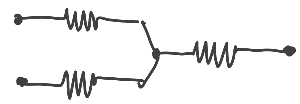
Contents of Week 3 and 4

Tuesday, February 18, 2025 3:40 PM

Plane wall :

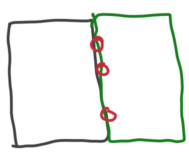


Conduction → $R_{cond} = \dfrac{L}{kA}$

Convection → $R_{conv.} = \dfrac{1}{hA_s}$

Radiation (+ Convection) → $R_{rad} = \dfrac{1}{h_{rad} A_s}$, $h_{rad} \to \epsilon\sigma (T_s^2 + T_\infty^2)(T_s + T_\infty)$

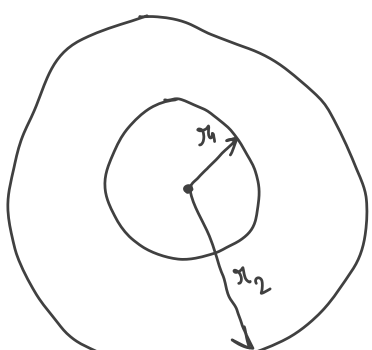
$R_{conv} = \dfrac{1}{h_{conv.} A_s}$ → $h_{comb} = h_{conv.} + h_{rad.}$



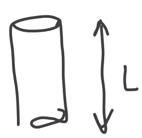
→ Contact Resistance → $\left[R_c = \dfrac{\Delta T_{int}}{\dot{Q}/A} \right]$

Cylinders

→ $R_{cyl.} = \dfrac{\ln(r_2/r_1)}{2\pi L k}$



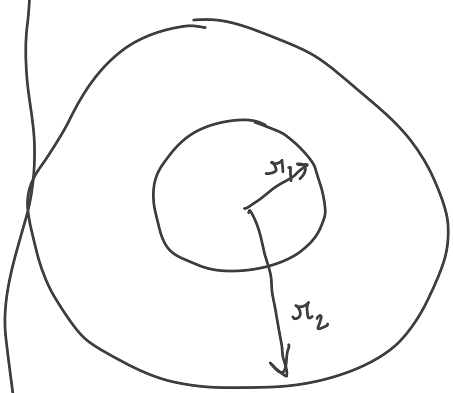
top view

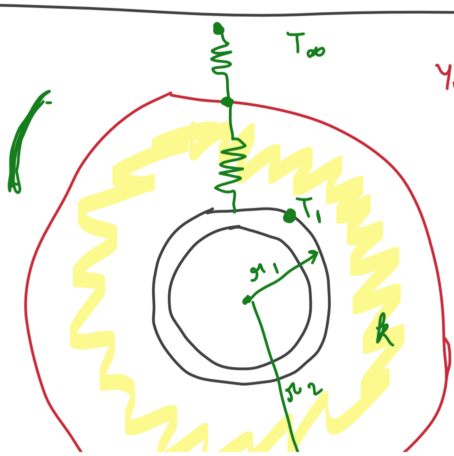


front view

Sphere

$R_{sph} = \dfrac{r_2 - r_1}{4\pi r_1 r_2 k}$





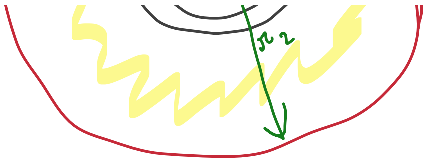
Yellow - insulation.

CRITICAL RADIUS

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{ins} + R_{conv.}} \Rightarrow \dot{Q} \text{ as a function of '} r_2 \text{'}$$

R_{ins} → cyl, $R_{conv.}$ → $\left(\frac{1}{hA}\right)$

$$\boxed{\frac{d\dot{Q}}{dr_2} = 0}$$



dr_2

for max heat transfer, find what is 'r_2'.

