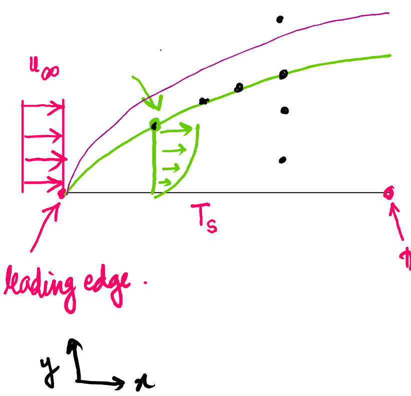
Forced Convection



$u = 0.99\, u_\infty \leftarrow$ Velocity Boundary Layer.

$$\boxed{\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0}} \Rightarrow \boxed{C_{f,x} = \frac{\tau_{s,x}}{\frac{1}{2}\rho u_\infty^2}}$$

Temp. Boundary Layer $\rightarrow \left[\frac{T_s - T(x,y)}{T_s - T_\infty} = (0.99)\right]$

$$Re_x = \frac{\rho u_\infty x}{\mu}$$ & For a flat plate, $\boxed{Re_{x,c} = 5\times10^5}$

Experimental results for the local heat transfer coefficient h_x for flow over a flat plate with an extremely rough surface were found to fit the relation

$$h_x(x) = ax^{-0.1}$$

where a is a coefficient ($W/m^{1.9}\cdot K$) and x (m) is the distance from the leading edge of the plate.

1. Develop an expression for the ratio of the average heat transfer coefficient $\bar{h}_x$ for a plate of length x to the local heat transfer coefficient h_x at x.
2. Plot the variation of h_x and $\bar{h}_x$ as a function of x.

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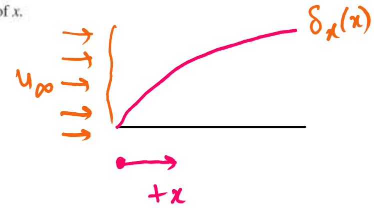


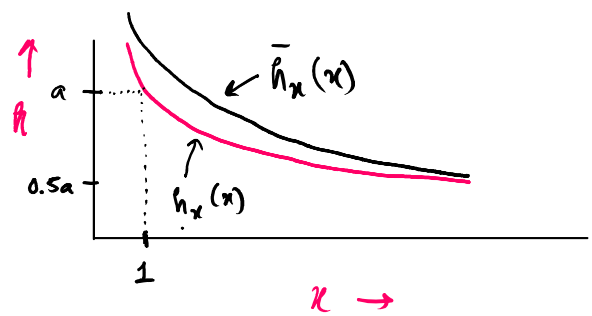
$$h_x(x) = ax^{-0.1}$$

$$\bar{h}_x = \left[\frac{\int_0^x h_x(x)\,dx}{x}\right]$$

$$\bar{h}_x = \frac{1}{x}\int_0^x ax^{-0.1} = \frac{1}{x}\cdot a \cdot \frac{x^{0.9}}{0.9}$$

$$\boxed{\bar{h}_x = 1.11\, a\, x^{-0.1}}$$

$$\frac{\bar{h}_x}{h_x} = \frac{1.11\,\cancel{ax^{-0.1}}}{\cancel{ax^{-0.1}}} = \boxed{1.11}$$



6.2 In flow over a surface, velocity and temperature profiles are of the forms

$$u(y) = Ay + By^2 - Cy^3 \quad \text{and}$$
$$T(y) = D + Ey + Fy^2 - Gy^3$$

where the coefficients A through G are constants. Obtain expressions for the friction coefficient C_f and the convection coefficient h in terms of u_∞, T_∞, and appropriate profile coefficients and fluid properties.

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$$T(0) = D + 0 + 0 - 0$$

$$\boxed{T_s = D}$$

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0}$$

$$u(y) = Ay + By^2 - Cy^3$$
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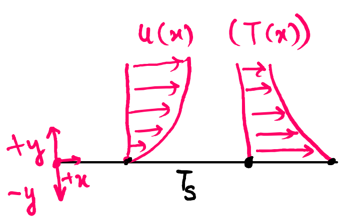

(ii) $$h = \frac{-k_f\,(\partial T/\partial y)|_{y=0}}{T_s - T_\infty} \quad — ①$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = \frac{\partial}{\partial y}\left[D + Ey + Fy^2 - Gy^3\right]\Big|_{y=0}$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = E + 2Fy - 3Gy^2\Big|_{y=0}$$

$$\left|\left.\frac{\partial T}{\partial y}\right|_{y=0} = E\right. \quad — ②$$

$$\tau_s = \mu \frac{\partial u}{\partial y}\bigg|_{y=0}$$

$$\tau_s = \mu\left[A + 2By - 3Cy^2\right]\bigg|_{y=0}$$

$$\boxed{\tau_s = \mu A} \qquad C_f = \frac{\tau_s}{\frac{1}{2}\rho u_\infty^2} = \boxed{\frac{2\mu A}{\rho u_\infty^2}}$$

$$\left|\frac{\partial T}{\partial y}\right|_{y=0} = E \quad \text{—②}$$

from —① & —②,

$$\boxed{h = \frac{-k_f \cdot E}{T_s - T_\infty}}$$

6.4 Water at a temperature of $T_\infty = 25°C$ flows over one of the surfaces of a steel wall (AISI 1010) whose temperature is $T_{s,1} = 40°C$. The wall is 0.35 m thick, and its other surface temperature is $T_{s,2} = 100°C$. For steady-state conditions what is the convection coefficient associated with the water flow? What is the temperature gradient in the wall and in the water that is in contact with the wall? Sketch the temperature distribution in the wall and in the adjoining water.

