110 \\ \times 01000 \\ \hline 0000000000 \\ 0000000000 \\ 0000000000 \\ 1110010 \\ 0000000 \\ \hline 1100100000 \\ \downarrow 2^c \\ 0011011111 \\ + 1 \\ \hline \end{array}$$

2's complement of Multiplicand

$$\begin{array}{r} 01110 \\ \downarrow 2^c \\ 10001 \\ + 1 \\ \hline 10010 \end{array}$$

$$\begin{array}{r} *0011100000 \\ = \underline{\underline{-112}} \end{array}$$

⑥ Apply Booth's Algorithm to multiply signed numbers +13 and -6.

Solⁿ - Refer to text book / Notes.

⑦ Multiply following using booth's algorithm of signed 2's complement Numbers.

$$\text{Multiplicand} = (010111)_2 = +23$$

$$\text{Multiplier} = (110110)_2 = -10.$$

Solⁿ Booth recoding of multiplier

$$\begin{array}{r} 110110 \\ \hline 0-1+10-10 \end{array}$$

$$\begin{array}{r} 010111 \\ \times 0-1+10-10 \\ \hline 000000000000 \\ 111111101001 \\ 000000000000 \\ 000010111 \\ 11101001 \\ 0000000 \\ \hline 0111100011010 \\ \downarrow 2'C \\ 000011100101 \\ + 1 \\ \hline 000011100110 \\ = -230 \end{array}$$

for -1 take 2's complement of multiplicand.

$$\begin{array}{r} 010111 \\ \downarrow 2'C \\ 101000 \\ + 1 \\ \hline 101001 \end{array}$$

③ Given $A = 10101$, $B = 00100$, perform A/B
using restoring division method.

$$\frac{A}{B} = \frac{10101}{00100} - \text{Dividend } (1) \quad 8 \\ 00100 - \text{Divisor } (2)$$

M	Step.	A	Q	
		00000	10101	Initialization
00100				
$\downarrow 2^4 C$				
11011	Shift	00001	0101□	
+ 1100	Sub.	<u>11100</u>		
	Set Q ₀	01101		
	Restore	<u>100100</u>	010110	I st cycle.
		00001		

Shift	00010	10110□	
Sub	<u>+ 11100</u>		
Set Q ₀	01110		
Restore	<u>00100</u>	1011010	II cycle.
	00010		

Shift	00101	01000□	
Sub	<u>+ 11100</u>		
Set Q ₀	00001		
Don't Restore	<u>10000</u>	010001	III cycle
	00001		

Shift	00100	100110	
Sub	<u>+ 11100</u>		
Set Q ₀	00010		
Restore	<u>00100</u>	100110	IV cycle.
	00010		

Shift	00101	00010□	
Sub	<u>+ 11100</u>		
Set Q ₀	00001		
Don't Restore	<u>10000</u>	00001	V cycle.
	00001		

Remainder Quotient

Non Restoring Division of (10101) Dividend (6)			
Sol?	M. steps	A	Q. Initialization
	$\begin{array}{r} 00100 \\ \times 11011 \\ \hline 11100 \end{array}$	$\begin{array}{r} 00000 \\ + 11100 \\ \hline 01101 \end{array}$	10101
	shift	00001	0101 \square
	sub	$\begin{array}{r} + 11100 \\ - 01101 \\ \hline 01010 \end{array}$	I cycle
	setq ₀		
	shift	11010	101 \square \square
	Add	$\begin{array}{r} 00100 \\ + 01110 \\ \hline 01110 \end{array}$	II cycle
	setq ₀		
	shift	11101	01 \square \square \square
	Add	$\begin{array}{r} 00100 \\ + 00001 \\ \hline 00001 \end{array}$	III cycle
	setq ₀		
	shift	00010	1 \square \square \square \square
	sub	$\begin{array}{r} 11100 \\ - 01110 \\ \hline 01010 \end{array}$	IV cycle
	setq ₀		
	shift	11101	0 \square \square 1 \square 1
	Add	$\begin{array}{r} 00100 \\ + 00001 \\ \hline 00001 \end{array}$	
	setq ₀		
		00001	0 \square \square 1 0 \square
			remainder quotient

4 Perform following division 1000/11

Sol? Refer to text book / Notes.