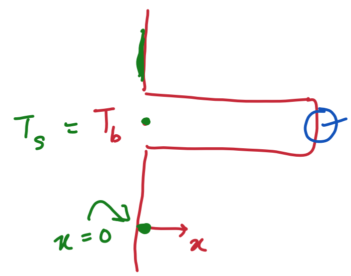
Cylinder $\rightarrow$ $r_{cr} = \frac{k}{h}$ Sphere $\rightarrow$ $r_{cr} = \frac{2k}{h}$.

Max heat transfer
min resistance.

Week 4 Summary

$$\dot{Q} = hA_s(T_s - T_\infty)$$

Add fins $\rightarrow$ increase A_s $\rightarrow$ increase $\dot{Q}$ } Fins



$$\boxed{\frac{d^2\theta}{dx^2} - m^2\theta = 0}$$

✓ $m^2 = \frac{hp}{kA_c}$

✓ $\theta = T - T_\infty$

$\theta(x) = c_1 e^{+mx} + c_2 e^{-mx}$ $\rightarrow$ general solution

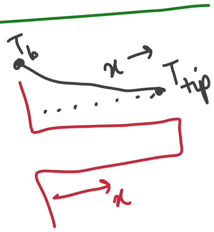
Put the boundary conditions:

1 $\rightarrow$ $\theta(0) = \theta_b = T_b - T_\infty$

Fin Tip $\rightarrow$ ∞ly long fin.
$\rightarrow$ Adiabatic fin Tip
$\rightarrow$ Fin tip is convecting heat.
$\rightarrow$ Specific Temp at End of the fin. $T_{fin,tip} = T_L$

I Fin efficiency:

$$\eta_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin,max}}$$



$\dot{Q}_{fin,max}$ $\rightarrow$ whole fin is at $T = T_b$.

II Fin Effectiveness

$$\epsilon_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\ fin}}.$$

3-57

$\dot{Q} = ?$ $T_{interface} = ?$ $\Delta T_F = ?$

$k_A = 2$

$\rightarrow A = \cancel{0.12m} \times \cancel{8m}. = \cancel{0.96} m^2$ (5m)

$k_A = 2$
$k_B = 8$
$k_C = 20$
$k_D = 15$
$k_E = 35$
$k_F = 2$

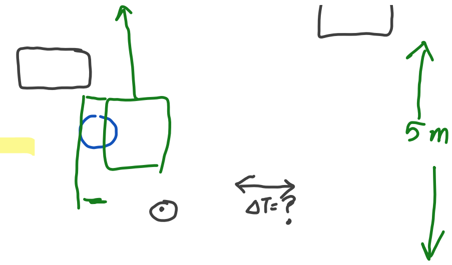
W/mK
W/m°C

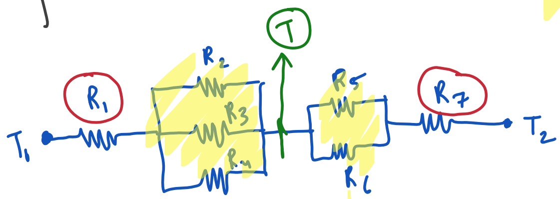
$\rightarrow A = \cancel{0.12m} \times \cancel{8A} = \cancel{0.96} m^2$

5m

$L \rightarrow$ 5m × 8m = 40 m^2

$L \rightarrow$





$R_1 = \left(\frac{L}{KA}\right)_A = \frac{0.01}{2 \times 0.96}$ — R_p $\quad R_Q \quad R_2 = \left(\frac{L}{KA}\right)_C = \frac{0.05}{20 \times 0.96}$

$R_3, R_4, R_5 \dots R_7.$ $\quad \left(\frac{L}{KA}\right) \rightarrow$

$\frac{1}{R_p} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$

R_p

$\frac{1}{R_Q} = \frac{1}{R_5} + \frac{1}{R_6}$

R_Q

$R_{total} = R_1 + R_p + R_Q + R_7 \rightarrow R_{total}$

$\rightarrow \dot{Q}_{total} = \frac{T_1 - T_2}{R_{total}} \rightarrow \boxed{1.91 \times 10^5 \text{ W}} \leftarrow$ complete