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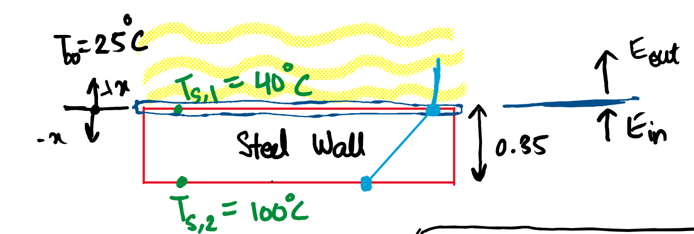
$h = ?$



$$\boxed{\begin{array}{l} k_{steel} = 61.7\ W/mK \\ k_{H_2O} = 0.62\ W/mK \end{array}}$$

$E_{in} = E_{out}$

$q''_{conduction} = q''_{convection}$

$$k_{steel} \times \frac{T_{s,2} - T_{s,1}}{L} = h\left(T_{s,1} - T_\infty\right)$$

$$61.7 \times \frac{(100-40)}{0.35} = h(40-25)$$

$\hookrightarrow \boxed{705\ W/m^2K}$

$\frac{dT}{dx}\Big|_{st,\,x=0}$ & in water $\frac{dT}{dx}\Big|_{H_2O,\,x=0}$

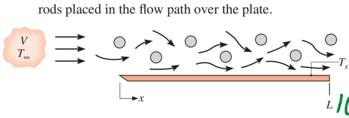
$$\frac{100-40}{0.35} = 171.4\ K/m$$

$$\frac{dT}{dx}\bigg|_{H_2O} = -\frac{h}{k_{st}}\left(T_{s_1} - T_\infty\right)$$

$$\frac{dT}{dx}\bigg|_{H_2O} = \frac{-705}{61.7}(40-25)$$

$$= \boxed{-17056\ K/m}$$

6.7 Parallel flow of atmospheric air over a flat plate of length $L = 3$ m is disrupted by an array of stationary rods placed in the flow path over the plate.



Laboratory measurements of the local convection coefficient at the surface of the plate are made for a prescribed value of V and $T_s > T_\infty$. The results are correlated by an expression of the form $h_x = 0.7 + 13.6x - 3.4x^2$, where h_x has units of $W/m^2 \cdot K$ and x is in meters. Evaluate the average convection coefficient $\bar{h}_L$ for the entire plate and the ratio $\bar{h}_L/h_L$ at the trailing edge.

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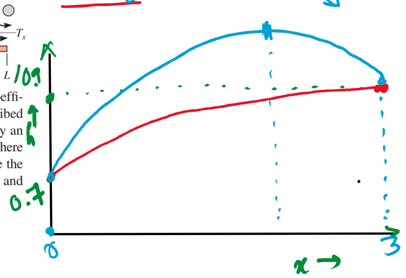
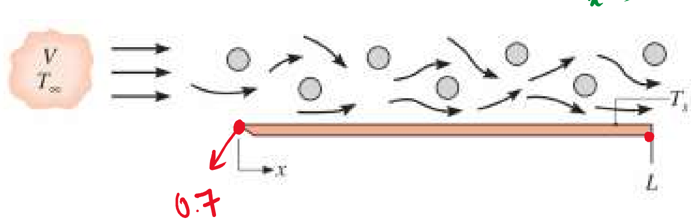
$$h_x = 0.7 + 13.6x - 3.4x^2$$





$$\bar{h}_L = \frac{1}{L} \times \int_0^L h_x(x)\,dx$$

$$\bar{h}_L = \frac{1}{L} \times \int_0^L \left[0.7 + 13.6x - 3.4x^2\right]dx$$

$$\bar{h}_L = \frac{1}{L} \times \left[0.7x + \frac{13.6x^2}{2} - \frac{3.4x^3}{3}\right]\bigg|_0^L$$

$$\bar{h}_L = \frac{1}{L}\left[0.7L + \frac{13.6L^2}{2} - \frac{3.4L^3}{3}\right]$$

$$\bar{h}_L = \ \& \ L = 3 \Rightarrow \boxed{\bar{h}_L = 10.9\ W/m^2K}$$

$$\frac{\bar{h}_L}{h_L(x)}\bigg|_{x=L} \Rightarrow \frac{10.9}{0.7 + 13.6(3) - 3.4(3)^2} = 1$$

6.8 For laminar free convection from a heated vertical surface, the local convection coefficient may be expressed as $h_x = Cx^{-1/4}$, where h_x is the coefficient at a distance x from the leading edge of the surface and the quantity C, which depends on the fluid properties, is independent of x. Obtain an expression for the ratio $\bar{h}_x/h_x$, where $\bar{h}_x$ is the average coefficient between the leading edge ($x = 0$) and the x-location. Sketch the variation of h_x and $\bar{h}_x$ with x.

$$h_x = Cx^{-1/4}$$

$$\bar{h}_x = \frac{1}{x}\int_0^x Cx^{-1/4}\,dx.$$

which depends on the fluid properties, is independent of x. Obtain an expression for the ratio $\bar{h}_x/h_x$, where $\bar{h}_x$ is the average coefficient between the leading edge ($x = 0$) and the x-location. Sketch the variation of h_x and $\bar{h}_x$ with x.

$$\bar{h}_x = \frac{1}{x}\int_0 C x^{-1/4}\, dx .$$

$$\bar{h}_x = \frac{1}{x} C x^{3/4} \times \frac{4}{3}$$

$$\bar{h}_x = \frac{4}{3} C x^{-1/4} \leftarrow$$

$$\frac{\bar{h}_x}{h_x} = \frac{\frac{4}{3} C \cancel{x^{-1/4}}}{C \cancel{x^{-1/4}}} = \boxed{4/3}$$