

**CS590 – Algorithms Assignment 4**

**Name: Sushant Bhat**

**CWID: 10474365**

**Q4.** Find the maximum alignment for  $X = \text{dcdcbacbbb}$  and  $Y = \text{acdccbdbbb}$  by using the Smith-Waterman algorithm. Execute the pseudocode algorithm and fill the necessary tables  $H$  and  $P$  in a bottom-up fashion. Reconstruct the strings  $X'$  and  $Y'$  using the tables  $H$  and  $P$ .

**Table for H:**

0	0	0	0	0	0	0	0	0	0	0
0	-1	-1	2	1	0	-1	-1	2	1	0
0	-1	1	1	4	3	2	1	1	1	0
0	-1	0	3	3	3	2	1	3	2	1
0	-1	1	2	5	5	4	3	2	2	1
0	-1	0	1	4	4	4	6	5	4	4
0	2	1	0	3	3	6	5	5	4	3
0	1	4	3	2	5	5	5	4	4	3
0	0	3	3	2	4	4	7	6	6	6
0	-1	2	2	2	3	3	6	6	8	8
0	-1	1	1	1	2	2	5	5	8	10

**Table for P:**

-	d	d	d	l	l	d	d	d	l	l
-	d	d	u	d	d	l	l	u	d	d
-	d	u	d	u	d	d	d	d	l	l
-	d	d	u	d	d	l	l	u	d	d
-	d	u	u	u	d	d	d	l	d	d
-	d	l	u	u	d	d	u	d	d	d
-	u	d	l	d	d	u	d	d	d	d
-	u	u	d	d	u	d	d	l	d	d
-	d	u	d	d	u	d	d	d	d	d
-	d	u	d	d	u	d	d	d	d	d

### 15. 1-2

Show, by means of a counterexample, that the following "greedy" strategy does not always determine an optimal way to cut rods. Define the **density** of a rod of length  $i$  to be  $p_i / i$ , that is, its value per inch. The greedy strategy for a rod of length  $n$  cuts off a first piece of length  $i$ , where  $1 \leq i \leq n$ , having maximum density. It then continues by applying the greedy strategy to the remaining piece of length  $n - i$ .

Let  $p_1 = 0$ ,  $p_2 = 4$ ,  $p_3 = 7$  and  $n = 4$ . The greedy strategy would first cut off a piece of length 3 since it has highest density. The remaining rod has length 1, so the total price would be 7. On the other hand, two rods of length 2 yield a price of 8.

### 15. 1-5

The Fibonacci numbers are defined by recurrence (3.22). Give an  $O(n)$ -time dynamic-programming algorithm to compute the  $n$ th Fibonacci number. Draw the subproblem graph. How many vertices and edges are in the graph?

#### Algorithm for Fibonacci:

FIBONACCI( $n$ )

  let  $\text{fib}[0..n]$  be a new array

$\text{fib}[0] = 1$

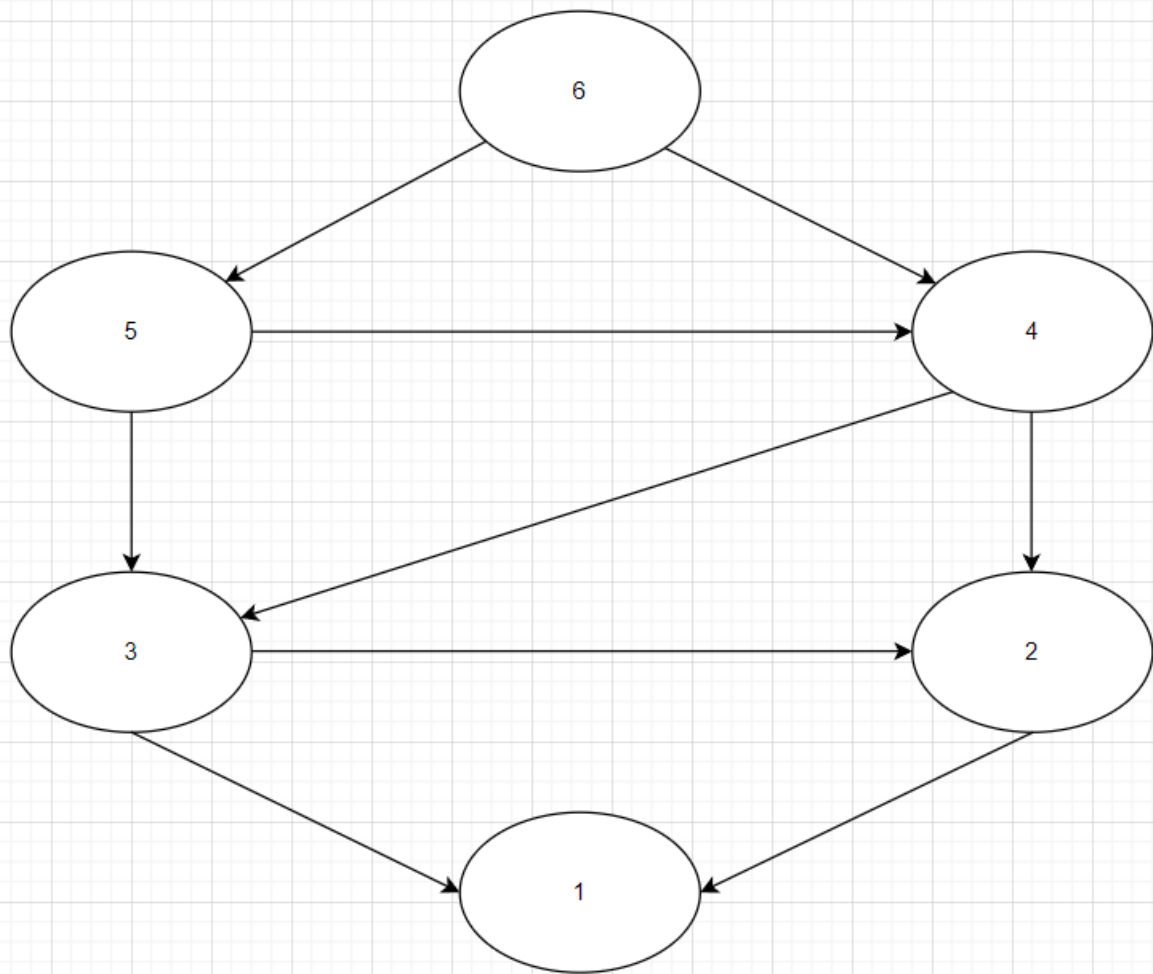
$\text{fib}[1] = 1$

  for  $i = 2$  to  $n$

$\text{fib}[i] = \text{fib}[i - 1] + \text{fib}[i - 2]$

  return  $\text{fib}[n]$

Graph for Fibonacci Numbers



### 15. 4-1

Determine an LCS of  $\langle 1,0,0,1,0,1,0,1 \rangle$  and  $\langle 0,1,0,1,1,0,1,1,0 \rangle$ .

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	1	1	1	1	1	1
2	0	1	1	2	2	2	2	2	2	2
3	0	1	1	2	2	2	3	3	3	3
4	0	1	2	2	3	3	3	4	4	4
5	0	1	2	3	3	3	4	4	4	5
6	0	1	2	3	4	4	4	5	5	5
7	0	1	2	3	4	4	5	5	5	6
8	0	1	2	3	4	5	5	6	6	6

The LCS is  $\langle 1,0,0,1,1,0 \rangle$  or  $\langle 1,0,1,0,1,0 \rangle$ .