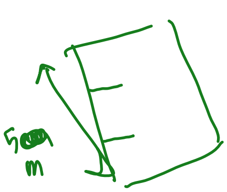
(b) → Total thermal resistance at

3-57

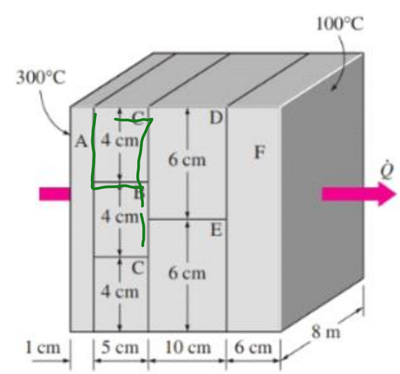
Small Section → Same height as given in this fig & width as 1 m.

[5m/12 cm → 41.6777 is not an int.]

$\downarrow$



$\frac{5m}{3} \rightarrow$ 1.666m. → actual height of 'C' & 'B'



D&E actual height is 2.5m.

Small Section

h = 12 cm $\quad$ w = 1 m $\quad$ & $\quad$ l = 22 cm

$R_A = 0.04 \qquad R_5 = 0.11 \qquad \rightarrow \qquad \dot{Q}_{total} = \frac{T_1 - T_2}{R_{total}}$

$$\begin{cases} R_1 = 0.04 & R_5 = 0.11 \\ R_2 = 0.06 & R_6 = 0.05 \\ R_3 = 0.16 & R_7 = 0.25 \\ R_4 = 0.06 & \end{cases} \rightarrow \dot{Q}_{total} = \frac{T_1 - T_2}{R_{total}}$$

is for a section (small)

$\rightarrow$ 572 W $\downarrow$ 12 cm high & 1 m thick section

$\dot{Q} \propto A$

$$\dot{Q}_{total}\text{(big section)} = 572 \times \frac{5m \times 8m}{0.12m \times 1m} = \underline{1.91 \times 10^5 W}$$

(b)

$$\dot{Q} = \frac{T_1 - T}{R_1 + R_p} = 1.91 \times 10^5 W.$$

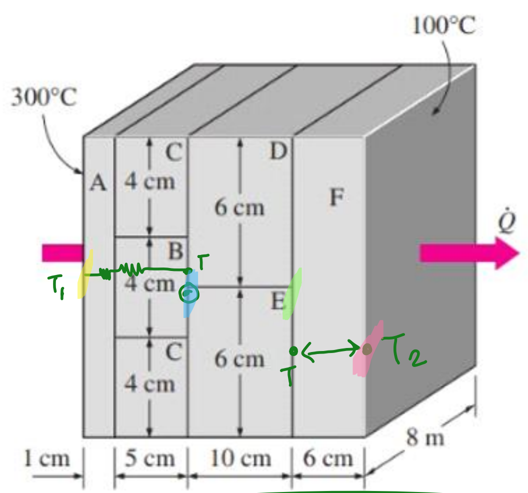
PLS TAKE CARE

$\underbrace{R_1 + R_p}$ small Section

$$T = -\dot{Q} \times (R_1 + R_p) + T_1$$

$$T = -572 \times (0.04 + 0.025) + 300^\circ C$$

$\hookrightarrow T_{interface} = \boxed{263^\circ C}$



in order $\begin{cases} 300 \\ 263 \\ 143 \\ 100 \end{cases}$

(c) Temp. drop across section F $\rightarrow$

$$\dot{Q} = \frac{T - T_2}{R_7} \Rightarrow \boxed{T = 143^\circ C}$$

for small section

for small section

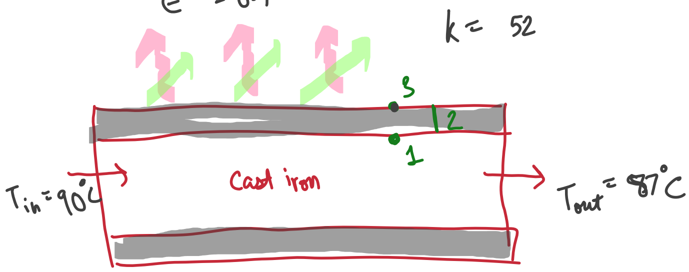
3.76

$T_\infty = 10^\circ C$

$\epsilon = 0.7$

$k = 52$

Convection + Radiation



① Convection inside
② conduction through pipe material
③ Convection + Radiation outside.

