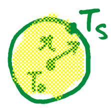
Contents of Week 5

Tuesday, February 25, 2025 6:42 PM

Let's say ball is hotter than the surrounding. T_∞

$T_0 > T_s > T_\infty$



When $\underbrace{\Delta T}_{(T_0 - T_s)}$ in the sphere is <<< $T_s - T_\infty$

↳ then we have a lumped capacity system.

Conduction inside the ball → v. fast.

Convection outside the ball → v. slow.

$$hA(T_s - T_\infty) << kA\frac{\Delta T}{L_c}$$

Cond → $kA\frac{dT}{dx}$

$L_c = \frac{V}{A}$

$$h\cancel{A}(\cancel{T_s - T_\infty}) << \frac{k\cancel{A}(\cancel{T_s - T_\infty})}{L_c}$$

$\frac{hL_c}{k} << 1$ } if $\boxed{\frac{hL_c}{k} < 0.1}$ → Lumped Capacity Model is Valid.

$L_c = \frac{V}{A_s}$ &

Plane Wall → $L_c = L/2$

Long Cylider → $L_c = r/2$

Sphere → $L_c = r/3$

$T_\infty > T_s$.

h, T_∞

in time dt.

$$hA_s[T_\infty - T_t] = -mC_p\, d[T_\infty - T_t]$$

m, V, ρ

T_i

A_s

$\dot{Q}$ → heat gained & T_t → temp. of box at any time 't'

$$\int_0^t \frac{-hA_s}{mC_p}\,dt = \int_{T_i}^{T_t} \frac{d(T_\infty - T_t)}{T_\infty - T_t}$$

m → 'ρV'

$$\frac{-hA_s}{\rho V C_p}\, t = \ln\left(\frac{T(t) - T_\infty}{T_i - T_\infty}\right)$$

$$e^{-bt} = \frac{T(t) - T(\infty)}{T_i - T(\infty)}$$

$b = \frac{hA_s}{\rho V C_p}$ sec^{-1}.

$$\frac{T_t - T_\infty}{T_i - T_\infty} = \frac{\theta_t}{\theta_i}$$

$$\left(\begin{array}{l}\theta_i = T_i - T_\infty \\ \theta_t = T_t - T_\infty\end{array}\right)$$

4-14

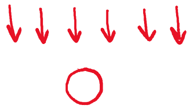
$k = 35\ W/mK$

$\rho = 8500\ kg/m^3$

$C_p = 320\ J/kg^\circ C$

$d = 0.0012\ m$.

Gas $[h, T_\infty]$



$\frac{T_t - T_\infty}{T_i - T_\infty} = e^{-bt}$ } $\frac{T_t - T_\infty}{T_i - T_\infty} = 0.01$

initial Temp → T_i

Final Temp → T_t

} 99% accuracy reaching time.

$C_p = 320\ J/kg^\circ C$

$d = 0.0012\,m$.

$$L_c = \frac{V}{A_s} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{0.0012\,m}{6} = 0.0002\,m$$

$$Bi = \frac{h.L_c}{k} = \frac{65 \times 0.0002}{35} = \boxed{0.0004 < 0.1}$$

Lumped Capacity Model is applicable

Initial Temp $\rightarrow T_i$
Final Temp $\rightarrow T_t$
} 99% accuracy reaching time.

$$0.01 = e^{-bt} \qquad b = \frac{hA_s}{\rho V C_p} = \frac{h}{\rho C_p L_c}$$

$$b = \frac{65}{8500 \times 320 \times 0.0002} = 0.1195\ s^{-1}$$

$$0.01 = e^{-0.1195 \times t} \rightarrow \boxed{t = 38.5 \text{ seconds}}.$$

4.33

$T_i = 900^\circ C$ $k = 54$ & $\rho = 7833$

$T_t = 100^\circ C$ & $\rightarrow C_p = 465\ J/kg$

$$L_c = \frac{V}{A_s} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{0.008}{6} = 0.0013\ m$$

$$Bi = \frac{hL_c}{k} = \frac{75 \times 0.0013}{54} = \boxed{0.0018 < 0.1}$$

LCM applicable.

$$\frac{T_t - T_\infty}{T_i - T_\infty} = e^{-bt} \qquad b = \frac{h}{\rho C_p L_c} = \frac{75}{7833 \times 465 \times 0.0013} = \boxed{0.01584}\ sec^{-1}$$

$$\frac{100 - 35}{900 - 35} = e^{-0.01584 \times t} \Rightarrow \boxed{t = 163 \text{ seconds}}$$

Q for 1 ball $= mC_p \Delta T = mC_p [T_{final} - T_{initial}]$

$= 0.0021 \times 465 \times [100 - 900]$

$m = \rho V = 7833 \times \frac{\pi D^3}{6}$

$\Downarrow$

$\boxed{m = 0.0021\ kg}$

$Q_{1\,ball} = -781\ J$

2500 balls / hour. Joules.

Per hour heat transfer rate $= -781 \times 2500$ (per hour)

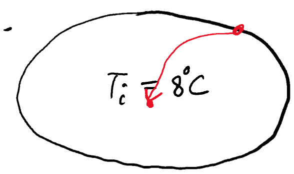
$= -781 \times \frac{2500}{60 \times 60}$ (per sec)

$$\dot{Q}_{2500\,balls} \Rightarrow \boxed{-543\ W \approx -543\ J/s}.$$

4.34

$h = 1400\ W/mK$ $T_\infty = 97^\circ C$

$D = 5.5\ cm \rightarrow D = 0.055\,m$.



$$Bi = \frac{hL_c}{k} \Rightarrow \frac{1400 \times \left(\frac{0.055}{6}\right)}{0.6}$$

$\boxed{64.2} >> 0.1$

Can't apply the lumped Capacity Model.

Fourier No.
Biot No
} $\rightarrow$ LCM

$\boxed{17.8 \text{ mins}} \leftarrow$ Answer.

$h \rightarrow$ very high
$k \rightarrow$ very low
} $\rightarrow$ Bad for LCM.