$A_i = \pi D_i L = 1.885 m^2$

$A_o = \pi D_o L = 2.168 m^2$

$R_i = \frac{1}{h_i A_i} = \frac{1}{120 \times 1.885} = 0.0044$ K/W

$R_p = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2.3/2)}{2\pi(52)\times(15)} \Rightarrow 0.00003$ K/W.

$R_o = ?$

$\downarrow$

$\underbrace{\text{conv.} + \text{rad.}}$

$h_{rad} = \epsilon\sigma(T_s^2 + T_\infty^2)(T_s + T_\infty)$

(Temp in K) $\quad \hookrightarrow$ let's assume that $\underline{T_s = 90°C}$.

$h_{rad} = 0.7 \times 5.67\times10^{-8} \times [(273.15+90)^2 + (273.15+10)^2][263.15 + 283.15]$

$\hookrightarrow 5.167$ W/m²K

$h_{conv.} = 15$ W/m² $\quad$ } $\quad h_{tot} = 5.167 + 15 = \boxed{20.167 \text{ W/m}^2}$

$R_o = \frac{1}{h_{tot} \times A_o} = \frac{1}{20.167 \times 2.168} = 0.0229$ K/W.

$\dot{Q} = \frac{T_1 - T_\infty}{R_{total}}$ $\quad$ } $\quad R_{tot} = R_i + R_p + R_o = 0.0273$ K/W.

$\hookrightarrow \dot{Q} = \left(\frac{90 - 10}{0.0273}\right) \rightarrow \boxed{2927.W}$

$\underline{\dot{Q} = \dot{m} C_p \Delta T}$

$\downarrow$

$\underline{4180 \text{ J/kgK.}}$

$2927 = \dot{m} \times 4180 \times 3$

$\hookrightarrow \dot{m} = 0.233$ kg/s.

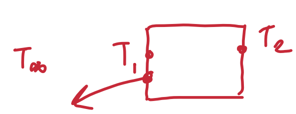
$\dot{m} = \rho A v \Rightarrow 0.233 = 998.2 \times \left(\frac{\pi \times 0.04^2}{4}\right) \times v$

$\rightarrow \boxed{v = 0.186 \text{ m/s}}$

$\underline{\text{Outer Surface Temp.}} \rightarrow \quad \dot{Q} = \frac{T_i - T_{surf}}{R_i + R_p}$

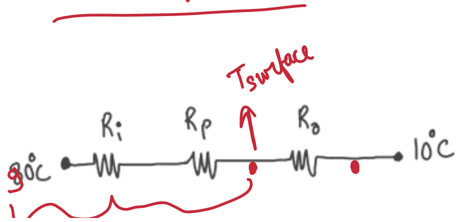
??

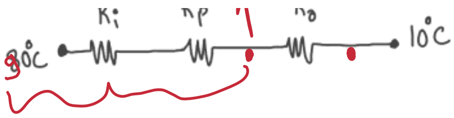
$\dot{Q} = \frac{90 - T_{surf}}{0.0044 + 0.0003} \Rightarrow \underline{2927}$



$T_1 > T_\infty$

$\dot{Q} + \rightarrow T_1$ to T_∞





0.0044 + 0.0003

$\dot{Q}+ \rightarrow T_1$ to T_∞

$\dot{Q}(+ve) = hA(\Delta T)$

$\dot{Q} = hA(T_1 - T_\infty)$

$T_\infty > T_1$

$\dot{Q}(+ve)$

$\dot{Q} = hA(T_\infty - T_1)$

$T_{surf} = 77°C$

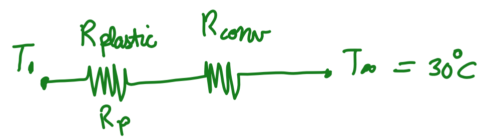
but we assumed $\underline{90°C}$

$h_{rad,prev} = 5.197$

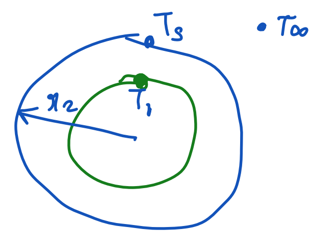
$h_{rad} = 5$

3-88

$I = 10\,A$, $V = 8V$ } $\dot{Q} = IV = 80\,W.$



$R_{conv} = \frac{1}{h_o A_o} = \frac{1}{24 \times \pi \times 0.004 \times 10} = 0.3316\ K/W$



$R_p = \frac{\ln(r_2/r_1)}{2\pi k L} = \frac{\ln(2/1)}{2\pi \times 0.15 \times 10} = 0.0735\ K/W$

$R_{total} = 0.0735 + 0.3316 = \boxed{0.4051\ K/W.}$

$\dot{Q} = \frac{T_1 - T_\infty}{R_{total}} \Rightarrow 80 = \frac{T_1 - 30}{0.4051} \Rightarrow \boxed{T_1 = 62.4°C}$

$r_{cr,cylinder} = \frac{k}{h} = \frac{0.15}{24} = 0.00625\ m \rightarrow \boxed{6.25\ mm}$

Critical thickness $\rightarrow$ min Resistance, max $\dot{Q}$ }

$t_{current} = 1mm$

$t_{ideal} = 5.25\ mm$

$x2 \rightarrow 2mm$

Voltage Diff → Constant

Res → if L↑ R↑

$V = IR$ $I = \frac{V}{R}$ | $P = VI$ $(P = V^2/R)$

3-166 Total Area of Surf = 1m × 1m = $\underline{1m^2}$

no. of fins ?? → $n = \frac{1m^2}{(0.006) \times (0.006)}$

no. of fins ?? → $n = \frac{1m}{(0.006)\times(0.006)}$

→ $n = \boxed{27777}$

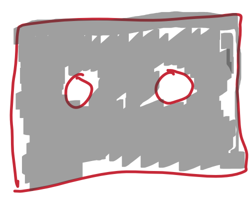
$$\eta_{fin} = \frac{\tanh aL}{aL}$$

→ $a = ?$ $\quad a = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{h\pi D}{k\pi D^2/4}} = \sqrt{\frac{4h}{kD}} = 15.37\ m^{-1}$

$$\eta_{fin} = \frac{\tanh(15.37\times 0.03)}{15.37\times 0.03} = \boxed{0.935}$$

→ $A_{unfinned}$

$$A_{fins} = 27777\times\left(\pi D L + \frac{\pi D^2}{4}\right) = 6.68\ m^2 \leftarrow$$



$$A_{unfinned} = (1m\times 1m) - 27777\times\left(\frac{\pi D^2}{4}\right) = 0.86\ m^2$$

$$\dot{Q}_{finned} = \eta_{fin}\times\dot{Q}_{fin,max} = \eta_{fin}\times hA_{fin}(T_b - T_\infty)$$

$$= 0.935\times 35\times 6.68\ m^2\times(100-30^\circ)$$

$$= \boxed{15300\ W}$$

$$\dot{Q}_{unfinned} = hA_{unfinned}(T_b - T_\infty) = 35\times 0.86\times(100-30) = \boxed{2107\ W}$$

$$\dot{Q}_{total} = 15000\ W + 2107\ W = \boxed{17407\ W}$$

$$A_{no\ fin} = 1m\times 1m = 1m^2$$

$$\dot{Q}_{no\ fin} = hA(\Delta T) = 35\times 1m^2\times(100-30) = \boxed{2450\ W}$$

$$\epsilon_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\ fin}} = \frac{17407}{2450} = \boxed{7.1}$$